PRODUCTION AND INVENTORY PLANNING WITH IMPERFECT QUALITY ITEMS AND SHORTAGES

BY

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بسم الله الرحمن الرحيم

In the name of Allah, the Most Gracious and the Most Merciful

Dedicated to

My family members

Father, Mother, Muna, Mohammad, Raed,

Grandfather, Grandmother,

All relatives and friends

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First of all, thanks to almighty Allah, the merciful and the compassionate, who guide us to the straight and the right way and for reasons too numerous to be mentioned. May peace and blessing be upon our leader, prophet Mohammed (PBUH), his family and his companions.

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THESIS ABSTRACT (ENGLISH)

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In this thesis we investigate a production and inventory system where imperfect quality items are produced and shortages are backordered. We point out and correct some of the literature errors in this area. On the other hand we developed new models that are realistic where we got rid of some unrealistic assumptions.

First, we consider a finite production rate model where imperfect items are reworked after the completion of production. Here we find the optimal production quantity and optimal shortage allowed. Next, we developed a similar system with two types of imperfect quality items. Third, we introduced a new model to integrate production and inventory control with process targeting. Finally, we consider the production planning problem with two types of imperfect quality items with the realistic assumption of stochastic breakdowns, where the machine may fail randomly. For each of these models there are numerical examples to show the solution methodology and sensitivity analysis conducted to show the effect of models parameters on the optimal decision variables.

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THESIS ABSTRACT (ARABIC)

ملخص الرسالة

الاسم: **عمر غالب عبدالله الصوافي**

عنوان الرسالة: **تخطيط الانتاج والمخزون بوجود قطع غير مطابقة مع السماح بالعجز**

التخصص: **هندسة الصناعة والنظم**

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هذه الرسالة تناقش مواضيع تخطيط وضبط الإنتاج والمخزون في النظم التي تحتوي على نسبة عشوائية غير مطابقة مع السماح بالعجز عن تلبية الطلب حيث يعاد تلبية العجز مع دورة الإنتاج التالية. هذه الرسالة تصحح بعض الاخطاء التي ارتكبها الباحثون في هذا المجال. ومن جهة أخرى, نقدم في هذه الرسالة نماذج جديدة و واقعية بعد التخلص من بعض الفرضيات غير الواقعية.

بداية, سنقدم نموذج لنظام تصنيعي يكون فيه معدل الإنتاج ثابت والهدف إيجاد كمية التصنيع المثلى وكمية العجز المثلى المسموح بها في حالة أن كل القطع غير المطابقة سوف يعاد تصنيعها بعد الانتهاء من عملية الإنتاج. ثم سنعرض نموذجاً مشابهاً للأول ولكن بوجود نوعان من عدم المطابقة وبالتالي درجتان من العشوائية. أيضا نقدم نموذجاً جديداً يدمج بين تخطيط وضبط الإنتاج عندما يحتوي الإنتاج على نسبة من القطع غير المطابقة وبين تحديد القيم المثلى للعملية التصنيعية. وأخيرا وليس آخرا سنناقش نموذج ضبط الإنتاج والمخزون مع فرضية واقعية وهي حدوث اعطال عشوائية لآلة الإنتاج. ولكل من هذه النماذج تم تقديم مثال تحليلي لتوضيح طريقة إيجاد القيم المثلى, وتم عمل دراسة حساسية لفهم تأثير باريمترات النماذج المختلفة على القيم المراد حسابها وايجادها.

شهادة ماجستير علوم

جامعة الملك فهد للبترول والمعادن

الظهران ، المملكة العربية السعودية

CHAPTER 1

INTRODUCTION

Before presenting the core of the work, it will be helpful to present general information related to the area of the work. So, at the beginning we will present a brief summary about economic order quantity, EOQ, and economic production quantity, EPQ, models without and with shortage. Next, we will discuss the motivation for this thesis and its main objectives.

1.1 CLASSICAL EOQ

EOQ Determines the quantity a company or vendor should order to minimize the total inventory costs by balancing the inventory holding cost and average fixed ordering cost. We consider the following main assumptions:

1. The demand rate is known and fixed.
2. The ordering cost is known and constant.
3. The lead time is known and constant.
4. The purchase price of the item is constant (no discount is available).
5. The whole batch is delivered at once.
6. Only one product is involved.
7. Items are perfect.
8. No shortages are allowed.

The total cost is given by:

Total Cost (TC) = purchase cost (C_p) + ordering cost (C_o) + carrying (holding) cost (C_c)

It is clear that if the ordered quantity is large, the ordering cost will be smaller while the holding cost will be high. The EOQ balances the two types of costs by ordering the quantity that gives minimum total cost as shown in Fig. 1.1 Purchase cost does not affect the solution since it is constant and independent on the quantity ordered.

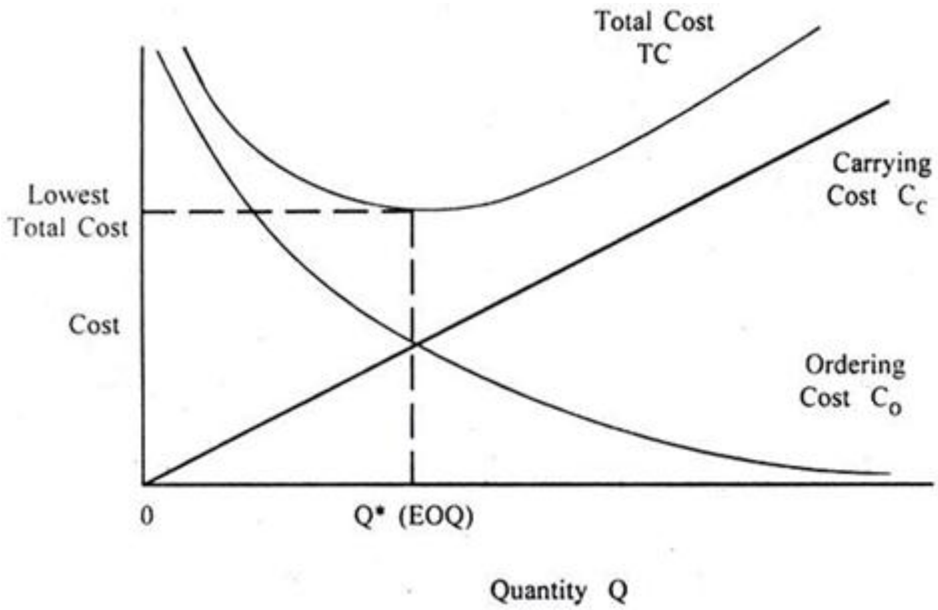

Figure 1.1 EOQ versus Total cost

1.2 CLASSICAL EPQ/ EMQ

The economic production quantity, EPQ, or economic manufacturing quantity, EMQ, model is an extension of EOQ, where the production rate is finite. Here, the inventory is build up in a continuous manner during production. Production starts again when the inventory is depleted. In the classical EPQ/EMQ we assume the following:

1. The demand rate is known and constant.

2. Production runs to replenish inventory are made at regular intervals.

3. The production rate is continuous and constant during the production run.

4. Production set-up cost is fixed (independent of quantity produced).

5. The lead time is fixed.

6. The production cost of an item is constant.

7. The replenishment is made incrementally.

8. Items are perfect.

9. No shortage is allowed.

1.3 SHORTAGE

Shortage occurs when on hand inventory is unable to satisfy the demand (stock out). For a moment one may imagine that a shortage is an evil and should be avoided. To the contrary, sometimes shortage is desirable, mainly in case where the holding cost is higher than the shortage cost. Shortages are either lost sales or backordered.

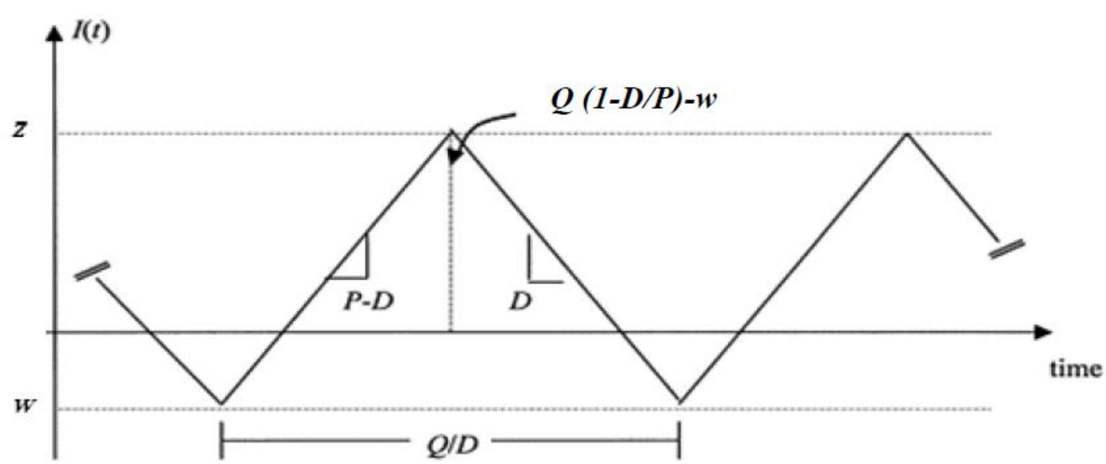

Figure 1.2 Inventory level for a production system with backordered shortage

1.4 MOTIVATION OF THESIS

EOQ and EPQ models are based on restrictive and often unrealistic assumptions that make them not representative of real life conditions.

The determination of the optimal economic order or production quantity is an important and active area of research for the following reasons:

1. The problem has practical impact and implication to industry.

2. The problem has many dimensions to address.

3. It is of multidisciplinary nature.

4. Unrealistic assumptions in the original models.

The classic finite production rate model assumes that all items produced are of perfect quality. However, real-life production systems are exposed to process deterioration and/or other factors, which make the generation of imperfect quality items inevitable. Therefore, studies are being carried out to enhance the classic finite production rate model by addressing the case of defective items. In real life, defective items are either reworked or scrapped.

1.5 THESIS OBJECTIVES

This thesis will focus on some production and inventory control models. According to the literature reviewed there are some errors in handling these models specifically when it accounts for shortage. See for example Hayek and Salameh (2001), Chiu et al. (2004), Rezaei (2005), Chiu and Chiu (2006), Wee et al. (2007), and Peter Chiu et al. (2010). So our main objectives are summarized in the following points:

1. Point out some common errors in the literature of production and inventory models considering shortage and constructing the correct models.

2. Develop two inventory control models where imperfect quality items are produced and backordering is allowed. In one model imperfect items are reworked after the end of the production process. The other model considers the generation of scrapped and re-workable items. Scrapped items are disposed of as they are produced and re-workable items are reworked at the end of the production process.

3. Combine the inventory model with random proportions of scrapped and re-workable items with process targeting to find the EPQ, optimal shortage allowed, and the optimal process mean.

4. Propose an inventory model to find the economic production run time and optimal shortage allowed when there is random interruption in the production process due to failure, preventive maintenance or any other disturbing event.

1.6 THESIS ORGANIZATION

The rest of the thesis is organized as follows. Chapter 2 summarizes literature related to our work. Chapter 3 presents an inventory model with imperfect quality items that are reworked after the production process ends. Chapter 4 presents a finite production rate model with two types of imperfect quality items under special case. Chapter 5 presents a finite production rate model with two types of imperfect quality items under general case. In Chapter 6, we investigate an integrated model that combines production and inventory control with process targeting. Chapter 7 studies an inventory model with stochastic breakdowns. Finally, chapter 8 summarizes the thesis and presents suggestions for further extensions and future work.

CHAPTER 2

LITERATURE REVIEW

One of the earliest papers that discussed imperfect production processes is Lee and Rosenblatt (1985) where they found the optimal production cycle time. Zhang and Gerchak (1990) studied the EOQ model with random proportion of defective items received. They focus on the joint lot sizing and inspection policy to be used. Cheng (1991) proposed an EOQ model, where the demand depends on unit production cost. The problem was formulated as a geometric program. A closed form optimal solution was found.

Khouja (1995) extended the Economic production quantity model by considering production cost as a function of production rate which in turn is as a decision variable. Liu and Yang (1996) discussed the lot-sizing problem in a single production stage. In their model; processing may generate two types of defective jobs: re-workable jobs; and non re-workable jobs that are disposed of immediately. Grubbstrom and Erdem (1999) presented a simple algebraic method instead of calculus for determining the optimal lot size with backlogging. Cárdenas-Barrón (2001) extended the algebraic approach to find the EPQ formula with shortage and under the condition of backlog cost per unit and time unit. Lin et al (2008) presented a straightforward algebraic approach to replace the calculus based method used in imperfect EMQ model of Wang Chiu (2007).

Goyaland Nebebe (2000) presented a method for solving the problem of production quantity and shipment policy of a product from one vendor to a single buyer. Salameh and Jaber (2000) generated an economic order quantity model where the received lot contains random proportion of defective items. After 100% inspection process, defective items are sold as a single batch in a secondary market. Chan et al. (2003) studied an EPQ model, where defective items are not disposed, but are sold at a discount price or it can be used in another production stage. Chang (2004) studied an inventory problem where fuzzy defective items received, the received lot screened 100% and defective items are sold as a single batch with discount prior to receiving the next shipment. He developed a model with fuzzy defective rate and fuzzy demand rate. Papachristos and Konstantaras (2006) discussed the issue of shortages in models with random proportion of imperfect quality items and shortage not allowed. They revisit the papers of Salameh and Jaber (2000) and Chan et al. (2003). They show that the conditions and constraints given in these papers are not sufficient to ensure that shortages will not occur. Then, they extended the model of Salameh and Jaber (2000) to the case in which withdrawing takes place at the end of the planning horizon.

Wee et al. (2007) developed an EOQ model for items with imperfect quality and shortages are backordered. The model is similar to Salameh and Jaber (2000) but assume that customers are willing to wait for new supply when there is a shortage. Eroglu and Ozdemir (2007) developed an EOQ model by allowing backorders and each received lot contains some defective items. They consider 100% inspection of each lot. Imperfect quality items are either sold in a secondary market or scrapped.

Maddah and Jaber (2008) restudied the EOQ model with random fraction of imperfect quality items and a screening process as in Salameh and Jaber (2000). Then they investigated the effect of screening speed and variability of the supply process on the order quantity. In addition, they extended the model by allowing for several batches of secondary market items to be collected and shipped in one lot. Khan et al. (2010) developed an EOQ model similar to Salameh and Jaber (2000). They include inspection, the inspector may commit type one or type two errors. The errors are random variables, and there are associated costs.

Jaber et al. (2008) extended the work of Salameh and Jaber (2000) by assuming a learning curve, where the percentage of imperfect quality items per received lot reduces according to the learning curve. Khan et al. (2010) extended Salameh and Jaber's work for the situation where there is learning in inspection. They considered two cases due to slow inspection effect, lost sales and backorders.

Hayek and Salameh (2001) considered the case where all imperfect items are stored and reworked at the end of the production process. They studied their effect on the optimal operating policy, where shortages are allowed and backordered. Chiu et al. (2007) investigated an inventory model to find the optimal production run time where there are random proportion of imperfect items and stochastic breakdowns. A fixed proportion of imperfect quality items is scrapped and the rest are reworked. They adopt the no resumption policy where the interrupted lot is aborted as breakdown take place. Peter Chiu et al. (2010) developed a finite production rate inventory model similar to Chiu et al. (2007) but they considered abort return policy under which the interrupted lot will be resumed immediately after maintenance.

Ben-Daya (2002) developed an integrated model to find out the economic production quantity and optimal preventive maintenance level for an imperfect process. Jamal et al (2004) developed an inventory model for determining the economic production quantity for a single product, which is manufactured in a single-stage manufacturing system that generates imperfect quality products. Defective products are reworked in the same cycle or after n cycles. Cárdenas-Barrón (2009) extended the work of Jamal et al (2004) by considering planned backorders.

Yoo et al. (2009) studied the economic production quantity model that includes imperfect inspection process, where the inspector may commit type one or two errors and the defective sales are returned. Sana (2010) investigated the Economic Production Lot sizing model with imperfect production system in which the production process may shift from an 'in-control' state to an 'out-of-control' state at random time. It is assumed that when the process is out of control defective items are produced and are reworked at some cost.

Chan and Tai (2006) developed an integrated model that combines inventory control and process targeting to find the best combination of economic manufacturing quantity, specification limits, and the process mean. They considered a rectifying inspection plan and used a symmetric quadratic quality loss function for inspection and measuring the product quality. Chan and Khoo (2009) investigated an integrated model that combines economic manufacturing quantity and the problem of quality loss model with k machines in a serial production system. Rectifying inspection plan with single sampling is adopted and Taguchi's symmetric quadratic quality loss function is used to estimate the product quality.

CHAPTER 3

DETERMINING OPTIMAL LOT SIZE WITH IMPERFECT QUALITY, REWORK, AND SHORTAGE

3.1 INTRODUCTION

This chapter considers the classical problem of finding the optimal production lot size that results in minimizing the total cost of inventory and setup while satisfying the demand. We consider a much more realistic version of the problem. First; we assume that the production process randomly generates non-conforming items with some probability. This probability, in turn, is a random variable. Second; we assume that defective items are reworked at a finite rate that may be slower than the demand rate. Finally; we assume that shortages in the form of backorders are allowed at a penalty cost. These realistic conditions are incorporated in a mathematical model. The mathematical model contains two integrals which we compute using a 12-node Gaussian quadrature method. An example is provided where we consider five probability density functions to model the probability of producing defectives. Finally, we study the effect of increasing the average probability of producing defectives on economic production quantity and shortage levels.

3.2 MODEL DESCRIPTION

This model is an extension of Hayek and Salameh (2001). We consider an inventory model of finite production rate, P, which is constant. The demand rate is continuous and constant. We assume that the proportion of non-conforming items,r, produced during the production process. This proportion is random variable with probability density function $g(r)$. Conforming items are used to fulfill the demand. On the other hand non-conforming items are reworked after production ends. The rework rate, P_R, may be different from the production rate, P. We assume that the production rate, P, exceeds the sum of the demand rate and the rate of generating re-workable items. Reworked items are assumed to be perfect and are used to fulfill the demand. Finally, shortages are backordered at a penalty.

3.3 MODEL FORMULATION

In this thesis we will examine the case where the rework rate of imperfect quality items is less than or equal to the demand rate, i.e. $P_R \leq D$. Figures 3.1 and 3.2 below show the on hand inventory during a production-inventory cycle. In Figure 3.1, the on hand inventory drops to zero after re-work is over. On the other hand, Fig. 3.2, we notice that the inventory level has dropped to zero and shortages are being built up during the rework stage. In this model, the inventory level after the rework stage may stay positive or it may drop to zero and shortages are built up. Depending on the proportion of the non-conforming items and then on the difference between the rework rate P_R and demand rate D. Therefore, in the subsequent analysis one has to combine the two cases in Figures 3.1 and 3.2.

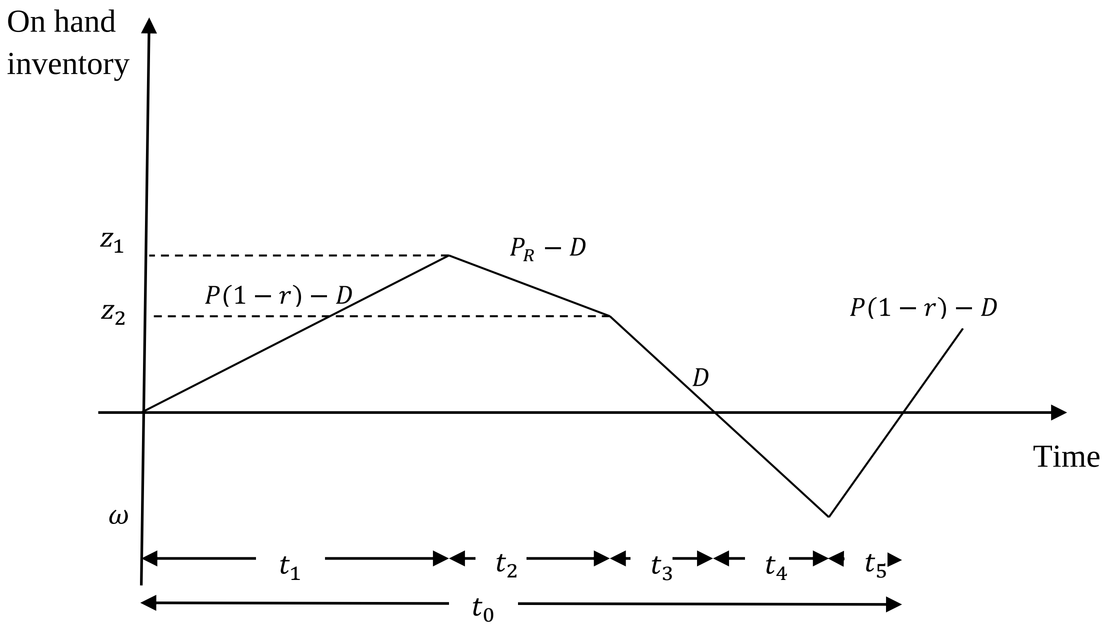

Figure 3.1 On- hand inventory of conforming items in case of positive Z_2

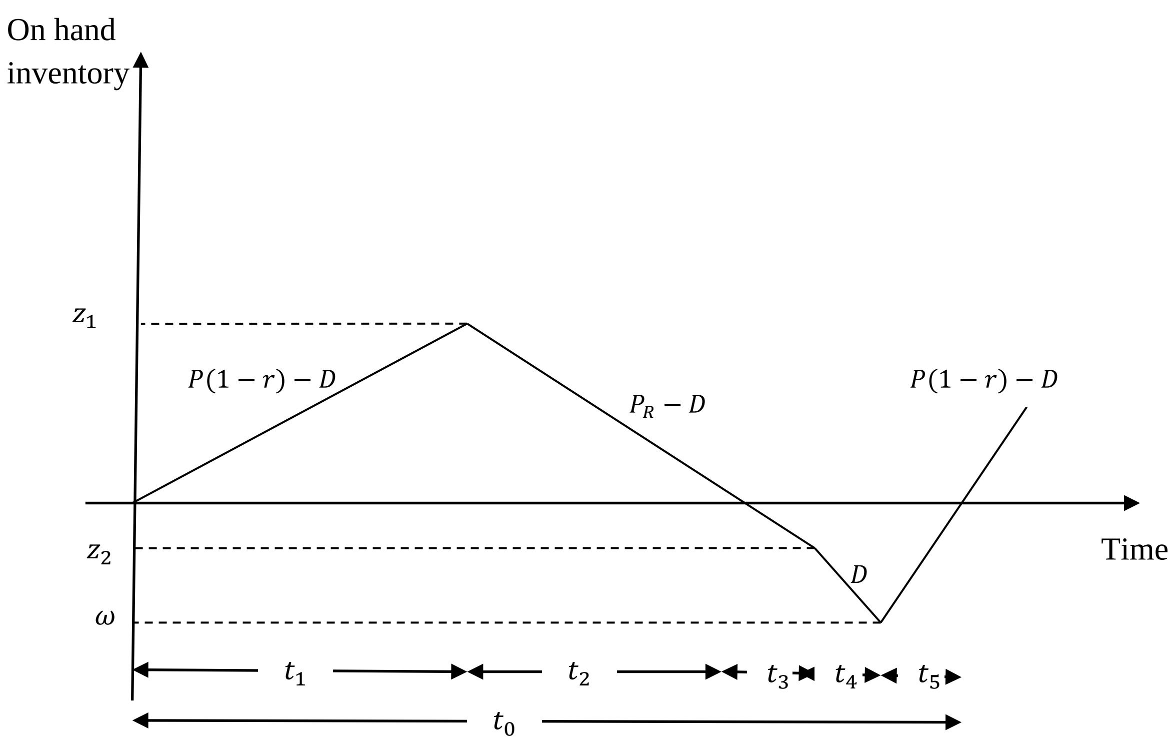

Figure 3.2 On- hand inventory of conforming items in case of negative Z_2

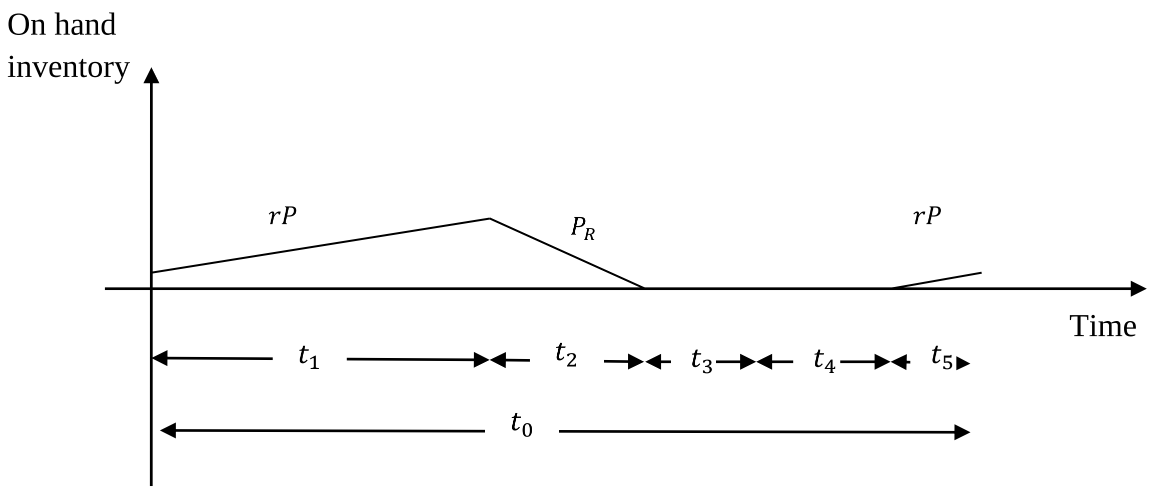

Figure 3.3 On- hand inventory of non-conforming items in case of positive z_2

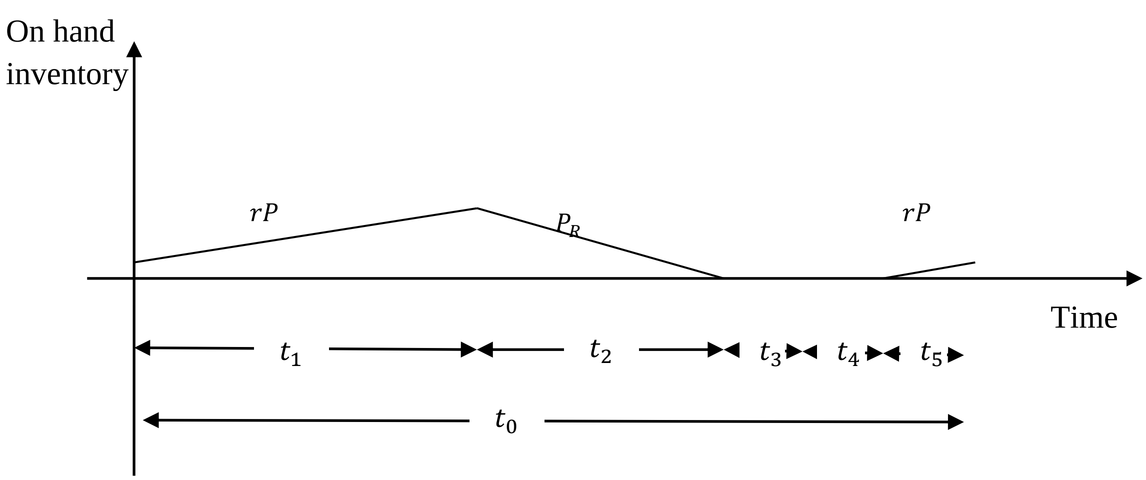

Figure 3.4 On- hand inventory of non-conforming items in case of negative z_2

There is no mention in Hayek and Salameh (2001) of the relative values of $P_R and\ D$. However, Figure 1 in their paper that shows the on hand inventory similar to Figure 3.1 above. We will denote the total cost per unit time derived using Figure 3.1 as TCU_{HS}. In the remainder of this section we derive the total cost expression for case shown in Fig. 3.2.

To avoid shortage during production time, the production rate should exceed the sum of the demand rate and the rate of generating nonconforming items. Since r is a random variable, then we must have $\mathrm{P}(1-\max(\mathrm{r})) \geq \mathrm{D}$. This model is valid if $\max(r) \leq \frac{P_R}{D}\left(1-\frac{D}{P}\right)$ as we show later. If this condition is violated, then the production system will never be able to satisfy the demand.

Production takes place during the intervals t_5 and t_1, therefore Q = (t_5+t_1)P. The quantity of conforming items produced is (t_5+t_1)[P(1–r) –D]. This quantity is used to satisfy the shortage and buildup the inventory to level z_1.

$$z_1 = (P(1-r)-D)\frac{Q}{P} - w \tag{3.1}$$

The time, t_1, needed to build up the inventory to level z_1 of perfect quality items is given by:

$$t_1 = \frac{z_1}{P(1-r)-D} \tag{3.2}$$

The time t_2 needed to rework the non-conforming items until inventory becomes zero.

$$t_2 = \frac{Z_1}{D-P_R} \tag{3.3}$$

The time t_3 needed to build up a shortage level of z_2 units is.

$$t_3 = \frac{z_2}{D - P_R} \tag{3.4}$$

The inventory level after rework stage, z_2 is:

$$z_2 = (t_2 + t_3)(D - P_R) - z_1 \tag{3.5}$$

Where:

$$(t_2 + t_3) = \frac{rQ}{P_R} \tag{3.6}$$

Substituting z_1 from (3.1) and $(t_2 + t_3)$ from (3.6) into (3.5), we get:

$$z_2 = \left(\frac{D(rP+P_R)}{PP_R} - 1\right)Q + w \tag{3.7}$$

The time t_4 needed to build up maximum shortage level of (w) units is:

$$t_4 = \frac{w - z_2}{D} \tag{3.8}$$

Since $t_4 \geq 0$, then $w \geq z_2$,

From (3.8) one must have:

$$r \leq \frac{P_R}{D}\left(1 - \frac{D}{P}\right) \tag{3.9}$$

Condition (3.9) should be true for all values of r, Hence,

$$max(r) \leq \frac{P_R}{D}\left(1 - \frac{D}{P}\right) \tag{3.10}$$

The time t_5 needed to eliminate the back order once production is started again is:

$$t_5 = \frac{w}{P(1-r)-D} \tag{3.11}$$

The time needed to consume all units Q at rate D is the cycle length t_0 expressed as

$$t_0 = \frac{Q}{D} \tag{3.12}$$

The relevant costs per cycle associated to this model are shown below:

(1) Production cost of all items (perfect and imperfect) = cQ.

(2) Repair cost of non-conforming items = $c_R Qr$.

(3) Setup cost = A.

(4) Holding cost of conforming items and items to be reworked.

Holding costs =$\mathrm{h}\left(\frac{\mathrm{z}_1\mathrm{t}_1}{2} + \frac{\mathrm{z}_1{}^2}{2(D-P_R)} + \frac{\mathrm{rP}}{2}(\mathrm{t}_1+\mathrm{t}_5)^2\right) + \mathrm{h_R P_R}\frac{(\mathrm{t}_2+t_3)^2}{2}$.

(5) Shortage cost =$\mathrm{b}\left(\frac{\mathrm{z}_2{}^2}{2(D-P_R)} + \frac{w+z_2}{2}\mathrm{t}_3 + \frac{\mathrm{w}}{2}\mathrm{t}_5\right)$

The total cost per production cycle is the sum of all above costs and is given by:

$$\begin{aligned} TC(Q,w|r) = c\,Q + c_R Qr + A + \mathrm{h}\left(\frac{\mathrm{z}_1\mathrm{t}_1}{2} + \frac{\mathrm{z}_1{}^2}{2(D-P_R)} + \frac{\mathrm{rP}}{2}(\mathrm{t}_1+\mathrm{t}_5)^2\right) \\ + \mathrm{h_R P_R}\frac{(\mathrm{t}_2+t_3)^2}{2} + \mathrm{b}\left(\frac{\mathrm{z}_2{}^2}{2(D-P_R)} + \frac{w+z_2}{2}\mathrm{t}_3 + \frac{\mathrm{w}}{2}\mathrm{t}_5\right) \end{aligned} \tag{3.13}$$

By dividing the total cost per cycle from (3.11) over the cycle length from (3.10), the total cost per unit time can be written as:

$$TCU_{SA}(Q,w|r) = \frac{TC(Q,w|r)}{t_0} = A_0 + \frac{A_1}{Q} + A_2 Q + A_3 w + A_4\frac{\mathrm{w}^2}{\mathrm{Q}} \tag{3.14}$$

Where:

$A_0 = D(c + c_R r)$

$A_1 = AD$

$A_2 = \frac{hD}{2}\left(\frac{a_1}{P} + \frac{{a_1}^2}{a_3} + \frac{r}{P}\right) + \frac{h_R D r^2}{2P_R} + \frac{bD}{2}\left(\frac{{a_2}^2}{a_3} - \frac{{a_2}^2}{D}\right)$

$A_3 = -hD\left(\frac{1}{\mathrm{P}} + \frac{a_1}{a_3}\right) + bD\left(\frac{a_2}{a_3} - \frac{a_2}{D}\right)$

$A_4 = \frac{hD}{2}\left(\frac{1}{P\mathrm{a}_1} + \frac{1}{a_3}\right) + \frac{bD}{2}\left(\frac{1}{a_3} + \frac{1}{Pa_1}\right)$

$a_1 = 1 - r - D/P$

$a_2 = \frac{D(rP+P_R)}{PP_R} - 1$

$a_3 = D - P_R$

To prove that $TCU_{SA}(Q,w)$ is a convex function, we examine the convexity of its components.

The second term is a positive constant times the reciprocal of a non-negative linear function hence it is convex. The third and fourth terms are linear. The last term is a positive constant times w^2/Q. It is straight forward to show that the Hessian of this function is positive semidefinite. Hence, the last term is also a convex function. Therefore, the expected annual cost function is convex function and a local solution is also global.

As we mentioned previously, the value of z_2 depends on the proportion of non-conforming items. The proportion of non-conforming items has a limit upon which z_2 will be negative. This limit can be calculated by (3.15).

$$L_r = \frac{P_R\left(1 - \frac{D}{P} - \frac{w}{Q}\right)}{D} \tag{3.15}$$

Let r_{min} and r_{max} be the minimum and maximum value of r. Then we have two cases. If $r_{min} \leq r \leq L_r$, then $z_2 \geq 0$ and the total cost per unit time is given by TCU_{HS}. On the other hand if $L_r \leq r \leq r_{max}$, then $z_2 \leq 0$ and the total cost per unit time is given by TCU_{SA}. Therefore, the expected cost per unit time is given by;

$$TCU(Q,w) = \int_{r_{min}}^{L_r} f(r)TCU_{HS}\,dr + \int_{L_r}^{r_{max}} f(r)\,TCU_{SA}\,dr \tag{3.16}$$

The minimum of $TCU(Q,w)$ can not be found in closed form. Therefore, we find the solution by performing exhaustive search for Q and w. The integration is carried out numerically using a 12-node Gaussian quadrature method

3.4 NUMERICAL EXAMPLE AND SENSITIVITY ANALYSES

In this Chapter we solve an example similar to that of Hayek and Salameh (2001). We use the same values of the data in their example except for P_R and b which we take as 500 unit/year and $ 25/unit/year, respectively.

The proportion of non-conforming items is distributed over the interval [0, 0.1].

Table 3.1 shows the optimal values of economic production quantity Q^*, optimal shortage quantity w^* , and the corresponding total cost per unit time $TCU(Q^*,w^*)$ for five probability distribution functions for r.

Table 3.1 Optimal EPQ and shortage quantity for different probability mass functions of r.

$g(r)$	Formula	Parameters		Q^*	w^*	$TCU(Q^*,w^*)$
Uniform	$g(r)=\frac{1}{b-a}$	a=0	b=0.1	1060	95	128672
Normal	$g(r)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\left(\frac{(r-\mu)^2}{2\sigma^2}\right)}$	μ=0.05	σ=0.015	1070	99	128535
Exponential	$g(r)=\lambda e^{-\lambda r}$	λ=55	------	1112	116	127684
Gamma	$g(r)=\frac{1}{\beta^\alpha\Gamma(\alpha)}r^{\alpha-1}e^{-\frac{r}{\beta}}$	σ=3	β=0.01	1097	110	128015
Weibull	$g(r)=\frac{P}{\beta^\alpha}r^{\alpha-1}e^{-\left(\frac{r}{\beta}\right)^\alpha}$	σ=4	β=0.06	1062	96	128650

To show the need for this model we will compare the answer obtained above for the case where proportion of nonconforming items follows the normal distribution with the solution generated using the classical model. The classical finite production model, where shortages are backordered gives $\mathrm{Q}^*=1138$ units and $\mathrm{w}^*=126$ units. If these values are substituted in the cost function given in (3.16), we get $\mathrm{TCU}(\mathrm{Q}^*,\mathrm{w}^*)=\$\,128{,}737$. This value exceeds the optimal value shown in Table 3.1.

Next, we study the effect of the imperfect quality items proportion on the optimal solution. We consider the case that *r* is normally distributed with fixed standard deviation $\sigma=0.015$ and varied mean μ. We choose μ such that $0\le\mu+3.5\sigma\le 1-\frac{D}{P}$. We use the same data of the above example except for the rework rate which we set at 800 units/year. Fig. 3.5 illustrates the behavior of the optimal production quantity Q^* and optimal shortage quantity w^* for the changes in mean of the non-conforming items proportion. One notices that when μ increases the optimal production quantity Q^* and optimal shortage quantity w^* decrease, Figure 3.6 shows behavior of $\mathrm{TCU}(\mathrm{Q}^*,\mathrm{w}^*)$ with changes in the mean of the non-conforming items proportion. The figure shows as the proportion of non-conforming items increase, TCU increases.

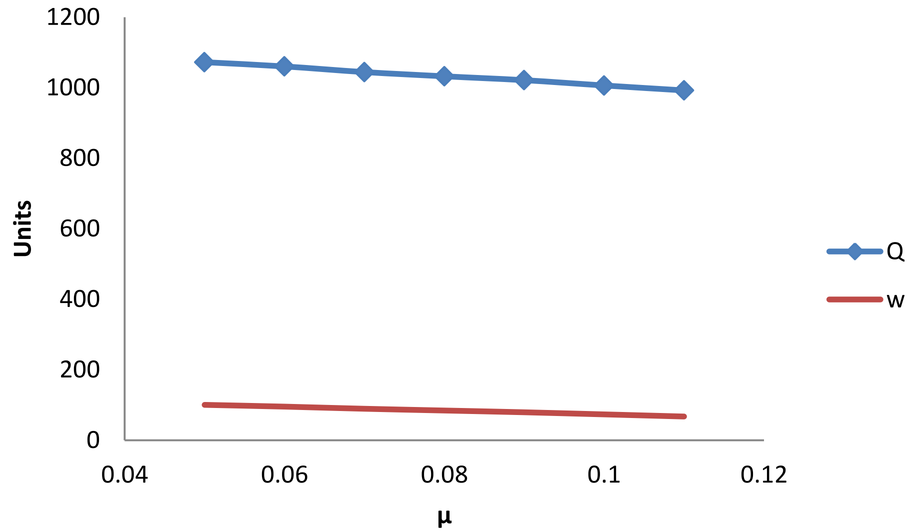

Figure 3.5 The effect of non-conforming items proportion on the optimal production quantity Q* and the optimal shortage quantity w*.

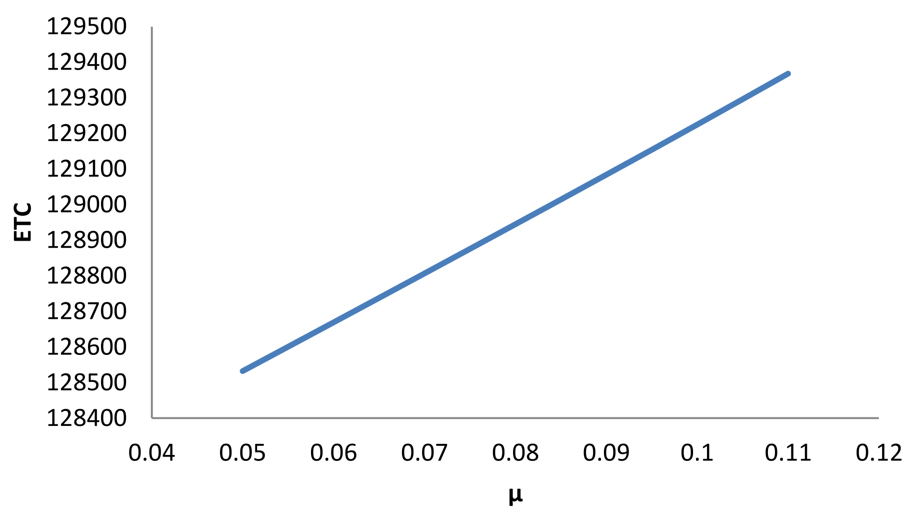

Figure 3.6 The effect of non-conforming items proportion on the total cost per unit time $\mathbf{TCU}(\mathbf{Q}^*, \boldsymbol{\omega}^*)$

3.5 SUMMARY

The classical EPQ model is inappropriate when production lots have non-conforming items. Therefore, new models are required for more realistic solutions. Such an EPQ model is developed when each production lot contains proportion of non-conforming items these items are reworked at constant rate, and shortages are allowed. It is assumed that imperfect quality items proportion is random variable. An example is provided for the developed model, and effects of individual changes in imperfect quality items proportion on optimal solution have been studied. From the model discussed in this chapter we conclude that shortages may occur depending on the percentage of the non-conforming items. So we develop solution for all cases of shortages for the model under study. One notices that when the imperfect quality items proportion increases, the optimal production quantity and optimal shortage quantity decreases, where the total cost per unit time increases.

CHAPTER 4

PRODUCTION PLANNING WITH TWO TYPES OF IMPERFECT QUALITY ITEMS WITH $P_R \geq D$

4.1 ITRODUCTION

This chapter considers the problem of satisfying constant and continuous demand through batch production at a finite rate. We assume that produced items may contain non-conforming ones that can be reworked and others that are scrapped. Costs are associated with these types of non-conforming items. The proportions of re-workable and scrapped items are random variables. In addition, we assume that shortages are permissible at some cost. This realistic scenario is modeled mathematically. We derive a closed form for the optimal batch size and maximum shortage quantity that result in minimizing the total cost of production, inventory and setup costs. It is shown that the global solution of the problem is obtained. An example is presented and sensitivity to changes in model parameters is studied.

4.2 MODEL DESCRIPTION

We assume that the production and demand rates, P and D respectively are constant and known. In this chapter we assume that there are two types of non-conforming items, re-workable items and scrapped items. The proportions of non-conforming items that are re-workable and that are scrapped are r and s respectively. Both of these proportions are random variables with probability density functions $g(r)$ and $f(s)$. Conforming items are used to fulfill the demand. Scrapped items are disposed of at a cost, while re-workable items are processed after production ends. The rework rate, P_R, may be different from the production rate, P. We assume that the production rate exceeds the sum of the demand rate and the rate of generating re-workable and scrapped items. Reworked items are assumed to be perfect and are used to fulfill the demand. Finally, shortages are backordered at a penalty.

This chapter is an extension of Hayek and Salameh (2001) in two directions. First we introduce the probability of having scrapped items. Second, we point to the proper assumptions underlying the model. Ignoring these assumptions, results in faulty application of the model.

4.3 MODEL FORMULATION

To avoid shortage during production time, the production rate should exceed the sum of the demand rate and the rate of generating nonconforming items. Since s and r are random variables, then we must have $P(1 - \max(s) - \max(r)) \geq D$. Also, we assume that during rework the inventory level is non-negative. A sufficient condition to achieve this case is that $P_R \geq D$.

Production takes place during the intervals t_5 and t_1, therefore $Q=(t_5+t_1)P$. The quantity of conforming items produced is $(t_5+t_1)[P(1–s–r) –D]$. This quantity is used to satisfy the shortage and buildup the inventory to level z_1.so the value of z_1 given by (4.1):

$$z_1 = (P(1-s-r)-D)\frac{Q}{P} - w \tag{4.1}$$

t_1, needed to build up the inventory to level z_1 of perfect quality items is given by:

$$t_1 = \frac{z_1}{P(1-s-r)-D} \tag{4.2}$$

The number of items that need rework is rQ, so the time t_2 needed to rework the re-workable items produced is:

$$t_2 = \frac{rQ}{P_R} \tag{4.3}$$

The maximum inventory level reached, z_2 is: $z_2 = z_1 + t_2(P_R – D)$. By substituting z_1 from (4.1) and t_2 from (4.3) we get:

$$z_2 = \left(1 - s - \frac{D(rP+P_R)}{PP_R}\right)Q - w \tag{4.4}$$

Hence the time to consume this inventory level, t_3. Where:

$$t_3 = \frac{z_2}{D} \tag{4.5}$$

The time t_4 needed to buildup a maximum shortage of w units is:

$$t_4 = \frac{w}{D} \tag{4.6}$$

The time t_5 needed to eliminate the backorders once production is started again is:

$$t_5 = \frac{w}{P(1-s-r)-D} \tag{4.7}$$

The cycle length t_0 which is the time needed to consume all perfect units (1–*s*) *y* at rate *D* can be expressed as.

$$t_0 = \frac{(1-s)Q}{D} \tag{4.8}$$

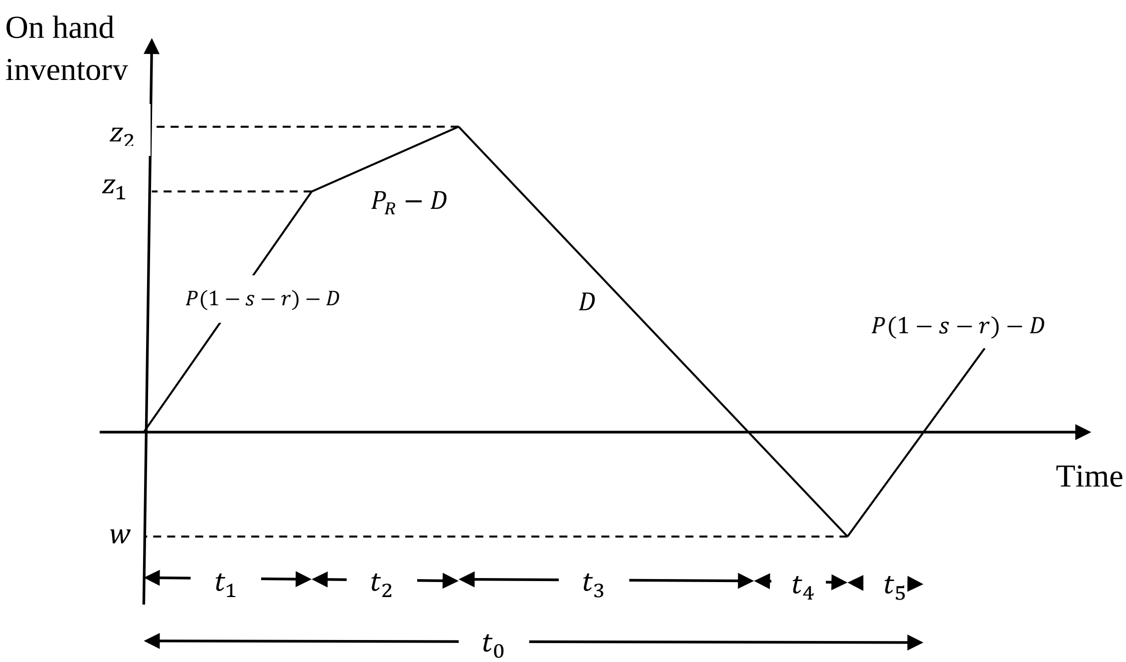

Figure 4.1 On-hand inventory of conforming items in case of $P_R \geq D$

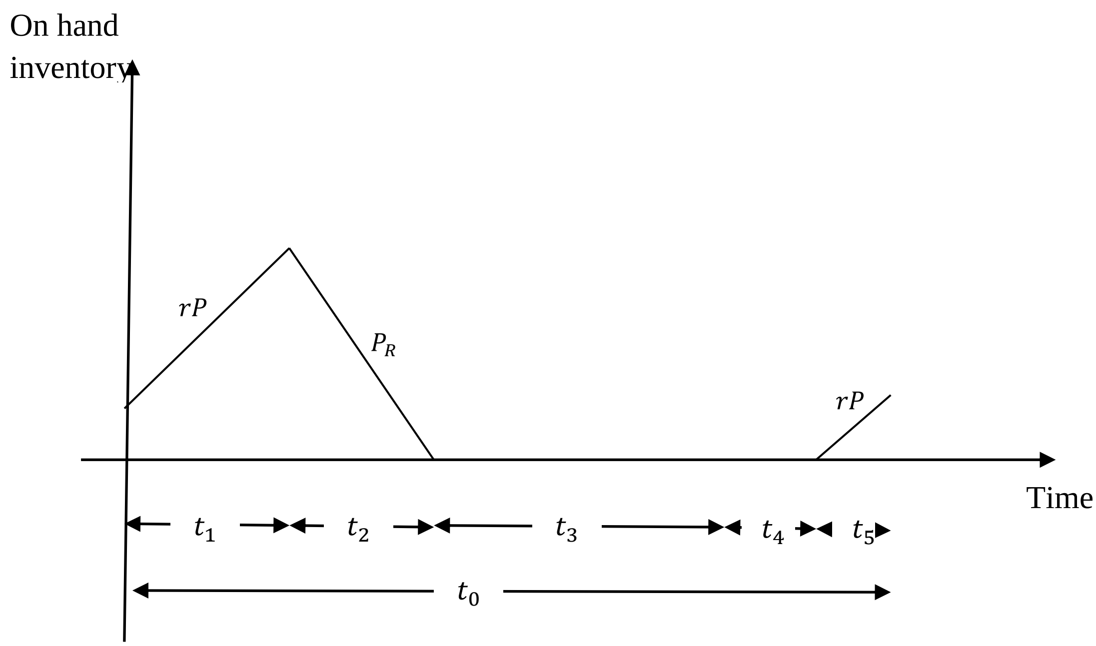

Figure 4.2 On-hand inventory of re-workable items in case of $P_R \geq D$

This model has six cost items that we list below;

(1) Production cost of all items = cQ.

(2) Rework cost of re-workable items = $c_R Q r$.

(3) Disposal cost for defective items =$c_d Q s$.

(4) Setup cost = A.

(5) Holding cost of conforming items and items to be reworked =

Holding costs =$h\left[\frac{z_1 t_1}{2} + \frac{z_1+z_2}{2} t_2 + \frac{z_2 t_3}{2}\right] + h\frac{rP}{2}(t_1+t_5)^2 + h_R P_R \frac{t_2^2}{2}$.

(6) Shortage cost =$b\frac{w}{2}(t_4+t_5)$

The total cost per production cycle is sum of all above costs and is given by

$$TC(Q,w|r,s) = c\,Q + c_R Qr + c_d Qs + A + h\left[\frac{z_1 t_1}{2} + \frac{z_1+z_2}{2}t_2 + \frac{z_2 t_3}{2}\right] + h\frac{rP}{2}(t_1+t_5)^2 + h_R P_R \frac{t_2^2}{2} + b\frac{w}{2}(t_4+t_5) \tag{4.9}$$

Total cost per unit time can be written as:

$$TCU(Q,w|r,s) = \frac{TC(Q,w|r,s)}{t_0} \tag{4.10}$$

Substituting the expression of z_1, z_2, t_i, i=0,1,..,5 into TCU results in the following expression:

$$TCU(Q,w|r,s) = \frac{cD}{1-s} + \frac{\mathrm{c_R rD}}{1-s} + \frac{c_d\,s\,D}{1-s} + \frac{DA}{Q(1-s)} + \frac{h}{2}\left[\left(1-s+\frac{Ds}{P(1-s)} - \frac{D}{P(1-s)}\right)Q - 2w\right] + \frac{(h_R-h)Q}{2}\left(\frac{Dr^2}{P_R(1-s)}\right) + (b+h)\frac{w^2}{2Q}\left(\frac{1-s-r}{(1-s)\left(1-s-r-\frac{D}{P}\right)}\right) \tag{4.11}$$

The expected value of $TCU(Q, w|r, s)$is:

$$ETC(Q,w) = \int_{s_{min}}^{s_{max}} \int_{r_{min}}^{r_{max}} TCU(Q,w|r,s)f(s)g(r)drds$$

$$= cDE\left(\frac{1}{1-s}\right) + \mathrm{c_R D}E\left(\frac{r}{1-s}\right) + c_d DE\left(\frac{s}{1-s}\right)$$

$$+ \frac{DA}{Q}E\left(\frac{1}{1-s}\right) + \frac{h}{2}\left[\left(1 - \frac{D}{P} - E(s)\right)Q - 2w\right] \tag{4.12}$$

$$+ \frac{(h_R - h)QD}{2P_R}E\left(\frac{r^2}{1-s}\right)$$

$$+ (p+h)\frac{w^2}{2Q}E\left(\frac{1-s-r}{(1-s)\left(1-s-r-\frac{D}{P}\right)}\right)$$

The expected total cost per unit time can be written as;

$$ETC(Q,w) = A_0 + \frac{A_1}{Q} + A_2 Q - h.w + A_3.\frac{\mathrm{w}^2}{\mathrm{Q}} \tag{4.13}$$

Where:

$$A_0 = cDE\left(\frac{1}{1-s}\right) + \mathrm{c_R D}E\left(\frac{r}{1-s}\right) + c_d DE\left(\frac{s}{1-s}\right)$$

$$A_1 = DAE\left(\frac{1}{1-s}\right)$$

$$A_2 = \frac{h}{2}\left(1 - \frac{D}{P} - E(s)\right) + \frac{(h_R - h)D}{2P_R}E\left(\frac{r^2}{1-s}\right)$$

$$A_3 = \frac{p+h}{2}E\left(\frac{1-s-r}{(1-s)\left(1-s-r-\frac{D}{P}\right)}\right)$$

To prove that ETC is a convex function; we examine the convexity of its components.

The second term is a positive constant times the reciprocal of a non-negative linear function hence it is convex. The third and fourth terms are linear. The last term is a positive constant times w^2/Q. It is straight forward to show that the Hessian of this function is positive semi definite. Hence the last term is also a convex function. Therefore, the expected annual cost function is convex function and a local solution is also global.

The optimal values of Q and w can be obtained by setting the partial derivatives of ETC equal to zero. This yields the two simultaneous equations $w^2[4A_2A_3^2/h^2 - A_3] = A_1$ and $Q = (2A_3\, w)/h$.

A solution exists if $A_2A_3 \geq h^2/4$.

This yields:

$$Q^* = \sqrt{\frac{A_1}{A_2 - \frac{h^2}{4A_3}}} \tag{4.14}$$

$$w^* = \frac{h}{2A_3} \times Q^* \tag{4.15}$$

The optimum expected total cost per unit time $ETC(Q,w)$ is obtained by direct substitution of Q^* and w^* given by (4.14) and (4.15), respectively.

Since inventory level after production process ends, z_1, must be greater or equal to zero, we should have:

$$1 - s_{max} - r_{max} - \frac{D}{P} \geq \frac{w}{Q} \tag{4.16}$$

Since the left side equal A_5, then $z_1 \geq 0$ if $A_5 \geq \frac{w}{Q}$

If Q^* and w^* given by (4.14) and (4.15) satisfy (4.16). Then Q^* and w^* are the solution of the inventory control problem.

If Q^* and w^* violate (4.16), then at an optimal solution we must have:

$$A_5 = \frac{w}{Q} \tag{4.17}$$

Substituting w from (4.17) into ETC given by (4.13) results in:

$$ETC'(Q) = A_0 + \frac{A_1}{Q} + A_2 Q - h\, A_5 Q + A_3 {A_5}^2 Q \tag{4.18}$$

The minimum of $ETC'(Q)$ is achieved at:

$$Q^{*\prime} = \sqrt{\frac{A_1}{A_2 - hA_5 + A_3 {A_5}^2}} \tag{4.19}$$

Then,

$$w^{*\prime} = A_5 Q^{*\prime} \tag{4.20}$$

If the production is always conforming, i.e. s =0 and r=0, we get the same equations of the classical finite production model, where shortages are allowed and backordered:

$$Q^* = \sqrt{\frac{2AD(b + \mathrm{h})}{bh(1 - \frac{D}{P})}} \quad (4.21)$$

$$w^* = \frac{h}{(b + h)}(1 - \frac{D}{P})Q^* \quad (4.22)$$

and

$$TCU(Q^*, w^*) = cD + \frac{AD}{Q} + \frac{bw^2 + h\left[w - Q\left(1 - \frac{D}{P}\right)\right]^2}{2Q\left(1 - \frac{D}{P}\right)} \quad (4.23)$$

4.4 NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

The example considered in this paper is taken from Hayek and Salameh (2001) with some changes in the data values. Specifically we take P_R = 2000 unit/year instead of 100 unit/year to satisfy the condition that $P_R \geq D$. By taking in consideration that in our model we assume that r is proportion of re-workable items produced as random variable with a known probability density function $g(r)$.

P = 1600 units/year,

D= 1200 units/year,

P_R = 2000 units/year,

c = \$104/unit,

c_R= \$8/unit,

c_d= \$5/unit,

A = \$1500,

h = \$20/unit/year,

h_R = \$22/unit/year,

b = \$25/unit,

$$f(s) = \begin{cases} 20 & for\ 0 < s < 0.05 \\ 0 & otherwise \end{cases}$$

$$g(r) = \begin{cases} 10 & for\ 0 < r < 0.1 \\ 0 & otherwise \end{cases}$$

Therefore,

$E(s) = 0.025; E(r) = 0.05, E\left(\frac{1}{1-s}\right) = 1.02587,\ E\left(\frac{s}{1-s}\right) = 0.02587, E\left(\frac{r}{(1-s)}\right) =$

$0.05129, E\left(\frac{r^2}{(1-s)}\right) = 0.00342,\ E\left(\frac{1-s-r}{(1-s)(1-s-r-\frac{D}{P})}\right) = 5.59026$

The optimum solution is Q^*=1126 units, w^*=90 and ETC(Q^*,w^*)=\$ 131956/year.

For the classical finite production model, where backorders are allowed and output is always conforming, we find values of $Q^* = 1138$ units, $w^* = 126$ units. By substituting these values in the cost function given in (4.13), we get $ETC(Q^*, w^*)$ =\$132095 /year. We notice that the resulted cost is greater than the one obtained using our formulas. So in the case of imperfect quality items produced, the optimum values should be obtained using the new equations developed.

Suppose that the probability density function of s and r are uniform random variables with ranges [0,S] and [0,R] respectively. Table 4.1, shows the effect of S and R on Q^*, $w^*$and $ETC(Q^*, w^*)$. One notices that as S and R increase, w^* decreases. While Q^* increases respect to proportion of scrapped items and decreases respect to re-workable items proportion, but ETC(Q^*,w^*) increases as the proportion of non-conforming items increases.

Table 4.1 Effect of S and R on Q^*,$\omega^*$and $ETC(Q^*, w^*)$

S	R														
	0			0.025			0.05			0.075			0.1		
	Q*	w*	TCU	Q*	w*	TCU	Q*	w*	TCU	Q*	w*	TCU	Q*	w*	TCU
0	1138	126	127962	1121	120	128131	1104	113	128302	1085	106	128477	1067	98	128655
0.025	1175	124	129566	1156	117	129738	1137	110	129914	1117	102	130092	1096	94	130276
0.05	1213	121	131227	1192	113	131404	1171	106	131584	1149	98	131767	1126	90	131956
0.075	1254	117	132950	1230	109	133131	1206	101	133317	1182	93	133506	1156	84	133702
0.1	1296	113	134739	1269	104	134926	1242	96	135118	1214	87	135315	1169	58	135561

Figures 4.3 and 4.4 illustrate the behavior of the optimal production quantity Q^* and optimal shortage quantity w^* against different expected values of re-workable and defective proportions, *s* and *r* respectively.

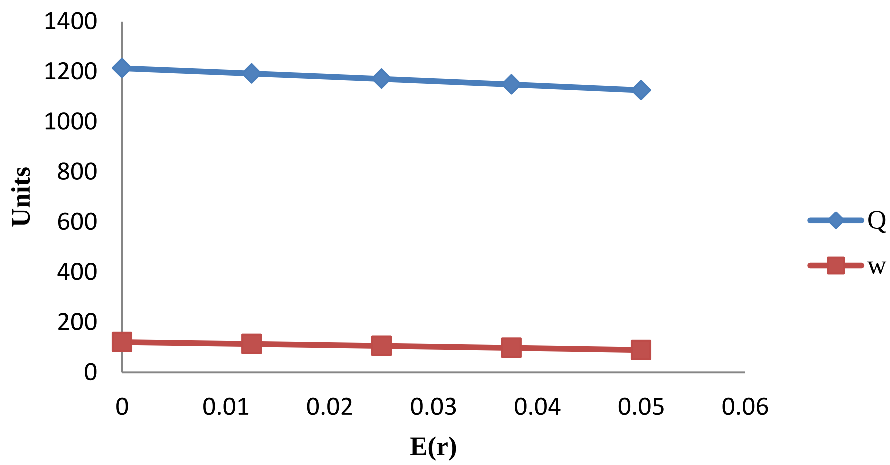

Figure 4.3 Proportion of re-workable items effects on optimal production quantity Q* and the optimal shortage quantity w* for E(s)=0.05

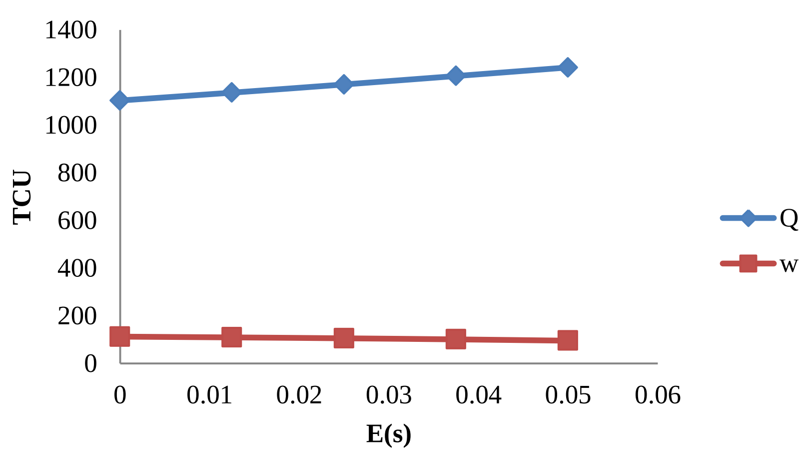

Figure 4.4 Proportion of scrapped items effects on optimal production quantity Q* and the optimal shortage quantity w*for E(r)=0.05.

4.5 SUMMARY

The classical EPQ model is inappropriate when the production contains non-conforming items. Therefore, new models are required for more realistic solutions. Such an EPQ model is developed when each ordered lot contains non-conforming items these non-conforming items either reworked after production ends, and become a good items with re-workable cost, or it will be scrapped and disposed with a disposal cost, and shortages backordered. It is assumed that defective and re-workable items proportions are a random variables uniformly distributed. One notices that, when proportion of non-conforming items increase individually, the economical production quantity and maximum shortage quantity will decrease respect to re-workable items proportion, economical production quantity increases respect to scrapped items proportion while the maximum shortage quantity will decrease respect to scrapped items proportion. Where the optimal total cost per unit time increases as proportion of non-conforming items increases.

CHAPTER 5

PRODUCTION PLANNING WITH TWO TYPES OF IMPERFECT QUALITY ITEMS

5.1 INTRODUCTION

In the classical economic production quantity we assume that all items produced are perfect quality items, where this is not the real case behavior because the process may deteriorate or get affected by the environment or any other factor. The finite production rate model with two types of imperfect quality items produced is examined in this chapter. Each produced lot contains proportion of non-conforming items which contains two types. First type is re-workable items which can be reworked after finishing of production and become conforming items. The portion of re-workable items considered being a random variable with known probability density function and re-workable cost, second type is a scrapped items which has to be disposed with disposal cost and has known probability density function. Backorders are allowed. In this chapter we also investigate the case when we have shortages due to the high proportion of non-conforming items and the fact that rework rate is less than the demand rate. The effect of producing non-conforming items on optimal solution is studied while numerical example is provided for the developed model.

The model discussed in this chapter is the same as Chapter 4 except that the condition $P_R \geq D$ is relaxed

5.2 MODEL FORMULATION

As shown in Chapter 3, when $P_R \leq D$ there is a possibility that the inventory level drops to zero before the rework is completed. This fact depends mainly on the percentage of the non-conforming items and then on the difference between the rework rate P_R and demand rate D. Figures 5.1 and 5.3 show the case when the inventory level after completion of rework is positive, i.e. $z_2 \geq 0$. On the other hand Figures 5.2 and 5.4 show the case when $z_2 < 0$.

We denote the total cost per unite time for the case $z_2 \geq 0$ and $z_2 < 0$ as $TCU_{Z_2>0}$ and $TCU_{Z_2<0}$, respectively. Note that $TCU_{Z_2>0}$ is the same as that *ETC* in Chapter 4.

To avoid shortage while producing, production rate of good items is always greater than or equal to the sum of the demand rate and the rate at which non-conforming items are produced {P(1 - max(s) - max(r)) $\geq$ D}. This model is valid if $\max(r) \leq \frac{P_R}{D}\left(1 - \max(s) - \frac{D}{P}\right)$ as will be shown later.

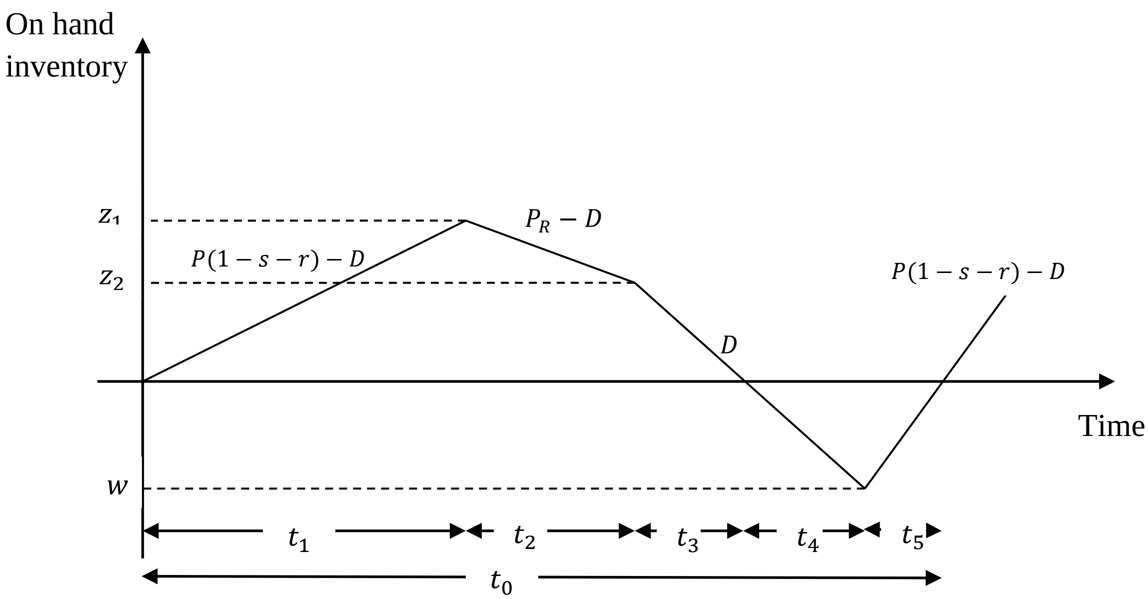

Figure 5.1 On- hand inventory of conforming items in case of positive Z_2

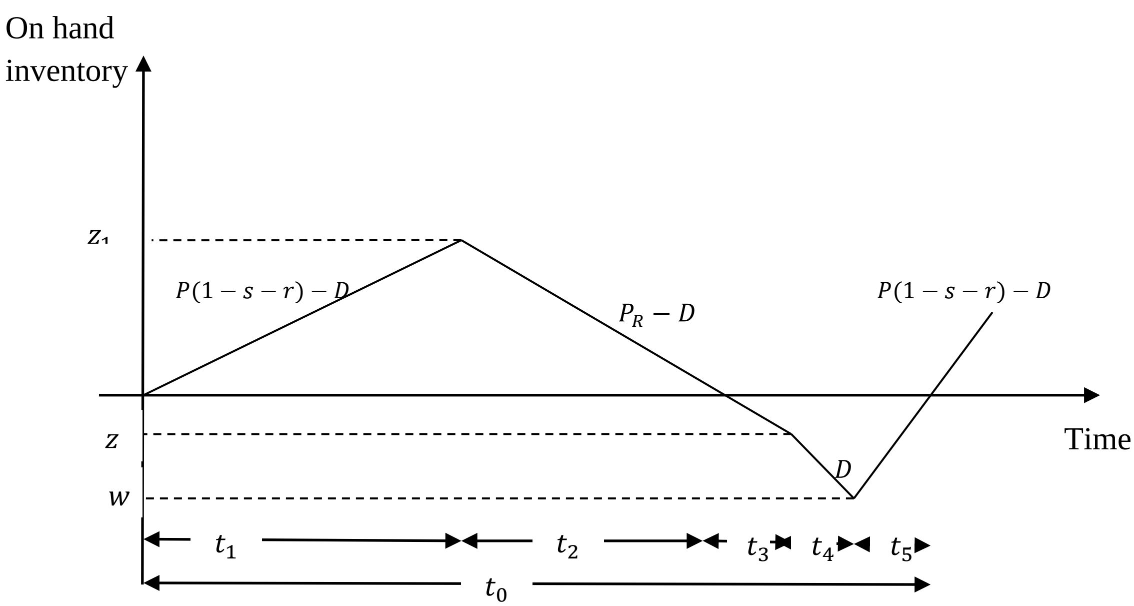

Figure 5.2 On- hand inventory of conforming items in case of negative Z_2

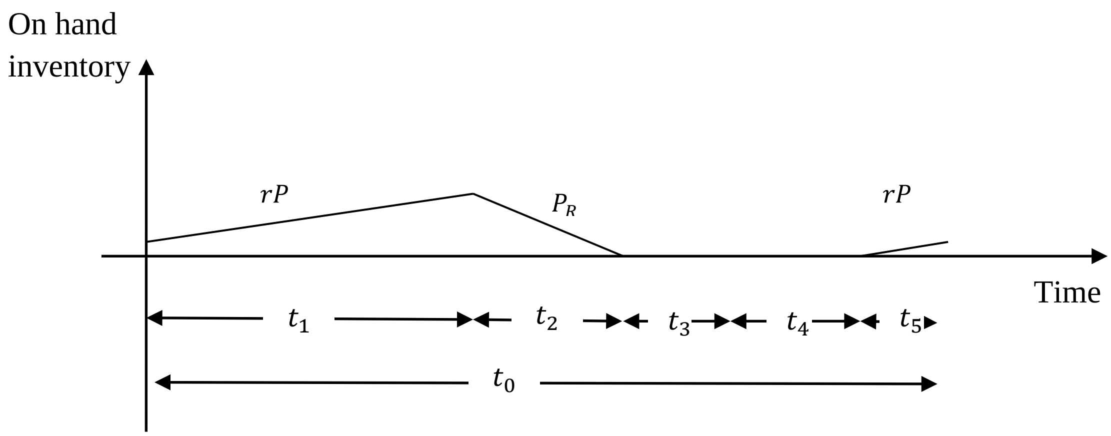

Figure 5.3 On- hand inventory of non-conforming items in case of positive z_2

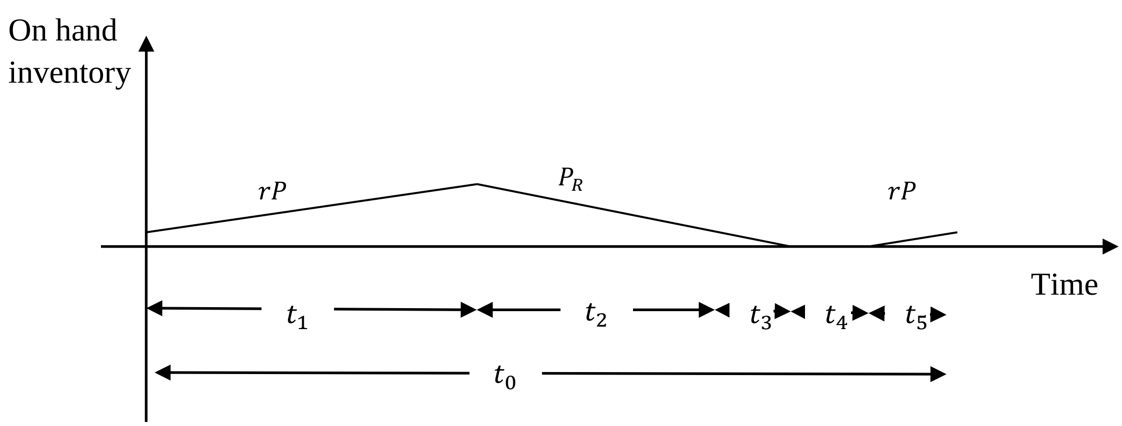

Figure 5.4 On- hand inventory of non-conforming items in case of negative z_2

From Fig. 5.1, we find the following:

The inventory level z_1 is:

$$z_1 = (1 - s - r - D/P)Q - w \tag{5.1}$$

The time t_1 needed to build up z_1 units of items that are perfect quality items

$$t_1 = \frac{z_1}{P(1-s-r)-D} \tag{5.2}$$

And the time t_2 needed to rework portion of the non-conforming items until inventory becomes zero,

$$t_2 = \frac{z_1}{D-P_R} \tag{5.3}$$

The time t_3 needed to build up a back order level of z_2 units is:.

$$t_3 = \frac{z_2}{D-P_R} \tag{5.4}$$

The inventory level after rework process z_2 is:

$$z_2 = (t_2+t_3)(P_R-D)-z_1 \tag{5.5}$$

Where:

$$(t_2+t_3) = \frac{rQ}{P_R} \tag{5.6}$$

Substituting z_1 from (5.1) and (t_2+t_3) from (5.6) into (5.5),we get:

$$z_2 = \left(s+\frac{D(rP+P_R)}{PP_R}-1\right)Q+w \tag{5.7}$$

The time t_4 needed to build up a back order level of w units is:

$$t_4 = \frac{w-z_2}{D} \tag{5.8}$$

Since $t_4 \geq 0$, then $w \geq z_2$,

From (5.8) one must have:

$$r \le \frac{P_R}{D}\left(1 - s - \frac{D}{P}\right) \tag{5.9}$$

Condition (5.9) should be true for all values of s and r, Hence,

$$\max(r) \le \frac{P_R}{D}\left(1 - \max(s) - \frac{D}{P}\right) \tag{5.10}$$

The time t_5 needed to eliminate the back order once production is started again is:

$$t_5 = \frac{w}{P(1 - s - r) - D} \tag{5.11}$$

The time needed to consume all units Qat rate D is the cycle length t_0 expressed as:

$$t_0 = \frac{(1 - s)\,Q}{D} \tag{5.12}$$

We have six related costs in our model:

(1) Production cost of all items = $c\,Q$.

(2) Rework cost of re-workable items = $c_R Q r$.

(3) Disposal cost for defective items =$c_d Q s$.

(4) Set-up cost = A.

(5) Holding cost: considering holding cost of perfect items and items reworked.

Holding costs =$\mathrm{h}\left(\frac{\mathrm{z_1 t_1}}{2} + \frac{\mathrm{z_1}^2}{2(D-P_1)} + \frac{\mathrm{rP}}{2}(\mathrm{t_1} + \mathrm{t_5})^2\right) + \mathrm{h_R P_R}\frac{(t_2+t_3)^2}{2}$.

(6) Shortage cost =$\mathrm{b}\left(\frac{\mathrm{z_2}^2}{2(D-P_R)} + \frac{w+z_2}{2}\mathrm{t_3} + \frac{\omega}{2}\mathrm{t_5}\right)$

Total cost is the summation of all above costs, so we get:

$$
\begin{aligned}
&TC(Q, w|r, s) \\
&\qquad = c\,Q + c_d Qs + c_R Qr + A \\
&\qquad + h\left(\frac{z_1 t_1}{2} + \frac{{z_1}^2}{2(D-P_R)} + \frac{rP}{2}(t_1+t_5)^2\right) + h_R P_R \frac{t_2^2}{2} \\
&\qquad + \mathrm{b}\left(\frac{{\mathrm{z}_2}^2}{2(D-P_R)} + \frac{w+z_2}{2}\mathrm{t}_3 + \frac{\mathrm{w}}{2}\mathrm{t}_5\right)
\end{aligned}
\tag{5.13}
$$

Total cost per unit can be written as:

$$TCU_{Z_2<0}(Q, w|r, s) = \frac{TC(Q, w|r, s)}{t_0} \tag{5.14}$$

As we mentioned previously that the value of z_2 depends on the proportion of non-conforming items, so the proportion of non-conforming items has a limit upon which z_2 will be negative. This limit can be calculated as follows:

$$s + \frac{D}{P_R} * r = 1 - \frac{D}{P} - \frac{w}{Q} \tag{5.15}$$

If RHS greater than or equal to LHS, the $TCU_{Z_2>0}$ can be used. If RHS less than or equal to LHS, then $TCU_{Z_2<0}$ will be used.

Let s_{min} and s_{max} be the minimum and maximum value of s. on the other hand, r_{min} and r_{max} are the minimum and maximum value of r. The total cost per unit time can be calculated using (5.16);

$$
\begin{aligned}
TCU(Q, w) = &\int_{s_{min}}^{U_s}\int_{r_{min}}^{U_r} f(s)g(r)TCU_{Z_2>0}\,ds\,dr \\
&+ \int_{L_s}^{s_{max}}\int_{L_r}^{r_{max}} f(s)g(r)\,TCU_{Z_2<0}\;ds\,dr
\end{aligned}
\tag{5.16}
$$

Where:

$$L_s = max\left\{s_{min}, \left(1 - \frac{D}{P} - \frac{w}{Q} - \frac{Dr_{max}}{P_R}\right)\right\}$$

$$U_s = min\left\{s_{max}, \left(1 - \frac{D}{P} - \frac{w}{Q} - \frac{Dr_{min}}{P_R}\right)\right\}$$

$$L = \frac{P_R\left(1 - s - \frac{D}{P} - \frac{w}{Q}\right)}{D}$$

$$L_r = max\{r_{min}, L\}$$

$$U_r = min\{r_{max}, L\}$$

The minimum of $TCU(Q, w)$ can not be found in closed form. Therefore, we find the solution by performing exhaustive search for Q and w. The integration is carried out numerically using a 12-node Gaussian quadrature method

5.3 NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

Taking in consideration that *s* is the proportion of scrapped items produced, which is random variable with a known probability density function $f(s)$, and r is the proportion of re-workable items produced, which as random variable with a known probability density function $g(r)$.

A manufactured product has a constant demand rate of 1200 unit/year. The machine used to manufacture this item has a production rate of 1600 unit/year. And the related costs as follows:

c = \$104/unit,

c_R= \$8/unit,

c_d= \$5/unit,

A = \$1500,

h = \$20/unit/year,

h_R = \$22/unit/year,

b = \$25/unit/year,

The proportion of non-conforming items is distributed over the interval [0, 0.1].The defective items are reworked at a rate of 1000 units/year.

Table 5.1 shows the optimal values of economic production quantity Q^* , optimal shortage quantity w^* , and related total cost per unit time $TCU(Q^*, w^*)$ for five probability distribution functions for s and r. We used the same distribution function for both s and r.

To show the need for this model we will compare the answer obtained above for the case where proportion of nonconforming items follows the normal distribution with the solution generated using the classical model. The classical finite production model, where shortages are backordered gives $\mathrm{Q}^* = 1138$ units and $\mathrm{w}^* = 126$ units. If these values are substituted in the cost function given in (3.16), we get $\mathrm{TCU}(\mathrm{Q}^*, \omega^*) = 135440\$$. This value exceeds the optimal value shown in Table 5.1.

Table 5.1 Optimal EPQ and shortage quantity for different probability mass functions for s and r.

f(s,r)	Formula	Parameters		Q^*	w^*	$TCU(Q^*,w^*)$
Uniform	$f(s)=\frac{1}{b-a}$	a=0	b=0.1	1166	54	135547
Normal	$f(s)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\left(\frac{(s-\mu)^2}{2\sigma^2}\right)}$	μ=0.05	σ=0.015	1170	59	135206
Exponential	$f(s)=\lambda e^{-\lambda s}$	λ=55	------	1070	54	129718
Gamma	$f(s)=\frac{1}{\beta^\alpha\Gamma(\alpha)}s^{\alpha-1}e^{-\frac{s}{\beta}}$	σ=3	β=0.01	1110	56	131744
Weibull	$f(s)=\frac{P}{\beta^\alpha}s^{\alpha-1}e^{-\left(\frac{s}{\beta}\right)^\alpha}$	σ=4	β=0.06	1190	60	135984

Next, we study the effect of the imperfect quality items proportion on the optimal solution. We consider the case that s and r is normally distributed with fixed standard deviation $\sigma = 0.015$ and varied mean. We choose μ such that $0 \leq \mu + 3.5\sigma \leq 1 - \frac{D}{P}$.

Table 5.2 shows the effect of non-conforming item on the optimal solution .Fig. 5.5 illustrates the behavior of optimal production quantityQ^* and optimal shortage quantity w^* for the deviation in mean of scraped items portion by fixing the mean of the re-workable items portion. Fig. 5.6 illustrates the behavior of optimal production quantity Q^* and optimal shortage quantity w^* for the deviation in mean of the portion of re-workable items produced by fixing the mean of the scraped items portion. One notices that when μ increases the optimal production quantity Q^* and optimal shortage quantity w^* decreases, where Figures 5.7 and 5.8 show the effect of non-conforming items on the total cost per unit time TCU. They show that as the portion of the two types of non-conforming items increase, TCU increases.

Table 5.2 The effect of non-conforming items on the optimal solution

μ_s	μ_r = 0.04			0.05			0.06		
	Q^*	w^*	TCU	Q^*	w^*	TCU	Q^*	w^*	TCU
0.04	1190	93	133685	1177	88	133832	1159	82	133980
0.05	1223	90	135074	1205	84	135224	1188	79	135375
0.06	1258	87	136493	1234	80	136645	1212	73	136799
0.07	1291	83	137940	1275	77	138096	1250	63	138261

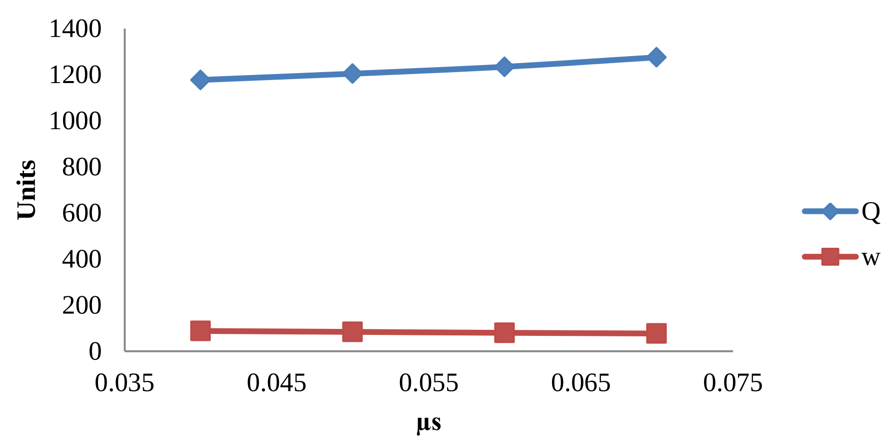

Figure 5.5 Effect of scrapped items proportion on the EPC and optimal shortage quantity ($\mu_r = 0.05$).

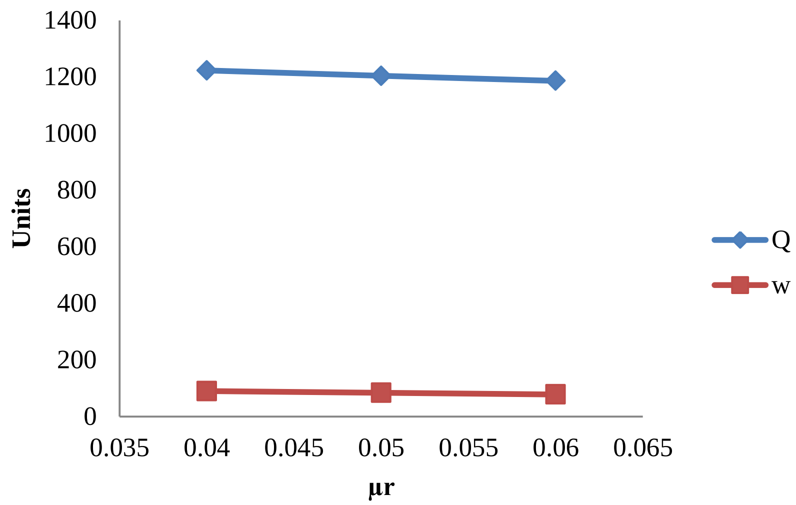

Figure 5.6 Effect of re-workable items proportion on the EPC and optimal shortage quantity ($\mu_s = 0.05$).

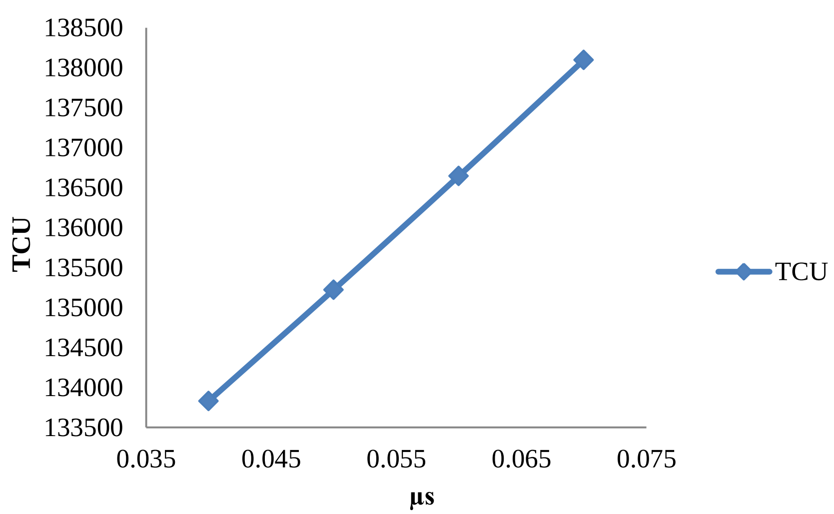

Figure 5.7 Effect of scraped items proportion on the TCU ($\mu_r = 0.05$).

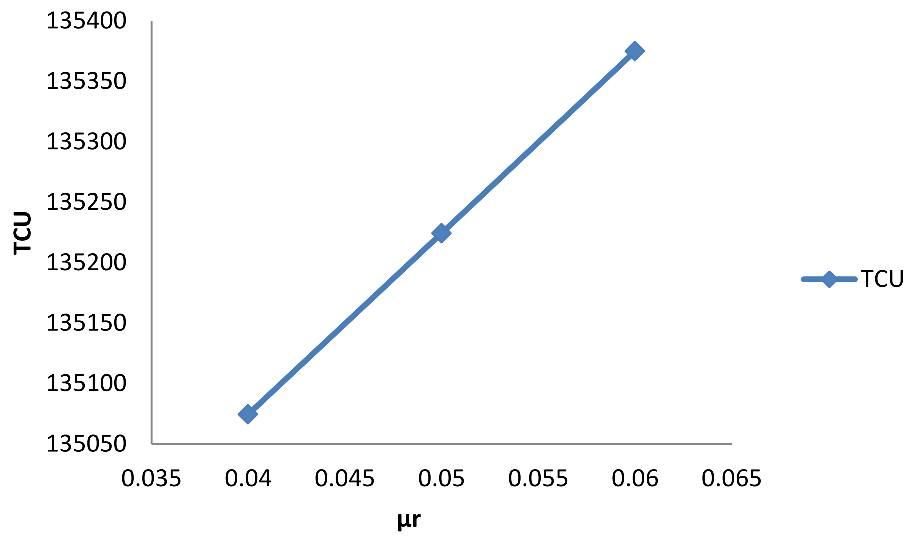

Figure 5.8 Effect of re-workable items proportion on the TCU ($\mu_s = 0.05$).

5.4 SUMMARY

The finite production rate model is presented where the process may generate non-conforming items. Where some of them are scrapped as they are produced, where the others are reworked with constant rework rate at the end of the production time.

The proportions of the scrapped and reworked items are random variables. So we apply different distribution functions for these random variables to test the results and the effect of these non-conforming items on the optimum values of the economic production quantity, maximum shortages allowed, and so for the total cost per unit time. One can conclude that if the proportion of the scrapped items produced increases, the economic production quantity increases while the maximum shortages allowed decreases. But if the percentage of re-workable items increases, both EPQ and maximum shortages allowed are decrease. And the total cost per unit time is increasing when non-conforming items portion increases.

CHAPTER 6

PRODUCTION PLANNING WITH PROCESS TARGETING

6.1 INTRODUCTION

Researchers almost deal with process targeting or production and inventory planning separately, while they are somehow related to each other and one of them affects the results of the other. This chapter presents a model that combines between production and inventory control with process targeting. The model in this chapter is similar to the model discussed in Chapter 5 in addition to determining the optimal mean of some quality characteristic of known probability density function. At the end of this chapter, numerical example is provided to illustrate the solution procedure. And sensitivity analysis conducted to show the effect of changes in the model parameter on the decision variables.

6.2 MODEL DESCRIPTION

The model discussed in this chapter is identical to that of Chapter 5 in addition to determining the mean of random quality characteristic. The value of the mean will affect the proportion of re-workable and scrapped items as follows.

As shown in Fig. 6.1 we assume that there are upper and lower specifications for the quality characteristic. If the quality characteristic exceeds the upper specification limit U, the item is reworked, where we assume that reworked items are always conforming. If the quality characteristic is between the lower, LSL, and upper, USL, specification limits, the item is conforming and it is used to satisfy the demand. Otherwise the item is scrapped. Figures 6.2 and 6.3 show the inventory level of conforming and non-conforming items, respectively, for production cycle with $z_2 \geq 0$. While Figures 6.4 and 6.5 show the inventory level of conforming and non-conforming items, respectively, for production cycle with $z_2 < 0$.

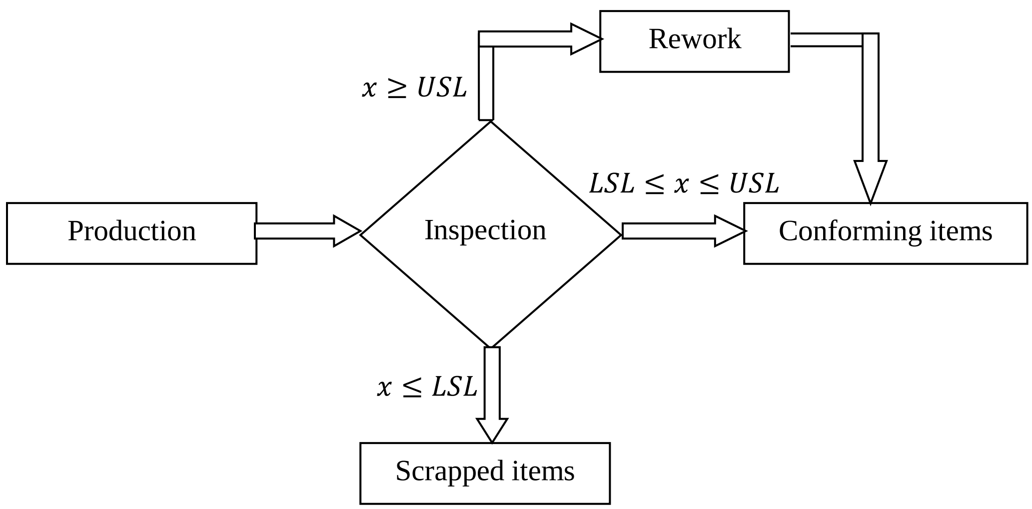

Figure 6.1 Items classification depending on the quality characteristic value.

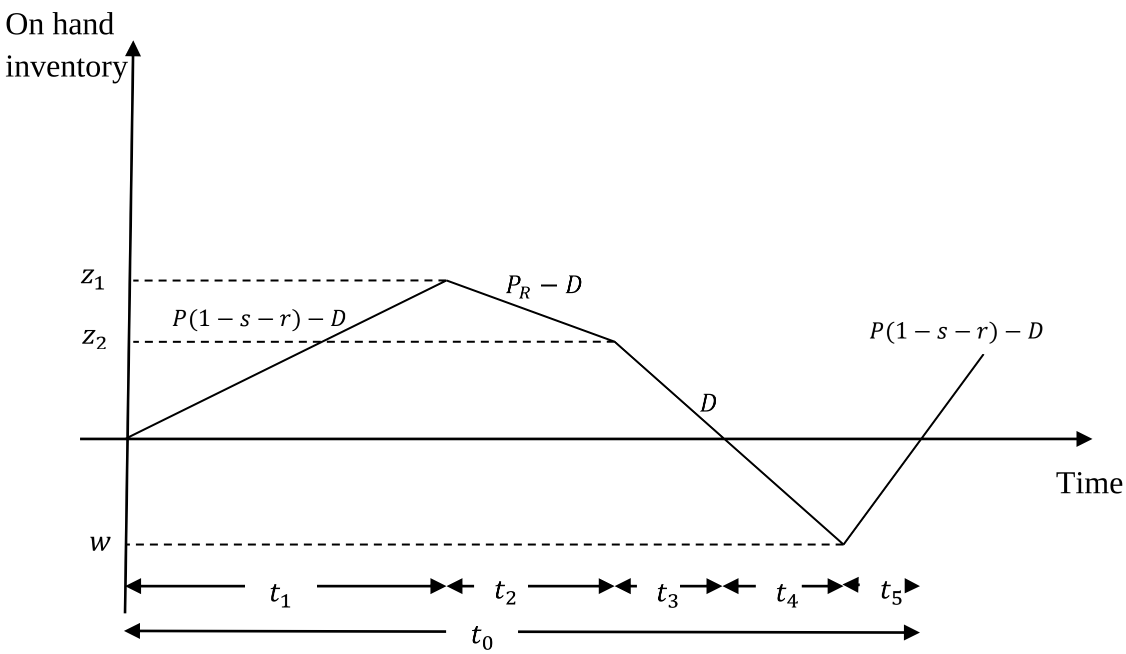

Figure 6.2 On- hand inventory of conforming items in case of positive z_2

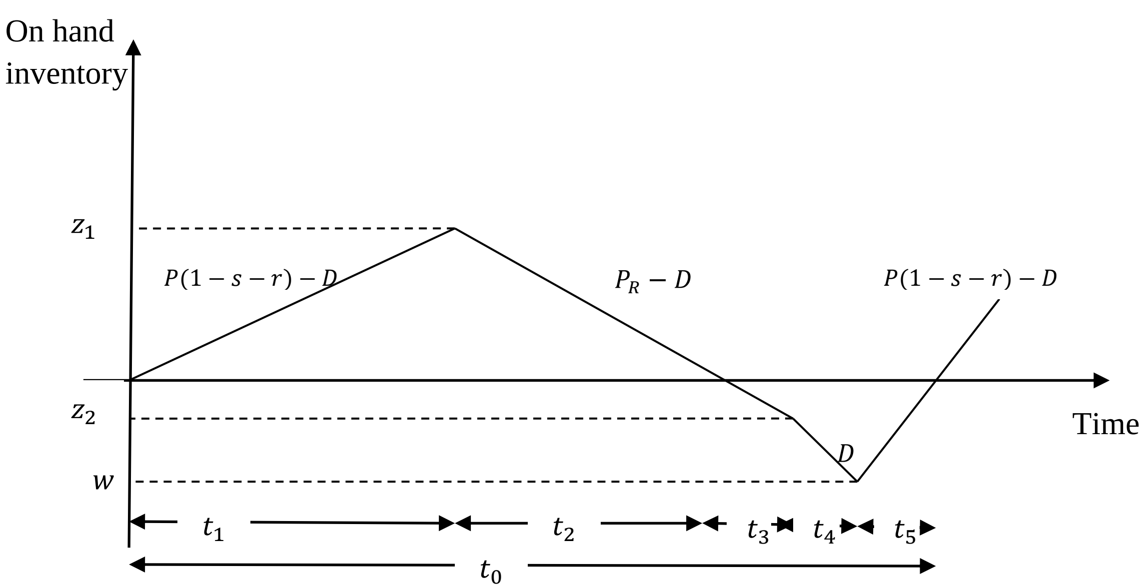

Figure 6.3 On- hand inventory of conforming items in case of negative z_2

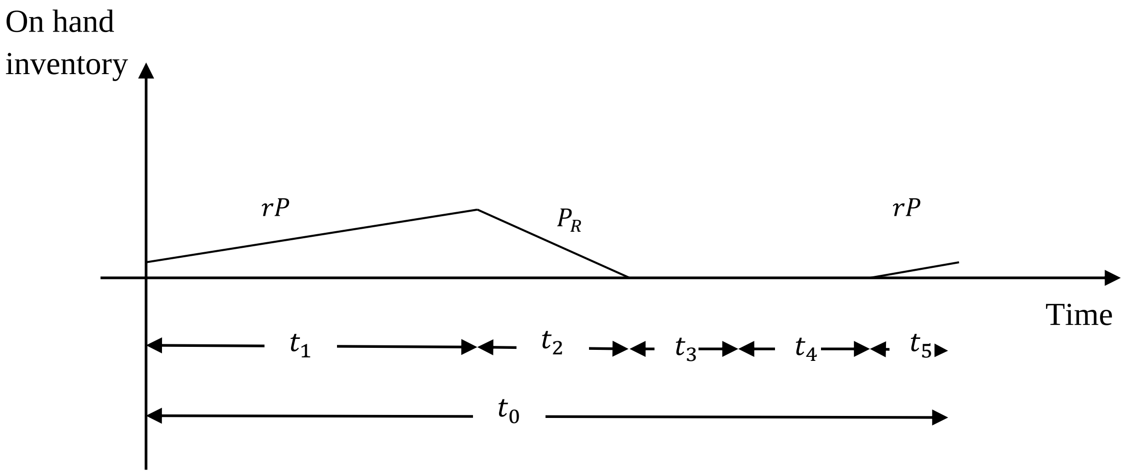

Figure 6.4 On- hand inventory of re-workable items in case of positive z_2

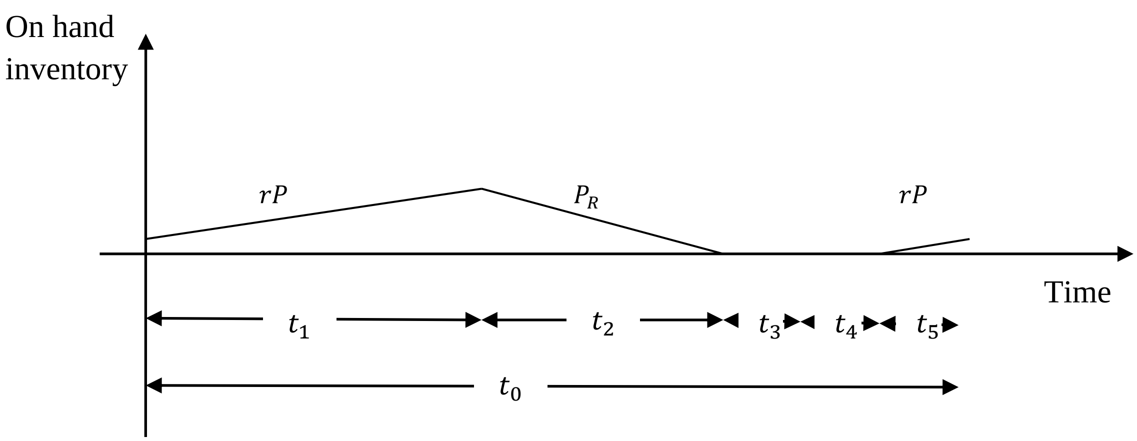

Figure 6.5 On- hand inventory of reworkable items in case of negative z_2

6.3 MODEL FORMULATION

The process mean will define the proportion of non-conforming items, so it affects the decision variables of the production and inventory. We aim to develop a mathematical model to find the optimal combination between the process mean, EPQ, and maximum shortage allowed.

Most of quality characteristics in real life follow the normal distribution. Hence, we will consider normal distribution as the probability density function of the quality characteristic under study.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)} \tag{6.1}$$

Where:

x: Quality characteristic value.

μ: Mean of the normal distribution

σ: Standard deviation of the normal distribution

Since the probability density function of the quality characteristic known and specification limits are defined, then the proportion of scrapped and re-workable items s and r can be calculated easily through the following formulas, depending on the previous definitions:

$$s = \int_{x_{min}}^{LSL} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \tag{6.2}$$

And

$$r = \int_{x=USL}^{x_{max}} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \tag{6.3}$$

To avoid shortage while producing and having feasible model, production rate of good items is always greater than or equal to the sum of the demand rate and the rate at which non-conforming items are produced $P(1-s-r) > D$. This model is valid if $s + \frac{D}{P_R} r \leq 1 - \frac{D}{P}$ as shown in Chapter 5.

Inventory level after rework process, z_2 will affect the total cost per cycle and it depends mainly on the proportion of non-conforming items, so the proportion of non-conforming items has a limit upon which z_2 will be negative. This limit can be calculated as follows:

$$s + \frac{D}{P_R} * r = 1 - \frac{D}{P} - \frac{w}{Q} \tag{6.4}$$

Since s and r depend on the process mean, which is a decision variable in this case, if (6.4) is not satisfied, we have two scenarios. First one, if the LHS is less than or equal to RHS, the case of $z_2 > 0$ can be used. Second scenario if LHS is greater than or equal to RHS, then $z_2 < 0$ case will be used.

The total cost per cycle consist of production cost of all items, rework cost of re-workable items, disposal cost for defective items, set-up cost, holding cost of perfect items and reworked items, and shortage cost. So the total cost per unit time for the case of $\mathrm{Z}_2 > 0$ can be written as:

$$TCU_{\mathrm{Z}_2>0} = e_1 + \frac{e_2}{Q} + e_3 Q + e_4 \mathrm{w} + e_5 \frac{\mathrm{w}^2}{Q} \tag{6.5}$$

Where:

$$e_1 = \frac{D}{(1-s)}(c \ + c_d s + c_R r)$$

$$e_2 = \frac{AD}{(1-s)}$$

$$e_3 = \frac{h}{2}\left(1 - \frac{D}{P} - s\right) + \frac{(h_R - h)Dr^2}{2P_R(1-s)}$$

$$e_4 = -h$$

$$e_5 = \frac{p+h}{2}\left(\frac{1-s-r}{(1-s)\left(1-s-r-\frac{D}{P}\right)}\right)$$

By setting the partial derivative of $TCU_{\mathrm{Z}_2>0}$ with respect to Q and w to zero, we get the optimal Q and w as follows:

$$Q_1 = \sqrt{\frac{e_2}{e_3 - \frac{{e_4}^2}{4e_5}}} \tag{6.6}$$

and

$$\mathrm{w}_1 = \frac{-e_4}{2e_5} Q_1 \tag{6.7}$$

It is easy to show the convexity of $TCU_{\mathrm{Z}_2>0}$ as we did in Chapter 4. Hence, Q_1 and w_1 are the optimum values.

The total cost per unit time for the case of $\mathrm{z}_2 < 0$ is:

$$\mathrm{TCU}_{\mathrm{Z}_2<0} = c_1 + \frac{c_2}{Q} + c_3 Q + c_4 \mathrm{w} + c_5 \frac{\mathrm{w}^2}{Q} \tag{6.8}$$

Where:

$a_1 = 1 - s - r - D/P$

$a_2 = s + \frac{D(rP+P_R)}{PP_R} - 1$

$a_3 = P(1-s-r) - D$

$a_4 = D - P_R$

$c_1 = \frac{D}{(1-s)}(c + c_d s + c_R r)$

$$c_2 = \frac{AD}{(1-s)}$$

$$c_3 = \mathrm{h}\frac{D}{2(1-s)}\left(\frac{{a_1}^2}{\mathrm{a}_3} + \frac{{a_1}^2}{a_4} + \frac{\mathrm{rP}{a_1}^2}{{a_3}^2}\right) + \frac{\mathrm{h_R P_R}D}{2(1-s)}\left(\frac{{a_1}^2}{{\mathrm{a}_4}^2}\right) + \mathrm{b}\frac{D}{2(1-s)}\left(\frac{2{a_2}^2}{a_4}\right)$$

$$c_4 = -\mathrm{h}\frac{D}{2(1-s)}\left(\frac{2a_1}{\mathrm{a}_3} + \frac{2a_1}{a_4}\right) - \frac{\mathrm{h_R P_R}D}{2(1-s)}\left(\frac{2a_1}{{\mathrm{a}_4}^2}\right) + \mathrm{b}\frac{D}{2(1-s)}\left(\frac{5a_2}{a_4}\right)$$

$$c_5 = \mathrm{h}\frac{D}{2(1-s)}\left(\frac{1}{\mathrm{a}_3} + \frac{1}{a_4}\right) + \frac{\mathrm{h_R P_R}D}{2(1-s){\mathrm{a}_4}^2} + \mathrm{b}\frac{D}{2(1-s)}\left(\frac{3}{a_4} + \frac{1}{a_3}\right)$$

By setting the partial derivative of $\mathrm{TCU}_{\mathrm{Z}_2<0}$ with respect to Q and w to zero, we get the optimal Q and w as follows:

$$Q_2 = \sqrt{\frac{c_2}{c_3 - \frac{{c_4}^2}{4c_5}}} \tag{6.9}$$

And

$$\mathrm{w}_2 = \frac{-c_4}{2c_5}Q_2 \tag{6.10}$$

It is easy to show the convexity of the $\mathrm{TCU}_{\mathrm{Z}_2<0}$ as we did in Chapter 5. Hence, $\mathrm{TCU}_{\mathrm{Z}_2<0}$ has optimum solution when $Q = Q_2$ and $\mathrm{w} = \mathrm{w}_2$.

6.4 SOLUTION PROCEDURE

To find the optimal process mean, EPQ, and maximum shortage allowed we develop a simple algorithm. This algorithm is summarized in the following steps:

1. Define the feasible range of the process mean such that production rate of perfect quality items is greater than the demand rate. and not allowing for shortage before fulfilling the shortage of the previous cycle

$$\int_{x=LSL}^{USL} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \geq \frac{D}{P} \tag{6.11}$$

and

$$1 - \int_{x=LSL}^{USL} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \leq \frac{P_R}{D}\left(1 - \frac{D}{P}\right) \tag{6.12}$$

Search for the lowest mean that satisfy the condition above and it is called μ_l. Then find upper one in the range μ_u through the following formula: $\mu_u = USL - \mu_l + LSL$. The optimal mean falls in the interval $[\mu_l, \mu_u]$.

2. Since the calculation of the total cost will be changed depending on the value of μ. For each μ in the feasible range we do the following:

$$IF \quad \frac{-e_4}{2e_5} \leq 1 - \frac{D}{P} - s - \frac{D}{P_R} * r$$

$$Q = Q_1 = \sqrt{\frac{e_2}{e_3 - \frac{e_4{}^2}{4e_5}}} \qquad \mathrm{w} = \mathrm{w}_1 = \frac{-e_4}{2e_5} Q_1$$

$$TCU = TCU_{\mathrm{Z}_2 > 0}$$

$$IF \quad \frac{-c_4}{2c_5} \geq 1 - \frac{D}{P} - s - \frac{D}{P_R} * r$$

$$Q = Q_2 = \sqrt{\frac{c_2}{c_3 - \frac{c_4{}^2}{4c_5}}} \qquad \mathrm{w} = \mathrm{w}_2 = \frac{-c_4}{2c_5} Q_2$$

$$\mathrm{TCU} = \mathrm{TCU}_{\mathrm{Z}_2 < 0}$$

$$IF \quad \frac{-e_4}{2e_5} \leq 1 - \frac{D}{P} - s - \frac{D}{P_R} * r \quad \& \quad \frac{-c_4}{2c_5} \geq 1 - \frac{D}{P} - s - \frac{D}{P_R} * r$$

$$\text{TCU} = \text{Min}\left\{\text{TCU}_{\text{Z}_2>0}, \text{TCU}_{\text{Z}_2<0}\right\}$$

$$Q = argmin\{\text{Q}_1, \text{Q}_2\}$$

$$w = argmin\{\text{w}_1, \text{w}_2\}$$

3. The optimal process mean, economic production quantity, and optimal shortage allowed are the ones related the minimum total cost per unit time.

6.5 NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

A specific product has a constant demand rate of 1200 unit/year. The machine used to manufacture this item has a production rate of 1600 unit/year. And related costs are as follows:

c = $104/unit,

c_R= $8/unit,

c_d= $5/unit,

A = $1500,

h = $20/unit/year,

h_R = $22/unit/year,

b = $25/unit/year,

The quality characteristic follows normal distribution with known standard deviation 0.152. The defective items are reworked at a rate of 1000 unit/year.

Using the same procedure illustrated before we found the optimal combination of the required decision variables as shown in table 6.1.

Table 6.1 Results for the given numerical data.

Feasible range of μ		μ	Q	w	TCU
μ_l	μ_u				
24.878	25.122	25.102	946	38	131606

To show the effect of the parameters on the optimal values we did sensitivity analyses. this sensitivity discuss the effect of the changes in standard deviation, production rate, demand rate, setup cost, holding cost of good items, holding cost of re-workable items, shortage cost, production cost, rework cost, and disposal cost.

Table 6.2 shows the effect of standard deviation on the optimal values. The feasible range of the process mean decreases, the optimal mean increases, the EPQ decreases, and optimal shortage quantity decreases as the standard deviation increases.

Table 6.2 Sensitivity analyses respect to standard deviation

σ	Feasible range of μ		μ	Q	w	TCU
	μ_l	μ_u				
0.125	24.852	25.148	25.07	1051	86	129655
0.13	24.857	25.143	25.075	1035	78	129980
0.135	24.861	25.139	25.081	1015	69	130325
0.14	24.866	25.134	25.087	996	60	130687
0.145	24.871	25.129	25.093	976	51	131063
0.15	24.876	25.124	25.099	955	42	131449
0.155	24.881	25.119	25.104	937	32	131845
0.16	24.887	25.113	25.108	918	22	132265
0.165	24.893	25.107	25.106	915	18	132720
0.17	24.901	25.099	25.098	930	19	133284

Table 6.3 shows the effect of production rate on the optimal values. As the production rate increases, the feasible range of the process mean increases, the optimal mean increases, the EPQ decreases, and optimal shortage quantity increases.

Table 6.3 Sensitivity analyses respect to production rate

P	Feasible mean range		μ	Q	w	TCU
	μ_l	μ_u				
1400	24.95	25.05	25.049	1312	15	131826
1450	24.923	25.077	25.076	1127	16	131402
1500	24.905	25.095	25.094	1023	17	131403
1550	24.89	25.11	25.099	980	27	131501
1600	24.878	25.122	25.102	946	38	131606
1650	24.867	25.133	25.103	920	47	131705
1700	24.858	25.142	25.103	900	56	131795
1750	24.849	25.151	25.104	880	64	131879
1800	24.842	25.158	25.105	862	71	131956
1850	24.835	25.165	25.105	848	78	132027

Table 6.4 shows the effect of demand rate on the optimal values. The feasible range of the process mean decreases, the optimal mean decreases, the EPQ increases, and optimal shortage quantity decreases as the demand rate increases.

Table 6.4 Sensitivity analyses respect to demand rate

D	Feasible mean range		μ	Q	w	TCU
	μ_l	μ_u				
800	24.751	25.249	25.108	613	104	89100
850	24.763	25.237	25.107	647	100	94453
900	24.775	25.225	25.106	682	94	99794
950	24.787	25.213	25.106	719	87	105124

1000	24.8	25.2	25.105	758	79	110443
1050	24.819	25.181	25.104	801	71	115751
1100	24.839	25.161	25.103	846	61	121048
1150	24.858	25.142	25.103	893	50	126333
1200	24.878	25.122	25.102	946	38	131606
1250	24.899	25.101	25.096	1014	25	136888

Table 6.5 shows the effect of setup cost on the optimal values. As the setup cost increases, the feasible range of the process mean doesn’t change, the optimal mean decreases, the EPQ increases, and optimal shortage quantity increases.

Table 6.5 Sensitivity analyses respect to setup cost

A	Feasible mean range		μ	Q	w	TCU
	μ_l	μ_u				
1000	24.878	25.122	25.105	767	29	130898
1100	24.878	25.122	25.104	806	31	131052
1200	24.878	25.122	25.103	844	33	131199
1300	24.878	25.122	25.103	879	34	131340
1400	24.878	25.122	25.102	914	36	131475
1500	24.878	25.122	25.102	946	38	131606
1600	24.878	25.122	25.101	979	40	131732
1700	24.878	25.122	25.101	1010	41	131854
1800	24.878	25.122	25.1	1041	43	131972
1900	24.878	25.122	25.1	1070	44	132087

Table 6.6 shows the effect of holding cost on the optimal values. The feasible range of the process mean doesn’t change, the optimal mean decreases, the EPQ decreases, and optimal shortage quantity increases as the holding cost increases.

Table 6.6 Sensitivity analyses respect to holding cost of good items

h	Feasible mean range		μ	Q	w	TCU
	μ_l	μ_u				
12.5	24.878	25.122	25.103	1110	33	131029
15	24.878	25.122	25.103	1044	34	131238
17.5	24.878	25.122	25.102	992	37	131429
20	24.878	25.122	25.102	946	38	131606
22.5	24.878	25.122	25.101	909	39	131771
25	24.878	25.122	25.1	877	40	131927
27.5	24.878	25.122	25.099	849	41	132075
30	24.878	25.122	25.098	824	42	132216
32.5	24.878	25.122	25.097	802	43	132351
35	24.878	25.122	25.096	782	44	132480

Table 6.7 shows the effect of holding cost of re-workable items on the optimal values. As the holding cost of re-workable items increases, the feasible range of the process mean doesn't change, the optimal mean decreases, the EPQ decreases, and optimal shortage quantity increases.

Table 6.7 Sensitivity analyses respect to holding cost of re-workable items

h_R	Feasible mean range		μ	Q	w	TCU
	μ_l	μ_u				
16	24.878	25.122	25.105	964	36	131509
19	24.878	25.122	25.103	956	37	131558
22	24.878	25.122	25.102	946	38	131606
25	24.878	25.122	25.1	940	39	131652
28	24.878	25.122	25.099	931	39	131696
31	24.878	25.122	25.098	924	39	131739
34	24.878	25.122	25.096	920	40	131780

37	24.878	25.122	25.095	913	41	131820
40	24.878	25.122	25.094	907	41	131859
43	24.878	25.122	25.093	902	41	131897

Table 6.8 shows the effect of shortage cost on the optimal values. The feasible range of the process mean doesn't change, the optimal mean increases, the EPQ decreases, and optimal shortage quantity decreases as the shortage cost increases.

Table 6.8 Sensitivity analyses respect to shortage cost

b	Feasible mean range		μ	Q	w	TCU
	μ_l	μ_u				
13	24.878	25.122	25.096	1000	57	131462
16	24.878	25.122	25.097	984	52	131508
19	24.878	25.122	25.099	968	46	131546
22	24.878	25.122	25.101	955	41	131578
25	24.878	25.122	25.102	946	38	131606
28	24.878	25.122	25.103	938	34	131629
31	24.878	25.122	25.103	933	32	131649
34	24.878	25.122	25.104	927	30	131667
37	24.878	25.122	25.105	922	27	131682
40	24.878	25.122	25.105	918	26	131696

Table 6.9 shows the effect of production cost on the optimal values. As the production cost increases, the feasible range of the process mean doesn't change, the optimal mean increases, the EPQ decreases, and optimal shortage quantity decreases.

Table 6.9 Sensitivity analyses respect to production cost

C	Feasible mean range		μ	Q	w	TCU
	μ_l	μ_u				
85	24.878	25.122	25.093	967	44	108549
90	24.878	25.122	25.095	962	43	114621
95	24.878	25.122	25.098	955	41	120689
100	24.878	25.122	25.1	951	39	126755
105	24.878	25.122	25.102	946	38	132818
110	24.878	25.122	25.104	941	36	138879
115	24.878	25.122	25.105	939	35	144939
120	24.878	25.122	25.107	934	33	150997
125	24.878	25.122	25.108	932	32	157053
130	24.878	25.122	25.109	929	31	163108

Table 6.10 shows the effect of rework cost on the optimal values. The feasible range of the process mean doesn't change, the optimal mean decreases, the EPQ increases, and optimal shortage quantity increases as the rework cost increases.

Table 6.10 Sensitivity analyses respect to rework cost

Cr	Feasible mean range		μ	Q	w	TCU
	μ_l	μ_u				
4	24.878	25.122	25.114	916	25	130757
6	24.878	25.122	25.108	932	32	131193
8	24.878	25.122	25.102	946	38	131606
10	24.878	25.122	25.095	962	43	131992
12	24.878	25.122	25.089	976	47	132354
14	24.878	25.122	25.084	987	50	132697
16	24.878	25.122	25.079	998	53	133022
18	24.878	25.122	25.075	1008	56	133333
20	24.878	25.122	25.071	1017	58	133630

22	24.878	25.122	25.067	1026	60	133916

Table 6.11 shows the effect of disposal cost on the optimal values. As the disposal cost increases, the feasible range of the process mean doesn't change, the optimal mean increases, the EPQ decreases, and optimal shortage quantity decreases. But it effects slightly.

Table 6.11 Sensitivity analyses respect to disposal cost

Cd	Feasible mean range		μ	Q	w	TCU
	μ_l	μ_u				
2	24.878	25.122	25.1	951	39	131568
3	24.878	25.122	25.101	948	38	131580
4	24.878	25.122	25.101	948	38	131593
5	24.878	25.122	25.102	946	38	131606
6	24.878	25.122	25.102	946	38	131618
7	24.878	25.122	25.103	944	37	131631
8	24.878	25.122	25.103	944	37	131643
9	24.878	25.122	25.103	944	37	131655
10	24.878	25.122	25.104	941	36	131667
11	24.878	25.122	25.104	941	36	131679

The total cost is increases as any parameter increase. Table 6.12 summarizes the sensitivity of the optimal solution against the increase in parameters value.

Table 6.12 Sensitivity of the optimal solution against model parameters

Parameter	Feasible range of μ		μ	Q	w	TCU
	μ_l	μ_u				
σ	↑	↓	↑	↓	↓	↑
P	↓	↑	↑	↓	↑	↑
D	↑	↓	↓	↑	↓	↑
A	—	—	↓	↑	↑	↑
h	—	—	↓	↓	↑	↑
hR	—	—	↓	↓	↑	↑
b	—	—	↑	↓	↓	↑
C	—	—	↑	↓	↓	↑
CR	—	—	↓	↑	↑	↑
Cd	—	—	↑	↓	↓	↑

6.6 SUMMARY

In this chapter we presented a new integrated model which combine between inventory and production control with process targeting. We introduced a simple algorithm to search for the optimal solution, which is a combination of optimal process mean, economic production quantity, and the optimal shortage allowed. We show that EPQ and optimal shortage allowed are highly affected by the value of the process mean. And we find out that the model parameters have significant effect on the model decision variables with different levels. Finally, to get correct and representative final solution the feasible boundaries should be determined through usage of suitable constrains.

CHAPTER 7

PRODUCTION PLANNING WITH STOCHASTIC BREAKDOWNS

7.1 INTRODUCTION

This chapter presents a finite production model where the machine or production equipment may breakdown randomly. It is assumed that imperfect quality items produced. These imperfect quality items are either scrapped or reworked. We consider that the time to failure (breakdown) is a random variable also we assume that the proportions of scrapped and re-workable items are random variables. These random variables follow different given probability density functions. In this research 'no-resumption' policy is adopted. The objective is to find the optimal production run time and the optimal shortage allowed, where the shortages are backordered. The effect of the stochastic breakdown and production of non-conforming items on the optimal solution is studied and numerical example is provided for the developed model.

7.2 MODEL DESCRIPTION

The proposed production model considers unreliable machine and imperfect quality items are produced. We assume that the production rate, P, and demand rate, D, are constant. There are two types of non-conforming items, re-workable items and scrapped items. The proportions of non-conforming items that are re-workable and that are scrapped are *s* and *r*, respectively. Both of these proportions are random variables with given probability density functions *f(s)* and *g(r)*. The time to breakdown t is random variable and follow known probability density function *l(t)*. The no-resumption policy is adopted in this model, where the production process is terminated as the breakdown occurs and after repairing the machine re-workable items are processed if any and no further production is performed till the optimal shortage allowed, then start another production cycle. Conforming items are used to fulfill the demand. Scrapped items are disposed of at a cost, while re-workable items are processed after production ends. The rework rate, P_R, may be different from the production rate, P. We assume that the production rate exceeds the sum of the demand rate and the rate of generating re-workable and scrapped items. We assume that the machine will not break down during the rework process or shortage period. Shortages are backordered.

7.3 MODEL FORMULATION

As mentioned previously that the time to break down is random variable, it means that we have two cases, one with breakdown and the other without. So we have to show each one separately, and then combine the two cases in final formulation.

7.3.1 BREAKDOWN OCCURS PRIOR THE END OF PRODUCTION

Let t_1 be the production run time. In this case, the machine breaks down before the end of the production run time, i.e. $t < t_1$ as shown in Fig. 7.1.

There are several cases to be considered in this section depending on the inventory level and shortage level at specific points of the production cycle. These are defined as follows.

Let z_1 be the on-hand inventory of perfect items when the machine breaks down.

z_2: be the on-hand inventory of perfect items or shortage level after the completion of the rework stage.

z_3: be the on-hand inventory of perfect items or shortage level after the completion of the machine repair.

There are four subcases to be considered:

Case 1: $z_3 \geq 0$ & $z_2 \geq 0$. We denote the corresponding total cost per unit time as TCU_1.

Case 2: $z_3 \geq 0$ & $z_2 \leq 0$. We denote the corresponding total cost per unit time as TCU_2.

Case 3: $z_3 \leq 0$ & $z_2 \geq 0$. We denote the corresponding total cost per unit time as TCU_3.

Case 4: $z_3 \leq 0$ & $z_2 \leq 0$. We denote the corresponding total cost per unit time as TCU_4.

Next, we derive the total cost per unit time formulas for all subcases as follows.

Case 1. TCU_1 → $z_3 \geq 0$ $z_2 \geq 0$

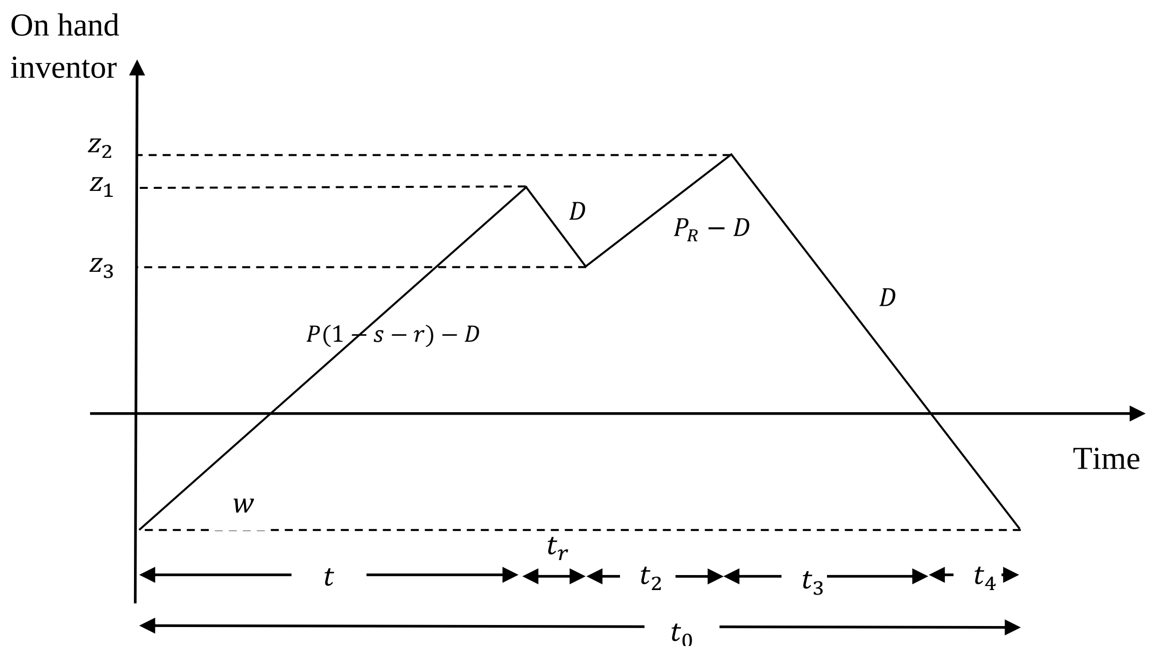

Figure 7.1 On- hand inventory of conforming items with breakdown

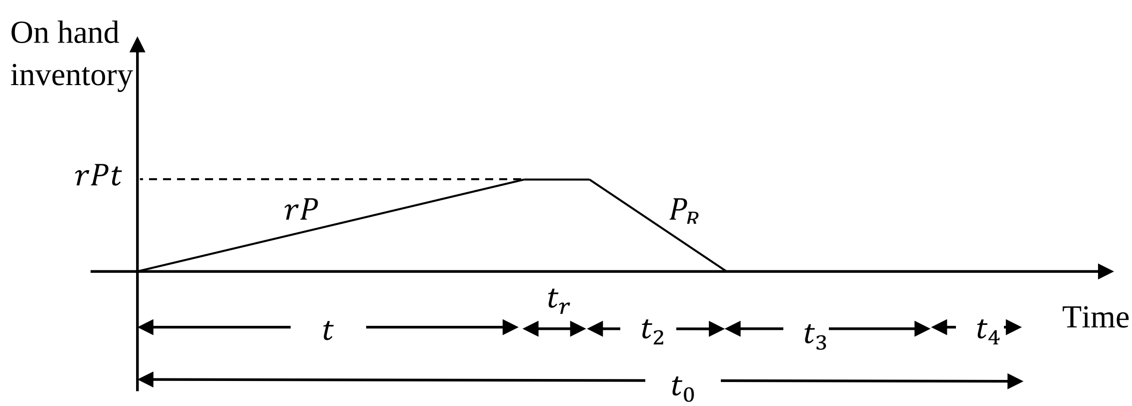

Figure 7.2 On- hand inventory of re-workable items with breakdown

From the above two Figures we find the following:

$$z_1 = (P(1-s-r)-D)t - w \tag{7.1}$$

$$z_3 = z_1 - t_r D = (P(1 - s - r) - D)t - w - nD \tag{7.2}$$

$$z_2 = z_3 + t_2(P_R - D) \tag{7.3}$$

$$t_2 = \frac{rtP}{P_R} \tag{7.4}$$

$$t_3 = \frac{z_2}{D} \tag{7.5}$$

$$t_3 = \frac{z_2}{D} \tag{7.6}$$

$$t_4 = \frac{w}{D} \tag{7.7}$$

$$t_0 = \frac{tP(1 - s)}{D} \tag{7.8}$$

The total cost per unit time for this kind of cycles will be:

$$TCU_1(Q, w|r, s) = \frac{TC_1(Q, w|t, r, s)}{t_0} \tag{7.9}$$

Where:

$$\begin{aligned} TC_1&(Q, w|t, r, s) \\ &= c\ tP + c_d\ tPs + c_R\ tPr + A \\ &+ h\left(\frac{{z_1}^2}{2(P(1 - s - r) - D)} + \frac{n(z_1 + z_3)}{2} + \frac{(z_3 + z_2)t_2}{2} + \frac{z_2 t_3}{2}\right. \\ &\left. + \frac{rP}{2}t^2 + nrPt\right) + h_R P_R \frac{t_2^2}{2} + b\left(\frac{\omega^2}{2(P(1 - s - r) - D)} + \frac{\omega^2}{2D}\right) \end{aligned} \tag{7.10}$$

Case 2. TCU_2 $\rightarrow$ $z_3 \geq 0$ $z_2 \leq 0$

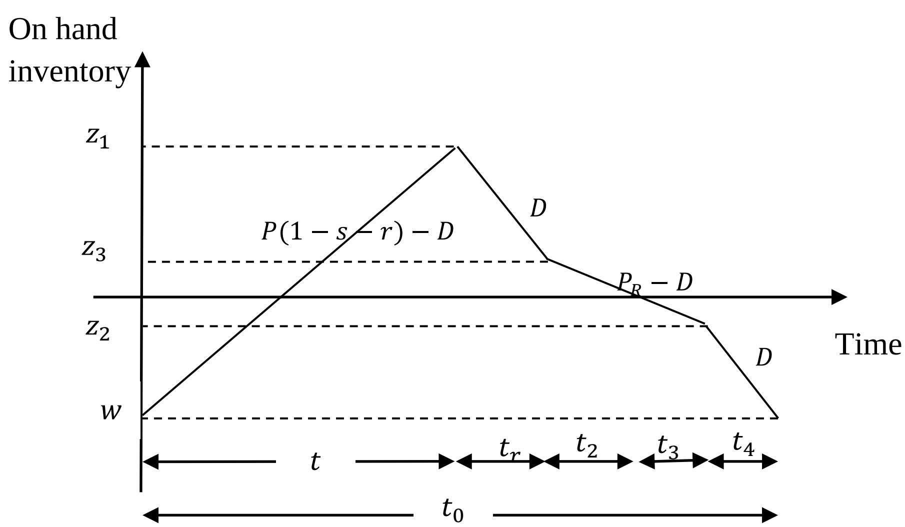

Figure 7.3 On- hand inventory of good items with breakdown for $\mathbf{z_3 > 0}$ & $\mathbf{z_2 < 0}$

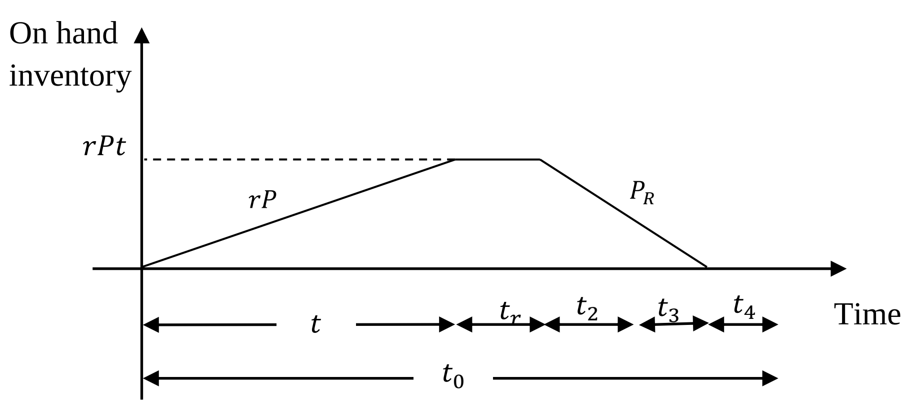

Figure 7.4 On- hand inventory of re-workable items with breakdown $\mathbf{z_3 > 0}$ & $\mathbf{z_2 < 0}$

The inventory level and the corresponding times for Figures 7.3 and 7.4 are:

$$z_1 = (P(1-s-r)-D)t - w \tag{7.11}$$

$$z_3 = z_1 - t_r D = (P(1-s-r)-D)t - w - nD \tag{7.12}$$

$$z_2 = (t_2 + t_3)(D - P_R) - z_3 \tag{7.13}$$

$$t_2 = \frac{z_3}{D - P_R} \tag{7.14}$$

$$t_3 = \frac{z_2}{D - P_R} \tag{7.15}$$

$$t_2 + t_3 = \frac{rtP}{P_R} \tag{7.16}$$

$$t_4 = \frac{w - z_2}{D} \tag{7.17}$$

$$t_0 = \frac{tP(1 - s)}{D} \tag{7.18}$$

The total cost and the total cost per unit time when $z_3 \geq 0$ and $z_2 \leq 0$ are:

$$\begin{aligned} TC_2(t_1, w|t, r, s) \\ &= c\ tP + c_d\ tPs + c_R\ tPr + A \\ &+ h\left(\frac{{z_1}^2}{2(P(1 - s - r) - D)} + \frac{n(z_1 + z_3)}{2} + \frac{z_3 t_2}{2} + \frac{rP}{2} t^2 \right. \\ &\left. + nrPt\right) + h_R P_R \frac{(t_2 + t_3)^2}{2} \\ &+ b\left(\frac{\omega^2}{2(P(1 - s - r) - D)} + \frac{\omega^2 - {z_2}^2}{2D} + \frac{z_2 t_3}{2}\right) \end{aligned} \tag{7.19}$$

$$TCU_2 = \frac{TC_2(t_1, w|t, r, s)}{t_0} \tag{7.20}$$

Case 3. TCU_3 → $z_3 \leq 0$ $z_2 \geq 0$

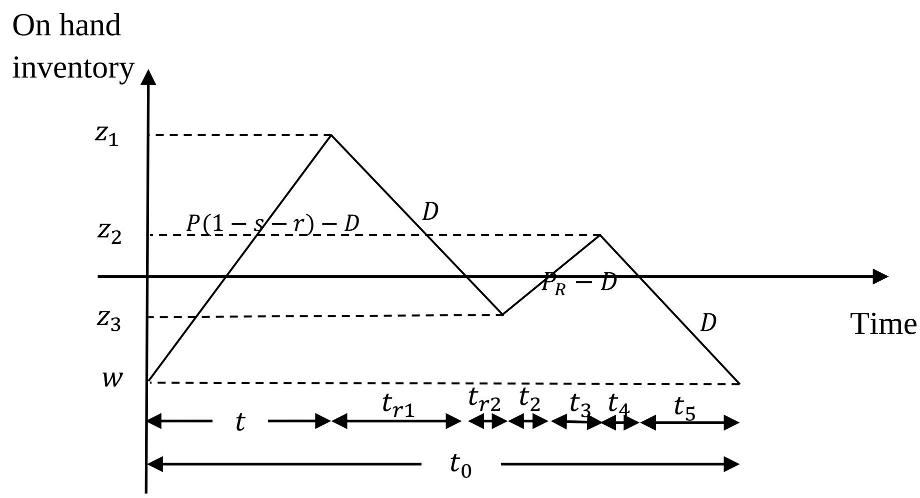

Figure 7.5 On- hand inventory of good items with breakdown for $\mathbf{z_3 < 0}$ & $\mathbf{z_2 > 0}$.

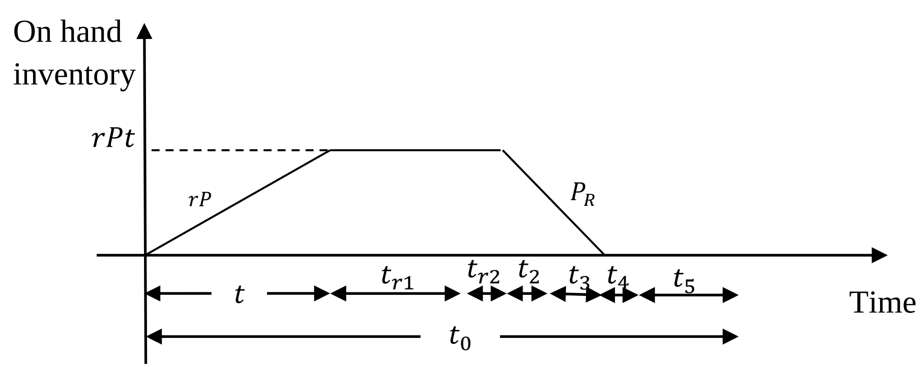

Figure 7.6 On- hand inventory of re-workable items with breakdown $\mathbf{z_3 < 0}$ & $\mathbf{z_2 > 0}$

The inventory level and the corresponding times for Figures 7.5 and 7.6 are:

$$z_1 = (P(1-s-r)-D)t - w \tag{7.21}$$

$$z_3 = t_r D - z_1 = nD - (P(1-s-r)-D)t + w \tag{7.22}$$

$$z_2 = (t_2 + t_3)(P_R - D) - z_3 \tag{7.23}$$

$$t_{r1} = \frac{z_1}{D} \tag{7.24}$$

$$t_{r2} = \frac{z_3}{D} \tag{7.25}$$

$$t_2 = \frac{z_3}{P_R - D} \tag{7.26}$$

$$t_3 = \frac{z_2}{P_R - D} \tag{7.27}$$

$$t_2 + t_3 = \frac{rtP}{P_R} \tag{7.28}$$

$$t_4 = \frac{z_2}{D} \tag{7.29}$$

$$t_0 = \frac{tP(1-s)}{D} \tag{7.30}$$

The total cost and the total cost per unit time when $z_3 \leq 0$ and $z_2 \geq 0$ are:

$$TC_3(t_1, w|t, r, s)$$

$$= c\ tP + c_d\ tPs + c_R\ tPr + A$$
$$+ h\left(\frac{{z_1}^2}{2(P(1-s-r)-D)} + \frac{z_1 t_{r1}}{2} + \frac{z_2 t_3}{2} + \frac{z_2 t_4}{2} + \frac{rP}{2} t^2 + nrPt\right) + h_R P_R \frac{(t_2+t_3)^2}{2} \tag{7.31}$$
$$+ b\left(\frac{\omega^2}{2(P(1-s-r)-D)} + \frac{\omega^2}{2D} + \frac{z_3 t_{r2}}{2} + \frac{z_3 t_2}{2}\right)$$

$$TCU_3 = \frac{TC_3(t_1, w|t, r, s)}{t_0} \tag{7.32}$$

Case 4. TCU_4 → $z_3 \leq 0$ $z_2 \leq 0$

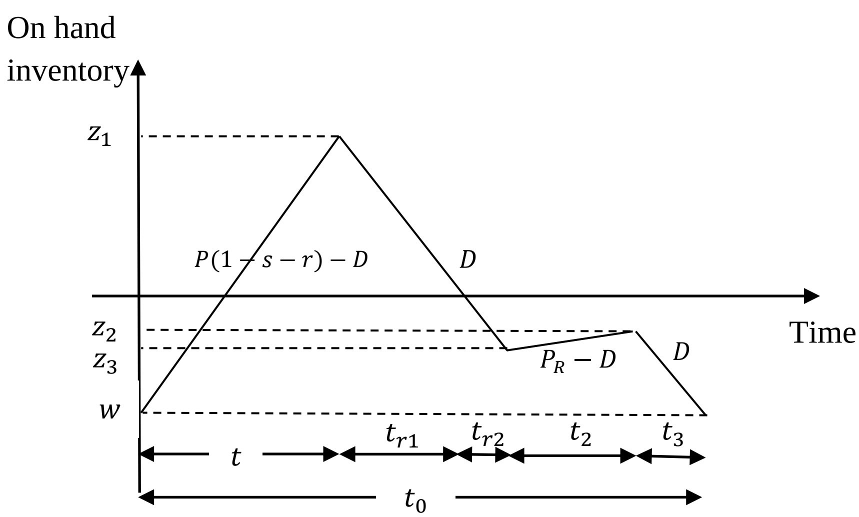

Figure 7.7 On- hand inventory of good items with breakdown $\mathbf{z_3 < 0}$ & $\mathbf{z_2 < 0}$

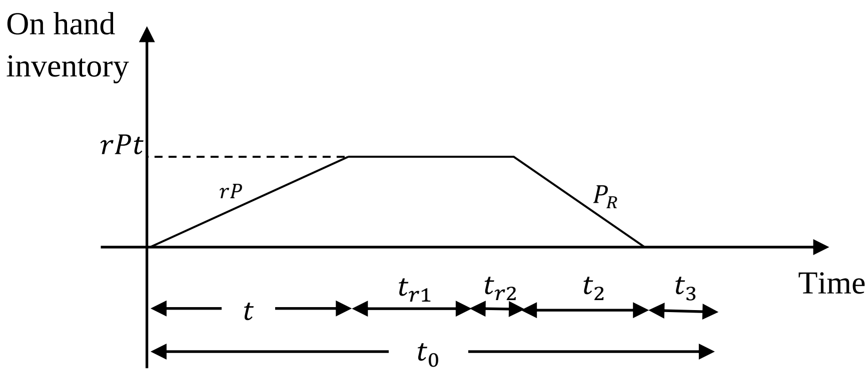

Figure 7.8 On- hand inventory of re-workable items with breakdown $z_3 < 0$ & $z_2 < 0$.

The inventory level and the corresponding times of Figures 7.7 and 7.8 are:

$$z_1 = (P(1 - s - r) - D)t - w \tag{7.33}$$

$$z_3 = t_r D - z_1 = nD - (P(1 - s - r) - D)t + w \tag{7.34}$$

$$z_2 = z_3 + t_2(P_R - D) \tag{7.35}$$

$$t_{r1} = \frac{z_1}{D} \tag{7.36}$$

$$t_{r2} = \frac{z_3}{D} \tag{7.37}$$

$$t_2 = \frac{rtP}{P_R} \tag{7.38}$$

$$t_3 = \frac{w - z_2}{D} \tag{7.39}$$

$$t_0 = \frac{tP(1 - s)}{D} \tag{7.40}$$

The total cost and the total cost per unit time when $z_3 \leq 0$ and $z_2 \leq 0$ are:

$$TC_4(t_1, w|t, r, s)$$

$$= c\ tP + c_d\ tPs + c_R\ tPr + A$$

$$+ h\left(\frac{{z_1}^2}{2(P(1-s-r)-D)} + \frac{z_1 t_{r1}}{2} + \frac{rP}{2}t^2 + nrPt\right)$$

$$+ h_R P_R \frac{t_2^2}{2} \tag{7.41}$$

$$+ b\left(\frac{\omega^2}{2(P(1-s-r)-D)} + \frac{z_3 t_{r2}}{2} + \frac{z_3 + z_2}{2}t_2\right.$$

$$\left. + \frac{z_2 + w}{2}t_3\right)$$

$$TCU_4 = \frac{TC_4(t_1, w|t, r, s)}{t_0} \tag{7.42}$$

7.3.1 NO BREAKDOWNS WITHIN THE PRODUCTION PERIOD

In this case, the production up time is smaller than the time t (time to breakdown). This case is similar to the model described in Chapter 5, but here we are interested in the production run time, t_1. Note that $Q = Pt_1$. Fig. 7.9 shows on hand inventory of perfect quality items without machine breakdown. And Fig 7.10 shows the on hand inventory of non-conforming items at the same conditions.

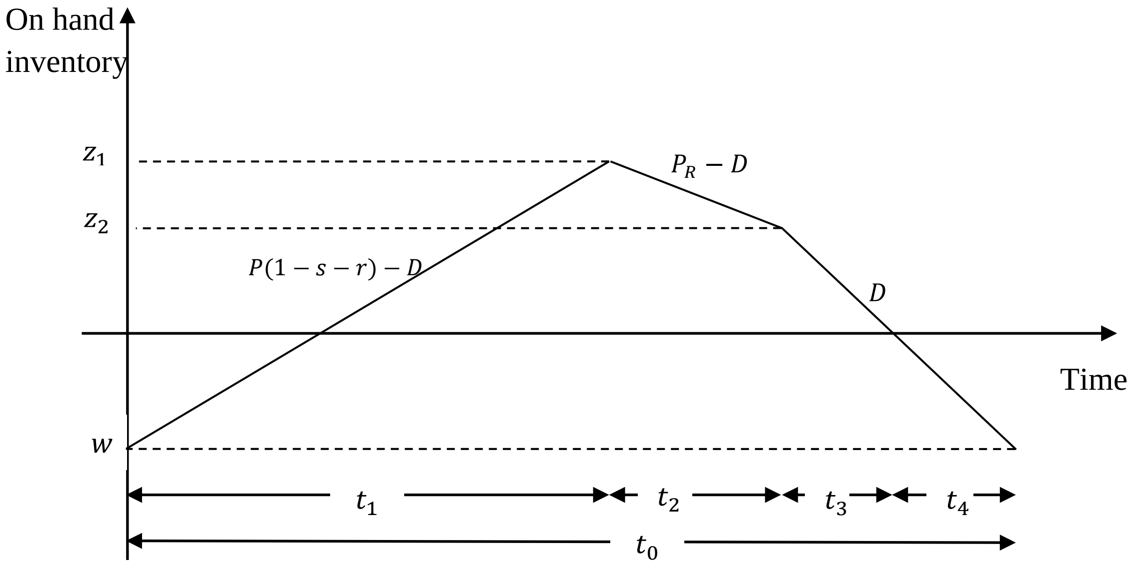

Figure 7.9 On- hand inventory of conforming items without breakdowns

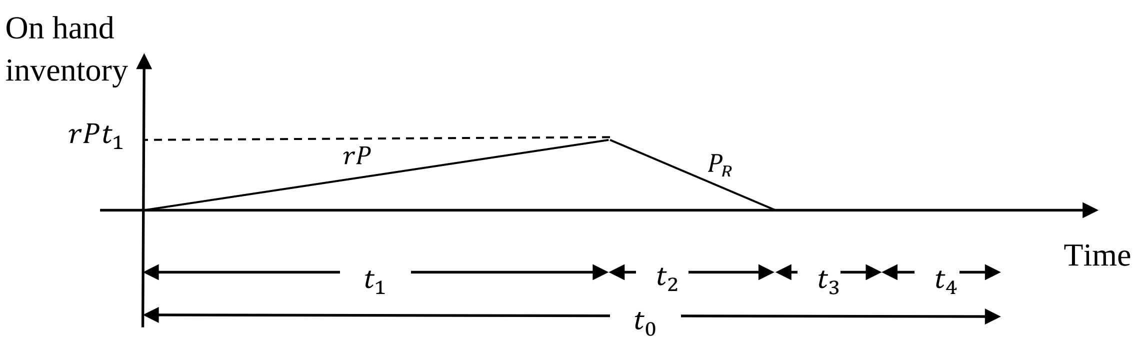

Figure 7.10 On- hand inventory of re-workable items without breakdowns

Similar to what have been done in Chapter 5, the corresponding total cost depends on the value of z_2 either positive or negative, so it can be described as follows:

The total cost per unit time when $z_2 \leq 0$ is:

$$TCU_4 = e_1 + \frac{e_2}{t_1} + e_3 t_1 + e_4 w + e_5 \frac{\mathrm{w}^2}{t_1} \qquad (7.43)$$

where:

$$e_1 = \frac{D}{(1-s)}(c \ + c_d s + c_R r)$$

$$e_2 = \frac{AD}{P(1-s)}$$

$$e_3 = \frac{hP}{2}\left(1 - \frac{D}{P} - s\right) + \frac{(h_R - h)PDr^2}{2P_R(1-s)}$$

$$e_4 = -h$$

$$e_5 = \frac{p+h}{2P}\left(\frac{1-s-r}{(1-s)\left(1-s-r-\frac{D}{P}\right)}\right)$$

and the total cost per unit time when $z_2 \geq 0$ is:

$$TCU_5 = c_1 + \frac{c_2}{t_1} + c_3 t_1 + c_4 w + c_5 \frac{w^2}{t_1} \tag{7.44}$$

where:

$$c_1 = \frac{D}{(1-s)}(c \ + c_d s + c_R r)$$

$$c_2 = \frac{AD}{P(1-s)}$$

$$c_3 = h\frac{DP}{2(1-s)}\left(\frac{{a_1}^2}{a_3} + \frac{{a_1}^2}{a_4} + \frac{rP{a_1}^2}{{a_3}^2}\right) + \frac{h_R D r^2}{2P_R(1-s)} + b\frac{D}{2(1-s)}\left(\frac{{a_2}^2}{a_4} - \frac{{a_2}^2}{D}\right)$$

$$c_4 = -\mathrm{h}\frac{D}{2(1-s)}\left(\frac{2a_1}{\mathrm{a}_3} + \frac{2a_1}{a_4}\right) + \mathrm{b}\frac{D}{2(1-s)}\left(\frac{2a_2}{a_4} - \frac{2a_2}{D}\right)$$

$$c_5 = \mathrm{h}\frac{D}{2P(1-s)}\left(\frac{1}{\mathrm{a}_3} + \frac{1}{a_4}\right) + \mathrm{b}\frac{D}{2(1-s)}\left(\frac{1}{a_4} + \frac{1}{a_3}\right)$$

$a_1 = 1 - s - r - D/P$

$a_2 = s + \frac{D(rP+P_R)}{PP_R} - 1$

$a_3 = P(1 - s - r) - D$

$a_4 = D - P_R$

7.3.2 COMBINE THE SIX CASES (WITH & WITHOUT BREAKDOWN)

There is a limits upon which the values of z_2 and z_3 positive or negative. Since we have three random variables, we will fix two of them and make the limit refers to the third one. In our model we make the limit for proportion of re-workable items r. According to this, the limit of *r* upon which z_3 become negative will be r_1 as follows:

$$r_1 = 1 - s - \frac{D}{P} - \frac{w}{tP} - \frac{nD}{tP} \tag{7.45}$$

And r_2 for z_2:

$$r_2 = \frac{P_R}{D}\left(1 - s - \frac{D}{P} - \frac{w}{tP} - \frac{nD}{tP}\right) \tag{7.46}$$

If the proportion of re-workable items is less than the limits r_1 and r_2, z_3 and z_2 are positives, but if the proportion of re-workable items is greater than the limits, z_3 and z_2 are negatives. So if $D \leq P_R$, the total cost per unit time can be written as follows:

$$TCU = \int_{t_{min}}^{t_1} \int_{s_{min}}^{U_{s1}} \int_{r_{min}}^{U_{r1}} TCU_1 \, g(r)f(s)l(t) \, drdsdt$$
$$+ \int_{t_{min}}^{t_1} \int_{L_{s1}}^{U_{s2}} \int_{L_{r1}}^{U_{r2}} TCU_3 \; g(r)f(s)l(t) \, drdsdt$$
$$+ \int_{t_{min}}^{t_1} \int_{L_{s2}}^{s_{max}} \int_{L_{r2}}^{r_{max}} TCU_4 \, g(r)f(s)l(t) \, drdsdt \tag{7.47}$$
$$+ \int_{t=t_1}^{t_{max}} \int_{s_{min}}^{s_{max}} \int_{r_{min}}^{r_{max}} TCU_5 \, g(r)f(s)l(t) \, drdsdt$$

Where:

$$L_{s1} = max\left\{s_{min}, \left(1 - \frac{D}{P} - \frac{w}{tP} - \frac{nD}{tP} - r_{max}\right)\right\}$$

$$U_{s1} = min\left\{s_{max}, \left(1 - \frac{D}{P} - \frac{w}{tP} - \frac{nD}{tP} - r_{min}\right)\right\}$$

$$L_{s2} = max\left\{s_{min}, \left(1 - \frac{D}{P} - \frac{w}{tP} - \frac{nD}{tP} - \frac{Dr_{max}}{P_R}\right)\right\}$$

$$U_{s2} = min\left\{s_{max}, \left(1 - \frac{D}{P} - \frac{w}{tP} - \frac{nD}{tP} - \frac{Dr_{min}}{P_R}\right)\right\}$$

$$L_1 = 1 - s - \frac{D}{P} - \frac{w}{tP} - \frac{nD}{tP}$$

$$L_2 = \frac{P_R}{D}\left(1 - s - \frac{D}{P} - \frac{w}{tP} - \frac{nD}{tP}\right)$$

$$L_{r1} = max\{r_{min}, L_1\}$$

$$U_{r1} = min\{r_{max}, L_1\}$$

$$L_{r2} = max\{r_{min}, L_2\}$$

$U_{r2} = min\{r_{max}, L_2\}$

On the other hand, if $D \geq P_R$, the total cost per unit time will be as follows:

$$TCU = \int_{t_{min}}^{t_1} \int_{s_{min}}^{U_{s2}} \int_{r_{min}}^{U_{r2}} TCU_1 \, g(r)f(s)l(t) \, drdsdt$$

$$+ \int_{t_{min}}^{t_1} \int_{L_{s2}}^{U_{s1}} \int_{L_{r2}}^{U_{r1}} TCU_2 \, g(r)f(s)l(t) \, drdsdt$$

$$+ \int_{t_{min}}^{t_1} \int_{L_{s1}}^{s_{max}} \int_{L_{r1}}^{r_{max}} TCU_4 \, g(r)f(s)l(t) \, drdsdt \tag{7.48}$$

$$+ \int_{t_1}^{t_{max}} \int_{s_{min}}^{U_s} \int_{r_{min}}^{U_r} TCU_5 \, g(r)f(s)l(t) \, drdsdt$$

$$+ \int_{t_1}^{t_{max}} \int_{L_s}^{s_{max}} \int_{L_r}^{r_{max}} TCU_6 \, g(r)f(s)l(t) \, drdsdt$$

Where:

$$L_s = max\left\{s_{min}, \left(1 - \frac{D}{P} - \frac{w}{tP} - \frac{Dr_{max}}{P_R}\right)\right\}$$

$$U_s = min\left\{s_{max}, \left(1 - \frac{D}{P} - \frac{w}{tP} - \frac{Dr_{min}}{P_R}\right)\right\}$$

$$L = \frac{P_R\left(1 - s - \frac{D}{P} - \frac{w}{tP}\right)}{D}$$

$L_r = max\{r_{min}, L\}$

$U_r = min\{r_{max}, L\}$

The minimum of TCU can't be found in closed form. Since the limits are not constant and it is function of the decision variables. Therefore, we find the solution by performing exhaustive search for t_1 and w. The integration is carried out numerically using a 12-node Gaussian quadrature method

In real life, the time to failure (breakdown) almost follows exponential distribution function. Since we assume that no breakdowns during the period of fulfilling the previous cycle shortage and the rework time, the shifted exponential distribution can be used. The formula for that function is similar to the original exponential distribution but it started from point greater than zero and the probability before that point equal zero. The pdf of this function is:

$$l(t) = \begin{Bmatrix} \lambda e^{-\lambda(t+\delta)} & \delta \leq t \leq \infty \\ 0 & otherwise \end{Bmatrix}$$

Where: λ is the rate parameter and δ is the shift value.

According to this, the minimum value of t is δ and no breakdowns occur before this point. So to guarantee that the inventory level z_1 is positive when the breakdown occurs, the time t_s needed to fulfill the shortage of the previous production cycle mustn't exceed the shift.

$$t_s = \frac{w}{P(1 - \mathrm{s} - \mathrm{r}) - D} \leq \delta \tag{7.49}$$

Since (7.49) should be hold for all the values of s and r, (7.49) can be written as:

$$\frac{w}{P(1 - \max(s) - \max(r)) - D} \leq \delta \tag{7.50}$$

From (7.50), the maximum shortage allowed must be constrained as follows:

$$w \le \delta(P(1 - \max(s) - \max(r)) - D) \tag{7.51}$$

Since our model depends on some assumptions. These assumptions must be defined as constraints to get precise and representative optimal values. Constraints are related to the inventory level after repair z_3 and inventory level after rework process z_2, where the both should be less than or equal to the maximum shortage allowed. So the following constraints should be met before searching for optimal values:

$$\delta \ge \frac{n}{\frac{P}{D}(1 - \max(s) - \max(r)) - 1} \quad if \quad D \le P_R \tag{7.52}$$

$$\delta \ge \frac{n}{\frac{P}{D}\left(1 - \max(s) - \frac{D}{P_R}\max(r)\right) - 1} \quad if \quad D \ge P_R \tag{7.53}$$

Finally, to prevent breakdown occurrence during the rework process we must have:

$$t_2 = \frac{rtP}{P_R} \le \delta \tag{7.54}$$

(7.54) should satisfy all the values of r, so (7.54) can be simplified to:

$$t_1 \le \frac{\delta P_R}{\max(r)\, P} \tag{7.55}$$

7.4 NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

Consider a single product model where the demand rate is 1200 unit/year and production rate of 1600 unit/year. Breakdowns may occur after a random period of time. This period follows a shifted exponential distribution function with rate parameter $\lambda = 1.2$ and shift $= 0.3$. The time required to repair is 0.0274 year. Other related costs are:

c = \$104/unit,

c_R= \$8/unit,

c_d= \$5/unit,

A = \$1500,

c_r=2000,

h = \$20/unit/year,

h_R = \$22/unit/year,

b = \$25/unit/year,

The proportion of scrapped items is uniformly distributed between [0, 0.05], and the proportion of re-workable items is uniformly distributed between [0, 0.1]. The re-workable items are reworked at a rate of 1000 unit/year.

Using the gauss quadrature method 12-node, and by holding the previous mentioned constraints, the optimal run time t_1 is 0.40412 year, the optimal shortage allowed w is 48 units, and the related total cost per unit time *TCU* is \$ 132991/year.

To show the effect of mean time between breakdowns on the decision variables, we consider the previous example but change the value of the rate parameter λ which is the reciprocal of the mean time between breakdowns. Table 7.1 shows that effect.

Table 7.1 The effect of mean time between breakdowns on the required decision variables

λ	Mean time between breakdowns($\frac{1}{\lambda}$)	t_1	w	TCU
0.8	1.25	0.46440	48	132196
0.9	1.11	0.44796	48	132562
1	1.00	0.42604	48	132773
1.1	0.91	0.42330	48	132904
1.2	0.83	0.40412	48	132991
1.3	0.77	0.39316	47	133054
1.4	0.71	0.37672	47	133102
1.5	0.67	0.36576	46	133138
1.6	0.63	0.35206	44	133167
1.7	0.59	0.33836	43	133189

One can conclude that as the mean between break downs increases, the optimal run time and optimal shortage allowed increase also but within the feasible region of the model. And as a result the total cost per unit time will increase also. Table 7.2 illustrates the effect of repair cost on the decision variable. It is clear that the production run time and optimal shortage allowed is decrease as the cost of repair increases. And by default the total cost per unit time will increase as repair cost increases.

Table 7.2 The effect of repair cost on the required decision variables

Repair cost	t_1	w	TCU
1600	0.42604	48	132876
1800	0.42330	48	132936
2000	0.40412	48	132991
2200	0.38494	48	133040
2400	0.37398	47	133083
2600	0.35754	45	133117
2800	0.33836	43	133144
3000	0.32740	41	133163
3200	0.31370	40	133174
3400	0.30274	39	133178

7.5 SUMMARY

This chapter investigated the finite production rate model when there are stochastic breakdowns. The model presented contains three levels of randomness, the proportion of scrapped items, proportion of re-workable items, and the time to breakdown. We showed the effect of stochastic breakdown on the optimal production run time and the optimal shortage allowed. The optimal production run time and the optimal shortage allowed are decrease as the mean time between breakdowns decreases. And through the result we figure out the importance of constraints and how to employ it to get optimum feasible solution.

CHAPTER 8

CONCLUSIONS AND FURURE RESEARCH

8.1 CONCLUSIONS

In this thesis we extended the economic production quantity model to the case where imperfect items are generated. We assumed that there are two types of nonconforming, scrapped and re-workable items with random proportion of each. Scrapped items are disposed of immediately as they are generated. On the other hand, re-workable items are processed soon after the main production is over. Re-workable items are always perfect. We find out that the change in the proportion of scrapped items is proportional to the optimal lot size but inversely proportional to optimal maximum shortage. On the other hand, the change in the proportion of re-workable items is inversely proportional to the change in the economic production quantity and the optimal maximum shortage. The optimal total cost increases as any of the two proportions increase.

In addition to the above case, we introduced a model where the mean of some quality characteristic is determined. This is an integration of production and inventory control with process targeting. We developed a solution procedure for this case and show the effect of model parameters on the decision variables.

In real life, the production may be interrupted either by a breakdowns or maintenance activities. This case has been studied in this thesis, resulting in a model that has three degrees of randomness; proportion of scrapped items, proportion of re-workable items, and stochastic breakdowns. We show that as the mean time between breakdowns increases, the optimal run time and the optimal shortage allowed increase. On the other hand, production run time and optimal shortage allowed decrease as the cost of repair increases. In this model, the assumptions were enforced by adding the proper constraints. It is unfortunate that the literature includes several papers that violate their own assumption.

8.2 FUTURE RESEARCH

This thesis opens a wide horizon for extensions, since it corrects some errors committed by previous researchers. Furthermore it presents a new way to deal with production and inventory control. Below are some suggestions for further extensions:

1. Consider the error in inspection process, where the inspector may commit type one or two errors when all the imperfect quality items are re-worked. Or he/she may commit an error in identifying good, scrapped, and re-workable items. This will result in six types of error.
2. Consider that breakdown may happen in the shortage period or in the rework process, or use another policy instead of NR when breakdown occurs.
3. Extend the integrated model of production planning and process targeting by using quadratic loss function or by adding another decision variables such as the lower, upper specification limits, or standard deviation.

NOMENCLATURE

A : set-up cost for each production run.

b : shortage cost per item per unit time.

c : production cost per item.

c_d: disposal cost per scrapped item ($/item),

c_R : rework cost per item of imperfect quality.

c_r : repair cost for fixing the breakdown.

D : demand rate in units per unit time.

$g(\mathrm{r})$: probability density function of re-workable items proportion.

$f(s)$: probability density function of scrapped items proportion.

h : holding cost of a perfect items per item per unit time ($/item/unit time).

h_R : holding cost of a re-workable item per unit time.

$l(t)$: probability density function of time to breakdown.

m : cost of repairing the machine.

μ : the process mean.

P : production rate in units per unit time.

P_R : rework rate of imperfect quality items in units per unit time.

Q : total items produced during a production cycle.

r : proportion of imperfect items produced (random variable).

s : proportion of defective items produced that are scrapped (random variable).

σ: process standard deviation.

t : time to breakdown (random variable)

t_1 : optimal production run time.

t_r, n : time of repairing the machine.

w : maximum back order level in a production cycle.

z_1 : inventory level of good items after the end of production process.

z_2 : inventory level of good items after the end of rework process.

z_3 : inventory level of good items after the end of repair process.

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