Integrating Production and Maintenance Scheduling Under Stochastic Production Conditions

BY

Laith Awni Abdul-Fattah Al-Hadidi

A Dissertation Presented to the
DEANSHIP OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

DOCTOR OF PHILOSOPHY

In

Industrial & Systems Engineering

May 2011
DEANSHIP OF GRADUATE STUDIES

This dissertation, written by Laith Awni Abdul-Fattah Al-Hadidi under the direction of his thesis advisor and approved by his thesis committee, has been presented and accepted by the Dean of Graduate Studies, in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Industrial & Systems Engineering.

Dissertation Committee

Dr. Umar Al-Turki
Dissertation Advisor

Dr. Mohammed Adur-Rahim
Dissertation Co-Advisor

Dr. Salih Duffuaa
Member

Dr. Abdulbasit Andijani
Member

Dr. Bekir Sami Yilbas
Member

Dr. Fouad Al-Sunni
Department Chairman

Dr. Salam Zummo
Dean of Graduate Studies

Date 9/7/2011
Dedication

To my

Dear parents, brothers & sisters,
I dedicate this work
Acknowledgment

I sincerely thank King Fahd University of Petroleum & Minerals for supporting this research, and especially the Systems Engineering Department for providing the help and facility to carry it out.

I wish to express my profound gratitude to my advisor, Dr. Umar Al-Turki, and my co-advisor Dr. Mohammed Abdur-Rahim, who supervised this research. I highly appreciate their valuable suggestions, continuous encouragement, extraordinary support, and patience in correcting my writing. They were always kind, understanding and sympathetic to me. I also wish to thank other members of my dissertation committee Dr. Salih Duffuua, Dr. Abdulbasit Andijani and Dr. Bekir Sami Yilbas for their cooperation and guidance. A great thanks goes to Dr. Christopher Garris for his careful proofreading.

I wish to extend my thanks to the anonymous referees of International Journal of Industrial and Systems Engineering (IJISE), International Journal of Mathematics in Operational Research (IJMOR) and, International Journal of Operational Research (IJOR) for their comments and insights which improved the content of this dissertation.

It goes without saying, I truly appreciate the love, sacrifices, prayers and understanding i received from my parents Dr. Awni & Mrs. Rima, brothers and sisters Sawsan, Hashem, Dr. Moayyed and Bushra. They were always supporting me and encouraging me with their best wishes.
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The goal of this dissertation is the integration between maintenance and production scheduling. The dissertation starts with a structured and comprehensive review of the literature on modelling the integration between maintenance and production scheduling. The review makes a distinction between interrelated and integrated models. Interrelated models superimpose the solution of one element on the model of the other element, while integrated models consider two or more elements of the production system simultaneously. Integrated models are proven to provide significant savings in operational cost and higher production system efficiency.

The second part of the dissertation models the integration of production scheduling and preventive maintenance scheduling on a single machine that is subject to random failures. Preventive maintenance that restores the machine to ‘as good as new’ condition is should be performed, if needed, before the start of job processing. The objective is to determine a schedule that
combines job processing and preventive maintenance activities that mini-
mizes the total weighted expected jobs completion times. A mixed integer
programming model is developed for the problem. Solving this model results
in the optimum schedule.

The third part of the dissertation studies the previous problem taking
into account the total cost of maintenance and production. Total cost in-
cludes costs of preventive maintenance, minimal repair and work in process
inventory. Again the problem is formulated as a mixed integer programming
model.

The dissertation concludes with suggestions for some future directions for
research in this area. In addition, a preliminary investigation is presented
regarding models that add quality to the integrated model, resulting in a
three way joint determination of an integrated schedule and an optimum
quality control chart design.
ملخص بحث
درجة الدكتوراة في الفلسفة

ليث عوني عبدالفتاح الحديدي

الإم: 

عنوان الابطروحة: الجدولة الشاملة للصيانة والإنتاج تحت ظروف إنتاجية متغيرة

التخصص الدقيق: هندسة صناعية ونظم

تاريخ التخرج: يونيو 2011

الموضوع الرئيسي لهذه الابطروحة هو الجدولة الشاملة المتزامنة للعمليات الصناعية وعمليات الصيانة الدورية بغرض زيادة الإنتاجية و الكفاءة التشغيلية. الجزء الأول من هذه الابطروحة يقدم مراجعة شاملة ذات بناء محدد لجدولة الصيانة والعمليات الصناعية. قامت هذه المراجعة بتقريب بين النماذج المتداخلة والنماذج المتزامنة. النماذج المتداخلة تقوم باعتبار أحد عناصر الإنتاج كشرط يقيد عمليات الإنتاج الأخرى. النماذج المتزامنة تقوم بنمذجة عنصرين أو أكثر من عناصر الإنتاج. أثبتت النماذج المتزامنة وفرًا لتكلفة الإنتاج وكفاءة أعلى لنظام الإنتاج.

الجزء الثاني من هذه الابطروحة يقوم بنمذجة العمليات الصناعية وصيانة الوقائية لآلة صناعية واحدة معرضة لأعمال غير متوقعة و المطلوب هو جدولة تشتمل العمليات الصناعية وعمليات الصيانة الوقائية مما يضمن الكفاءة الأقل من حيث مجموع أوقات إكمال المهام الإنتاجية. تقوم الصيانة الوقائية بإعادة حالة الآلة للحالة 

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الإبتدائية إذا تم عمل الصيانة الوقائية. تم في هذه الأطروحة تطوير نموذج رياضي
لإيجاد الجدول الزمني الأمثل للعمليات الصناعية وعمليات الصيانة الوقائية.
يقوم الجزء الثالث من هذه الأطروحة بدراسة المسألة السابقة من حيث كلفة
الإنتاج وتكلفة الصيانة الوقائية. تكون الكلفة الكاملة من الصيانة الوقائية، الإصلاح
وكلفة التخزين. يقوم هذا الجزء من الأطروحة بتقديم نموذج رياضي مشابه
للمؤثر السابق.

هذه الأطروحة تتبجي بتقديم مقترحات لأفكار محتملة مستقبلية لهذه المسألة و
كذلك بتبقي أولي لبحث إمكانية إضافة التصميم الأمثل لأدوات ضبط الجودة
للمؤثر المزارع لينتج نموذج ثلاثي بين جدولة العمليات الصناعية، الصيانة الوقائية
و ضبط الجودة.
Chapter 1

INTRODUCTION

1.1 INTRODUCTION

Production scheduling, maintenance and quality are the fundamental elements for any production system. Since the 1950’s, research in these elements has sustained a growing attention of researchers, as well as practitioners. This interest was driven by the increasing competition in global markets. As such, the theory in this area has nurtured and became mature. Many researchers believe that treating these elements independently would result in a suboptimal solution. Hence, research in the integrated models has recently gained momentum.

Modern production systems rely on optimal and effective planning and scheduling for their operations. It is a usual practice to plan for one element of the production system, independent of the others and, to ignore their possible mutuality. Furthermore, this independent planning is done through
separate functional teams. As a result, plans of a specific function may disrupt other function plans. For example, the maintenance function schedules a major shutdown and communicates it to the production. The suggested maintenance may maximize the machine availability, but will disrupt production plans. Similarly, production schedulers may have the tendency to utilize machines to their full capacity to meet demand disregarding maintenance requirements. In this case, productivity may increase, but machine availability will decrease due to more frequent breakdowns. Figure 1.1 shows the possible interactions between different elements of a production system that will be clearly visible at the shop floor level where conflicts between the three plans arise.

Figure 1.1: Classical disjoint planning for production, maintenance and quality

Independent planning may provide optimal performance at the level of a specific function. A global optimal can only be achieved by models integrat-
ing all functional elements of the system. Integrated production models are expected to deal with multiple elements of conflicting nature; hence, planning these elements independently will cause conflicts between their corresponding functions. This disturbance can be reduced through coordination between two or more elements of the production system. Figure 1.2 shows functional coordination in real life practice at the two levels of planning and scheduling, where production planning is done and then that plan goes to the shop floor and quality planning for implementation. Meanwhile, maintenance planning and scheduling develop their own plans and schedules to be implemented on the shop floor.

![Diagram showing interrelation between planning of production, quality and maintenance on the shop floor](image)

Figure 1.2: Interrelation between planning of production, quality and maintenance on the shop floor

Integrated models are usually not easy to develop, because of their multidimensional complexity. As such, the level of integration in planning between functional elements in production setting is minimal. Planners may give higher priority to a certain function and plan for that solely. The output
plan will be taken as an input to the second in priority function. For that function, a plan will be built taking the input of the other function as a constraint. For example, production schedules can be generated given that the machine will be out of service for a specific duration. This situation can be seen as coordination or superimposition rather than real integration. These types of models can be referred to as interrelated models.

1.2 AN OVERVIEW

The objective of production planning is to identify the optimal production plan that satisfies demand with a minimum cost. This cost is usually composed of inventory holding and production setup costs. Production planning usually assumes a perfect environment in terms of resource availability and process quality. Resource unavailability during the production process will increase production costs and affect inventory levels needed to satisfy customer demand. Production planning is done as part of a hierarchical planning process, where the production plan is cascaded down to a more detailed production schedule. The objective of scheduling is to sequence production tasks, in order to minimize a certain performance measure of customer satisfaction. Performance measures may include average flow time, maximum production time or meeting due dates. These measures are minimized under different configurations of production resources, such as single machine, parallel machines, flowshop or jobshop. For each configuration, dif-
Different scheduling rules are presented to optimize different performance measures. These rules are used to schedule production tasks on machines over time. Available production scheduling models are mostly deterministic models based on methodologies varying from traditional integer programming to Branch-Bound techniques to Lagrangean relaxation and optimization-based heuristics. These models and techniques have been implemented in a variety of manufacturing systems.

Customers’ satisfaction of delivery, offered by successful production planning and scheduling, can be affected by quality deterioration resulting from shifts in process mean, variance or both. Preventive Maintenance (PM) is utilized to delay or prevent these shifts, which can be detected by Statistical Process Control (SPC) tools. To control a production process, proper planning is needed for PM activities as well as effective quality control tools. PM studies focus on inspection and replacement intervals or address equipment condition. PM planning models are typically stochastic in nature (either mathematical or simulation) accompanied by optimization techniques designed to maximize equipment availability or minimize equipment maintenance costs.

Preventive maintenance is usually done in two different ways. The first is time-based or preventive, where maintenance tasks are scheduled for the production unit when it reaches a certain age. The second is condition-based or predictive where maintenance is scheduled based on machine conditions.
The ultimate goal of condition-based maintenance is to perform maintenance when the maintenance activity is most appropriate and before the equipment loses optimum performance. This is in contrast to time-based maintenance, where a piece of equipment gets maintained at predetermined points of time whether it needs it or not.

1.3 DISSERTATION OBJECTIVES AND MOTIVATION

Although, the interest in integrated modeling is increasing, review papers on integrated models are limited (Pandey et al., 2010; Budai et al., 2008; Ben-Daya and Rahim, 2001). Motivated by the lack in review papers, this dissertation provides, at first, a state of the art model for the integration between planning and scheduling of production, maintenance, and quality. It is proven that integrated models offer savings in operating costs and a higher utilization of resources.

After reviewing the literature, this dissertation considers three integrated problems:

1. Integrating job scheduling and preventive maintenance on a single machine to minimize expected total job completion times.

2. Integrating job scheduling and preventive maintenance on a single machine to minimize costs associated with maintenance, minimal repair
and work in process inventory.

3. Investigating the effect of quality rejection rate on delaying job completion time and its effect on decisions related to job scheduling and preventive maintenance.

This chapter only provides a brief introduction for each problem. A detailed introduction is given at the beginning of each of the coming chapters.

The motivation to study the integrated job scheduling and preventive maintenance problem, in addition to cost savings, comes from the need to overcome conflicts arising between production and maintenance functions in most manufacturing systems. While the production unit has an interest in keeping a continuous production run to satisfy customer needs, the maintenance function is committed to long life asset management and optimum maintenance tasks and activities. These two objectives, in many cases, cause conflicts when planned or unplanned shutdowns cause a serious delay in production schedules. Solving the production scheduling and PM planning problems independently ignores these inherent conflicts. Even when the conflict is managed, the result is not usually optimized globally, because both schedules are developed independently from each other and then combined over the planning horizon.

To our knowledge, there are no integrated models between job scheduling and quality control in the literature. Meanwhile, the integration between preventive maintenance and quality control is well established in the literature.
Fewer models exist in the integration between job scheduling and preventive maintenance. The three way integration between job scheduling, preventive maintenance and quality control is highly challenging. This dissertation attempts to address this gap in the literature, which simultaneously captures the effect of job scheduling, preventive maintenance and quality control on the overall system performance.

1.4 ORGANIZATION OF THE DISSERTATION

Chapter 2 provides a literature review focusing on the integration between planning and scheduling of production, maintenance and quality. The review identifies some of the gaps that needed to be addressed. As a result, Chapters 3 and 4 discuss integration between job scheduling and preventive maintenance for two different objectives. Chapter 3 addresses the minimization of expected total weighted job completion times. Chapter 4 addresses the objective of minimizing total costs associated with maintenance and inventory.

Finally, Chapter 6 summarizes the research in this dissertation, and briefly discusses some potential future work.
Chapter 2

LITERATURE REVIEW OF INTEGRATED MODELS IN PRODUCTION PLANNING AND SCHEDULING, MAINTENANCE AND QUALITY

2.1 INTRODUCTION

In recent years, researchers have enriched the literature in integrated planning. Much research has investigated integrating production and maintenance planning (Aghezzaf et al., 2007; Chelbi et al., 2008; El-Ferik, 2008). Research also has investigated integration production and maintenance scheduling (Sortrakul and Cassady, 2007; Kuo and Chang, 2007; Yulan et al., 2008; Naderi et al., 2009a,b; Berrichi et al., 2009, 2010). Integrating maintenance and quality control was investigated too (Zhou and Zhu, 2008; Yeung et al., 2008; Wu and Makis, 2008; Panagiotidou and Tagaras, 2007, 2008; Panagiotidou and Nenes, 2009; Radhoui et al., 2010).
Despite increasing interest, existing work reviewing integrated models are limited (Pandey et al., 2010a; Budai et al., 2008; Ben-Daya and Rahim, 2001). This chapter can be seen as an extension to their work. The key feature distinguishing the work in this chapter from other reviews is the differentiation between the concept of interrelation and integration. In addition, this review considers the integrated models of production scheduling. In section 2.2, interrelated models are presented in production planning and scheduling, maintenance and quality. Section 2.3 discusses two way integration between production elements. Section 2.4 discusses production planning, maintenance and quality integrated models. Section 2.5, presents gaps in the literature. Finally, section 2.6 presents conclusions.

2.2 INTERRELATED MODELS

Interrelated models consider optimizing a certain component of a production system, taking the requirement of another component as a constraint. For example, models exist for scheduling jobs with machine unavailability due to maintenance operations (arrow 4 in Figure 2.1). The inverse is also common where maintenance operations are scheduled while taking into consideration the production schedule (arrow 3 in Figure 2.1). Production schedules and maintenance schedules are cascaded down from production plans and maintenance plans. In this example, interrelation can occur at the planning level (arrows 1 and 2 in Figure 2.1). Section 2.2.1 presents interrelated models
between production and maintenance scheduling. Section 2.2.2 presents interrelated models between maintenance and inventory control. Section 2.2.3 presents interrelated models between quality control and inventory control where EMQ is assumed to have defectives.

![Interrelated models between production and maintenance](image)

**Figure 2.1:** Interrelated models between production and maintenance

### 2.2.1 PRODUCTION AND MAINTENANCE SCHEDULING INTERRELATED MODELS

Most papers in the scheduling field are based on the assumption that machines are continuously available. Adiri et al. (1989) studied the single machine nonpreemptive scheduling problem of minimizing total completion time
of jobs for both stochastic and deterministic cases. The single machine is not available for the entire scheduling horizon. To solve the problem, a shortest processing time (SPT) heuristic algorithm was proposed. In Schmidt (2000), a review was presented, related to deterministic scheduling problems, where machines are not continuously available for processing. Sadfi et al. (2005) studied the single machine total completion scheduling problem subject to a period of maintenance. In Ji et al. (2007), the objective was to find a schedule that minimizes the makespan, subject to periodic maintenance and nonresumable jobs. Low et al. (2008) studied a single machine scheduling problem, with an availability constraint, under simple linear deterioration for both preemptive and nonpreemptive cases. The objective was to minimize the makespan in the system.

The maintenance task can be seen as a special job that should not be processed with longer than a predefined interval. Qi et al. (1999) studied scheduling maintenance on a machine that must be maintained after it continuously works for a period of time. The objective was to minimize total completion time of jobs. They presented two metaheuristics to solve the problem. Raza et al. (2007) extended this work with the objective to minimize total earliness and tardiness with a common due date. In Low et al. (2010), a several maintenance periods modeled on a single machine. The machine should be stopped for maintenance after a periodic time interval or to change tools after a fixed amount of jobs are processed simultaneously. Yang et al. (2002) scheduled flexible maintenance on a single machine, where the
machine should be stopped for a constant maintenance time interval, during the scheduling period. Liao and Chen (2003) studied scheduling periodic maintenance jobs on a single machine. Several maintenance periods were deliberated, where each maintenance type was required after a periodic time interval. Specifically, the problem was to minimize the maximum tardiness with periodic maintenance and nonresumable jobs. Chen (2009) scheduled periodic maintenance to minimize the number of tardy jobs. Based on the Moores algorithm, an effective heuristic was developed to provide a near-optimal schedule for the problem.

The multiple machine scheduling problems with unavailability constraints have also been studied. Schmidt (1988) studied the parallel machine preemptive scheduling problem, where each job had a deadline and each machine had different availability intervals. Lee (1991) considered the problem of minimizing the makespan for parallel machines, where machines may not be available in a certain time interval. To solve this problem, he proposed a modified longest processing time (LPT) algorithm and showed a tight worst-case bound. Kellerer (1998) introduced an algorithm for a parallel machine problem with different starting times. Ho and Wong (1995) studied minimizing makespan on m parallel machines and proposed a two machine optimization (TMO) algorithm. Liao et al. (2005, 2007) introduced a fixed known unavailable period for the TMO. Their objective was to minimize makespan on two machines, where only one machine has a fixed known unavailable period. Lin and Liao (2007) extended previous work and presented a model
minimizing makespan on two machines, where each machine has a fixed and known unavailable period. Lee and Liman (1993) studied minimizing total completion time for two parallel machines where one machine is available for some period, after which the machine is no longer available. Their work was extended by Mosheiov (1994) to m parallel machines with different starting times. Lee and Chen (2000) studied minimizing total completion time on m parallel machines, where each machine should be maintained at least once in the planning horizon. They identified the problem as having two versions. In first, more than one machine can be maintained simultaneously. In second, no more than one machine can be maintained at a time.

The flowshop problem with unavailability constraint was also studied in the literature (Lee, 1997; Espinouse et al., 1999; Blazewicz et al., 2001; Espinouse et al., 2001; Kubiak et al., 2002; Aggoune, 2003; Allaoui and Artiba, 2004; Kubzin and Strusevich, 2005; Aggoune and Portmann, 2006; Allaoui and Artiba, 2006; Yang et al., 2008; Allaoui et al., 2008; Liao and Tsai, 2009; Gholami et al., 2009).
Table 2.1: Maintenance and scheduling interrelated models

<table>
<thead>
<tr>
<th>Category</th>
<th>Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single machine</td>
<td>Adiri et al. (1989), Qi et al. (1999), Schmidt (2000), Yang et al. (2002), Liao and Chen (2003), Sadfi et al. (2005), Raza et al. (2007), Low et al. (2008), Chen (2009), Low et al. (2010)</td>
</tr>
</tbody>
</table>

2.2.2 PRODUCTION AND MAINTENANCE PLANNING INTERRELATED MODELS

In the following, we consider maintenance models under the direct effect of production requirements model but that don’t provide changes to the original production plan. The production system may be planned to have
an excess amount of production (buffer) to overcome shortages due to unexpected production interruption due to machine breakdowns. There are a number of models that explored production systems with buffer (Das and Sarkar, 1999; Iravani and Duenyas, 2002; Yao et al., 2005; Kyriakidis and Dimitrakos, 2006; Chelbi and Ait-Kadi, 2004).

Some models in the literature presented the economic manufacturing quantity (EMQ) or lot sizing with the presence of stochastic machine breakdowns and corrective maintenance (Groenevelt et al., 1992a,b; Sarper, 1993; Anily et al., 1998; Vassiliadis and Pistikopoulos, 2001; Cavory et al., 2001; Ben-Daya, 2002; Chung, 2003; Lee, 2005; Lin and Gong, 2006; Kenne et al., 2006; Lodree and Geiger, 2010).

Finch and Gilbert (1986) presented an integrated conceptual framework for maintenance and production, in which they focus especially on manpower issues during corrective and preventive work. Duffuaa and Al-Sultan (1997, 1999), extended Finch and Gilbert (1986) and casted the maintenance scheduling problem in a stochastic framework. For a multipurpose plant, Lou et al. (1992) and Dedopoulos and Shah (1995) considered a multi-product manufacturing system, with random breakdowns and random repair time. They offered an interrelated production and maintenance model that determines the relationship between failure and profitability, as well as the costs of different maintenance policies. Vaurio (1999) developed unavailability and cost rate functions for components whose failures can occur randomly. Di-
jkuizen (2001) discussed the problem of clustering preventive maintenance jobs in a multi-component production system that has multiple setups. Cassady et al. (2000) presented the concept of selective maintenance, where production systems are required to perform a sequence of operations with finite breaks between each operation. Rishel and Christy (1996) studied the impact of incorporating maintenance policies into the material requirement planning (MRP) system. Four performance measures were used to evaluate the impact of merging maintenance policies with the MRP system: number of on-time orders, number of scheduled maintenance actions, number of equipment failures and the total maintenance costs. Six different MRP systems were used to determine if integrating scheduled maintenance with the production schedule would improve the performance measures. Brandolese et al. (1996) considered the problem of planning a multi-product made up for flexible machines operating in parallel. They developed a model that determined the optimal schedule for both production and maintenance check points. Cheung et al. (2004) considered a plant with several units of different types, where there are different shutdown periods for maintenance. The problem was to allocate units to these periods in a way that production was least affected. Maintenance was not modeled in detail, but incorporated through frequency or period restrictions.
Table 2.2: Maintenance and inventory control interrelated models

<table>
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<th>Category</th>
<th>Models</th>
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2.2.3 PRODUCTION AND QUALITY INTERRELATED MODELS

Classical production planning models assume all produced items have an acceptable quality level (i.e. production process mean and variance) remains in-control during the entire production time. In the literature, some of the production planning models can be found with the assumption to have rejected items. This type of production is called an imperfect production process (Rosenblatt and Lee (1986a,b) and Lin et al. (1991) and Hariga and
Ben-Daya (1998) and Salameh and Jaber (2000)).


Lee and Zipkin (1992) considered quality issues in Kanban production systems. Rahim and Banerjee (1988) and Schneider et al. (1990) developed an optimal decision rule for tool re-setting for a process with random linear drift to minimize the cost of rejected items. Gunasekaran et al. (1995) addressed issues related to optimal investments and lot sizing policies for
improved productivity and quality. For a deteriorating production system, Yeh et al. (2000) studied optimal run length where products are sold under warranty.

Table 2.3: Quality control and inventory control interrelated models

<table>
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<tr>
<th>Category</th>
<th>Models</th>
</tr>
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<tbody>
<tr>
<td>Other</td>
<td>Rahim and Banerjee (1988), Schneider et al. (1990), Lee and Zipkin (1992), Gunasekaran et al. (1995), Yeh et al. (2000)</td>
</tr>
</tbody>
</table>

2.3 INTEGRATED MODELS

Integrated models between inventory control, maintenance and quality control are models where decision variables are simultaneously serving more than one of them as shown in Figure 2.2 (areas 1, 2, 3, 4 and 5). Section 2.3.1 discusses integration between maintenance and inventory control (area 1 in Figure 2.2). Section 2.3.2 discusses integration between maintenance and scheduling (area 2 in Figure 2.2). Section 2.3.3 discusses integration between...
quality control and inventory control (area 3 in Figure 2.2). Section 2.3.4 discusses integration between quality control and maintenance (area 4 in Figure 2.2).

![Figure 2.2: Integrated models in production, maintenance and quality](image)

2.3.1 INTEGRATING MAINTENANCE AND INVENTORY CONTROL

Srinivasan and Lee (1996) modeled PM as a decision of the inventory level. PM operation is initiated when inventory reaches a certain level then it restores production system to an ‘as-good-as new’ condition. After the PM
operation, the production system will remain without production until the inventory level drops down to or below a predefined value. At that inventory level, the facility continues to produce stock until inventory is raised back to the inventory level where maintenance is applied. A minimal repair is conducted if the facility breaks down. Okamura et al. (2001) generalized the model with the assumption that both the demand and the production process are a continuous-time renewal counting process. Machine breakdowns follow non-homogeneous Poisson process. Pistikopoulos et al. (2000) described an optimization framework for general multi-purpose process models, which determines both the optimal design as well as the production and maintenance plans simultaneously. Goel et al. (2003) presented a reliability allocation model coupled with the existing design, production and maintenance optimization framework. The aim was to identify the optimal size and initial reliability for each unit of equipment at the design stage.

A properly integrated production and maintenance plan can be achieved by optimizing the production rate and maintenance intervals for a production system. This system consists of multiple identical machines with random breakdowns, repairs and PM activities. The objective of the control problem is to find the production rate and PM frequency of the machines, so as to minimize the total cost of inventory/backlog, repair and PM. This line of research can be found (Gharbi and Kenné, 2000; Kenné and Boukas, 2003; Kenné et al., 2003; Gharbi and Kenné, 2005).
Production and maintenance planning were integrated for a given set of items that must be produced in lots on a capacitated production system throughout a specified finite planning horizon (Aghezzaf et al., 2007; Dieulle et al., 2003). Ben-Daya and Noman (2006) developed an integrated model that considers simultaneous inventory production decisions, maintenance schedule, and warranty policy. This model was presented for a deteriorating system that experiences quality shifts. El-Ferik (2008) dealt with the problem of joint determination for both economic production quantity and imperfect PM schedules. The production system undergoes PM, either upon failure or after having reached a predetermined age, whichever occurs first.

Production planning is affected by machine breakdowns due to aging. Proper PM planning will reduce the age of the production unit and, hence, number of defective items produced in the lot. Chelbi et al. (2008) focused on finding, simultaneously, the optimal values of the lot size \( Q \) and the age \( T \) at which preventive maintenance must be performed. They considered a single unit production system, which must satisfy a constant and continuous demand \( D \), shifts to an out-of-control state and produces non-conforming units at a given rate, after a random period of operation. The system is submitted to an age-based preventive maintenance policy. A PM action is performed as soon as the system reaches age \( T \) without having shifted to the out-of-control state. If the shift to the out-of-control state is detected, a restoration action of the system is planned for \( L \) time units later.
Total Cost Rate \( = \frac{Production Cost + Maintenance Cost}{Expected Cycle Time}\)

Production Cost = Inventory Cost + Nonconforming Items Cost

+ Shortage Cost + Setup Cost
Table 2.4: Maintenance and inventory control integrated models

<table>
<thead>
<tr>
<th>Category</th>
<th>Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimizing the production rate and maintenance intervals</td>
<td>Gharbi and Kenné (2000) considered a multiple-identical-machine manufacturing system with random breakdowns, repairs and PM activities. Kenné and Boukas (2003) dealt with the production and PM planning control problem for a multi-machine flexible manufacturing system (FMS). Kenné et al. (2003) presented the analysis of the optimal production control and corrective maintenance planning problem for a failure prone manufacturing system consisting of several identical machines, Gharbi and Kenné (2005) presented a two-level hierarchical control model that find the production and PM rates minimize the total cost of inventory/backlog, repair and PM.</td>
</tr>
</tbody>
</table>
2.3.2 INTEGRATING MAINTENANCE AND SCHEDULING

Production planning is very dependent on machine conditions. Machine breakdowns may disturb the production process and cause delay in schedules. Despite this dependency, classical scheduling models ignored machine breakdowns. However, recently, researchers spot out the need to integrate planning for production and maintenance. This integration is justified by expected cost savings and better resource utilization. Maintenance scheduling might delay the production schedule, but will reduce expected number of machine failures.

For the objective of total weighted tardiness, Cassady and Kutanoglu (2003) investigated the benefits of integration through a numerical study of small problems. In Yulan et al. (2008) five objectives were simultaneously considered to optimize the integrated problem of PM and production scheduling. The five objectives included minimizing maintenance cost, makespan, total weighted completion time (TWC) of jobs, total weighted tardiness (TWT), and maximizing machine availability. Sotrakul et al. (2005) developed three algorithms to solve the integrated problem. Algorithms performance was evaluated using multiple instances of small, medium, and large size. Sotrakul and Cassady (2007) developed heuristics based on genetic algorithms to solve the integrated model. In order to minimize the total
tardiness, Kuo and Chang (2007) proposed a solution method to find the optimal integrated production schedule and preventive maintenance plan for a single machine under a cumulative damage process. Naderi et al. (2009a) presented a job shop scheduling environment that has sequence-dependent setup times, which is subjected to preventive maintenance policies. The optimization criterion was to minimize the makespan.

Cassady and Kutanoglu (2005) investigated the value of integrating production and PM scheduling. This model was developed for a single machine that has increasing hazard rate and is subject to failure. Each time the machine fails, it needs a fixed time to repair $t_r$. Expected number of failures can be minimized by performing preventive maintenance before the start of the job which will restore the machine to an ‘as-good-as-new’ condition. This PM will delay the start of the job by fixed time to maintain $t_p$, nevertheless. If the machine is required to process $n$ jobs with the objective to minimize their expected total completion times then the scheduler is required to provide simultaneously, optimal sequence and, when to perform PM’s. To represent the problem in the form of mathematical program, a binary variable $y_{[i]}$ is defined where $y_{[i]} = 1$ if PM is conducted and $y_{[i]} = 0$ if PM is not conducted. Let $P_{[i]}$ be the processing time for job $i$. The expected completion time for job $i$ will be

$$E(c_{[i]}) = \sum_{k=1}^{i} [y_{[k]} \times t_p + P_{[k]} + t_r \times \text{number of failures}]$$
If each job has a given weight $w[i]$ then the objective function would be to minimize the total weighted expected completion time

$$Total \ Weighted \ Expected \ completion \ Time = \sum_{i=1}^{n} (w[i]E(c[i]))$$

Identifying the sequence, mathematically, can be done by introducing job assignment binary variable $x_{ij}$. This variable is two dimensional; one is for jobs domain $i = \{1, 2, \ldots, n\}$ and the other is for position domain $j = \{1, 2, \ldots, n\}$.

$$x_{ij} = \begin{cases} 1 & \text{if job } i \text{ is assigned to position } j \\ 0 & \text{if job } i \text{ is not assigned to position } j \end{cases}$$

Two logical sets of constraints will constrain the objective function; first set of constraints states that job $i$ can not seize two positions at the same time i.e.

$$\sum_{j=1}^{n} x_{ij} = 1 \quad \forall \quad i = 1, \ldots, n$$

second set of constraints states that one position can not hold more than one job i.e.

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \forall \quad j = 1, \ldots, n$$
Table 2.5: Maintenance and scheduling integrated models

<table>
<thead>
<tr>
<th>Category</th>
<th>Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single machine</td>
<td>Cassady and Kutanoglu (2003) formulated the integrated problem for a single machine with the objective to minimize expected total weighted tardiness, Sortrakul et al. (2005) developed heuristics based on genetic algorithms to solve Cassady and Kutanoglu (2003), Cassady and Kutanoglu (2005) formulated the integrated problem for a single machine with the objective to minimize expected total weighted completion times, Sortrakul et al. (2005) developed heuristics based on genetic algorithms to solve Cassady and Kutanoglu (2005), Kuo and Chang (2007) developed an integrated model to minimize total tardiness for a single machine under a cumulative damage process, In Yulan et al. (2008), five objectives where examined on a single machine,</td>
</tr>
<tr>
<td>Flowshop</td>
<td>Naderi et al. (2009b) investigated flexible flow line problems with sequence dependent setup times and different PM policies to minimize the makespan,</td>
</tr>
<tr>
<td>Parallel</td>
<td>machines</td>
</tr>
<tr>
<td>Berrichi et al. (2009) proposed a model to simultaneously optimize two criteria: the makespan for the production part and the system unavailability, Berrichi et al. (2010) presented an algorithm based on Ant Colony optimization paradigm to solve the joint production and maintenance scheduling problem,</td>
<td></td>
</tr>
<tr>
<td>Jobshop</td>
<td>Naderi et al. (2009a) considered job shop scheduling with sequence-dependent setup times and PM policies to minimize the makespan. Four metaheuristics based on simulated annealing and genetic algorithms were employed to solve the problem.</td>
</tr>
</tbody>
</table>
2.3.3 INTEGRATING QUALITY CONTROL AND INVENTORY CONTROL

In classical production planning, it is a common assumption that the start of production will be in an in-control state. After some time the production process may then shift to an out-of-control state. Lee and Rosenblatt (1987, 1989) and Tseng (1996) showed that the derived optimal lot size was smaller than the classical EMQ if the in-control-state time follows an exponential distribution. Wang and Sheu (2003) extended Lee and Rosenblatt (1987) in which they assumed that periodic inspections were imperfect with two types of error: Type I and Type II. They used a Markov chain to jointly determine the production cycle, process inspection intervals and maintenance levels.

For the economic design of control charts, Rahim (1994) developed an economic model for joint determination of production quantity, inspection schedule and control chart design for a production process which is subject to a non-Markovian random shock. Rahim and Ben-Daya (1998) generalized the previous model by assuming that the production stops for a fixed amount of time, not only for a true alarm, but also whenever there is a false alarm during the in-control state. Liou et al. (1994) and Ohta and Ogawa (1991) determined optimal EPQ and inspection frequency for a single item, where there is an inspection error. Peters et al. (1988) determined a joint optimal inventory and quality control policy for a production system. Goyal et al.
(1993) reviewed models that integrate production, lot sizing, inspection and rework. Ben-Daya and Makhdoum (1998) studied joint optimization of the economic production quantity (EPQ) and the economic design of control chart under different preventive maintenance policies.

The integrated model’s objective is to minimize the expected cost rate for production and quality costs. Sampling cost, cost of running the process under out-of-control state, cost of false alarm and cost of repairing the process are the four components of quality costs. Production costs are composed of setup cost, average holding cost and shortage costs. Since production system may be disrupted by sampling and repairing then production cycle time is a stochastic variable that $E[T_c]$

$$Total Cost Rate = Production Cost + Quality Cost =$$

$$\begin{align*}
= & \frac{K}{E[T_c]} + K_1 \bar{I} + K_2 \bar{S} \\
& + \frac{f(n,f)+C_2f(f,P_f)+C_3f(f,\alpha)+C_4f(P_f)}{E[T_c]}
\end{align*}$$

Where $K$ is setup cost, $K_1$ is holding cost, $\bar{I}$ is the average inventory, $K_2$ is cost of shortage and $\bar{S}$ is the average shortage. For the quality cost let $n$ be the sample size and let $f$ the number of samples taken during production time, $\alpha$ is probability of type I error, $C_2$ is cost per unit time of running process under out-of-control, $C_3$ is cost per false alarm, $C_4$ is cost of detecting and correcting process. $P_f$ probability system will shift out-of-control.
2.3.4 INTEGRATING QUALITY CONTROL AND MAINTENANCE

The production process is usually considered to follow a deteriorating scheme where the in-control period follows a general probability distribution with increasing hazard rate. Ben-Daya and Duffuaa (1995) pointed out possibility of integration between PM and quality control in two ways. In the first approach, proper maintenance is expected to increase the time between failures that the machine can have. The second approach is based on Taguchi’s approach for quality where a quadratic function (called Taguchi loss function) is defined (Taguchi, 1986). This function measures deviation of product quality characteristics. The economic design of control charts and the optimization of preventive maintenance policies are two research areas that have recently received increasing attention in the quality and reliability literature. More and more researchers recognize the strong relationship between product quality, process quality and equipment maintenance.

Pate-Cornell et al. (1987) addressed the inspection-maintenance problem which was extended by Tagaras (1988). Chiu and Huang (1996, 1995) developed models that introduce preventive maintenance into the design of control charts. In Cassady et al. (2000), a combined control chart-preventive maintenance strategy is defined. The process can shift to an out-of-control condition due to a manufacturing equipment failure. Collani (1999) addressed the rela-
tionship between tool wear and continuous monitoring process. He assumed
the quality to be a function of a machine’s degradation state. He developed
an economic model that incorporates both process control and maintenance
policies. Linderman et al. (2005) demonstrated the value of integrating sta-
tistical process control and maintenance by jointly optimizing their policies
to minimize the total costs associated with quality, maintenance, and inspec-
tion. Zhou and Zhu (2008) discussed the integration of statistical process
control and maintenance, and provides an integrated model of control chart
and maintenance management. A mathematical model is given to analyze
the cost of the integrated model and the grid-search approach is used to find
the optimal values of policy variables that minimize hourly cost. Yeung et al.
(2008) formulated a combined preventive maintenance and SPC policy using
partially observable, discrete-time Markov decision process. Wu and Makis
(2008) considered the economic-statistical design of a control chart for a
maintenance application. The machine deterioration process is described by
a three-state continuous time Markov chain. The machine state is unobserv-
able, except for the failure state. Panagiotidou and Tagaras (2008) developed
an economic model for the optimization of maintenance procedures in a pro-
duction process with two quality states. In addition to deteriorating with
age, the equipment may experience a jump to an out-of-control state (qual-
ity shift), which is characterized by lower production revenues and higher
tendency to failure. Panagiotidou and Nenes (2009) proposed a model for
the economic design of a variable-parameter Shewhart control chart. This
model is used to monitor the process mean, where, apart from quality shifts, failures may also occur.

A new cost named obsolescence cost was incorporated in Sun and Xi (2010) where the impact of tool-degraded state on the product quality was studied. The degraded state of a tool is set back to its nominal value through preventive or corrective maintenance. In general, most maintenance policies assume the replacement occurs with identical parts to the replaced tools but a new technology may appear and replace the old part even if this part was functioning well. The replacement can be done each time a corrective maintenance is undertaken. The old technology can be salvaged and the incurred cost for it and the new technology is called obsolescence cost. The Sun and Xi (2010) model minimized costs of maintenance, quality loss, and tool obsolescence cost.

Ben-Daya and Rahim (2000) developed a model that jointly optimize the economic design of process mean control chart, and the optimal maintenance level. The objective was to minimize the total cost rate.

$$Total\ Cost\ Rate = \frac{E(PM) + E(QC)}{E(T)}$$

In the formula, $E(PM)$ is the expected preventive maintenance cost, $E(QC)$ is the expected quality cost and $E(T)$ is the expected cycle time. The problem is then to simultaneously determine the optimal production run time, the optimal preventive maintenance level, and the optimal design parameters of
the process mean control chart (namely \( h_1; h_2; \cdots ; h_m \), the sample size \( n \), and the control limit coefficient \( k \)).

### 2.4 INTEGRATING MAINTENANCE, INVENTORY CONTROL AND QUALITY CONTROL

Three way integration between production, maintenance and quality comes as a natural extension to the two way integration. In the literature, some attempts have been made to model the integration of the three functions, production planning, maintenance and quality. Many models discussed joint integration between production and quality under different PM policies (Huang and Chiu, 1995; Makis and Fung, 1995; Tseng et al., 1998). Makis and Fung (1995) presented a model for the joint determination of the lot size, inspection interval and preventive replacement time for a production facility that was subject to a random failure. The time when the system was in-control was exponentially distributed and once it was out-of-control, a certain amount of defective items were produced. Inspecting the unit on a periodic basis was done to review the production process. After a certain number of production-runs, the production facility was replaced. This model was studied under the effect of machine failures in Makis and Fung (1998). Rahim and Ben-Daya (1998) presented a generalized model for a continuous production process for simultaneous determination of production quantity, inspection schedule
and control chart design, with a non-zero inspection time for false alarms. Rahim and Ben-Daya (2001) looked at the effect of deteriorating products and a deteriorating production process, on the optimal production quantity, inspection schedule and control chart design parameters. Radhoui et al. (2010) developed a joint quality control and preventive maintenance policy for a production system producing conforming and nonconforming units. Chelbi et al. (2008) proposed an integrated production-maintenance strategy for unreliable production systems producing conforming and non-conforming items that links EMQ, quality and an age-based preventive maintenance policy. Ben-Daya (1999) developed an integrated model for the joint optimization of the economic production quantity, the economic design of process mean control chart, and the optimal maintenance level. The system is a deteriorating process with increasing hazard rate and the in-control period follows a general probability distribution.

\[
\text{Total Cost Rate} = \frac{\text{Production Cost} + \text{Maintenance Cost} + \text{Quality Cost}}{\text{Expected Cycle Time}}
\]

\[
\text{Total Cost Rate} = \frac{S_0 + E(HC) + E(PM) + E(QC)}{E(T)}
\]

In the formula, \(S_0\) is the setup cost, \(E(HC)\) is the expected holding cost, \(E(PM)\) is the expected preventive maintenance cost, \(E(QC)\) is the expected quality cost, \(E(T)\) is the expected cycle time. The problem is then to simul-
taneously determine the optimal production run time, the optimal preventive maintenance level, and the optimal design parameters of the process mean control chart (namely sampling frequencies $h_1; h_2; \cdots ; h_m$, the sample size $n$, and the control limit coefficient $k$).

\section{2.5 Missing Integrated Models}

In the literature, production scheduling was integrated with maintenance scheduling resulting in cost savings. Also, quality was integrated with maintenance. As observed, there is a missing gap in the literature of integrating production scheduling and quality. The suggested integrated modeling will provide, simultaneously, optimal schedule for inspection and production. Inspectors may spend some time in testing products. This time will affect the production schedule. The higher number of inspections, the more delay production schedule will have. Moreover, scheduling models can be studied under the effect of deteriorating jobs, where the machine have the probability to shift to an out-of-control state during job processing.

As a natural evolution in the integrated models literature, researchers can look into the models where production, PM scheduling and quality can be integrated. In these models job processing times can be deteriorated due to the probability that system can shift to out-of control state. PM should be scheduled before the start of a job and will reduce the age of the machine.
This reduction in age will reduce probability of the machine to shift to out-of-control state during job processing. However, it will delay the start of the job due to the time needed to perform PM. This suggested integrated model will provide production, maintenance and inspection schedules.

2.6 CONCLUSIONS

This review emphasizes providing a better understanding of the interrelation and integration between production, maintenance and quality models. The key feature distinguishing this chapter from other reviews is the differentiation between the concept of interrelation and integration. Interrelated models are meant to describe models that optimize solely one function taking other function as a constraint. Another feature distinguishing this review is the inclusion of production scheduling in addition to production planning, maintenance and quality. To the best of our knowledge, there is no integrated model between scheduling and quality or scheduling, quality and maintenance.
Chapter 3

JOINT JOB SCHEDULING AND PREVENTIVE MAINTENANCE ON A SINGLE MACHINE

3.1 INTRODUCTION

Production scheduling focuses on allocating machine capacity to job processing, while maintenance scheduling focuses on maintaining machine capacity. These two problems are interrelated, in that machine interruptions cause delay in production schedules. However, this interrelatedness seems to be overlooked in the literature. Classical production models assume continuous machine availability, which might not be true in most real life manufacturing systems. A machine may become unavailable during the production process, due to preventive maintenance (PM), which can be scheduled in advance or due to random breakdowns. Recently, researchers addressed the need to integrate the scheduling of both production and maintenance. Kenne
et al. (2009) stated that the integrated production planning and preventive maintenance problem is concerned with coordinating production and maintenance operations to meet customer demand with the aim of minimizing cost. Pandey et al. (2010) pointed out that production scheduling and maintenance have been treated as separate issues. In real life situations, machines do fail or need to be maintained and hence may become unavailable during certain periods. Thus, the interdependency of scheduling and maintenance has resulted in a considerable amount of interest in developing models.

In recent years, few researchers have addressed this conflict by developing integrated models of production and maintenance scheduling. Many models have discussed the integrated optimization problem of production scheduling and PM for a single machine. Cassady and Kutanoglu (2003) compared the optimal total weighted tardiness under integrated production scheduling and PM planning. Their results indicated that there was an average of a 30% reduction in the expected total weighted tardiness when the production scheduling and PM were integrated. Thus, the need to integrate production scheduling with PM planning was clearly demonstrated. Similarly, Cassady and Kutanoglu (2005) considered an integrated model of job scheduling on a single machine and its preventive maintenance. The objective was to minimize the total weighted expected completion times. They assumed that the uptime of a machine follows a Weibull distribution such that the machine is minimally repaired when it fails and the maintenance restores the machine to an ‘as good as new’ state. Using weighted shortest processing
time (WSPT) they minimized the total weighted expected completion times for large size problems. An optimal maintenance interval was provided by maximizing the availability of the machine. They integrated job scheduling and PM on a single machine by introducing a binary variable representing whether or not to schedule PM before each job. The above mentioned-papers (Cassady and Kutanoglu, 2003, 2005) used the total enumeration method as the optimization technique. Aghezzaf et al. (2007) presented an integrated lot sizing and preventive maintenance strategy of the system that satisfies the demand for all items over the entire time horizon without backlogging, and which minimizes the expected sum of production and maintenance costs. They assumed that any maintenance action carried out on the system, within a period, reduces the system’s available production capacity during that period. Berrichi et al. (2009, 2010) introduced a new bi-objective approach for the joint production and maintenance-scheduling problem. Their aim was to simultaneously optimize two criteria: the minimization of the makespan for the production part and the minimization of the system unavailability for the maintenance side.

Clearly, integrated models have proven expected cost reductions over independent solutions of the two problems. Similar to Cassady and Kutanoglu (2005), this chapter finds the optimal integrated production schedule and PM plan to minimize the total weighted expected completion times of a fixed number of jobs processed on a single machine, and investigates how the optimal PM plan interacts with the optimal production schedule. Cassady and
Kutanoglu (2005) used the full enumeration procedure to solve the integrated problem. In their procedure, they fixed a production job sequence and then enumerated all possible maintenance schedules, which are $2^n$ maintenance schedules (where $n$ is the number of jobs). The minimum maintenance cost was found for the given job sequence. Then, the job sequence was altered and again all maintenance schedules were evaluated for the new job sequence to find the minimum maintenance cost. They continued in the previous method until all production sequences are evaluated. The minimum of all evaluated job sequences was considered to be the optimal solution. However, they did not fully utilize the integrated model developed in their paper.

The model in this chapter provides, simultaneously, the optimal schedules for production jobs and PM. The key feature distinguishing our model is the development of an explicit mathematical formulation for the problem that jointly integrates production scheduling and preventive maintenance scheduling. The decision variables of the model include both scheduling problems, which once solved, yield an optimum solution of the whole problem.

### 3.2 PROBLEM DEFINITION

Consider $n$ independent jobs ready for processing on a single machine that is subject to interruptions due to preventive maintenance and unexpected random failures. PM policy is adopted to increase machine availability that
restores the machine to an ‘as good as new’ state. When the machine fails, it is assumed that it is minimally repaired, i.e. the machine is restored to an operating condition, but the machine age is not altered. This implies that, upon machine failure, the machine operator does just enough maintenance to resume machine function. The assumption that PM restores the machine to ‘as good as new’ implies that PM is a more comprehensive action than repair. This may include the replacement of many key parts in the machine. It is assumed that jobs are not interrupted by PM. However, jobs interrupted by failure can be resumed after repair without any additional time penalty. The maintenance and repair times are assumed to be constant, $t_p$ and $t_r$ respectively. Time to failure for the machine $T$ is assumed to follow a Weibull probability distribution having shaping parameter $\beta$ greater than 1.

Some particular applications of the Weibull distributions associated with the process-failure mechanism can be seen in real life. The distribution of time to leakage failure of dry cell batteries can be approximated by a Weibull distribution (Berrittoni, 1964). For example, consider a typical manufacturing process with several mechanical and electrical control devices. Certain control devices require the continuous direct current output of dry cell batteries. When the battery fails, the process can drift into an out-of-control state and needs a repair that is assumed to be minimal. When choosing a job sequence, a decision is made whether or not to perform PM prior to each job. The integrated problem is further complicated by the fact that completion times for the jobs are stochastic, because the machine may or may not fail
during each job and PM decisions change the stochastic process governing machine failure.

3.3 FORMULATION

The single machine scheduling, with the objective of minimizing the weighted mean completion times, is modeled in classical scheduling theory as follows:

\[
\text{Min} \sum_{j=1}^{n} \left( \sum_{i=j}^{n} w_i \right) P_j
\]  

(3.1)

\[
\sum_{j=1}^{n} x_{ij} = 1 \quad \forall \quad i = 1, \ldots, n
\]  

(3.2)

\[
\sum_{i=1}^{n} x_{ij} = 1 \quad \forall \quad j = 1, \ldots, n
\]  

(3.3)

Where

\[
x_{ij} = \begin{cases} 
1 & \text{if the } i^{th} \text{ job performed is job } j \\
0 & \text{Otherwise}
\end{cases}
\]

To introduce the effect of random machine failures and the scheduling of
PM activities we followed an approach similar to Cassady and Kutanoglu (2005).

\[ y_i = \begin{cases} 
1 & \text{if PM is performed before job } i \\
0 & \text{Otherwise} 
\end{cases} \quad i = 1, 2, \cdots, n \]

Let \( N(\tau) \) be the number of failures that occur during a time interval \( \tau \) and \( E[N(P_i)] \) be the expected number of failures that occur while processing job \( i \).

Figure 3.1 presents a Gantt chart for the sequence \( j_1, j_2, \cdots, j_{n-1}, j_n \) with hatched area representing PM activity of times \( t_p \) and random machine breakdowns. It presents the notations used for modeling completion times in a schedule.

Figure 3.1: Gantt chart for the effect of preventive maintenance on a production schedule

Taking randomness into consideration for equation 3.1, the objective func-
tion becomes

$$\sum_{i=1}^{n} [w_i E(c_i)] = w_1 E(c_1) + w_2 E(c_2) \cdots + w_{n-1} E(c_{n-1}) + w_n E(c_n) \quad (3.4)$$

Since $c_i$ is a function of machine breakdowns, which is in turn a function of the age of the machine, we need to introduce a variable $a_{[i]}$ to represent the age at the completion of the $i^{th}$ job. Since PM renews the machine, the age of the machine at the end of processing the $i^{th}$ job is

$$a_{[i]} = (1 - y_i) a_{[i-1]} + P_{[i]} \quad i = 1, 2, \cdots, n \quad (3.5)$$

Given that job $i$ starts at age $(1 - y_i) a_{[i-1]}$ and ends at age $a_{[i]}$, the expected number of breakdowns during the processing of the $i^{th}$ job, assuming Weibull hazard rate, is

$$E[N(P_{[i]})] = \int_{(1-y_i) a_{[i-1]}}^{a_{[i]}} z(t) \, dt = \int_{1-y_i a_{[i-1]}}^{a_{[i]}} \frac{\beta}{\eta} t^{\beta-1} \, dt$$

which can be simplified to be

$$= m(a_{[i]}) - m((1 - y_i) a_{[i-1]}) = \left(\frac{a_{[i]}}{\eta}\right)^{\beta} - \left(\frac{(1-y_i) a_{[i-1]}}{\eta}\right)^{\beta}$$
The expected job completion time can be written as

\[ E[c_{[i]}] = \sum_{k=1}^{i} \{ t_p y_k + P_{[k]} + t_r [m(a_{[k]}) - m((1 - y_i)a_{[k-1]})] \} \]

\[ i = 1, 2, \ldots, n \] \hspace{1cm} (3.6)

The deterministic scheduling model presented earlier can now be modified to model the expected total completion time of the integrated problem as follows:

\[ \text{Minimize} \]

\[ \sum_{i=1}^{n} (w_{[i]} E [c_{[i]}]) = \sum_{i=1}^{n} \left\{ W_i \left( t_p y_i + \sum_{j=1}^{n} (P_j x_{ij}) + t_r [m(a_{[i]}) - m((1 - y_i)a_{[i-1]})] \right) \right\} \]

Subject to
\[
\sum_{j=1}^{n} x_{ij} = 1 \quad i = 1, 2, \ldots, n
\]
\[
\sum_{i=1}^{n} x_{ij} = 1 \quad j = 1, 2, \ldots, n
\]
\[
W_i = \sum_{k=i}^{n} \left( \sum_{j=1}^{n} (w_j x_{ij}) \right) \quad i = 1, 2, \ldots, n
\]
\[
a_{[i]} = (1 - y_i) a_{[i-1]} + \sum_{j=1}^{n} (P_j x_{ij}) \quad i = 1, 2, \ldots, n
\]
\[
a_{[0]} = C
\]
\[
x_{ij} \quad \text{binary} \quad i = 1, 2, \ldots, n \quad j = 1, 2, \ldots, n
\]
\[
y_i \quad \text{binary} \quad i = 1, 2, \ldots, n
\]

Where \(C\) is an arbitrary constant that represents the initial age of the machine at the beginning of the scheduling period.

The model has \(2n\) constraints and \(n(n + 1)\) variables. The solution of this model can be obtained using one of the available solvers that run under optimization programs such as GAMS and AMPL.

### 3.4 Example for Solving the Integrated Problem

Consider processing 3 jobs of processing times: 8, 48 and 41 units of time. Each job has a different weight: \(w_1 = 2\), \(w_2 = 10\), \(w_3 = 10\). Processing
starts when the machine has an age of $a_{[0]} = 88$, preventive maintenance time $t_p = 5$ and corrective maintenance time $t_r = 15$ units. The machine failure rate follows a Weibull distribution with parameters $\beta = 2, \eta = 100$. Upon failure a minimal repair is conducted with repair time $t_r = 15$.

GAMS language was used to input the model (12 variables and 6 constraints) and the BARON solver was used to find the optimal solution. The BARON solver is a computational system designed for solving non-convex NLP optimization problems to global optimality. As a result, the optimal joint solution will give $x_{13} = x_{21} = x_{32} = 1$ (corresponding to sequence $J_3 - J_1 - J_2$) and $y_1 = y_3 = 1$ (PM before first and last jobs) with $\sum_{i=1}^{3} w_i E[c_{[i]}] = 1740.993$ as shown in Figure 3.2.

![Gantt chart and the corresponding machine age for the optimal joint solution](image)
In the following, we will solve the same problem independently by first solving for optimal PM timing and then superimpose the solution on the scheduling problem. First, we will find the optimal PM interval that would maximize the availability. Machine availability is defined as the ratio of machine uptime $t$ to the total time that includes uptime, repair time and PM time.

\[
\text{Availability} = \frac{\text{uptime}}{\text{uptime} + \text{maintenance time} + \text{repair time}} = \frac{t}{t + t_r + t_c N(t)}
\]

(3.7)

For the Weibull distribution it can be shown that $t^*$ (the optimal time for PM) is

\[
t^* = \eta \left[ \frac{t_p}{t_r (\beta - 1)} \right]^{\left( \frac{1}{\beta} \right)}
\]

For the problem in the example, $t^* = 57.7$. This indicates that the machine should be maintained at age 57.7.

For the scheduling problem, it is well known that arranging jobs according to the weighted shortest processing time (WSPT) results in the optimal schedule ($J_1, J_3, J_2$). Since the machine initial age is 88, that is higher than
$t^*$, then PM is scheduled at time 0 and $J_1$ starts processing after the initial PM and completes at time 48.5. Similarly, $J_3$ and $J_2$ complete at 62.6 and 114.06, respectively. The total expected cost is 1746, which is higher than the integrated solution.

![Figure 3.3: Gantt chart and the corresponding machine age for independent solution](image)

Independent planning may provide optimal performance at the level of a specific function. Management usually looks at the production system as a whole, and separate optimal solutions may not provide an optimal solution for the whole system. Usually there is a global optimal that includes all major functions in the production system. This global optimal can only be achieved by integrating models for all different functions. Integrated production models are expected to deal with multiple objectives with a conflicting nature.
Hence, planning these elements independently will cause conflicts between functions. This disturbance can be avoided through integrated modeling.

3.5 CONCLUSIONS

Preventive maintenance is typically used to reduce random machine breakdowns while processing a scheduled job. Optimal scheduling of PM activities maximizes the availability of the machine for production process. However, PM schedules often conflict with the production schedule that is subject to adjustments. This yields a suboptimal schedule. The approach that emerged recently is to integrate these two scheduling problems to get a global optimal solution. In this work, production scheduling and maintenance operations, for a single machine, were integrated at the shop floor level. The objective was to find the job order sequence and maintenance decisions that would minimize the total expected weighted completion times.

A few researchers has developed an integrated model, including Cassady and Kutanoglu (2005). However, those models need to be further modified to give an integrated solution. The main contribution of this chapter is to develop a complete integrated model. An example is used to demonstrate the application of the model and the integrated solution. Research in integrated modeling still has great potential to contribute to, justified by the expected savings provided.
Chapter 4

AN INTEGRATED COST MODEL FOR PRODUCTION SCHEDULING AND PERFECT MAINTENANCE

4.1 INTRODUCTION

The integration between production elements has received attention in recent years. This integration is strongly justified by significant savings in operations costs; however, integrated models are not easy to solve. This difficulty is due the fact that integrated models deal with multiple conflicting objectives. In this work, production scheduling and maintenance operations, for a single machine, are integrated at the shop floor level. It is a common practice to schedule both of them independently, which is done through separate functional teams. The resulting plans of a specific function may disrupt the other function plans. For example, the maintenance function assigns scheduled shut-down intervals. These intervals will be communicated to the
production unit. The suggested maintenance intervals may maximize the machine availability, but they will affect production plans. Similarly, production schedulers may have the tendency to utilize machines to their full capacity to meet demand. Under this condition, productivity may increase, but machine availability will decrease due to having more breakdowns.

This chapter integrates, simultaneously, the decisions of preventive maintenance and job order sequencing for a single machine. The motivation for this work is encouraged by the need of many real life applications, such as the automotive industry (Aksoy and Ozturk, 2010). In addition, this integration is expected to provide a reduction in the total expected cost. Each job order $i$ consists of serving $Q_i$ work pieces on a single machine. The manufacturer is expected to deliver $n$ job orders with different processing times $P_i$. The machine is subjected, upon failure, to minimal repair action where each repair will cost the manufacturer $c_m$ and restores the machine to work with no improvement in its condition. The manufacturer can do a preventive maintenance major action only before the start of serving a job order. This action will cost the manufacturer $c_p$ and will restore the machine to an ‘as good as new’ condition, reducing the chance of machine breakdown during operation. All job order work pieces are released to the shop floor at time 0, hence; the manufacturer should consider the holding cost during the scheduling horizon. The holding cost (HC), minimal repair cost (MRC) and preventive maintenance cost (PMC) compromise the expected cost. Each job order sequence will change the average expected cycle costs. The objective is to find the job
order sequence and maintenance decisions that would minimize the expected cost.

As observed in Cassady and Kutanoglu (2003), integrated models had proved an expected reduction in cost that might reach up to a 30% savings, over treating the two problems independently. However, no analytical solutions were provided for these integrated models. Instead, a full enumeration for small size problems was suggested to solve the integrated problem (Cassady and Kutanoglu, 2003, 2005). Hence, the need for more analytical or near optimal solutions is highly appreciated. The previous models do not capture the direct costs associated with machine downtime (e.g. labor, parts, etc.) of machine failures and machine maintenance. Hence, the need for cost based models of the integrated problem is justified (Gribkovskaia et al., 2010; Khanra et al., 2010; Goswami et al., 2010; Sun-Lee and Yoon, 2010; Karamatsoukis and Kyriakidis, 2010). The use of mathematical modeling for the purpose of production scheduling or preventive maintenance planning is well established in the literature (Diaby, 2010a,b). Typically, preventive maintenance planning models are stochastic models designed to either maximise equipment availability or minimize equipment maintenance costs.

The focus of this chapter is to formulate a cost model that simultaneously considers maintenance, minimal repair and holding costs for several production jobs, with the objective to minimize the expected total costs. Total expected cycle cost includes expected maintenance cost, expected minimal
repair cost and expected holding cost. The organization of the rest of this article is as follows. Section 4.2 defines, in detail, the integrated problem and its assumptions. Sections 4.3 provide formulation of the integrated model as a mathematical program. Section 4.4 presents and solves an example for the integrated problem. Section 4.5 derives independent maintenance model and compares it with the integrated solution found in Section 4.4. Finally, concluding remarks are shown in Section 4.6.

4.2 PROBLEM DEFINITION

Consider \( n \) job orders to be processed on a single machine that is subject to a preventive maintenance (PM) requirement. These job orders are available at time zero with no precedence constraints. Each job order consists of processing \( Q_i \) work pieces. Moreover, the machine is available continuously along the time horizon unless a machine breakdown occurs during job processing. If so, a minimum repair is conducted that restores the machine to its condition prior to breakdown. Also, the interrupted job should resume the job after machine repair. Each breakdown will delay completion time of successive jobs by the needed time to repair \( t_r \) (assumed to be constant). To reduce the chance for machine breakdown, a PM activity can be performed before starting any job. If so, it will restore machine condition to ‘as good as new’. This PM activity will delay successive jobs by the time of PM \( t_p \) (assumed to be constant). The machine may or may not fail, causing the
completion time for each job to be stochastic. The PM decisions affect the stochastic process governing machine failure and hence change the expected value of job completion time $E(c_i)$.

Each maintenance action costs a fixed PM cost $c_p$. Similarly, each breakdown will cost a fixed minimal repair cost $c_m$. It is assumed that all jobs are released to the shop floor ready to process at the start of the schedule. Hence, given that $\bar{Q}_i$ is the average work-in-process inventory, each job order $i$ waiting on the shop floor will incur a holding cost per unit time $h\bar{Q}_i$ until the time when the machine completes job order $i$ that is, $E(c_i)$ and the total holding cost will be $hE(c_n)\sum_{i=1}^{n} \bar{Q}_i$. The details of these costs will be discussed in Section 4.3. The problem would be to identify a set of PM decisions, as well as a set of job sequencing decisions, in a way that reduces the expected total weighted completion times. Prior to the job starting, a decision has to be made whether to perform PM or not. If so, PM will take a constant time that will delay consequent jobs by such time $t_p$. If not, the job will start at the completion time of the previous job. However, the single machine can fail during job processing. The number of failures during job processing is strongly affected by the machine age, i.e. when the machine ages it has a higher probability to fail.

Furthermore, failures are randomly distributed over machine operation. Let $N(\tau)$ be the number of machine failures in $\tau$ time units of machine operation. Given that $\tau_{P_i}$ is the time units of machine operation over $P_i$, 

then \( E[N(\tau_{P_i})] \) is the expected number of failures during machine operation for job \( i \). For job \( i \), the PM will reduce \( E[N(\tau_{P_i})] \), but it will delay the start of job \( i \) by \( t_p \). This situation can be represented by a binary variable \( y_i \). Let

\[
y_i = \begin{cases} 
1 & \text{if PM is performed before job } i \\
0 & \text{otherwise} 
\end{cases} \quad i = 1, 2, \ldots, n \quad (4.1)
\]

Without loss of generality, Figure 4.1 represents a Gantt chart for job sequence \( j_1, j_2, \ldots, j_{n-1}, j_n \) with hatched area representing PM decisions \( y_i t_p \).

![Gantt Chart](image)

**Figure 4.1:** Gantt chart for the integrated problem

The following assumptions are considered in the problem

1. Jobs cannot be preempted for PM.
2. Jobs interrupted by failure can be resumed after repair without additional time penalty.
3. The machine has increasing hazard rate.
4. Upon failure, minimum repair is conducted and machine will resume with the same age.

5. PM restores the machine to a ‘as good as new’ condition.

6. Repair times are deterministic and known in advance.

7. The number of machine breakdowns is unknown (random variable).

8. The number of breakdowns does not depend on job type.

9. Machine breakdowns are independent.

10. Raw material for all job orders are released at the start of the schedule.

### 4.3 FORMULATION

Consider a single machine in a manufacturing system that is required to process a set of $n$ jobs, and suppose that preempting one job for another is not permitted. The purpose of production scheduling is to choose an optimal sequence for the jobs. Let

$$x_{ij} = \begin{cases} 
1 & \text{if the } i^{th} \text{ job performed is job } j \\
0 & \text{Otherwise}
\end{cases}$$

(4.2)
then

\[ P[i] = \sum_{j=1}^{n} (P[j]x_{ij}) \quad \forall \quad i = 1, \ldots, n \] (4.3)

Two logical sets of constraints emerge, the first set of constraints states that job \( i \) can not seize two positions at the same time, i.e.

\[ \sum_{j=1}^{n} x_{ij} = 1 \quad \forall \quad i = 1, \ldots, n \] (4.4)

The second set of constraints states that one position can not hold more than one job, i.e.

\[ \sum_{i=1}^{n} x_{ij} = 1 \quad \forall \quad j = 1, \ldots, n \] (4.5)

Example

Suppose a single machine has two jobs needed to be scheduled with parameters \( P_1 = 2 \) and \( P_2 = 4 \) then the single machine can have two possible sequences: either \( J_1 \) following \( J_2 \) or \( J_2 \) following \( J_1 \).

- Sequence 1-2, where \( J_1 \) follows \( J_2 \) is expressed with following variables:
  \( x_{11} = 1, \ x_{12} = 0, \ x_{21} = 0, \ x_{22} = 1 \) \( P[1] = 2 \) \( P[2] = 4 \) (see Figure 4.2)

- Sequence 2-1, where \( J_2 \) follows \( J_1 \) is expressed with following variables:
In addition to choosing a job sequence, it is also necessary to decide whether or not to perform PM prior to each job. The integrated problem is further complicated by the fact that completion times for the jobs are stochastic, because the machine may or may not fail during each job, and PM decisions change the stochastic process governing machine failure. The completion time of a job is a random variable that depends on the following:

- the age of the machine prior to processing the job
- the completion time for previous jobs
- the time to complete PM, and the PM decision
the job’s processing, the minimal repair time

the number of machine failures during the job

PM is assumed to restore the machine to an ‘as good as new’ condition i.e. machine age after PM will reduce to 0. Let

\[ a_{[i]} \] be the machine age at the completion of \( i^{th} \) job

then

\[ a_{[i]} = (1 - y_i)a_{[i-1]} + P_{[i]} \quad i = 1, 2, \cdots, n \]

Suppose the machine used to process the jobs is subject to failure, and the time to failure for the machine is governed by a Weibull probability distribution, having shaping parameter \( \beta \) greater than 1. When the machine fails, it is assumed that the machine is minimally repaired, i.e. the machine is restored to an operating condition, but machine age is not altered. This implies that, upon machine failure, the machine operator does just enough maintenance to resume machine function. The assumption that PM restores the machine to ‘as good as new’ implies that PM is a more comprehensive action that may include the replacement of many key parts in the machine. Also, the operation and maintenance of the machine (between two successive PM’s) can be modeled as a renewal process. Due to the minimal repair, the occurrence of failures during each cycle of the renewal process can be mod-
eled using a non homogeneous Poisson process. Given that the job \( i \) starts at age \((1 - y_i)a_{i-1}\) and ends at age \( a_i \) then

\[
E[N(\tau_{P[i]})] = \int_{(1-y_i)a_{i-1}}^{a_i} z(t) \, dt
\]

\[
= \int_{(1-y_i)a_{i-1}}^{a_i} \frac{\beta}{\eta} t^{\beta-1} \, dt
\]

\[
= m(a_i) - m((1 - y_i)a_{i-1})
\]

\[
= \left( \frac{a_i}{\eta} \right)^\beta - \left( \frac{(1-y_i)a_{i-1}}{\eta} \right)^\beta
\]

(4.6)

where \( z(t) \) corresponds to the hazard function to the underlying Weibull probability distribution and \( m(\tau) = \int_0^\tau z(t) \, dt = \int_0^\tau \frac{\beta}{\eta} t^{\beta-1} \, dt = \left( \frac{z}{\eta} \right)^\beta \).

Figures 4.4 and 4.5 present the effect of maintenance decision \( y_i \) on the \( E[N(\tau_{P[i]})] \), where it will be reduced from \( \int_{(1-y_i)a_{i-1}}^{a_i} z(t) \, dt = m(a_i) - m((1 - y_i)a_{i-1}) \) (if \( y_i = 0 \)) down to \( \int_0^{a_i} z(t) \, dt = m(P_i) \) (if \( y_i = 1 \)). However, the PM decision, \( y_i = 1 \), will delay completion time of job \( i \) by \( t_p \) time units.

The objective is to provide jobs sequence, as well as PM schedules to minimize the expected cost \( E(Cost) \).
The expected cycle cost includes expected holding cost \((HC)\), expected minimal repair cost \((MRC)\), and PM cost \((PMC)\). Each job order will represent processing one batch of size \(Q_i\). Raw materials for all job orders
are assumed to be released to the shop floor at time 0. Figure 4.6 shows the inventory level for job order $i$. The average inventory for a job order $i$ is a sequence dependent. Hence

$$Q_{[i]} = \sum_{j=1}^{n} (Q_{j} x_{ij}) \quad \forall \quad i = 1, \cdots, n$$  \hspace{1cm} (4.7)

The average holding quantity is equal to the sum of areas I and II (in Figure 4.6) over the $E(C_{[i]})$. i.e. $\bar{Q}_{[i]} = \frac{I+II}{E(C_{[i]})}$

Figure 4.6: Gantt chart and the corresponding inventory level
\[ \bar{Q}_{[i]} = \frac{Q_{[i]} \left( E(c_{[i]}) - \frac{[P_{[i]} + t_r m(a_{[i]}) - m((1 - y_k)a_{[i-1]})]}{2 E(c_{[i]})} \right)}{E(c_{[i]})} = \frac{Q_{[i]} \left( 1 - \frac{[P_{[i]} + t_r m(a_{[i]}) - m((1 - y_k)a_{[i-1]})]}{2 E(c_{[i]})} \right)}{} \] (4.8)

Figure 4.7 shows the expected completion times for job 1, 2, \( \cdots \), n.

\[ \begin{align*}
E[c_{[1]}] &= t_p y_1 + P_{[1]} + t_r \left[ m(a_{[1]}) - m((1 - y_1)a_{[0]}) \right] \\
E[c_{[2]}] &= E[c_{[2]}] + t_p y_2 + P_{[2]} + t_r \left[ m(a_{[2]}) - m((1 - y_2)a_{[1]}) \right] \\
&\vdots \\
E[c_{[n]}] &= E[c_{[n]}] + t_p y_n + P_{[n]} + t_r \left[ m(a_{[n]}) - m((1 - y_n)a_{[n-1]}) \right] \quad (4.9)
\end{align*} \]

The expected completion times for (Job \( i = 1, 2, \cdots, n \)) can be found by substituting in a recursive manner as follows.
A general formula for \( E[c_{[i]}] \) can be found as follows:

\[
E[c_{[i]}] = \sum_{k=1}^{i} \left\{ t_{py} + P_k + t_r E[N(t_{P_k})] \right\}
\]

\[
= \sum_{k=1}^{i} \left\{ t_{py} + P_k + t_r \left[ m(a_k) - m((1 - y_k)a_{k-1}) \right] \right\}
\]

\[
i = 1, 2, \cdots, n
\]

Hence,

\[
\bar{Q}_{[i]} = \frac{Q_{[i]} \left( E(c_{[i]}) - \frac{[P_i + t_r [m(a_{[i]}) - m((1 - y_{[i]}a_{[i-1]})]]]}{2} \right)}{E(c_{[i]})}
\]

\[
= Q_{[i]} \left\{ 1 - \frac{[P_i + t_r [m(a_{[i]}) - m((1 - y_{[i]}a_{[i-1]})]]]}{2 \sum_{k=1}^{i} (t_{py} + P_k + t_r [m(a_k) - m((1 - y_k)a_{k-1})])} \right\}
\]

(4.10)

The holding cost is usually estimated by holding cost unit \( h \), which is expressed in monetary unit per workpiece unit per time unit. Let \( E(c_{[n]}) \) be the expected time to complete all \( n \) jobs. Then the expected holding cost will be

\[
HC = h \times E(c_{[n]}) \times \sum_{i=1}^{n} \bar{Q}_{[i]}
\]

(4.11)

(4.12)

Given that \( c_m \) is the expected minimal repair cost per failure, then expected minimal repair will be

\[
MRC = c_m \sum_{i=1}^{n} \left[ m(a_{[i]}) - m((1 - y_i)a_{[i-1]}) \right]
\]

(4.13)
Given that $c_p$ is the cost for each PM then expected PM cost

$$PMC = c_p \sum_{i=1}^{n} y_i$$  \hspace{1cm} (4.14)

The expected total cost will be

$$E(Cost) = HC + MRC + PMC$$

$$= hE(c_{[a]}) \sum_{i=1}^{n} \bar{Q}_{[a]} + c_m \sum_{i=1}^{n} [m(a_{[i]}) - m((1 - y_i)a_{[i-1]})] + c_p \sum_{i=1}^{n} y_i$$ \hspace{1cm} (4.15)

Finally, figure 4.8 shows the resulting mathematical program for the integrated problem
Min \( \{ hE(c_{[n]} \sum_{i=1}^{n} Q_{[i]} + c_m \sum_{i=1}^{n} [m(a_{[i]}) - m((1-y_i)a_{[i-1]})]) + c_p \sum_{i=1}^{n} y_i \} \)

Subject to

\( P_{[i]} = \sum_{j=1}^{n} (P_j x_{ij}) \quad i = 1, 2, \ldots, n \)

\( Q_{[i]} = \sum_{j=1}^{n} (Q_j x_{ij}) \quad i = 1, \ldots, n \)

\( a_{[0]} = \) Initial machine age

\( a_{[i]} = (1-y_i)a_{[i-1]} + P_{[i]} \quad i = 1, 2, \ldots, n \)

\( E(c_{[n]}) = \sum_{k=1}^{n} \left\{ t_p y_k + P_{[k]} + t_r \left[ m(a_{[k]}) - m((1-y_k) a_{[k-1]}) \right] \right\} \)

\( Q_{[i]} = Q_{[i]} \left\{ 1 - \frac{[P_{[i]} + t_r \left[ m(a_{[i-1]}) - m((1-y_i) a_{[i-1]}) \right]]}{2 \sum_{k=1}^{i} (t_p y_k + P_{[k]} + t_r \left[ m(a_{[k]}) - m((1-y_k) a_{[k-1]}) \right])} \right\} \quad i = 1, 2, \ldots, n \)

\( \sum_{j=1}^{n} x_{ij} = 1 \quad i = 1, 2, \ldots, n \)

\( \sum_{i=1}^{n} x_{ij} = 1 \quad i = 1, 2, \ldots, n \)

\( x_{ij} \) binary \( i = 1, 2, \ldots, n \) \( j = 1, 2, \ldots, n \)

\( y_i \) binary \( i = 1, 2, \ldots, n \)

Figure 4.8: Integrated cost model
4.4 SOLVING THE INTEGRATED PROBLEM

The formulation shown in Section 4.3 can be solved through one of the mathematical programming languages. In the following example GAMS language was used to input the model and the BARON solver was used to reach the optimal solution. The BARON solver is a computational system designed for solving non-convex NLP optimization problems to global optimality. When $\beta > 1$, it may be practical to perform preventive maintenance on the machine in order to reduce the increasing risk of machine failure.

Table 4.1 considers processing three job orders consisting of $Q_1 = Q_2 = Q_3 = 500$ work piece. Job order 1 needs six minutes for each work piece. Job order 2 needs three minutes for each work piece. Job order 3 needs two minutes for each work piece. The machine age is $a_0 = 88$ hours. The preventive maintenance time is $t_p = 5$ hours. The machine failure rate follows a Weibull distribution with the following parameters $\beta = 2, \eta = 100$. Upon failure, a minimal repair is conducted with a repair time $t_r = 15$ hours. For the Weibull distribution $m(t) = \int_0^{t_1} \frac{\beta}{\eta^\beta} t^{(\beta-1)} dt = \left(\frac{t}{\eta}\right)^\beta. h = 1.50$ monetary unit/work piece/hour, $c_m = 500$ and $c_p = 500$. 
Table 4.1: Example parameters for the integrated cost model

<table>
<thead>
<tr>
<th>Job order</th>
<th>Job order size (work piece)</th>
<th>Processing time per work piece (minute)</th>
<th>Processing time per job order (hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500</td>
<td>6</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>500</td>
<td>3</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>500</td>
<td>2</td>
<td>16.66</td>
</tr>
</tbody>
</table>

\[
\text{Min} \left\{ hE(c_{3}) \sum_{i=1}^{3} Q_{[i]} + c_{m} \sum_{i=1}^{3} \left[ m(a_{[i]}) - m((1 - y_{i})a_{[i-1]}) \right] + c_{p} \sum_{i=1}^{3} y_{i} \right\}
\]
Subject to

\[ P_{[1]} = 53x_{11} + 26x_{12} + 18.67x_{13} \]
\[ P_{[2]} = 53x_{21} + 26x_{22} + 18.67x_{23} \]
\[ P_{[3]} = 53x_{31} + 26x_{32} + 18.67x_{33} \]
\[ Q_{[1]} = 500x_{11} + 500x_{12} + 500x_{13} \]
\[ Q_{[2]} = 500x_{21} + 500x_{22} + 500x_{23} \]
\[ Q_{[3]} = 500x_{31} + 500x_{32} + 500x_{33} \]
\[ a_{[0]} = 88 \]
\[ a_{[1]} = P_{[1]} + (1 - y_1)88 \]
\[ a_{[2]} = P_{[2]} + (1 - y_2) \left[ P_{[1]} + (1 - y_1)88 \right] \]
\[ a_{[3]} = P_{[3]} + (1 - y_3) \left[ P_{[2]} + (1 - y_2)P_{[1]} + (1 - y_1)88 \right] \]
\[ E(c_{[3]}) = \sum_{k=1}^{3} \left\{ tp'y_k + P_{[k]} + tr \left[ m(a_{[k]}) - m((1 - y_k)a_{[k-1]}) \right] \right\} \]
\[ \bar{Q}_1 = Q_1 \left\{ 1 - \frac{[P_1 + t_r (m(P_1) + (1 - y_1)) - m((1 - y_1))]}{2(t_p y_1 + t_r (m(P_1) + (1 - y_1)) - m((1 - y_1)))} \right\} \]

\[ \bar{Q}_2 = Q_2 \left\{ 1 - \frac{[P_2 + t_r (m(a_2) - m((1 - y_2) a_{[2]}))]}{2 \sum_{k=1}^2 (t_p y_k + t_r (m(a_k) - m((1 - y_k) a_{[k]})))} \right\} \]

\[ \bar{Q}_3 = Q_3 \left\{ 1 - \frac{[P_3 + t_r (m(a_3) - m((1 - y_3) a_{[3]}))]}{2 \sum_{k=1}^3 (t_p y_k + t_r (m(a_k) - m((1 - y_k) a_{[k]})))} \right\} \]

\[ x_{11} + x_{12} + x_{13} = 1 \]

\[ x_{21} + x_{22} + x_{23} = 1 \]

\[ x_{31} + x_{32} + x_{33} = 1 \]

\[ x_{11} + x_{21} + x_{31} = 1 \]

\[ x_{12} + x_{22} + x_{32} = 1 \]

\[ x_{13} + x_{23} + x_{33} = 1 \]

\[ \text{binary variables} \quad x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}, y_1, y_2, y_3 \]

Solution: \( x_{13} = x_{22} = x_{31} = 1 \) and \( x_{11} = x_{12} = x_{21} = x_{23} = x_{32} = x_{33} = 0 \) and \( y_1 = y_3 = 1 \) and \( y_2 = 0 \) with a total expected cost that is equal to $178,030. Hence, the optimal job and PM sequences result in Job order sequence \( J_3 - J_2 - J_1 \) and PM decisions \( y_1 = 0, y_2 = 1, y_3 = 0 \) will give the minimal cost. This means that to achieve the minimum expected cost, production should start with Job 3, PM, Job 2 then Job 1.

The effect of six parameters \( (\beta, t_p, t_r, c_m, c_p, h) \) over the expected cost is shown in the two tables 4.2 and 4.3. A \( 2^6 = 64 \) factorial design was used to generate 64 trials (32 trials for \( \beta = 2 \) and another 32 trials for \( \beta = 3 \)). Table 4.2 shows that the cost will increase with \( c_p, c_m \) and \( h \) increase. The
need for more PM can be seen with \( t_r = 30 \) hours compared to \( t_r = 15 \) hours. The machine will have more failures with the increase of \( \beta \), hence, PM is more economically justified with higher machine failures. This can be shown by comparing the costs with \( \beta = 3 \) and \( \beta = 2 \) where costs are less in Table 4.3.

The use of mathematical modeling for the purpose of production scheduling or preventive maintenance planning is well established in the literature. Typically, preventive maintenance planning models are stochastic models designed to either maximize equipment availability or minimize equipment maintenance costs. The next section will show how the integrated solution compares to the independent solution. The PM problem will be treated independently of the production sequence. The optimal PM interval is derived such that the machine availability will be maximized. The PM interval will be then superimposed on the production schedule that was found in example 1. The resulting cost will be compared to the integrated scheduling cost.
Table 4.2: Parameters effect on expected cost ($\beta = 2$)

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Table 4.3: Parameters effect on expected cost ($\beta = 3$)

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4.5 SOLUTION ANALYSIS

In Section 4.3, it was shown how maintenance decision $y_i$ can affect the $E[cost]$. This section will investigate the benefits of integrating the job scheduling and PM decisions by solving independently for the optimal PM, such that the availability of the machine is maximized. This is will show how do it compares to the optimal integrated job sequence.

Machine availability $A(\tau)$ is defined as the ratio of machine uptime $[0, \tau]$ to the total time that includes uptime, repair time and PM time.

$$A(\tau) = \frac{\text{uptime}}{\text{uptime+maintenance time+repair time}}$$

$$= \frac{\tau}{\tau + t_p + t_r N(\tau)}$$

$$= \frac{\tau}{\tau + t_p + t_r \left( \frac{x}{\eta} \right)^{\beta} - 0} \quad (4.16)$$

Hence, to find $\tau^*$, $A(\tau)$ is differentiated with respect to $\tau$ and equal it to zero.

$$\frac{dA(\tau)}{d\tau} = \frac{(\tau + t_p + t_r \left( \frac{x}{\eta} \right)^{\beta})^2 - \tau \left( 1 + \frac{dt_p}{d\tau} \tau^{(\beta-1)} \right)}{(\tau + t_p + t_r \left( \frac{x}{\eta} \right)^{\beta})^3}$$

$$= 0$$
\[ \tau + t_p + t_r \left( \frac{\tau}{\eta} \right)^\beta - \left( \tau + \beta t_r \left( \frac{\tau}{\eta} \right)^\beta \right) = 0 \]

\[ \tau = \eta \left[ \frac{t_p}{t_r(\beta-1)} \right]^{\left(\frac{1}{\beta}\right)} \]

For Example 1, \( \tau^* = 100 \left[ \frac{5}{15} \right]^{\left(\frac{1}{2}\right)} = 57.7 \) hours. This indicates that the machine should be maintained at the age of 57.7 hours. Since the initial machine age of 88 is higher than \( \tau^* \), then PM is scheduled at time 0 then it will start processing job 3 at age 0. The machine will finish job 3 at age 16.66 and job 2 at age 41.66. Now, if the machine reaches the age of 57.7 hours during job 1. There are two options:

- Schedule PM after job 1. The machine will finish job 1 at age 91.66 hours (see Figure 4.9).

- Schedule another PM before job 1. The machine will finish job 1 at age 53.75 hours (see Figure 4.10).

For the given sequence \((J_3 - J_2 - J_1)\) there are \(2^3 = 8\) PM decisions that are shown in the following table.
Figure 4.9: Gantt chart and the corresponding machine age for scheduling preventive maintenance before $J_3$ only

Figure 4.10: Gantt chart and the corresponding machine age for scheduling preventive maintenance before $J_1$ and before $J_3$

Table 4.4: PM effect job order sequence $J_3 - J_2 - J_1$

<table>
<thead>
<tr>
<th></th>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$y_3$</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>181,930</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>178,030</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>181,130</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>182,480</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>181,250</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>187,300</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>190,470</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>202,220</td>
</tr>
</tbody>
</table>
In table 4.4, the cost for the schedule in Figure 4.9 is 182,480 which is \( \frac{|182,480 - 178,030|}{178,030} = 2.50\% \) higher than the optimal integrated solution found. The cost for the schedule in Figure 4.10 is 181,250, which is \( \frac{|181,250 - 178,030|}{178,030} = 1.8\% \) higher than the optimal integrated solution found.

Independent planning may provide optimal performance at the level of a specific function. Usually there is a global optimal that includes all major functions in the production system. This global optimal can only be achieved by integrating models for all different functions. Integrated production models are expected to deal with multiple objectives with a conflicting nature. Hence, planning these elements independently will cause conflicts between functions. This disturbance can be avoided through integrated modeling.

\section*{4.6 CONCLUSIONS}

In this Chapter, production scheduling and maintenance operations for a single machine were integrated at the shop floor level. The objective was to find the job order sequence and maintenance decisions that would minimize the expected cost. The contribution of this work was to study the effect of production and maintenance scheduling on WIP inventory. The problem was formulated as a mixed integer program. The integrated solution simultaneously provided the production and maintenance schedule.

Integrated modeling is expected to provide better savings over indepen-
dent models; hence, integrated modeling has recently gained momentum. However, integrated models are sophisticated and are not easy to solve. Research in integrated modeling still has great potential to contribute, justified by the expected savings provided by integrated modeling.
In this chapter, we provide an overview for some of the recent work that combines costs of maintenance, scheduling and quality. Section 5.1 briefly discusses the work of Pandey et al. (2010c, 2011b) that integrates maintenance and quality. Then, Section 5.2 discusses the work of Pandey et al. (2010a, b, 2011a) which investigates the effect of integrating maintenance and quality over scheduling.

5.1 MAINTENANCE AND QUALITY INTEGRATION

Pandey et al. (2010c) included rejection cost for nonconforming items into corrective maintenance cost. In their model, a machine has a single component that is subject to two random failure modes, namely $FC_1$ and
Machine ceases production immediately after $FC_1$ while it keeps producing nonconforming items when $FC_2$ occurs. For example, in case of a computer numerical controlled (CNC) grinding machine, if work head belt is broken, then it will stop the machine completely, while if loosening of ball screw chuck nut occurs, then it will continue producing undesired oval-shaped components. After the occurrence of $FC_2$, a shift in the process mean will continue increasing until it is detected and maintenance conducted for restoring the process back to normal. Amount of defective products are estimated based on the deviation ($\delta$) from a standard mean $\mu_0$ for a certain quality characteristic. The quality of the machine is defined by its ability to produce products with minimum deviation from the standard mean which is a function of its age. A control chart mechanism is used to detect the process shift due to machine aging. For the quality characteristic, the upper specification limit, lower specification limit and target value are known. $\bar{X}$-chart is used to detect the production process which operates with a known and constant standard deviation $\sigma$. After a PM, production starts in a in-control state and with time the process mean $\bar{x}$ starts to deviate away from $\mu_0$. The deviation (process shift) is expressed by the amount $\delta \sigma$ and it affects the proportion of nonconforming items.

Preventive maintenance is introduced to the machine to minimize the cost per unit time (CPUT). The planner should define time between PM’s (TBM) which minimize (CPUT). Total cost includes preventive maintenance costs
in addition to corrective maintenance cost.

\[
CPUT = \frac{N_{CM} \times C_{CM} + N_{PM} \times C_{PM}}{T}
\]

Where \( T \) is the evaluation period, \((C_{PM})\) is the cost of PM and \((C_{CM})\) is the cost of corrective maintenance. \( N_{PM} \) is the number of PM’s and \( N_{CM} \) is the number of corrective maintenance.

Pandey et al. (2010c) studied two models \( M_1 \) and \( M_2 \):

1. Model \( M_1 \): Expected total cost model based on independent models for calculating maintenance and rejection costs.

2. Model \( M_2 \): Expected total cost model based on integrating rejection cost in the optimal maintenance model.

They assumed the maintenance to be imperfect. In this section we relaxed this assumption and reproduced the results with the assumption that PM restores the machine to “as good as new” condition. The machine age will be zero after PM and increase till the age of \( TBM \), which occurs before the next PM.

In \( M_1 \), CPUT is calculated based on cost of repair/replacement and downtime cost, as follows:

\[
E[C_{CM}]_{M_1} = MTTR_{CM} \cdot [PR.C_{lp} + LC] + C_{Rep} \quad (5.1)
\]

\[
E[C_{PM}]_{M_1} = MTTR_{PM} \cdot [PR.C_{lp} + LC] + C_{Res} \quad (5.2)
\]
Where $MTTR_{CM}$ and $MTTR_{PM}$ are the mean time to repair for corrective and for preventive maintenance. $PR$ is production rate, $C_{lp}$ is cost of lost production, $LC$ is labor cost, $C_{Rep}$ is repair cost, and $C_{Res}$ is restoration cost.

Let $TBM^*$ be the time between maintenance that maximize machine availability.

$$TBM^* = \eta \left[ \frac{MTTR_{PM}}{MTTR_{CM} (\beta - 1)} \right]^{\frac{1}{\beta}}$$  \hspace{1cm} (5.3)$$

$$E[N_{PM}] = \frac{T}{TBM^*}$$
$$E[N_{CM}] = \frac{T \beta}{\eta^{\beta - 1}}$$  \hspace{1cm} (5.4)$$

The expected annual total cost of maintenance action during planning period will be

$$E[C_f]_{M1} = E[N_{CM}] \times E[C_{CM}]_{M1} + E[N_{PM}] \times E[C_{PM}]_{M1}$$
$$= E[N_{CM}] \times [MTTR_{CM} \times PR.C_{lp} + LC] + C_{Rep} + E[N_{PM}] \times [MTTR_{PM} \times PR.C_{lp} + LC] + C_{Res}$$  \hspace{1cm} (5.5)$$
Thus the expected total cost of model $M_1$

$$[ETC_{M+Q}]_{M_1} = CPUT_{TBM} \times T + C_{Rejection} \cdot E[N_{defect}] \cdot E[N_{CM}] \cdot P_{FM_2} \quad (5.6)$$

Where $P_{FM_2}$ is the probability to experience failure mode that causes quality drift and $C_{Rejection}$ is the cost of rejection.

In $M_1$, rejection cost was added independently to $CPUT_{TBM}$. In $M_2$, $CPUT$ will be calculated through integrating directly the rejection cost to corrective maintenance cost $E[C_{CM}]_{M_2}$ as follows:

$$E[C_{CM}]_{FM1} = \text{MTTR}_{CM}(PR.C\text{lp} + LC) + C_{Rep}$$

$$E[C_{CM}]_{FM2} = E[C_{Rej}] + \text{MTTR}_{CM}(PR.C\text{lp} + LC) + C_{Rep}$$

$$E[C_{CM}]_{M_2} = E[C_{CM}]_{FM1} \times P_{FM_1} + E[C_{CM}]_{FM2} \times P_{FM_2} \quad (5.7)$$

Hence,

$$CPUT_{M_2} = \frac{E[N_{CM}] \times E[C_{CM}] + E[N_{PM}] \times E[C_{PM}]}{T}$$

$$= \frac{E[N_{CM}] \times E[C_{CM}] + E[N_{PM}] \times [\text{MTTR}_{PM}(PR.C\text{lp} + LC) + C_{Res}]}{T} \quad (5.8)$$

$$[ETC_{M+Q}]_{M_2} = CPUT_{M_2} \cdot T \quad (5.9)$$
Let $P_{FM_2} = 0.4$, $\beta = 2$, $\eta = 1000$, $MTTR_{CM} = 12h$ and $MTTR_{PM} = 7h$. The machine is supposed to operate three 7-hour-shifts a day and 6 days a week.

$$TBM = \eta \left( \frac{MTTR_{PM}}{MTTR_{CM}(1-\beta)} \right)^{(1/\beta)} = 764$$

$$E[N_{PM}] = \frac{3 \times 7 \times 6 \times 52}{764} = 8.6$$

$$E[N_{CM}] = E[N_{PM}] \left( \frac{TBM}{\eta} \right)^{\beta} = 5$$

Table 5.2 shows $[ETC_{M+Q}]_{M_1}$ and $[ETC_{M+Q}]_{M_2}$ for different quality policies shown in Table 5.1.

<table>
<thead>
<tr>
<th>Policy</th>
<th>$\delta$</th>
<th>LCL</th>
<th>UCL</th>
<th>n</th>
<th>TBS</th>
<th>$\beta$</th>
<th>$E[N_{Out-of-control}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>PR=45</td>
</tr>
<tr>
<td>P1</td>
<td>0.6</td>
<td>-2</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>78.75%</td>
<td>14</td>
</tr>
<tr>
<td>P2</td>
<td>0.6</td>
<td>-2</td>
<td>2</td>
<td>6</td>
<td>8</td>
<td>70.18%</td>
<td>10</td>
</tr>
<tr>
<td>P3</td>
<td>0.6</td>
<td>-2</td>
<td>2</td>
<td>4</td>
<td>16</td>
<td>78.75%</td>
<td>28</td>
</tr>
<tr>
<td>P4</td>
<td>0.6</td>
<td>-2</td>
<td>2</td>
<td>6</td>
<td>16</td>
<td>70.18%</td>
<td>20</td>
</tr>
<tr>
<td>P5</td>
<td>0.6</td>
<td>-3</td>
<td>3</td>
<td>4</td>
<td>8</td>
<td>96.41%</td>
<td>83</td>
</tr>
<tr>
<td>P6</td>
<td>0.6</td>
<td>-3</td>
<td>3</td>
<td>6</td>
<td>8</td>
<td>93.70%</td>
<td>47</td>
</tr>
<tr>
<td>P7</td>
<td>0.6</td>
<td>-3</td>
<td>3</td>
<td>4</td>
<td>16</td>
<td>96.41%</td>
<td>167</td>
</tr>
<tr>
<td>P8</td>
<td>0.6</td>
<td>-3</td>
<td>3</td>
<td>6</td>
<td>16</td>
<td>93.70%</td>
<td>95</td>
</tr>
</tbody>
</table>
Table 5.2: Total cost for model 1 and 2 under different quality policies

<table>
<thead>
<tr>
<th>PR</th>
<th>[ETC_{M+Q}]_{M_1}</th>
<th>[ETC_{M+Q}]_{M_2}</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>11,369,480</td>
<td>15,132,618</td>
</tr>
<tr>
<td>60</td>
<td>22,738,961</td>
<td>22,418,694</td>
</tr>
<tr>
<td>90</td>
<td>10,504,759</td>
<td>10,344,626</td>
</tr>
<tr>
<td>45</td>
<td>11,209,347</td>
<td>14,952,885</td>
</tr>
<tr>
<td>60</td>
<td>22,419,361</td>
<td>23,219,641</td>
</tr>
<tr>
<td>90</td>
<td>10,744,960</td>
<td>10,744,960</td>
</tr>
<tr>
<td>45</td>
<td>14,131,784</td>
<td>18,855,723</td>
</tr>
<tr>
<td>60</td>
<td>28,303,601</td>
<td>33,259,733</td>
</tr>
<tr>
<td>90</td>
<td>17,702,761</td>
<td>22,186,500</td>
</tr>
<tr>
<td>45</td>
<td>12,690,582</td>
<td>16,934,120</td>
</tr>
<tr>
<td>60</td>
<td>25,421,197</td>
<td>33,259,733</td>
</tr>
<tr>
<td>90</td>
<td>15,781,159</td>
<td>23,691,755</td>
</tr>
<tr>
<td>45</td>
<td>17,494,588</td>
<td>23,339,461</td>
</tr>
<tr>
<td>60</td>
<td>34,989,175</td>
<td>23,339,461</td>
</tr>
<tr>
<td>90</td>
<td>16,629,867</td>
<td>22,186,500</td>
</tr>
<tr>
<td>45</td>
<td>14,612,184</td>
<td>19,496,257</td>
</tr>
<tr>
<td>60</td>
<td>29,264,402</td>
<td>29,264,402</td>
</tr>
<tr>
<td>90</td>
<td>13,747,463</td>
<td>22,186,500</td>
</tr>
<tr>
<td>45</td>
<td>14,131,784</td>
<td>18,855,723</td>
</tr>
<tr>
<td>60</td>
<td>28,303,601</td>
<td>33,259,733</td>
</tr>
<tr>
<td>90</td>
<td>17,702,761</td>
<td>22,186,500</td>
</tr>
<tr>
<td>45</td>
<td>12,690,582</td>
<td>16,934,120</td>
</tr>
<tr>
<td>60</td>
<td>25,421,197</td>
<td>33,259,733</td>
</tr>
<tr>
<td>90</td>
<td>15,781,159</td>
<td>23,691,755</td>
</tr>
<tr>
<td>45</td>
<td>17,494,588</td>
<td>23,339,461</td>
</tr>
<tr>
<td>60</td>
<td>34,989,175</td>
<td>23,339,461</td>
</tr>
<tr>
<td>90</td>
<td>16,629,867</td>
<td>22,186,500</td>
</tr>
<tr>
<td>45</td>
<td>14,612,184</td>
<td>19,496,257</td>
</tr>
<tr>
<td>60</td>
<td>29,264,402</td>
<td>29,264,402</td>
</tr>
<tr>
<td>90</td>
<td>13,747,463</td>
<td>22,186,500</td>
</tr>
</tbody>
</table>

**Note:** The table entries are costs in thousands. C_{rej} = 20,000 and C_{lp} = 2,000 for the first set of costs, and C_{rej} = 25,000 and C_{lp} = 1,000 for the second set of costs. The last set of costs has C_{rej} = 25,000 and C_{lp} = 700.
By comparing results in table 5.2, $[ETC_{M+Q}]_{M_2}$ shows savings over $[ETC_{M+Q}]_{M_1}$. Same as Pandey et al. (2010c), it is observed that the proposed approach by them still gives better results compared to the conventional approaches that do not consider the cost of rejection and when the maintenance is perfect as we assumed.

5.2 MAINTENANCE, QUALITY CONTROL DECISIONS, AND SCHEDULING

In Pandey et al. (2010a,b), the decision of when to do maintenance $t_{pm}$ was integrated with the decisions of SPC quality control chart design parameters (Duncan’s mode) i.e. sample size $n_s$, time between samples $h$, and the number of standard deviations of the sampling distribution between the center line of the control chart and the control limits $k$. The objective is to simultaneously determine optimal values of $n_s$, $h$, $k$, and $t_{pm}$ that minimizes the cost of maintenance and quality control. The total expected cost is associated with poor quality, inspection/sampling, corrective/preventive maintenance and process downtime. Pandey et al. (2010a,b) superimposed the optimal preventive maintenance interval obtained onto the production-schedule to determine optimal job sequences that will minimize penalty cost incurred due to schedule delay.

In Pandey et al. (2010a,b), PM was assumed to be imperfect. We relaxed this assumption and reproduced the results. The details of the relaxed model
are described in the following.

The expected process quality control cost for the evaluation period is given as

$$E[TCQ]_{Process\ Failure} = E[C_{process}] \times \frac{T_{eval}}{E[T_{cycle}]} \quad (5.10)$$

We need to calculate the $E[C_{process}]$ and $E[T_{cycle}]$ for the quality control chart. The cycle length consists of the in-control $E[T_1]$ and the out-of-control time and process resetting or machine restoration time.

$$E[T_1] = \frac{1}{\lambda} + T_0 \alpha \sum_{i=0}^{\infty} i[e^{-\lambda h_i} - e^{-\lambda h_{i+1}}] \quad (5.11)$$

Where $\alpha$ is type I error and $\beta_\delta$ is type II error for $\delta$ shift in mean. $T_0$ is the time spent to search for false alarm.

The out of control time $E[T_2]$ consists of the expected time of the following events:

- the time between occurrence of an assignable cause and the next sample
- the expected time to trigger an out-of-control signal
- the expected time to plot and chart a sample $nT_s$ where $T_s$ is time
needed to sample and chart one item.

- the expected time to validate the assignable cause $T_1$
- the expected time to reset the process if failure is due to external reasons $T_{\text{reset}}$ or the expected time to restore the machine $MT_{CM}$ if failure is due to $FM_2$

The out-of-control time $E[T_2]$ can be expressed as follows

$$E[T_2] = h \times \left( ARL_{2mc} \frac{\lambda_2}{\bar{X}} + ARL_{E} \frac{\lambda_1}{\bar{X}} \right) - \tau$$
$$+ n_s T_s + T_1 + \left( T_{\text{reset}} \frac{\lambda_1}{\bar{X}} + MT_{CM} \frac{\lambda_2}{\bar{X}} \right)$$  \hspace{1cm} (5.12)

For the evaluation period $T_{\text{eval}}$, the expected process quality control cost will be

$$E[T_{\text{cycle}}] = E[T_1] + E[T_2]$$
$$= \frac{1}{\lambda} + T_0 \times \frac{s}{ARL_{1}} + h \times \left( ARL_{2mc} \frac{\lambda_2}{\bar{X}} + ARL_{E} \frac{\lambda_1}{\bar{X}} \right) - \tau +$$
$$n_s T_s + T_1 + \left( T_{\text{reset}} \frac{\lambda_1}{\bar{X}} + MT_{CM} \frac{\lambda_2}{\bar{X}} \right)$$  \hspace{1cm} (5.13)
\[ E[TCQ] = E[C_{cycle}] \times \frac{T_{eval}}{E[T_{cycle}]} \] (5.14)

The process quality cost \( E[TCQ] \) consists of the following components: the cost of rejection incurred while operating process in-control \( C_I \) and out of control \( C_o \), the cost of sampling, and the cost of evaluating the false alarm and true alarm and cost of resetting or restoring (through corrective maintenance) the process.

The expected total cost per unit time of preventive maintenance and process quality control chart policy \( ECPUT_{M*Q} \) is the ratio of the sum of the expected total cost of the process quality control \( E[TCQ] \), expected total cost of preventive maintenance \( E[C_{pm}] \) and expected total cost of machine failure \( E[C_{CM}]_{FM_1} \) to the evaluation time.

\[ ECPUT_{M*Q} = \frac{E[C_{CM}]_{FM_1} + E[C_{pm}] + E[TCQ]}{T_{eval}} \] (5.15)

Example:

A case of camshaft from an industrial context is presented. Most automotive manufactures want to have a camshaft that optimizes the performance level they need at a given revolution per minute. This requires proper ma-
chining of the camshaft as per the specifications. Cam shaft diameter is an important quality characteristic. Hence, an $\bar{x}$ control chart is used to monitor the manufacturing process. The initial values of relevant parameters are presented in table 5.3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>value</th>
<th>Parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_E$</td>
<td>1.5</td>
<td>$\delta_{mc}$</td>
<td>0.6</td>
</tr>
<tr>
<td>$T_s$</td>
<td>1/3</td>
<td>$C_F$</td>
<td>100</td>
</tr>
<tr>
<td>$C_{v}$</td>
<td>50</td>
<td>$C_f$</td>
<td>1200</td>
</tr>
<tr>
<td>$C_{rej}$</td>
<td>3000</td>
<td>$C_{FCPCM}$</td>
<td>10000</td>
</tr>
<tr>
<td>$C_{reset}$</td>
<td>5000</td>
<td>$T_0$</td>
<td>1</td>
</tr>
<tr>
<td>$T_1$</td>
<td>1</td>
<td>$T_{reset}$</td>
<td>2</td>
</tr>
<tr>
<td>$PR$</td>
<td>10</td>
<td>$C_{lp}$</td>
<td>400</td>
</tr>
<tr>
<td>$LC$</td>
<td>500</td>
<td>$C_{FCPPM}$</td>
<td>1000</td>
</tr>
</tbody>
</table>

The global optimization tool box of Maple 12 has been used to solve the optimization problem. The optimal values of decision variables that minimize the expected total cost of system per unit time $ECPUT_{M*Q}$ are as follows $h^* = 22.47$, $k^* = 5.63$, $n^*_s = 8.78$, $t^*_{PM} = 285.35$ and $ECPUT_{M*Q}^* = 108.74$. The main decision parameters are to determine inspection timing $h$ and preventive maintenance timing $t_{PM}$.

After finding the optimal time between maintenance, it will be superimposed on the batch sequence. A full enumeration will be performed before finding the optimal sequence. Consider a single machine that processes three batches using the presented (in table 5.3) processing time, setup time, penalty cost, due date (DD), and other production parameters.
Table 5.4: Parameters for batches

<table>
<thead>
<tr>
<th>Batch</th>
<th>P (min)</th>
<th>size</th>
<th>setup time</th>
<th>total time</th>
<th>Release</th>
<th>DD</th>
<th>P_i</th>
<th>holding cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>500</td>
<td>3</td>
<td>53</td>
<td>0</td>
<td>100</td>
<td>75</td>
<td>1.71</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>500</td>
<td>1</td>
<td>26</td>
<td>0</td>
<td>50</td>
<td>50</td>
<td>1.71</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>500</td>
<td>2</td>
<td>18.66</td>
<td>0</td>
<td>40</td>
<td>45</td>
<td>1.71</td>
</tr>
</tbody>
</table>

The following costs are occurring and results in $CPUT_S$ as follows:

Tardiness = $\sum P_i \left[ \max(0, C_i - d_i) \right]$

Inventory cost = $\sum h_i \bar{Q}_i C_i$

$$CPUT_S = \frac{Tardiness\text{cost} + Inventory\text{cost}}{C_{\text{max}}}$$

Table 5.5 shows the cost of the different six sequences where sequence 3 − 2 − 1 gives the least cost.

Table 5.5: Possible batches sequences

<table>
<thead>
<tr>
<th>sequence</th>
<th>C_1</th>
<th>C_2</th>
<th>C_3</th>
<th>T_1</th>
<th>T_2</th>
<th>T_3</th>
<th>Penalty</th>
<th>Inventory</th>
<th>CPUT_S</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2-3</td>
<td>53</td>
<td>79</td>
<td>98</td>
<td>0</td>
<td>29</td>
<td>58</td>
<td>4060</td>
<td>333450</td>
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<tr>
<td>1-3-2</td>
<td>53</td>
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<td>72</td>
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<td>0</td>
<td>0</td>
<td>275310</td>
<td>2809.29</td>
</tr>
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</table>

This section reproduced the work of Pandey et al. (2010b,c) with the assumption of a perfect maintenance. The optimal PM interval that resulted from the integrated model of maintenance and quality is superimposed on the production schedule. A full enumeration for all possible combinations was conducted to find where to place the PM job. The results with perfect PM,
same as Pandey et al. (2010b,c), still advocates for the three way integration between scheduling, maintenance and quality control.

5.3 CONCLUSIONS

In this chapter, we tested the integration between maintenance and quality control. In the first model, we tested the inclusion of maintenance into the rejection cost due to quality deviation. The model studies the effect of rejection cost on optimal maintenance planning decision. In the second model, we tested the integration between economic design of control chart with determining optimal timing for PM. The timing was superimposed on a three jobs schedule and a full enumeration for the six sequences has been explored to determine the minimal cost. In the two sections, the integration is highly justified between scheduling, maintenance and quality control.
Chapter 6

CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

6.1 SUMMARY

This dissertation considered the integration of scheduling of production, maintenance and quality control functions. Chapter 2 of this dissertation presented a comprehensive review for the state-of-the-art framework of the literature and identified gaps for further investigation. A framework for classifying integrated models into interrelated and integrated models is presented. Models that consider one function with the effect of other function as a constraint is classified as “interrelated”. Models that jointly considers the decisions of two functions or more are classified as “integrated” models.

Chapter 3 of the dissertation studied integrating job scheduling and preventive maintenance (PM) on a single machine so that the expected total job completion times is minimized. The machine is subjected to random
breakdowns that follow an increasing failure hazard rate. If the machine fails while processing a job, it is subjected to a minimal repair action that brings the machine back to operation without altering its age. The problem was formulated as a mixed integer program that once solved provides a schedule that combines job processing and PM.

The same problem is considered in the Chapter 4 of the dissertation with the objective of minimizing the total cost of PM, repair, and work in process inventory. Maintenance cost is affected by the frequency of performing preventive maintenance, where in each time a fixed cost occurs. The repair cost and inventory cost were considered. The problem was formulated as a mixed integer program that once solved provides a schedule that combines PM and job processing.

6.2 DISSERTATION CONTRIBUTION

The main contributions of this dissertation are the following:

1. A comprehensive review for integrated modeling literature is provided. The review differentiates between the models depending on the integration level. This review provides researchers with a well structured overview of the literature that helps in identifying gaps that are still open for research and their relation to other well studied areas.

2. A mixed integer program was proposed to solve the integrated schedul-
ing of job processing and preventive maintenance operations. The objective function was to minimize the expected total weighted completion times. An explicit formula was proposed to the objective function that include decision variables related to job scheduling as well as preventive maintenance. The results of this part of the dissertation advance research of integration of production and maintenance one step ahead.

3. A model that captures costs of preventive maintenance, minimal repair and work in process holding cost was proposed. The problem was formulated and solved as a mixed integer program. This model brings integration in the context of economic optimization which is a major concern in real life applications.
6.3 FUTURE RESEARCH DIRECTIONS

The area of integrated modeling is still wide open for further research in different dimensions. From a production scheduling point of view, it would be of great interest to consider multiple machines with different types of shops like jobshop, flowshop and parallel machines. From a maintenance point of view, different polices or different maintenance actions can be explored. Such an enhancement to the current models would help well with the advanced problems faced by todays manufacturers. The following suggestions are also worth exploring:

1. With the rise of globalization and the concept of open markets, most plants are producing different types of products for different markets. As such, many issues can be considered such as:

   (a) The pricing of these products becomes an issue and should be connected to the production, maintenance and quality costs.

   (b) Logistics can be included and the maintenance of the fleet can be part of the model.

   (c) Integrated models can be studied with the trends of outsourcing policies for maintenance and productions of subcomponents.

2. Integrated models can also be extended to other job related performance measures, such as meeting due dates or multi performance criteria.
3. The assumption of constant maintenance and repair times can be generalized to be random variables following a certain distribution. The model can be studied under the effect of different repair and preventive maintenance time random distributions.

4. In many cases, the machine is drastically affected by the type of job it performs. This effect can be investigated in future work.

5. The inventory consumption was assumed to be linearly affected by processing times. The model should perhaps be tested with non linear inventory consumption, as well.
Appendix A

GAMS code for minimizing $EWCT$

sets
   i positions  / 1*3/  
   j jobs        / 1*3/  
alias(i,f);
Parameters
   p(j) process time j
       /  
       1  8  
       2  48  
       3  41  
       / 
   w(j) weight j
       / 
       1  2  
       2  10
3 10 
/
Scalar beta shape factor in failure distribution /2/;
Scalar theta scale factor in failure distribution /100/;
Scalar age starting machine age /88/;
Scalar tp preventive maintenance time /5/;
Scalar tr repair time /15/;
Variables
   x(i,j) sequence decision
   y(i) maintenance decision
   a(i) machine ages
   a_bar_0 machine starting effective age
   a_bar(i) machine effective ages
   m_a(i) integrated hazard for machine ages
   m_a_bar_0 integrated hazard for machine starting effective ages
   m_a_bar(i) integrated hazard for machine effective ages
   ex_p(i) expected processing time
   pos_p(i) process time in position i
   pos_w(i) weight in position i
   z total expected completion time;

Binary Variable x;
Binary Variable y;
Positive Variable a;
Positive Variable a_bar;
Positive Variable a_bar_0;
Positive Variable m_a;
Positive Variable m_a_bar;
Positive Variable m_a_bar_0;
Positive Variable ex_p;
Positive Variable pos_p;
Positive Variable pos_w;

Equations

cost define objective function

c1(i) position process

c2(i) position weight

c3 age after first job

c4(i) age after each job after i=1

c5 effective age before job 1

c6(i) age after each job

c7 integrate hazard for effective initial machine age

c8(i) integrate hazard for effective age before each job

c9(i) integrate hazard for effective age after each job

c10 expected processing time for first job

c11(i) expected processing time for jobs after first job
c12(j) one job only will occupy each position
c13(i) one position only for one job;

cost .. z =e= \sum(i, \sum(f$(\text{ord}(f)\geq \text{ord}(i)), pos_w(f)) * (y(i) * tp + ex_p(i))) ;

c1(i) .. pos_p(i) =e= \sum(j, x(i,j) * p(j));
c2(i) .. pos_w(i) =e= \sum(j, x(i,j) * w(j));
c3 .. a('1') =e= \text{pos}_p('1') + (1-y('1'))*age;
c4(i-1) .. a(i) =e= \text{pos}_p(i) + \text{a_bar}(i-1);
c5 .. \text{a_bar}_0 =e= (1-y('1'))*age;
c6(i-1) .. \text{a_bar}(i-1) =e= (1-y(i)) * a(i-1);
c7 .. \text{m_a_bar}_0 =e= \text{(a_bar}_0/\theta)\text{**beta};
c8(i-1) .. \text{m_a_bar}(i-1) =e= \text{(a_bar}(i-1)/\theta)\text{**beta};
c9(i) .. \text{m_a}(i) =e= \text{(a(i)/\theta)\text{**beta};}
c10 .. \text{ex_p('1')} =e= \text{pos}_p('1') + \text{tr}*(\text{m_a('1')-m_a_bar_0});
c11(i-1) .. \text{ex_p(i)} =e= \text{pos}_p(i) + \text{tr}*(\text{m_a(i)-m_a_bar(i-1))};
c12(j) .. \text{sum}(i, x(i,j)) =e= 1;
c13(i) .. \text{sum}(j, x(i,j)) =e= 1;

model transport /all/;
Solve transport using minlp minimizing z;
Display x.l, y.l, z.l;
Appendix B

Full enumeration code

clear all
l=1;f=0;
%for nn=1:l
N=3; %Number of jobs
% %for i=1:2^N*factorial(N)
sequence=perms(1:1:N);
% PP =randint(1,N,[1,50]);
% WW = randint(1,N,[1,10]);
% age=randint(1,1,[1,100]);
age=88;
PP=[8 48 41];
PP_j=PP;
WW=[2 10 10];
WW_j=WW;
et=eta=100;
beta=2;
t_r=15;
t_p=5;
age_th=eta^2*t_p./(2*t_r*PP);

%end
%for i=1:size(sequence,2)
% perms(0:1:1)
% nchoosek(0:1:3,4)
% dec2bin(1)
% dec2bin(7,N)
y=zeros(2^N,N);%%maintenance decisions
for i=1:2^N
        y(i,:)=bitget(uint8(i), N:-1:1);
end

for i=1:size(sequence,1)%% filling the sequence
        for j=1:size(sequence,2)
                for m=1:N
                        if sequence(i,j)==m
                                P(i,j)=PP(m);
                                W(i,j)=WW(m);
                                WSPT(i,j)=W(i,j)/P(i,j);
                        end
                end
        end
end
\%age_{th}(i,j)=\eta^2 t_p/(2t_r P(i,j));
end
end
end
end

a={}; \%age after job completion
for i=1:size(sequence,1)
a{1}=zeros(2^N,N);
for h=1:2^N
a{i}(h,1)=age;
end
end
af={}; \%effective age
for i=1:size(sequence,1)
af{i}=zeros(2^N,N);
end
for k=1:size(sequence,1)
for i=1:2^N
for j=1:N
if j==1
\% \text{af}(i,j) = \text{age} \n\% \text{a}_k(i,j) = \text{age} \times (1 - y(i,j)) 
\text{af}_k(i,j) = (1 - y(i,j)) \times \text{a}_k(i,j); 
\text{else} 
\text{a}_k(i,j) = \text{af}_k(i,j-1) + P(k,j-1); 
\text{af}_k(i,j) = (1 - y(i,j)) \times \text{a}_k(i,j); 
\text{end} 
\text{end} 
\text{end} 
C_j = \{}; \text{ completion time} 
\text{for } k = 1 : \text{size(sequence,1)} 
\quad C_j\{k\} = \{}; 
\quad \text{for } i = 1 : 2^N 
\quad \quad C_j\{k\}\{i\} = 0; 
\quad \text{end} 
\quad \text{end} 
\text{end} 

\text{for } k = 1 : \text{size(sequence,1)} 
\quad \text{for } i = 1 : 2^N 
\quad \quad \text{for } j = 1 : N 
\quad \quad \quad \text{if } j = 1
\[ C_{j{}^{i}}(j) = y_{i,j}t_p + P_{k,j} + \ldots + t_r((a_{k}^{i}(i,j+1)/\eta)^{\beta} - (a_{f}^{k}{}^{i}(i,j)/\eta)^{\beta}); \]

\text{elseif} j == N
\[ C_{j{}^{i}}(j) = C_{j{}^{i}}(j-1) + y_{i,j}t_p + P_{k,j} + \ldots + t_r(((a_{f}^{k}{}^{i}(i,j)+P_{k,j})/\eta)^{\beta} - (a_{f}^{k}{}^{i}(i,j)/\eta)^{\beta}); \]

\text{else}
\[ C_{j{}^{i}}(j) = C_{j{}^{i}}(j-1) + y_{i,j}t_p + P_{k,j} + \ldots + t_r((a_{k}^{i}(i,j+1)/\eta)^{\beta} - (a_{f}^{k}{}^{i}(i,j)/\eta)^{\beta}); \]
\text{end}
\text{end}
\text{end}
\text{end}

cc = 1; % c_count
C = 0;
for k = 1:size(sequence,1)
    for i = 1:2^N
        C(cc) = sum(W(k,:).*C_{j{}^{i}}{}^{i});
        cc = cc + 1;
    end
end
[as,bs] = min(C);
bs1 = ceil(bs/2^N);
bs2=mod(bs,2^N);
sequenceo=sequence(bs1,:);
yo=bitget(str2num(dec2bin(bs2,N)), N:-1:1);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%TWEC

WSPT_min=sort(WSPT(1,:),'descend');
for i=1:size(sequence,1)
    if WSPT(i,:)==WSPT_min
        sequence_TWEC=sequence(i,:);
        s1=(i-1)*2^N+1;
        s2=i*2^N;
        TWEC=zeros(1,2^N);
        r=0;
        for s=s1:s2
            r=r+1;
            TWEC(r)=C(s);
        end
        COST_WSPT=min(TWEC);
        for d=1:2^N
            if TWEC(d)==COST_WSPT
                y_WSPT=bitget(str2num(dec2bin(d,N)), N:-1:1);
            end
        end
    end
end
end

end

a_2=age;
Ni=N;
y_2={};
indexes=1:size(PP,2);
indexes_j=0;%indexes;
WSPT_remain={};
af_2c={};
for j=1:N
    for i=1:N
        % if sum(i==indexes_j)==1
        %     y_2{j}(i)=[];
        % else
        if a_2(j)<age_th(i)
            y_2{j}(i)=0;
        else
            y_2{j}(i)=1
        end

        % end
        af_2(i)=(1-y_2{j}(i))*a_2(j);
if j==1
    WSPT_remain{j}(i)=WW_j(i)/(y_2{j}(i)*t_p+PP_j(i)+...
    t_r*(((af_2(i)+PP_j(i))/eta)^beta-(af_2(i)/eta)^beta ));
else
    if sum(i==indexes_j)==1
        %for t=1:length(indexes_j)
        WSPT_remain{j}(i)=0; %
        %end
    else
        WSPT_remain{j}(i)=WW_j(i)/(y_2{j}(i)*t_p+PP_j(i)+...
        t_r*(((af_2(i)+PP_j(i))/eta)^beta-(af_2(i)/eta)^beta ));
    end
end
end

[J_value,J_max(j)]=max(WSPT_remain{j});
indexes_j(j)=J_max(j);

% Ni=ni-1;

a_2(j+1)=af_2(J_max(j))+PP(J_max(j));

% PP_j(J_max(j))

% PP_j = PP(indexes_j);
% WW_j = WW(indexes_j);
af_2c{j}=af_2;
y_{2c}(j) = y_{2\{j\}}(J_{\text{max}}(j));
    clear af_2;
end

y_{2c}
indexes_j

%%%%%%%%%%%%%%%%%%%%%

COST_{WSPT\_remain} = \min(C);
sequence_{TWEC\_remain} = indexes_j;
y_{WSPT\_remain} = y_{2c};
output = \{ \min(C), \text{COST}_{WSPT}, \text{COST}_{WSPT\_remain}; ... \\
    \text{sequence}_o, \text{sequence}_{TWEC}, \text{sequence}_{TWEC\_remain}; ... \\
    yo, y_{WSPT}, y_{WSPT\_remain} \};

% m = {};
if \text{COST}_{WSPT} > \min(C)
%     f = f + 1;
    \text{COST}_{WSPT};
    \text{sequence}_{TWEC};
    y_{WSPT};
    \min(C);
    \text{sequence}_o;
yo;
end

\% end

\% f\_percent=(1-f/l)*100
Nomenclature

PM  Preventive maintenance

\(n\)  Number of job to be scheduled

\(P_j\)  Processing time of job \(j\)

\(w_j\)  Weight of job \(j\)

\(P_{[i]}\)  Processing time of the \(i^{th}\) job in the sequence

\(w_{[i]}\)  Weight of \(i^{th}\) job in the sequence

\(c_{[i]}\)  Expected completion time of \(i^{th}\) job

\(T\)  Time to failure

\(\beta\)  Weibull shape parameter for probability distribution of \(T\)

\(\eta\)  Weibull scale parameter for probability distribution of \(T\)

\(a_{[0]}\)  Age of the machine prior to job sequencing-PM planning

\(a_{[i]}\)  Age of the machine after the \(i^{th}\) job in the sequence

\(t_r\)  Time required to repair the machine

\(t_p\)  Time required to perform PM on the machine

\(\tau\)  Time elapsed since last PM

\(N(\tau)\)  Number of machine failures in \(\tau\) time units of machine operation

\(m(\tau)\)  Expected number of failures in period of \(\tau\)

\(z(\tau)\)  Hazard function of \(\tau\)

\(y_i\)  PM binary decision variable
\[ c_p \quad \text{PM cost} \]
\[ c_m \quad \text{Minimal repair cost} \]
\[ Q_i \quad \text{Job order size} \]
\[ h_i \quad \text{Holding cost for one working piece of job order} \]
\[ FC_1 \quad \text{Failure mode 1} \]
\[ FC_2 \quad \text{Failure mode 2} \]
\[ \delta_E \quad \text{Shift in the mean due to external factor} \]
\[ \delta_{mc} \quad \text{Shift in the mean due to } FM_2 \]
\[ \beta_\delta \quad \text{type II error} \]
\[ \alpha \quad \text{Type I error} \]
\[ \text{LCL} \quad \text{Lower control limit} \]
\[ \text{UCL} \quad \text{Upper control limit} \]
\[ \text{LSL} \quad \text{Lower specifications limit} \]
\[ \text{USL} \quad \text{Upper specifications limit} \]
\[ n_s \quad \text{sample size} \]
\[ C_{CM} \quad \text{cost of corrective maintenance} \]
\[ CPU_T \quad \text{Cost per unit time} \]
\[ N_{CM} \quad \text{Number of corrective maintenance} \]
\[ MTTR_{CM} \quad \text{Mean time to repair} \]
\[ MTTR_{PM} \quad \text{Mean time to PM} \]
\[ PR \quad \text{Production rate} \]
\[ C_{lp} \quad \text{Cost of lost production} \]
\[ LC \quad \text{Labor cost} \]
$C_{Rep}$  Cost of repair
$C_{Res}$  Cost of restoration
$TBM$  Time between maintenance
$C_{Rej}$  Cost of rejection
$R_\delta$  Percentage of nonconforming items
$E[N_{\text{defect}}]$  Expected number of defectives
$T_s$  Time to sample and chart one item
$T_0$  Time spent to search for false alarm
$T_1$  Time to validate the assignable cause
$c_f$  False alarm cost
$c_F$  Fixed cost per sample
$c_v$  Variable cost per sample
$ARL_1$  Average run length when process in control
$ARL_2$  Average run length when process out of control
$C_{FCPCM}$  Fixed cost per CM
$C_{FCPPM}$  Fixed cost per PM
$P_{FM_2}$  Is the probability to have failure mode
$h$  Time between samples
$k$  Control chart width
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Laith A. Hadidi received his B.Sc. in Mechanical Engineering and M.Sc. in Industrial Engineering from the University of Jordan, Amman, Jordan, in 2002 and 2005, respectively. Then he joined, as a Ph.D. candidate and lecturer-B, the Systems Engineering Department at King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia. Laith completed his Ph.D. in Industrial and Systems Engineering in June, 2011. His areas of research interests include preventive maintenance planning, operations sequencing and scheduling, production systems design and simulation.

Laith is originally from the Hashemite Kingdom of Jordan. He lives in Amman, Jordan and can be reached by Email: laith.alhadidi@gmail.com and Mobile: 00962796507017. Laith stayed within KFUPM campus during his doctoral studies. His contact details for KFUPM are: Email: lhadidi@kfupm.edu.sa and Mobile: 00966501277088.