Modeling and Analysis of Two-Flow Interactions in Wireless Networks

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***Abstract*—Interference plays a complex and often defining role in the performance of wireless networks, especially in multi-hop scenarios. In the presence of interference, Carrier Sense Multiple Access MAC protocols are known to suffer from the hidden terminal and exposed terminal problems, which can cause poor performance and unfairness. In this paper, we examine the possible interference modes arising among two interfering one-hop connections under a *Two-Disc model* of interference. We classify the large set of resulting configurations into five categories and develop closed form expressions to compute their probability of occurrence. The analysis exposes two new categories, whose occurrence is common, and whose behavior differs significantly from the three known interference categories. Further, the frequency of occurrence of the categories differ significantly from existing results (obtained with a simpler unit disc model of interference). We develop throughput estimation models for the different categories and validate them using simulation.**

I. Introduction

In Multi-Hop Wireless Networks (MHWNs) that use Carrier Sense Multiple Access (CSMA), interference is manifested in different modes of operation, which can lead to poor performance and short term or long term unfairness. Complex interactions occur between interfering links based on the relative location of the senders and receivers (more accurately the state of the channels among them). These interactions play an important role in determining performance, and give rise to long or short term unfairness. Understanding these interactions is critical for understanding and characterizing behavior in MHWNs and for designing effective protocols for them.

Recent work has analyzed and classified the different behaviors that arise between two interfering links that use the IEEE 802.11 protocol [6], [13]. Understanding and characterizing interactions at this level using formal techniques is a promising first step towards an understanding of the effect of interference from first principles, and in designing protocols that more effectively account for it.

This paper makes several contributions for improving the analysis of two-flow interference using more realistic assumptions, identifying additional types of interactions, and analytically modeling their behavior. Specifically, we make the following contributions:

1) *Generalizing existing analysis by allowing Interference range to be different from reception range (Section IV).* The generalization leads to a larger number of individual scenarios compared to those identified by previous work [6], [13].
More importantly, we identify two new categories of interactions that arise commonly (over 10% of all the cases), and whose behavior differs significantly from those known in literature.
2) *Geometric analysis, leading to closed form expressions, for the probability of occurrence of the scenarios (Section V).* In contrast to the existing geometric models [6], we use a new simpler approach using a recent geometric result [4], that allows direct evaluation of the probability of the grouped categories, avoiding the need to model each of the individual scenarios (5 categories instead of 53 individual scenarios). The geometric models are validated against a Monte Carlo characterization of the probability.
3) *Analytical performance models for the different categories of interactions, including the two newly identified categories (Section VI).* The models are validated using simulation.

We also present preliminary results with extensions of the model (e.g., to model the effect of changing the carrier sense threshold). We believe that these contributions collectively enhance the understanding of the causes and impact of interference. However, several important steps remains towards a generalization of this understanding, including the use of a more realistic channel model and experimental validation of the results. We present our conclusions and areas of future work in Section VII.

II. Related work

Carrier sense multiple access (CSMA) [9] MAC protocols such as IEEE 802.11 [14] are commonly used in wireless networks. Despite extensive research in protocol design [17] [8] [1], collisions cannot be eliminated in CSMA MAC protocols. Specifically, depending on the relative location of the senders and receivers (more accurately, the characteristics of the channels between them), a number of interaction scenarios with distinct behavior occur. Bharghavan et al identify several of these scenarios and propose modifications to the MACAW protocol to address them individually [1] in a network where the interference range is equal to the reception range.

Our work is most related to the following two efforts that attempt to methodically characterize and analyze the performance of the different modes of interactions that occur between two interfering links. Rogers and Abu-Ghazaleh [13] conduct a simulation study of all the possible configurations of two interfering links under saturation traffic, and discover a number of cases with destructive interactions. A formal analysis of two-flows was first studied by Garetto et al [6]. They enumerated the types of interactions that occur under assumptions of transmission range equal to interference range, and developed geometric models for analyzing their expected frequency [6]. The work in this paper, generalizes this analysis, leading to a more accurate characterization of the impact of interference on CSMA protocols, and in the process discovering two new modes of interaction.

Models for computing throughput in CSMA networks were studied initially by Boorstyn et al. [3] and Tobagi et al. [15]. Sophisticated models for calculating the throughput in IEEE 802.11 based networks have been recently proposed [5], [10], [16].Even though these works account for the effect of interference, they do so for given topologies and using iterative methods. In contrast, the modeling component of this paper targets constructive analytical models for the special case of two contending flows under the identified categories of interactions.

III. Background and Existing Models

Garetto et al [6] categorize the two-flow interactions using a boolean physical model where the transmission radius is equal to the interference radius. In a two flow scenario, two senders S_1 and S_2 communicate with two receivers D_1 and D_2 respectively. There exist four secondary (or cross-flow) channels that lead to the different modes of interactions; these are S_1S_2 S_1D_2, S_2D_1 and D_1D_2. The nodes for each secondary link can be either in range or out of range, leading to 2^4 different scenarios corresponding to the different combinations of states that each of the four secondary links can be in. The 16 scenarios can be reduced to 13 by eliminating the dual scenarios (mirror scenarios that are identical by re-labelling the connections). They compute the occurrence probability of each of the scenarios conditioned on a fixed distance between the primary senders and receivers. More interestingly, they recognize that the individual scenarios can be grouped into three basic categories described below.

Sender-Connected (SC): This category includes all scenarios where the two senders are in range. Thus, CSMA prevents senders from concurrent transmission, and no collisions other those arising when the two senders start transmission at the same time will occur. Such collisions are unavoidable, and their probability is low due to the randomization of the backoff period.

Asymmetric Incomplete State (AIS): In the remaining scenarios the senders are not connected (Incomplete State). A distinguishing attribute is whether the state of the S_1D_2 and S_2D_1 links are identical (Symmetric) or different (Asymmetric). In Asymmetric Incomplete State, only one of the senders interferes with the other destination and only one the flows experiences collisions.

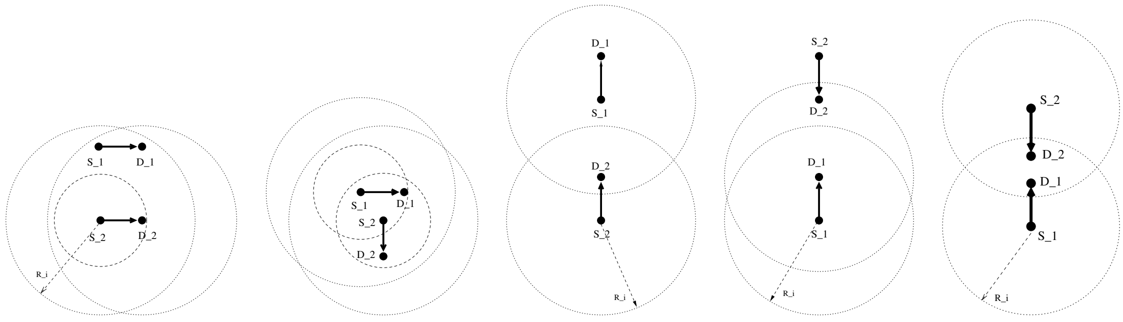
Symmetric Incomplete State (SIS): In this category, the senders are not connected. However, either both the senders can interfere with the other destination, or they cannot. In these scenarios, short term unfairness may arise, but no bias exists to lead to long term unfairness.

IV. Categorizing Two-flow Interactions

This section presents the categories of interactions that arise when the assumption of the interference range being equal to the communication range is relaxed. We assume the IEEE 802.11 basic mode (without RTS/CTS), which is the default mode in most of the IEEE 802.11 network cards.

The possible states of the four secondary flows (S_1S_2, S_1D_2, D_1S_2 and D_1D_2) now become: (1) in communication range; (2) in interference range, but not in communication range; (3) out of range. Each of the four cross links can be in one of the above 3 states relative to each other for a total of 3^4, or 81 enumerable scenarios. After removing the dual scenarios, which are identical other than relabelling of the connections, a total of 53 distinct scenarios remain. Furthermore, the categories of scenarios exhibiting different interference behavior grow from the three described in the previous section, to five. In the following we discuss the five categories in more detail.

(1) Senders Connected Symmetric Interference (SCSI): SCSI represents sender connected scenarios where there is symmetric interference between opposite source and destination. For example, if link S_1D_2 is in interference range then D_1S_2 is also in interference



(a) SCSI (b) SCAI (c) AIS (d) IDIS (e) SIS

Fig. 1. Sample scenarios in each category

range. Figure 1(a) shows a sample SCSI scenario. Flows in this group share the medium fairly due to symmetry.
(2) Senders Connected Asymmetric Interference (SCAI): this subset of scenarios represent the first new category of interaction that we identify. In SCAI: (1) the senders are within communication range of each other; (2) One sender and the opposite receiver (belonging to the other flow) are in interference range (e.g., $S_1 D_2 \leq R_i$ in Figure 1(b)); and (3) The other sender and receiver are not in interference range of each other.

Figure 1(b) shows a SCAI scenario where S_1 and D_2 are in interference, but not in communication, range. Under IEEE 802.11, S_1 can sense the channel busy when D_2 sends an ACK to S_2, but cannot decode the packet. It perceives such a busy signal as an ongoing transmission. In order to avoid a possible collision, S_1 waits for the channel to be idle for an EIFS period (a significantly larger period than the standard DIFS interframe separation) to ensure completion of the ongoing transaction. S_2 receives the ACK from D_1 and waits for DIFS before decrementing its backoff. As a result, S_2 wins the channel again and long term unfairness occurs.
(3) Asymmetric Incomplete State (AIS): This category is identical to the AIS category in the original classification. Specifically, (1) the senders are out of range (not connected); (2) One source and the opposite receiver are in interference range; and (3) The second source and its opposite receiver are out of range. Figure 1(c) shows a sample AIS scenario. Many of the packets sent to D_2 are lost because of interference from S_1, while D_1 receives all packets from S_1 successfully.
(4) Interfering Destinations Incomplete State (IDIS): This is the second newly identified category of interactions which is a subset of the originally classified SIS cases. This group includes scenarios where all the secondary links are out of range except the two destinations. Figure 1(d) shows one such scenario. Since both the sources are out of range (not sender connected), they transmit packets simultaneously. The destination that receives its packet sends an ACK, thus causing a collision for the ongoing packet transmission at the other destination. This causes short term unfairness for each link. IDIS is a *Sender Unconnected*, *Symmetric* and *Incomplete state* scenario that experiences drops due to ACK packets.
(5) Symmetric Incomplete State (SIS): The senders are out of range and both sets of opposite source and destination are within communication or interference range. Figure 1(e) shows a scenario with SIS. Since the two senders are out of range, they can transmit simultaneously. Since each destination can be interfered by the opposite source, there is a packet drop at both the destination. This will cause significant throughput degradation for both links.

We show in Section VI that the performance of the two flows is strongly influenced by the category they fall in. The two newly identified cases account for more than 10% of the scenarios; therefore it is important to identify and study them. In addition, we show in Section V that the frequency of occurrence of all the categories (including the three original ones) differ significantly as the interference range increases. Thus, the presented model allows us to characterize the probability of occurrence of the different scenarios more accurately under typical conditions.

V. Determining Scenario Probability

In this section, geometric models are developed to predict the probability of occurrence of the categories identified in the previous section. Due to the increased number of cases, and the increased complexity of each case due to the addition of a separate interference range, we develop an alternative (and simpler) approach to the one used by Garetto et al [6]. The problem is one of calculating different regions of intersection of the circles forming the communication and interference ranges of the different nodes, which correspond to the interactions scenarios. The existing approach [6] would require a complex and case by case treatment of the 53 scenarios; in contrast, our model captures the 5 categories directly.

A. Preliminaries and Assumptions

We define the interference range and communication range as r_i and r_c respectively. The radius of the whole network is represented by r_s. From the structure of the scenario, since D_1 is the destination of S_1 for one flow these two nodes are always within r_c of each other, and similarly D_2 is always within r_c of S_2.

We use a two-disc binary model of interference where a node inside the communication range will receive a message without any errors in the absence of interference. A node transmitting from interference range will cause all packets to be dropped at the receiver. While this model improves on the existing approaches, we continue to pursue more accurate models that use Signal to Interference and Noise ratio SINR as future work.

We assume a network of a size sufficiently large to account for all the interaction configurations. However, the area would then include configurations where the two flows do not interact. The computed probabilities then have to be normalized to eliminate the non-interacting cases (which our model does). We use a network radius r_s equal to $2r_c$ + r_i centered around one of the sources; this r_s represents the minimum network radius that captures all scenarios. Increasing the network size further only increases non-interacting configurations (which are removed in the normalization step).

To simplify presentation, we assume that Carrier Sense and interference ranges are equal – a common assumption in network simulators. However, we already extended the models to support decoupling carrier sense range from interference range, and later present some analysis of the impact of changing carrier sense range.

The derivation uses the following terminology: $C(X)$ refers to the area of communication range of X (circle of radius r_c around X) and $T(X)$ refers to the interference range of X (circle of radius r_i around X).

The models compute the probability of the presence of a node within r_c or r_i from other nodes concurrently as appropriate for the category being modeled. By modeling the categories directly, our methodology differs from that of existing studies [6] which model the individual cases. The approach requires computing the area of intersection of two or three circles of different radii. Fewell recently derived expressions for the intersection of three circles–a surprisingly difficult problem [4].

B. Example: IDIS Probability Derivation

We developed models for the five categories; in the interest of space, we show the derivation only for IDIS. Rest of the models are derived in a similar fashion.

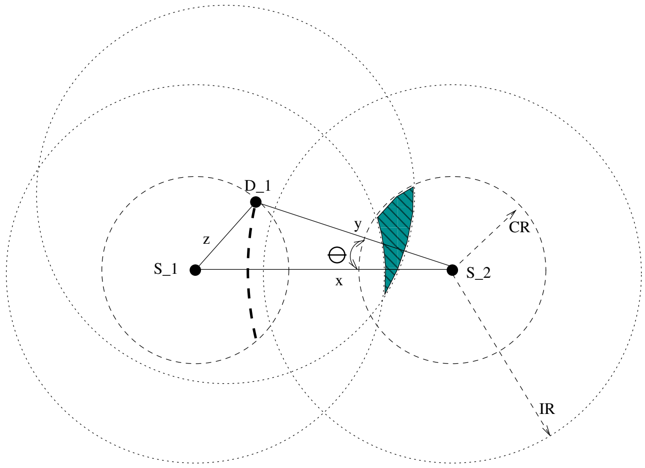


Fig. 2. IDIS Group Example

To compute the probability of IDIS we have to calculate *(a)* The probability that the two sources are out of range of each other. *(b)* The probability that both destinations are out of range of the opposite sources and *(c)* Given the constraints of (b), the two destinations are in range. To compute (a), the probability that S_2 is a distance x from S_1 in a network of radius r_s is given by $\frac{2x}{r_s^2}$, integrating from r_i to r_s yields the probability p_1 of S_2 being out of range of S_1. More precisely,

$$p_1 = \int_{r_i}^{r_s} \frac{2x}{r_s^2}\ dx \qquad (1)$$

We divide (b) in two parts: the probability that D_1 is out of range of S_2 and the probability that D_2 is out of range of S_1. First we solve the first part and then combine the second part with (c). Let us assume that D_1 is at a distance y from S_2, we find the probability that D_1 is on an arc at a radius of y from S_2 as

$$\frac{y\ dy\ d\theta}{C(S_1)} \qquad (2)$$

where dy is the width of the arc and θ is the angle $\angle D_1\ S_2\ S_1$ as shown in the figure. Because of symmetry we will only consider positive θ and double the result to get the lower half. Since D_1 is in communication range of S_1, θ varies from 0 to $\theta_{\max}$, computed as follows.

$$\theta_{\max} = \arccos \frac{x^2 + y^2 - r_c^2}{2xy} \qquad (3)$$

Since we are interested in D_1 being out of range of S_2 the distance y has a lower limit of r_i. It is possible that for larger values of x, the arc of radius y around S_2 will not intersect circle of radius r_c around S_1. To take care of this case we take the lower limit of y to be the maximum of r_i and $x - r_c$. The maximum value that y can take is $x + r_c$. Integrating eq 2 with respect to θ and y yields the probability of D_1 being out of range of S_2

$$p_2 = \int_{max(r_i, x-r_c)}^{x+r_c} \int_0^{\theta_{\max}} \frac{y d\theta dy}{C(S_1)} \tag{4}$$

To find the probability of D_1 and D_2 being in range we find the area of intersection $(A(S_2 \cap D_1))$ of the circle with radius r_i around D_1 and the area of the circle with radius r_c around S_2. Dividing this area by $C(S_2)$ gives the probability that D_1 and D_2 are in range. This probability includes the cases where S_1 and D_2 are in range. To remove these cases we subtract from $(A(S_2 \cap D_1))$ the area of intersection of circles of radii r_i around S_1, r_i around D_1, and r_c around S_2.

$$p_3 = \frac{(C(S_2) \cap T(D_1)) - C(S_2) \cap T(S_1) \cap T(D_1)}{C(S_2)} \tag{5}$$

The expression for the area of intersection of three circles (Equation 16 in [4]) requires that the distances between the center of the circles and their radii be known. The distance between S_1 and D_1 is the only unknown, which is calculated by using the law of cosines (Figure 2).

$$z^2 = x^2 + y^2 - 2xy\cos\theta \tag{6}$$

Combining Equations 1, 4, and 5 the probability of IDIS is

$$P(IDIS) = \int_{r_i}^{r_s} \int_{l_y}^{x+r_c} \int_0^{\theta_{\max}} p_3 \frac{2xy}{r_s^2 C(S_1)} d\theta dy dx \tag{7}$$

where $l_y = max(r_i, x - r_c)$.

C. Validation and Analysis

We validate the geometric models for the five categories by comparing against exhaustive enumeration of the cases. Specifically, S_1 is placed at a fixed location. D_1 is moved around S_1 in the entire area of a circular disc with radius equal to the communications range. For every placement of S_1 and D_1, we move S_2 around S_1 in an area of circular disc of radius $(r_i + 2r_c)$. For each location of S_2, we place D_2 in the circular area of radius r_c around S_2. For each of the scenarios we categorize the interaction between the links to produce the total number of times each scenario will occur.

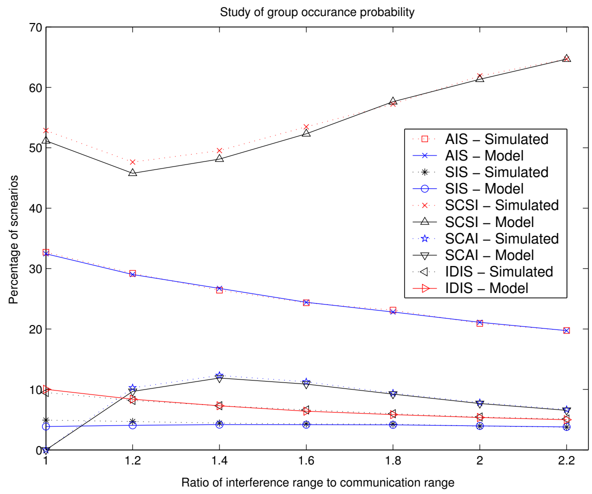


Fig. 3. Occurrence Probability of the Groups

Figure 3 shows that the geometric models closely match the results obtained by exhaustive enumeration, as the ratio of interference (and carrier sense) range to communication range is increased. We plot the values until a ratio of 2.2 – a ratio corresponding to the standard use of 250m/550m as communication/interference range. If we increase the carrier sense range further, the groups where senders are connected (SCSI and SCAI) increase while the other groups become more rare. Increasing the carrier sense range reduces channel reuse as more senders become unnecessarily connected.

We note that IDIS and SCAI comprise over 10% of the scenarios. In addition, as the interference ratio grows, IDIS always has a percentage higher than SIS. Thus, these newly identified interactions are important and require careful analysis. Also, note that the ratios of the cases at typical interference to communication ratios (e.g., 2.2) are significantly different from those at ratio 1.

VI. Throughput Estimation Model

This section develops models for the throughput of the two links under the five categories. The channel capacity is denoted by C. The minimum and maximum backoff window is represented by $\mathrm{CW}_{\min}$ and $\mathrm{CW}_{\max}$, respectively. The *conditional collision probability* (p) is the probability of collision given that the link transmitted a packet. The probability that a source node starts transmission during an idle slot is the *conditional transmission probability* (τ). Bianchi [2] derived the expression for τ under Binary Exponential Backoff (BEB) as a function of p (Equation 8) for WLANs.

$$\tau = \frac{2q(1-p^{m+1})}{q(1-p^{m+1}) + W[1-p-p(2p)^{m'}(1+p^{m-m'}q)]} \quad (8)$$

where $W = \mathrm{CW}_{\min}$, $q = 1-2p$, m is maximum number of retries and m' is the number of stages to reach $\mathrm{CW}_{\max}$ ($m' \leq m$).

We make the following assumptions: (1) The traffic on both links is saturated. Under less than saturated conditions, the interactions play a less important role; and (2) The nodes use the basic mode of IEEE 802.11 (without RTS/CTS), which is the the default mode in the network cards. Extension of the model for relaxing the above assumptions is an area of future work.

We first model the throughput for the sender connected categories (SCSI and SCAI) where the challenge is to derive the share of the channel obtained by each sender. We later model the hidden terminal categories (AIS,SIS and IDIS) where the problem is to obtain the effect of hidden terminals with disconnected sources.

Sender connected categories: For SCSI, the nodes arbitrate the channel successfully and the throughput can be directly estimated using Bianchi's model [2]. Under the SCAI category (refer to Figure 1(b)), the EIFS effect causes one of the links (which we refer to as the 'weaker link') to wait for longer times before decrementing the backoff, thus causing throughput degradation and unfairness.

Let τ_1 and τ_2 be the conditional transmission probability of weaker and stronger links respectively. Since the senders are connected, the probability of winning the channel by the weaker and the stronger link are in the ratio $\tau_1 : \tau_2$. Both the links suffer no hidden terminals ($p = 0$ for both links). Hence, the throughput of the link i is given by Equation 9. l_i and o_i denote the payload size and the overhead size per packet, respectively.

$$T_i = C\frac{\tau_i}{\tau_1+\tau_2} \cdot \frac{l_i}{l_i+o_i} \quad (9)$$

The stronger link always transmits with the same probability *when the channel is idle*. Hence, τ_2 is calculated by Equation 8. What remains is estimating τ_1.

Since we are interested in calculating the *transmission probability conditioned on the channel being idle*, we ignore the time during which the channel is busy. An idle slot can be in one of the backoff/EIFS states (a countable state space). And, the weak link will transmit when the backoff counter is zero (a subset of the state space). Hence, we use a discrete time Markov chain to calculate the probability of transmission at an idle slot (τ_1).

Rule	From	To	Probability
1	$B(i),\ i \neq 0$	$B(i-1)$	$1-\tau_2$
2	$B(i)$	$E(i,M)$	τ_2
3	$E(i,j),\ j \neq 0$	$E(i,j-1)$	$1-\tau_2$
4	$E(i,j)$	$E(i,M)$	τ_2
5	$E(i,0),\ i \neq 0$	$B(i)$	$1-\tau_2$
6	$B(0)$	$B(i)$	$\frac{1-\tau_2}{\mathrm{CW}_{\min}+1}$
7	$E(0,0)$	$B(i)$	$\frac{1-\tau_2}{\mathrm{CW}_{\min}+1}$

TABLE I
TRANSITION PROBABILITIES

In order to compute the state space, we observe that the source may be decrementing its backoff or experiencing an EIFS wait period during an idle slot (ignoring the DIFS period because DIFS $\ll$ EIFS). Let B(i) represent the i^{th} backoff stage where $0 \geq i \geq \mathrm{CW}_{\min}$. EIFS duration is approximated by M discrete slots. Let $E(i,j)$ denote the j^{th} EIFS slot during the i^{th} backoff stage. Then, $B(i)$ and $E(i,j)$ represents the states of the chain.

The channel becomes busy for the weaker link when the stronger link transmits during an idle slot. Note that τ_2 is independent of τ_1. The transition probabilities between the states are represented in Table I.

The weaker link starts transmitting the packet only when the channel is idle at the slot boundary when: (1) the backoff counter is zero (state $B(0)$); or (2) The EIFS period is completed and backoff counter is zero (state $EIFS(0,0)$). Hence, the probability with which the node starts transmitting a packet at an idle time slot (τ_1) is given by Equation 10.

$$\tau_1 = (1-\tau_2)(\Pi_{B(0)} + \Pi_{EIFS(0,0)}) \quad (10)$$

where Π are the limiting probabilities of the above chain.

Figure 5(a) validates the model by comparing it with simulation (with standard MAC parameters). The simulation was conducted using the QualNet simulator [11]. Packet size was varied from 200 bytes to 1024 bytes. Since the links compete with a ratio $\tau_1 : \tau_2$, a constant ratio of the throughput between the weak and the strong link *independent of the packet size* is observed. Thus, the fairness of the links cannot be altered by altering the packet sizes of both the links.

Hidden Terminal Categories: We develop a general throughput model for scenarios with hidden terminals and specialize it for capturing AIS, SIS and IDIS. Due to lack of space, we explain the throughput model for the AIS category.

General Hidden Terminal Scenario: The transmissions on a link can be abstracted by cycles of successful

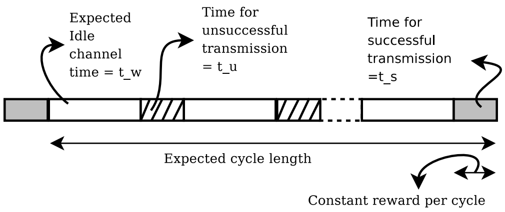


Fig. 4. Packet transmission attempts

transmissions by a source. Let t_s and t_u represent the constant packet transmission durations for a successful and unsuccessful attempt, respectively. A single cycle for a successful transmission is shown in Figure 4. Let t_w be the expected value of the channel idle times between transmissions and let n_u be the expected value of the number of transmissions before a single successful transmission. Let p and τ be the conditional collision and transmission probabilities. The expected value of n_u is $\frac{1}{1-p}$ and the expected value of t_w is $\frac{1}{\tau}$. Hence, the expected wait time of a cycle is given by $n_u(t_w + t_u) - t_u + t_s$. The long-term throughput can be found by recognizing that the behavior represents a *Renewal Reward process* [7]. The overall throughput of the link i (T_i) is then given by Equation 11.

$$T_i = \frac{Ct_s}{n_u(t_w + t_u) - t_u + t_s} \tag{11}$$

The variables that need to be computed are p and τ, which vary based on the category being modeled.

AIS formulation:

Recall that in AIS, a source of one link can cause collision at the destination of the other, but not vice versa. Figure 1(c) shows this scenario where the transmission of S_2D_2 will succeed only during the idle periods of the link S_1D_1. Let the conditional transmission probability and the conditional loss probability of the link S_1D_1 be τ_1 and p_1 respectively (similarly τ_2 and p_2 represents these probabilities for link S_2D_2). The estimates for the derivation of S_1D_1 are straightforward since it does not experience any hidden terminals. Hence, $p_1 = 0$ and $\tau_1 = \frac{2}{\mathrm{CW_{min}}}$.

The packet transmission of S_2D_2 is successful only if the complete packet is transmitted when S_1 is inactive. A single slot of overlap between S_1D_1 and S_2D_2 can cause a packet collision at D_1. By this rule, it can be shown that:

$$p_2 = 1 - \frac{\sum_{i=l_2}^{\mathrm{CW_{min1}}} \frac{i-l_2}{i}}{\mathrm{CW_{min1}} + 1} \tag{12}$$

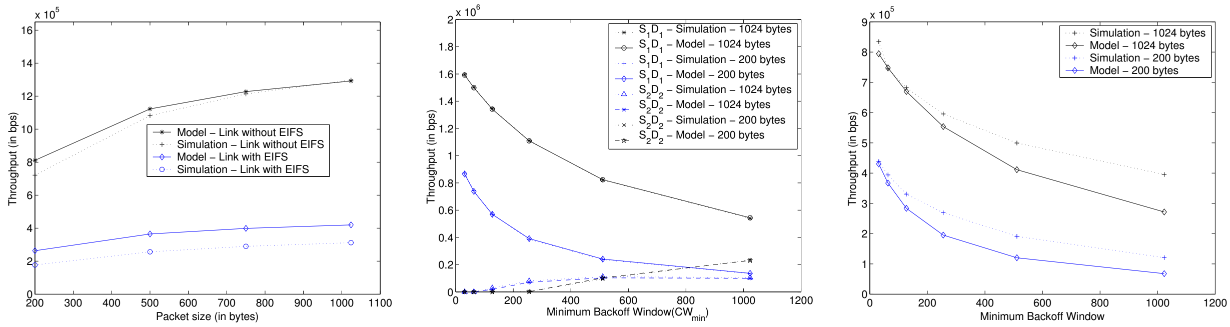
The value of τ_2 can be calculated by Equation 8. This completes the calculation of all the variables (p's and τ's) for throughput estimation of the links.

We now compare the effectiveness of AIS formulation. The weaker link S_2D_2 will get non-zero throughput only when it is able to fit the packet between S_1's transmission. Since this primarily depends upon the value of $\mathrm{CW_{min}}$, we validate the model for different $\mathrm{CW_{min}}$ and packet sizes. Figure 5(b) shows that the model matches closely with the simulations. Fair operation for the weaker link occurs only at larger values of $\mathrm{CW_{min}}$. Under low $\mathrm{CW_{min}}$, the effect of AIS can be reduced by decreasing the packet size (or increasing transmission rate).

Symmetric categories (SIS, IDIS): In the symmetric categories, the conditional collision probability (p) of both the links are dependent on the each other. This coupling makes independence assumptions inaccurate, thus complicating the model. An accurate model of these cases would require modeling the combined states of the two senders (each of which may take any of the states in the Bianchi model), leading to a very large Markov chain. We develop an approximate model to compute the throughput under symmetric cases. The simulation results indicate our model accurately predicts the throughout under the IDIS category (Figure 5(c)). However, the accuracy of the model is limited under SIS. We are working on several extensions to the model and have preliminary results for some of them.

Effect of carrier sense: First, we are updating the model to allow the carrier sense range (CSR) to be decoupled from the interference range [18]. Preliminary results show that if CSR is low, the occurrence of SIS group dominates as more senders transmit while they are in interference range with each other. Conversely, Sender connected groups increase as CSR increases. Also as CSR increases, SCAI occurrence also increases, causing a more pronounced effect of EIFS and exposed terminals (which we do not analyze). AIS and IDIS occurrence remains constant until CSR becomes greater than interference range, when they start becoming more rare as more sources become connected.

Interference Effect in chains: We are in the process of analyzing the interactions that arise in a single multi-hop chain connection. Links in a chain topology can exhibit the different modes of interference, leading to significant impact on the expected performance of these chains. However, the nature of the chain, and the expected geometry cause SCSI and AIS categories to become more prevalent. Analyzing ways of detecting these situations



(a) SCAI (b) AIS (c) IDIS

Fig. 5. Throughput estimation

and designing routing protocols that take advantage of this information is part of our future research.

More Accurate Physical Interference Models: Our most immediate future work include using the more realistic Signal to Interference and Noise Ratio (SINR) interference model in place of the two-disc model. We believe that the proposed geometric framework becomes more important as the number of possible interactions explodes under the SINR model (an estimated 20736 individual interactions between two-flows under the SINR model [12]).

VII. Concluding Remarks

The paper makes several contributions to the analysis of two single hop wireless flows. In contrast to the existing studies that use simplistic interference model, we use a more realistic interference model to approximate the link behavior. As demonstrated by the paper, this leads to additional types of categories that were absent under the simplistic interference model. The paper categorizes the interactions and develops closed form expressions to compute the probability of occurrence of each category. The frequency of occurrence is analyzed as a function of the interference/carrier sense range. The results demonstrate a significant variation of the occurrence probabilities when compared with the existing simplistic interference models. The paper also contributes constructive models for the throughput in presence of hidden terminals, although the models for SIS remain approximate.

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