Proof of $\frac{dC_M}{dC_L} = \frac{X_{cg}}{\bar{c}} - \frac{X_{NP}}{\bar{c}}$

$$\because C_{m\alpha} = C_{L\alpha w}\left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}}\right) + C_{m\alpha f} - \eta V_H C_{L\alpha t}\left(1 - \frac{d\epsilon}{d\alpha}\right) \quad (1)$$

$$\because \frac{X_{NP}}{\bar{c}} = \frac{X_{ac}}{\bar{c}} - \frac{C_{m\alpha f}}{C_{L\alpha w}} + \eta V_H \frac{C_{L\alpha t}}{C_{L\alpha w}}\left(1 - \frac{d\epsilon}{d\alpha}\right) \quad (2)$$

$$-\frac{X_{ac}}{\bar{c}} = \frac{-X_{NP}}{\bar{c}} - \frac{C_{m\alpha f}}{C_{L\alpha w}} + \eta V_H \frac{C_{L\alpha t}}{C_{L\alpha w}}\left(1 - \frac{d\epsilon}{d\alpha}\right)$$

multiply by $C_{L\alpha w}$

$$\therefore -C_{L\alpha w}\frac{X_{ac}}{\bar{c}} = -C_{L\alpha w}\frac{X_{NP}}{\bar{c}} - C_{m\alpha f} + \eta V_H C_{L\alpha t}\left(1 - \frac{d\epsilon}{d\alpha}\right) \leftarrow$$

substitute into (1)

$$C_{m\alpha} = C_{L\alpha w}\left(\frac{X_{cg}}{\bar{c}} - \frac{X_{NP}}{\bar{c}}\right)$$

$$\frac{C_{m\alpha}}{C_{L\alpha}} = \frac{X_{cg}}{\bar{c}} - \frac{X_{NP}}{\bar{c}}$$

$$\frac{C_{m\alpha}}{C_{L\alpha}} = \frac{dC_m/d\alpha}{dC_L/d\alpha} \doteq \boxed{\frac{dC_m}{dC_L} = \frac{X_{cg}}{\bar{c}} - \frac{X_{NP}}{\bar{c}}}$$

Problem 2-13

Solution:

The wing-fuselage contribution to the dirctional stability is given by

$$C_{n_{\beta wf}} = -k_n k_{R_l} \frac{S_{f_s}}{S_w} \frac{l_f}{b}$$

From the data in Figure P2.9 we can calculate the parameters needed to estimate k_n from Figure 2.28.

$$\frac{x_m}{l_f} = \frac{8.0m}{13.7m} = 0.58$$

$$\frac{l_f^2}{S_{fs}} = \frac{(13.7m)^2}{15.4m^2} = 12.2$$

$$\sqrt{\frac{h_1}{h_2}} = \sqrt{\frac{1.6m}{1.07m}} = 1.22$$

$$\frac{h}{w_f} = \frac{1.6m}{1.6m} = 1.0$$

$$k_n = 0.0016$$

The constant k_{Rl} is found from Figure 2.29.

$$V = 150 \text{ m/s}$$

$$R_{l_f} = \frac{(150m/s)\,(13.7m)}{1.46 \times 10^{-5} m^2/s} \qquad \frac{VL}{\nu}$$

$$R_{l_f} = 141 \times 10^6$$

$$k_{Rl} = 2.04$$

$$C_{n_{\beta wf}} = -k_n k_{Rl} \frac{S_{f_s}}{S_w} \frac{l_f}{b}$$

$$= -(0.0016)(2.04)\left[\frac{15.4m^2}{20.27m^2}\right]\frac{13.7m}{10.4m}$$

$$= -0.0033/\text{deg}$$

$$= -0.187/\text{rad}$$

The vertical tail contribution is given by

$$C_{n_{\beta_v}} = V_v \eta_v C_{L_{\alpha_v}} \left(1 + \frac{d\sigma}{d\beta}\right)$$

It is desired that the airplane have a $C_{n\beta} = 0.1$/rad. Therefore the vertical tail must provide a value of 0.29/rad to overcome the destabilizing contribution of the wing-fuselage.

$$C_{n_{\beta_v}} = 0.29/\text{rad} = V_v C_{L_{\alpha_v}} \eta_v \left(1 + \frac{d\sigma}{d\beta}\right)$$

where

$$V_v = \frac{l_v S_v}{Sb}$$

$$C_{L_{\alpha_v}} = \frac{C_{\ell_\alpha}}{1 + C_{\ell_\alpha} / (\pi AR_v)}$$

$$\eta_v \left(1 + \frac{d\sigma}{d\beta}\right) = 0.724 + 3.06 \frac{S_v/S}{1 + \cos \Lambda_{c/4}} + 0.4 \frac{Z_w}{d} + 0.009\, AR_w$$

The area of the vertical tail affects each of the terms in the above equation. If we fix the tail moment arm l_v and aspect ratio then we can by trial and error determine the magnitude of S_v. Assuming $l_v = 4.0$m, $z_w = 0.4$, D = 1.6, AR and a two-dimensional section lift coefficient of 0.1/deg.

$S_v \simeq 3.9\ m^2$

Problem 2.3:

$$M = M_c + M_w$$

$$M_c = L_c X_{cg} + M_{ac_c} \quad , \quad M_w = -L_w(L_t - X_{cg}) + M_{ac_w}$$

$$M = L_c X_{cg} - L_w(L_t - X_{cg}) + M_{ac_c} + M_{ac_w}$$

divide by $\frac{1}{2}\rho V^2 S \bar{c}$ where $L_c = \frac{1}{2}\rho V_c^2 S_c$

$$\boxed{C_M = \eta C_{L_c} \frac{X_{cg}}{\bar{c}} \frac{S_c}{S} - C_{L_w}\left(\frac{L_t - X_{cg}}{\bar{c}}\right) + C_{M_{ac_c}} + C_{M_{ac_w}}}$$

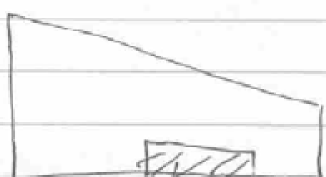
$$C_{L_c} = C_{L_{c\alpha}} \alpha \quad , \quad C_{L_w} = C_{L_{w\alpha}} \alpha$$

$$X_{NP} = X_{cg}\Big|_{C_{M_\alpha}=0} \Rightarrow \eta C_{L_{c\alpha}} \frac{X_{NP}}{\bar{c}} \frac{S_c}{S} + \frac{C_{L_{w\alpha}} X_{NP}}{\bar{c}} - C_{L_{w\alpha}} \frac{L_t}{\bar{c}} = 0$$

$$\boxed{\frac{X_{NP}}{\bar{c}} = C_{L_w} \frac{L_t}{\bar{c}} \Big/ \left(\eta C_{L_{c\alpha}} \frac{S_c}{S} + C_{L_{w\alpha}}\right)}$$

Problem 2.14:

$$\frac{\text{Flap area}}{\text{Wing area}} = \frac{0.575}{10.65} \simeq 0.054$$



From fig 2.21 $\tau \simeq 0.19$

Flap area

$$C_{l_{\delta a}} = \frac{2 C_{L_{\alpha w}}}{S b} \int_{y_1}^{y_2} c y \, dy \qquad c = -0.27 y + 2.75$$

$$C_{l_{\delta a}} = 0.0058 \left[\frac{2.75}{2} y^2 - \frac{0.27}{3} y^3\right]_{3.4}^{4.8}$$

$$C_{l_{\delta a}} \simeq 0.054 \text{ /rad}$$