SE-514 (OPTIMAL CONTROL)

OPTIMAL CONTROL FOR SINGLE AND DOUBLE INVERTED PENDULUM

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Abstract

The project aimed at designing a linear controller for a single inverted pendulum taking it from its stable position to the inverted upright position. A single inverted pendulum consists of a pole mounted on a motor driven cart in such a way that the pole can swing freely in vertical plane. The controller achieves the aim by moving the cart the back and forth on a rail of limited length by applying the appropriate of force. The force is applied on the drive by means of compensating the input voltage to the power amplifier of the driving motor. This is done by introducing the kalman gain and varying it for effective control. The designed controller is also used on a non-linear system but with a linear relationship between the input and the kalman gain, in a real time implementation. Furthermore, a similar linear controller is designed to control a double inverted pendulum. In this instance, a double inverted pendulum consists of a two-stage pendulum attached to a motor-driving cart. The controller designed for both single and double inverted pendulum gave satisfactory results as shown in the state trajectories of both systems.

Introduction

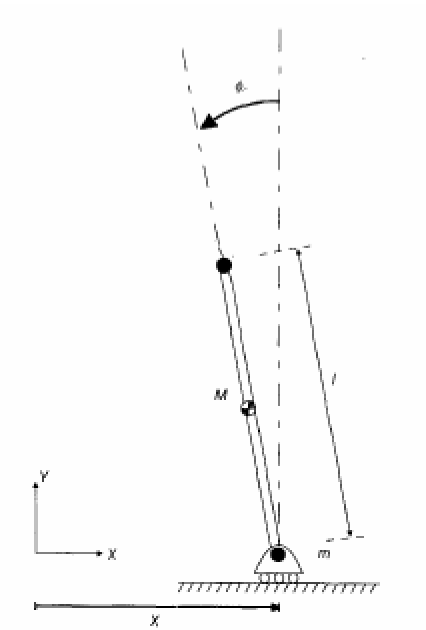
The control of an inverted pendulum is one of the fundamental problems in control field. The process is non-linear and unstable with one input signal and several output signals. The output of the system depends on the system variables and the degree of freedom of the system. The aim is to balance a pendulum vertically on a motor driven cart. The position of the cart is steered to different positions with a position reference signal.

A control strategy used to stabilize the pendulum at the upright inverted position is the linear quadratic regulator (LQR) technique. This technique is effective and it is used in the project. The control of an inverted pendulum is of two folds with two different controllers. The swing-up controller, which diverge the pendulum from the stable position is one and the stabilization controller is the other, which stabilize the pendulum in the unstable inverted position. In the case of using a control technique in a linear model like LQR, both controllers have to be designed individually. In this project, only the stabilization controller was designed. However, the Feedback pendulum system's swing-up controller was used for the purpose of the real time implementation.

In this project, the equations of motion for the system is derived and linearized based on certain assumptions and reference position. This is followed by the design of the controller and simulation of the single inverted pendulum offline using a MATLAB Simulink linear model. A real time implementation of the Feedback system with the linear controller is also performed and results shown. As a comparison, a similar controller was designed for a double inverted pendulum and simulated in the same manner.

Single inverted pendulum

Description of the System



Where

M	mass of the pendulum
m	mass of the cart
l	length of the pendulum
ø	angle of pendulum from vertical
x	cart position coordinate (horizontal)
g	gravitation constant, 9.81 m/s^2
F	force applied to the cart

The figure above shows an inverted pendulum. The aim is to move the cart along the x direction to a desired point with the pendulum in a vertical upright position. The cart is driven by a DC motor, which is controlled by a Kalman filter. The cart's horizontal position (x) and the pendulum's angle (ϕ), are measured and supplied to the control system. A disturbance force (F), can be supplied can be supplied on top of the pendulum.

Modeling of the system

The cart, on which the pendulum is supported, is able to move back and forth according to the dc motor. We are assuming the movement of the cart will be smooth and surface friction between the cart and rail is assumed to be minimal enough to be neglected. Also the equilibrium position of the cart is the middle of the track and the corresponding equilibrium angle of the pendulum is at its upright vertical position.

A model of the inverted pendulum is used as the basis for control design of the real-time system. The dynamic equations and values of the theoretical model are calculated to be as close as possible to the actual process.

In order to obtain the inverted pendulum's model, the system's dynamics is analyzed using the Lagrange Method. Our system is a Two Degree of Freedom System (i.e. x and ϕ)

Potential Energy:

$$V = Mg\left(\tfrac{1}{2} l \cos\phi\right)$$

Kinetic Energy

$$T = \tfrac{1}{2} m\dot{x}^2 + \tfrac{1}{2} M\left[(\dot{x} - \tfrac{1}{2} l\dot{\phi}\cos\phi)^2 + (\tfrac{1}{2} l\dot{\phi}\sin\phi)^2\right] + \tfrac{1}{2} I\dot{\phi}^2$$

Lagrange Equations:

- $$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) - \frac{\partial T}{\partial x} - \frac{\partial V}{\partial x} = F$$
- $$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\phi}}\right) - \frac{\partial T}{\partial \phi} - \frac{\partial V}{\partial \phi} = 0$$

So we get.

- $$[m + M]\ddot{x} - \left[\tfrac{1}{2} Ml\cos\phi\right]\ddot{\phi} = F$$
- $$-\left[\tfrac{1}{2} Ml\cos\phi\right]\ddot{x} + \left[M(\tfrac{1}{2} l)^2 + I\right]\ddot{\phi} + \tfrac{1}{2} Mgl\sin\phi = 0$$

Linearization

These two equations will be linearized about $\phi = \pi$. Assume that $\phi = \pi + \theta$ (θ represents a small angle from the vertical upward direction).

Therefore,

$\cos\phi = -1$, $\sin\phi = -\theta$, $F = u$ (where u represents the input)

So that the Equations of Motion become:

- $$[m + M]\ddot{x} + [\tfrac{1}{2}Ml]\ddot{\theta} = u$$
- $$[\tfrac{1}{2}Ml]\ddot{x} + [M(\tfrac{1}{2}l)^2 + I]\ddot{\theta} - \tfrac{1}{2}Mgl\theta = 0$$

State-Space Representation

A state-space representation of the inverted pendulum dynamics system can be derived from the two previously linearized equations. Using these parameters of the Pendulum-Cart setup.

$g = 9.81 m/s^2$, $l = 0.017m$, $m = 1.12kg$, $M = 0.11kg$, $I = 0.013kg - m^2$

We get

$$\dot{x} = Ax + Bu$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -0.6586 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 433.209 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0.8676 \\ 0 \\ -35.892 \end{bmatrix} u$$

$$y = Cx$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix}$$

Controllability

The system described by the matrices (**A**,**B**) can said to be controllable if there exists an unconstrained control **u** that can transfer any initial state **x**(0) to any other desired location **x**(*t*). For the system $\dot{x} = Ax + Bu$, the system can be determined to be controllable if the determinant of the controllability matrix is nonzero.

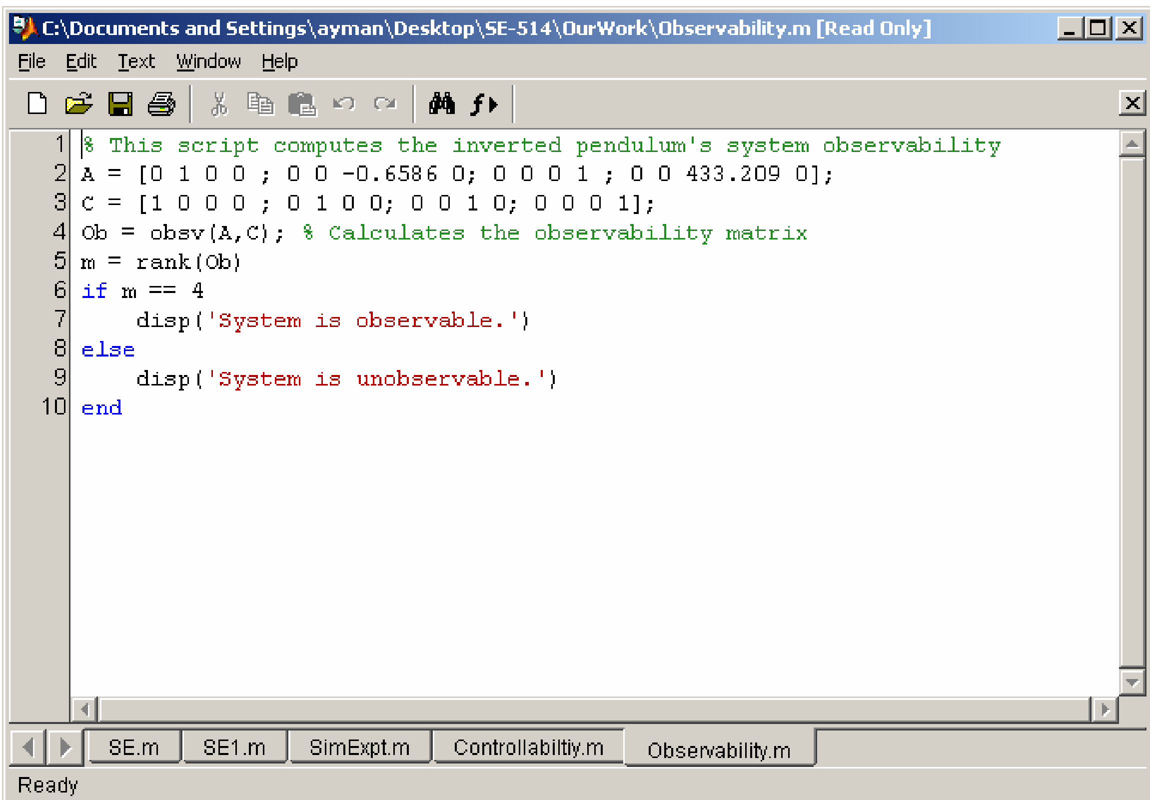
C:\Documents and Settings\ayman\Desktop\SE-514\OurWork\Controllabiltiy.m [Read Only]

```
% This script computes the inverted pendulum's system controllability
A = [0 1 0 0 ; 0 0 -0.6586 0; 0 0 0 1 ; 0 0 433.209 0];
B =[0; 0.8676; 0; -35.892];
Cn = ctrb(A,B); % Calculates the controllability matrix
n = det(Cn)
if abs(n) < eps
    disp('System is uncontrollable.')
else
    disp('System is controllable.')
end
```

```
>>
n =
  1.5981e+008

System is controllable.
```

Observability

Observability refers to the ability to estimate a state variable. A system is observable if, and only if, there exists a finite time T such that the initial state $\mathbf{x}(0)$ can be determined from the observation history $y(t)$ given the control $u(t)$. For the same system with output $y = Cx$, the system is observable if the rank of the observability matrix is 4, which is the full length of the observability matrix.




```
>>
m =
    4

System is observable.
```

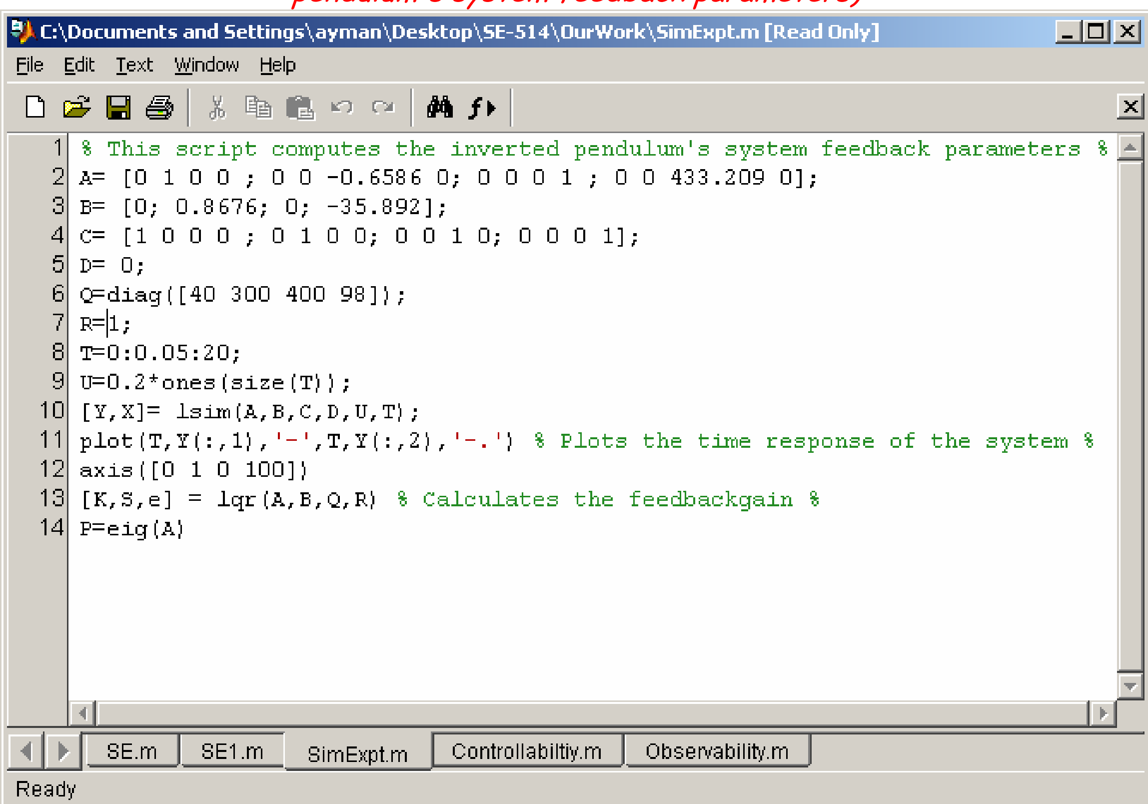
Simulation (off-line)

The open-loop behavior of the system can be observed by simulating a step response to the system. And It is observed with a step input, the system is unstable. Thus, a controller needs to be designed and implemented to improve and stabilize the system.

In order to stabilize the inverted pendulum system, a state feedback approach is considered Shown in the block diagram.

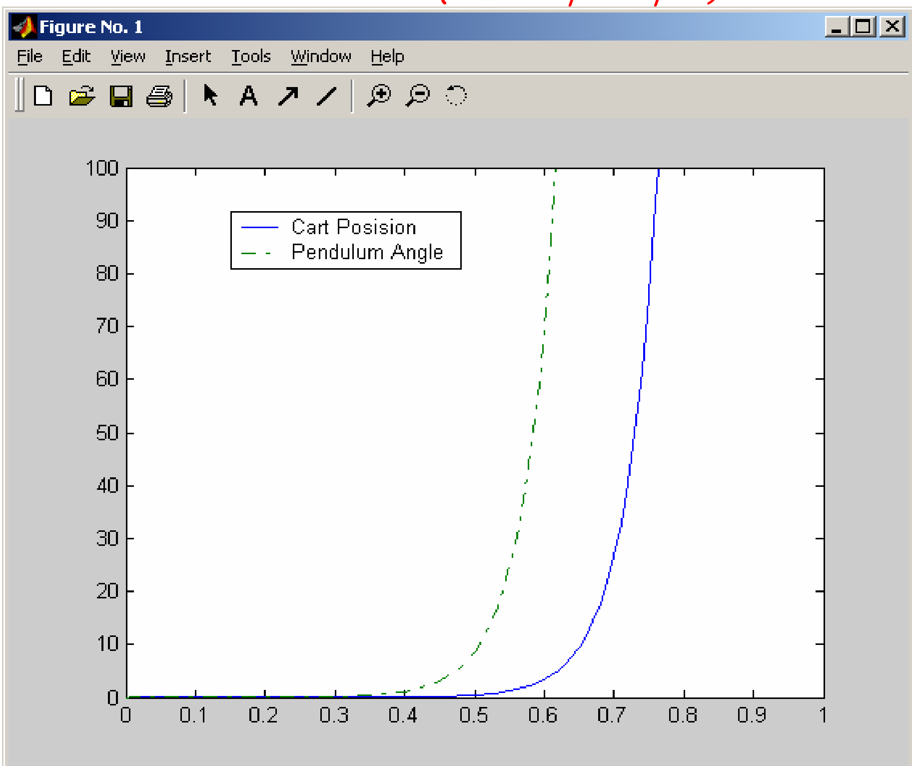
A full-state feedback condition is assumed and the feedback gain, K of the system is to be determined. The feedback matrix gain can be calculated by using the LQR method, which will provide with the optimal controller values.

(This script Plots the time response of the system and computes the inverted pendulum's system feedback parameters)




```
% This script computes the inverted pendulum's system feedback parameters %
A= [0 1 0 0 ; 0 0 -0.6586 0; 0 0 0 1 ; 0 0 433.209 0];
B= [0; 0.8676; 0; -35.892];
C= [1 0 0 0 ; 0 1 0 0; 0 0 1 0; 0 0 0 1];
D= 0;
Q=diag([40 300 400 98]);
R=1;
T=0:0.05:20;
U=0.2*ones(size(T));
[Y,X]= lsim(A,B,C,D,U,T);
plot(T,Y(:,1),'-',T,Y(:,2),'-.') % Plots the time response of the system %
axis([0 1 0 100])
[K,S,e] = lqr(A,B,Q,R) % Calculates the feedbackgain %
P=eig(A)
```

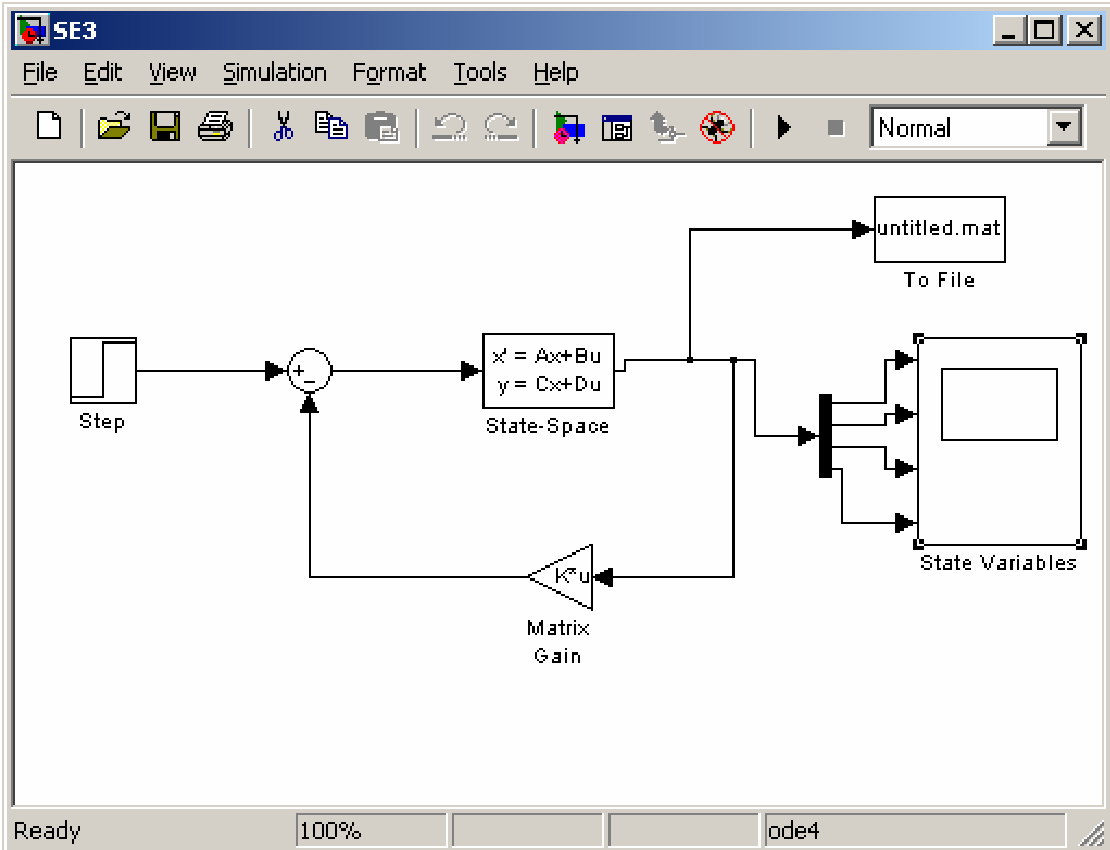
(Time response plot)



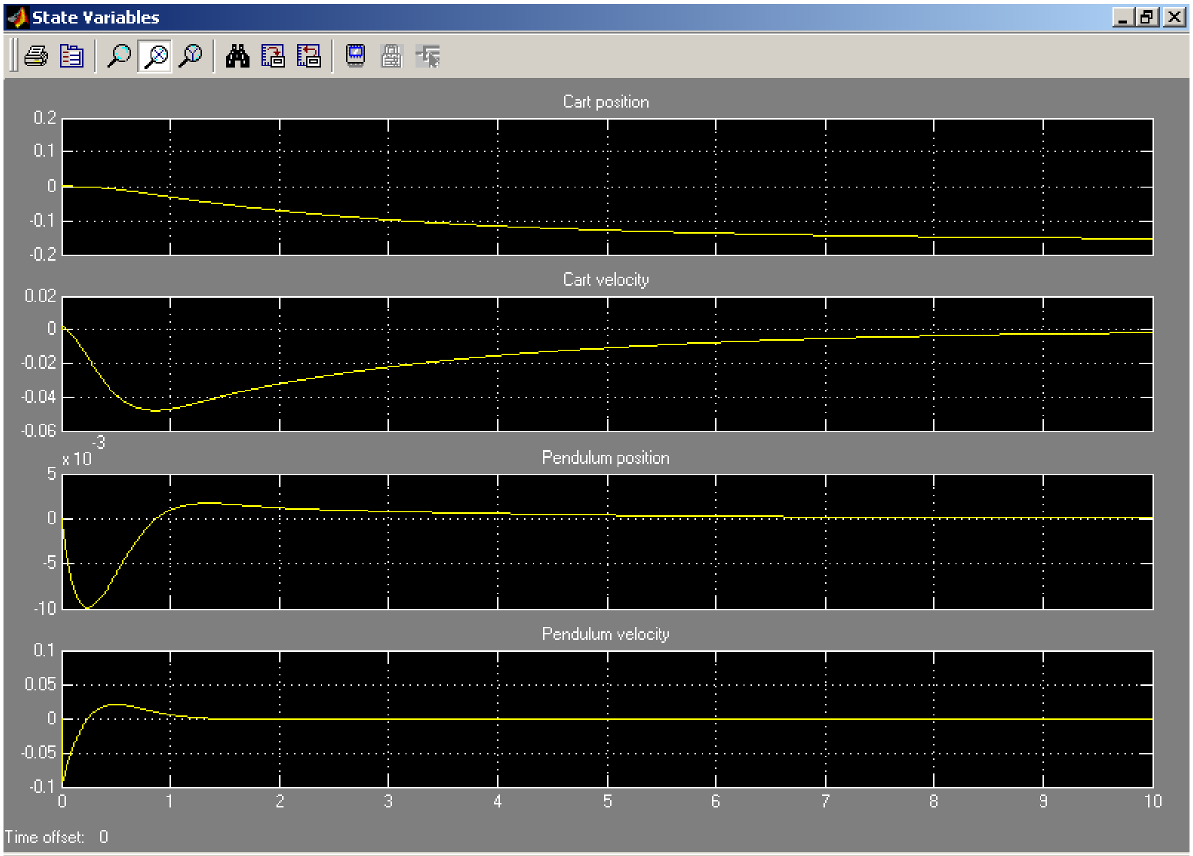
(Matlab output)

```
>>
K =
   -6.3246  -19.6878  -80.0385  -10.6064

S =
  124.5169   43.8057   67.0807    1.2351
   43.8057  129.5279  207.5819    3.6795
   67.0807  207.5819  684.7947    7.2478
    1.2351    3.6795    7.2478    0.3845

e =
  1.0e+002 *
  -3.5684
  -0.0320 + 0.0261i
  -0.0320 - 0.0261i
  -0.0037

P =
         0
         0
   20.8137
  -20.8137
```

(Block Diagram of Full State Feedback System)

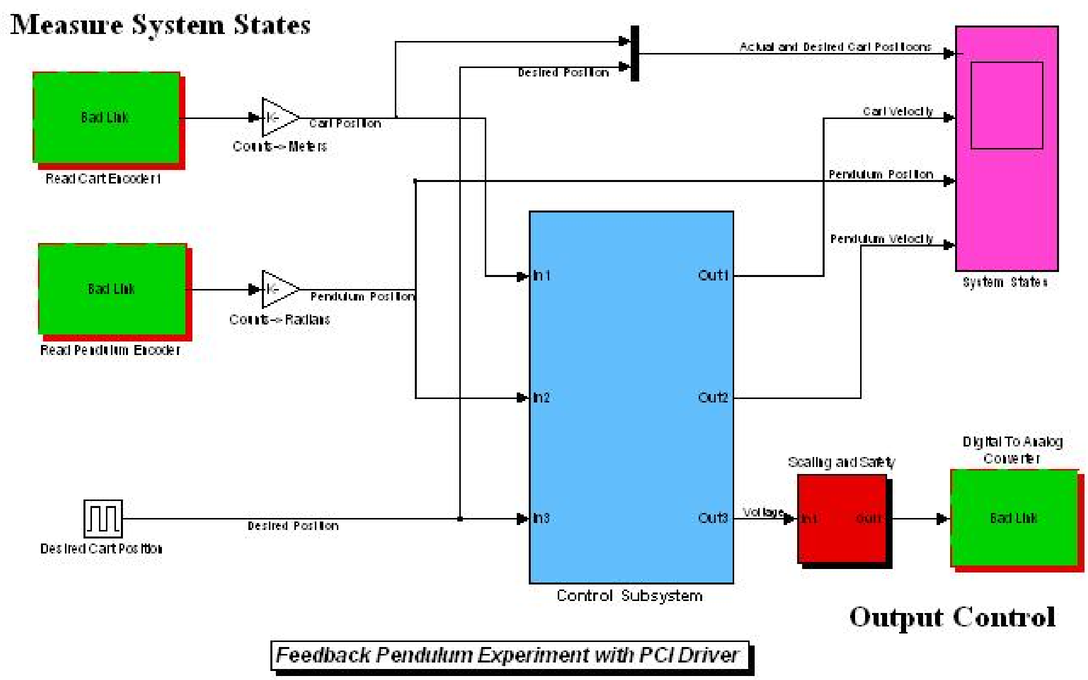


(Simulaink output)

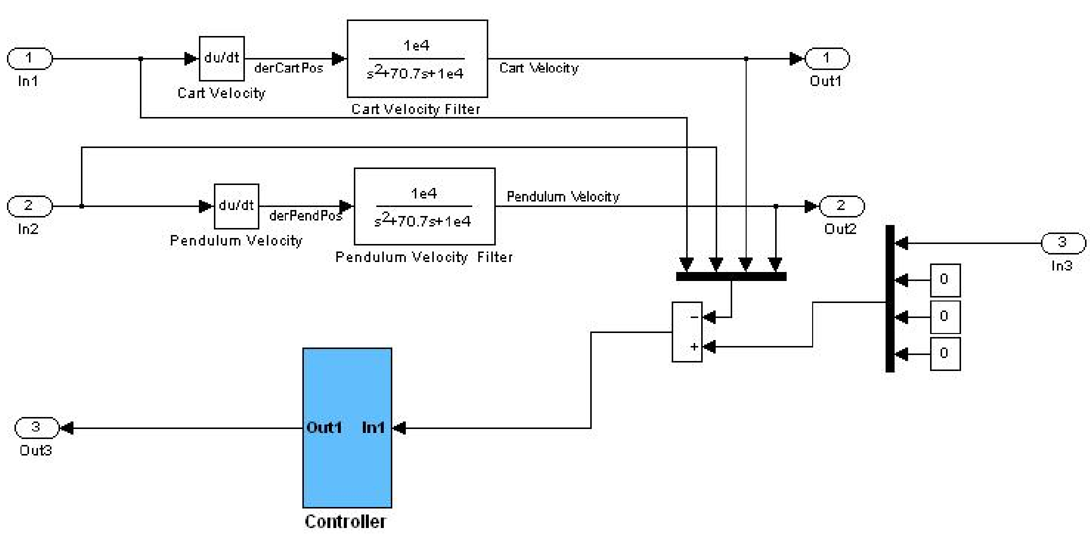

State Variables
Cart position
0.2
0.1
0
-0.1
-0.2
Cart velocity
0.02
0
-0.02
-0.04
-0.06
x 10
-3
Pendulum position
5
0
-5
-10
Pendulum velocity
0.1
0.05
0
-0.05
-0.1
0
1
2
3
4
5
6
7
8
9
10
Time offset: 0

Simulation (Real-Time)

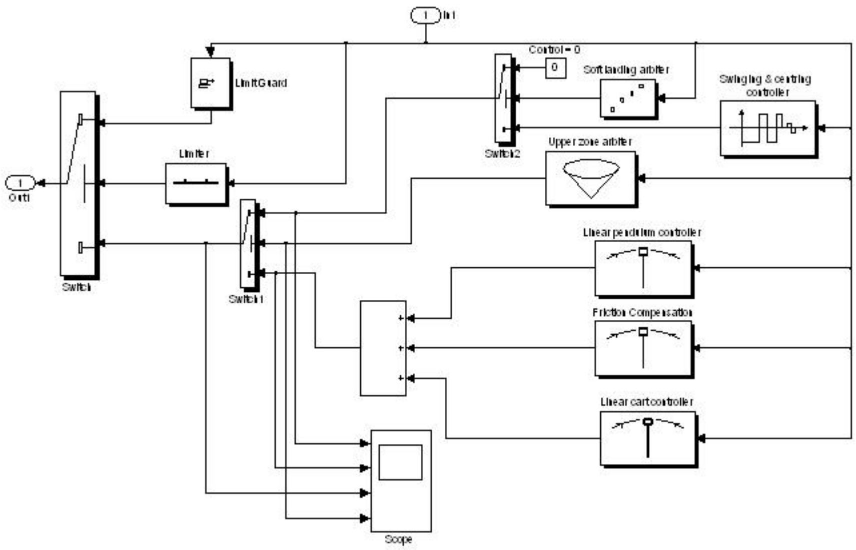
(Simulink block diagram for inverted pendulum)



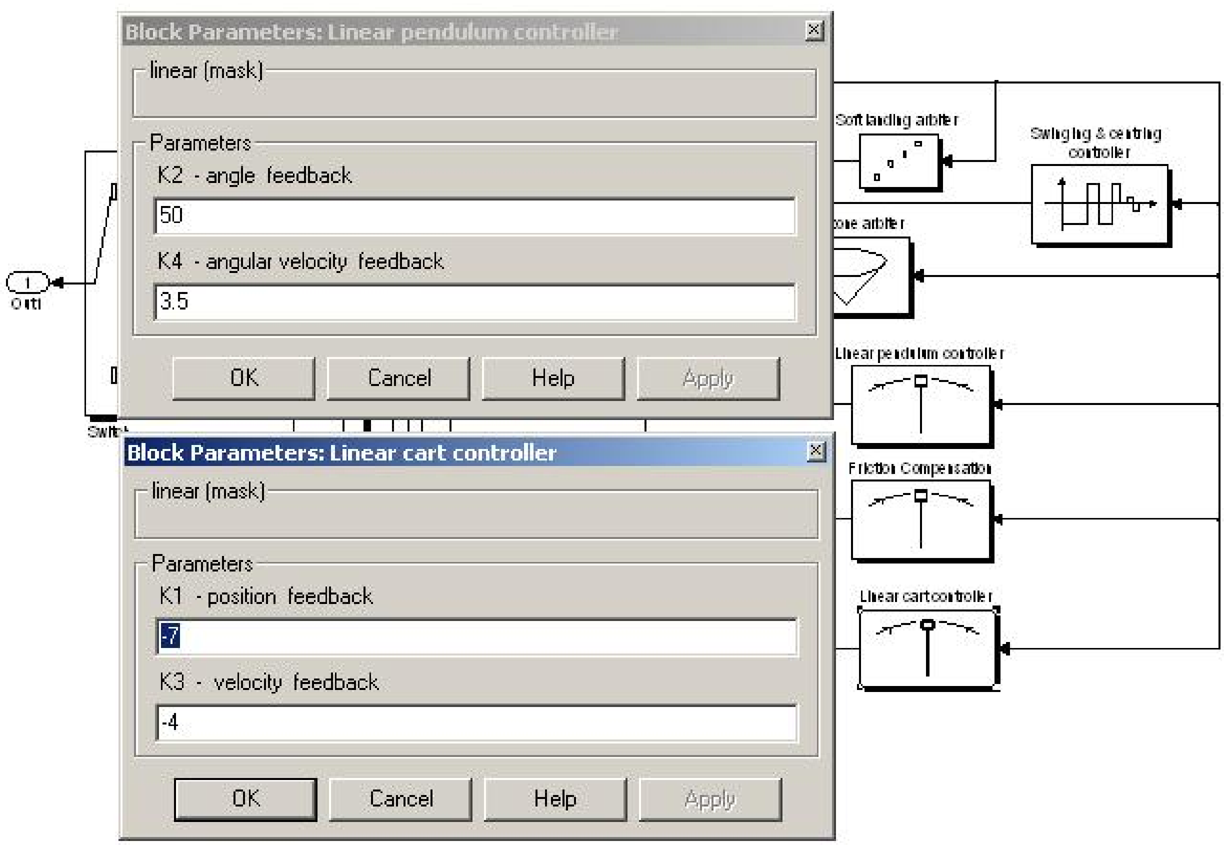
(When clicking on the blue control system)



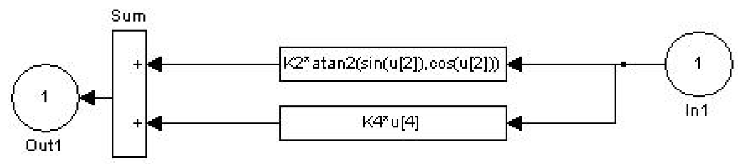
(Inverted pendulum controller)

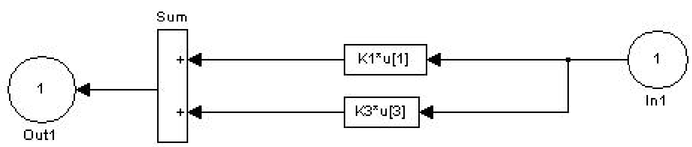


(Cart and pendulum control Gains given by the Feedback)

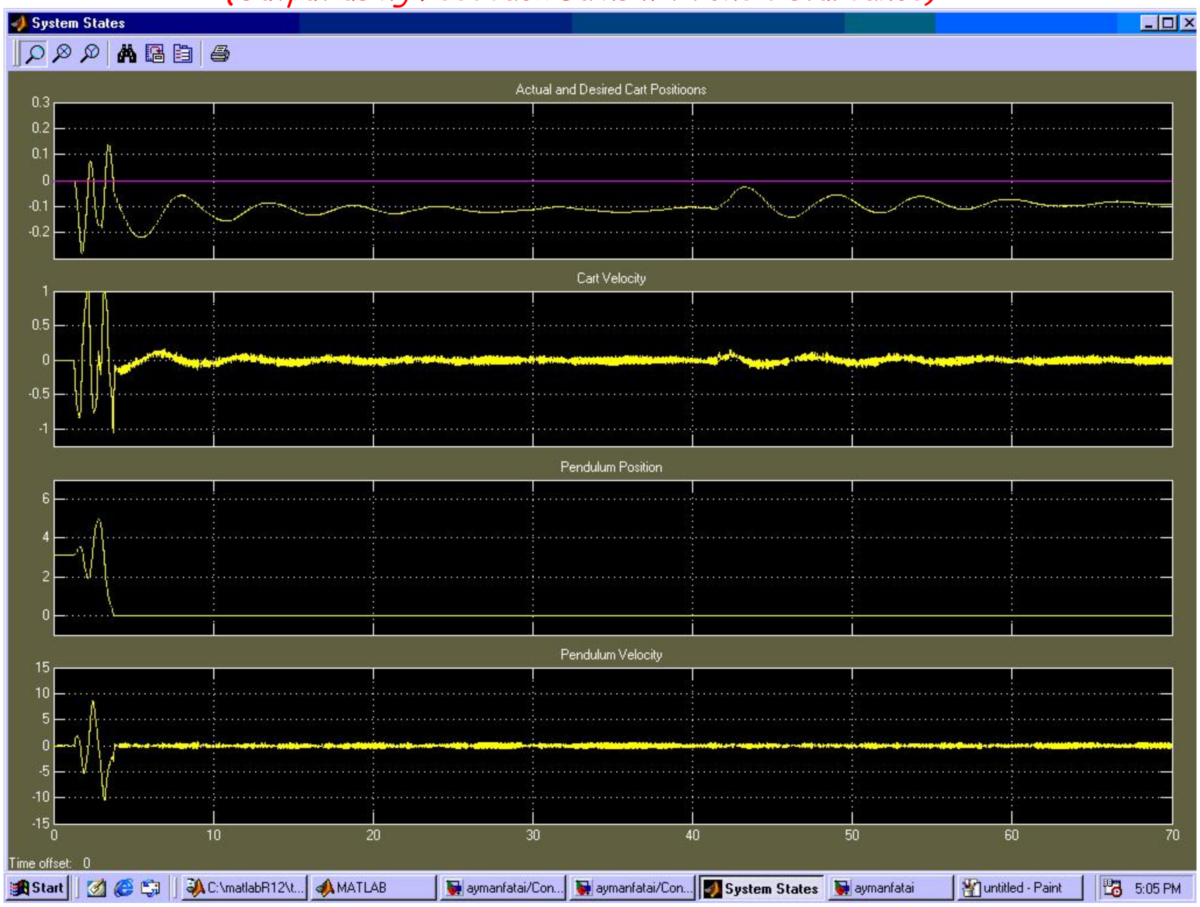


(For angle feedback the atan2(sin(u[2],cos(u[2])) to take care of quadrant)

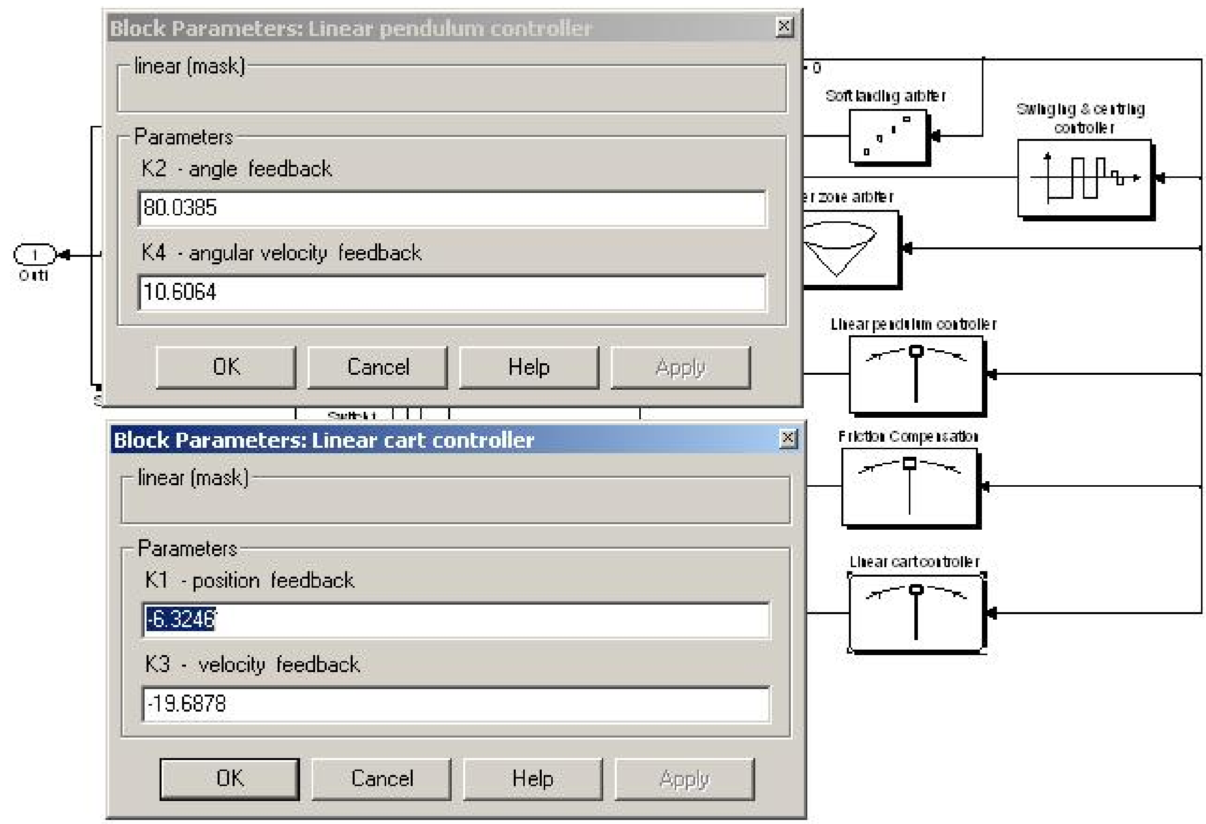




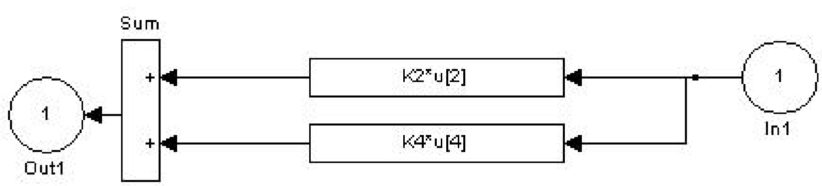
(Output using Feedback Gains with one disturbance)

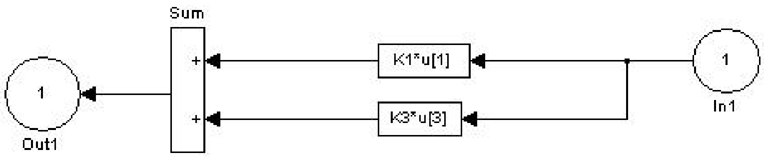


(Our Cart and pendulum control Gains)

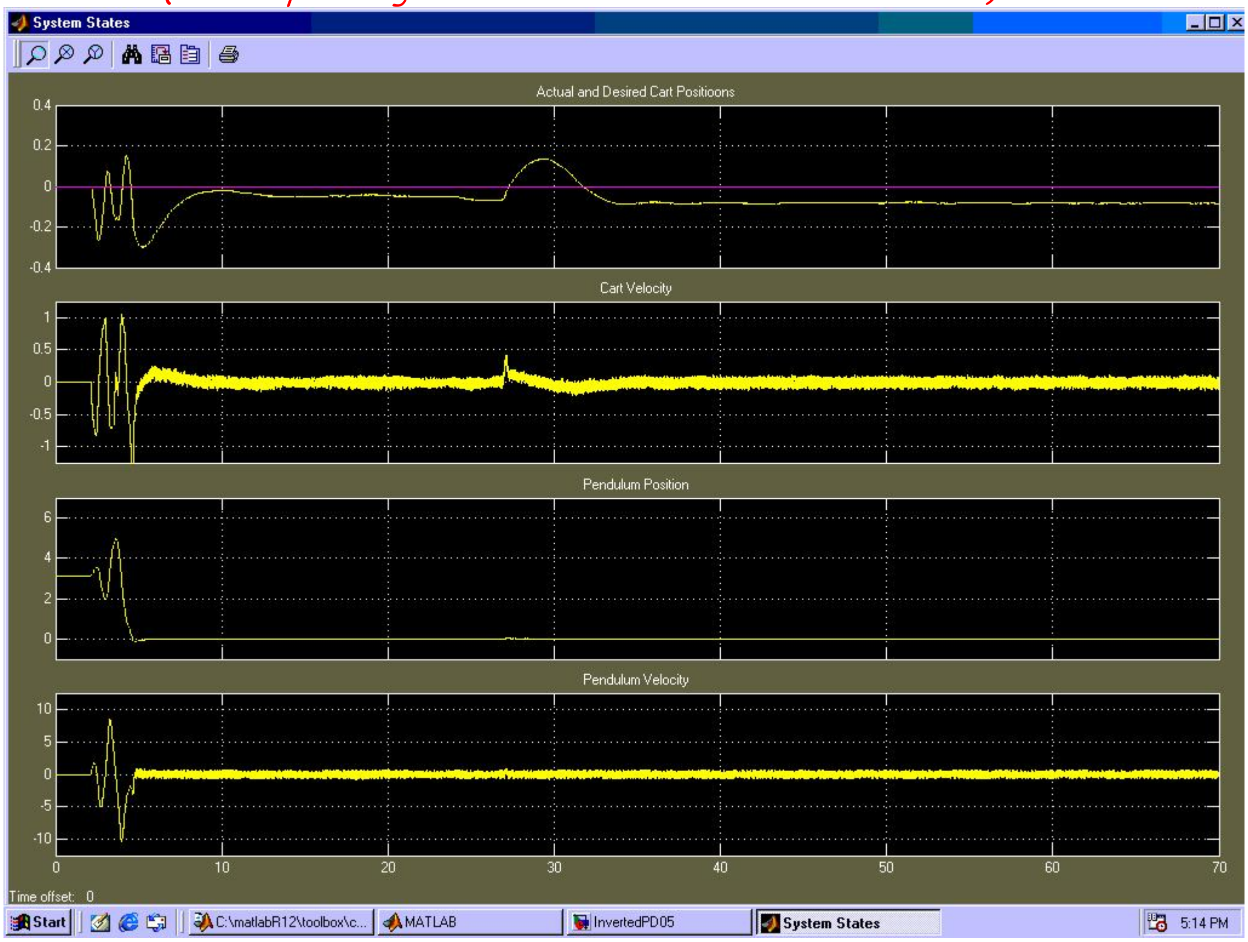


(Since our model was linearized around the upvertical we don't need to take care of quadrant)

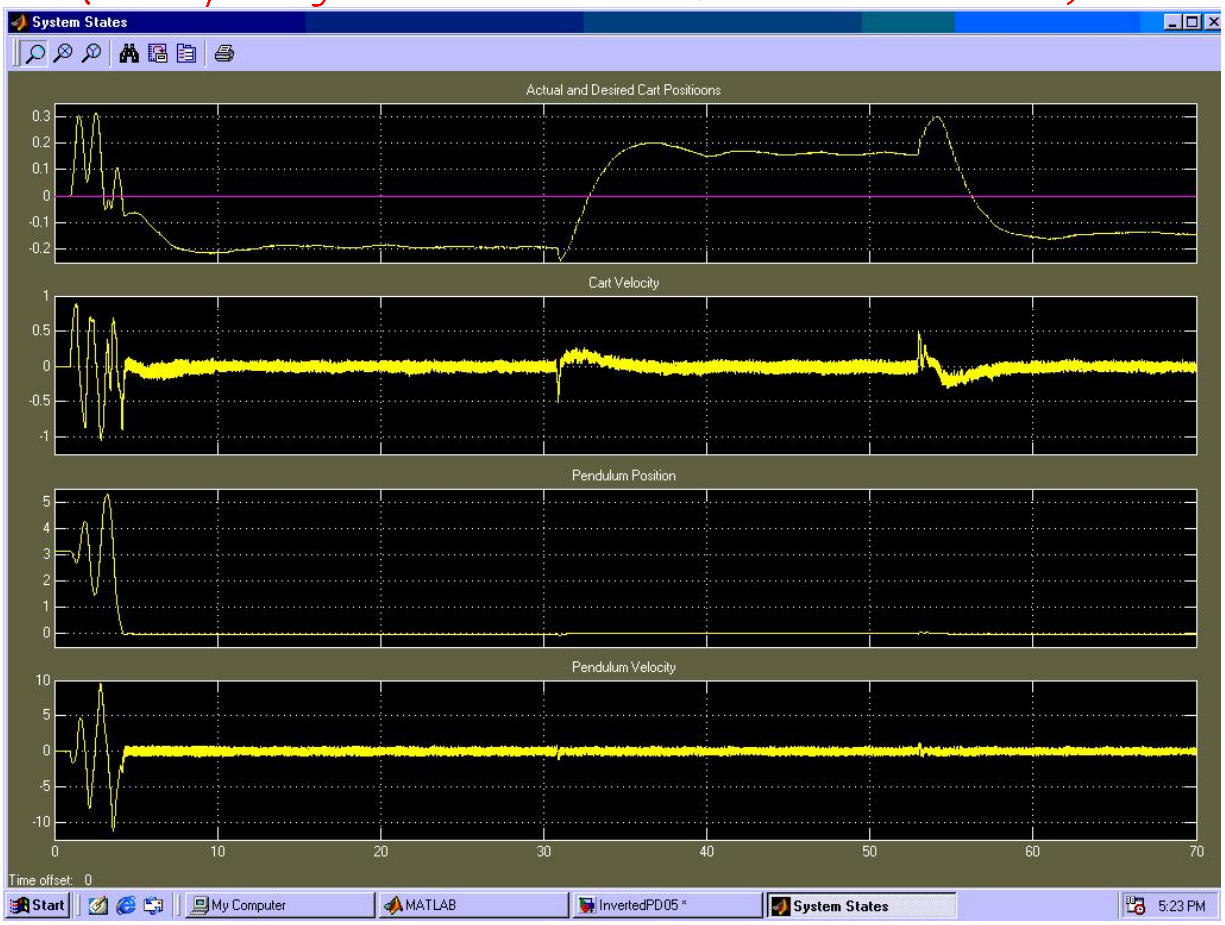




(Our Output using our Model and Gains with one disturbance)



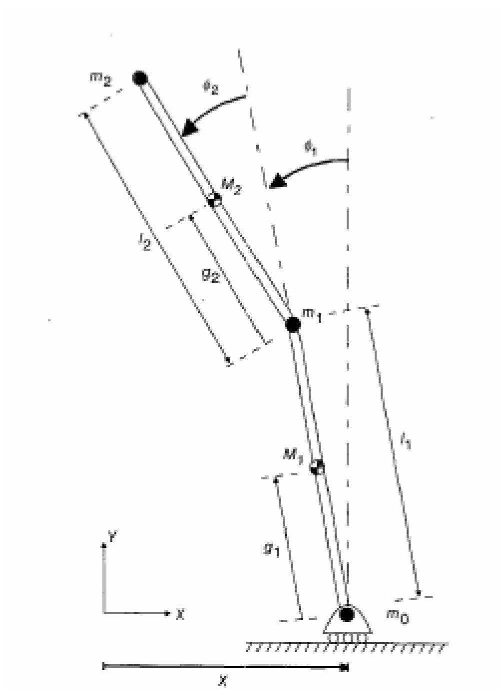
(Our Output using our Model and Gains with more than one disturbances)



Double inverted pendulum

(Here we are going to follow the same procedures of single inverted pendulum)

Description of the System



Where

M_1	mass of the 1st pendulum
M_2	mass of the 2nd pendulum
m_0	mass of the cart
l_1	length of the 1st pendulum
l_2	length of the 2nd pendulum
$\varnothing_1$	angle of 1st pendulum from vertical
$\varnothing_2$	angle of 2nd pendulum with respect to the 1st pendulum
x	cart position coordinate (horizontal)
g	gravitation constant, 9.81 m/s^2
F	force applied to the cart

Modeling of the system

In order to obtain the double inverted pendulum's model, the system's dynamics is analyzed using the Lagrange Method. Our system is a Three Degree of Freedom System (i.e. x, ϕ_1 and ϕ_2)

Potential Energy:

$$V = (M_1 g_1 + m_1 l_1 + M_2 l_1 + m_2 l_1) g \cos\phi_1 + (M_2 g_2 + m_2 l_2)\cos(\phi_1 + \phi_2)$$

Kinetic Energy

$$T = \tfrac{1}{2} m_0 \dot{x} 2^2 + \tfrac{1}{2} M_1 \left[(\dot{x} - g_1 \dot{\phi}_1 \cos\phi_1)^2 + (g_1 \dot{\phi}_1 \sin\phi_1)^2 \right] + \tfrac{1}{2} I_1 \dot{\phi}_1^{\,2}$$
$$+ \tfrac{1}{2} m_1 \left[(\dot{x} - l_1 \dot{\phi}_1 \cos\phi_1)^2 + (l_1 \dot{\phi}_1 \sin\phi_1)^2 \right]$$
$$+ \tfrac{1}{2} M_2 \left[\left(\dot{x} - l_1 \dot{\phi}_1 \cos\phi_1 - g_2 (\dot{\phi}_1 + \dot{\phi}_2) \cos(\phi_1 + \phi_2) \right)^2 + \left(l_1 \dot{\phi}_1 \sin\phi_1 + g_2 (\dot{\phi}_1 + \dot{\phi}_2) \sin(\phi_1 + \phi_2) \right)^2 \right]$$
$$+ \tfrac{1}{2} I_2 (\dot{\phi}_1^{\,2} + \dot{\phi}_2^{\,2})$$
$$+ \tfrac{1}{2} m_2 \left[\left(\dot{x} - l_1 \dot{\phi}_1 \cos\phi_1 - g_2 (\dot{\phi}_1 + \dot{\phi}_2) \cos(\phi_1 + \phi_2) \right)^2 + \left(l_1 \dot{\phi}_1 \sin\phi_1 + g_2 (\dot{\phi}_1 + \dot{\phi}_2) \sin(\phi_1 + \phi_2) \right)^2 \right]$$

Lagrange Equations:

- $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) - \frac{\partial T}{\partial x} - \frac{\partial V}{\partial x} = F$
- $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\phi}}\right) - \frac{\partial T}{\partial \phi} - \frac{\partial V}{\partial \phi} = 0$
- $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\phi}}\right) - \frac{\partial T}{\partial \phi} - \frac{\partial V}{\partial \phi} = 0$

So we get.

$$
\begin{aligned}
&(m_0 + M_1 + m_1 + M_2 + m_2)\ddot{x} \\
&-\left[(M_1 g_1 + m_1 l_1 + M_2 l_1 + m_2 l_1)\cos\phi_1 + (M_2 g_2 + m_2 l_2)\cos(\phi_1 + \phi_2)\right]\ddot{\phi}_1 \\
&-(M_2 g_2 + m_2 l_2)\cos(\phi_1 + \phi_2)\ddot{\phi}_2 \\
&+(M_1 g_1 + m_1 l_1 + M_2 l_1 + m_2 l_1)\sin\phi_1\dot{\phi}_1^{\,2} \\
&+\left[M_2 g_2 (\dot{\phi}_1 + \dot{\phi}_2)^2 + m_2 l_2 (\dot{\phi}_1 + \dot{\phi}_2)^2\right]\sin(\phi_1 + \phi_2) = F
\end{aligned}
$$

$$
\begin{aligned}
&-\left[(M_1 g_1 + m_1 l_1 + M_2 l_1 + m_2 l_1)\cos\phi_1 + (M_2 g_2 + m_2 l_2)\cos(\phi_1 + \phi_2)\right]\ddot{x} \\
&-\left[M_1 g_1^{\,2} + I_1 + (m_1 + M_2 + m_2) l_1^{\,2} + 2(M_2 g_2 + m_2 l_2) l_1 \cos\phi_2 + M_2 g_2^{\,2} + I_2 + m_2 l_2^{\,2}\right]\ddot{\phi}_1 \\
&+\left[M_2 g_2^{\,2} + I_2 + m_2 l_2^{\,2} + (M_2 g_2 + m_2 l_2) l_1 \cos\phi_2\right]\ddot{\phi}_2 \\
&-2(M_2 g_2 + m_2 l_2) l_1 \sin\phi_2 \dot{\phi}_1 \dot{\phi}_2 - (M_2 g_2 + m_2 l_2) l_1 \sin\phi_2 \dot{\phi}_2^{\,2} \\
&-g\left[(M_2 g_2 + m_2 l_2)\sin(\phi_1 + \phi_2) + (M_1 g_1 + m_1 l_1 + M_2 l_1 + m_2 l_1)\sin\phi_2\right] = 0
\end{aligned}
$$

$$
\begin{aligned}
&-\left[(M_2 g_2 + m_2 l_2)\cos(\phi_1 + \phi_2)\right]\ddot{x} \\
&+\left[(M_2 g_2 + m_2 l_2) l_1 \cos\phi_2 + M_2 g_2^{\,2} + I_2 + m_2 l_2^{\,2}\right]\ddot{\phi}_1 + \left[M_2 g_2^{\,2} + I_2 + m_2 l_2^{\,2}\right]\ddot{\phi}_2 \\
&+(M_2 g_2 + m_2 l_2) l_1 \sin\phi_2 \dot{\phi}_1^{\,2} - g(M_2 g_2 + m_2 l_2)\sin(\phi_1 + \phi_2) = 0
\end{aligned}
$$

Linearization

These three equations will be linearized about $\phi = \pi$. Assume that $\phi_1 = \pi + \theta_1$ and $\phi_2 = \pi + \theta_2$ (θ represents a small angle from the vertical upward direction).

Therefore,

$$\cos\phi_1 = -1,\ \sin\phi_1 = -\theta_1,\ \cos\phi_2 = -1,\ \sin\phi_2 = -\theta_2,$$
$$\cos(\phi_1 + \phi_2) = 1 - \theta_1\theta_2,\ \sin(\phi_1 + \phi_2) = \theta_1 + \theta_2$$
$$\dot{\phi}_1^{\,2} = 0,\ \dot{\phi}_2^{\,2} = 0,\ \dot{\phi}_1\dot{\phi}_2 = 0,\ F = u \text{ (where u represents the input)}$$

So that the Equations of Motion become:

- $$(m_0 + M_1 + m_1 + M_2 + m_2)\ddot{x} + [(M_1 g_1 + m_1 l_1 + M_2 l_1 + m_2 l_1)]\ddot{\theta}_1 - (M_2 g_2 + m_2 l_2)\ddot{\phi}_2 = u$$

- $$\begin{aligned}&-[(M_2 g_2 + m_2 l_2) - (M_1 g_1 + m_1 l_1 + M_2 l_1 + m_2 l_1)]\ddot{x}\\ &-\left[M_1 g_1^{\,2} + I_1 + (m_1 + M_2 + m_2)l_1^{\,2} - 2(M_2 g_2 + m_2 l_2)l_1 + M_2 g_2^{\,2} + I_2 + m_2 l_2^{\,2}\right]\ddot{\theta}_1\\ &+\left[M_2 g_2^{\,2} + I_2 + m_2 l_2^{\,2} - (M_2 g_2 + m_2 l_2)l_{12}\right]\ddot{\theta}_2\\ &-g[(M_2 g_2 + m_2 l_2)(\theta_1 + \theta_2) - (M_1 g_1 + m_1 l_1 + M_2 l_1 + m_2 l_1)\theta_1] = 0\end{aligned}$$

- $$\begin{aligned}&-[M_2 g_2 + m_2 l_2]\ddot{x}\\ &+\left[M_2 g_2^{\,2} + I_2 + m_2 l_2^{\,2} - (M_2 g_2 + m_2 l_2)l_1\right]\ddot{\theta}_1 + \left[M_2 g_2^{\,2} + I_2 + m_2 l_2^{\,2}\right]\ddot{\theta}_2\\ &-g(M_2 g_2 + m_2 l_2)(\theta_1 + \theta_2) = 0\end{aligned}$$

State-Space Representation

A state-space representation of the double inverted pendulum dynamics system can be derived from the two previously linearized equations. Using these parameters of the Pendulum-Cart setup.

m_0	1.1 kg	l_1	0.39 m
m_1	0.12 kg	l_2	0.395 m
m_2	0.02 kg	g_1	0.195 m
M_1	0.08 kg	g_2	0.1975 m
M_2	0.08 kg	I_i	$M_i l_i^2/12$

We get

$$\dot{x} = Ax + Bu$$

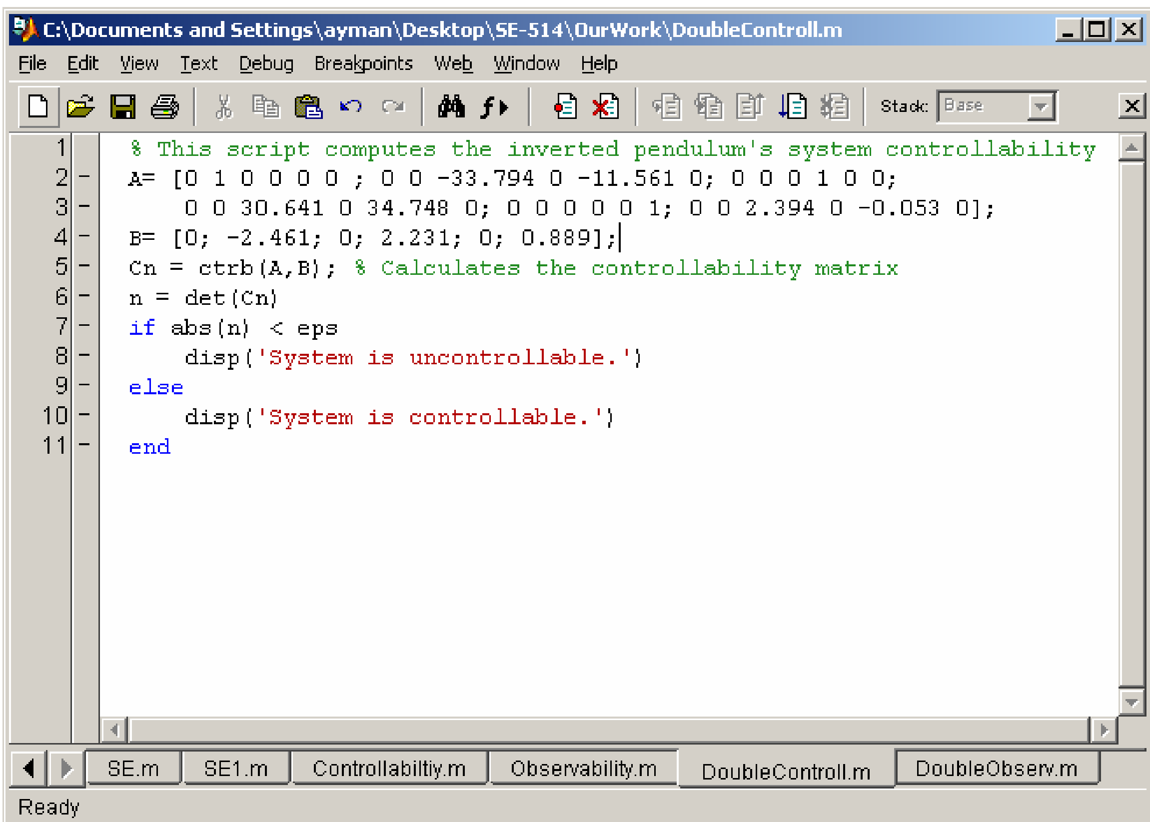
$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta}_1 \\ \ddot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -33.794 & 0 & -11.561 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0.641 & 0 & 34.748 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2.394 & 0 & -0.053 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -2.461 \\ 0 \\ 2.231 \\ 0 \\ 0.889 \end{bmatrix} u$$

$$y = Cx$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 00 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \end{bmatrix}$$

Controllability

The system described by the matrices (**A**,**B**) can said to be controllable if there exists an unconstrained control **u** that can transfer any initial state **x**(0) to any other desired location **x**(*t*). For the system $\dot{x} = Ax + Bu$, the system can be determined to be controllable if the determinant of the controllability matrix is nonzero.

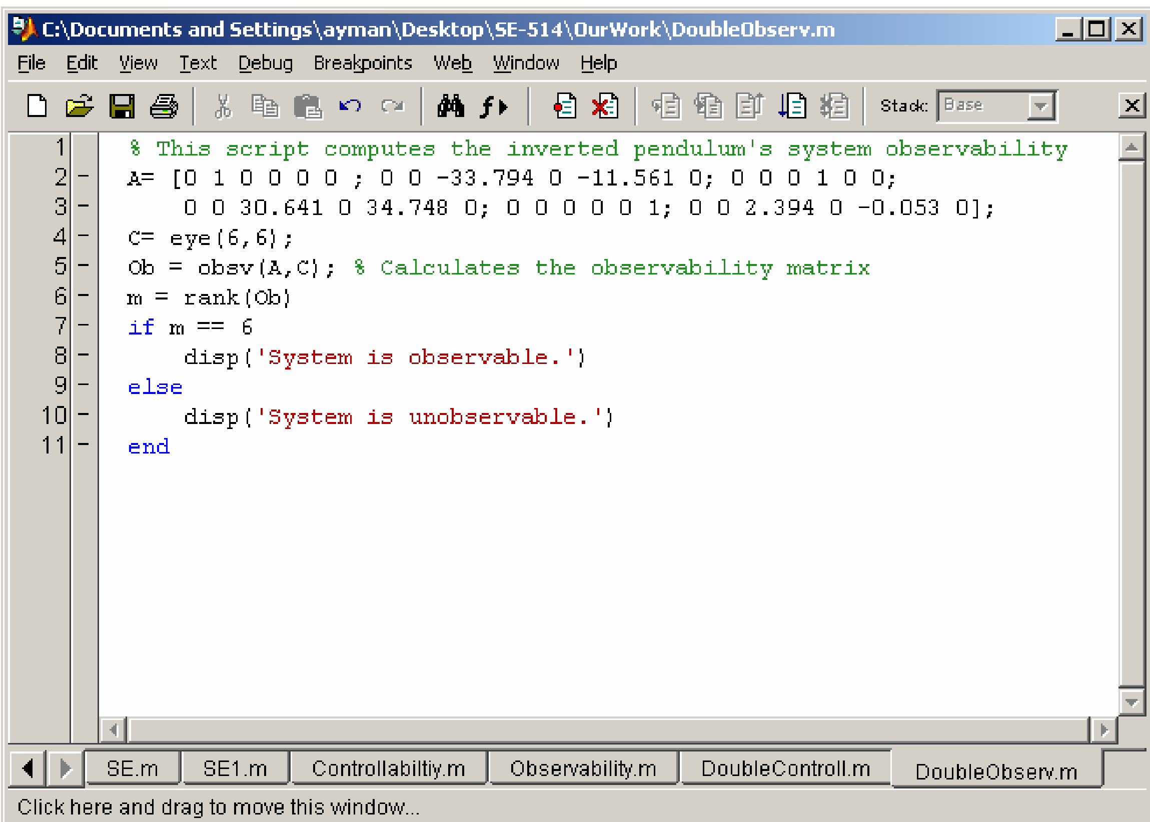



```
>>
n =
```

```
-2.0059e+009

System is controllable.
```

Observability

observability refers to the ability to estimate a state variable. A system is observable if, and only if, there exists a finite time *T* such that the initial state **x**(0) can be determined from the observation history *y*(*t*) given the control *u*(*t*). For the same system with output $y = Cx$, the system is observable if the rank of the observability matrix is 4, which is the full length of the observability matrix.




```
% This script computes the inverted pendulum's system observability
A= [0 1 0 0 0 0 ; 0 0 -33.794 0 -11.561 0; 0 0 0 1 0 0;
    0 0 30.641 0 34.748 0; 0 0 0 0 0 1; 0 0 2.394 0 -0.053 0];
C= eye(6,6);
Ob = obsv(A,C); % Calculates the observability matrix
m = rank(Ob)
if m == 6
    disp('System is observable.')
else
    disp('System is unobservable.')
end
```

```
>>
m =
```

6 System is observable.

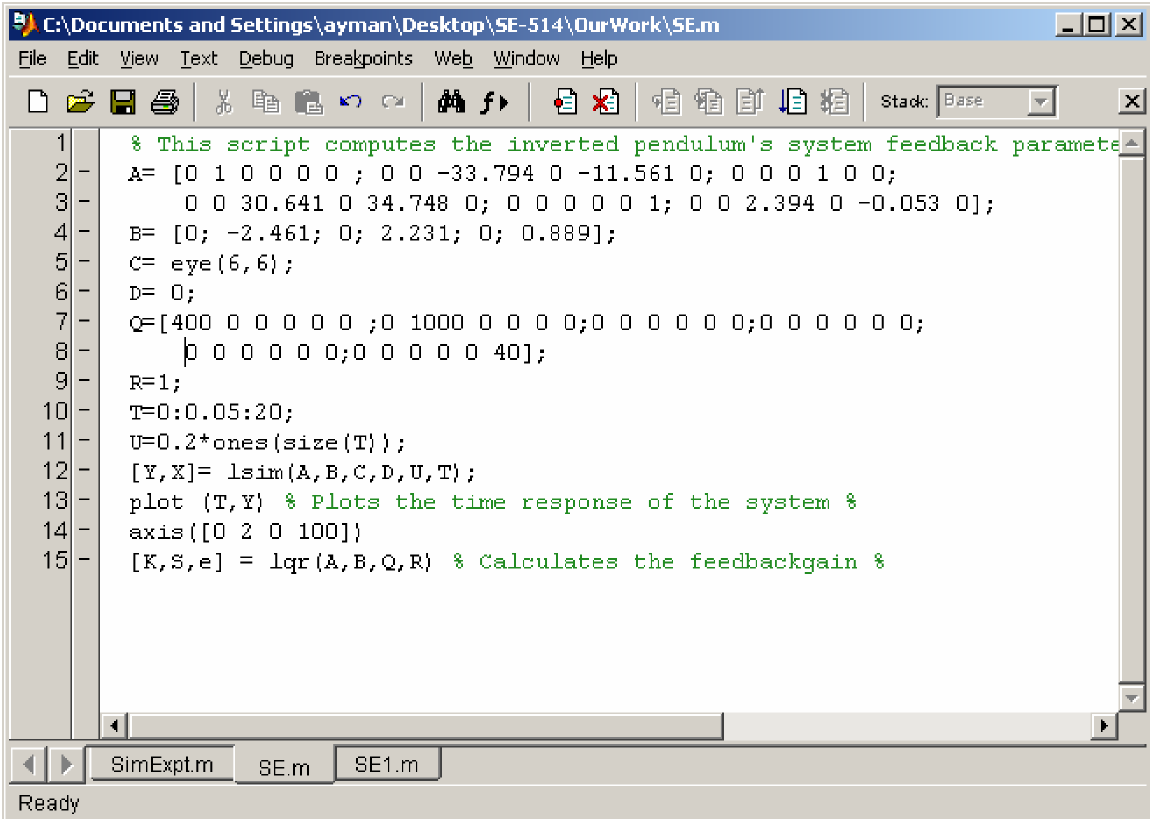
Simulation (off-line)

The open-loop behavior of the system can be observed by simulating a step response to the system. And It is observed with a step input, the system is unstable. Thus, a controller needs to be designed and implemented to improve and stabilize the system.

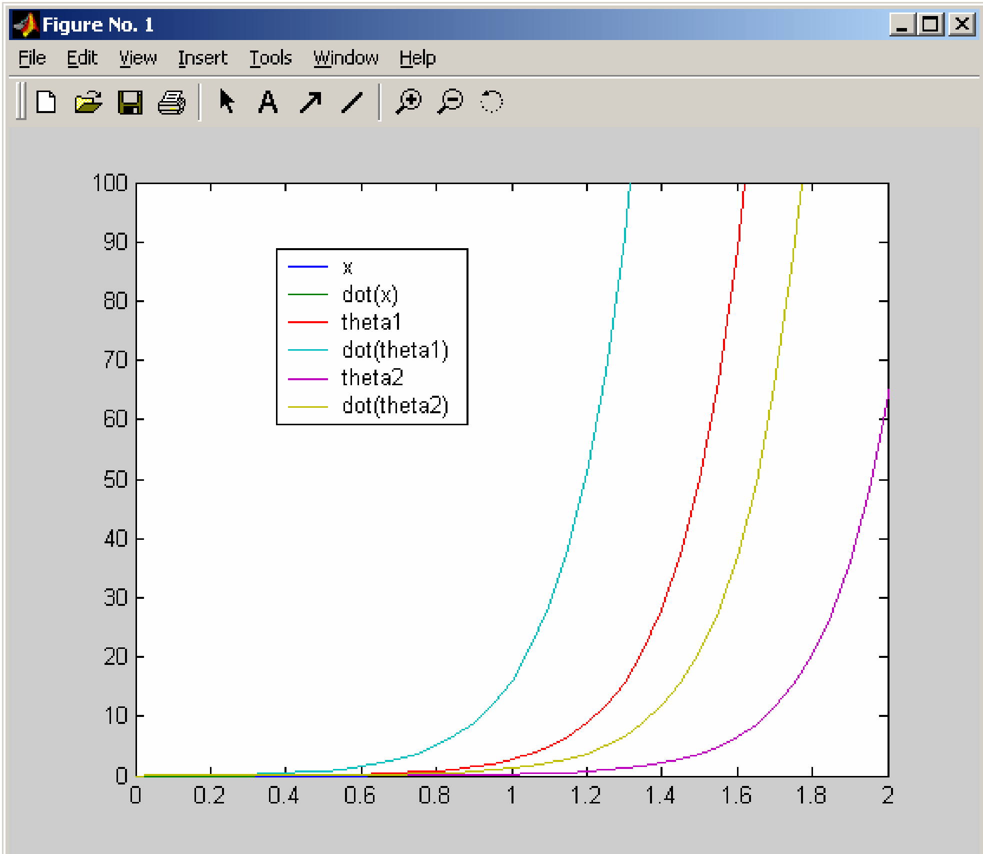
In order to stabilize the double inverted pendulum system, a state feedback approach is considered Shown in the block diagram.

A full-state feedback condition is assumed and the feedback gain, K of the system is to be determined. The feedback matrix gain can be calculated by using the LQR method, which will provide with the optimal controller values.

(This script Plots the time response of the system and computes the inverted pendulum's system feedback parameters)



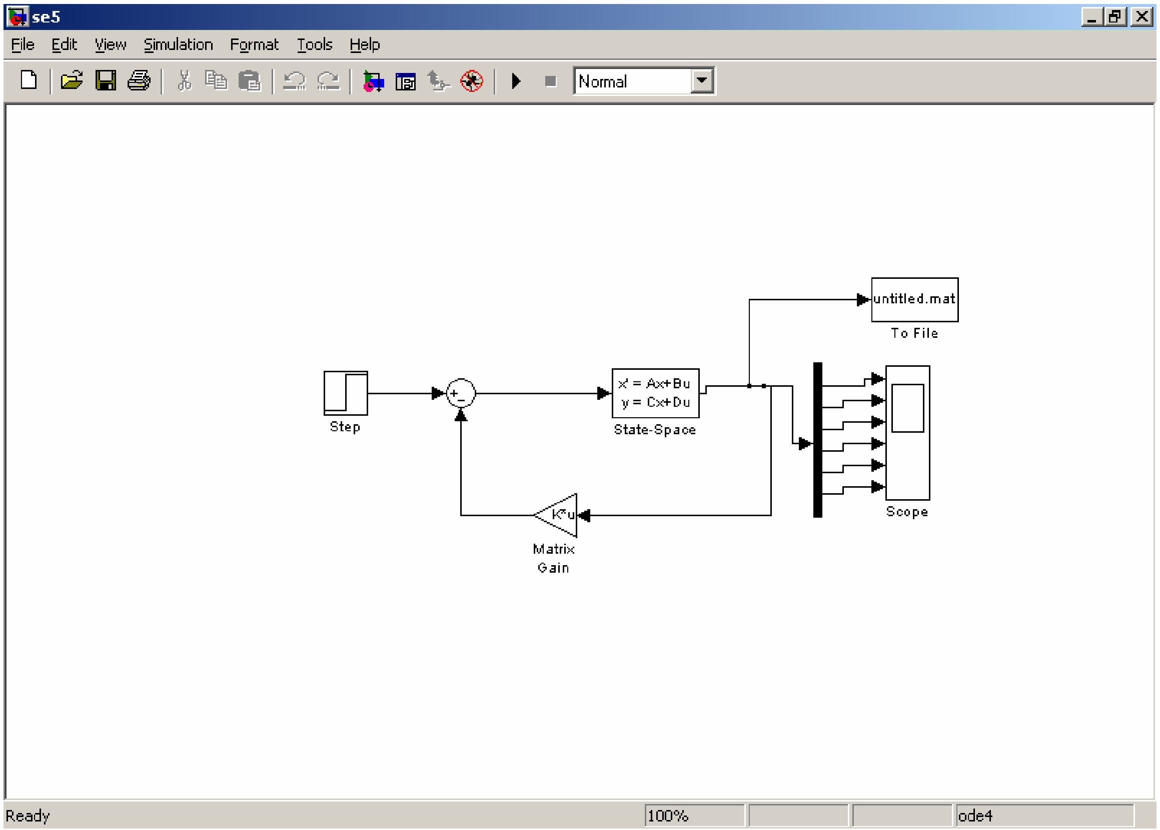
(Time response plot)



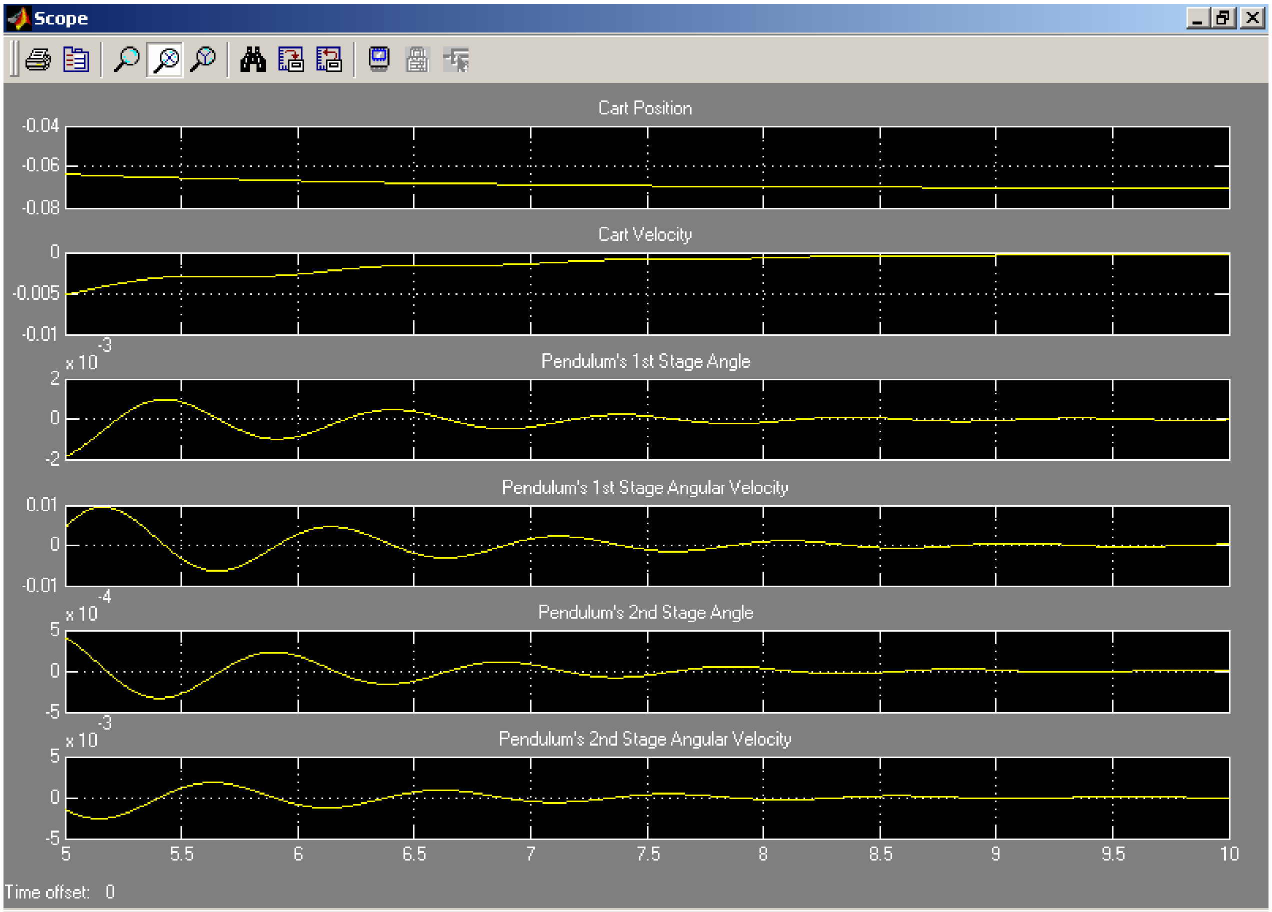
(Matlab output)

```
>>
K =
  -20.0000  -45.2943  439.6200   48.7811  -84.8873 -147.2131

S =
  1.0e+004 *

    0.0906    0.0526   -0.0976    0.0225    0.2944    0.0868
    0.0526    0.1029   -0.2435    0.0371    0.5800    0.1866
   -0.0976   -0.2435    2.1084    0.1190   -1.4396   -0.9231
    0.0225    0.0371    0.1190    0.0419    0.2050    0.0032
    0.2944    0.5800   -1.4396    0.2050    3.3235    1.0816
    0.0868    0.1866   -0.9231    0.0032    1.0816    0.4922

e =
 -78.4661
  -2.7411 + 2.9499i
  -2.7411 - 2.9499i
  -2.4232 + 2.9484i
  -2.4232 - 2.9484i
  -0.6325
```

(Block Diagram of Full State Feedback System)



(Simulaink output)


Scope
Cart Position
-0.04
-0.06
-0.08
Cart Velocity
0
-0.005
-0.01
x 10^-3
Pendulum's 1st Stage Angle
2
0
-2
Pendulum's 1st Stage Angular Velocity
0.01
0
-0.01
x 10^-4
Pendulum's 2nd Stage Angle
5
0
-5
x 10^-3
Pendulum's 2nd Stage Angular Velocity
5
0
-5
5 5.5 6 6.5 7 7.5 8 8.5 9 9.5 10
Time offset: 0

Simulation (Real-Time)

Not accomplished (equipments not available)

Conclusion

From the results of the simulations offline of the single and double inverted pendulums, it can be concluded that the unstable trajectory of both systems with just a step input to the system, was controlled with the use of the kalman gain to stabilize the trajectory. However, the controllability and observability of the system was verified and found satisfactory before a feedback closed loop system was created from an open loop system.

During implementation in real time situation, it was noticed that the cart responds to a small disturbance applied to the pendulum at the stable upright position. The cart moves to the opposite direction to the applied force, to compensate for the applied force and maintain the stable upright position of the pendulum.

Finally, it was observed that the designed controllers performed satisfactorily. Both offline and real time results as shown by the plots of the state variables, converged after the introduction of the controller.

REFERENCES

1. Furuta, K., Yamakita, M., and Kobayashi, S. ‘Swing-up control of inverted pendulum using pseudo-state feedback’, *J. System. Control Eng.*, 1992, 206, pp.263-269.

2. Wicklund, M., Kristenson, A., and Astrom, K.J. ‘A new strategy fro swinging up and inverted pendulum’. Proceedings of IFAC 12th World Congress, 1992, Vol. 9, pp. 151-154.

3. Astrom, K.J., Furuta, K. ‘Swinging up a pendulum by energy control’. IFAC 13th World Congress, San Francisco, California, 1996.

4. Rubi, J., Rubio, A., and Avello, A. ‘Swing-up problem for a self-erecting double inverted pendulum’, *IEE Proc.-Control Theory Appl*., Vol. 149, No. 2, March 2002.

5. CTM Example: Inverted Pendulum Modeling, http://www.engin.umich.edu/group/ctm/examples/pend/invpen.html.

6. Control in a MATLAB Environment – Digital Pendulum (33-005-1C), Feedback, England, 2001.

7. Kawashima, Takeshi. ‘Swing-up and stabilization of inverted pendulum using only one sliding mode controller with nonlinear model observer’.

8. Furuta, T.T.K., Suzuki, S.H.S., and Sugiki, A. ‘Swing-up control of inverted pendulum by periodic input’ Proceedings of IFAC 15th World Congress, Barcelona, Spain, 2002.