KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

AEROSPAC ENGINEERING DEPARTMENT

SEMESTER (041)

AE-540

(Flight Dynamics & Control I)

Project Final Report

Simulation and Auto Pilot Design for BEAVER Airpalne

Prepared For

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Sec # 01

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Introduction

The objective here is to simulate a 3-Dimentional Aircraft in motion specifically the Beaver Aircraft. Then, finding the trim conditions required to keep the aircraft at steady level, and using this values to simulate again the motion. After that, these nonlinear equations of motion will be linearized and the stability derivatives will be estimated for the longitudinal motion, finally, comes auto pilot and velocity hold design applying both the classical and the modern control theories.

Equations of Motion Simulation

The general nonlinear equations of motion of any aircraft are represented by 12 equations:

$$\dot{U} = \frac{1}{m}\left(F_x - mg\sin\theta\right) - QW + RV$$
$$\dot{V} = \frac{1}{m}\left(F_y + mg\cos\theta\sin\phi\right) - RU + PW$$
$$\dot{W} = \frac{1}{m}\left(F_z + mg\cos\theta\cos\phi\right) - PV + QU$$

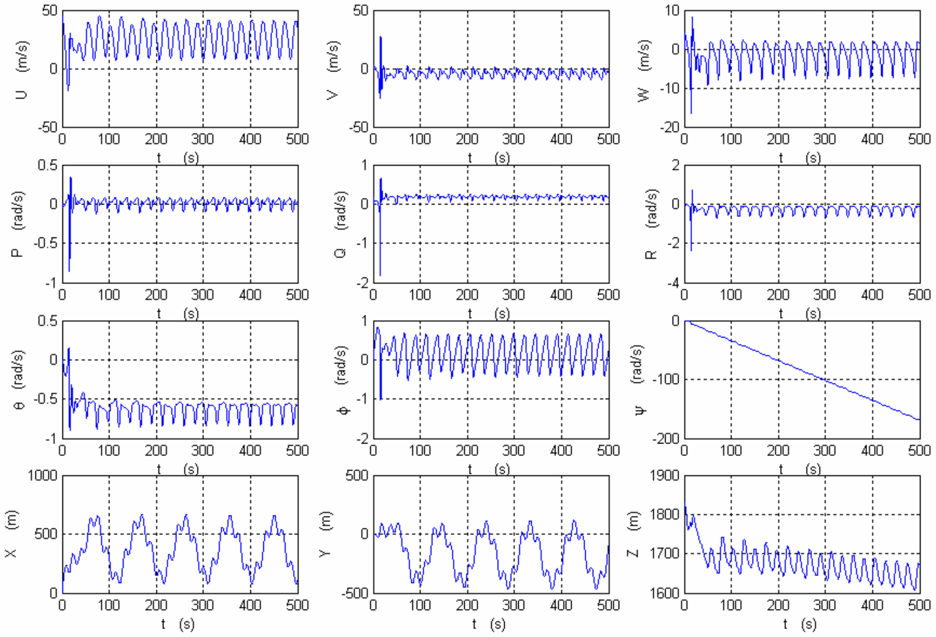
$$\dot{P} = \frac{1}{I_{xx}}\left[L + \dot{R}\,I_{xz} - QR\left(I_{zz} - I_{yy}\right) + PQI_{xz}\right]$$
$$\dot{Q} = \frac{1}{I_{yy}}\left[M - RQ\left(I_{xx} - I_{zz}\right) - I_{xz}\left(P^2 - R^2\right)\right]$$
$$\dot{R} = \frac{1}{I_{zz}}\left[N + \dot{P}\,I_{xz} - PQ\left(I_{yy} - I_{xx}\right) + QRI_{xz}\right]$$

$$\dot{\theta} = Q\cos\phi - R\sin\phi$$
$$\dot{\phi} = P + Q\sin\phi\tan\theta + R\cos\phi\tan\theta$$
$$\dot{\psi} = Q\sin\phi\sec\theta + R\cos\phi\sec\theta$$

$$\dot{X} = \left(\cos\theta\cos\psi\right)U + \left(\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi\right)V + \left(\cos\psi\sin\theta\cos\psi + \sin\phi\sin\psi\right)W$$
$$\dot{Y} = \left(\cos\theta\sin\psi\right)U + \left(\sin\phi\sin\theta\sin\psi - \cos\phi\cos\psi\right)V + \left(\cos\psi\sin\theta\sin\psi + \sin\phi\cos\psi\right)W$$
$$\dot{Z} = \left(-\sin\theta\right)U + \left(\sin\phi\cos\theta\right)V + \left(\cos\phi\cos\theta\right)W$$

Where the forces and moments are provided for the Beaver as nonlinear equations

Beaver's Motion Simulation



Trim Conditions

Trim conditions are found by assuming that there is no motion in the lateral direction besides, all angular rates are zero.

By using these assumptions the equations of motion turn to be:

$$F_x - mg\sin\theta = 0$$
$$F_y + mg\cos\theta\sin\phi = 0$$
$$F_z + mg\cos\theta\cos\phi = 0$$

$$L = 0$$
$$M = 0$$
$$N = 0$$

By using Matlab we get the trim values as:

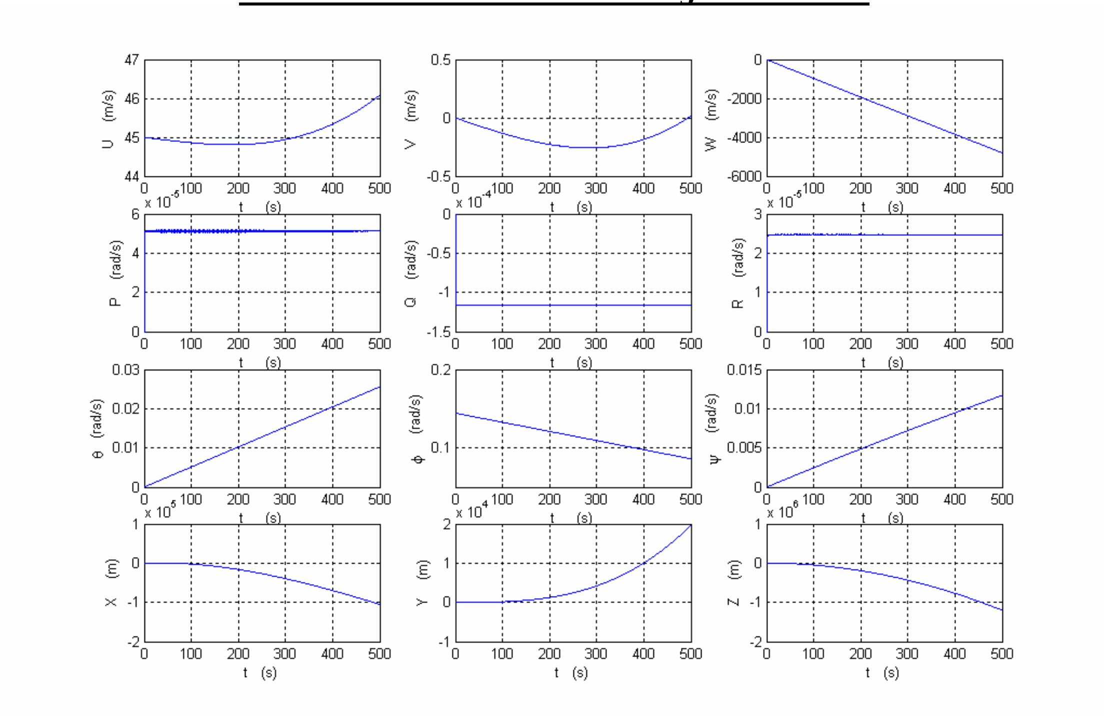
$$\alpha = 0.1931$$
$$\delta_a = 0.0109$$
$$\delta_r = 0.0117$$
$$\delta_e = -0.0074$$
$$\theta = 0.0404$$
$$\text{dpt} = -1.1092$$

Equations of Motion Simulation Using Trim Conditions

Using the trim values to simulate the Beaver motion (trim values used here which are given by the instructor)

$$V = 45$$
$$\alpha = 0.1444$$
$$\delta_a = 0.0091$$
$$\delta_r = \text{-}0.046$$
$$\delta_e = \text{-}0.0425$$
$$\theta = 0.1444$$
$$dpt = 0.56578$$
$$\beta = \text{-}0.0147$$

Beaver's Motion Simulation Using Trim Values



Equations of Motion Linearization

If we assume the motion consists of small deviations we can get the following two types of motion:

Longitudinal Equations of Motion

$$\begin{pmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{pmatrix} + \boldsymbol{B} \begin{pmatrix} \Delta\delta e \\ \Delta\delta T \end{pmatrix}$$

$$\boldsymbol{A} = \begin{pmatrix} X_u & X_w & 0 & -g \\ Z_u & Z_w & u_0 & 0 \\ M_u + M_{\dot{w}} Z_u & M_w + M_{\dot{w}} Z_w & M_q + M_{\dot{w}} u_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\boldsymbol{B} = \begin{pmatrix} X_{\delta e} & X_{\delta T} \\ Z_{\delta e} & Z_{\delta T} \\ M_{\delta e} + M_{\dot{w}} Z_{\delta e} & M_{\delta T} + M_{\dot{w}} Z_{\delta T} \\ 0 & 0 \end{pmatrix}$$

And

Lateral-Directional Dynamics

$$\begin{bmatrix} \Delta\dot{v} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{bmatrix} = \begin{bmatrix} Y_v & Y_p & -(u_0 - Y_r) & g \\ L_v^* + \frac{I_{xz}}{I_x} N_v^* & L_p^* + \frac{I_{xz}}{I_x} N_p^* & L_r^* + \frac{I_{xz}}{I_x} N_r^* & 0 \\ N_v^* + \frac{I_{xz}}{I_z} L_v^* & N_p^* + \frac{I_{xz}}{I_z} L_p^* & N_r^* + \frac{I_{xz}}{I_z} L_r^* & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta\phi \end{bmatrix} + \begin{bmatrix} 0 & Y_{\delta_r} \\ L_{\delta_a}^* + \frac{I_{xz}}{I_x} N_{\delta_a}^* & L_{\delta_r}^* + \frac{I_{xz}}{I_x} N_{\delta_r}^* \\ N_{\delta_a}^* + \frac{I_{xz}}{I_z} L_{\delta_a}^* & N_{\delta_a}^* + \frac{I_{xz}}{I_z} L_{\delta_a}^* \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{bmatrix}$$

where

$$L_v^* = \frac{L_v}{1 - \left(I_{xz}^2 / I_x I_z\right)}, \quad N_v^* = \frac{N_v}{1 - \left(I_{xz}^2 / I_x I_z\right)}$$

Stability Derivatives Estimation

Here we are going to estimate the longitudinal derivatives.

Using the Matlab code written to find matrix A and B:

$$A = \begin{bmatrix} -0.0249 & -0.0100 & 0 & -9.81 \\ -0.1871 & -1.3130 & 45 & 0 \\ 0.0688 & 0.3860 & -59.4777 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.3591 & 1.2220 \\ -4.1892 & -1.6451 \\ -9.0765 & 0.1693 \\ 0 & 0 \end{bmatrix}$$

Auto Pilot Hold Design

Using Classical Control Approach

Here Root Locus method will be implemented.

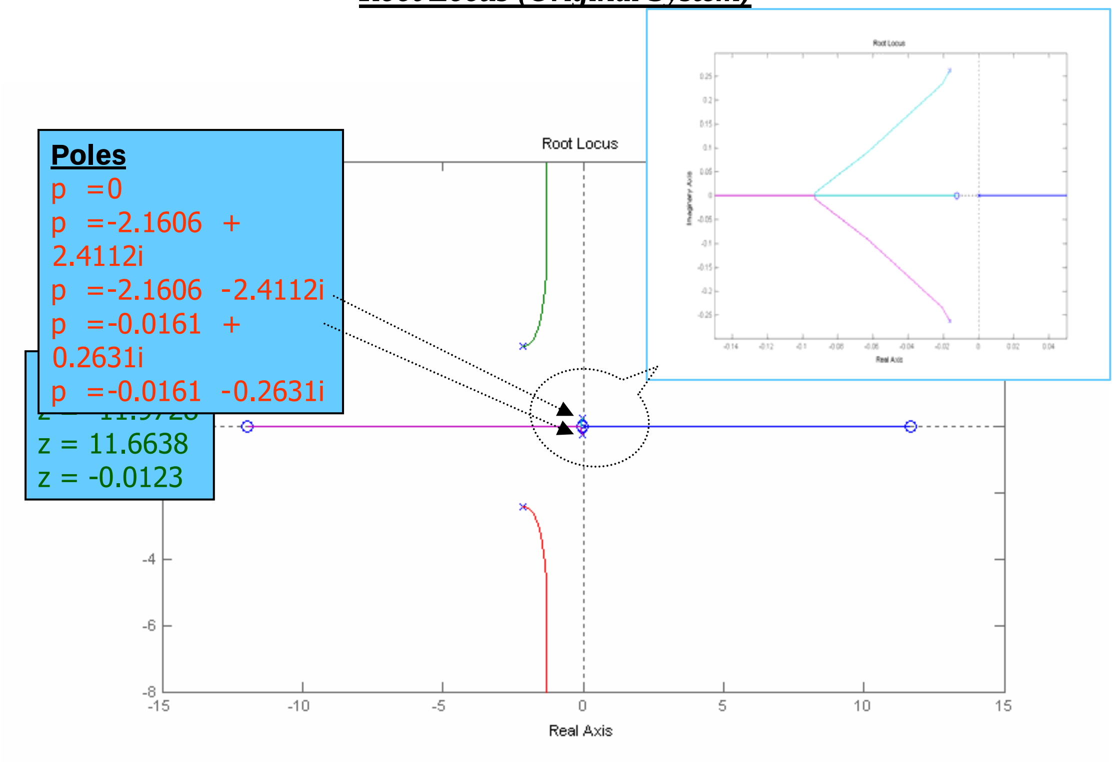
Altitude hold transfer function is:

$$\frac{\Delta h}{\Delta \delta_e} = \frac{u_0}{s}\left(\frac{\Delta q}{s\Delta \delta_e} - \frac{\Delta \alpha}{\Delta \delta_e}\right)$$

$$\frac{\Delta h}{\Delta \delta_e} = \frac{4.182\, s^3 + 1.344\, s^2 - 584\, s - 7.187}{s^5 + 4.354\, s^4 + 10.69\, s^3 + 0.6385\, s^2 + 0.7282\, s}$$

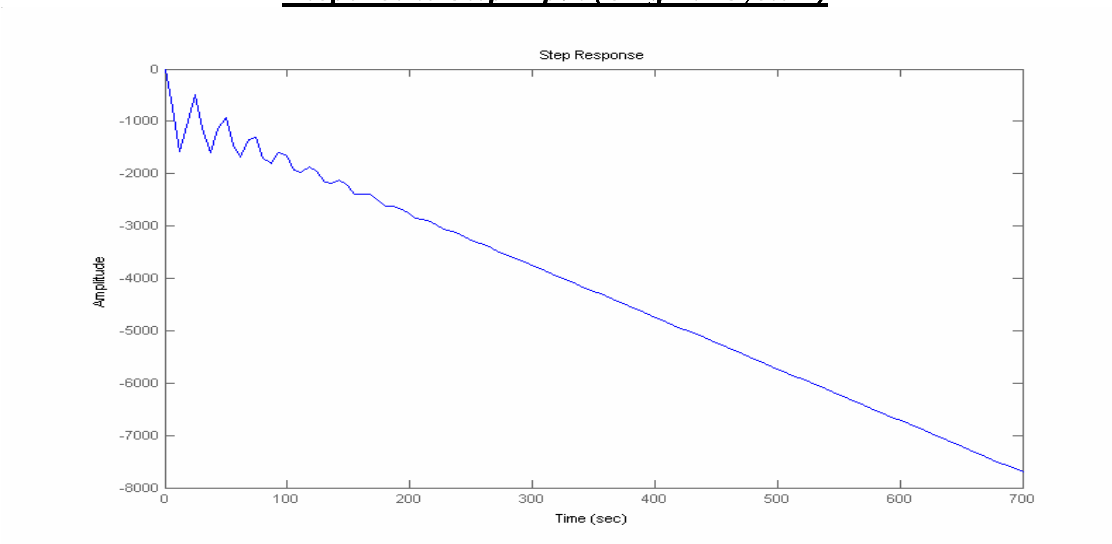
$$\frac{\Delta h}{\Delta \delta_e} = \frac{4.1822\,(s + 11.97)\,(s - 11.66)\,(s + 0.01231)}{s\,(s^2 + 0.03228 s + 0.06947)\,(s^2 + 4.321 s + 10.48)}$$

Root Locus (Original System)



We can see that there is a pole at 0 which makes the system marginally stable and there is a zero at 11.6638 which attracts the pole at 0 to the left side.

Response to Step Input (Original System)

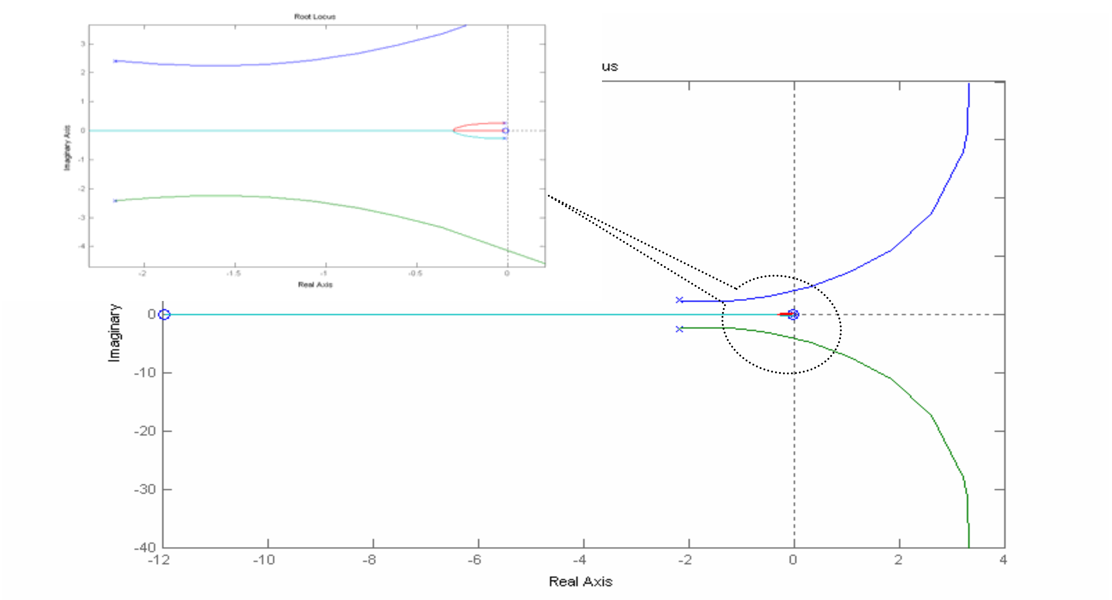


We can see below if we can get rid of the pole at 0 and also get rid of the zero at 11.66 we observe that the root locus on the right plan is gone which stabilizes the system as we can see from the plot of compensated system

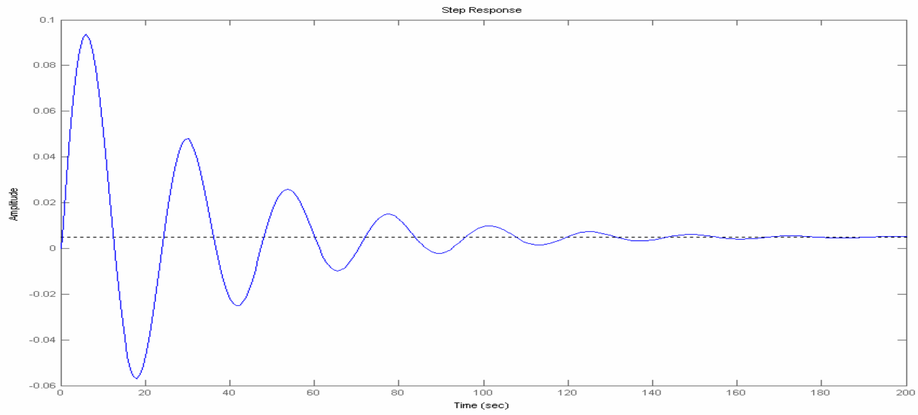
$$\frac{\Delta h}{\Delta \delta_e} = \left(\frac{s+z}{s+p}\right) \times \frac{u_0}{s}\left(\frac{\Delta q}{s\Delta \delta_e} - \frac{\Delta \alpha}{\Delta \delta_e}\right)$$

$$\frac{\Delta h}{\Delta \delta_e} = \left(\frac{s}{s-11.66}\right) \times \left(\frac{4.1822\,(s+11.97)\,(s-11.66)\,(s+0.01231)}{s\,(s^2+0.03228s+0.06947)\,(s^2+4.321s+10.48)}\right)$$

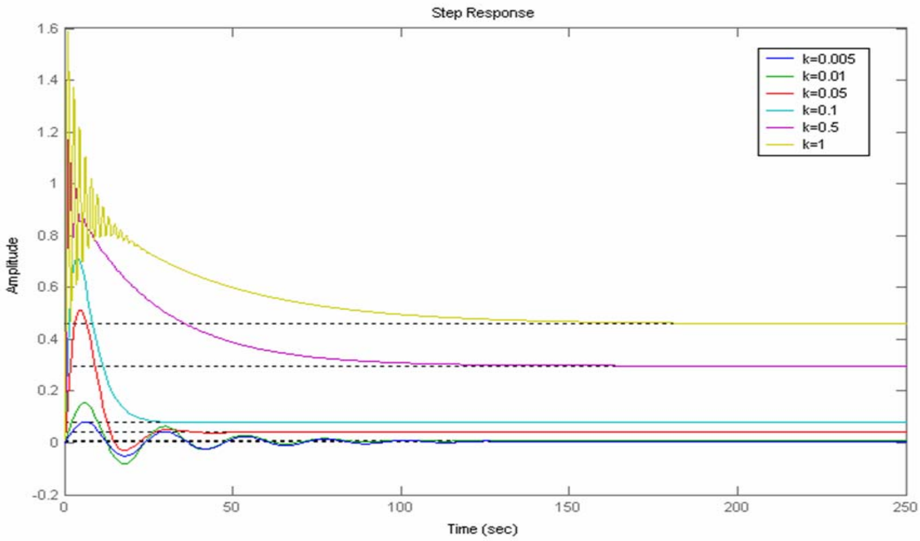
Root Locus (Compensator Added to System)



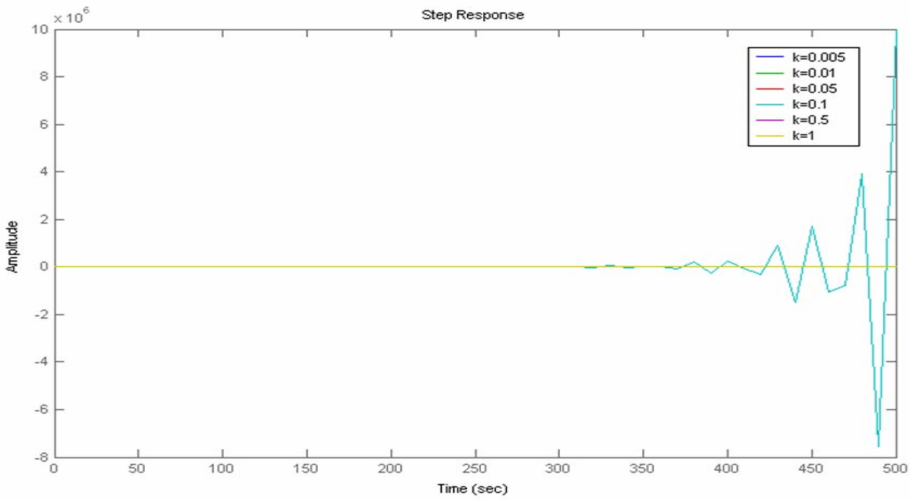
Response to Step Input (Adding Compensator)



Response to Step Input (Adding Compensator) with different k



Response to Step Input (Adding Compensator) at k=1.4



Using Modern Control Approach

Beaver according to flying quality requirements is classified as:

- Level 2
- Class II
- Category B

Which should has:

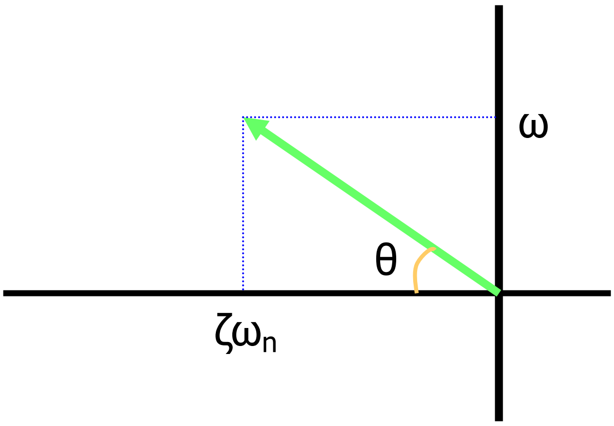
- Long Period (Phugoid) Mode

  $\zeta_{ph} > 0$

- Short Period Mode

  $0.2 < \zeta_{sp} < 2$

Here by specifying the settling time t_{ss} and the damping ratio ζ we can find the pole location directly:



$$\theta = \cos^{-1}\zeta$$

$$t_{ss} = 4T$$

$$\eta = \zeta\omega_n = \frac{1}{T} = \frac{4}{t_{ss}}$$

$$\omega = \zeta\omega_n \tan\theta = \frac{4}{t_{ss}}\tan\theta = \frac{4}{t_{ss}}\tan\left(\cos^{-1}\zeta\right)$$

$$s = -\zeta\omega_n \pm i\omega$$

$$s = -\frac{4}{t_{ss}} \pm i\frac{4}{t_{ss}}\tan\left(\cos^{-1}\zeta\right)$$

$$s = \frac{4}{t_{ss}}\left[-1 \pm i\tan\left(\cos^{-1}\zeta\right)\right]$$

My design requirements are:

For altitude hold we have to modify our system such that the altitude

Long Period (Phugoid) Mode:

$$t_{ss} = 200 \text{ seconds} \qquad \zeta_{ph} = 0.1 \quad \Longrightarrow \quad \boxed{s = -0.02 \pm i0.2}$$

Short Period Mode:

$$t_{ss} = 1.5 \text{ seconds} \qquad \zeta_{sp} = 0.9 \quad \Longrightarrow \quad \boxed{s = -2.67 \pm i1.29}$$

Is directly related, we assumed also that we velocity is held constant by some other system.

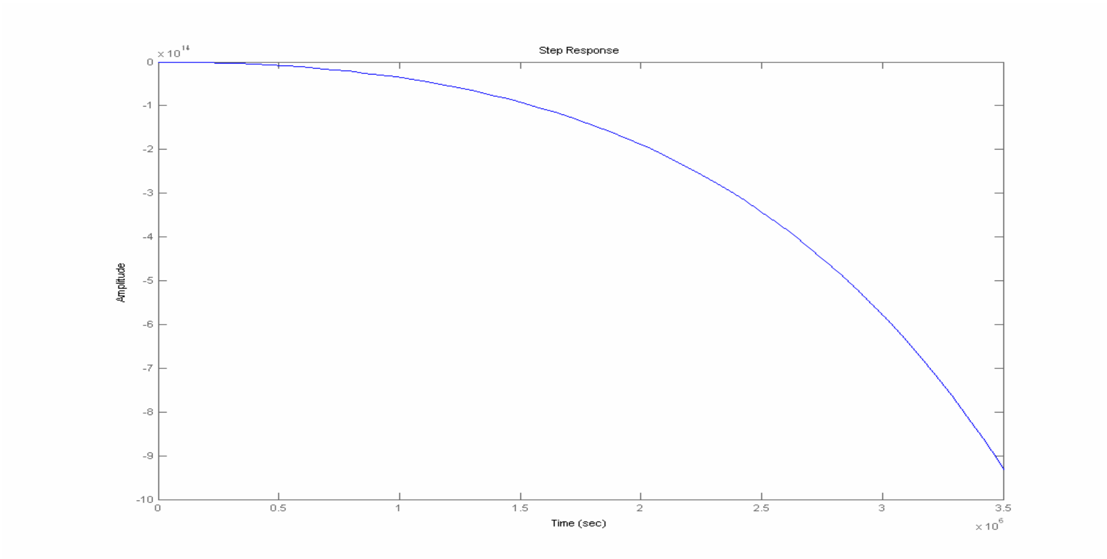
The first row and column will canceled and replaced by $\Delta\dot{h} = u_0(\Delta q - \Delta\alpha)$

$$\begin{bmatrix}\Delta\dot{u}\\ \Delta\dot{\alpha}\\ \Delta\dot{q}\\ \Delta\dot{\theta}\end{bmatrix} = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}\begin{bmatrix}\Delta u\\ \Delta\alpha\\ \Delta q\\ \Delta\theta\end{bmatrix} + \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}[\Delta\delta_e] \longrightarrow \begin{bmatrix}\Delta\dot{\alpha}\\ \Delta\dot{q}\\ \Delta\dot{\theta}\\ \Delta\dot{h}\end{bmatrix} = \begin{bmatrix} & & & 0\\ & & & 0\\ & & & 0\\ -u_0 & 0 & u_0 & 0\end{bmatrix}\begin{bmatrix}\Delta\alpha\\ \Delta q\\ \Delta\theta\\ \Delta h\end{bmatrix} + \begin{bmatrix} \\ \\ \\ 0\end{bmatrix}[\Delta\delta_e]$$

We can see in the next page the step response of the system wiyhout and with feedback gain (using Matlab)

Step Response (No FeedBack)

$$\frac{\Delta h(s)}{\Delta \delta_e(s)} = C[sI - A]^{-1} B \qquad C = [0 \quad 0 \quad 0 \quad 1]$$



Step Response (With FeedBack)

$$\frac{\Delta h(s)}{\Delta \delta_e(s)} = C[sI - (A - Bk)]^{-1} B$$

$s = -0.02 \pm i0.2$ $\quad$ $s = -2.67 \pm i1.29$ $\quad$ $k = [-0.0010 \quad 0.2793 \quad -0.0992 \quad 0.0005]$