FUNDAMENTAL ASPECTS OF RAREFIED GAS DYNAMICS AND HYPERSONIC FLOW

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Abstract: The project focuses on the understanding of the special character of very high speed flows (hypersonic) such as encountered in spacecraft entry to a planetary atmosphere, and launch to orbit, in the high density (continuum) and low density (rarefied) regimes.

1. INTRODUCTION

The study of rarefied gas dynamics examines the underlying physical processes of gas dynamics in terms of the motion of the individual molecules of which the gas is composed. Investigate the limits of the continuum gas assumptions of classical fluid mechanics.

In hypersonic flow study, hypersonic equivalence, small perturbations, blunt body analysis, viscous flows, oblique shock waves, Newtonian related surface pressure theories, friction and heat transfer and real gas effects (chemical and non-equilibrium) shall be discussed.

2. RAREFIED GAS DYNAMICS

Rarefied gases mechanics has been the subject of many investigations but until comparatively recently, these studies were confined to the case of very slow speeds and in general to internal flow geometries associated typically with vacuum installations. Since then, researches in this field expanded due to the possibility of flight at very high altitudes and at very high speeds.

2.1 RAREFIED GAS BACKGROUND

A rarefied gas flow is a flow in which the length of the molecular mean free path $\bar{l}$ is comparable to some significant dimension L of the flow field. The gas then does not behave entirely as a continuous fluid but rather exhibits some characteristics of its coarse molecular structure. The dimensionless ratio l/L is denoted by K and is called the Knudsen number. A rarefied gas flow is thus one for which K is not negligibly small. For some considerations, L may be a characteristic dimension of the body itself or the diameter of an internal flow conduit. For other considerations, L may be the boundary layer thickness, the diameter of a wind tunnel probe, or the thickness of the shock transition zone. In particular, one may expect to encounter rarefied gas effects in those regions of the flow possessing very sharp gradients, i.e. regions in which the velocity, pressure, or temperature change appreciably in the space of a few mean free paths regardless of whether or not the absolute density of the gas flow is especially low.

The Knudsen number K is related to the more familiar parameters of fluid mechanics, the Mach number M and the Reynolds number Re. From kinetic theory,

$$\nu = \frac{1}{2}\bar{l}\bar{v}_m \qquad (1)$$

Where ν is the kinematic viscosity and $\bar{v}_m$ is the mean molecular speed. The mean speed $\bar{v}_m$ is related to the sound speed a as follows:

$$a = \bar{v}_m\sqrt{\frac{\pi\gamma}{8}} \qquad (2)$$

Where γ is the isentropic exponent; hence from equations (1) and (2)

$$\bar{l} = 1.26\sqrt{\gamma}\frac{\nu}{a} \qquad (3)$$

Combining these results yields the fundamental relation

$$K = 1.26\sqrt{\gamma}\frac{M}{\mathrm{Re}} \qquad (4)$$

Where K and Re are both based on the same characteristic length L.

2.2 GAS DYNAMICS FLOW REGIMES

The characterization of gas dynamics into various regimes, based on characteristic ranges of values of an appropriate Knudsen number. The various criteria which have been suggested are in some disagreement. In the present treatment the flow is divided into four basic types i.e. "continuum flow", "slip flow", "transition flow" and "free molecule flow" are defined so that they correspond to flows in which, roughly speaking, the density levels are, respectively, ordinary, slightly rarefied, moderately rarefied, and highly rarefied.

For flows of high Reynolds number, i.e. Re » 1, the significant characteristic dimension of the flow field, which is of importance in the determination of viscous effects, is the boundary layer thickness δ rather than a dimension L typical of the body itself. Since

$$\frac{\delta}{L} \sim \frac{1}{\sqrt{\mathrm{Re}}} \qquad (5)$$

The corresponding Knudsen number K is given by

$$K \sim \frac{M}{\sqrt{\mathrm{Re}}} \qquad (6)$$

Ordinary gas dynamics hence prevails for $M\big/\sqrt{\mathrm{Re}} \ll 1$ and $\mathrm{Re} \gg 1$. On the other hand, for very small Reynolds numbers, the Stokes type "slow flow" occurs and the characteristic dimension itself is the significant parameter. Also, for any internal flow, only the diameter L of the conduit is of significance. Here the appropriate Knudsen number is simply K based on the body dimension and ordinary low speed continuum dynamics prevails for $M\big/\sqrt{\mathrm{Re}} \ll 1$. For flows in which the value of the appropriate Knudsen number is small but not negligible, some departure from continuum gas dynamics phenomena may be expected to occur. As shown in more detail below, one of the more striking of these effects is the phenomenon of "slip"; the layer of gas immediately adjacent to a solid surface is no longer at rest but has a finite tangential velocity. The term "slip flow" is thus appropriate for flows of small but not negligible Knudsen number. The change from ordinary continuum gas dynamics to this regime is of course gradual; but to make the ideas clear, the slip flow regime on the basis of our present experimental evidence is defined by the following limits:

$$\begin{array}{ll} 0.01 < \dfrac{M}{\sqrt{\mathrm{Re}}} < 0.1 & \mathrm{Re} > 1 \\ & , \qquad (7) \\ 0.01 < \dfrac{M}{\mathrm{Re}} < 0.1 & \mathrm{Re} < 1 \end{array}$$

These boundaries are shown in Fig. 1.

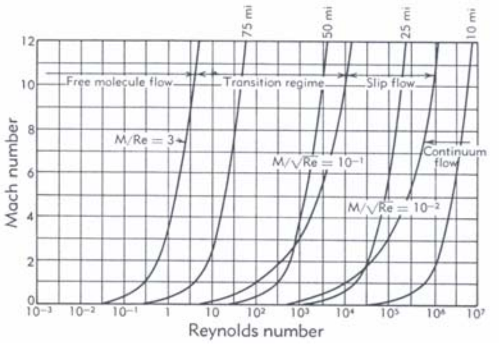


Fig. 1 Gas dynamics flow regimes

In the flow slip regime, so defined, the mean free path is of the order of 1 to 10 percent of the boundary layer thickness or the characteristic dimension of the flow field. Slip flow effects may thus be expected to be approximately of this same order. Thus true rarefaction effects such as slip occur only in coincidence with either strong viscous or compressibility effects. In the slip flow regime, as defined above, these latter phenomena very often dominate rarefaction effects associated with the coarse molecular structure of the gas, and really large scale deviations from continuum behavior are then not apparent until the " transition " regime, defined below, is reached. It might also be observed that in the hypersonic range, the boundary layer thickness is no longer given by Equation (5), so that appropriate modifications in the parameter $M\big/\sqrt{\mathrm{Re}}$ should be made here.

For extremely rarefied flows, the mean free path 1 is much greater than a characteristic body dimension L. Under these circumstances no boundary layer is formed. Molecules reemitted from a surface do not collide with free stream molecules until far away from the body. One may consequently neglect any distortion of the free stream velocity distribution due to the presence of the body: Here the flow phenomena are mostly governed by molecule-surface interaction. This regime of fluid mechanics is termed "free molecule flow" and may be defined on the basis of present experimental evidence by

$$\frac{M}{\mathrm{Re}} > 3 \tag{8}$$

In the transition regime between the slip flow and free molecule regimes, the mean free path is of the same order as a typical body dimension. Surface collisions and free stream intermolecular collisions are of more or less equal importance, and the analysis becomes extremely complicated. Present knowledge about this transition regime is very much more limited than that in the free molecule or the slip regime. Except for a few special theoretical results which are presented in subsequent articles, the information which is available is mostly empirical. For some theoretical considerations, that portion of the transition regime immediately adjacent to the free molecule flow regime becomes of special importance. It corresponds physically to the region in which the effects of a few intermolecular collisions begin to distort the free stream velocity distribution. This regime has not been given a separate designation, however, and in the present section it is not considered separately.

These various flow regimes are shown in Fig. 1, together with the corresponding altitudes in miles above sea level, provided that the characteristic body dimension is taken as 1 foot. The calculations are based on the NACA "proposed" standard atmosphere tables. The results are given in Fig. 2.

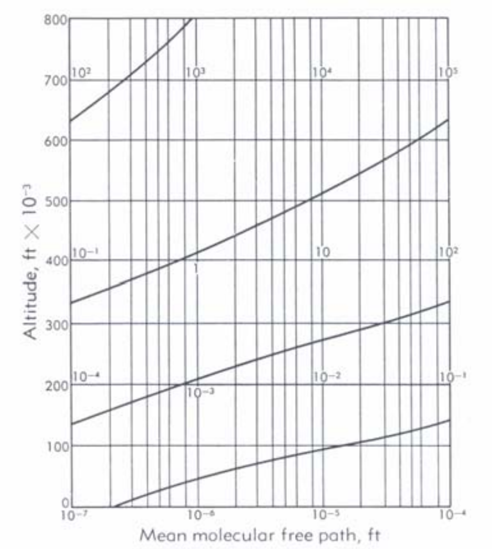


Fig. 2 Mean molecular free path of the adopted atmosphere

2.3 FREE MOLECULE FLOW

The free molecule flow regime is the regime of extreme rarefaction. The molecular mean free path is by definition many times the characteristic dimension of the body which is assumed to be located in a gas flow of infinite extent. The molecules which hit the surface of the body and are then reemitted on the average travel very far before colliding with other molecules. It is consequently valid to neglect the effect of the reemitted particles on the incident stream, at least so far as effects on the body itself are concerned. The incident flow is therefore assumed to be entirely undisturbed by the presence of the body. This is the basic assumption of free molecule flow theory. It is a consequence of this basic assumption that no shock waves are expected to form in the vicinity of the object. The "boundary layer" will be very diffuse and has no effect on the flow incident on the body.

Theoretical calculations of the external heat transfer and aerodynamic characteristics of bodies submerged in a free molecule flow field may be carried out by treating the flows of incident and reflected or reemitted molecules separately. In calculating the flux of momentum or energy incident on the surface it is assumed that the approaching gas is in local Maxwellian equilibrium. The results should hence be applied to very high altitude considerations with some care, since present knowledge as to the state and composition of the upper atmosphere is limited.

3. HYPERSONIC FLOW

The term 'hypersonic' was first used in a paper by Tsien (1946), and implies that the flight velocity is very much greater than the ambient speed or sound. The term 'hypervelocity' is also employed. No precise definition may be given of the velocity at which a supersonic flow becomes a hypersonic flow because the onset of those effects characteristic of hypersonic flow is in fact gradual, and varies with the geometry of the vehicle and with the nature of the surrounding atmosphere as well as with the flight velocity. Nevertheless, an approximate classification of the flow regimes of aerodynamics may be based on the value of the Mach number M, that is to say on the ratio of the flow velocity relative to the vehicle to the ambient speed of sound, but generally, hypersonic flows are defined as those flows where the Mach number is greater than 5.

It may be noted here that in the hypervelocity flight of a typical blunt-nosed vehicle, there will be parts of the flow field round the body in which the flow has become decelerated and in which subsonic, transonic, or supersonic flow may exist. Thus the study of the hypersonic flow past such a body may involve the knowledge of the aerodynamics of all the flight regimes. The types of problem which arise in these local regions of subsonic or supersonic flow may, however, be different from those which are normally considered for flight at lower velocities; for instance, curved shock waves are often present in hypersonic flow and it is necessary to consider the influence on the flow field of the vortices generated at the shock.

There are, broadly speaking, two main effects associated with hypersonic flows-the purely fluid dynamic effects arising from the high Mach number, and the physico-chemical or 'real-gas' effects due to the high temperature developed in the flow. At the same time hypersonic flight often takes place at high altitudes where the atmospheric density is so low that the mean free path may become comparable with a characteristic dimension of the flow field; under these conditions continuum flow no longer exists and the methods of rarefied gas-dynamics must be used as discussed earlier.

3.1 THE EFFECT OF HIGH MACH NUMBER

Probably the most striking difference between flow at subsonic and at supersonic speeds is the formation of a shock ahead of bodies and wings when the flight velocity is above the ambient sonic velocity. Upstream of such a shock the flow is completely undisturbed, and it is only downstream of the shock that the influence of the body is felt. The shock may be either attached, as in the case of a slender pointed body or a wing with a sharp leading edge, or detached, i.e. standing off from the surface, if the nose of the body or the wing leading edge is blunt. The behavior of the shock for the flow past a body of revolution as the Mach number, M_1, increases from supersonic to hypersonic values may be illustrated by considering the flow of a perfect gas past two different body shapes-a slender cone and a sphere. (The term 'perfect' gas is used to mean an inviscid gas obeying the law $p/\rho = RT$, and for which the ratio of specific heats, γ, has a constant value.)

Fig. 3 shows the position of the attached shock at different values of the Mach number for the flow about a slender cone with $\gamma = 1.4$. At low supersonic velocities there is a large angle between the shock and the body. As M_I increases, the angle between the shock and the body decreases, until at M_i = 10 it is only a few degrees. This is of some consequence to the theoretical treatment of hypersonic flows because one of the main approximations used in linearized supersonic theory is based on the shock angle being large relative to the body angle, and this is evidently no longer true at high Mach numbers.

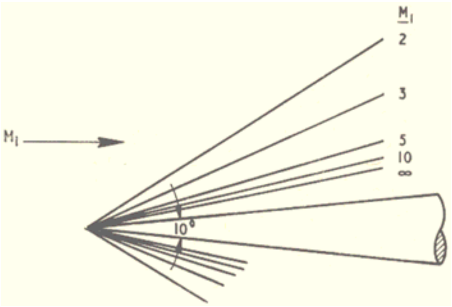
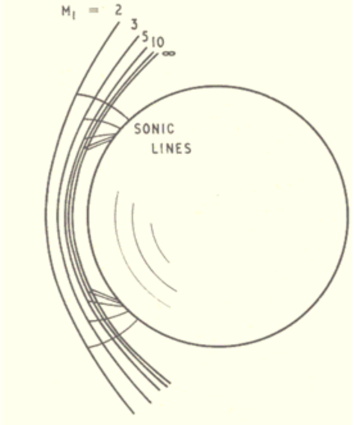


Fig. 3 Shock positions for flow past slender cones at different Mach numbers

The variation of the bow shock with Mach number for the flow past a sphere is shown in Fig. 4. The shock stands off from the body by an amount which decreases as the Mach number increases. The limiting shock lies fairly close to the surface of the sphere (it is very nearly concentric with it over most of the front of the sphere) and the region between the shock and the surface is referred to as the 'shock layer.' Just behind the front part of the shock the flow has been brought nearly to rest and is subsonic; as it passes round the sphere it accelerates, passes through the local velocity of sound at the sonic line and then becomes supersonic. In contrast to the flow past the sharp cone, the shock is highly curved; this means that there are large transverse gradients of the flow quantities and these gradients play an important part in determining the flow field. For the corresponding plane flows past a wedge and a cylinder the shock behaves in a similar manner.



Shock shapes for flow past sphere at different Mach numbers (γ = 1·4).

Fig. 4 Shock shapes for flow past sphere at different Mach number

3.2 REAL GAS EFFECTS

At the high temperatures which exist behind the shock for hypervelocity flight, the temperature energy of the gas becomes comparable with the energies associated with various molecular and atomic processes, such as excitation of the vibrational modes of the molecules, dissociation and ionization. Under these conditions the gas no longer behaves as a perfect gas having a constant value of the ratio of specific heats, and the energy which is involved in these processes must be taken into account when calculating the flow field.

Fig. 5 shows how the chemical composition of air behind a normal shock wave varies with the flight velocity for flight at 200,000 feet altitude. Oxygen starts to dissociate for velocities above about 5,000 feet/second, and Nitrogen above 12,000 feet/second. The number of free electrons produced becomes appreciable when the velocity exceeds 18,000 feet/second. The electrical conductivity of the gas may exceed 1 mho/cm, and this makes attractive the possibility of controlling the flow field by electric or magnetic fields; in addition the presence of free electrons surrounding the body alters the dielectric properties of the medium, and can have a large effect on radio communications to and from a vehicle.

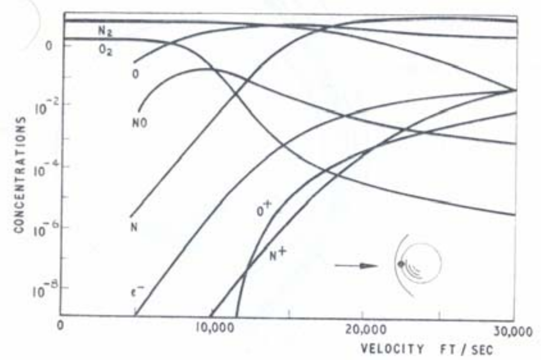


Fig. 5 Mole fraction concentration behind normal shock (300,000 ft altitude)

As a result of these real gas effects, the effective specific heat ratio, γ behind the shock wave can change significantly from the normal value of 1.4 for air. As the velocity increases γ first decreases from 1.4 as vibrational modes are excited and then decreases further as dissociation and ionization take place. Many of the approximate analytical approaches to hypersonic flow are based on having a small value of $(\gamma - 1)$ - an approximation which because of real gas effects can hold for a wide range of hypersonic flight conditions.

A change in γ can have a significant effect on the flow field. For instance the variation with γ of the shock stand-off distance for the flow past a sphere at $M = \infty$ is plotted in Fig. 6 (it is assumed that γ is constant between the shock and the body). We see that the stand-off distance depends quite markedly on the value of γ, and since the stand-off distance is relatively easy to measure experimentally, it gives a good indication of the influence of the real gas effects on the flow field. Another feature of the high temperature layer of gas surrounding a body is that radiation from the gas into the body, or to the atmosphere can become important at hypervelocities. The light emitted from the nose region of such a body can become quite intense and is readily observable in experiments in high enthalpy wind tunnels.

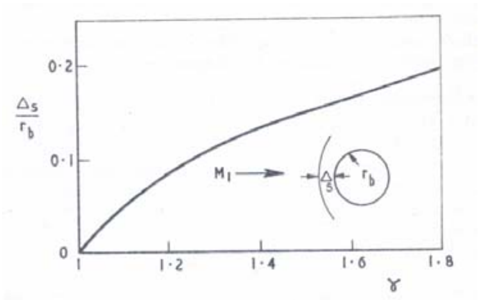


Fig. 6 Variation of shock stand-off distance with γ for flow past sphere ($M_l = \infty$).

3.3 HEAT TRANSFER AND THE BOUNDARY LAYER IN HYPERSONIC FLOW

At high Mach numbers very high temperatures will be developed in regions where the flow is decelerated, such as in the boundary layers close to surfaces. Because of the high temperatures, the viscosity is high, and the density is low, and the boundary layer thickness may be very much greater than at the same free-stream Reynolds number in subsonic or supersonic flow. Since, as we have seen the shock wave will lie relatively close to the body in hypersonic flow, the boundary layer thickness may no longer be negligible compared with the distance between the body and the shock. The additional displacement effect of this boundary layer can result in an appreciable modification of the whole flow-field-an example of viscous interaction. Thus the pressure along a flat plate with even a very sharp leading edge can differ significantly from the free stream value because of the effective blunting caused by the thick boundary layer.

Separation effects, too, may become important in hypersonic flows, because of the prevalence of thick laminar boundary layers which do not readily withstand adverse pressure gradients. Another feature of hypersonic boundary layers not normally present in the types of body used at supersonic velocities is the vorticity interaction arising from the presence of curved shock waves. If a boundary layer is formed on a surface which is supporting a curved shock, the vorticity gradient behind the shock influences the rate of growth of the boundary layer.

The process of heat transfer through boundary layers in hypersonic flows does not differ in principle from that in supersonic flows, although certain extra simplifying assumptions become possible in hypersonic boundary layers with cooled walls because the density in the hot outer part of the layer may be much less than that near the wall. The main differences between supersonic and hypersonic heat transfer are that very much greater temperature differences between the stream and the wall may have to be considered, and that because of the high temperature gradients, there will be large changes in the viscosity and thermal conductivity across boundary layers.

In addition real-gas and non-equilibrium effects can have a large influence on the boundary layer growth and on heat transfer rates, and it is necessary to consider the influence of the diffusion and convection of the various species produced by dissociation and ionization through the boundary layer. The nature of the wall can play an important part in determining the local recombination rates as it can act as a third body for recombination processes. The boundary layer problem is further complicated for bodies during atmospheric re-entry if the heating rates are so high that melting or sublimation of the surface takes place. The effect of the injection of some form of coolant into the boundary layer must also be considered.

3.4 LOW DENSITY EFFECTS

At normal altitudes and velocities the air flowing past a vehicle can be treated as if it were a continuum. At high altitudes, the air becomes less ,dense, and the motion of the individual gas particles becomes important. The parameter which determines the onset of low density effects is the Knudsen number, *K*, as mentioned. Continuum flow starts to break down when *K* is of the order unity. As the density is reduced, the main effect is that the shock ahead of a body becomes merged with the flow close to the body. At very low densities ($K \sim 10-100$) free molecular flow conditions are approached in which the individual air particles reflect from the surface, but do not undergo collisions with other particles until they are outside the region of influence of the body. The variation of the atmospheric mean free path with altitude is shown in Table 1.

Table 1 Variation of the atmospheric mean free path with altitude

Altitude (*ft*)	Mean Free Path (*ft*)	Density ($\rho/\rho_{sea-level}$)
0	2.18×10^{-7}	1
1×10^{5}	1.61×10^{-5}	1.35×10^{-1}
2×10^{5}	8.45×10^{-4}	2.57×10^{-2}
3×10^{5}	1.25×10^{-1}	1.75×10^{-5}
4×10^{5}	2.18×10^{1}	9.90×10^{-7}
5×10^{5}	1.65×10^{2}	1.28×10^{-9}

It is only for altitudes above 200,000 feet that low density effects must be taken into account for typical hypersonic vehicles.

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