CHAPTER 1

INTRODUCTION

1.1 Introduction

Studies have shown that 6 to 40 percent of the total cost of sales can be attributed to the cost of quality in a typical company [24]. For this reason, many companies have turned to improving the processes of achieving quality in order to reduce costs. The new perspective has led companies to reexamine the traditional assumptions and approaches used to achieve quality improvement. The classical approach of quality control, which focused on screening and correction of defects, is giving way to new methodologies that emphasize prevention. Unlike the classical approach, which assumed that the process settings (mean and variance) were given, there is a new approach which views the process settings as variables that can be controlled through investments in improved raw materials, worker training, and process capabilities. To effectively carry out the new approach, companies need methods to evaluate investments that are aimed at changing process settings. This new approach is called "process targeting". In process targeting, it is assumed that process parameters or machine settings are variables, so the problem will focus on the determination of the optimum values of the process parameters or machine settings that will achieve some economical objectives. The process targeting is one of the important problems in economic and quality control.

The initial process targeting problem addressed is the "can filling problem". The first real attempt to tackle the can filling problem was in Springer [1951]. In general, the can filling problem is described as follows: Consider a filling problem in which cans are produced continuously. The quality characteristic of interest is the net weight of the filled can. The value of this quality characteristic is a random variable X. A lower specification limit L exists for X. A can is accepted if $X \geq L$ and rejected otherwise. Accepted cans are sold at a fixed price a, while rejected cans are sold at a reduced price r, where $r < a$. In this problem, it was assumed that X follows a normal distribution with mean μ and standard deviation σ. Moreover, 100 percent inspection was used for product quality control and inspection is assumed to be error free. The objective of this problem is to find the optimal mean (target) μ so that the net income for the process is maximized. It is assumed that μ is a parameter that can be controlled by the filling machine setting

Many research papers have addressed the process targeting problem. Each paper considers the problem, with different assumptions. As a result, different models and solution methods exist in the literature see chapter 2 for more details. Despite the wide spectrum of variation of the process targeting problem that have been addressed, very few have considered the case where the product goes through two processes instead of one. Considering such a problem gives another dimension to the classical process targeting problem. Moreover, such a problem widely exists in multistage serial production systems, such as electronic industry.

A related work to the problem considered in this thesis is the work of Al-Sultan (1994). In his work, he considered the targeting problem in which a product is processed by two machines in series. The product is assumed to have two quality characteristics. Each quality characteristic has a lower specification limit, L_1 and L_2 for the first and second quality characteristics respectively. The problem can be described as follows: A lot of N items are sequentially processed by the two machines. After being processed by the first machine, it was assumed that the value of the first product quality characteristic is a random variable, denoted by X_1. After processing by the first machine the lot is inspected by a sampling plan, in which a sample of size n_1 is drawn from the lot. If more than d_1 (fixed number) items have their first quality characteristic less than L_1, then the whole lot is rejected. Otherwise, the lot passes to the second machine. Then the lot is processed by the second machine and the value of the second product quality characteristic is assumed to be a random variable, denoted by X_2. A sample of size n_2 is drawn from the lot and inspected. If greater than d_2 items are found to have their second attribute less than L_2, then the lot is rejected, otherwise the lot is characterized as acceptable. The items that are rejected after machine 1 are sold at a price of a_1, while those rejected after machine 2 are sold at a price of a_2. Accepted items are sold at a price of a_3. The objective of this problem is to find the optimal set points μ_1 and μ_2 for the two quality characteristics such that the total profit is maximized. The total profit is the sum of the revenues from selling rejected items (after machine 1 and 2) and acceptable items minus the cost of processing by machine 1 and 2.

This problem can be extended further by relaxing or changing some of the assumptions. A variation of the problem has been considered by Al-sultan and Pulak (2000). They developed a mathematical model to find the optimum μ_1 and μ_2 for the same problem described above except that 100 % inspection is used for product quality control. Another variation and extension of the above problem is to consider quality characteristics that are a product of the two processes or are composite of two quality characteristics. In this case, dependency between the two processes will be considered.

In this thesis, several variations of the problem addressed by Al-Sultan and Pulak are considered. The first variation is to assume that the product has two quality characteristics that are related in an additive manner. The first quality characteristic is assumed to be a random variable X_1 and the second quality characteristic is the sum of $X_1 + X_2$ where X_2 is the output of the second process. An example of this is the painting of fire extinguishers in which they go through two successive processes. The first process makes the coating on the surface and the second process makes the final external coating. Specification limits on the thickness of the first coating and on the final thickness determine the quality of the product. The quality of the product is ensured by 100 % inspection in the first problem, while a sampling plan is used for that purpose in the second version of the problem. For each of the two versions inspection error is taken into consideration to form the third and the fourth versions of the problem. The objective is to determine the optimal means, μ_1 and μ_2 for both processes that maximizes the profit. A mathematical model is developed for each of the four problems in chapters 3, 4, 5 and 6.

1.2 Key Factors in Process Targeting Problems

Process targeting problems are affected by many important factors. The main factors include quality characteristic distribution, product specification, process costs and market prices.

- Quality characteristic distribution:

The quality characteristic distribution plays an important role in modeling the process targeting problem. In many papers in the literature the quality characteristic is assumed to be normally distributed with known variance. It is highly recommended to test the distribution using standard goodness of fit tests such as the Chi-square test and Kolmogorov test to ensure that this assumption is satisfied.

- Product specification:

The specification limits on the quality characteristic determines the product acceptance criteria. The specification limits are usually determined by market and technical fitness and accurate information about them is necessary for realistic process targeting model.

- Costs:

There are many costs involved in the process targeting problem. These costs are production costs, material costs, inspection costs and rework costs. Knowledge about these costs is essential for obtaining realistic solutions for the process targeting problem.

- Market prices:

Sometimes selling prices are included in the process targeting model and the objective in this case would be to maximize the expected profit. Therefore, market study is essential in developing the process targeting model. The market study determines the selling prices for all kind of items. Accepted items are sold at their regular market price a_1, while defective items are sold at a lower secondary market price a_2, where $a_1 > a_2$.

1.3 Quality Control Schemes

Various quality control schemes have evolved over time. These schemes include: product control, process control and process capability analysis.

- *Product control* can be achieved by two techniques. The first technique is called acceptance sampling, which is concerned with inspection and decision making regarding products based on a sample taken from the lot. While the other technique is called 100% inspection, in which a decision regarding products is made based on inspecting the whole lot. This area of quality control is utilized for developing the models in the thesis and described in section 1.3.1 in more details.
- *Statistical Process Control (SPC)* is a diagnostic tool that allows you to determine "assignable" versus "common" causes of variation. Common causes of variation are normal and affect every process while assignable causes of variation occur when something happens that is not usually part of the process. SPC allows you to identify when these assignable causes occur so that you can eliminate them and bring predictability, or "control" to a process without overreacting to normal variability. Control charts are one of the most effective SPC tools. Other SPC tools include Histogram, Check Sheet, Scatter Diagram and Pareto Chart.
- *Process capability analysis* is an engineering study to estimate process capability. The estimate of process capability may be in the form of a probability distribution having a specified shape, center (mean), or spread (standard deviation).

1.3.1 Inspection

Inspection is often used to appraise the quality of purchased and manufactured items. Inspections can be divided into: sampling plans, 100 % inspections and repeat inspections. In this thesis, we focus on 100 % inspections and sampling plans. A brief description about the two types is given below:

1. **100 % Inspection:** This method involves inspecting every product received and usually it is applicable in situations where the component is extremely critical and passing any defectives would result in an unacceptably high failure cost. It has many disadvantages such as: it is expensive; it can not be used for destructive testing; it may cause a delay in the production schedule.

2. **Sampling Plan:** This method involves inspecting a sample of products drawn from a lot. The whole lot will be judge based on the sample. If the sample meets specifications, then we accept the whole lot otherwise lot will be rejected. This method has many advantages over the 100 % inspection. It costs less, involves less damage to the products, applicable to destructive testing and involves fewer personnel.

In this thesis, we focus on product control through inspection. A 100% inspection and a sampling plan are utilized in this thesis to test the conformity of the products. Moreover, a more realistic inspection plan in which inspection is assumed to be error prone is implemented. In this case we assume that inspectors make errors during inspection process. Section 1.3.2 describes inspection with error.

1.3.2 Inspection Error

The manufacture of quality products demands measurements that are both high precision and high accuracy, because inspection is used to determine whether or not a product meets specifications. The inspection results are commonly used to influence the operation in making the current part or the production of the next part, thereby correcting a potential quality problem before a product is completed. Hence, the accuracy and effectiveness of the inspection procedures and equipments are essential for precision manufacturing.

Unfortunately, there are always sources of errors in measuring equipments and measurement systems. The sources of errors that come from the measuring equipments include imperfect mechanical structure, errors in control systems, and environmental disturbances. As measurement error is defined as the discrepancy between actual and measured dimensions, it will be affected not only by the error resulted from the measuring equipment and the repeatability of the measurement, but also by the error resulted from the compound effect of machine errors and the geometric characteristic of the measured surfaces. A variety of techniques have been developed to deal with machine error modeling and compensation as well as uncertainty in inspection. Another source of error is the error coming from the sampling inspection. This type of error exists when the product's quality is controlled by sampling plan instead of 100% inspection. In this thesis, an attempt is made to study the effect of inspection errors on process targeting models.

1.4 Problem Statement

The problem considered in this thesis can be described as follows: Consider a manufacturing environment in which products go through two different processes. The quality of the final product is determined by two quality characteristics, one is based on the first process and the other is based on the composite of the two processes. Specification limits are set on both quality characteristics. Let us assume that the quality characteristic of the first process is a random variable denoted by X_1, and the quality characteristic of the second process is another random variable denoted by X_2. The final quality characteristic is denoted by X, where $X=X_1+X_2$. Quality requirement is that $X_1 \geq L_1$ and $X \geq L$, where L_1 and L are predetermined constants set by product designer. A product that meets both specifications is sold at a regular market with a price a_1. A product that meets the first specification and fails to meet the second specification is sold at a_2, where $a_1 > a_2$. An item that fails to meet the first specification is reworked (scraped) at a cost that includes variable processing cost and fixed rework cost.

A real example for such a problem is the painting of fire extinguishers in which an extinguisher goes through two successive processes. The first process makes the coating on the surface and the second process makes the final external coating. Specification limits on the thickness of the first coating and on the final thickness determine the quality of the product. Chapter 3, 4, 5 and 6 considers this problem in details including assumptions, notations, models development and solution methodology.

1.5 Thesis Objectives

The objectives of the thesis are:

1. Develop a process targeting model to maximize the profit from two processes producing a product with two quality characteristics. The first quality characteristic is determined by the first process only and the final quality characteristic depends on the setting of both processes using acceptance sampling as a mean for product quality control assuming perfect inspection.

2. Develop a process targeting model to maximize the profit from two processes producing a product with two quality characteristics. The first quality characteristic is determined by the first process only and the final quality characteristic depends on the setting of both processes using 100% inspection as a mean for product quality control assuming perfect inspection.

3. Extend the models resulting from objectives 1 and 2 for the case of measurement systems with error.

4. Study the effect of inspection errors for the models developed in objective 3.

1.6 Thesis Organization

The thesis is organized as follows: Literature review in the area of process targeting is presented in chapter 2. Chapter 3 includes a process targeting model that maximizes the profit for two successive processes producing a product with two quality characteristics.

The first quality characteristic is determined by the first process only and the final quality characteristic depends on the setting of both processes using sampling plan. The same model is extended in chapter 4 for the case where inspection error exists and a sensitivity analysis is conducted to study the effect of the error on the model. Chapter 5 includes a process targeting model that maximizes the profit for two successive processes producing a product with two quality characteristics and the final quality characteristic depends on the setting of both processes using 100% inspection. Chapter 6 includes a process targeting model that maximizes the profit for two successive processes producing a product with two quality characteristics and the final quality characteristic depends on the setting of both processes using 100% inspection with inspection error. Moreover, sensitivity analysis to study the effect of the error on the model is conducted on the same chapter. Finally, conclusion and recommendations for further research are outlined in chapter 7.

CHAPTER 2

LITERATURE REVIEW

2.1 Overview

The purpose of this chapter is to present the literature in the area of process targeting with emphasis on models extended in this thesis. The extensions made in this thesis are based on the models developed by Al-Sultan (1994) and Al-Sultan and Pulak (2000).

2.2 Literature Review

This section includes a brief literature review in the area of "Process Targeting". It covers most of the papers that concerned with the targeting problem from 1951 to 2004 in chronological order.

C. H. Springer (1951) firstly concerned with the problem of process targeting. He considered the problem of finding the optimal process mean for a canning process when both upper and lower control limits are specified. He assumed that the cost of producing under-limit and over-limit products is fixed.

D. C. Bettes (1962) studied the same problem as in Springer (1951) except that only the lower limit was specified. He found the optimal process mean and the upper limit for a fixed lower limit using an empirical method that depends on trial and error.

R. Collins, K. Case and G. Bennett (1973) considered the effects of inspection error on probability of acceptance, average outgoing quality and average total inspection in a single sampling plan. These measures are examined under both replacement and non-replacement assumptions.

G. Bennett, K. Case and J. Schmidt (1974) considered the effects of inspector error on a cost-based quality control system. The system examined is of a single sampling plan design involving several cost components.

W. Hunter and C. Kartha (1977) addressed the problem of finding the optimal process mean with only a specified lower limit and in which under-filled items are sold at reduced prices. They also assumed that conforming items are sold at a fixed price with a penalty cost due to excess in quality. This paper considered as a base for later papers concerned with targeting problems.

K. Case and G. Bennett (1977) concerned with the adverse monetary effects of imperfect measurement by incorporating measurement error into a cost based variables acceptance sampling model. They showed that attributes inspection error rates, known to be typical from industry studies, cause unexpectedly high increase in cost.

L. S. Nelson (1978) considered the same problem by Hunter and Kartha (1977). The objective of the paper was to find the best target value that will balance the give-away cost and the loss associated with rejected items so as to maximize net income. A four-cycle arithmetic graph is provided for determining the target value.

L. S. Nelson (1979) considered the same problem by Springer (1951). A nomograph is provided to set the process mean so that scrap cost is minimized.

S. Bisgaard, W. Hunter and L. Pallensen (1984) extended the model in W. Hunter and C. Kartha (1977) such that cans filled below specification limit are sold in a secondary market at a price proportional to the filled quantity.

D. Y. Golhar (1987) investigated the problem of selecting the optimum process mean in a canning process in which cans filled above the lower limit are sold at a fixed price, while the underfilled cans are emptied and refilled at a reprocessing cost. He determined, without measurement error, the optimum process mean that maximizes the expected profit per container.

K. Tang and H. Schneider (1987) investigated the economic and statistical effects of inspection error on a complete inspection plan. They assumed that if the inspection result indicates that an item fails to meet predetermined specification limits, the item is reworked so that its quality characteristic is exactly equal to the target value. Two models with considerations of inspection error are developed under different rework schemes, then compared with the model without inspection error consideration.

R. V. Vidal (1988) considered the same problem stated in S. Bisgaard, W. Hunter and L. Pallensen (1984). A simple graphical approach was developed to find stationary solutions that might be global maxima.

M. A. Rahim and P. K. Banerjee (1988) firstly considered the problem of selecting the optimal production run for a process with random linear drifts (e.g. tool wear). A cost function per unit of finished product is derived. A search algorithm as well as a graphical method are suggested to find the optimal production run.

D. Golhar and S. Pollock (1988) extended D. Golhar (1987) model to a process where both the process mean and the upper limit can be controlled. Underfilled and overfilled cans are emptied and refilled. Simple approximate analytical expressions relating the optimal values to fundamental process parameters are given.

D. Y. Golhar (1988) considered the same problem stated in Golhar and Pollock (1988). However, since the computations were time-consuming, a computer program is developed that calculates the desired optimal values.

K. Tang (1988) presented an economic model to determine the most profitable specifications for the complete inspection plan with the considerations of economic loss caused by quality deviations, rework cost and inspection cost.

R. L. Schmidt and P. E. Pfeifer (1989) analyzed the savings that result from a reduction in the process variance in a single-level canning problem. They presented an approximation equation that provides a simple linear relationship between the percentage cost reduction and the percentage reduction in standard deviation.

G. Taguchi, E. A. Elsayed and T. C. Hsiang (1989) concerned with quality engineering in production systems. They considered the quality concept and quality cost through all phases of a product's life cycle. They proposed a loss function approach as a measure of quality, and its use in determining product specification, target values of product characteristics and desired tolerances relevant to each target value.

Y. V. Hui (1989) considered complete inspection plans for bicharacteristic products. A decision approach is presented to minimize the cost involved after inspection.

F. J. Arcelus and M. A. Rahim (1990) presented an economic model which incorporates the joint control of both variable and attribute quality characteristics of a product. Items are acceptable if they meet the specifications for both types of characteristics at the same time. The objective is to simultaneously select the appropriate target values for the characteristics, so as to maximize the expected income per lot.

T. O. Boucher and M. A. Jafari (1991) extended the line of research by evaluating the problem of finding the optimum target value under a sampling plan as opposed to 100% inspection. Two conditions are examined, (1) when sampling results in destructive testing and (2) when the testing is nondestructive.

R. L. Schmidt and P. E. Pfeifer (1991) extended the model by Golhar and Pollock (1988) to a capacitated (bottleneck) production process). A closed-form expression for the optimal upper control limit is developed, and a one-way table and an approximating equation are provided for the optimal mean.

B. J. Melloy (1991) formulated the problem of a uniform filling of an item under compliance testing. The objective was to determine the economically optimal settings of the mean and the screening limits, subject to an acceptable level of risk.

K. Tang and J. Lo (1993) considered a situation where inspection is based on surrogate variable instead of the quality characteristic of interest. They determined the optimum process mean and the screening limits when a correlated variable is used in inspection. Since a correlated variable is not perfectly correlated with the quality characteristic, acceptance cost may be incurred by accepting non conforming items for shipment.

D. S. Bai and M. K. Lee (1993) presented the problem of selecting the process mean and the cut-off value of a correlated variable for a filling process in which inspection is based on the correlated variable rather than the process mean itself. A profit model is constructed which involves selling price and filling, rework, inspection and penalty costs.

K. S. Al-Sultan (1994) extended the model of Boucher and Jafari (1991) to the case of two machines in series. He developed an algorithm for finding optimum target values for the two machines when a sampling plan is used.

J. Liu, K. Tang and Y. H. Chun (1995) considered the two-level problem, in which a lower specification limit is used to screen out nonconforming items and an artificial upper limit is used to screen out overfilled items. Both nonconforming and overfilled items are reprocessed until they become accepted items. They also considered a capacity constraint that requires that the total number of conforming items produced by the production process meet a specified demand.

D. P. Mihalko and D. Y. Golhar (1995) firstly considered the problem of estimating the expected profit for an automatic filling operation when the standard deviation of the filling process is unknown. A procedure is developed for obtaining the estimated confidence interval for the expected profit under 100% inspection.

K. S. Al-Sultan and M. A. Al-Fawzan (1997) considered the same model of Rahim and Banerjee (1988) for a process with random linear drift with some modifications. They used the same approach of Golhar and Pollock (1988) to study the effect of variance on the expected total cost per good item for the modified version. The objective was to find the optimal initial mean and cycle length.

W. Liu and M. Raghavachari (1997) extended the model given by Schmidt and Pfeifer (1991) to the case where the amount of fill follows an arbitrary continuous distribution. The best process mean setting as well as the best upper specification limit are sought to maximize the expected profit per fill attempt. They found that the optimal upper limit is given by a very simple formula regardless of the shape of the distribution, while the optimal process mean is determined using a general condition.

M. F. Pulak and K. S. Al-Sultan (1997) provided a FORTRAN based computer package for solving nine selected targeting models. They also included schemes for finding an initial starting solution for each model which will help the user to obtain optimal solutions quickly.

M. Cain and C. Janssen (1997) presented the model where the cost is asymmetric across the target. A linear cost below lower specification limit and a quadratic cost above specification limit are assumed.

Y. H. Chun and D. B. Rinks (1998) firstly defined the producer's risk and the consumer's risk in acceptance sampling based on the assumption that the proportion defective of incoming lots is a random variable that follows a beta distribution. The modified producer's risk and consumer's risk are derived.

S. M. Pollock and D. Golhar (1998) considered the canning process with constant demand and capacity constraint for the production process. They assumed that there is a penalty for producing a nonconforming cans.

K. S. Al-Sultan and M. A. Al-Fawzan (1998) considered a multistage manufacturing system where processing at each stage is performed by a process that deteriorates with time. A mathematical model is developed for this problem to minimize the cost of maintenance, quality and penalties for unfulfilled demand for items. The model finds optimal initial settings of the process means and optimal cycle length.

S. H. Hong and E. A. Elsayed (1999) extended the model given by Golhar (1987) for the case with normally distributed measurement error. They developed a model for determining jointly the optimum process mean and the cutoff value on the observed characteristic.

S. H. Hong, E. A. Elsayed and M. K. Lee (1999) considered the problem of jointly determining the optimum process mean and screening limits for each market in situations where there are several markets with different price/cost structure. It is assumed that quality characteristic is normally distributed with an unknown mean and a known variance.

P. E. Pfeifer (1999) provided a general canning problem model consisting of a piecewise linear profit function. This paper give a simple relationship between two competing objective functions for the canning problem: expected profit per fill-attempt and expected profit per can to be filled.

J. Roan, L. Gong and K. Tang (2000) they incorporated the issues associated with production setup and raw material procurement into the classical process mean problem. The product of interest is assumed to have a lower specification limit, and the items that do not conform to the specification limit are scrapped with no salvage value. A two-echelon model is formulated for a single-product production process, and an efficient algorithm is developed for finding the optimal solution.

K. S. Al-Sultan and M. F. S. Pulak (2000) considered a manufacturing system with two machines in series. The manufactured product is assumed to have two attributes which are related to the processing of the product, by machine 1 and machine 2 respectively. Each attribute has a lower specification limit (LSL) set for it, and if the measured attribute for a certain product is less than its LSL, the product is recycled at a certain cost. A mathematical model is developed for finding the optimum setting point for each machine, and a numerical approach is suggested for solving this model.

W. W. Williams, K. Tang and L. Gong (2000) considered three types of process improvement actions for a container-filling operation: reducing the process setup cost, reducing the arrival rate of the out-of-control state, and reducing the process variance. Models are formulated to determine the optimal process improvement and production parameters that minimize the unit time expected cost across a given planning horizon.

M. A. Rahim, K. S. Al-Sultan (2000) considered the problem of simultaneously determining the optimal target mean and target variance for a process. This might result in a reduction in variability and in the total cost of the production process. A reduction in variability upholds the modern concept of Taguchi's loss function. The objective is to maximize the expected net profit per item with respect to the parameters of interest (the target mean and the target variance).

M. A. Rahim, J. Bhadury and K. S. Al-Sultan (2001) considered the problem of selecting the most economical target mean and variance for a continuous production process. They suggested three new approaches for the economic selection of a target variance integrated with a target mean. In the first approach, an expected profit maximization criterion is used to obtain the target mean and variance simultaneously. In the second approach, a minimum cost criterion based on the Taguchi loss function is used. In the third approach, an economic model for the selection of the target variance is developed using both customer and producer costs to minimize societal loss independent of the product quality characteristic distribution.

M. K. Lee, S. H. Hong and E. A. Elsayed (2001) considered the problem of determining the optimum target value of the quality characteristic of interest and the screening limits for a correlated variable under single and two-stage screenings. The optimum process mean and screening limits of the correlated variable are jointly determined by maximizing the profit function, which involves selling and discounted prices as well as production, inspection, and penalty costs.

J. Teeravaraprug and B. R. Cho (2002) studied a multivariate quality loss function as an extension of the Taguchi loss function to capture the overall loss to the customer when multiple quality characteristics are present. In addition to, they developed an optimization scheme to determine the most economical process target levels for multiple quality characteristics.

W. G. Ferrell Jr. and A. Chhoker (2002) focused on designing economically optimal acceptance sampling plans assuming the Taguchi hypotheses are appropriate. Inspection error is explicitly included in the model as is the ability to mitigate the consequences by expending resources.

M. K. Lee and E. A. Elsayed (2002) considered the problem of determining the optimum process mean and screening limits of a surrogate variable associated with product quality under a two-stage screening procedure. The surrogate variable is inspected first to decide whether an item should be accepted, rejected or additional observations should be taken. They assumed that the performance and surrogate variables are jointly normally distributed. The optimum process mean and screening limits are obtained by maximizing the expected profit which includes selling price, production, reprocessing, inspection and penalty costs.

S. O. Duffuaa and M. Khan (2002) presented a new inspection plan for critical multicharacteristic components. A mathematical model that depicts the plan is developed. The objective is to find the optimal number of repeat inspections and the sequence of characteristics for inspection that minimizes expected total cost per accepted component.

The expected cost consists of the cost of inspection and the cost of misclassifications. An algorithm is proposed to achieve this objective.

S. O. Duffuaa and A. W. Siddiqui (2002) developed a process targeting model for three class screening problem. They extended the work in the literature by incorporating product uniformity which introduced through a Taguchi type quadratic loss function. In addition, they studied the effect of model parameters on expected profit and optimal process mean.

E. P. Markowski and C. A. Markowski (2002) considered an attribute acceptance sampling problem in which inspection errors can occur. Unlike many common situations, the source of the inspection errors is the uncertainty associated with statistical sampling. Alternative sampling plans are designed to address the risk of statistical classification error.

S. O. Duffuaa and A. W. Siddiqui (2003) developed a process targeting model for a three-class screening problem in which measurement errors exist. To reduce the effect of measurement errors, they introduced the concept of cut-off points. These cut-off points are considered to be the decision variables. In addition, they studied the effect of various model parameters on expected profit, optimal process mean and cut-off points.

Chung-Ho Chen (2003) modified Cho and Leonard’s piecewise linear loss function for measuring the product quality and determining the optimum process mean for the larger-the-better Weibull quality characteristic. Also, the discussed the application of determining the optimum process mean for the modified Cho and Leonard’s model with piecewise linear loss function.

R. C. Quinino and L. Lee Ho (2004) obtained the optimal procedure for repeated and independent classifications of products with diagnosis errors. The objective is to minimize the mean total inspection cost. The procedures are implemented in a program using the software Matlab.

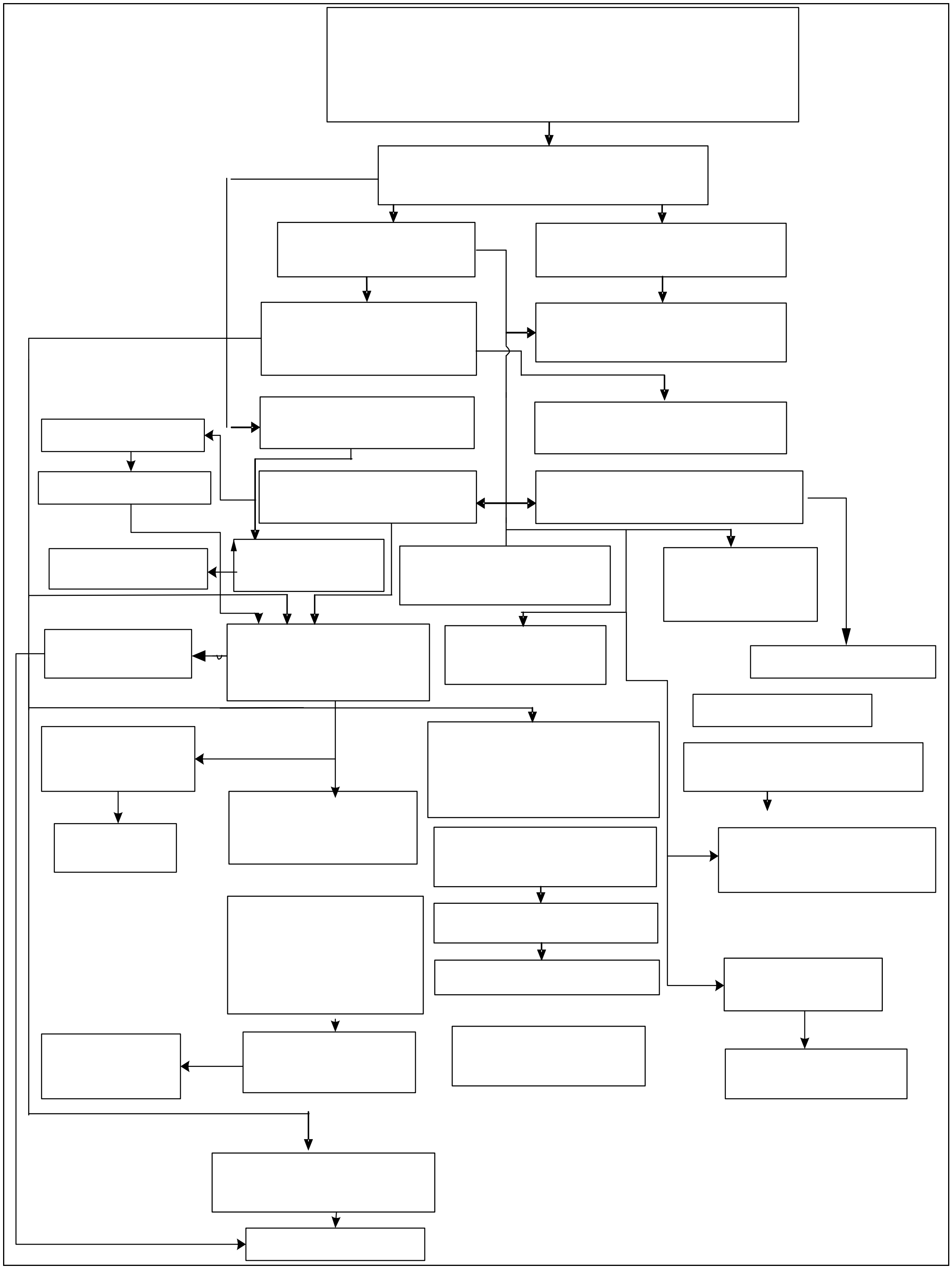

Figure (2-1) Process targeting literature review

2.3 Conclusion

In this thesis, a related work to Al-Sultan (1994) and Al-Sultan and Pulak (2000) is considered. In Al-Sultan (1994), a targeting problem in which a product is processed by two machines in series is considered. The quality of the product is controlled through sampling plan. The product is assumed to have two quality characteristics. Each quality characteristic has a lower specification limit, L_1 and L_2 for the first and second quality characteristics respectively. The objective of the problem is to find optimal mean settings of the two processes. In Al-Sultan and Pulak (2000), the same problem is considered except that a 100% inspection is used for the product quality control.

In this thesis, a new problem that is related to the previous two models has been considered. The same problem discussed earlier is considered except that it is assumed that the product has two quality characteristics that are related in an additive manner. The first quality characteristic is assumed to be a random variable X_1 and the second quality characteristic is the sum of $X_1 + X_2$ where X_2 is the output of the second process. The objective is to determine the optimal means μ_1 and μ_2 for both processes in order to maximize profit. In the next four chapters, different approaches are considered to tackle this problem.

CHAPTER 3

PROCESS TARGETING WITH TWO INDEPENDENT PROCESSES IN SERIES USING SAMPLING PLAN

3.1 Introduction

The purpose of this chapter is to develop a process targeting model for two independent processes in series using sampling plan as a mean for product quality control. Inspection is assumed to be error free in this chapter.

Consider a manufacturing environment in which products go through two different painting processes. The first painting process involves coating the product with an initial layer that will prevent corrosion. The second painting process involves painting the product with colorful layer that will give the desired color. A product quality characteristic could be thickness of the paint, number of spots on the surface of the paint, and the weight of the product.

In this chapter, product quality control is done through a sampling plan. At the end of each process a sample is taken from the lot. In this case, lot sentencing depends on the number of defectives in the sample. If the sample meets the quality requirements, the whole lot is accepted; otherwise the lot will be reworked or sold at a secondary market. Notice that, the quality characteristic of interest after the second process depends on the quality characteristic of interest after the first process. This means that the two processes contribute to the final quality characteristic. The objective of the model in this chapter is to maximize the expected profit by finding the optimal target values for the two processes.

This chapter is organized as follows; the problem under consideration is stated in section 3.2. Section 3.3 presents the model development and section 3.4 contains solution and results. Sensitivity analysis on the model is given in section 3.5 and finally section 3.6 concludes this chapter.

3.2 Statement of the Problem

Consider a manufacturing environment in which products go through two different processes. The product has two quality characteristics. The first quality characteristic is determined by the first process. The second quality characteristic is determined by both processes. Specification limits are set on both quality characteristics.

Let us assume that the quality characteristic of the first process is a random variable denoted by X_1, and the quality characteristic of the second process is another random variable denoted by X_2. The final quality characteristic is denoted by X, where $X=X_1+X_2$. Quality requirement is that $X_1 \geq L_1$ and $X \geq L$, where L_1 and L are predetermined constants set by product designer or market information. A sampling plan is used for product quality control. A lot of N products get processed by the first process, and then a sample of size n_1 is taken from the lot and tested. If the number of nonconforming items in the sample, D_1, is greater than d_1, the total lot is inspected with a cost of I per item. Conforming items are sent to the second process whereas the nonconforming items are reworked at a fixed cost (r). After being processed by the second process, a sample of size n_2 is taken from the lot and tested. If the number of nonconforming items, D_2, is greater than d_2, the lot is rejected and sold at a secondary market, otherwise the lot is accepted and sold at a regular market, see figure (3-1). The objective is to find the optimal set points μ_1^* and μ_2^* for the two quality characteristics such that the total profit is maximized.

A real example is painting of fire extinguishers in which an extinguisher goes through two successive processes. The first process makes the coating on the surface and the second process makes the final external coating. Specification limits on the thickness of the first coating and on the final thickness determine the quality of the product. So, the objective is to find the optimal thicknesses for both coating layers to be determined such that the total profit is maximized.

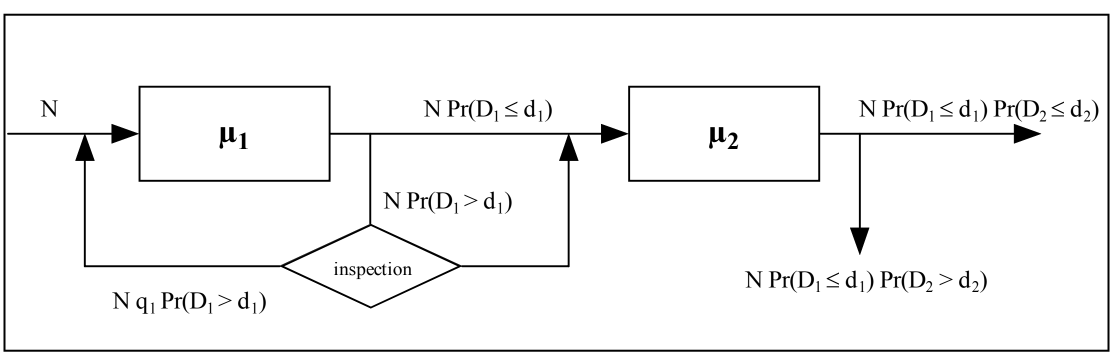

Figure (3-1) Quality targeting model for two processes in series using sampling plan.

3.3 Model Development

In this section, a process targeting model is developed for the problem stated. Notations and assumptions are presented prior to model development.

The following notations are adopted:

X_i A random variable that represent the value of the quality characteristic of the product after finishing process i, where i = 1, 2.

X Sum of the values of both quality characteristics after being processed by both processes.

L_1 Given lower specification limit for the first product quality characteristic.

L Given lower specification limit for the composite (sum) of the product's two quality characteristics.

a_1 Selling price for conforming items.

a_2 Selling price for items rejected after the second process.

c_1 Cost of material for the first process.

c_2 Cost of material for the second process.

I Inspection cost /item rejected after machine 1.

r Rework cost /item rejected after machine 1.

μ_1 Mean setting for the first process.

μ_2 Mean setting for the second process.

σ_1 Standard deviation for the first process.

σ_2 Standard deviation for the second process.

N Lot size.

n_i Sample size for sampling after process i, i = 1,2.

d_i Allowable number of nonconforming items found in a sample of size n_i, i=1,2.

D_i A random variable that represent the number of nonconforming units found in a sample of size n_i, i = 1,2.

$E\{TP(\mu_1,\mu_2)\}$ Expected total profit as a function of μ_1 and μ_2.

The following assumptions are used to develop the model:

1. X_1 and X_2 are independent and normally distributed with mean μ_1 and μ_2 respectively and standard deviation σ_1 and σ_2 respectively.
2. The actual number of nonconforming items in a sample of size n_i follows binomial distribution with parameters n_i and q_i, where i = 1, 2.
3. The processing order is fixed.
4. Costs of processing are assumed to be directly proportional to the values of the product quality characteristics.
5. The secondary selling price is less than the regular selling price and the lower specification limit for the first process is less than the lower specification limit of the second process.
6. Reprocessing operation, after the first machine, involves removing the paint and starting the first process from the beginning at a fixed cost (r).
7. There is no drift or shift in the means of the processes.
8. Sampling plan is used for quality control.
9. Items are processed by lot.

The revenue for a lot of size N can be stated as:

$$R = \begin{cases} a_1 N & if \quad D_1 \le d_1 \quad and \quad D_2 \le d_2 \\ a_2 N & if \quad D_1 \le d_1 \quad and \quad D_2 > d_2 \\ -(i + rq_1)N & if \quad D_1 > d_1 \end{cases}$$

The cost for each lot includes: material cost, and inspection cost for both cases, when a lot is accepted or not. The expected total profit is given by:

$$E\{TP(\mu_1, \mu_2)\} = a_1.N.\Pr(D_1 \le d_1).\Pr(D_2 \le d_2) + a_2.N.\Pr(D_1 \le d_1).\Pr(D_2 > d_2) -$$
$$I.N.\Pr(D_1 > d_1) - r.q_1.N.\Pr(D_1 > d_1) - c_1.\mu_1.N - c_2.\mu_2.N.\Pr(D_1 \le d_1)$$

Where, $a_1.N.\Pr(D_1 \le d_1).\Pr(D_2 \le d_2)$ is the expected revenue from selling items in the first market, and $a_2.N.\Pr(D_1 \le d_1).\Pr(D_2 > d_2)$ is the expected revenue from selling items in the secondary market. $I.N.\Pr(D_1 > d_1)$ is the expected cost of inspecting the lot rejected after the first process, $r.q_1.N.\Pr(D_1 > d_1)$ represents the expected cost of reworking nonconforming items in the lot. For the first process, the expected value of the processing cost per lot is given by $c_1.\mu_1.N$, whereas for the second process, the expected value of the processing cost per lot is given by $c_2.\mu_2.N.\Pr(D_1 \le d_1)$.

The expected profit per item can be written as:

$$E\{P(\mu_1, \mu_2)\} = a_1.\Pr(D_1 \le d_1).\Pr(D_2 \le d_2) + a_2.\Pr(D_1 \le d_1).\Pr(D_2 > d_2) -$$
$$(rq_1 + I)\Pr(D_1 > d_1) - c_1.\mu_1 - c_2.\mu_2.\Pr(D_1 \le d_1)$$
$$.........................(3-1)$$

A necessary condition to maximize the profit with respect to μ_1 and μ_2 is to set the partial derivatives to zero.

$$\frac{\partial E\{P(\mu_1^*, \mu_2^*)\}}{\partial \mu_1} = 0$$

$$\frac{\partial E\{P(\mu_1^*, \mu_2^*)\}}{\partial \mu_2} = 0$$

Notice that,

$$\Pr(D_1 \le d_1) = \sum_{i=0}^{d_1} \binom{n_1}{i} q_1^i (1-q_1)^{n_1-i} \qquad , where$$

$$q_1 = \Pr(X_1 < L_1) = \Phi\left(\frac{L_1 - \mu_1}{\sigma_1}\right)$$

$$\Pr(D_2 \le d_2) = \sum_{i=0}^{d_2} \binom{n_2}{i} q_2^i (1-q_2)^{n_2-i} \qquad , where$$

$$q_2 = \Pr(X < L) = \Pr(X_1 + X_2 < L) = \Phi\left(\frac{L - \mu_1 - \mu_2}{\sqrt{\sigma_1^2 + \sigma_2^2 + 2Cov(X_1, X_2)}}\right) = \Phi\left(\frac{L - \mu_1 - \mu_2}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)$$

To take partial derivative of equation (3-1) with respect to μ_1 we need to use the chain rule:

$$\frac{\partial E\{P(\mu_1, \mu_2)\}}{\partial \mu_1} = \frac{\partial E\{P(\mu_1, \mu_2)\}}{\partial q_1} \cdot \frac{\partial q_1}{\partial \mu_1} + \frac{\partial E\{P(\mu_1, \mu_2)\}}{\partial q_2} \cdot \frac{\partial q_2}{\partial \mu_1} \qquad ....(3-2)$$

Where,

$$\frac{\partial E\{P(\mu_1, \mu_2)\}}{\partial q_1} = a_1 \Pr(D_2 \le d_2) . \frac{\partial \Pr(D_1 \le d_1)}{\partial q_1} + a_2 \Pr(D_2 > d_2) . \frac{\partial \Pr(D_1 \le d_1)}{\partial q_1} +$$

$$r\left[\Pr(D_1 \le d_1) + q_1 . \frac{\partial \Pr(D_1 \le d_1)}{\partial q_1}\right] + I . \frac{\partial \Pr(D_1 \le d_1)}{\partial q_1} - c_2 \mu_2 . \frac{\partial \Pr(D_1 \le d_1)}{\partial q_1}$$

$$= \frac{\partial \Pr(D_1 \le d_1)}{\partial q_1} . \left[a_1 \Pr(D_2 \le d_2) + a_2 \Pr(D_2 > d_2) + r q_1 + I - c_2 \mu_2\right] + r \Pr(D_1 \le d_1)$$

Given that:

$$\Pr(D_1 \le d_1) = \sum_{i=0}^{d_1} \binom{n_1}{i} q_1^i (1-q_1)^{n_1-i}$$

then $$\frac{\partial \Pr(D_1 \le d_1)}{\partial q_1} = \sum_{i=0}^{d_1} \binom{n_1}{i} \left[i q_1^{i-1} (1-q_1)^{n_1-i} - q_1^i (n_1 - i)(1-q_1)^{n_1-i-1} \right]$$

$$= -n_1(1-q_1)^{n_1-1} + \left[n_1(1-q_1)^{n_1-1} - n_1(n_1-1)q_1(1-q_1)^{n_1-2} \right] +$$

$$\left[n_1(n_1-1)q_1(1-q_1)^{n_1-2} - \frac{n_1(n_1-1)(n_1-2)}{2} q_1^2 (1-q_1)^{n_1-3} \right] + \ldots\ldots. +$$

$$\left[\frac{n_1!}{(n_1-d_1)!\,d_1!} n_1 q_1^{d_1-1} (1-q_1)^{n_1-d_1} - \frac{n_1!}{(n_1-d_1)!\,d_1!} (n_1-d_1) q_1^{d_1} (1-q_1)^{n_1-d_1-1} \right]$$

$$= -\frac{n_1!}{(n_1-d_1)!\,d_1!} (n_1-d_1) q_1^{d_1} (1-q_1)^{n_1-d_1-1}$$

$$= -\frac{n_1!}{(n_1-d_1-1)!\,d_1!} q_1^{d_1} (1-q_1)^{n_1-d_1-1}$$

So,

$$\frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial q_1} =$$

$$\left[-\frac{n_1!}{(n_1-d_1-1)!\,d_1!} q_1^{d_1} (1-q_1)^{n_1-d_1-1} \right] . \left[a_1 \Pr(D_2 \le d_2) + a_2 \Pr(D_2 > d_2) + r q_1 + I - c_2 \mu_2 \right] + r \Pr(D_1 \le d_1) \qquad \ldots\ldots\ldots\ldots\ldots\ldots.(3-3)$$

$$\frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial q_2} = a_1 \Pr(D_1 \le d_1) . \frac{\partial \Pr(D_2 \le d_2)}{\partial q_2} + a_2 \Pr(D_1 > d_1) . \frac{\partial \Pr(D_2 \le d_2)}{\partial q_2}$$

$$= \frac{\partial \Pr(D_2 \le d_2)}{\partial q_2} . \left[(a_1 - a_2) \Pr(D_1 \le d_1) \right]$$

Since, $\Pr(D_2 \le d_2) = \sum_{i=0}^{d_2} \binom{n_2}{i} q_2^i (1-q_2)^{n_2-i}$

then $$\frac{\partial \Pr(D_2 \le d_2)}{\partial q_2} = \sum_{i=0}^{d_2} \binom{n_2}{i} \left[i q_2^{i-1} (1-q_2)^{n_2-i} - q_2^i (n_2 - i)(1-q_2)^{n_2-i-1} \right]$$

Expanding the previous expression, gives:

$$= -n_2 (1-q_2)^{n_2-1} + \left[n_2 (1-q_2)^{n_2-1} - n_2 (n_2-1) q_2 (1-q_2)^{n_2-2} \right] +$$

$$\left[n_2 (n_2-1) q_2 (1-q_2)^{n_2-2} - \frac{n_2 (n_2-1)(n_2-2)}{2} q_2^2 (1-q_2)^{n_2-3} \right] + ... +$$

$$\left[\frac{n_2!}{(n_2-d_2)!\,d_2!} n_2 q_2^{d_2-1} (1-q_2)^{n_2-d_2} - \frac{n_2!}{(n_2-d_2)!\,d_2!} (n_2-d_2) q_2^{d_2} (1-q_2)^{n_2-d_2-1} \right]$$

$$= -\frac{n_2!}{(n_2-d_2)!\,d_2!} (n_2-d_2) q_2^{d_2} (1-q_2)^{n_2-d_2-1}$$

$$= -\frac{n_2!}{(n_2-d_2-1)!\,d_2!} q_2^{d_2} (1-q_2)^{n_2-d_2-1}$$

So,

$$\frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial q_2} = \left[-\frac{n_2!}{(n_2-d_2-1)!\,d_2!} q_2^{d_2} (1-q_2)^{n_2-d_2-1} \right] . \left[(a_1 - a_2) \Pr(D_1 \le d_1) \right] \qquad ....(3-4)$$

Substituting expressions (3-3) and (3-4) in the chain rule equation (3-2), yields

$$\frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial \mu_1} =$$

$$\left\{\begin{array}{l}\left[-\frac{n_1!}{(n_1-d_1-1)!d_1!}q_1^{d_1}(1-q_1)^{n_1-d_1-1}\right].\left[\frac{-1}{\sigma_1}\phi\left(\frac{L_1-\mu_1}{\sigma_1}\right)\right] \\ \left[a_1\Pr(D_2\le d_2)+a_2\Pr(D_2>d_2)+rq_1+I-c_2\mu_2\right].\end{array}\right\} + \left\{r\Pr(D_1\le d_1).\left[\frac{-1}{\sigma_1}\phi\left(\frac{L_1-\mu_1}{\sigma_1}\right)\right]\right\} +$$

$$\left\{\left[-\frac{n_2!}{(n_2-d_2-1)!d_2!}q_2^{d_2}(1-q_2)^{n_2-d_2-1}\right].\left[(a_1-a_2)\Pr(D_1\le d_1)\right].\left[\frac{-1}{\sqrt{\sigma_1^2+\sigma_2^2}}\phi\left(\frac{L-\mu_1-\mu_2}{\sqrt{\sigma_1^2+\sigma_2^2}}\right)\right]\right\}$$

$$......................(3-5)$$

To take partial derivative of equation (3-1) with respect to μ_2 we also need use chain rule:

$$\frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial \mu_2} = \frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial q_1}.\frac{\partial q_1}{\partial \mu_2} + \frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial q_2}.\frac{\partial q_2}{\partial \mu_2}$$

Notice that,

$\frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial q_1}.\frac{\partial q_1}{\partial \mu_2} = 0$, because q_1 is independent of μ_2.

As a result,

$$\frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial \mu_2} = \left[-\frac{n_2!}{(n_2-d_2-1)!d_2!}q_2^{d_2}(1-q_2)^{n_2-d_2-1}\right].\left[(a_1-a_2)\Pr(D_1\le d_1)\right].$$

$$\left[\frac{-1}{\sqrt{\sigma_1^2+\sigma_2^2}}\phi\left(\frac{L-\mu_1-\mu_2}{\sqrt{\sigma_1^2+\sigma_2^2}}\right)\right]$$

$$.............(3-6)$$

To find the necessary conditions, we need to equate expressions (3-5) and (3-6) to zero:

Setting $\frac{\partial E\{P(\mu_1 , \mu_2)\}}{\partial \mu_1} = 0, and \quad \frac{\partial E\{P(\mu_1 , \mu_2)\}}{\partial \mu_2} = 0$, and simplifying, yields

The necessary condition associated with (3-5) can be written as:

$$[a_1 \Pr(D_2 \le d_2) + a_2 \Pr(D_2 > d_2) + rq_1 + I - c_2\mu_2].\left[-\frac{n_1!}{(n_1 - d_1 - 1)! d_1!} q_1^{d_1} (1 - q_1)^{n_1 - d_1 - 1} \right] +$$

$$r \Pr(D_1 \le d_1) = 0 \qquad \text{.....................}(3-7)$$

The necessary condition associated with (3-6) can be written as:

$$\left[-\frac{n_2!}{(n_2 - d_2 - 1)! d_2!} q_2^{d_2} (1 - q_2)^{n_2 - d_2 - 1} \right].[\Pr(D_1 \le d_1)] = 0 \qquad \text{.........................}(3-8)$$

Optimal values of μ_1 and μ_2 can be obtained by solving (3-7) and (3-8) simultaneously. To solve the system of equations (3-7) and (3-8) simultaneously, multi-dimensional search techniques are used such as Newton's method and Hooke and Jeeve's method. Newton's method requires the calculation and inversion of the second order derivative (Hessian matrix). Working out analytical second order derivatives is not easy but can be done and their calculation is relatively fast. The real problem comes in first storing and more importantly inverting the matrix for large systems. Although clever methods exist for inverting matrices they are slow for large matrices.

3.4 Example

In this section, a real example from industry is presented to test the model developed in section 3.3. This example is considered as a case study for a particular problem that exists in the industry, particularly in Heba Fire Fighting Equipment Factory. Heba factory is one of the biggest factories in the gulf area that manufactures fire and safety equipments. In the first place, Heba factory concerns with manufacturing different kinds of fire extinguishers. However, when it comes to fire protection and safety requirements, it is important to provide customers with the best quality and reliability. Therefore, the company has invested more time and resources in making its customer satisfaction its main objective. From this prospective, the company welcomed our visit and provided us with all necessary information that may help in completing our thesis and solving their problem.

The case study that we are considering can be described as follows: Fire extinguishers (cylinders) go through different processes before they become ready-to-use final products. The most important processes are the ones that could affect the quality of the fire extinguishers. We found that the most important processes are the last two which are, coating the cylinders with zinc phosphate and then coating them with powder paint. The process of coating the cylinder with zinc phosphate is called process 1 while the process of coating the cylinder with powder paint is called process 2. The quality characteristic of interest is the thickness of the coating after process 1 and the thickness of the coating after process 2. Notice that, the coating thickness after process 1 is the thickness of the zinc phosphate while the coating thickness after process 2 is the sum of zinc phosphate and powder paint thicknesses. Specifications have to be met such that, the coating thickness after the first process should always be greater than or equal to 10 µm and the coating thickness after the second process should always be greater than or equal to 110 µm. A sampling inspection is conducted after each process. The sampling plan used after process 1 is: $n_1 = 13$, $d_1 = 1$. The sampling plan used after process 2 is: $n_2 = 13$, $d_2 = 1$. The cost of coating 1 m^2 of steel by zinc phosphate is SR 0.75 at 20 µm thickness, and the cost of powder paint of 1 m^2 of steel is SR 2.2 at 100 μm thickness. A fire extinguisher has an external surface area equals to 0.4 m^2. Fire extinguishers that satisfy both specification limits are sold at a regular market at a price of SR 35.64 per extinguisher. Whereas, fire extinguishers that satisfy only the first specification limit are sold at a secondary market with a price of SR 32.67 per extinguisher.

A sample of data is collected from the factory. Observations of the thicknesses of both layers of coating have been obtained. The normal distribution has been fit to the observed data to be used as an input to model developed in Section 3.3. The adequacy of the fit has been assessed by goodness-of-fit tests, i.e. chi-square test. Fitting the normal distribution to the observed data from both coating layers has shown an acceptable p-value for the chi-square test, which means that the normal distribution can be used to represent the observed data from both layers. As a result, it has been found that the first layer of coating is normally distributed with mean 22.2 and variance 5.13, and the second layer of coating is normally distributed with mean 126 and variance 11.14.

Now, if coating thicknesses are much greater than the lower specification limits, then we may maintain the quality of fire extinguishers but in this case the cost of materials will increase. On the other hand, if coating thicknesses are close to the lower specification limits, then we may incur less material cost but the quality of the fire extinguishers will be affected. So, the problem is to decide where to set the means of the two machines (processes) based on the tradeoff among production cost, payoff of nondefective items and the costs incurred by the disposition of the defective items. The obtained information can be summarized as follows:

L_1 = 10 μm, L = 110 μm, a_1 = 35.64 SR/item, a_2 = 32.67 SR/item, c_1 = 0.015 SR/μm, c_2 = 0.0088 SR/μm, r = 1.2 SR/item, I = 0.025 SR/item, $n_1 = n_2 = 13$, $d_1 = d_2 = 1$, $\sigma_1 = 5.13$, $\sigma_2 = 11.14$.

To solve this problem, we optimized the model developed in Section 3.3 using Microsoft Excel Solver tool. This tool uses the Generalized Reduced Gradient (GRG2) nonlinear optimization code developed by Leon Lasdon, University of Texas at Austin, and Allan Waren, Cleveland State University. The results below show the values of the optimal process means and expected profit for the targeting problem using sampling plan inspection.

Optimal mean for the first machine = 25.3913

Optimal mean for the second machine = 113.203

Expected profit per item = 34.2371

These results show that the optimal thickness for zinc phosphate is 25.3913 μm and the optimal thickness for the powder paint is 113.203 μm. If we set the two machines such that they produce items with optimal thicknesses then our expected profit will be SR 34.2371 per item. More experiments are conducted to see the effect of sample sizes and the allowable number of nonconforming items in the samples on the optimal process means and expected profit. For example, table 3.1 shows optimal process means and expected profit using sampling plans composed of (n_1=n_2=10, d_1=1, 2, 3 and d_2=1, 2, 3). Remaining tables show the optimal process means and expected profit for different sample sizes.

Table 3.1 Expected profit and means for the case study using n_1=n_2=10.

	d_1 = 1			d_1 = 2			d_1 = 3		
d_2	μ_1	μ_2	EP	μ_1	μ_2	EP	μ_1	μ_2	EP
1	24.9365	112.2859	34.2511	21.916	115.3064	34.2707	19.8552	117.3672	34.284
2	24.9378	106.5921	34.305	21.9171	109.6128	34.3247	19.8561	111.6737	34.3379
3	24.9388	102.4251	34.3436	21.9179	105.446	34.3632	19.8568	107.5071	34.3765

Table 3.2 Expected profit and means for the case study using n_1=n_2=13.

	d_1 = 1			d_1 = 2			d_1 = 3		
d_2	μ_1	μ_2	EP	μ_1	μ_2	EP	μ_1	μ_2	EP
1	25.3913	113.2029	34.2371	22.4842	116.11	34.2561	20.539	118.0552	34.2687
2	25.3925	107.8114	34.2885	22.4852	110.719	34.3074	20.5398	112.6641	34.32
3	25.3934	103.9585	34.3243	22.4859	106.866	34.3433	20.5404	108.8115	34.3559

Table 3.3 Expected profit and means for the case study using n_1=n_2=15.

	d_1 = 1			d_1 = 2			d_1 = 3		
d_2	μ_1	μ_2	EP	μ_1	μ_2	EP	μ_1	μ_2	EP
1	25.632	113.6809	34.2298	22.7794	116.5335	34.2484	20.8868	118.426	34.2607
2	25.633	108.431	34.2799	22.7803	111.2835	34.2985	20.8877	113.1761	34.3108
3	25.634	104.7172	34.3146	22.781	107.5702	34.3332	20.8882	109.4629	34.3455

Table 3.4 Expected profit and means for the case study using n_1=n_2=20.

	d_1 = 1			d_1 = 2			d_1 = 3		
d_2	μ_1	μ_2	EP	μ_1	μ_2	EP	μ_1	μ_2	EP
1	26.1024	114.6011	34.2155	23.347	117.356	34.2335	21.5448	119.1586	34.2452
2	26.1034	109.5969	34.2635	23.3479	112.353	34.2815	21.5455	114.1549	34.2932
3	26.1042	106.1163	34.2961	23.3485	108.872	34.3142	21.5461	110.6745	34.3259

The tables above show that the sampling plan currently used by the company is not the best plan in terms of the total profit, rather than the sampling plan (n_1=n_2=10, d_1=d_2=3), which result in the maximum profit.

Also, the tables show that when the acceptance numbers d_1 and d_2 are fixed, the expected profit decreases as samples sizes increase. This is because when we increase sample sizes, number of nonconforming items will be increased as a result probability of rejection will be increased and hence, expected profit decreases. For the same samples sizes, if d_1 or d_2 is increased, number of nonconforming items decrease and hence the expected profit increases. However, when the samples sizes and d_1 are fixed, we found that the expected mean of the first machine is almost constant. This is because the expected mean of the first machine depends on d_1 which is fixed in this case. However, the expected mean of the second machine decreases as d_2 increase. The reason for this is that when d_2 increase, the probability of rejection after the second machine decreases. As a result, the expected profit increases. The optimization model decreases the value of μ_2 in order to reduce material and processing costs so that the total expected profit is increased. In the same manner, fixing the samples sizes and d_2, we found that the expected profit increases as d_1 increase. Also, in this case the optimization model attempts to keep μ_1 small in a way that balances between reducing material cost and reducing probability of rejection.

3.5 Sensitivity Analysis

In this section, sensitivity analysis on the model developed in Section 3.3 is conducted to investigate the effect of changing model parameters on model results. Four types of sensitivity analysis are considered using different sampling plans: (n_1=n_2=10, d_1=d_2=1,2,3), (n_1=n_2=13, d_1=d_2=1,2,3), (n_1=n_2=15, d_1=d_2=1,2,3) and finally (n_1=n_2=20, d_1=d_2=1,2,3).

3.5.1 The effect of d_1 and d_2 on the expected profit

In this analysis, the effect of acceptance numbers d_1 and d_2 on the expected profit is studied and shown in figures 3-2 and 3-3.

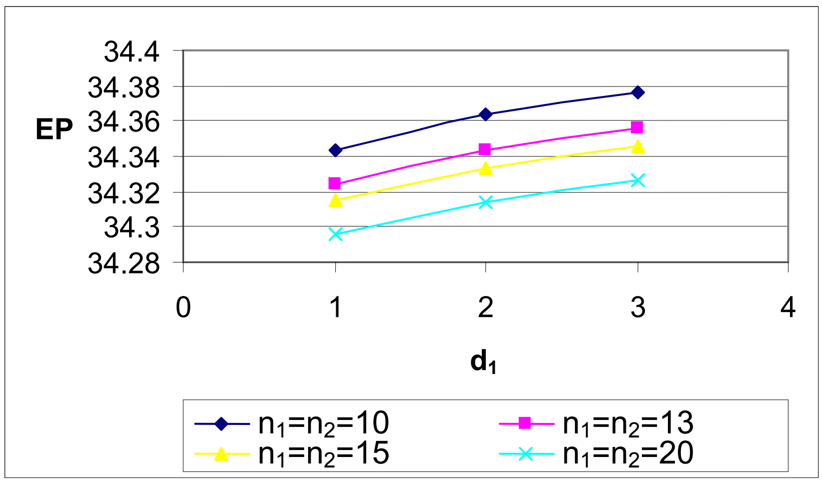

Figure (3-2) Expected profit versus d_1 at $d_2 = 1$.

Figure 3-2 shows that the expected profit increases as d_1 increases. The reason for this is that when d_1 increases, number of nonconforming items decrease and hence the expected profit increases. Moreover, for the same d_1, the expected profit decreases as sample sizes increase, because when sample size increases, the probability of having nonconforming items in the sample will also increase, hence the expected profit will decrease.

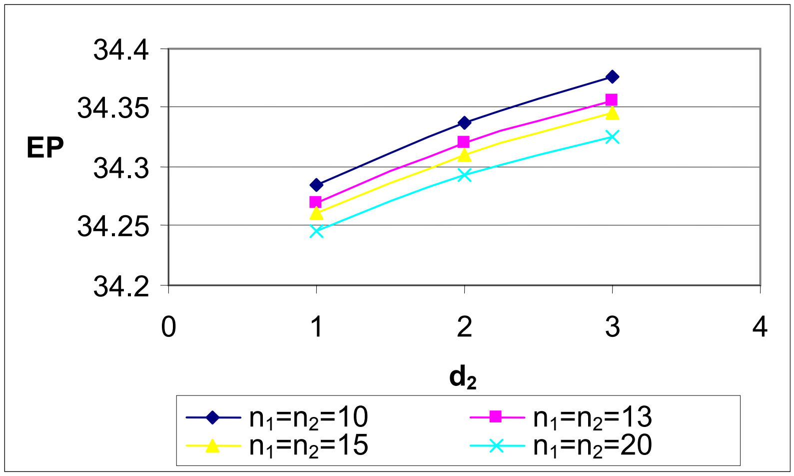

Figure (3-3) Expected profit versus d_2 at $d_1 = 1$.

Figure 3-3 shows that the expected profit increases as d_2 increases. The reason for this is that when d_2 increases, number of nonconforming items decrease and hence the expected profit increases. Moreover, for the same acceptance number, the expected profit decreases as sample sizes increase, because when sample size increases, the probability of having nonconforming items in the sample will also increase, hence the expected profit will decrease.

3.5.2 The effect of d_1 and d_2 on μ_1 and μ_2

In this case, the effect of acceptance numbers d_1 and d_2 on the optimal means of both processes is studied. Changing d_1 will have an effect on both of μ_1 and μ_2, while changing d_2 will affect only μ_2. Figures 3-4 and 3-5 show that as samples sizes increase for the same value of acceptance numbers d_1 and d_2; the set points (μ_1 or μ_2) are increased, because for any value of lot fraction defective q, the OC curve of the sampling plan with higher n will show a lower probability of acceptance. Another explanation is that sampling more items of a lot containing nonconforming items increases the likelihood of finding a nonconforming items and rejecting the lot. This will drive the producer to increase the expected set points.

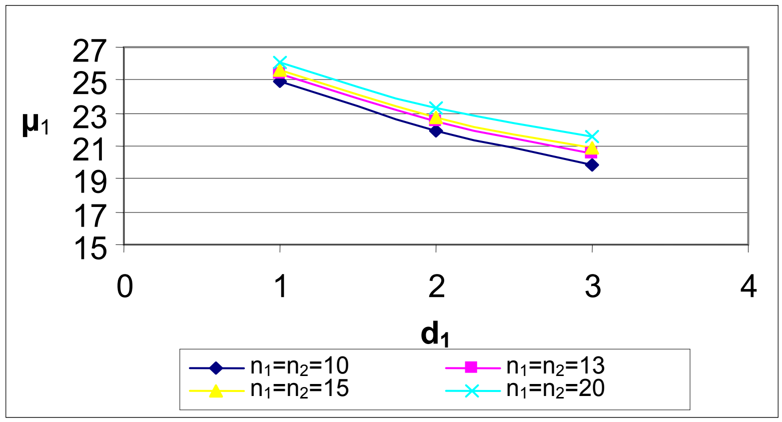

Figure (3-4) μ_1 versus d_1 at $d_2 = 1$.

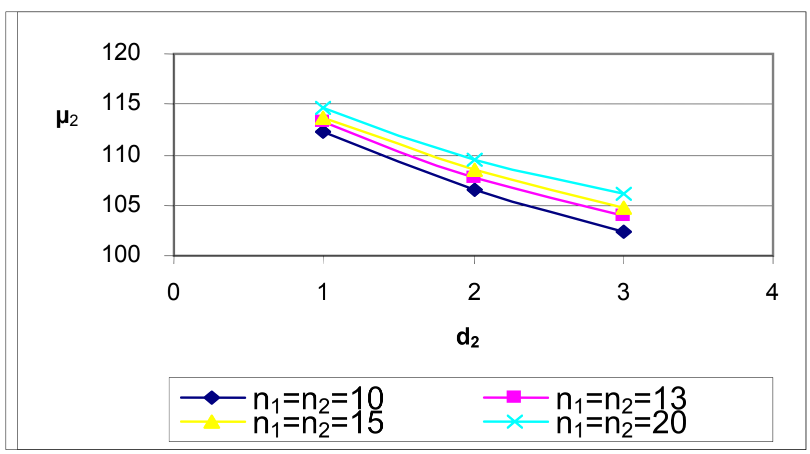

Figure (3-5) μ_2 versus d_2 at $d_1 = 1$.

Figure 3-6 shows that that as samples sizes increase for the same value of acceptance numbers d_1 and d_2; the mean of the second process is increased. The reason for this is that sampling more items of a lot containing nonconforming items increases the likelihood of finding a nonconforming items and rejecting the lot. This will drive the producer to increase the expected set point (μ_2).

Moreover, as the acceptance number increases, the optimum set point μ_2 increases. This result is to be expected since allowing more nonconforming items to occur in the first sample gives the producer more latitude in producing nonconforming items in the first process. As a result of the decrease in μ_1, there is going to be an increase in μ_2, since the probability of the acceptance in the second sample depends on the value of $\mu_1 + \mu_2$.

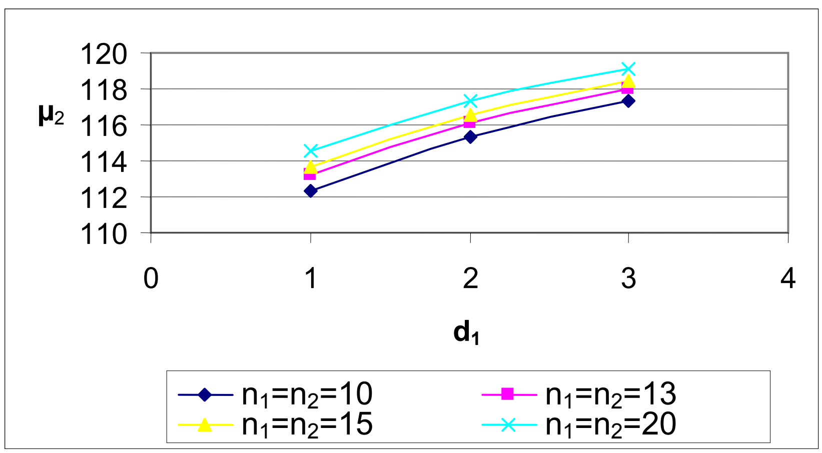

Figure (3-6) μ_2 versus d_1 at $d_2 = 1$.

3.5.3 The effect of cost parameters on the expected profit

The effect of different cost parameters on the expected profit are studied by taking the partial derivatives of the expected profit function with respect to selling prices (a_1 and a_2), processing costs (c_1 and c_2), rework cost (r) and inspection cost (I). The rate of change of the expected profit with respect to theses parameters are as follows:

$$\frac{\partial}{\partial a_1} E\{P(\mu_1, \mu_2)\} = \Pr(D_1 \le d_1)\Pr(D_2 \le d_2)$$

$$\frac{\partial}{\partial a_2} E\{P(\mu_1, \mu_2)\} = \Pr(D_1 \le d_1)\Pr(D_2 > d_2)$$

$$\frac{\partial}{\partial c_1} E\{P(\mu_1, \mu_2)\} = -\mu_1$$

$$\frac{\partial}{\partial c_2} E\{P(\mu_1, \mu_2)\} = -\mu_2 \Pr(D_2 \le d_2)$$

$$\frac{\partial}{\partial r} E\{P(\mu_1, \mu_2)\} = -q_1 \Pr(D_1 > d_1)$$

$$\frac{\partial}{\partial I} E\{P(\mu_1, \mu_2)\} = -\Pr(D_1 > d_1)$$

The rate of change of the expected profit with respect to the selling price a_1 is the product of the probability of accepting the lot after the first process and probability of accepting the lot after the second process. The rate of change of the expected profit with respect to the selling price a_2 is the product of probability of accepting the lot after the first process and probability of rejecting the lot after the second process. The negative value of the mean of the first process represents the rate of change of the expected profit with respect to c_1. However, the rate of change of the expected profit with respect to the processing cost c_2 is represented by the product of the negative value of the mean of the second process and probability of accepting the lot after the first process. The product of the negative value of the lot fraction defective after the fist process and probability of rejecting the lot after the first process represents the rate of change of the expected profit with respect to the rework cost r.

Finally, the rate of change of the expected profit with respect to the inspection cost I is the negative value of the probability of rejecting the lot after the first process.

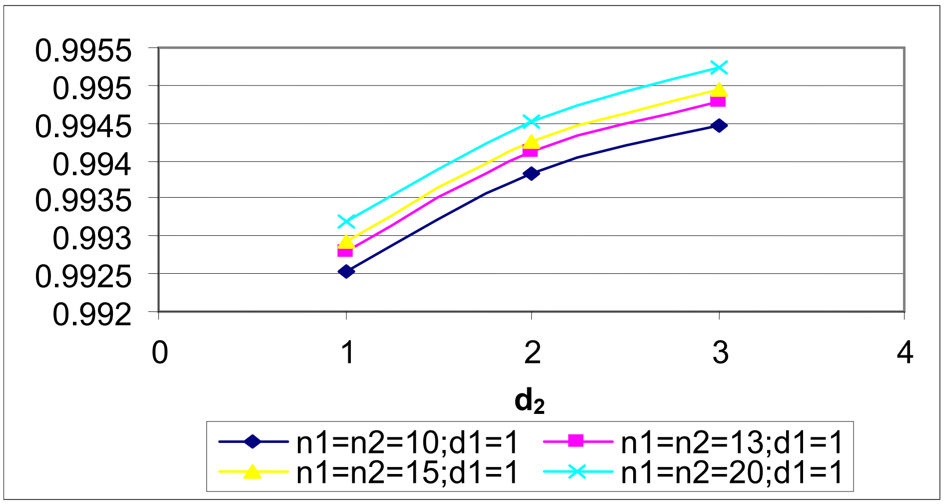

Figure (3-7) Rate of change of expected profit with respect to a_1 vs d_2

Figure 3-7 shows the rate of change of expected profit with respect to a_1 versus the acceptance number of the second sample d_2 when $d_1 = 1$. The rate of change of expected profit increases with the increase in the acceptance number d_2. This is because the increase of d_2 will decrease the number of nonconforming items in the second sample and hence probability of accepting the lot will increase.

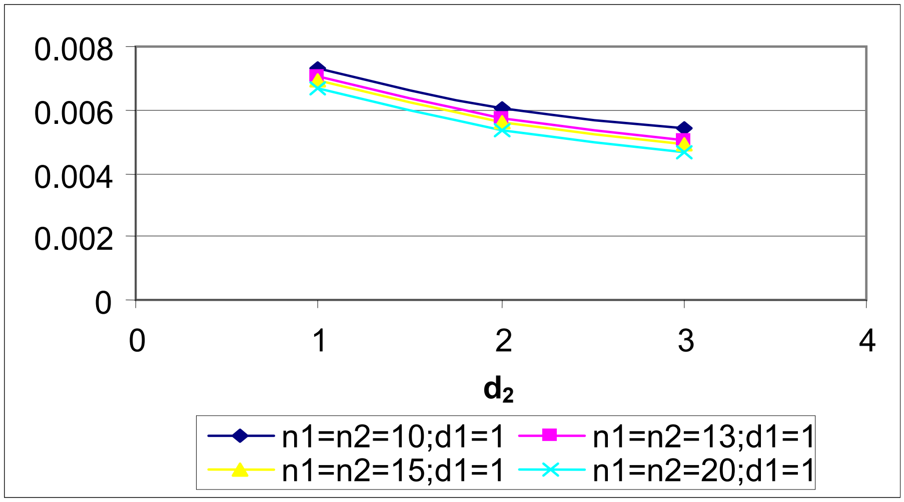

Figure (3-8) Rate of change of expected profit with respect to a_2 vs d_2

Figure 3-8 shows the rate of change of expected profit with respect to a_2 versus the acceptance number of the first sample d_2 when $d_1 = 1$. The rate of change of expected profit decreases with the increase in the acceptance number d_2. This is because the increase of d_2 will decrease the number of nonconforming items in the second sample and hence probability of rejecting the lot will decrease.

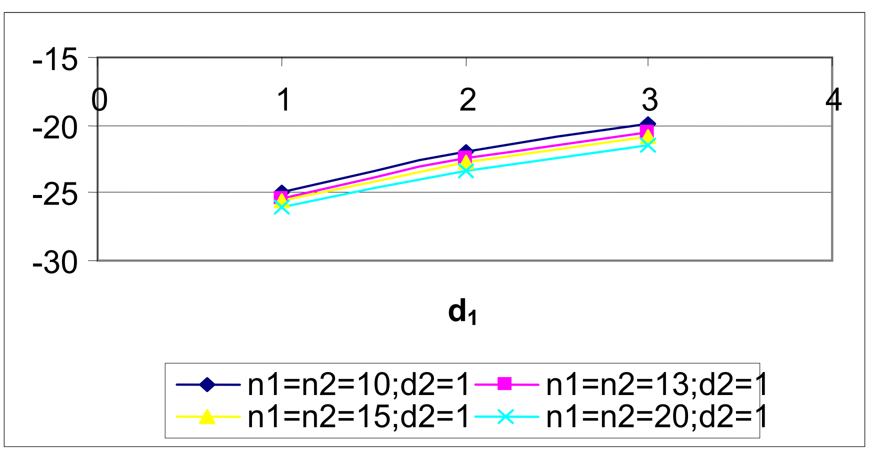

Figure (3-9) Rate of change of expected profit with respect to c_1 vs d_1

The rate of change of expected profit with respect to c_1 versus d_1 is shown in figure 3-9. The rate of change of expected profit is the negative of the mean of the first process. Increasing the acceptance number d_1 will result in decreasing the mean of the first process because raising the value of d_1 has the effect of enlarging the area of acceptance under the OC curve corresponding to the first sampling plan, i.e. allowing more nonconforming items to occur gives the producer more latitude in producing nonconforming units, but as seen in the figure the rate of change of expected profit with respect to c_1 increases because of the negative sign associated with the mean.

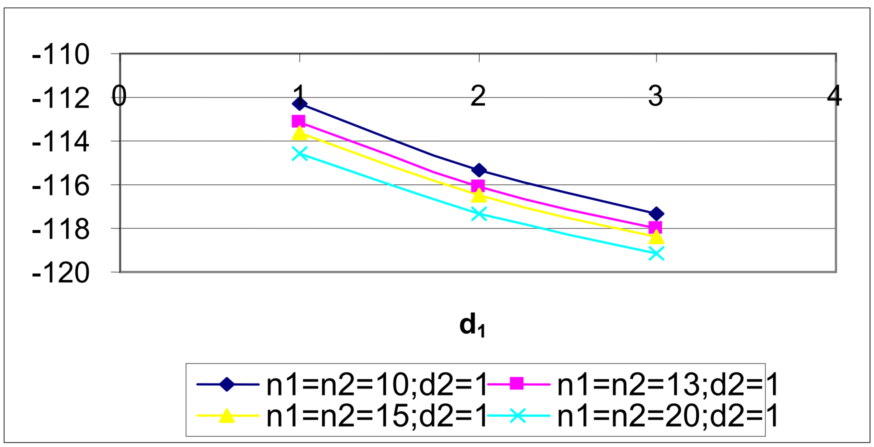

Figure (3-10) Rate of change of expected profit with respect to c_2 vs d_1

The rate of change of expected profit with respect to c_2 versus d_1 is shown in figure 3-10. The rate of change of expected profit is the negative product of the mean of the second process and probability of accepting the lot based on the first sample. Increasing the acceptance number d_1 will result in increasing the probability of accepting the lot and decreasing the mean of the first process and hence the mean of the second process will increase. As seen in the figure the rate of change of expected profit with respect to c_2 decreases because of the negative sign.

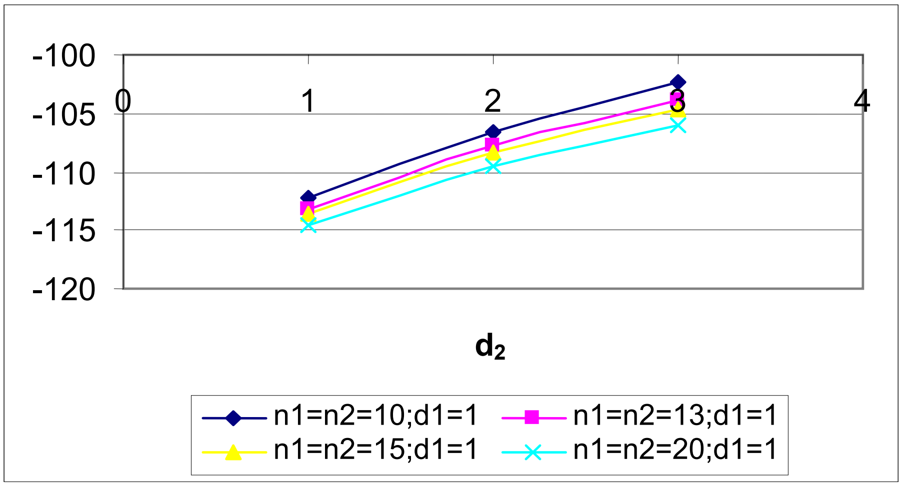

Figure (3-11) Rate of change of expected profit with respect to c_2 vs d_2

However, the rate of change of expected profit with respect to c_2 versus d_2, as shown in figure 3-11, increases as d_2 increases. This is because, increasing the acceptance number d_2 will result in decreasing the mean of the second process because raising the value of d_2 has the effect of enlarging the area of acceptance under the OC curve corresponding to the second sampling plan, i.e. allowing more nonconforming items to occur gives the producer more latitude in producing nonconforming units, but as seen in the figure the rate of change of expected profit with respect to c_2 increases because of the negative sign associated with the mean.

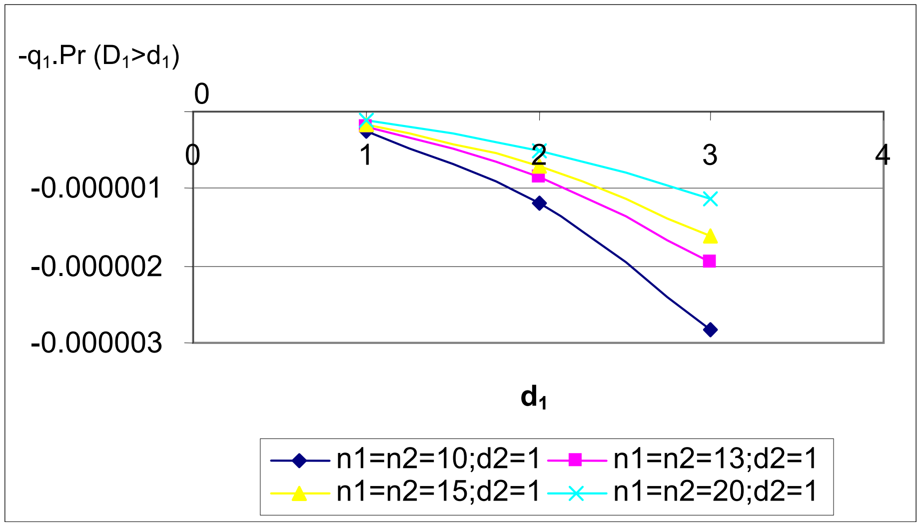

Figure (3-12) Rate of change of expected profit with respect to r vs d_1

Figure 3-12 shows the rate of change of expected profit with respect to r versus the acceptance number of the first sample d_1 when $d_2 = 1$. The rate of change of expected profit decreases with the increase in the acceptance number d_1.

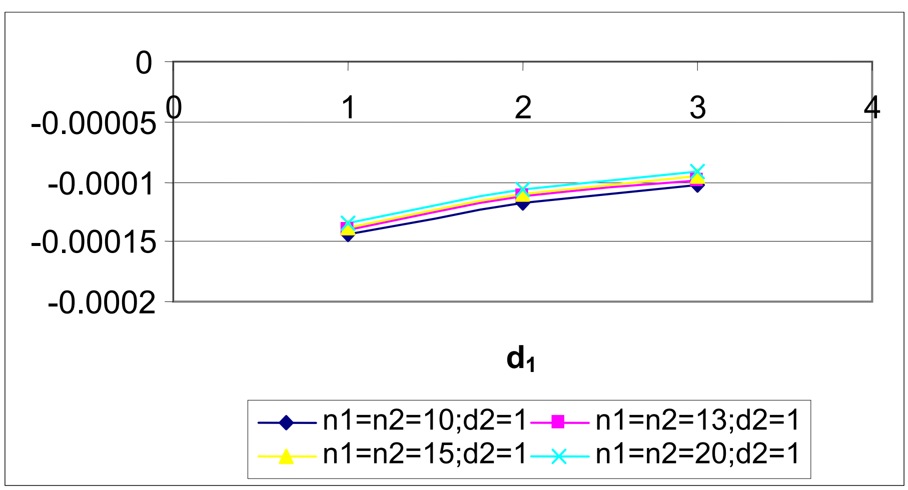

Figure (3-13) Rate of change of expected profit with respect to I vs d_1

Figure 3-13 shows the rate of change of expected profit with respect to I versus the acceptance number of the first sample d_1 when $d_2 = 1$. The rate of change of expected profit increases with the increase in the acceptance number d_1. This is because the increase of d_1 will decrease the number of nonconforming items and hence probability of rejecting the lot will decrease, but as seen in the figure the rate of change of expected profit with respect to I increases because of the negative sign.

3.6 Conclusion

In this chapter, a model is developed for two processes producing a single product with two quality characteristics. The first quality characteristic is determined by the first process and the second quality characteristic is determined by both processes. Specification limits are set on both quality characteristics. A sampling plan has been used as a mean for product quality control assuming perfect inspection. A real case has been considered to test the model developed in this chapter followed by the sensitivity analysis which has been conducted to investigate the effect of changing model parameters on model results.

The optimal process settings for the case study were found using the model developed in this chapter. Also, it is found that the currently sampling plan used by the company is not the best in terms of the total profit, rather than the sampling plan ($n_1=n_2=10$, $d_1=d_2=3$) which had shown better results.

Three cases have been considered in the sensitivity analysis: the effect of d_1 and d_2 on the expected profit, the effect of d_1 and d_2 on the means of both processes and the effect of cost parameters on the expected profit. In the model developed in this chapter, inspection is assumed to be error free. This assumption is relaxed in chapter 4.

CHAPTER 4

PROCESS TARGETING WITH TWO INDEPENDENT PROCESSES IN SERIES USING SAMPLING PLAN WITH INSPECTION ERROR

4.1 Introduction

The purpose of this chapter is to develop a process targeting model for two independent processes in series using sampling plan as a mean for product quality control. Inspection is assumed to be error prone in this chapter. Basically in this chapter, we are extending the model developed in chapter 3 by incorporating inspection error in the sampling plan.

Classically, sampling plans have assumed that the inspection process is perfect, with no errors in judgments being made by the inspector. This assumption is, in many cases, not realistic due to human error and/or measuring instrument's error. In reality, an inspector (human or machine) is subjected to make two types of errors. These errors are:

1. Type I error: Classifying a non-defective item as defective, it means inspectors reject a conforming item.
2. Type II error: classifying a defective item as non-defective, it means inspectors accept a nonconforming item.

Thus, inspection error may cause considerable loss due to misclassification of the product. The loss could be in the form of loss of goodwill, or loss of profit by selling a first market product as a secondary market product. The objective of this model is to maximize the expected profit by finding the optimal target values for two independent processes in series with quality sampling plans by considering the effect of inspection error.

This chapter is organized as follow; the problem under consideration is stated in section 4.2. Section 4.3 presents the model development and section 4.4 contains solution and results. Sensitivity analysis on the model is given in section 4.5 and finally section 4.6 concludes this chapter.

4.2 Statement of the Problem

Consider a manufacturing environment in which products go through two different processes. The product has two quality characteristics. The first quality characteristic is determined by the first process, and the second quality characteristic is determined by both processes. Specification limits are set on both quality characteristics.

Let us assume that the quality characteristic of the first process is a random variable denoted by X_1, and the quality characteristic of the second process is another random variable denoted by X_2. The final quality characteristic is denoted by X, where $X=X_1+X_2$. Quality requirement is that $X_1 \geq L_1$ and $X \geq L$, where L_1 and L are predetermined constants set by product designer or market information. A sampling plan is used for product quality control, however, inspection is assumed to be error prone. Therefore, if type I and type II errors are considered as part of the sampling plan, then the number of nonconforming items appears to the inspector is not the same as the actual number of nonconforming items in both samples. Thus, the expected apparent number of nonconforming items in n_1 and n_2, are given respectively by:

$$D_{e1} = (n_1 - D_1)e_{11} + (1 - e_{12})D_1$$

$$D_{e2} = (n_2 - D_2)e_{21} + (1 - e_{22})D_2$$

In the presence of error, the decision to accept the lot is based on the apparent number of nonconforming items in both samples, D_{e1} and D_{e2}. In this case the sampling plan is described as follows: a lot of N item gets processed by the first machine, and then a sample of size n_1 is taken from the lot and tested. If more than d_1 products have their first quality characteristic less than L_1, then we inspect the lot item by item (at a cost of I per item) to separate conforming and nonconforming items. The conforming items are sent to the second process whereas the nonconforming items are reworked at a fixed cost (r) per item. Otherwise, the lot proceeds to the second machine. After being processed by the second machine, a sample of size n_2 is taken from the lot and tested. If more than d_2 items have their second quality characteristic less than L, then the lot is rejected and sold at a secondary market, otherwise the lot is accepted and sold at a regular market, see figure (4-

1). The same fire extinguisher example used in chapter 3 is used for the case in this chapter. The objective of the model is to maximize the expected profit by finding the optimal target values of the two processes using sampling plans with inspection errors.

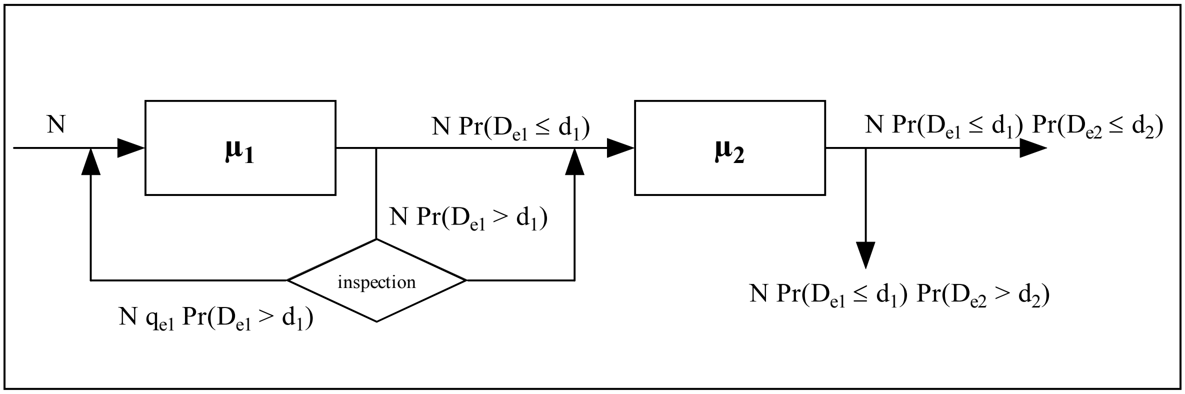

Figure (4-1) Targeting model for two processes using imperfect sampling plan.

4.3 Model Development

In this section, a process targeting model is developed for the problem stated. Notations and assumptions are presented prior to model development.

The following notations are adopted:

X_i A random variable that represents the value of the quality characteristic of the product after finishing process i, where i = 1, 2.

X Sum of the values of both quality characteristics, $X = X_1 + X_2$.

L_1 Lower specification limit for the first product quality characteristic.

L Lower specification limit for the composite (sum) of the product's two quality characteristics.

a_1 Selling price /item for accepted items after process 2.

a_2 Selling price /item for items rejected after process 2, (secondary selling price).

c_i Cost of material consumed in process i per unit of thickness, i = 1, 2.

I Inspection cost per item in the rejected lot after the first process.

r Rework cost per defective item in the rejected lot after process 1.

μ_1 Mean setting for process 1.

μ_2 Mean setting for process 2.

σ_1 Standard deviation for process 1.

σ_2 Standard deviation for process 2.

N Lot size.

n_i sample size for sampling after process i, i = 1,2.

d_i Allowable number of nonconforming items in the sample after process i, i=1,2.

D_i A random variable that represents the actual number of nonconforming units in the sample after process i, i = 1,2.

D_{ei} A random variable that represents the apparent number of nonconforming units in the sample after process i, i = 1,2.

q_i Lot fraction defective after process i, i = 1, 2.

q_{ei} Apparent lot fraction defective after process i, i = 1, 2.

e_{i1} Probability of type I error associated with the inspection after process i, i =1, 2.

e_{i2} Probability of type II error associated with the inspection after process i, i=1, 2.

TP Total profit.

P Profit per item.

The following assumptions are used to develop the model:

1. X_1 and X_2 are independent and normally distributed with mean μ_1 and μ_2 respectively and standard deviation σ_1 and σ_2 respectively.
2. The actual number of nonconforming items in a sample of size n_i follows binomial distribution.
3. The apparent number of nonconforming items in a sample of size n_i follows binomial distribution.
4. The processing order is fixed.
5. Costs of processing are assumed to be directly proportional to the mean setting of the process.
6. The secondary selling price is less than the regular selling price and the lower specification limit for the first process is less than the lower specification limit of the second process.
7. The rework after the first process involves removing the paint and starting the first process from the beginning at a fixed cost (r).
8. Sampling plan is used for product quality control and it is error prone.
9. There is no shift or drift in the processes over time.

The revenue for a lot of size N can be stated as:

$$R = \begin{cases} a_1 N & if \quad D_{e1} \le d_1 \quad and \quad D_{e2} \le d_2 \\ a_2 N & if \quad D_{e1} \le d_1 \quad and \quad D_{e2} > d_2 \\ -(I + rq_{e1})N & if \quad D_{e1} > d_1 \end{cases}$$

The cost for each lot includes: material cost, and inspection cost. The expected total profit can be represented by the following expression:

$$E\{TP(\mu_1, \mu_2)\} = a_1.N.\Pr(D_{e1} \le d_1).\Pr(D_{e2} \le d_2) + a_2.N.\Pr(D_{e1} \le d_1).\Pr(D_{e2} > d_2) -$$

$$I.N.\Pr(D_{e1} > d_1) - r.q_{e1}.N.\Pr(D_{e1} > d_1) - c_1.\mu_1.N - c_2.\mu_2.N.\Pr(D_{e1} \le d_1)$$

where, $a_1.N.\Pr(D_{e1} \le d_1).\Pr(D_{e2} \le d_2)$ is the expected revenue from selling items in the first market, and $a_2.N.\Pr(D_{e1} \le d_1).\Pr(D_{e2} > d_2)$ is the expected revenue from selling items in the secondary market. $I.N.\Pr(D_{e1} > d_1)$ is the expected cost of inspecting the lot rejected after the first process. The term $r.q_1.N.\Pr(D_1 > d_1)$ represents the expected cost of reworking nonconforming items in the lot. For the first process, the expected value of the processing cost per lot is given by $c_1.\mu_1.N$. Whereas, for the second process, the expected value of the processing cost per lot is given by $c_2.\mu_2.N.\Pr(D_{e1} \le d_1)$.

The expected profit per item can be obtained by dividing E(TP) by N.

$$E(P) = a_1.\Pr(D_{e1} \le d_1).\Pr(D_{e2} \le d_2) + a_2.\Pr(D_{e1} \le d_1).\Pr(D_{e2} > d_2) -$$

$$I.\Pr(D_{e1} > d_1) - r.q_{e1}.\Pr(D_{e1} > d_1) - c_1.\mu_1 - c_2.\mu_2.\Pr(D_{e1} \le d_1) \quad \text{.....................}(4-1)$$

Necessary conditions to maximize the profit with respect to μ_1 and μ_2 are to set the partial derivatives to zero.

$$\frac{\partial E\{P(\mu_1, \mu_2)\}}{\partial \mu_1} = 0$$

$$\frac{\partial E\{P(\mu_1, \mu_2)\}}{\partial \mu_2} = 0$$

Notice that:

$$D_{e1} = (n_1 - D_1)e_{11} + (1 - e_{12})D_1$$

$$\Pr(D_{e1} \le d_1) = \sum_{i=0}^{d_1} \binom{n_1}{i} q_{e1}^i (1 - q_{e1})^{n_1 - i} \qquad , where$$

$$q_{e1} = q_1(1 - e_{12}) + (1 - q_1)e_{11}$$

$$q_1 = \Pr(X_1 < L_1) = \Phi\left(\frac{L_1 - \mu_1}{\sigma_1}\right)$$

Similarly,

$$D_{e2} = (n_2 - D_2)e_{21} + (1 - e_{22})D_2$$

$$\Pr(D_{e2} \le d_2) = \sum_{i=0}^{d_2} \binom{n_2}{i} q_{e2}^i (1 - q_{e2})^{n_2 - i} \qquad , where$$

$$q_{e2} = q_2(1 - e_{22}) + (1 - q_2)e_{21}$$

$$q_2 = \Pr(X < L) = \Pr(X_1 + X_2 < L) = \Phi\left(\frac{L - \mu_1 - \mu_2}{\sqrt{\sigma_1^2 + \sigma_2^2 + 2Cov(X_1, X_2)}}\right) = \Phi\left(\frac{L - \mu_1 - \mu_2}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)$$

To take partial derivative of equation (4-1) with respect to μ_1 we need to use the chain rule:

$$\frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial \mu_1} = \frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial q_{e1}} \cdot \frac{\partial q_{e1}}{\partial q_1} \cdot \frac{\partial q_1}{\partial \mu_1} + \frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial q_{e2}} \cdot \frac{\partial q_{e2}}{\partial q_2} \cdot \frac{\partial q_2}{\partial \mu_1}$$

Where,

$$\frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial q_1} = a_1 \Pr(D_{e2} \le d_2) . \frac{\partial \Pr(D_{e1} \le d_1)}{\partial q_{e1}} + a_2 \Pr(D_{e2} > d_2) . \frac{\partial \Pr(D_{e1} \le d_1)}{\partial q_{e1}} +$$

$$r\left[\Pr(D_{e1} \le d_1) + q_{e1} . \frac{\partial \Pr(D_{e1} \le d_1)}{\partial q_{e1}}\right] + I . \frac{\partial \Pr(D_{e1} \le d_1)}{\partial q_{e1}} - c_2 \mu_2 . \frac{\partial \Pr(D_{e1} \le d_1)}{\partial q_{e1}}$$

$$= \frac{\partial \Pr(D_{e1} \le d_1)}{\partial q_{e1}} . \left[a_1 \Pr(D_{e2} \le d_2) + a_2 \Pr(D_{e2} > d_2) + r q_{e1} + I - c_2 \mu_2\right] + r \Pr(D_{e1} \le d_1)$$

Give that:

$$\Pr(D_{e1} \le d_1) = \sum_{i=0}^{d_1} \binom{n_1}{i} q_{e1}^{i} (1 - q_{e1})^{n_1 - i}$$

$$\Rightarrow \frac{\partial \Pr(D_{e1} \le d_1)}{\partial q_{e1}} = \sum_{i=0}^{d_1} \binom{n_1}{i} \left[i q_{e1}^{i-1} (1 - q_{e1})^{n_1 - i} - q_{e1}^{i} (n_1 - i)(1 - q_{e1})^{n_1 - i - 1} \right]$$

$$= -n_1 (1 - q_{e1})^{n_1 - 1} + \left[n_1 (1 - q_{e1})^{n_1 - 1} - n_1 (n_1 - 1) q_{e1} (1 - q_{e1})^{n_1 - 2} \right] +$$

$$\left[n_1 (n_1 - 1) q_{e1} (1 - q_{e1})^{n_1 - 2} - \frac{n_1 (n_1 - 1)(n_1 - 2)}{2} q_{e1}^{2} (1 - q_{e1})^{n_1 - 3} \right] + \ldots\ldots + $$

$$\left[\frac{n_1!}{(n_1 - d_1)! d_1!} n_1 q_{e1}^{d_1 - 1} (1 - q_{e1})^{n_1 - d_1} - \frac{n_1!}{(n_1 - d_1)! d_1!} (n_1 - d_1) q_{e1}^{d_1} (1 - q_{e1})^{n_1 - d_1 - 1} \right]$$

$$= -\frac{n_1!}{(n_1 - d_1)! d_1!} (n_1 - d_1) q_{e1}^{d_1} (1 - q_{e1})^{n_1 - d_1 - 1}$$

$$= -\frac{n_1!}{(n_1 - d_1 - 1)! d_1!} q_{e1}^{d_1} (1 - q_{e1})^{n_1 - d_1 - 1}$$

So,

$$\frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial q_{e1}} =$$

$$\left[-\frac{n_1!}{(n_1-d_1-1)!\,d_1!}q_{e1}^{d_1}(1-q_{e1})^{n_1-d_1-1}\right].\left[a_1\Pr(D_{e2}\le d_2)+a_2\Pr(D_{e2}>d_2)+rq_{e1}+I-c_2\mu_2\right]+$$

$$r\Pr(D_{e1}\le d_1) \qquad \qquad ....(4-2)$$

Similarly,

$$\frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial q_{e2}} = a_1\Pr(D_{e1}\le d_1).\frac{\partial \Pr(D_{e2}\le d_2)}{\partial q_{e2}} + a_2\Pr(D_{e1}>d_1).\frac{\partial \Pr(D_{e2}\le d_2)}{\partial q_{e2}}$$

$$= \frac{\partial \Pr(D_{e2}\le d_2)}{\partial q_{e2}}.\left[(a_1-a_2)\Pr(D_{e1}\le d_1)\right]$$

Since,

$$\Pr(D_{e2}\le d_2)=\sum_{i=0}^{d_2}\binom{n_2}{i}q_{e2}^{i}(1-q_{e2})^{n_2-i}$$

So,

$$\frac{\partial \Pr(D_{e2}\le d_2)}{\partial q_{e2}}=\sum_{i=0}^{d_2}\binom{n_2}{i}\left[iq_{e2}^{i-1}(1-q_{e2})^{n_2-i}-q_{e2}^{i}(n_2-i)(1-q_{e2})^{n_2-i-1}\right]$$

$$=-n_2(1-q_{e2})^{n_2-1}+\left[n_2(1-q_{e2})^{n_2-1}-n_2(n_2-1)q_{e2}(1-q_{e2})^{n_2-2}\right]+$$
$$\left[n_2(n_2-1)q_{e2}(1-q_{e2})^{n_2-2}-\frac{n_2(n_2-1)(n_2-2)}{2}q_{e2}^{2}(1-q_{e2})^{n_2-3}\right]+.......+$$
$$\left[\frac{n_2!}{(n_2-d_2)!\,d_2!}n_2q_{e2}^{d_2-1}(1-q_{e2})^{n_2-d_2}-\frac{n_2!}{(n_2-d_2)!\,d_2!}(n_2-d_2)q_{e2}^{d_2}(1-q_{e2})^{n_2-d_2-1}\right]$$

$$= -\frac{n_2!}{(n_2-d_2)!\,d_2!}(n_2-d_2)q_{e2}^{d_2}(1-q_{e2})^{n_2-d_2-1}$$

$$= -\frac{n_2!}{(n_2-d_2-1)!\,d_2!}q_{e2}^{d_2}(1-q_{e2})^{n_2-d_2-1}$$

then, $$\frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial q_2} = \left[-\frac{n_2!}{(n_2-d_2-1)!\,d_2!}q_{e2}^{d_2}(1-q_{e2})^{n_2-d_2-1}\right].\left[(a_1-a_2)\Pr(D_{e1}\le d_1)\right]$$

$$\text{......................}(4-3)$$

Substituting expressions (4-2) and (4-3) in the chain rule equation, yields

$$\frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial \mu_1} =$$

$$\left\{\begin{matrix}\left[-\frac{n_1!}{(n_1-d_1-1)!\,d_1!}q_{e1}^{d_1}(1-q_{e1})^{n_1-d_1-1}\right].\left[\frac{-1}{\sigma_1}\phi\left(\frac{L_1-\mu_1}{\sigma_1}\right)\right].(1-e_{12}-e_{11}) \\ \left[a_1\Pr(D_{e2}\le d_2)+a_2\Pr(D_{e2}>d_2)+rq_{e1}+I-c_2\mu_2\right].\end{matrix}\right\} +$$

$$\left\{r(1-e_{12}-e_{11})\Pr(D_{e1}\le d_1).\left[\frac{-1}{\sigma_1}\phi\left(\frac{L_1-\mu_1}{\sigma_1}\right)\right]\right\} +$$

$$\left\{\begin{matrix}\left[-\frac{n_2!}{(n_2-d_2-1)!\,d_2!}q_{e2}^{d_2}(1-q_{e2})^{n_2-d_2-1}\right].\left[\frac{-1}{\sqrt{\sigma_1^2+\sigma_2^2}}\phi\left(\frac{L-\mu_1-\mu_2}{\sqrt{\sigma_1^2+\sigma_2^2}}\right)\right] \\ \left[(a_1-a_2)\Pr(D_{e1}\le d_1)\right].(1-e_{22}-e_{21})\end{matrix}\right\}$$

$$\text{......................}(4-4)$$

To take partial derivative of equation (4-1) with respect to μ_2 we also need to use the chain rule:

$$\frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial \mu_2} = \frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial q_{e1}}\cdot\frac{\partial q_{e1}}{\partial q_1}\cdot\frac{\partial q_1}{\partial \mu_2} + \frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial q_{e2}}\cdot\frac{\partial q_{e2}}{\partial q_2}\cdot\frac{\partial q_2}{\partial \mu_2}$$

Notice that,

$\frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial q_{e1}} \cdot \frac{\partial q_{e1}}{\partial q_1} \cdot \frac{\partial q_1}{\partial \mu_2} = 0$, because q_1 is independent of μ_2.

As a result,

$$\frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial \mu_2} = \left[-\frac{n_2!}{(n_2-d_2-1)!d_2!} q_{e2}^{d_2}(1-q_{e2})^{n_2-d_2-1}\right].\left[(a_1-a_2)\Pr(D_{e1} \le d_1)\right].$$
$$\left[\frac{-1}{\sqrt{\sigma_1^2+\sigma_2^2}} \phi\left(\frac{L-\mu_1-\mu_2}{\sqrt{\sigma_1^2+\sigma_2^2}}\right)\right].(1-e_{22}-e_{21})$$

.............(4 – 5)

To find the necessary conditions, we need to equate expressions (4-4) and (4-5) to zero.

The necessary condition associated with (4-4) can be written as:

$$\left[a_1 \Pr(D_{e2} \le d_2) + a_2 \Pr(D_{e2} > d_2) + rq_1 + I - c_2\mu_2\right]\left[-\frac{n_1!}{(n_1-d_1-1)!d_1!} q_{e1}^{d_1}(1-q_{e1})^{n_1-d_1-1}\right] +$$

$$r\Pr(D_{e1} \le d_1) = 0$$

....................(4 – 6)

The necessary condition associated with (4-5) can be written as:

$$\left[-\frac{n_2!}{(n_2-d_2-1)!d_2!} q_{e2}^{d_2}(1-q_{e2})^{n_2-d_2-1}\right].\left[\Pr(D_{e1} \le d_1)\right] = 0 \qquad ....(4-7)$$

Optimal values of μ_1 and μ_2 can be obtained by solving (4-6) and (4-7) simultaneously. To solve the system of equations (4-6) and (4-7) simultaneously, multi-dimensional search techniques are used such as Newton's method and Hooke and Jeeve's method. Newton's method requires the calculation and inversion of the second order derivative (Hessian matrix). Working out analytical second order derivatives is not easy but can be done and their calculation is relatively fast. The real problem comes in first storing and more importantly inverting the matrix for large systems. Although clever methods exist for inverting matrices they are slow for large matrices.

4.4 Example

In this section, the same case study described in chapter 3 is considered with some modifications. The case study is described as follows: Fire extinguishers (cylinders) go through different processes before they become ready-to-use final products. The most important processes are the ones that could affect the quality of the fire extinguishers. It was found that the most important processes are the last two which are, coating the cylinders with zinc phosphate and then coating them with powder paint. The process of coating the cylinder with zinc phosphate is called process 1 while the process of coating the cylinder with powder paint is called process 2. The quality characteristic of interest is the thickness of the coating after process 1 and the thickness of the coating after process 2. Notice that, the coating thickness after process 1 is the thickness of the zinc phosphate while the coating thickness after process 2 is the sum of zinc phosphate and powder paint thicknesses. Specifications have to be met such that, the coating thickness after the first process should always be greater that or equal to 10 µm and the coating thickness after the second process should always be greater than or equal to 110 µm. A sampling inspection is conducted after each process. We assume that the inspection process is error prone where two types of errors associated with each inspection plan. The inspector could classify a conforming item as nonconforming or a nonconforming item as conforming. The sampling plan used after process 1 is: n_1 =13, d_1 = 1 and the corresponding sampling errors are; e_{11} = 0.01 and e_{12} = 0.05. The sampling plan used after process 2 is: n_2 =13, d_2

= 1 and the corresponding sampling errors are; e_{21} = 0.01 and e_{22} = 0.05. The cost of coating 1 m^2 of steel by zinc phosphate is 0.75 SR at 20 μm thickness, and the cost of powder paint of 1 m^2 of steel is 2.2 SR at 100 μm thickness. An extinguisher has an external surface area equals to 0.4 m^2. Extinguishers that satisfy both specification limits are sold at a regular market at a price of 35.64 SR per extinguisher. Whereas, cylinders that satisfy only the first specification limit are sold at a secondary market with a price of 32.67 SR per extinguisher.

Data for the thickness of both layers of coating is obtained from Heba factory. The normal distribution has been fit to the data to be used as an input to model developed in 4.3. The adequacy of the fit is assessed by goodness-of-fit tests, i.e., chi-square test. Fitting the normal distribution to the observed data from both coating layers has shown an acceptable p-value for the chi-square test, which means that the normal distribution can be used to represent the observed data from both layers. As a result, it has been found that the first layer of coating is normally distributed with mean 22.2 and variance 5.13, and the second layer 'of coating is normally distributed with mean 126 and variance 11.14. The information and data collected to run the model are:

L_1 = 10 μm, L = 110 μm, a_1 = 35.64 SR/item, a_2 = 32.67 SR/item, c_1 = 0.015 SR/μm, c_2 = 0.0088 SR/μm, r = 1.2 SR/item, I = 0.025 SR/item, $n_1 = n_2 = 13$, $d_1 = d_2 = 1$, $\sigma_1 = 5.13$, σ_2 = 11.14, e_{11} = 0.01, e_{12} = 0.05, e_{21} = 0.01 and e_{22} = 0.05.

To solve this problem, we optimized the model developed in section 4.3 using Microsoft Excel Solver tool. The tables below show the values of the optimal process mean for both processes and the expected profit for the targeting problem with different error prone sampling plans.

Table 4.1 Expected profit and means for the case study using n_1=n_2=10.

	$d_1 = 1$			$d_1 = 2$			$d_1 = 3$		
d_2	μ_1	μ_2	EP	μ_1	μ_2	EP	μ_1	μ_2	EP
1	27.5434	111.091	34.0484	23.2575	115.3769	34.2194	20.3931	118.2413	34.244
2	27.5467	104.6068	34.1265	23.2602	108.8932	34.2979	20.3948	111.7588	34.3224
3	27.5486	100.0382	34.1713	23.2617	104.3248	34.3428	20.3958	107.191	34.3674

Table 4.2 Expected profit and means for the case study using n_1=n_2=13.

	$d_1 = 1$			$d_1 = 2$			$d_1 = 3$		
d_2	μ_1	μ_2	EP	μ_1	μ_2	EP	μ_1	μ_2	EP
1	28.28334	112.1508	33.9157	24.2867	116.1475	34.1823	21.312	119.1221	34.2126
2	28.28683	105.8421	34.0002	24.2899	109.8391	34.2674	21.314	112.8149	34.2978
3	28.28845	101.4972	34.0442	24.2915	105.4942	34.3117	21.3151	108.4705	34.3421

Table 4.3 Expected profit and means for the case study using n_1=n_2=15.

	$d_1 = 1$			$d_1 = 2$			$d_1 = 3$		
d_2	μ_1	μ_2	EP	μ_1	μ_2	EP	μ_1	μ_2	EP
1	28.66682	112.7445	33.8153	24.8702	116.5409	34.1575	21.8191	119.5921	34.1931
2	28.67076	106.5136	33.9051	24.8738	110.3105	34.2482	21.8215	113.3629	34.2839
3	28.67256	102.2454	33.9493	24.8755	106.0424	34.2927	21.8225	109.0955	34.3284

Table 4.4 Expected profit and means for the case study using n_1=n_2=20.

	$d_1 = 1$			$d_1 = 2$			$d_1 = 3$		
d_2	μ_1	μ_2	EP	μ_1	μ_2	EP	μ_1	μ_2	EP
1	29.39545	113.9663	33.5225	26.0711	117.2907	34.0909	22.8871	120.4747	34.1459
2	29.39958	107.8981	33.6284	26.0756	111.222	34.1986	22.8901	114.4074	34.2536
3	29.40147	103.7062	33.6741	26.0775	107.0301	34.245	22.8914	110.2162	34.3001

Tables (4.1-4.4) show that increasing n while keeping d fixed will increase the probability of rejecting the lot resulting in a more tight inspection plan. To compensate for this change the process is shifted further from the lower limit, i.e., the mean is increased. Also, we can see that the expected profit decrease as samples sizes increase for a fixed value of d_1 and d_2. This is because, as we mentioned earlier, sampling more units in a lot shows higher chances of detecting the defect and rejecting the lot and hence, the producer increases the mean. The increase of the mean will cause a reduction in the chance of producing nonconforming items at the expense of cost of manufacturing. Hence, the expected profit decreases.

Moreover, the tables show that the sampling plan currently used by the factory is not the best in terms of total profit, instead, the sampling plan ($n_1 = n_2 = 10$, $d_1 = d_2 = 3$) gives the maximum total profit.

4.5 Sensitivity Analysis

In this section, the effect of type I and type II errors on the expected profit and the mean of the two processes are investigated. Four sampling plans are considered with the same acceptance numbers $d_1 = d_2 = 1$ and different sample sizes $n_1 = n_2 = 10$, 13, 15 and 20. Inspection errors can take one of the following values: 0.01, 0.03, and 0.05.

The change in the value of the profit due to these errors is measured by computing the difference between expected profit without error and expected profit with error. Thus, the percentage change in profit is given by,

$$\text{Percentage change in profit} = \frac{EP(without\ \ error) - EP(with\ \ error)}{EP(without\ \ error)} \times 100$$

Type I and II errors associated with the first sampling inspection are denoted as e_{11} and e_{12} respectively. However, type I and II errors associated with the second sampling inspection are denoted as e_{21} and e_{22} respectively. Table 4-5, shows the optimal mean for the two processes and the expected profit for different combinations of inspection errors. To compute the values in the table, we considered the sampling plan currently used by the factory ($n_1 = n_2 = 13$ and $d_1 = d_2 = 1$). The last column in table 4-5 is the percentage of reduction in the expected profit due to the existence of the error.

Table 4.5 Expected profit and optimal means at different values of errors.

Combination #	$(e_{11}, e_{12}, e_{21}, e_{22})$	μ^*_1	μ^*_2	EP	% change in profit
0	(0, 0, 0, 0)	25.3913	113.203	34.23711	0
1	(0.01, 0.01, 0.01, 0.01)	28.3431	112.2902	33.91376	0.944432367
2	(0.01, 0.01, 0.01, 0.03)	28.3431	112.1919	33.91454	0.942142288
3	(0.01, 0.01, 0.01, 0.05)	28.3432	112.0911	33.91535	0.93979784
4	(0.01, 0.01, 0.03, 0.01)	28.3362	114.7172	33.74504	1.437245123
5	(0.01, 0.01, 0.03, 0.03)	28.3362	114.6219	33.74579	1.435047732
6	(0.01, 0.01, 0.03, 0.05)	28.3362	114.5243	33.74656	1.432797658
7	(0.01, 0.01, 0.05, 0.01)	28.3262	115.6823	33.5037	2.142142588
8	(0.01, 0.01, 0.05, 0.03)	28.3262	115.5872	33.50445	2.139950352
9	(0.01, 0.01, 0.05, 0.05)	28.3262	115.4898	33.50522	2.137704641
10	(0.01, 0.03, 0.01, 0.01)	28.3135	112.3197	33.91393	0.943925603
11	(0.01, 0.03, 0.01, 0.03)	28.3135	112.2215	33.91472	0.941635524
12	(0.01, 0.03, 0.01, 0.05)	28.3136	112.1207	33.91552	0.939291078
13	(0.01, 0.03, 0.03, 0.01)	28.3066	114.7468	33.74521	1.436738188
14	(0.01, 0.03, 0.03, 0.03)	28.3066	114.6515	33.74596	1.434540798
15	(0.01, 0.03, 0.03, 0.05)	28.3066	114.5539	33.74673	1.432290725
16	(0.01, 0.03, 0.05, 0.01)	28.2966	115.7119	33.50387	2.141635406
17	(0.01, 0.03, 0.05, 0.03)	28.2966	115.6168	33.50462	2.139443172
18	(0.01, 0.03, 0.05, 0.05)	28.2966	115.5194	33.50539	2.137197461
19	(0.01, 0.05, 0.01, 0.01)	28.2833	112.35	33.91411	0.943407421
20	(0.01, 0.05, 0.01, 0.03)	28.2833	112.2517	33.9149	0.941117344
21	(0.01, 0.05, 0.01, 0.05)	28.2833	112.1509	33.9157	0.938772898
22	(0.01, 0.05, 0.03, 0.01)	28.2763	114.7771	33.74539	1.436219832
23	(0.01, 0.05, 0.03, 0.03)	28.2763	114.6818	33.74614	1.434022442
24	(0.01, 0.05, 0.03, 0.05)	28.2763	114.5841	33.74691	1.43177237
25	(0.01, 0.05, 0.05, 0.01)	28.2663	115.7422	33.50405	2.141116797
26	(0.01, 0.05, 0.05, 0.03)	28.2663	115.6471	33.5048	2.138924563
27	(0.01, 0.05, 0.05, 0.05)	28.2663	115.5497	33.50557	2.136678854
28	(0.03, 0.01, 0.01, 0.01)	29.4228	111.2106	32.20351	5.939740059
29	(0.03, 0.01, 0.01, 0.03)	29.4228	111.1122	32.20426	5.937563342
30	(0.03, 0.01, 0.01, 0.05)	29.4228	111.0114	32.20502	5.935334948
31	(0.03, 0.01, 0.03, 0.01)	29.4161	113.6373	32.04314	6.408157886
32	(0.03, 0.01, 0.03, 0.03)	29.4161	113.542	32.04385	6.406069269
33	(0.03, 0.01, 0.03, 0.05)	29.4162	113.4443	32.04459	6.403930577
34	(0.03, 0.01, 0.05, 0.01)	29.4066	114.6021	31.81375	7.078161849

35	(0.03, 0.01, 0.05, 0.03)	29.4066	114.5068	31.81446	7.076078133
36	(0.03, 0.01, 0.05, 0.05)	29.4066	114.4094	31.81519	7.073943588
37	(0.03, 0.03, 0.01, 0.01)	29.3941	111.2391	32.20369	5.939214364
38	(0.03, 0.03, 0.01, 0.03)	29.3941	111.2391	32.20443	5.937069504
39	(0.03, 0.03, 0.01, 0.05)	29.3942	111.0401	32.2052	5.934809254
40	(0.03, 0.03, 0.03, 0.01)	29.3876	113.6659	32.04332	6.407632028
41	(0.03, 0.03, 0.03, 0.03)	29.3876	113.5705	32.04403	6.405543412
42	(0.03, 0.03, 0.03, 0.05)	29.3876	113.4729	32.04477	6.403404721
43	(0.03, 0.03, 0.05, 0.01)	29.378	114.6307	31.81393	7.077635758
44	(0.03, 0.03, 0.05, 0.03)	29.378	114.5354	31.81464	7.075552043
45	(0.03, 0.03, 0.05, 0.05)	29.378	114.438	31.81537	7.073417499
46	(0.03, 0.05, 0.01, 0.01)	29.3649	111.2683	32.20388	5.938676647
47	(0.03, 0.05, 0.01, 0.03)	29.365	111.17	32.20462	5.936499931
48	(0.03, 0.05, 0.01, 0.05)	29.365	111.0692	32.20538	5.934271539
49	(0.03, 0.05, 0.03, 0.01)	29.3583	113.6951	32.0435	6.407094145
50	(0.03, 0.05, 0.03, 0.03)	29.3583	113.5998	32.04422	6.40500553
51	(0.03, 0.05, 0.03, 0.05)	29.3584	113.5021	32.04495	6.402866839
52	(0.03, 0.05, 0.05, 0.01)	29.3487	114.6601	31.81411	7.077097636
53	(0.03, 0.05, 0.05, 0.03)	29.3487	114.5646	31.81483	7.075013921
54	(0.03, 0.05, 0.05, 0.05)	29.3488	114.4672	31.81556	7.072879378
55	(0.05, 0.01, 0.01, 0.01)	29.6409	110.9921	29.4573	13.96089914
56	(0.05, 0.01, 0.01, 0.03)	29.6409	110.8941	29.45798	13.95890488
57	(0.05, 0.01, 0.01, 0.05)	29.6409	110.7933	29.45868	13.95686326
58	(0.05, 0.01, 0.03, 0.01)	29.6343	113.4191	29.31037	14.39005509
59	(0.05, 0.01, 0.03, 0.03)	29.6343	113.3238	29.31102	14.38814153
60	(0.05, 0.01, 0.03, 0.05)	29.6344	113.2261	29.31169	14.3861821
61	(0.05, 0.01, 0.05, 0.01)	29.6247	114.3843	29.10021	15.00390041
62	(0.05, 0.01, 0.05, 0.03)	29.6249	114.2885	29.10086	15.00199134
63	(0.05, 0.01, 0.05, 0.05)	29.6249	114.1911	29.10153	15.00003571
64	(0.05, 0.03, 0.01, 0.01)	29.612	111.0212	29.4575	13.96031225
65	(0.05, 0.03, 0.01, 0.03)	29.6121	110.9229	29.45818	13.95831798
66	(0.05, 0.03, 0.01, 0.05)	29.6121	110.8221	29.45888	13.95627637
67	(0.05, 0.03, 0.03, 0.01)	29.6055	113.4479	29.31057	14.38946801
68	(0.05, 0.03, 0.03, 0.03)	29.6055	113.3526	29.31122	14.38755446
69	(0.05, 0.03, 0.03, 0.05)	29.6055	113.255	29.3119	14.38559503
70	(0.05, 0.03, 0.05, 0.01)	29.5958	114.4131	29.10041	15.00331308
71	(0.05, 0.03, 0.05, 0.03)	29.596	114.3174	29.10106	15.00140401
72	(0.05, 0.03, 0.05, 0.05)	29.596	114.22	29.10173	14.99944839
73	(0.05, 0.05, 0.01, 0.01)	29.5825	111.0507	29.45771	13.95971165
74	(0.05, 0.05, 0.01, 0.03)	29.5825	110.9525	29.45839	13.95771739
75	(0.05, 0.05, 0.01, 0.05)	29.5826	110.8517	29.45909	13.95567577
76	(0.05, 0.05, 0.03, 0.01)	29.5759	113.4775	29.31078	14.38886724
77	(0.05, 0.05, 0.03, 0.03)	29.5759	113.3822	29.31143	14.38695368
78	(0.05, 0.05, 0.03, 0.05)	29.576	113.2845	29.3121	14.38499426
79	(0.05, 0.05, 0.05, 0.01)	29.5663	114.4427	29.10061	15.00271204
80	(0.05, 0.05, 0.05, 0.03)	29.5664	114.347	29.10127	15.00080298
81	(0.05, 0.05, 0.05, 0.05)	29.5665	114.2495	29.10194	14.99884735

Table 4-5 can be used to conduct cost/benefit analysis to improve the inspection process. One way to do this is to ask the inspector to estimate the error in his inspection process. Having that and using the table, we can estimate how much it costs or saves the factory if they work hard in reducing this error. So, it will help in conducting cost/benefit analysis for improving inspection error by purchasing new instruments or training workers.

Also, it is clear from the table that error combination number 61, (e_{11} = 0.05, e_{12} = 0.01, e_{21} = 0.05 and e_{22} = 0.01), has more effect on the total profit than other error combinations. This means that if we concentrate on controlling type I error then we can reduce its effect on the total profit.

Mainly, two types of sensitivity analysis are considered. The first analysis studies the effect of type I and type II errors, associated with both sampling inspections, on the expected profit. However, the second analysis studies the effect of type I and type II errors, associated with both sampling inspections, on the optimal means.

4.5.1 The effect of type I errors e_{11} and e_{21} on expected profit

In this section, the effect of type I errors associated with both processes on the expected profit is studied. This analysis is considered for different sampling plans: n_1=n_2=10, 13, 15, 20 and d_1=d_2=1. Each combination is solved for the optimum profit using Excel Solver.

Figures 4-2 to 4-4 show expected profit versus type I error associated with the first process e_{11} at different levels of e_{12}, e_{21} and e_{22}. Each figure contains four plots each with different sampling plans. As the sample size increases the expected profit decreases because the sampling plan with a higher sample size will show a smaller probability of acceptance.

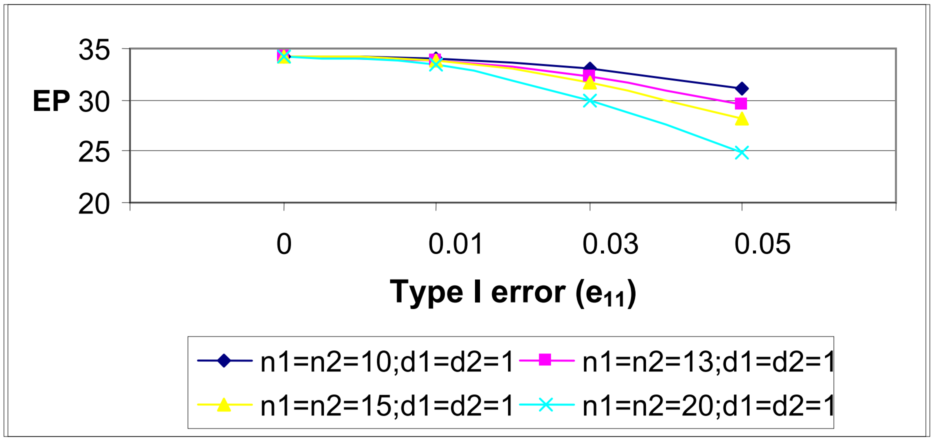

Figure (4-2) Expected profit versus e_{11} at $e_{12}=e_{21}=e_{22}=0.01$

Also, these figures show that at a given level of e_{12}, e_{21} and e_{22}, the expected profit decreases with the increase in the value of e_{11}. This result is expected because, high type I error means classifying more good items as defectives resulting in unnecessary reworks. Hence, the expected profit is reduced. As you notice, the reduction in the expected profit becomes higher as type I error e_{11} tends to 0.05.

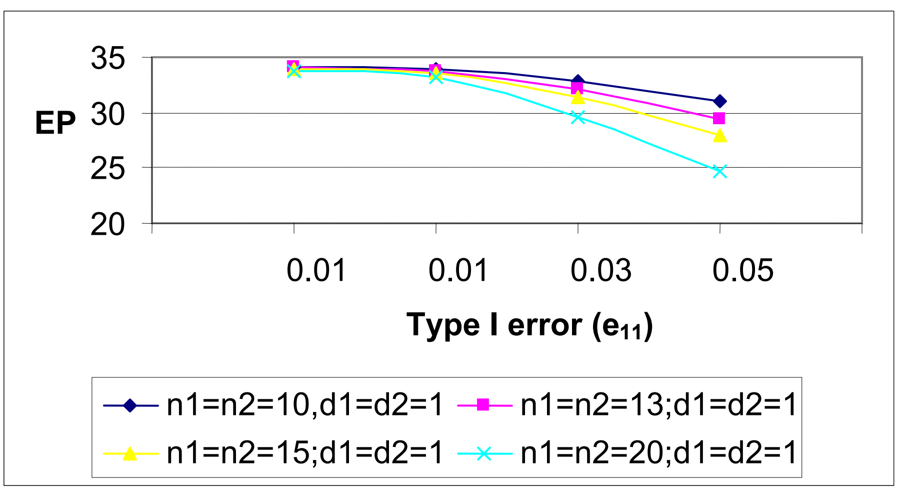

Figure (4-3) Expected profit versus e_{11} at $e_{12}=e_{21}=e_{22}=0.03$.

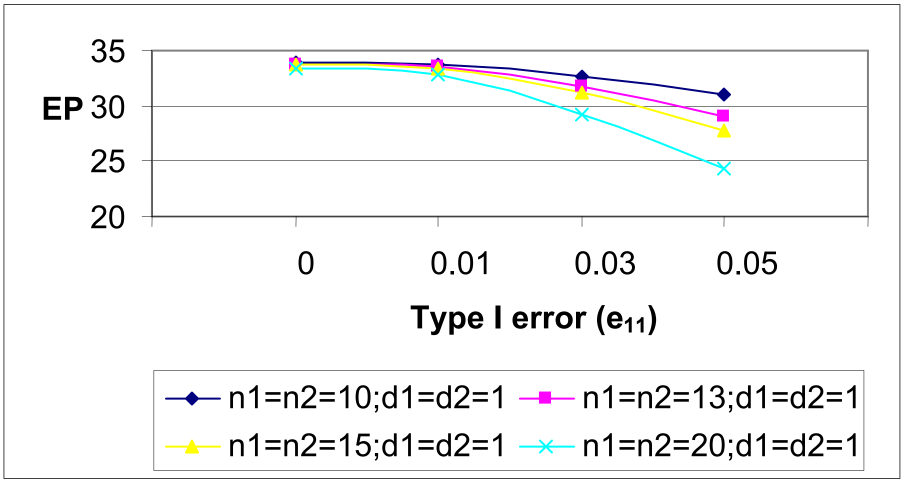

Figure (4-4) Expected profit versus e_{11} at e_{12}=e_{21}=e_{22}=0.05.

Figures 4-5 to 4-7 show expected profit versus type I error associated with the second process e_{21} at different levels of e_{11}, e_{12} and e_{22}. Each figure contains four plots each with different sampling plan. As the sample size increases the expected profit decreases because the sampling plan with a higher sample size will show a smaller probability of acceptance.

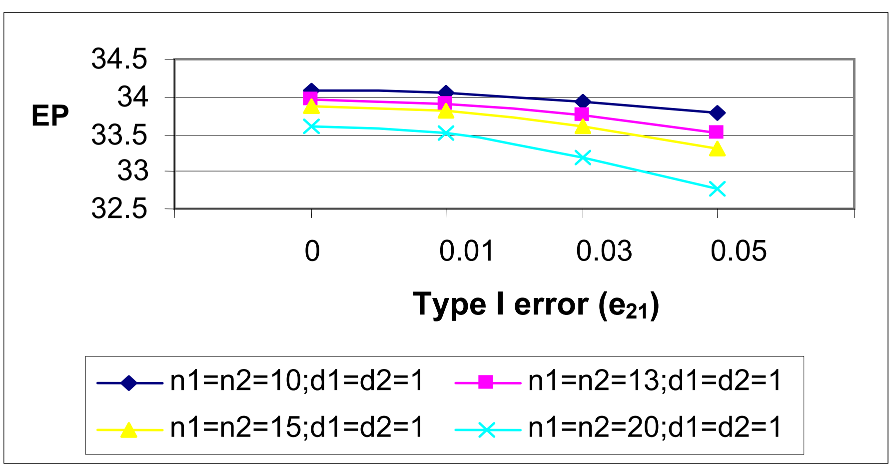

Figure (4-5) Expected profit versus e_{21} at e_{11}=e_{12}=e_{22}=0.01.

Also, these figures show that at a given level of e_{11}, e_{12} and e_{22}, the expected profit decreases with the increase in the value of type I error e_{21}. This result is expected because, high type I error means classifying more good items as defectives resulting secondary market products. Hence, the expected profit is reduced. As you notice, the reduction in the expected profit becomes higher as type I error e_{21} tends to 0.05.

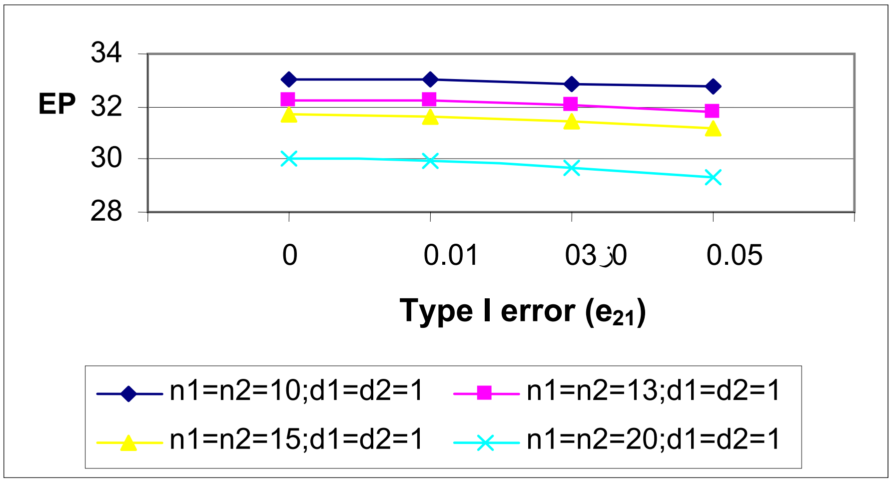

Figure (4-6) Expected profit versus e_{21} at $e_{11}=e_{12}=e_{22}=0.03$.

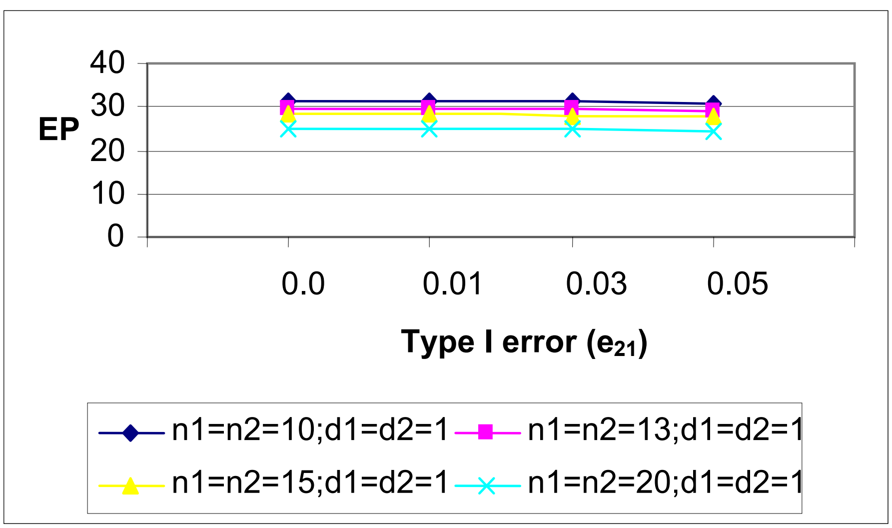

Figure (4-7) Expected profit versus e_{21} at $e_{11}=e_{12}=e_{22}=0.05$.

4.5.2 The effect of type II errors e_{12} and e_{22} on expected profit

The effect of type II errors associated with both processes on the expected profit using different levels of e_{12} and e_{22} (0, 0.01, 0.03, and 0.05) is studied in this section. This analysis is considered for different sampling plans: $n_1=n_2=10$, 13, 15, 20 and $d_1=d_2=1$. Each combination is solved for the optimum profit using Excel Solver.

Figures 4-8 to 4-10 show expected profit versus type II error e_{12} at different levels of e_{11}, e_{21} and e_{22}. Each figure contains four plots each with different sampling plan. As the sample size increases the expected profit decreases because the sampling plan with a higher sample size will show a smaller probability of acceptance.

These figures show that at a given level of e_{11}, e_{21} and e_{22}, the expected profit is almost constant with the increase in the value of type II error e_{12}. This means that committing type II error in the first process has little impact on the expected profit. Knowing that, we can put more effort to control type I error in order to maximize the profit.

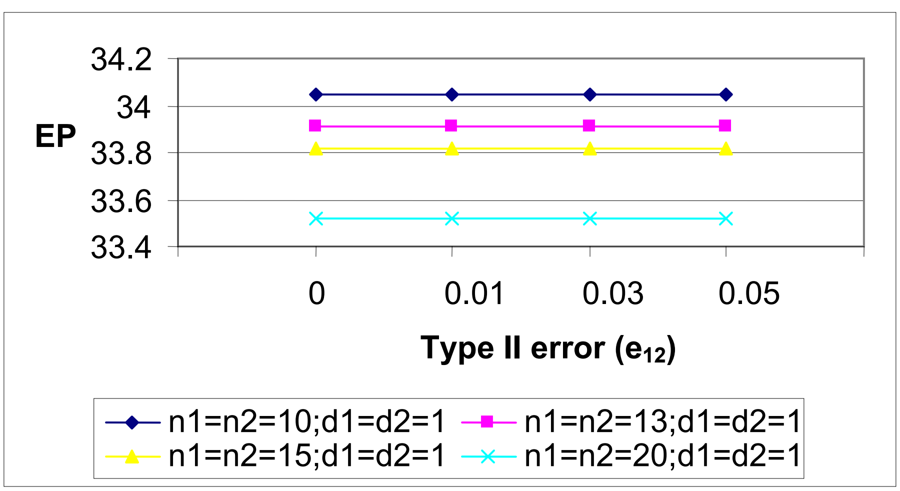

Figure (4-8) Expected profit versus e_{12} at $e_{11}=e_{21}=e_{22}=0.01$

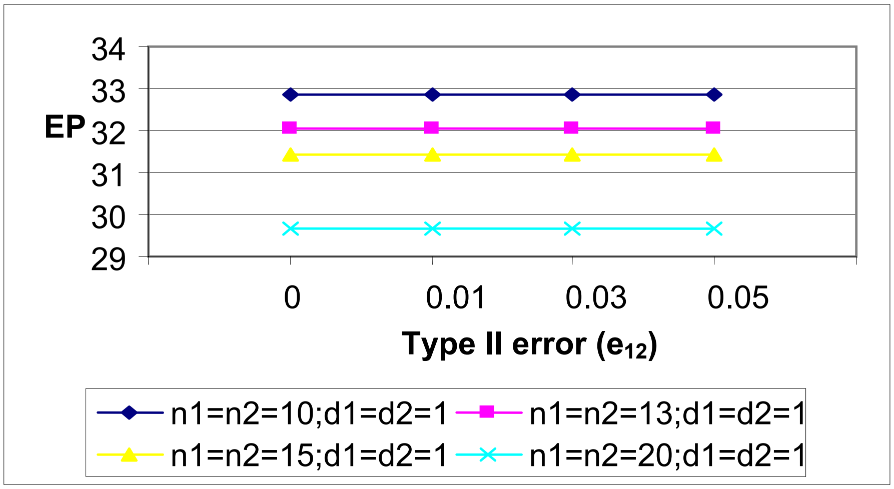

Figure (4-9) Expected profit versus e_{12} at $e_{11}=e_{21}=e_{22}=0.03$.

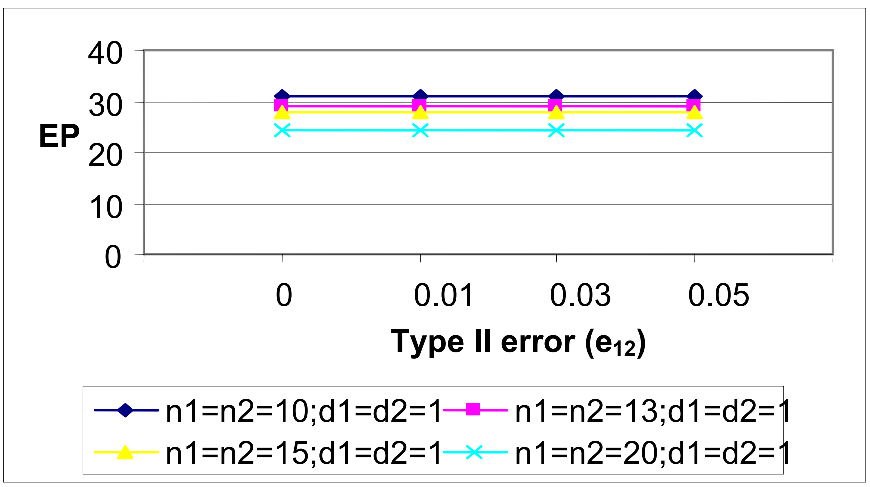

Figure (4-10) Expected profit versus e_{12} at $e_{11}=e_{21}=e_{22}=0.05$.

Figures 4-11 to 4-13 show expected profit versus type II error, associated with the second process. As you can see, when the sample size increases the expected profit decreases because the sampling plan with a higher sample size will show a smaller probability of acceptance. Meaning that, sampling more items in a lot increases number of nonconforming items found in the sample which will increase the probability of rejecting the lot. Hence, expected profit is expected to decrease.

Moreover, these figures show that at a given level of e_{11}, e_{21} and e_{12}, the expected profit is almost constant with the increase in the value of type II error e_{22}. This means that committing type II error in the second process has no impact on the expected profit. Knowing that, we can put more effort to control type I error in order to maximize the profit.

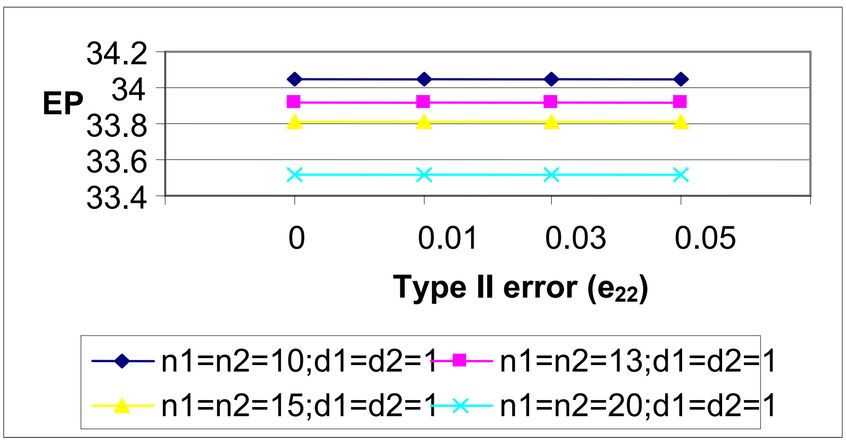

Figure (4-11) Expected profit versus e_{22} at $e_{11}=e_{21}=e_{12}=0.01$.

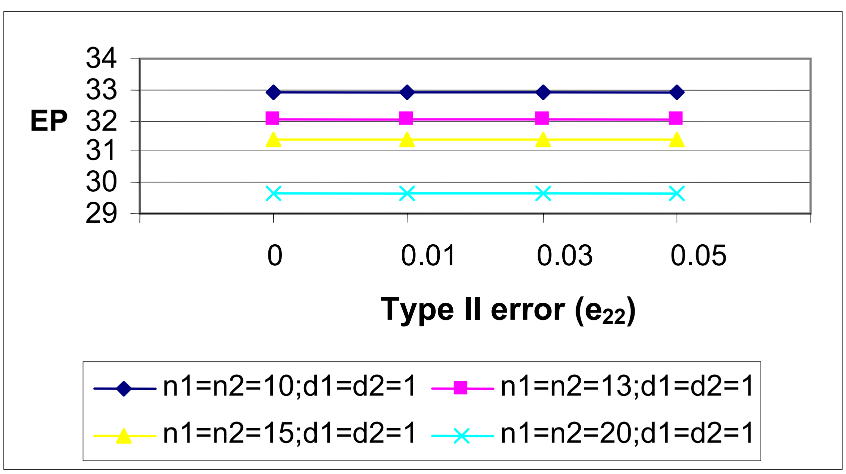

Figure (4-12) Expected profit versus e_{22} at $e_{11}=e_{21}=e_{12}=0.03$.

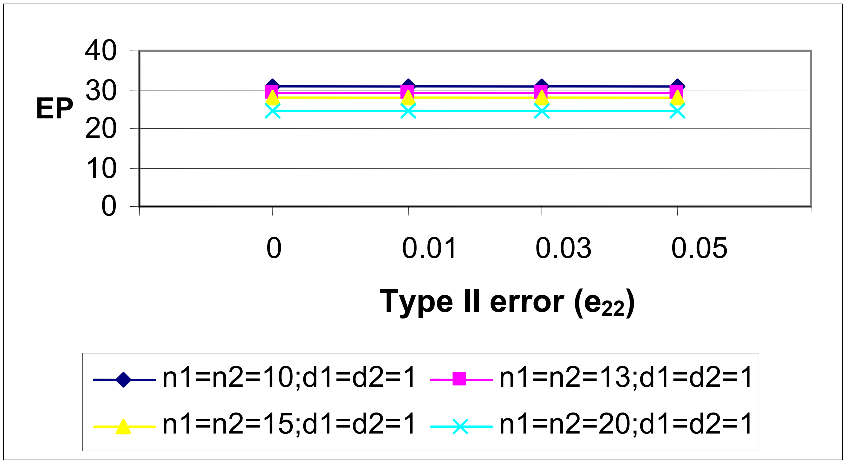

Figure (4-13) Expected profit versus e_{22} at e_{11}=e_{21}=e_{12}=0.05.

When comparing the effect of type I errors and type II errors on the expected profit, we found that type I errors result in the reduction of the expected profit while type II errors have no considerable impact on the expected profit. This will lead us to pay more attention to type I error than type II error and to concentrate on reducing type I error in order to maximize the total profit.

4.5.3 The effect of type I errors e_{11} and e_{21} on optimal means μ_1 and μ_2

In this section, the effect of type I errors associated with both processes on the optimal mean of both processes using different levels of e_{11} and e_{21} (0, 0.01, 0.03, and 0.05) is studied. This analysis is considered for different sampling plans: n_1=n_2=10, 13, 15, 20 and d_1=d_2=1. Each combination is solved for the optimum profit using Excel Solver.

All figures contain four plots each with different sampling plans. It can be seen that, as the sample size increases the mean of the process increases because for any value of observed lot fraction defective the OC curve of a sampling plan with higher n will show a lower probability of acceptance which is going to force the producer to increase the set point μ_1 or μ_2.

Figure 4-14 shows the optimal mean associated with the first process versus type I error associated with the first process. The figure shows that the mean μ_1 tends to increase with the increase in type I error e_{11}. This is because, the increase in e_{11} means rejecting more good items, so the model tends to increase the process mean to avoid the continuation of such error.

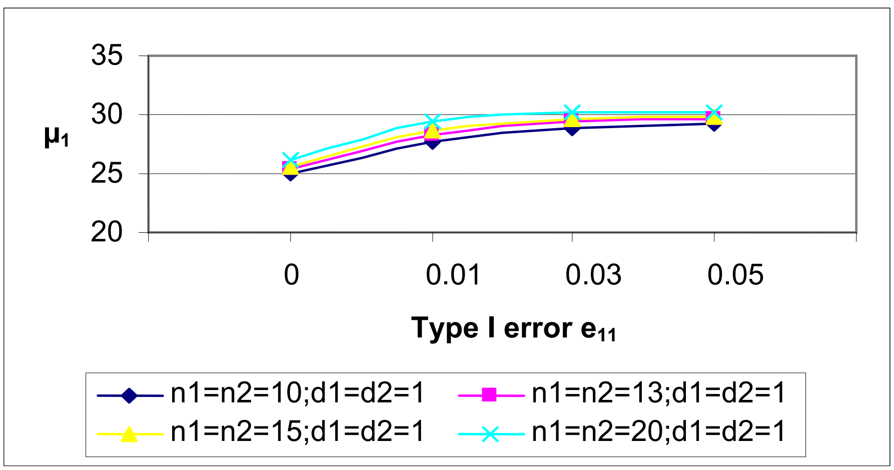

Figure (4-14) μ_1 versus e_{11} at $e_{21}=e_{12}=e_{22}=0.01$.

Figure 4-15 shows the optimal mean for the first process versus type I error associated with the second process. The mean μ_1 tends to decrease with the increase in type I error e_{21}. This is because, the increase in e_{21} means rejecting more good items at the second process, so the model tends to increase the process mean μ_2 to avoid the continuation of such error. As μ_2 increases cost of materials will also increase which will affect the expected profit. In this case, the model will reduce the material cost associated with the first process in order balance the expected profit.

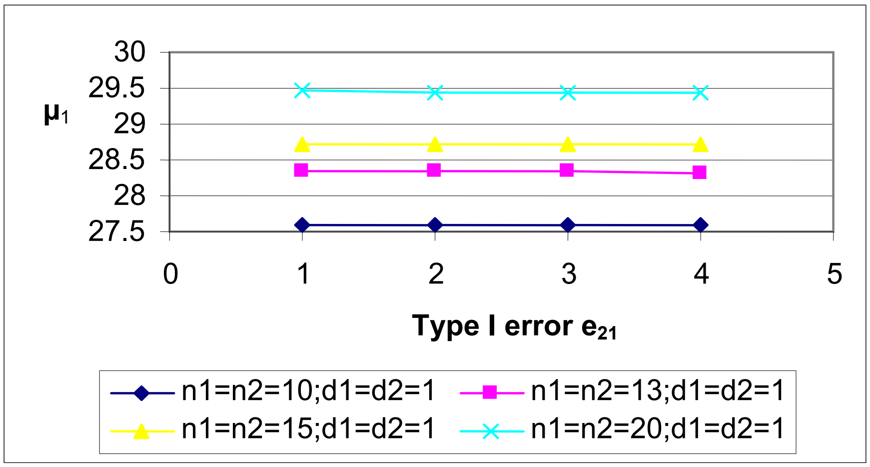

Figure (4-15) μ_1 versus e_{21} at $e_{11}=e_{12}=e_{22}=0.01$.

Figures 4-16 shows the optimal mean associated with the second process versus type I error associated with the first process. The figure shows that, the mean μ_2 tends to decrease with the increase in type I error e_{11}. This is to be expected, since the increase in e_{11} means rejecting more good items at the first process, so the model tends to increase the process mean to avoid the continuation of such error. As μ_1 increases, cost of materials will also increase which will affect the expected profit. In this case, the model will reduce the material cost associated with the second process in order keep the expected profit unchanged.

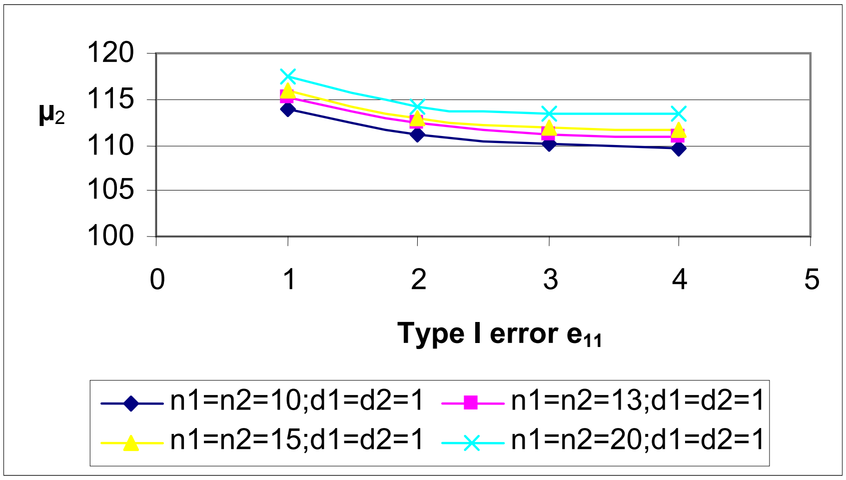

Figure (4-16) μ_2 versus e_{11} at $e_{211}=e_{12}=e_{22}=0.01$.

Figure 4-17 shows the optimal mean associated with the second process versus type I error associated with the second process. The mean μ_2 tends to increase with the increase in type I error e_{21}. This is because, the increase in e_{21} means rejecting more good items, so the model tends to increase the process mean to protect against the effect of the error.

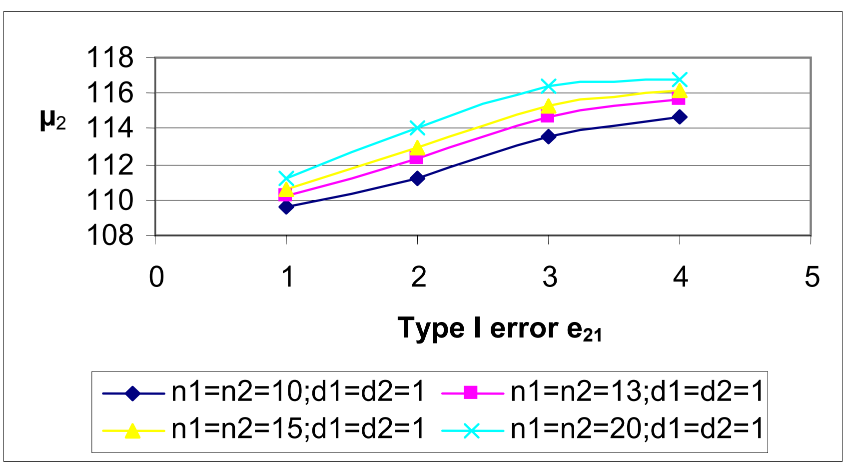

Figure (4-17) μ_2 versus e_{21} at $e_{11}=e_{12}=e_{22}=0.01$.

4.5.4 The effect of type II errors e_{12} and e_{22} on optimal means

In this section, the effect of type II errors associated with both processes on the optimal mean of both processes using different levels of e_{12} and e_{22} (0, 0.01, 0.03, and 0.05) is studied. This analysis is considered for different sampling plans: $n_1=n_2=10$, 13, 15, 20 and $d_1=d_2=1$. Each combination is solved for the optimum profit using Excel Solver.

All the figures contain four plots each with different sampling plan. It can be seen that, as the sample size increases the mean of the process increases because for any value of observed lot fraction defective the OC curve of a sampling plan with higher n will show a lower probability of acceptance which is going to force the producer to increase the set point μ_1 or μ_2.

Figure 4-18 shows the optimal mean associated with the first process versus type II error associated with the first process. The figure shows that the mean μ_1 tends to decrease with the increase in type II error e_{12}. This is because, the increase in e_{12} means accepting more defective items, so the model tends to decrease the process mean to counter the effect of the error.

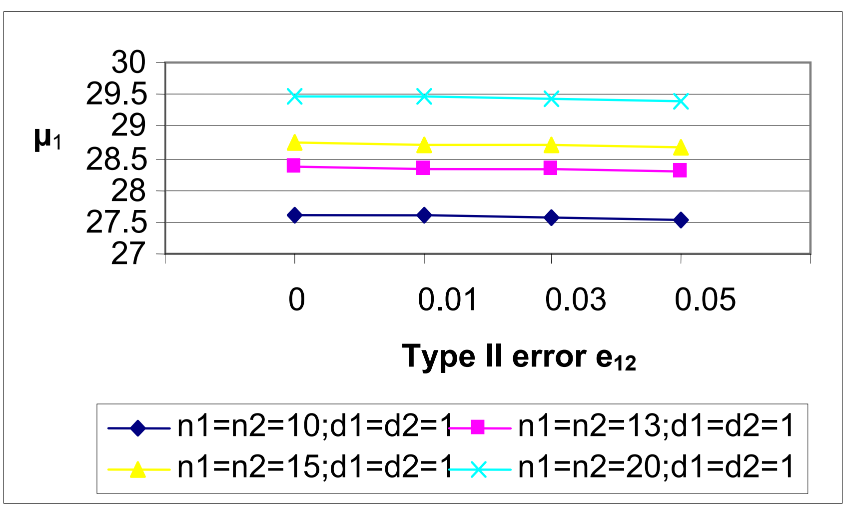

Figure (4-18) μ_1 versus e_{12} at $e_{11}=e_{21}=e_{22}=0.01$.

In figure 4-19, the mean μ_1 tends to increase with the increase in type II error e_{22}. This is because, the increase in e_{22} means accepting more defective items in the second sampling inspection, so the model tends to decrease the second process mean to counter the effect of the error. However, this could lead to rejecting good items. To cover the loss that could happen, the model tends to increase the first process mean in order to ensure that most of the items at the first inspection will not be rejected.

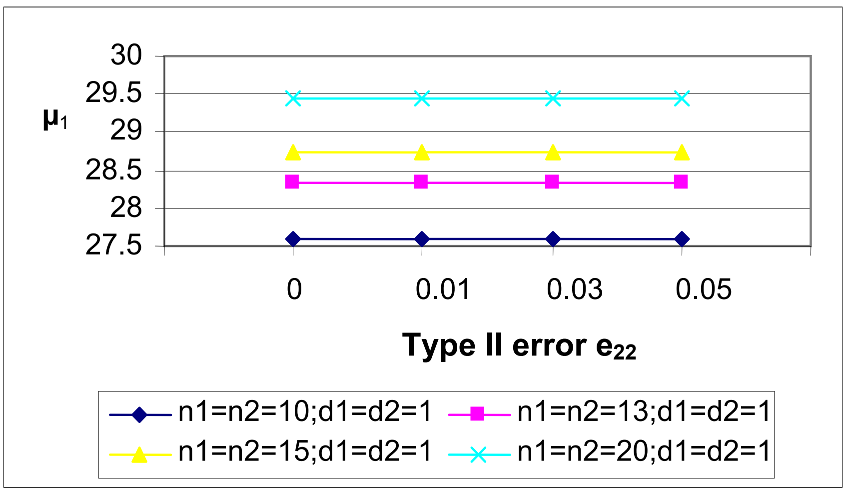

Figure (4-19) μ_1 versus e_{22} at $e_{11}=e_{21}=e_{12}=0.01$.

Figure 4-20 shows the optimal mean associated with the second process versus type II error associated with the first process. It can be seen that, the mean μ_2 tends to increase with the increase in type II error e_{12}. This is because, the increase in e_{12} means accepting more defective items, so the model tends to decrease the first process mean to counter the effect of the error. However, this could lead to rejecting good items. To cover the loss that could happen, the model tends to increase the second process mean in order to ensure that most of the items at the second inspection will not be rejected.

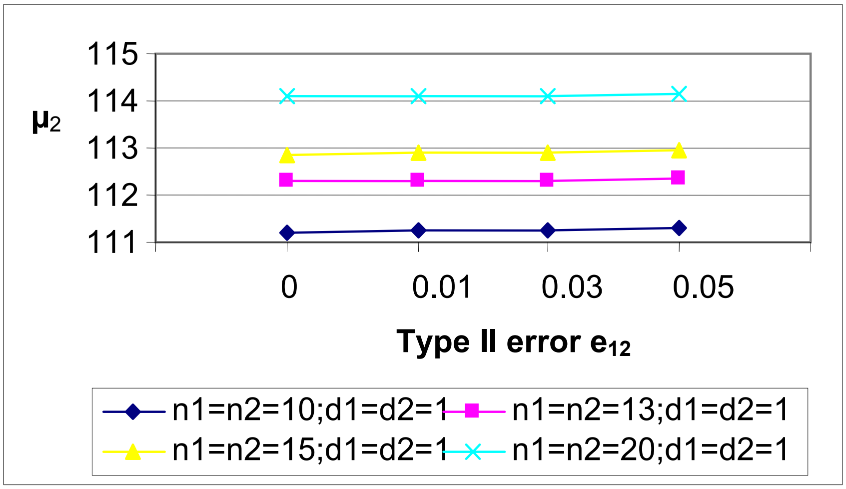

Figure (4-20) μ_2 versus e_{12} at e_{11}=e_{21}=e_{22}=0.01.

Figure 4-21 shows the optimal second process mean versus type II error associated with the second process. The figure shows that, the mean μ_2 tends to decrease with the increase in type II error e_{22}. This is because, the increase in e_{22} means accepting more defective items, so the model tends to decrease the process mean to counter the effect of type II error.

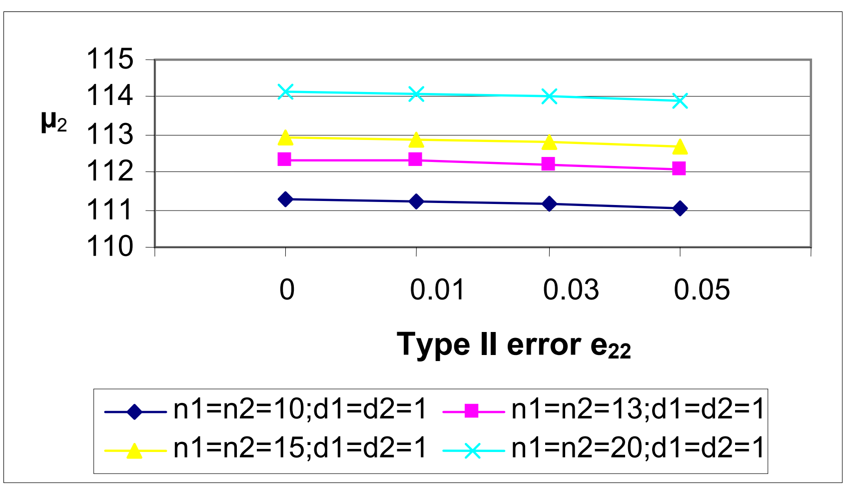

Figure (4-21) μ_2 versus e_{22} at e_{11}=e_{21}=e_{12}=0.01.

4.6 Conclusion

In this chapter, a model has been developed for two processes producing a single product with two quality characteristics. The first quality characteristic is determined by the first process and the second quality characteristic is determined by both processes. Specification limits are set on both quality characteristics. A sampling plan has been used as a mean for product quality control assuming that the inspection is error prone. A real case has been considered to test the model developed in this chapter followed by the sensitivity analysis which has been conducted to investigate the effect of changing model parameters on model results. From the case study used in this chapter, it can be concluded that type I error has more impact on the total profit than type II error. So, we must pay more attention to control or reduce type I error in order to maximize the total profit. Four cases have been considered in the sensitivity analysis: the effect of type I and type II errors on the expected profit, the effect of type I and type II errors on the optimal means. In chapters 3 and 4, sampling plans are used as a mean for product quality control. The next two chapters use 100% inspection as a mean for product quality control.

CHAPTER 5

PROCESS TARGETING WITH TWO INDEPENDENT PROCESSES IN SERIES USING 100 % INSPECTION

5.1 Introduction

The purpose of this chapter is to develop a process targeting model for the problem stated in chapter 3 except in this chapter a 100% inspection is used as a mean for product quality control.

The problem is described in the next section followed by model development. The fire extinguisher case study is used to illustrate the solution to the problem under a 100% inspection as a mean for quality control followed by some sensitivity analysis.

5.2 Statement of the Problem

Consider a manufacturing environment in which products go through two different processes in series. The product has two quality characteristics. The first quality characteristic is determined by the first process. The second quality characteristic is determined by both processes. Lower specification limits are set on both quality characteristics.

Let us assume that the quality characteristic of the first process is a random variable denoted by X_1, and the quality characteristic of the second process is another random variable denoted by X_2. The final quality characteristic is denoted by X, where $X=X_1+X_2$. Quality requirement is that $X_1 \geq L_1$ and $X \geq L$, where L_1 and L are predetermined constants set by product designer or market information. All items are inspected, i.e., 100% inspection is used for product quality control. A product that meets both specifications is sold at a regular market with a price a_1. A product that meets the first specification and fails to meet the second specification is sold at a_2, where $a_2 < a_1$. An item that fails to meet the first specification is reworked at a cost that includes a variable processing cost and a fixed rework cost (r), see figure (5-1). The objective is to determine the optimal process means, μ_1^* and μ_2^* for the two quality characteristics such that the total profit is maximized. The total profit is the sum of the revenues from selling final products minus the total processing cost and the possible rework cost.

A real example is painting of fire extinguishers in which an extinguisher goes through two successive processes. The first process makes the coating on the surface and the second process makes the final external coating. Specification limits on the thickness of the first coating and on the final thickness determine the quality of the product. So, the objective is to find the optimal thicknesses setting for both coating layers such that the total profit is maximized.

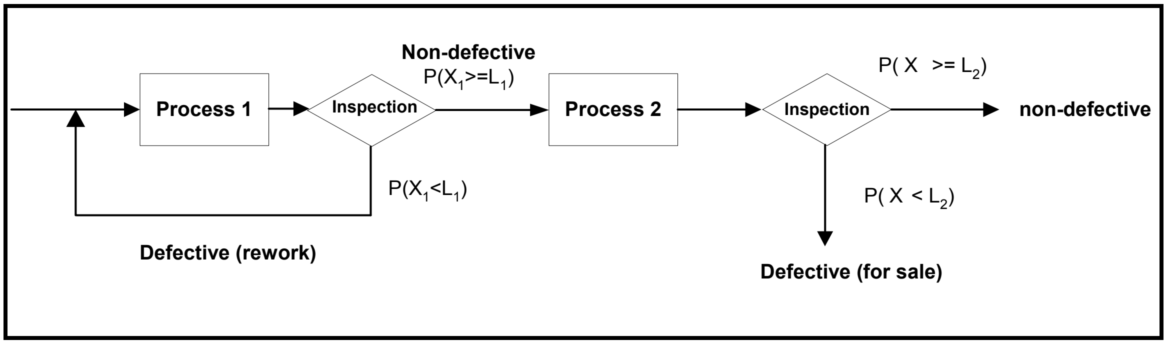

Figure (5-1) Process targeting model for two machines in series with 100% inspection.

5.3 Model Development

In this section, a process targeting model is developed for the problem stated in section 5.2. Notations and assumptions are presented prior to model development.

The following notations are adopted:

X_i A random variable that represents the value of the quality characteristic of the product after finishing process i, where i = 1, 2.

X Sum of the values of both quality characteristics, $X = X_1 + X_2$.

L_1 Lower specification limit for the first product quality characteristic.

L Lower specification limit for the composite (sum) of the product's two quality characteristics.

a_1 Selling price /item for accepted items after process 2.

a_2 Selling price /item for items rejected after process 2 (secondary selling price).

c_i Cost of material consumed in process i per unit of thickness, i = 1 ,2.

r Rework cost per defective item in the rejected lot after process 1.

μ_1 Mean setting for process 1.

μ_2 Mean setting for process 2.

σ_1 Standard deviation for process 1.

σ_2 Standard deviation for process 2.

P Profit per item.

The following assumptions are used to develop the model:

1. X_1 and X_2 are independent and normally distributed with mean μ_1 and μ_2 respectively and standard deviation σ_1 and σ_2 respectively.
2. The processing order is fixed.
3. Costs of processing are assumed to be directly proportional to the mean setting of the process.
4. The secondary selling price is less than the regular selling price and the lower specification limit for the first process is less than the lower specification limit of the second process.
5. The rework after the first process involves removing the paint and starting the first process from the beginning.
6. Rework cost consist of: variable cost and fixed cost.
7. 100 % inspection is used as a mean for quality control.
8. There is no shift or drift in the processes overtime.

The cost for each item includes, material costs for the two processes and a rework cost for items rejected after the first process. Let $P(X_1, X_2)$ be the profit per item for paint thicknesses X_1 for process 1 and X_2 for the second process. For given process means, μ_1 and μ_2, the expected value of the production cost per item is $c_1\mu_1$ and $c_2\mu_2$ respectively. If the first quality characteristic of an item is less than its lower specification limit, then the item is reworked at a cost $c_1X_1 + r$. The profit function per item can be stated as:

$$P(X_1, X_2) = \begin{cases} a_1 - c_1X_1 - c_2X_2 & if \quad X_1 \geq L_1 \quad and \quad X_2 \geq L - X_1 \\ a_2 - c_1X_1 - c_2X_2 & if \quad X_1 \geq L_1 \quad and \quad X_2 < L - X_1 \\ -c_1X_1 - r & if \quad X_1 < L_1 \end{cases}$$

Hence, the expected profit is given by:

$$\begin{aligned} E\{P(X_1, X_2)\} = & \int_{L_1}^{\infty} \int_{L-x_1}^{\infty} (a_1 - c_1x_1 - c_2x_2) f(x_1, x_2) dx_2 dx_1 + \\ & \int_{L_1}^{\infty} \int_{-\infty}^{L-x_1} (a_2 - c_1x_1 - c_2x_2) f(x_1, x_2) dx_2 dx_1 - \\ & \int_{-\infty}^{L_1} \int_{-\infty}^{\infty} (c_1x_1 + r) f(x_1, x_2) dx_2 dx_1 \end{aligned}$$

Assuming independence and rearranging the equation we get,

$$\begin{aligned} E\{P(\mu_1, \mu_2)\} = & a_1 \int_{L_1}^{\infty} \int_{L-x_1}^{\infty} f(x_1) f(x_2) dx_2 dx_1 + a_2 \int_{L_1}^{\infty} \int_{-\infty}^{L-x_1} f(x_1) f(x_2) dx_2 dx_1 - \\ & c_1 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_1 f(x_1) f(x_2) dx_2 dx_1 - c_2 \int_{L_1}^{\infty} \int_{-\infty}^{\infty} x_2 f(x_1) f(x_2) dx_2 dx_1 - \\ & r \int_{-\infty}^{L_1} \int_{-\infty}^{\infty} f(x_1) f(x_2) dx_2 dx_1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5-1) \end{aligned}$$

Assuming that the resulting thickness of both processes are normally distributed with means μ_1 and μ_2, and variances σ_1^2 and σ_2^2, the probability density function of X_1 and X_2 are represented by:

$$f(x_1) = \frac{1}{\sigma_1\sqrt{2\pi}} e^{-\frac{(x_1-\mu_1)^2}{2\sigma_1^2}}$$

$$f(x_2) = \frac{1}{\sigma_2\sqrt{2\pi}} e^{-\frac{(x_2-\mu_2)^2}{2\sigma_2^2}}$$

The standard normal density function ø can be expressed as:

$$\phi(z_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z_1^2}, \quad \text{where} \quad z_1 = \frac{x_1-\mu_1}{\sigma_1}$$

$$\phi(z_2) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z_2^2}, \quad \text{where} \quad z_2 = \frac{x_2-\mu_2}{\sigma_2}$$

Under this normality assumption the first element in equation (5-1) can be rewritten as:

$$a_1\int_{L_1}^{\infty}\int_{L-x_1}^{\infty} f(x_1)f(x_2)dx_2dx_1 = a_1\int_{L_1}^{\infty}\left[1-\Phi\left(\frac{L-x_1-\mu_2}{\sigma_2}\right)\right]f(x_1)dx_1$$

$$= a_1\int_{L_1}^{\infty} f(x_1)dx_1 - a_1\int_{L_1}^{\infty}\Phi\left(\frac{L-x_1-\mu_2}{\sigma_2}\right)f(x_1)dx_1$$

$$= a_1\left[1-\Phi\left(\frac{L_1-\mu_1}{\sigma_1}\right)\right] - a_1\int_{\frac{L_1-\mu_1}{\sigma_1}}^{\infty}\Phi\left(\frac{L-\sigma_1 z_1-\mu_1-\mu_2}{\sigma_2}\right)\phi(z_1)dz_1$$

$$= a_1\left[\Phi\left(\frac{\mu_1-L_1}{\sigma_1}\right) - \int_{\frac{L_1-\mu_1}{\sigma_1}}^{\infty}\Phi\left(\frac{L-\sigma_1 z_1-\mu_1-\mu_2}{\sigma_2}\right)\phi(z_1)dz_1\right]$$

Similarly, the second element can be rewritten as:

$$a_2\int_{L_1}^{\infty}\int_{-\infty}^{L-x_1} f(x_1)f(x_2)dx_2dx_1 = a_2\int_{L_1}^{\infty} f(x_1)dx_1\int_{-\infty}^{L-x_1} f(x_2)dx_2$$

$$= a_2\int_{L_1}^{\infty}\Phi\left(\frac{L-x_1-\mu_2}{\sigma_2}\right)f(x_1)dx_1$$

$$= a_2\int_{\frac{L_1-\mu_1}{\sigma_1}}^{\infty}\Phi\left(\frac{L-\sigma_1 z_1-\mu_1-\mu_2}{\sigma_2}\right)\phi(z_1)dz_1$$

The third element can be rewritten as:

$$c_1\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 f(x_1)f(x_2)dx_2dx_1 = c_1\mu_1$$

The fourth element can be rewritten as:

$$c_2\int_{L_1}^{\infty}\int_{-\infty}^{\infty} x_2 f(x_1)f(x_2)dx_2dx_1 = c_2\int_{L_1}^{\infty} f(x_1)dx_1\int_{-\infty}^{\infty} x_2 f(x_2)dx_2$$

$$= c_2\int_{L_1}^{\infty}\mu_2 f(x_1)dx_1$$

$$= c_2\mu_2\left[1-\Phi\left(\frac{L_1-\mu_1}{\sigma_1}\right)\right]$$

$$= c_2\mu_2\Phi\left(\frac{\mu_1-L_1}{\sigma_1}\right)$$

Finally, the fifth element can be rewritten as:

$$r\int_{-\infty}^{L_1}\int_{-\infty}^{\infty} f(x_1)f(x_2)dx_2dx_1 = r\int_{-\infty}^{L_1} f(x_1)dx_1 = r.\Phi\left(\frac{L_1-\mu_1}{\sigma_1}\right)$$

Putting all components back into equation (5-1) yields,

$$E\{P(\mu_1,\mu_2)\} = a_1\left[\Phi\left(\frac{\mu_1 - L_1}{\sigma_1}\right) - \int_{\frac{L_1-\mu_1}{\sigma_1}}^{\infty} \Phi\left(\frac{L - \sigma_1 z_1 - \mu_1 - \mu_2}{\sigma_2}\right)\phi(z_1)dz_1\right] +$$

$$a_2 \int_{\frac{L_1-\mu_1}{\sigma_1}}^{\infty} \Phi\left(\frac{L - \sigma_1 z_1 - \mu_1 - \mu_2}{\sigma_2}\right)\phi(z_1)dz_1 - c_1\mu_1 -$$

$$c_2\mu_2\Phi\left(\frac{\mu_1 - L_1}{\sigma_1}\right) - r\Phi\left(\frac{L_1 - \mu_1}{\sigma_1}\right)$$

or it can be written as,

$$E\{P(\mu_1,\mu_2)\} = (a_2 - a_1).\int_{\frac{L_1-\mu_1}{\sigma_1}}^{\infty} \Phi\left(\frac{L - \sigma_1 z_1 - \mu_1 - \mu_2}{\sigma_2}\right)\phi(z_1)dz_1 +$$

$$(a_1 + r - c_2\mu_2).\Phi\left(\frac{\mu_1 - L_1}{\sigma_1}\right) - c_1\mu_1 - r \qquad \text{..........................}(5-2)$$

Equation (5-2) represents the expression for the expected profit per item.

A necessary condition to maximize the profit with respect to μ_1 and μ_2 is to set the partial derivatives to zero.

$$\frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial \mu_1} = 0$$

$$\frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial \mu_2} = 0$$

The partial derivatives of equation (5-2) with respect to μ_1 and μ_2 are obtained by using the formula from Hunter and Kartha (1977).

$$\frac{\partial}{\partial \delta}\left[\int_{a(\delta)}^{b(\delta)} I(\delta,z)dz\right] = \int_{a(\delta)}^{b(\delta)} \frac{\partial}{\partial \delta} I(\delta,z)dz + I(\delta,b)\frac{\partial b(\delta)}{\partial \delta} - I(\delta,a)\frac{\partial a(\delta)}{\partial \delta}$$

Applying this formula to the model, we get:

$$\frac{\partial}{\partial \mu_1} \int_{\frac{L_1-\mu_1}{\sigma_1}}^{\infty} \Phi\left(\frac{L-\sigma_1 z_1-\mu_1-\mu_2}{\sigma_2}\right)\phi(z_1)dz_1 =$$

$$\int_{\frac{L_1-\mu_1}{\sigma_1}}^{\infty} \left[0+\Phi\left(\frac{L-\sigma_1 z_1-\mu_1-\mu_2}{\sigma_2}\right)\phi(z_1).\left(-\frac{1}{\sigma_1}\right)\right]dz_1 + 0 + \left[\Phi\left(\frac{L-L_1-\mu_2}{\sigma_2}\right)\phi\left(\frac{L_1-\mu_1}{\sigma_1}\right)\left(\frac{1}{\sigma_1}\right)\right]$$

$$= -\frac{1}{\sigma_1}\left[\int_{\frac{L_1-\mu_1}{\sigma_1}}^{\infty} \Phi\left(\frac{L-\sigma_1 z_1-\mu_1-\mu_2}{\sigma_2}\right)\phi(z_1)dz_1 - \Phi\left(\frac{L-L_1-\mu_2}{\sigma_2}\right)\phi\left(\frac{L_1-\mu_1}{\sigma_1}\right)\right]$$

So, the partial derivative of equation (5-2) with respect to μ_1 can be written as:

$$\Rightarrow \frac{\partial E\{P(X_1, X_2, \mu_1, \mu_2)\}}{\partial \mu_1} =$$

$$-\frac{(a_2-a_1)}{\sigma_1}\left[\int_{\frac{L_1-\mu_1}{\sigma_1}}^{\infty} \Phi\left(\frac{L-\sigma_1 z_1-\mu_1-\mu_2}{\sigma_2}\right)\phi(z_1)dz_1 - \Phi\left(\frac{L-L_1-\mu_2}{\sigma_2}\right)\phi\left(\frac{L_1-\mu_1}{\sigma_1}\right)\right] +$$

$$\frac{(a_1+r-c_2\mu_2)}{\sigma_1}\phi\left(\frac{\mu_1-L_1}{\sigma_1}\right) - c_2 = 0 \qquad \ldots\ldots\ldots\ldots(5-3)$$

Similarly, the partial derivative of equation (5-2) with respect to μ_2 is obtained.

$$\frac{\partial}{\partial \mu_1} \int_{\frac{L_1-\mu_1}{\sigma_1}}^{\infty} \Phi\left(\frac{L-\sigma_1 z_1-\mu_1-\mu_2}{\sigma_2}\right)\phi(z_1)dz_1 =$$

$$-\frac{1}{\sigma_2} \int_{\frac{L_1-\mu_1}{\sigma_1}}^{\infty} \phi\left(\frac{L-\sigma_1 z_1-\mu_1-\mu_2}{\sigma_2}\right)\phi(z_1)dz_1 + 0 - 0$$

$$\Rightarrow \frac{\partial E\{P(X_1, X_2, \mu_1, \mu_2)\}}{\partial \mu_2} =$$

$$-\frac{(a_2-a_1)}{\sigma_2} \int_{\frac{L_1-\mu_1}{\sigma_1}}^{\infty} \phi\left(\frac{L-\sigma_1 z_1-\mu_1-\mu_2}{\sigma_2}\right)\phi(z_1)dz_1 - c_2\Phi\left(\frac{\mu_1-L_1}{\sigma_1}\right) = 0 \quad \text{...............}(5-4)$$

The optimal values of μ_1 and μ_2 can be obtained by solving equations (5-3) and (5-4) simultaneously. One way to solve this system of equations is to use numerical methods.

5.4 Example

Consider the same case study described in section 3.4 with some modifications. The case study considered in this section can be described as follows: Fire extinguishers (cylinders) go through different processes before they become ready-to-use final products. The most important processes are the ones that could affect the quality of the fire extinguishers. The most important processes are the last two which are, coating the cylinders with zinc phosphate and then coating them with powder paint. The process of coating the cylinder with zinc phosphate is called process 1 while the process of coating the cylinder with powder paint is called process 2. The quality characteristic of interest is the thickness of the coating after process 1 and the thickness of the coating after process 2. Notice that, the coating thickness after process 1 is the thickness of the zinc phosphate while the coating thickness after process 2 is the sum of zinc phosphate and powder paint thicknesses. Specifications have to be met such that, the coating thickness after the first process should always be greater than or equal to 10 μm and the coating thickness after the second process should always be greater than or equal to 110 μm. A 100% inspection is conducted after each process. Three cases are considered. Cylinders that do not satisfy the first specification limit are reworked at a fixed cost, r. Cylinders that satisfy both specification limits are sold at a regular market price of SR 35.64 per cylinder. Whereas, cylinders that satisfy only the first specification limit are sold at a secondary market with a price of SR 32.67 per cylinder. The cost of coating 1 m^2 of steel by zinc phosphate is SR 0.75 at 20 μm thickness, and the cost of powder paint of 1 m^2 of steel is SR 2.2 at 100 μm thickness. A cylinder has an external surface area equals to 0.4 m^2. The cost of coating increases proportionally with the increase in thickness.

A sample of data is collected from the factory. Observations for both layers of coating are obtained and the normal distribution is fit to the observed data to be used as an input to model developed in equation (5.3). The adequacy of the fit has been assessed by goodness-of-fit tests, i.e. chi-square test. As a result, it has been found that the first layer of coating is normally distributed with mean 22.2 μm and standard deviation 5.13 μm, and the second layer of coating is normally distributed with mean 126 μm and standard deviation 11.14 μm. Given information are summarized as follows:

L_1 = 10 μm, L = 110 μm, a_1 = 35.64 SR/item, a_2 = 32.67 SR/item, c_1 = 0.015 SR/μm, c_2 = 0.0088 SR/μm, r = 1.2 SR/item, σ_1 = 5.13, σ_2 = 11.14.

Now, if coating thicknesses are much greater than the lower specification limits, then we may maintain the quality of fire extinguishers but the cost of materials will increase. On the other hand, if coating thicknesses are close to the lower specification limits, then we may incur less material cost but the quality of the fire extinguishers will be affected. So, the problem is to decide where to set the means of the two machines (processes) based on the tradeoff among production cost, payoff of nondefective items and the costs incurred by the disposition of the defective items.

To solve this problem, we optimized the model developed in Section 5.3 using Mathematica program. This program uses Nelder-Mead method for nonlinear optimization. The table below shows the values of the optimal process means and expected profit for the targeting problem using 100% inspection.

Optimal mean for the first machine = 19.9259 μm

Optimal mean for the second machine = 114.132 μm

Expected profit per item = SR 32.965

The above results show that the optimal thickness for zinc phosphate is 19.9259 μm and the optimal thickness for the powder paint is 114.132 μm. If we set the two machines such that they produce items with optimal thicknesses then our expected profit will be SR 32.965 per item. More experiments are conducted to see the effect of cost parameters and variances on the expected profit and optimal process means and.

5.5 Sensitivity Analysis

In this section, sensitivity analysis on the model developed in section 5.3 has been conducted to investigate the effect of changing model parameters on model results. Three types of sensitivity analysis have been performed in this section. In section 5.5.1, the effect of different cost parameters on the expected profit has been studied. Section 5.5.2 includes analysis to investigate the effect of the variance of both processes on the expected profit.

5.5.1 The effect of cost parameters on the expected profit

In this section, parametric analysis is used to study the effect of different cost parameters on the expected profit. Studying the rates of change of the profit function with different parameters will give a better understanding to the model behavior. For this reason, partial derivatives of the expected profit function with respect to selling prices a_1 and a_2, processing costs c_1 and c_2 and rework cost r are considered.

The rate of change of the expected profit with respect to the regular market selling price is:

$$\frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial a_1} = -\int_{\frac{L_1-\mu_1}{\sigma_1}}^{\infty} \Phi\left(\frac{L-\sigma_1 z_1-\mu_1-\mu_2}{\sigma_2}\right)\phi(z_1)dz_1 + \Phi\left(\frac{\mu_1-L_1}{\sigma_1}\right)$$

This equation shows that as a_1 increases the expected profit decreases at rate of:

$$\int_{\frac{L_1-\mu_1}{\sigma_1}}^{\infty} \Phi\left(\frac{L-\sigma_1 z_1-\mu_1-\mu_2}{\sigma_2}\right)\phi(z_1)dz_1 + \Phi\left(\frac{\mu_1-L_1}{\sigma_1}\right)$$

Figures 5-2 and 5-3 show the rate of change of the expected profit with the first market selling price a_1 versus σ_1 and σ_2 respectively. Both figures show a decrease in the rate of change of expected profit as σ_1 and σ_2 increase.

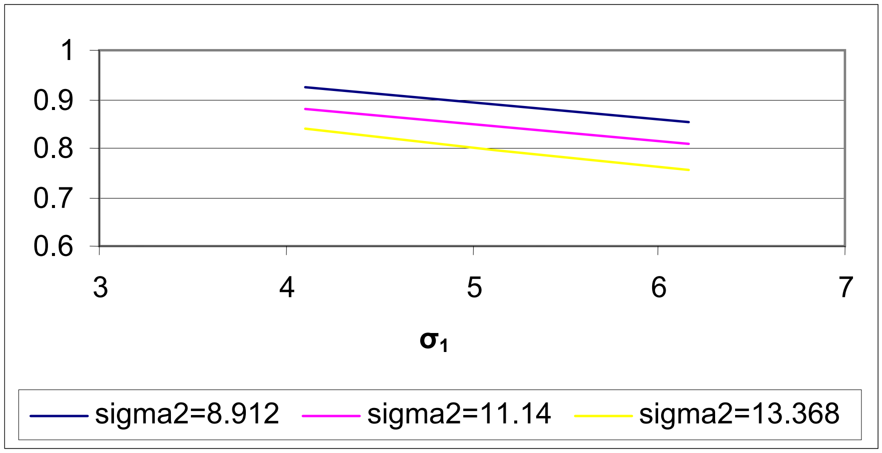

Figure (5-2) Rate of change of expected profit with respect to a_1 vs σ_1

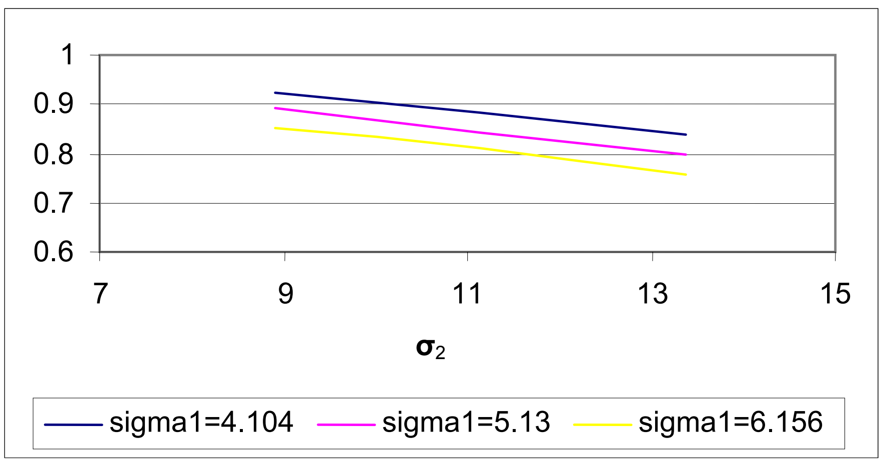

Figure (5-3) Rate of change of expected profit with respect to a_1 vs σ_2

The rate of change of the expected profit with respect to the secondary market selling price is:

$$\frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial a_2} = \int_{\frac{L_1-\mu_1}{\sigma_1}}^{\infty} \Phi\left(\frac{L-\sigma_1 z_1-\mu_1-\mu_2}{\sigma_2}\right).\phi(z_1)dz_1$$

This equation shows that as a_2 increases the expected profit increases with rate of:

$$\int_{\frac{L_1-\mu_1}{\sigma_1}}^{\infty} \Phi\left(\frac{L-\sigma_1 z_1-\mu_1-\mu_2}{\sigma_2}\right)\phi(z_1)dz_1$$

Figures 5-4 and 5-5 show the rate of change of the expected profit with the secondary market selling price a_2 versus σ_1 and σ_2 respectively. Both figures show an increase in the rate of change of expected profit as σ_1 and σ_2 increase. This means, as the variances increase the contribution from the secondary market increase. This is expected because as the variance increases, more items will be sent to the secondary market because they will not meet the specifications of the first market.

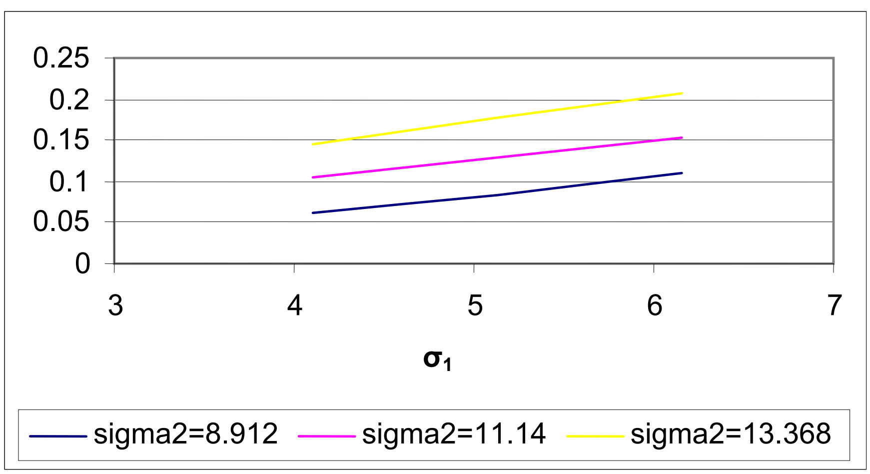

Figure (5-4) Rate of change of expected profit with respect to a_2 vs σ_1

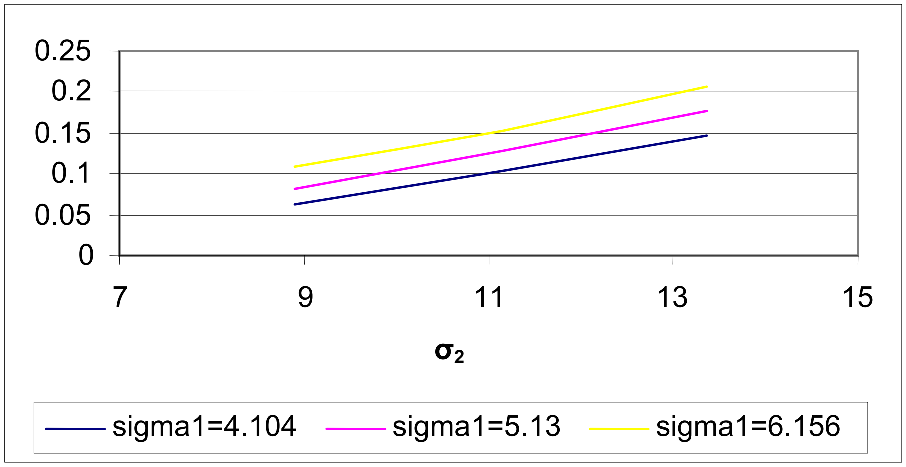

Figure (5-5) Rate of change of expected profit with respect to a_2 vs σ_2

The rate of change of the expected profit with respect to cost of materials consumed in the first process is:

$\frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial c_1} = -\mu_1$, which means that as c_1 increases the expected profit decreases with rate of μ_1, i.e., with one unit increase in c_1 the expected profit decreases with μ_1 units. The rate of change of the expected profit with respect to cost of materials consumed in the second process is:

$$\frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial c_2} = -\mu_2 \Phi\left(\frac{\mu_1 - L_1}{\sigma_1}\right)$$

This equation shows that as c_2 increases the expected profit decreases with rate of: $\mu_2 \Phi\left(\frac{\mu_1 - L_1}{\sigma_1}\right)$

The rate of change of the expected profit with respect to the rework cost is:

$\frac{\partial E\{P(\mu_1,\mu_2)\}}{\partial r} = \Phi\left(\frac{\mu_1 - L_1}{\sigma_1}\right)$, which indicates that as r increases the expected profit increases with rate of $\Phi\left(\frac{\mu_1 - L_1}{\sigma_1}\right)$.

5.5.2 The effect of σ_1 and σ_2 on the expected profit

In this section, the effect of σ_1 and σ_2 on the expected profit is studied. Increasing the variance of the process means that the variation between products becomes higher. Table 5.1 shows the effect of the variations in both processes on the expected profit and optimal means. Changing the original values of σ_1 and σ_2 by ± 10, ± 20 and ± 30, the following values of optimal process means and expected profit are obtained.

Table 5.1 Expected profit and optimal means for different values of σ_1 and σ_2

σ1	σ2	μ1	μ2	EP
3.591	7.798	18.6613	111.542	33.7984
	8.912	18.6746	112.419	33.7723
	10.026	18.6849	113.344	33.7442
	11.14	18.6921	114.311	33.7153
	12.254	18.6967	115.312	33.6865
	13.368	18.6997	116.346	33.6585
	14.482	18.7012	117.405	33.6314
4.104	7.798	18.8989	111.421	33.4189
	8.912	18.9071	112.38	33.3939
	10.026	18.9134	113.32	33.367
	11.14	18.9182	114.3	33.3393
	12.254	18.9214	115.302	33.3115
	13.368	18.9229	116.314	33.2839
	14.482	18.924	117.363	33.2574
4.617	7.798	19.3516	111.243	33.0765
	8.912	19.3586	112.298	33.0532
	10.026	19.3643	113.267	33.0278
	11.14	19.3687	114.246	33.0015
	12.254	19.3716	115.232	32.9747
	13.368	19.3734	116.241	32.9482

	14.482	19.3749	117.292	32.9227
5.13	7.798	19.8741	110.974	32.7669
	8.912	19.8804	112.154	32.7452
	10.026	19.8859	113.16	32.7213
	11.14	19.8901	114.142	32.6962
	12.254	19.8933	115.134	32.6708
	13.368	19.8955	116.139	32.6453
	14.482	19.9007	117.38	32.623
5.643	7.798	20.4339	110.655	32.4898
	8.912	20.4396	111.962	32.4694
	10.026	20.4452	113.044	32.4472
	11.14	20.4497	114.053	32.4236
	12.254	20.4453	114.277	32.3916
	13.368	20.4554	116.022	32.3746
	14.482	20.4573	117.046	32.3505
6.156	7.798	21.0181	110.329	32.2434
	8.912	21.0232	111.712	32.2241
	10.026	21.0285	112.855	32.203
	11.14	21.0335	113.933	32.181
	12.254	21.0372	114.925	32.1578
	13.368	21.04	115.914	32.1343
	14.482	21.0412	116.846	32.1101
6.669	7.798	21.6199	109.947	32.0247
	8.912	21.6245	111.462	32.0064
	10.026	21.6298	112.701	31.9868
	11.14	21.6346	113.758	31.9655
	12.254	21.6389	114.8	31.9437
	13.368	21.6423	115.82	31.9214
	14.482	21.6516	116.08	31.9015

Table 5.1 helps in conducting cost/benefit analysis for reducing the variances for process 1 and process 2. For example, reducing σ_2 from its current level to 7.798 will increase the profit by SR 0.0707 per item, and reducing σ_1 from 5.13 to 3.591 will increase the profit by SR 0.0191 per item. Reducing both variances will increase the profit by 1.1022 SR per item.

There are many practical ways to reduce the current variances, such as training the workers to be in the same level of knowledge and experience and using machines in the same condition to produce similar products.

5.6 Conclusion

In this chapter, a model is developed for two processes producing a single product with two quality characteristics. The first quality characteristic is determined by the first process and the second quality characteristic is determined by both processes. Specification limits are set on both quality characteristics. A 100% inspection has been used as a mean for product quality control assuming perfect inspection. A real case has been considered to test the model developed in this chapter followed by the sensitivity analysis which has been conducted to investigate the effect of changing model parameters on model results. Two cases have been considered in the sensitivity analysis: the effect of cost parameters on the expected profit, the effect of the variations on the expected profit. In the model developed in this chapter, inspection is assumed to be error free. This assumption is relaxed in chapter 6.

CHAPTER 6

PROCESS TARGETING WITH TWO INDEPENDENT PROCESSES IN SERIES USING 100 % INSPECTION WITH INSPECTION ERROR

6.1 Introduction

The purpose of this chapter is to develop a process targeting model for two independent processes in series using a 100% inspection as a mean for product quality control. In addition, the inspection process is assumed to be error prone. The model in this chapter extends the model in chapter 5 by incorporating inspection error. This will make the model more realistic since many studies have shown that inspectors as well as instrument devices are subject to error.

The motivation behind this extension stems from the fact that measurement system can cause considerable loss due to misclassification of the product. The loss could be in terms of replacement and warranty costs, loss of good will, and loss of profit by selling a higher grade product as a lower grade due to misclassification. The per unit loss in profit due to this error may seem small, however in some cases, the overall loss of profit (considering millions of units produced per year) may be in millions.

In this chapter, it is assumed that the measured quality characteristic has an observed value which is different from the true value due to the presence of measurement error. However, to reduce the effect of the error, cut off points for acceptance and rejection are modified. In this case, the product will be inspected and based on these cut off points the product will be considered as meeting specifications or not as shown in figure (6-1). The location of these cut off points depends on many factors, such as: the loss in profit due to misclassifying a higher grade product into a lower grade, the penalty associated with misclassifying a lower grade product with a higher grade, and the position of the mean, etc.

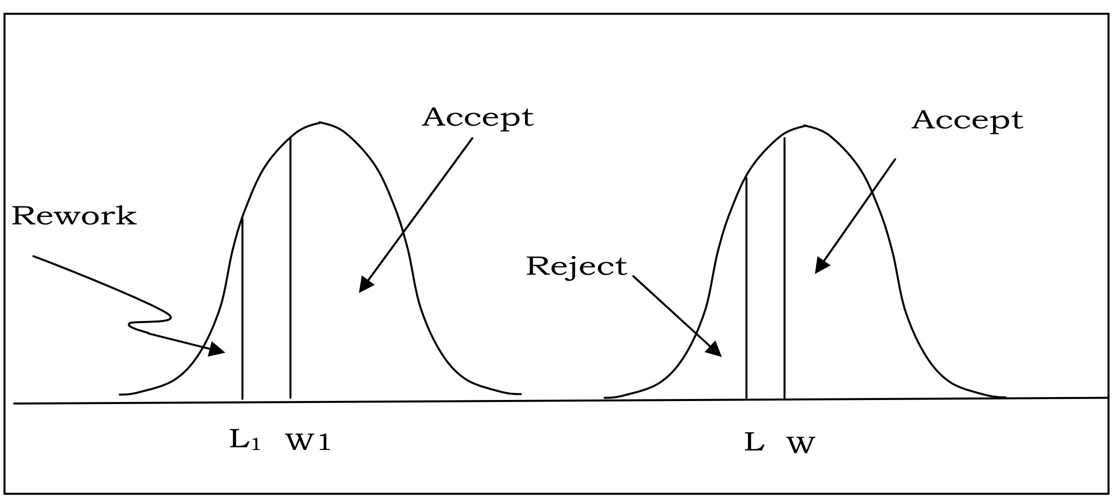

Figure (6-1) Cut off points values for the inspection.

The objective of the model is to determine the mean of both processes and the cut off points to maximize the expected profit.

6.2 Statement of the Problem

Consider a manufacturing environment in which products go through two different processes. The product has two quality characteristics. The first quality characteristic is determined by the first process. The second quality characteristic is determined by both processes. Specification limits are set on both quality characteristics.

Let us assume that the quality characteristic of the first process is a random variable denoted by X_1, and the quality characteristic of the second process is another random variable denoted by X_2. The final quality characteristic is denoted by X, where $X=X_1+X_2$. Quality requirement is that $X_1 \geq L_1$ and $X \geq L$, where L_1 and L are predetermined constants set by product designer or market information. All items are inspected, i.e., 100% inspection is used for product quality control.

The extension here is that inspection is assumed to be error prone. In this case, the observed value of the quality characteristic is not the same as the true value due to the presence of measurement error. Therefore, the inspector will observe the quality characteristic after process 1 as Y_1 instead of X_1, and the quality characteristic after process 2 as Y instead of X. However, to reduce the effect of the inspection error we introduce cut off points. These cut off points will be considered as the criteria of classification instead of lower specification limits, see figure (6-2). If the inspector observes that the first quality characteristic of an item less than its lower specification limit (or in this case its cut off value), then the item is reworked at a cost $c_1x_1 + r$.

Whereas, if he observes the second quality characteristic of an item less than its cut off value then this item will be sold at a secondary market with a price a_2. Finally, if he observes both quality characteristics of an item greater than their cut off values then this item is sold at a regular market with a price a_1.

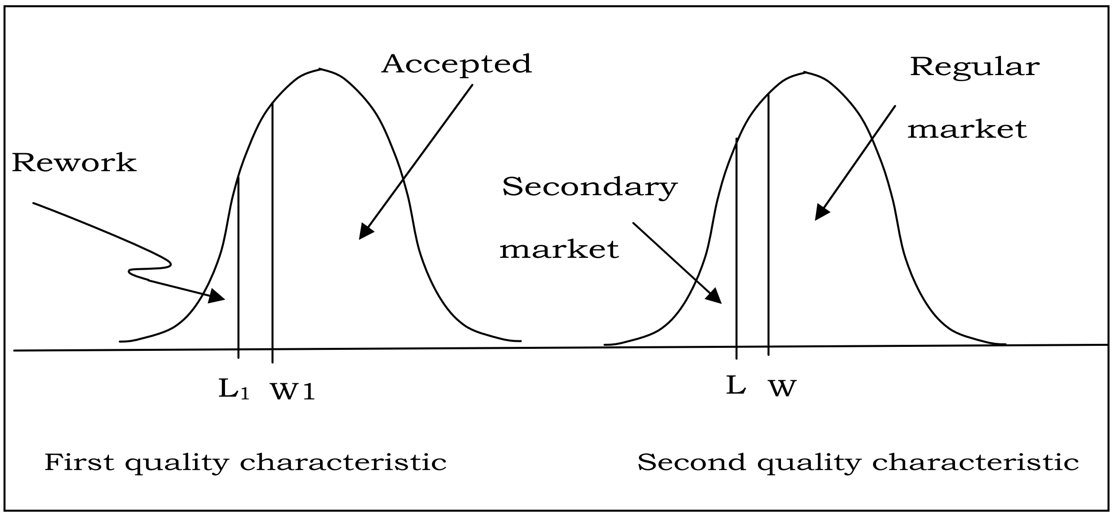

Figure (6-2) A production process with 100% inspection and inspection error.

The products classification is based on the observed values Y_1 and Y and the cut off values W_1 and W. It may happen that the product is classified as meeting the specifications for both quality characteristic but actually it does not due to measurement errors. The following penalties are introduced to minimize misclassifications:

b_1 is the penalty when $Y_1 \geq W_1$ and $Y \geq W$, while in fact $X_1 < L_1$ and $X \geq L$

b_2 is the penalty when $Y_1 \geq W_1$ and $Y \geq W$, while in fact $X_1 \geq L_1$ and $X < L$

b_3 is the penalty when $Y_1 \geq W_1$ and $Y \geq W$, while in fact $X_1 < L_1$ and $X < L$

b_4 is the penalty when $Y_1 \geq W_1$ and $Y < W$, while in fact $X_1 < L_1$ and $X < L$

b_5 is the penalty when $Y_1 \geq W_1$ and $Y < W$, while in fact $X_1 < L_1$ and $X \geq L$

The objective is to find the optimal W_1, W and the processes means to maximize the expected profit.

6.3 Model Development

In this section, a process targeting model is developed for the problem stated in section 6.2. Notations and assumptions are presented prior to model development.

The following notations are adopted:

X_i A random variable that represents the true value of the quality characteristic of the product after finishing process i, where i = 1, 2.

X Sum of the values of both quality characteristics, $X = X_1 + X_2$.

Y_i A random variable that represent the observed value of the quality characteristic of the product after finishing process i, where i = 1, 2.

Y Sum of the observed values of both quality characteristics after being processed by both processes.

L_1 Lower specification limit for the first product quality characteristic.

L Lower specification limit for the composite (sum) of the product's two quality characteristics.

W_1 Cut off value for Y_1.

W Cut off value for Y_2.

a_1 Selling price /item for accepted items after process 2.

a_2 Selling price /item for items rejected after process 2 (secondary selling price).

c_i Cost of material consumed in process i per unit of thickness, i = 1 ,2.

r Rework cost per defective item in the rejected lot after process 1.

μ_1 Mean setting for process 1.

μ_2 Mean setting for process 2.

σ_1 Standard deviation for process 1.

σ_2 Standard deviation for process 2.

ε_1 Error in X_1 measurement.

ε_2 Error in X_2 measurement.

P Profit per item.

The following assumptions are used to develop the model:

1. X_1 and X_2 are independent and normally distributed with mean μ_1 and μ_2 respectively and standard deviation σ_1 and σ_2 respectively.
2. The measurements "Y_1 and Y_2" are assumed to be unbiased and distributed normally across their true values.
3. ε_1 and ε_2 are assumed to be normally distributed with mean zero and zero respectively and standard deviation $\sigma_{\varepsilon 1}$ and $\sigma_{\varepsilon 2}$ respectively.
4. The processing order is fixed.
5. Costs of processing are assumed to be directly proportional to the mean setting of the process.
6. The secondary selling price is less than the regular selling price and the lower specification limit for the first process is less than the lower specification limit of the second process.
7. The rework after the first process involves removing the paint and starting the first process from the beginning.
8. Reprocessing cost consist of: variable cost and fixed cost.
9. 100 % inspection is used for quality control and it is error prone.
10. There is no shift or drift in the processes over time.

The relationship between (X_1 and Y_1); (X_2 and Y_2) can be described as follows:

Y_1 is the observed value of X_1 and Y_2 is the observed value of X_2, so we can write this relationship as:

$$Y_1 = X_1 + \varepsilon_1$$

$$Y_2 = X_2 + \varepsilon_2$$

Where, $\varepsilon_1 \sim N\ (0, \sigma_{\varepsilon 1}^2)$ and $\varepsilon_2 \sim N\ (0, \sigma_{\varepsilon 2}^2)$

The expected value of the observed valuesY_1 will be:

$$E(Y_1) = E(X_1 + \varepsilon_1) = E(X_1) + E(\varepsilon_1) = E(X_1) = \mu_1$$

The variances of the observed valuesY_1 will be:

$$Var(Y_1) = Var(X_1) + Var(\varepsilon_1) = \sigma_1^2 + \sigma_{\varepsilon_1}^2$$

Since X_1 and ε_1 are independent, $Cov(X_1, \varepsilon_1) = 0$

Similarly, the expected value of the observed valuesY_2 will be:

$$E(Y_2) = E(X_2 + \varepsilon_2) = E(X_2) + E(\varepsilon_2) = E(Y_2) = \mu_2$$

The variances of the observed valuesY_2 will be:

$$Var(Y_2) = Var(X_2) + Var(\varepsilon_2) = \sigma_2^2 + \sigma_{\varepsilon 2}^2$$

Since X_2 and ε_2 are independent, $Cov(X_2, \varepsilon_2) = 0$

Assuming Y_1, Y_2, X_1 and X_2 are normally distributed, the joint density function of (Y_1 and X_1); (Y_2 and X_2) can be written as:

$$\psi_1(x_1,y_1)=\frac{1}{2\pi\sigma_1\sigma_{Y1}\sqrt{1-\rho_1^{\ 2}}}\exp^{-\frac{1}{2\sqrt{1-\rho_1^{\ 2}}}\left\{\left(\frac{y_1-\mu_1}{\sigma_{Y1}}\right)^2+\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2-\frac{2\rho_1(y_1-\mu_1)(x_1-\mu_1)}{\sigma_{Y1}\sigma_1}\right\}}$$

$$\psi_2(x_2,y_2)=\frac{1}{2\pi\sigma_2\sigma_{Y2}\sqrt{1-\rho_2^{\ 2}}}\exp^{-\frac{1}{2\sqrt{1-\rho_2^{\ 2}}}\left\{\left(\frac{y_2-\mu_2}{\sigma_{Y2}}\right)^2+\left(\frac{x_2-\mu_2}{\sigma_2}\right)^2-\frac{2\rho_2(y_2-\mu_2)(x_2-\mu_2)}{\sigma_{Y2}\sigma_2}\right\}}$$

In standard form:

$$\psi_1(u_1,v_1)=\frac{1}{2\pi}\exp^{-\frac{1}{2}\left\{v_1^2+u_1^2-2\rho v_1 u_1\right\}}$$

$$\psi_2(u_2,v_2)=\frac{1}{2\pi}\exp^{-\frac{1}{2}\left\{v_2^2+u_2^2-2\rho v_2 u_2\right\}}$$

Where, ρ is the correlation co-efficient and it is equal to:

$$\rho_1=1-\frac{\sigma_{\varepsilon 1}^2}{\sigma_{Y1}^2}=\frac{\sigma_1^2}{\sigma_{Y1}^2}=\frac{\sigma_{\varepsilon 1}^2}{\sigma_1^2+\sigma_{\varepsilon 1}^2}$$

$$\rho_2=1-\frac{\sigma_{\varepsilon 2}^2}{\sigma_{Y2}^2}=\frac{\sigma_2^2}{\sigma_{Y2}^2}=\frac{\sigma_{\varepsilon 2}^2}{\sigma_2^2+\sigma_{\varepsilon 2}^2}$$

Let $P(X_1, X_2)$ be the profit for an item that has a paint thickness after process 1 equals to X_1 and a pint thickness after process 2 equals to X_2 and let $E\{P(\mu_1,\mu_2,W_1,W)\}$ be the expected profit. Notice that, since X is the composite thickness of the two painting layers, Y is also the composite observed thickness of the two painting layers, $Y = Y_1 + Y_2$. Given this description, we can construct the following profit function per item:

$$
P(X_1, X_2) = \begin{cases}
a_1 - c_1 X_1 - c_2 X_2 & if \quad Y_1 \geq W_1, \quad X_1 \geq L_1, \quad Y_2 \geq W - Y_1, \quad X_2 \geq L - X_1 \\
a_1 - c_1 X_1 - c_2 X_2 - b_1 & if \quad Y_1 \geq W_1, \quad X_1 < L_1, \quad Y_2 \geq W - Y_1, \quad X_2 \geq L - X_1 \\
a_1 - c_1 X_1 - c_2 X_2 - b_2 & if \quad Y_1 \geq W_1, \quad X_1 \geq L_1, \quad Y_2 \geq W - Y_1, \quad X_2 < L - X_1 \\
a_1 - c_1 X_1 - c_2 X_2 - b_3 & if \quad Y_1 \geq W_1, \quad X_1 < L_1, \quad Y_2 \geq W - Y_1, \quad X_2 < L - X_1 \\
a_2 - c_1 X_1 - c_2 X_2 & if \quad Y_1 \geq W_1, \quad X_1 \geq L_1, \quad Y_2 < W - Y_1, \quad X_2 < L - X_1 \\
a_2 - c_1 X_1 - c_2 X_2 - b_4 & if \quad Y_1 \geq W_1, \quad X_1 < L_1, \quad Y_2 < W - Y_1, \quad X_2 < L - X_1 \\
a_2 - c_1 X_1 - c_2 X_2 & if \quad Y_1 \geq W_1, \quad X_1 \geq L_1, \quad Y_2 < W - Y_1, \quad X_2 \geq L - X_1 \\
a_2 - c_1 X_1 - c_2 X_2 - b_5 & if \quad Y_1 \geq W_1, \quad X_1 < L_1, \quad Y_2 < W - Y_1, \quad X_2 \geq L - X_1 \\
-c_1 X_1 - r & if \quad Y_1 < W_1, \quad X_1 < L_1, \quad -\infty < Y_2 < \infty, \quad -\infty < X_2 < \infty \\
-c_1 X_1 - r & if \quad Y_1 < W_1, \quad X_1 \geq L_1, \quad -\infty < Y_2 < \infty, \quad -\infty < X_2 < \infty
\end{cases}
$$

The profit function shows four possibilities for items sold at a regular market and four possibilities for items sold at a secondary market and two possibilities for items that require rework.

- The possibilities for items sold at a regular market are:
  1. Correctly classified.
  2. Originally rework but misclassified as regular market items.
  3. Originally secondary market items but misclassified as regular market items.
  4. Originally rework but misclassified as good items and then misclassified as regular market items.
- The possibilities for items sold at a secondary market are:
  1. Correctly classified.
  2. Originally rework but misclassified as secondary market items.
  3. Originally regular market items but misclassified as secondary market items.
  4. Originally rework but misclassified as good items and then misclassified as secondary market items.
- The possibilities for items to be reworked:

1. Correctly classified.
2. Originally regular or secondary market items but misclassified as rework.

The expected profit per item can be written as:

$$E\{P(\mu_1, \mu_2, w_1, w)\} =$$

$$\int_{w_1}^{\infty}\int_{L_1}^{\infty}\int_{w-y_1}^{\infty}\int_{L-x_1}^{\infty}(a_1 - c_1x_1 - c_2x_2)\psi_1(x_1, y_1).\psi_2(x_2, y_2)dx_2dy_2dx_1dy_1 +$$

$$\int_{w_1}^{\infty}\int_{-\infty}^{L_1}\int_{w-y_1}^{\infty}\int_{L-x_1}^{\infty}(a_1 - c_1x_1 - c_2x_2 - b_1)\psi_1(x_1, y_1).\psi_2(x_2, y_2)dx_2dy_2dx_1dy_1 +$$

$$\int_{w_1}^{\infty}\int_{L_1}^{\infty}\int_{w-y_1}^{\infty}\int_{-\infty}^{L-x_1}(a_1 - c_1x_1 - c_2x_2 - b_2)\psi_1(x_1, y_1).\psi_2(x_2, y_2)dx_2dy_2dx_1dy_1 +$$

$$\int_{w_1}^{\infty}\int_{-\infty}^{L_1}\int_{w-y_1}^{\infty}\int_{-\infty}^{L-x_1}(a_1 - c_1x_1 - c_2x_2 - b_3)\psi_1(x_1, y_1).\psi_2(x_2, y_2)dx_2dy_2dx_1dy_1 +$$

$$\int_{w_1}^{\infty}\int_{L_1}^{\infty}\int_{-\infty}^{w-y_1}\int_{-\infty}^{L-x_1}(a_2 - c_1x_1 - c_2x_2)\psi_1(x_1, y_1).\psi_2(x_2, y_2)dx_2dy_2dx_1dy_1 +$$

$$\int_{w_1}^{\infty}\int_{-\infty}^{L_1}\int_{-\infty}^{w-y_1}\int_{-\infty}^{L-x_1}(a_2 - c_1x_1 - c_2x_2 - b_4)\psi_1(x_1, y_1).\psi_2(x_2, y_2)dx_2dy_2dx_1dy_1 +$$

$$\int_{w_1}^{\infty}\int_{L_1}^{\infty}\int_{-\infty}^{w-y_1}\int_{L-x_1}^{\infty}(a_2 - c_1x_1 - c_2x_2)\psi_1(x_1, y_1).\psi_2(x_2, y_2)dx_2dy_2dx_1dy_1 +$$

$$\int_{w_1}^{\infty}\int_{-\infty}^{L_1}\int_{-\infty}^{w-y_1}\int_{L-x_1}^{\infty}(a_2 - c_1x_1 - c_2x_2 - b_5)\psi_1(x_1, y_1).\psi_2(x_2, y_2)dx_2dy_2dx_1dy_1 +$$

$$\int_{-\infty}^{w_1}\int_{-\infty}^{L_1}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(-c_1x_1 - r)\psi_1(x_1, y_1).\psi_2(x_2, y_2)dx_2dy_2dx_1dy_1 +$$

$$\int_{-\infty}^{w_1}\int_{L_1}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(-c_1x_1 - r)\psi_1(x_1, y_1).\psi_2(x_2, y_2)dx_2dy_2dx_1dy_1$$

Simplifying the above expression,

$$E\{P(\mu_1,\mu_2,w_1,w)\}=$$

$$a_1\int_{w_1}^{\infty}\int_{-\infty}^{\infty}\int_{w-y_1}^{\infty}\int_{-\infty}^{\infty}\psi_1(x_1,y_1).\psi_2(x_2,y_2)dx_2dy_2dx_1dy_1+$$

$$a_2\int_{w_1}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{w-y_1}\int_{-\infty}^{\infty}\psi_1(x_1,y_1).\psi_2(x_2,y_2)dx_2dy_2dx_1dy_1-$$

$$c_1\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}x_1\psi_1(x_1,y_1).\psi_2(x_2,y_2)dx_2dy_2dx_1dy_1-$$

$$c_2\int_{w_1}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}x_2\psi_1(x_1,y_1).\psi_2(x_2,y_2)dx_2dy_2dx_1dy_1-$$

$$b_1\int_{w_1}^{\infty}\int_{-\infty}^{L_1}\int_{w-y_1}^{\infty}\int_{L-x_1}^{\infty}\psi_1(x_1,y_1).\psi_2(x_2,y_2)dx_2dy_2dx_1dy_1-$$

$$b_2\int_{w_1}^{\infty}\int_{L_1}^{\infty}\int_{w-y_1}^{\infty}\int_{-\infty}^{L-x_1}\psi_1(x_1,y_1).\psi_2(x_2,y_2)dx_2dy_2dx_1dy_1-$$

$$b_3\int_{w_1}^{\infty}\int_{-\infty}^{L_1}\int_{w-y_1}^{\infty}\int_{-\infty}^{L-x_1}\psi_1(x_1,y_1).\psi_2(x_2,y_2)dx_2dy_2dx_1dy_1-$$

$$b_4\int_{w_1}^{\infty}\int_{-\infty}^{L_1}\int_{-\infty}^{w-y_1}\int_{-\infty}^{L-x_1}\psi_1(x_1,y_1).\psi_2(x_2,y_2)dx_2dy_2dx_1dy_1-$$

$$b_5\int_{w_1}^{\infty}\int_{-\infty}^{L_1}\int_{-\infty}^{w-y_1}\int_{L-x_1}^{\infty}\psi_1(x_1,y_1).\psi_2(x_2,y_2)dx_2dy_2dx_1dy_1-$$

$$r\int_{-\infty}^{w_1}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\psi_1(x_1,y_1).\psi_2(x_2,y_2)dx_2dy_2dx_1dy_1$$

$$\ldots\ldots\ldots\ldots\ldots(6-1)$$

Simplifying expression (6-1) term by term using standard normal and bivariate normal distribution and using relationships between random variables, the first term can be simplified as follows:

$$a_1 \int_{w_1}^{\infty}\int_{-\infty}^{\infty}\int_{w-y_1}^{\infty}\int_{-\infty}^{\infty} \psi_1(x_1, y_1).\psi_2(x_2, y_2)dx_2 dy_2 dx_1 dy_1$$

$$= a_1 \int_{w_1}^{\infty}\int_{-\infty}^{\infty}\int_{w-y_1}^{\infty} \psi_1(x_1, y_1).f_2(y_2)dy_2 dx_1 dy_1$$

$$= a_1 \int_{w_1}^{\infty}\int_{w-y_1}^{\infty} f_2(y_2) f_1(y_1)dy_2 dy_1$$

$$= a_1 \int_{w-y_1}^{\infty} f_2(y_2)[1 - F_1(w_1)]dy_2$$

$$= a_1[1 - F_1(w_1)][1 - F_2(w - y_1)]$$

$$= a_1\left[1 - \Phi\left(\frac{w_1 - \mu_1}{\sqrt{\sigma_1^2 + \sigma_{\varepsilon 1}^2}}\right)\right]\left[1 - \Phi\left(\frac{w - y_1 - \mu_2}{\sqrt{\sigma_2^2 + \sigma_{\varepsilon 2}^2}}\right)\right]$$

$$= a_1\Phi\left(\frac{\mu_1 - w_1}{\sqrt{\sigma_1^2 + \sigma_{\varepsilon 1}^2}}\right)\Phi\left(\frac{\mu_2 - w + y_1}{\sqrt{\sigma_2^2 + \sigma_{\varepsilon 2}^2}}\right) \text{.............................} (6-2)$$

The second term can be simplified as follows:

$$a_2 \int_{w_1}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{w-y_1}\int_{-\infty}^{\infty} \psi_1(x_1, y_1).\psi_2(x_2, y_2)dx_2 dy_2 dx_1 dy_1$$

$$= a_2 \int_{w_1}^{\infty}\int_{-\infty}^{w-y_1} f_1(y_1) f_2(y_2)dy_2 dy_1$$

$$= a_2 \int_{-\infty}^{w-y_1} f_2(y_2)[1 - F_1(w_1)]dy_2$$

$$= a_2[1 - F_1(w_1)]F_2(w - y_1)$$

$$= a_2\Phi\left(\frac{\mu_1 - w_1}{\sqrt{\sigma_1^2 + \sigma_{\varepsilon 1}^2}}\right)\Phi\left(\frac{w - y_1 - \mu_2}{\sqrt{\sigma_2^2 + \sigma_{\varepsilon 2}^2}}\right) \text{...................} (6-3)$$

The third term can be simplified as follows:

$$c_1 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 \psi_1(x_1, y_1).\psi_2(x_2, y_2) dx_2 dy_2 dx_1 dy_1$$

$$= c_1 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 \psi_1(x_1, y_1).f_2(y_2) dy_2 dx_1 dy_1$$

$$= c_1 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 \psi_1(x_1, y_1) dx_1 dy_1$$

$$= c_1 \int_{-\infty}^{\infty} x_1 f_1(x_1) dx_1$$

$$= c_1 \mu_1 \qquad \text{..}(6-4)$$

The fourth term can be simplified as follows:

$$c_2 \int_{w_1}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_2 \psi_1(x_1, y_1).\psi_2(x_2, y_2) dx_2 dy_2 dx_1 dy_1$$

$$= c_2 \int_{w_1}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_2 \psi_1(x_1, y_1).f_2(x_2) dx_2 dx_1 dy_1$$

$$= c_2 \int_{w_1}^{\infty}\int_{-\infty}^{\infty} \mu_2 \psi_1(x_1, y_1) dx_1 dy_1$$

$$= c_2 \mu_2 \int_{w_1}^{\infty} f_1(y_1) dy_1$$

$$= c_2 \mu_2 \Phi\left(\frac{\mu_1 - w_1}{\sqrt{\sigma_1^2 + \sigma_{\varepsilon 1}^2}}\right) \qquad \text{......................}(6-5)$$

The fifth term can be simplified as follows:

$$r\int_{-\infty}^{w_1}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\psi_1(x_1,y_1).\psi_2(x_2,y_2)dx_2dy_2dx_1dy_1$$

$$=r\int_{-\infty}^{w_1}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\psi_1(x_1,y_1).f_2(y_2)dy_2dx_1dy_1$$

$$=r\int_{-\infty}^{w_1}\int_{-\infty}^{\infty}\psi_1(x_1,y_1)dx_1dy_1$$

$$=r\int_{-\infty}^{w_1}f(y_1)dy_1$$

$$=r\Phi\left(\frac{w_1-\mu_1}{\sqrt{\sigma_1^2+\sigma_{\varepsilon 1}^2}}\right) \qquad \text{...}(6-6)$$

Substituting expressions from (6-2) to (6-6) in the main expression of the expected profit per item (6-1), yield

$$E\{P(\mu_1,\mu_2,w_1,w)\}=$$

$$\Phi\left(\frac{\mu_1-w_1}{\sqrt{\sigma_1^2+\sigma_{\varepsilon 1}^2}}\right)\left[a_1\Phi\left(\frac{\mu_2-w+y_1}{\sqrt{\sigma_2^2+\sigma_{\varepsilon 2}^2}}\right)+a_2\Phi\left(\frac{w-y_1-\mu_2}{\sqrt{\sigma_2^2+\sigma_{\varepsilon 2}^2}}\right)-c_2\mu_2\right]-c_1\mu_1-r\Phi\left(\frac{w_1-\mu_1}{\sqrt{\sigma_1^2+\sigma_{\varepsilon 1}^2}}\right)-$$

$$b_1\int_{w_1}^{\infty}\int_{-\infty}^{L_1}\int_{w-y_1}^{\infty}\int_{L-x_1}^{\infty}\psi_1(x_1,y_1).\psi_2(x_2,y_2)dx_2dy_2dx_1dy_1-$$

$$b_2\int_{w_1}^{\infty}\int_{L_1}^{\infty}\int_{w-y_1}^{\infty}\int_{-\infty}^{L-x_1}\psi_1(x_1,y_1).\psi_2(x_2,y_2)dx_2dy_2dx_1dy_1-$$

$$b_3\int_{w_1}^{\infty}\int_{-\infty}^{L_1}\int_{w-y_1}^{\infty}\int_{-\infty}^{L-x_1}\psi_1(x_1,y_1).\psi_2(x_2,y_2)dx_2dy_2dx_1dy_1-$$

$$b_4\int_{w_1}^{\infty}\int_{-\infty}^{L_1}\int_{-\infty}^{w-y_1}\int_{-\infty}^{L-x_1}\psi_1(x_1,y_1).\psi_2(x_2,y_2)dx_2dy_2dx_1dy_1-$$

$$b_5\int_{w_1}^{\infty}\int_{-\infty}^{L_1}\int_{-\infty}^{w-y_1}\int_{L-x_1}^{\infty}\psi_1(x_1,y_1).\psi_2(x_2,y_2)dx_2dy_2dx_1dy_1$$

....(6 – 7)

Expression (6-7) represents the expected profit under error prone 100% inspection.

Now, consider a special case of the above problem in which all penalties are assumed to be equal. In this case, all the items that found to be nonconforming will be reworked regardless whether they were discovered after the first process or after the second process.

The model associated with the special case can be expressed as follows:

$$E\{P(\mu_1, \mu_2, w_1, w)\} =$$

$$\Phi\left(\frac{\mu_1 - w_1}{\sqrt{\sigma_1^2 + \sigma_{\varepsilon 1}^2}}\right)\left[a_1\Phi\left(\frac{\mu_2 - w + y_1}{\sqrt{\sigma_2^2 + \sigma_{\varepsilon 2}^2}}\right) + a_2\Phi\left(\frac{w - y_1 - \mu_2}{\sqrt{\sigma_2^2 + \sigma_{\varepsilon 2}^2}}\right) - c_2\mu_2\right] - c_1\mu_1 - r\Phi\left(\frac{w_1 - \mu_1}{\sqrt{\sigma_1^2 + \sigma_{\varepsilon 1}^2}}\right) -$$

$$b\int_{w_1}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\psi_1(x_1, y_1).\psi_2(x_2, y_2)dx_2dy_2dx_1dy_1$$

$$= \Phi\left(\frac{\mu_1 - w_1}{\sqrt{\sigma_1^2 + \sigma_{\varepsilon 1}^2}}\right)\left[a_1\Phi\left(\frac{\mu_2 - w + y_1}{\sqrt{\sigma_2^2 + \sigma_{\varepsilon 2}^2}}\right) + a_2\Phi\left(\frac{w - y_1 - \mu_2}{\sqrt{\sigma_2^2 + \sigma_{\varepsilon 2}^2}}\right) - c_2\mu_2\right] - c_1\mu_1 - r\Phi\left(\frac{w_1 - \mu_1}{\sqrt{\sigma_1^2 + \sigma_{\varepsilon 1}^2}}\right) -$$

$$b\int_{w_1}^{\infty}\int_{-\infty}^{\infty}\psi_1(x_1, y_1)dx_1dy_1$$

$$= \Phi\left(\frac{\mu_1 - w_1}{\sqrt{\sigma_1^2 + \sigma_{\varepsilon 1}^2}}\right)\left[a_1\Phi\left(\frac{\mu_2 - w + y_1}{\sqrt{\sigma_2^2 + \sigma_{\varepsilon 2}^2}}\right) + a_2\Phi\left(\frac{w - y_1 - \mu_2}{\sqrt{\sigma_2^2 + \sigma_{\varepsilon 2}^2}}\right) - c_2\mu_2\right] - c_1\mu_1 - r\Phi\left(\frac{w_1 - \mu_1}{\sqrt{\sigma_1^2 + \sigma_{\varepsilon 1}^2}}\right) -$$

$$b\left[1 - \Phi\left(\frac{w_1 - \mu_1}{\sqrt{\sigma_1^2 + \sigma_{\varepsilon 1}^2}}\right)\right]$$

This can be further simplified to get:

$$E\{P(\mu_1,\mu_2,w_1,w)\} = \Phi\left(\frac{\mu_1 - w_1}{\sqrt{\sigma_1^2 + \sigma_{\varepsilon 1}^2}}\right)\left[a_1\Phi\left(\frac{\mu_2 - w + y_1}{\sqrt{\sigma_2^2 + \sigma_{\varepsilon 2}^2}}\right) + a_2\Phi\left(\frac{w - y_1 - \mu_2}{\sqrt{\sigma_2^2 + \sigma_{\varepsilon 2}^2}}\right) - c_2\mu_2 - b\right] -$$

$$c_1\mu_1 - r\Phi\left(\frac{w_1 - \mu_1}{\sqrt{\sigma_1^2 + \sigma_{\varepsilon 1}^2}}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots(6-8)$$

From Expression (6-8), optimal process means and cut off points can be obtained for the special case where all penalties are equal.

6.4 Example

In this section, a real example is studied and solved using expression (6-8). Consider the same case study described in section 5.4.2 except that the 100% inspection is assumed to be error prone. The case study considered in this section can be described as follows: Fire extinguishers (cylinders) go through different processes before they become ready-to-use final products. The most important processes are the last two which are, coating the cylinders with zinc phosphate and then coating them with powder paint. The process of coating the cylinder with zinc phosphate is called process 1 while the process of coating the cylinder with powder paint is called process 2. The quality characteristic of interest is the thickness of the coating after process 1 and the thickness of the coating after process 2. Notice that, the coating thickness after process 1 is the thickness of the zinc phosphate while the coating thickness after process 2 is the sum of zinc phosphate and powder paint thicknesses. Specifications have to be met such that, the coating thickness after the first process should always be greater that or equal to 10 µm and the coating thickness after the second process should always be greater than or equal to 110 µm. A 100% inspection is conducted after each process and it is assumed to be error prone. In this case, the observed value of the quality characteristic is not the same as the true value due to the presence of inspection error. To reduce the effect of the inspection error cut off points are considered as the criteria of classification instead of lower specification limits. If the inspector observes that the thickness of the zinc coating is less than its cut off value W_1, then the item is reworked at a cost $c_1x_1 + r$. Whereas, if he observes the thickness of the paint coating to be less than its cut off value then the item will be sold at a secondary market with a price a_2. Finally, if he observes both coating thicknesses to be greater than their cut off values then this item is sold at a regular market with a price a_1. The problem is to find the optimal process means and cut off values that maximize the expected profit. The given information are summarized as follows:

$L_1 = 10$ µm, $L = 110$ µm, $a_1 = 35.64$ SR/item, $a_2 = 32.67$ SR/item, $c_1 = 0.015$ SR/µm, $c_2 = 0.0088$ SR/µm, $r = 1.2$ SR/item, $\sigma_1 = 5.13$, $\sigma_2 = 11.14$, $\sigma_{\varepsilon 1} = 0.6$, $\sigma_{\varepsilon 2} = 0.8$, $b = 3$ SR/item.

To solve this problem, we optimized the model developed in Section 6.3 using Microsoft Excel Solver tool. This tool uses the Generalized Reduced Gradient (GRG2) nonlinear optimization code developed by Leon Lasdon, University of Texas at Austin, and Allan Waren, Cleveland State University. Table 6.1 shows the values of the optimal process means, cut off points and expected profit for the targeting problem using 100% inspection with inspection error.

Table 6.1 Optimal process means and cut off values for the case study

μ_1^*	μ_2^*	W_1	W	**Expected profit**
26.164	104.957	8.25	106.2	31.277

Comparing the results in table 6.1 with the results in the case of error free inspection in table 5.1 we notice that the mean of the first process is increased while the mean of the second process is decreased. The reason for this is that when the designer knows that there will be errors in the inspection he is going to be more conservative and in this case he is going to set the mean of the first process high to avoid scraps, while in the second process, nonconforming items will be sold at a secondary market therefore the designer is less conservative in this case. Moreover, cut off points are set less than the specification limits since the penalty associated with the quality loss is not very high.

6.5 Sensitivity Analysis

In this section, sensitivity analysis is conducted to study the effect of error variability on the optimal means, cut off points and expected profit.

6.5.1 The effect of $\sigma_{\varepsilon 1}$ on the optimal means, cut off points and expected profit

In this section, the effect of $\sigma_{\varepsilon 1}$ on the optimal means, cut off points and expected profit is studied. Increasing the variance of the ε_1 means that the variation between inspection errors becomes higher. In this case, the designer will be more conservative regarding the first process so he is going to set the mean higher. Table 6.2 shows the effect of the variations of $\sigma_{\varepsilon 1}$ on the expected profit, optimal means and cut off values.

Table 6.2 Optimal means, cut off points and expected profit for different $\sigma_{\varepsilon 1}$

$\sigma_{\varepsilon 1}$	μ_1	μ_2	W_1	W	EP
0.1	26.05589	105.0656	8.25	106.2	31.27794
0.2	26.06483	105.0562	8.25	106.2	31.27787
0.3	26.08018	105.0407	8.25	106.2	31.27777
0.4	26.10234	105.0191	8.25	106.2	31.27762
0.5	26.13014	104.9913	8.25	106.2	31.27744
0.6	26.1641	104.9573	8.25	106.2	31.27785
0.7	26.20403	104.9175	8.25	106.2	31.27694
0.8	26.25004	104.8714	8.25	106.2	31.27662
0.9	26.302	104.8195	8.25	106.2	31.27627
1	26.35987	104.7616	8.25	106.2	31.27588

Figure (6-3) shows that as the variability in ε_1 increases the mean of the first process is also increases. This is because the high variability in the error will make the designer more conservative as a result he will set the means higher to avoid scraps.

While in figure (6-4), it can be seen that the mean of the second process decreases as $\sigma_{\varepsilon 1}$ increases, this is because when the designer set the mean of the first process high the material cost will increase as a result the designer tend to reduce the material cost in the second process by setting the mean of the second process low. Figure (6-4) shows that, in general, the expected profit decreases as $\sigma_{\varepsilon 1}$ increases.

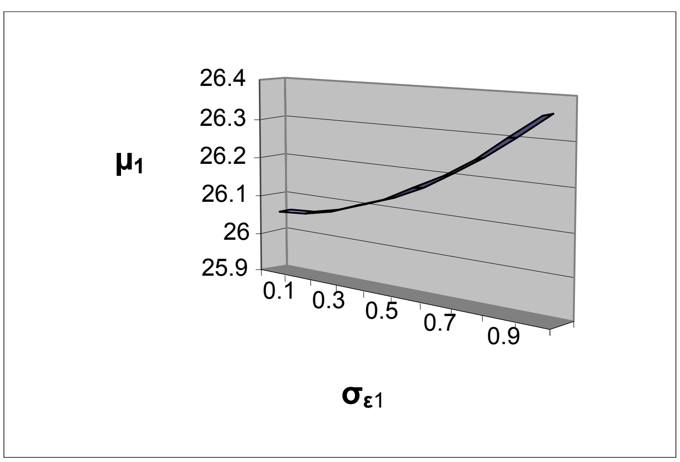

Figure (6-3) μ_1 vs $\sigma_{\varepsilon 1}$.

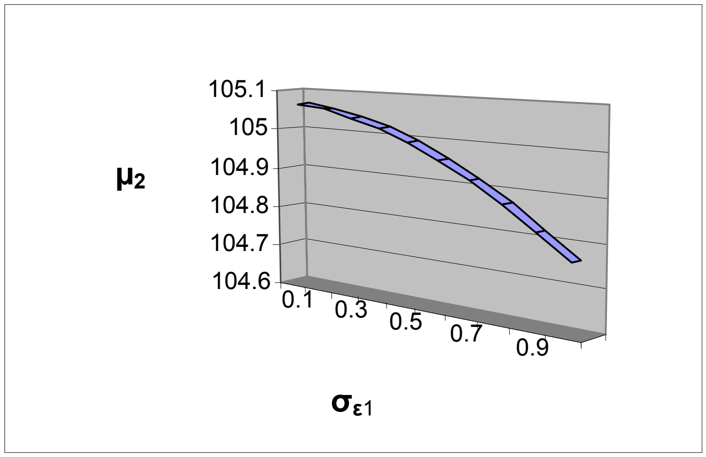

Figure (6-4) μ_2 vs $\sigma_{\varepsilon 1}$.

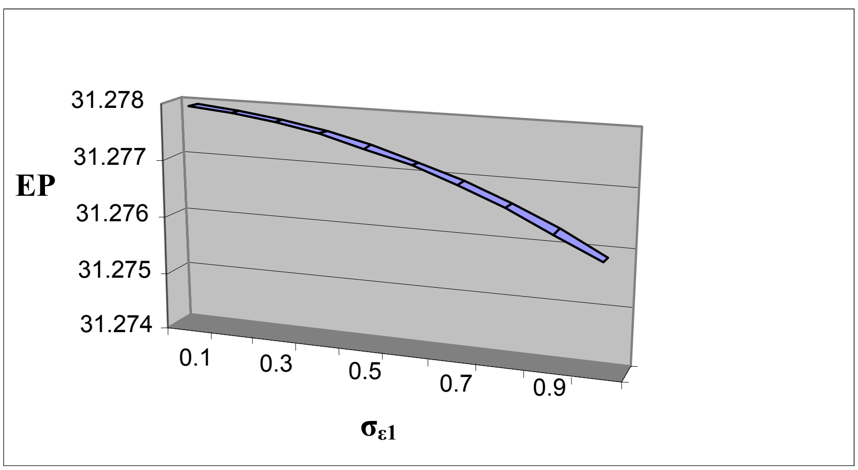

Figure (6-5) Expected profit vs $\sigma_{\varepsilon 1}$.

6.5.2 The effect of $\sigma_{\varepsilon 2}$ on the optimal means, cut off points and expected profit

In this section, the effect of $\sigma_{\varepsilon 2}$ on the optimal means, cut off points and expected profit is studied. Increasing the variance of the ε_1 means that the variation between inspection errors becomes higher. In this case, the designer will be more conservative regarding the second process so he is going to set the mean higher. Table 6.3 shows the effect of the variations of $\sigma_{\varepsilon 2}$ on the expected profit, optimal means and cut off values.

Table 6.3 Optimal means, cut off points and expected profit for different $\sigma_{\varepsilon 2}$

$\sigma_{\varepsilon 2}$	μ_1	μ_2	W_1	W	EP
0.1	26.16424	104.9067	8.25	106.2	31.27776
0.2	26.16424	104.9067	8.25	106.2	31.27773
0.3	26.16624	104.9088	8.25	106.2	31.27769
0.4	26.16727	104.9125	8.25	106.2	31.27763
0.5	26.16388	104.926	8.25	106.2	31.27755
0.6	26.16689	104.9289	8.25	106.2	31.27745
0.7	26.16385	104.9453	8.25	106.2	31.27734
0.8	26.16783	104.9491	8.25	106.2	31.27721
0.9	26.16389	104.9709	8.25	106.2	31.27706
1	26.16369	104.986	8.25	106.2	31.27689

Figure (6-6) shows that as $\sigma_{\varepsilon 2}$ increases the mean of the second process is also increases. This is because the high variability in the error will make the designer more conservative regarding the second process as a result he will set the mean higher to avoid secondary market products. Figure (6-7) shows that, in general, the expected profit decreases as $\sigma_{\varepsilon 2}$ increases.

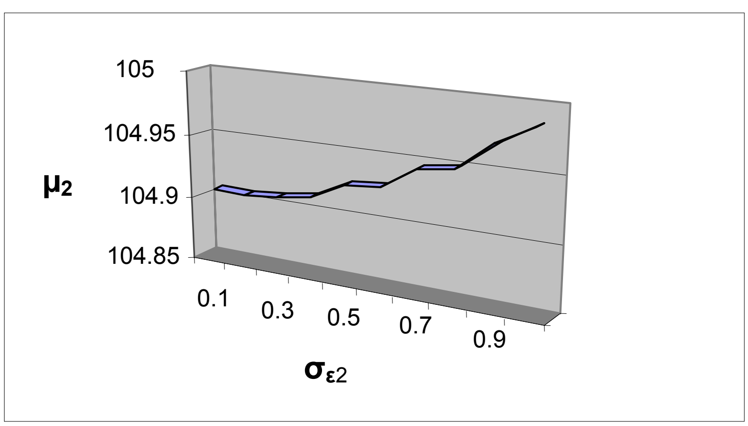

Figure (6-6) μ_2 vs $\sigma_{\varepsilon 2}$.

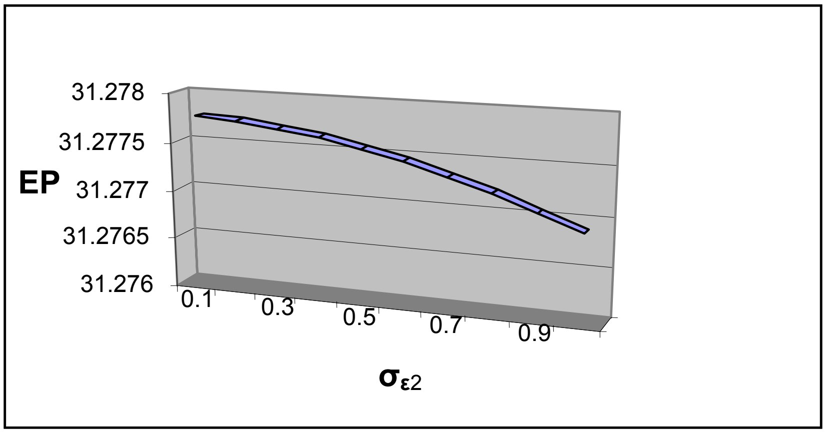

Figure (6-7) Expected profit vs $\sigma_{\varepsilon 2}$.

6.6 Conclusion

In this chapter, a model has been developed for two processes producing a single product with two quality characteristics. The first quality characteristic is determined by the first process and the second quality characteristic is determined by both processes. Specification limits are set on both quality characteristics. A 100% inspection has been used as a mean for product quality control and it is assumed to be error prone. A real case has been considered to test the model developed in this chapter followed by the sensitivity analysis which has been conducted to investigate the effect of variability in the inspection error on model results. It has been shown that as $\sigma_{\varepsilon 1}$ increases the mean of the first process is also increases while the mean of the second process decreases. If $\sigma_{\varepsilon 2}$ increases then the mean of the second process is also increases while the mean of the first process will not be affected. Moreover, the expected profit decreases as the error variability increases.

CHAPTER 7

CONCLUSION

7.1 Summary

The problem considered in this thesis is the determination of the optimal targets for two processes in series producing a product with two quality characteristics. The first quality characteristic is determined by the first process and the second quality characteristic is determined by both processes. Several variations of the problem are addressed. The variations in the problem are determined based on the product quality control approach and whether inspection is error free or not. The formulation of this problem is an extension of the process targeting problem in the literature. It considers quality characteristics that are affected by two processes instead of one process. The major contributions of this thesis are:

- Four different process targeting models for two processes in series producing a single product with two dependent quality characteristics are developed.
- The first model is developed for the above stated problem where product is controlled by sampling plans and inspection is error free.
- The second model is developed for the stated problem where product is controlled by sampling plans and inspection is error prone.
- The third model is developed for the stated problem where product is controlled by 100% inspection and inspection is error free.
- The fourth model is developed for the stated problem where product is controlled by 100% inspection and inspection is error prone.
- A real example from local industry of painting cylinders for manufacturing fire extinguishers is solved using the four process targeting models.
- Sensitivity analysis for all process targeting models has been conducted. It has been shown that the models are very sensitive to the variance of the thickness resulting from the two processes, selling prices and rework cost and insensitive to the inspection cost.
- The effect of inspection has been studied for models where inspection is error present. It has been shown that the models are very sensitive to type I error (rejecting conforming items) and insensitive to type II error (accepting nonconforming items).

7.2 Models Comparison

Table 7.1 summarizes the results obtained by solving the case study using the four targeting models developed in this thesis. On one hand, if inspection is assumed to be perfect, then model 1 provides better results in terms of the total profit. This implies if the company uses sampling plan for controlling the product's quality instead of 100% inspection, it will save SR 1.272 per item, which is a substantial amount of saving per lot. As an example, if the company's yearly production is 250,000 items, then the total amount of savings will be SR 318,000 per year.

Table 7.1 Comparison between the four targeting models.

Model Number	Inspection Type	Optimal mean for the first process	Optimal mean for the second process	Expected profit per item
1	Error free sampling plan	25.391	113.203	34.237
2	Error prone sampling plan	28.283	112.151	33.916
3	Error free 100% inspection	19.926	114.132	32.965
4	Error prone 100% inspection	26.164	104.957	31.277

The company now is using the sampling plan inspection, corresponds to model 1 in table 7.1, for product quality control. However, it has been shown that the used sampling plan does not provide the maximum total profit. Table 3.1 shows that if the company changes the current sampling plan to the sampling plan ($n_1=n_2=10$, $d_1=d_2=3$) then it will save about SR 0.14 per item. Moreover, table 5.1 indicates that reducing the variances of both processes could result in a considerable amount of saving, for example, reducing the variances of the two processes by 30% will increase the profit by 1.1022 SR per item.

On the other hand, if we assume that inspection is error prone then, model 2 and 4 can be compared. Table 7.1 shows that if the company uses the model with error prone sampling plan instead of 100% inspection then it will save about SR 2.639 per item. Moreover, reducing the inspection errors will also increase the total profit, for example table 4.5 helps in conducting a cost/benefit analysis which aids the company to calculate the amount of saving corresponding to error reduction. Out of this analysis, the main recommendations for the company to maintain high profit are: adopt the sampling plan inspection instead of 100% inspection for the product quality control in order to maintain high profit. The company can even increase the profit by improving the sampling plan. This can be done either by optimizing the sampling plans or reducing the inspection error which can be achieved by training the inspectors.

The company can implement the model in chapter 4 if the inspection errors are very small. However, if the errors are not very small, an experiment must be designed to estimate type I and type II errors. Then, the model should be run to determine optimal mean of both processes. The model is expected to improve the profitability of the company.

7.3 Further Research

The work done in this thesis can be extended in several directions. The following points list some of the possible extensions:

- Extend the models to the situation where the two processes deteriorate over time.
- Extend the models to the situation where constraints for meeting certain production demands exist.
- Generalize the models to the situation where the distribution of quality characteristics has unknown variances.

- Generalize the models to multi-stage processes in series.
- Extend the models by integrating production decisions.
- Extend the models to the situation where sampling plan parameters are unknown, and are determined together with process parameters.
- Extend the models to the situation where different cost functions are considered, such as Taguchi quadratic loss function and nonlinear cost function.
- Extend the models to the situation where the final product has three quality characteristics. The first quality characteristic is affected by the first process; the second quality characteristic is affected by both processes and the third quality characteristic (attribute) is affected by the second process.
- Extend the models to the situation where processes are assumed to be statistically dependent.