A New Correlation for Two-Phase Flow Through Chokes

by

Asaad Ibrahim Al-Towailib

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
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This thesis, written by Asaad Ibrahim Al-Towailib under the direction of his Thesis Advisor and approved by his Thesis Committee, has been presented to and accepted by the Dean of the College of Graduate Studies, in partial fulfilment of the requirments for the degree of MASTER OF SCIENCE in petroleum engineering.

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DEDICATION

To My Loving Family

ACKNOWLEDGEMENT

Praise and gratitude be to the Almighty, the creator and sustainer of the Universe, and peace be upon Prophet Muhammad, his progeny and faithful companions.

Acknowldgement due to King Fahd University of Petroleum and Minerals for support of this research. I wish to express my appreciation to Professor Muhammad Al-Marhoun as my major advisor. I also wish to thank the other members of my Thesis Committee Dr. Khalid Al-Fossail and Dr. Mohammed Aggour. I am also greatful for Abdulazeem Al-Towailib, Haider Al-Awami, and Naelah Al-Mousli of Saudi ARAMCO for their encouragment and support.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

خلاصة الرسالة

إسم الطالب الكامل : اسعد ابراهيم علوي الطويلب

عنـوان الدراسـة : علاقه رياضية جديدة لتدفق الغاز والزيت عبر صمامات التحكم

التخصـص : هندسة البترول

تاريخ الشهـادة : مارس ١٩٩٢ م

صمامات التحكم في التدفق واحدة من اهم وسائل التحكم ان لم تكن الأهم في إنتاجية أبار الزيت . إختيار المقاس المناسب لفتحة صمام التحكم في التدفق هو الخطوة الاولى في صناعة هذه الصمامات. طورت في هذا البحث علاقة رياضية عملية تربط بين مقاس فتحة الصمام وعوامل اخرى. هذه العلاقة تغطي مجالاً واسعاً من معدلات التدفق ومقاسات كثيرة من فتحات الصمام، وقد استنتجت باستخدام ٣٥٥٤ اختبارانتاجية لابار من عشرة حقول في الشرق الاوسط وتنطبق على حالات التدفق الحرج (فوق الصوتي) مما يجعلها مفيدة في وضع التصاميم الفنية لصمامات التحكم في التدفق، وبتطبيق عمليات الإحصاء الرياضية لمجموعة المعلومات المستخدمة، برهنت المعادلة الجديدة على افضليتها مقارنة بكل المعادلات المنشورة في هذا الموضوع.

درجة الماجستير في العلوم

جامعة الملك فهد للبترول والمعادن

الظهران، المملكة العربية السعودية

مارس ١٩٩٢ م

THESIS ABSTRACT

STUDENT NAME : Assad I. Al-Towailib

TITLE OF STUDY : A New Correlation for Two-phase Flow Through Chokes

MAJOR FIELD : Petroleum Engineering

DATE OF DEGREE : March 1992

Chokes are one if not the most important flow controllers in the oil producing wells. Selecting the optimum choke size is the first step in choke design. An empirical correlation that relates the choke size to other parameters was developed. The correlation covers wide ranges of flow rates and choke sizes. It was based on 3554 production test data from ten fields in the Middle East. The correlation applies for critical flow conditions which makes it useful for choke design purposes. Statistical error analysis shows that for the data used in this study, the new correlation outperforms all the published correlations in literature.

MASTER OF SCIENCE DEGREE

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

Dhahran, Saudi Arabia

March 1992

CHAPTER 1

CHAPTER 1

INTRODUCTION

All flowing wells utilize some kind of surface restriction, such as choke, in order to regulate the flow rate. Chokes serve many useful functions:

-Maintaining desirable flow rate

-Maintaining sufficient back pressure to prevent sand entry

-Preventing gas or water coning through regulation of rate.

Selecting the proper bean size is one of the most important tasks in the procedure of choke design. Simply specifying a valve size to match an existing pipeline size leaves much to chance and will likely create an impractical situation in terms of initial investment and adequacy of control.

The importance of correct sizing of control valves cannot be over-emphasized. The most expensive, sensitive, and accurate controller is of little value if the control valve cannot correct the flow properly to maintain a desired control point. Oversized valves provide poor control and can lead to system instability, excessive wear, and cycling of internal trim parts. Undersized valves generally cannot pass required flows and thus starve the process.

From a purely economic point of view, an undersized valve cannot do the job and must be replaced. On the other hand, too large a control valve costs more initially and may have higher maintenance costs because it likely will be required to operate closer to the seat, causing the seating surfaces to wear rapidly, especially when the process fluid is erosive or can flash or cavitate.

In this study, the relation of choke size to other pertinent parameters will be studied. Reported data from different Middle East fields will be used to develop an empirical correlation for multiphase flow through chokes.

1.1 Background

Selecting a control valve of the proper size for a given application is of paramount importance if the best possible performance is to be expected. Several correlations for single and multi phase flow across chokes have been developed to make the selection of the proper valve size handy and convenient.

1.2 Multiphase Flow

Multiphase flow is inherent to the daily production process of gas or oil. Multiphase flow is generaliy experienced when:

1. Liquid entrainment is carried by a gaseous flow.

2. Liquid at or near its bubble point vaporizes in the tubing causing a continuous two phase flow through the choke.

In fact, multiphase flow appears in all situations before the flow arrives at the system of separation of different phases. In particular, the presence of gas is responsible for many misunderstandings while analyzing the different situations encountered in the flow.

The types of flow encountered, for example, in vertical multiphase flow are different to those encountered in horizontal multiphase flow due to the tendency of segregation of the liquid phase. The gas slippage makes it difficult to study multiphase flow without the consideration of the type of flow which is always changing along the conductors.

Some fluid properties (as viscosity, for instance), are impossible to be determined in virtue of the presence of free gas. Simultaneous flow of oil, water, and gas imply three variables for each physical property and, at least, one more for the combination of them. Viscosity, for instance, yields the following variables: Oil viscosity, Water viscosity, Gas viscosity and viscosity of the mixture. On the other hand, each fluid has its own absolute and relative volume, temperature, pressure, etc. The percentage of each component in the mixture changes continuously, since gas is always coming out of solution, increasing the gas liquid ratio. All factors, as a consequence, are always changing, in each section of the conductors and, sometimes, even in the same section, the conditions of flow vary with time. The amount of free gas is responsible for different types of flow occuring in different localities along the flow line.

Viscous flow may occur in the liquid phase, while the gas phase is in the turbulent region. Slippage always results when different velocities between the phases are encountered and it is responsible for additional pressure loss.

In view of the difficulty of a complete mathematical analysis of such a complicated system, most of the studies are based on practical experiments in the laboratory or in the field.

Thus, given a multiphase flow problem, the solution is obtained by the use of suitable equations; but some or all variables are obtained from correlations based on field or laboratory experiments.

Many authors have studied multiphase flow and published the results of their investigations; thus, sufficient literature is available to show how much disagreement exists.

The basic difference in treating the problems is the consideration of the different types of flow occuring in a system. Some authors developed equations and correlations that can be applied for any flow condition. Others divided the flow in different regions in which a different type of flow is encountered, equations and correlations are developed for each particular situation. In the latter case it is also necessary to develop parameters that characterize the regions where each type of flow is encountered, and generally there are some regions that remain undetermined, creating difficulties in the choice of a suitable equation.

1.3 Thesis Objectives

The objective of this thesis is to develop a new empirical correlation for multiphase flow through chokes relating the choke size, flow rate, upstream and downstream tubing pressures, gas oil ratio and other pertinent variables.

Different statistical error analysis will be made to validate the correlation and compare it to the existing ones.

The final correlation will help the oil industry by providing better estimates of the required choke sizes for choke design purposes and will aid in the estimation of well flow rates where direct rate measurement is not feasible.

CHAPTER 2

CHAPTER 2

LITERATURE REVIEW

2.1 Tangren et al. Analysis (1)

Tangren, Dodge, and Seifert have done the first significant study on multi-phase flow in 1949. They developed an equation of state and an equation of motion for a gas-water mixture flowing through a de Laval nozzle (convergent - divergent nozzle), and assumed that the water-gas mixture behaves like a compressible fluid. The basic law of continuity, momentum, energy and ideal gas equations of state were applied in developing the equations for the mixture. The following assumptions were made:

1. The liquid is an incompressible fluid, and effects due to viscosity, surface tension, and vapor pressure are unimportant.

2. The gas is an ideal gas, with negligible effects of specific heat and is insoluble in the liquid.

3. The mixture is "homogeneous" in the sense that the bubbles of gas are so small and uniformly distributed that an arbitrarily

small samples contain the same mass ratio of gas to liquid as a whole mixture does.

4. The gas and liquid are always at the same temperature, and the flow is insulated (i.e., adiabatic).

5. The flow is one-dimensional, laminar, and expands or compresses slowly enough that inertial transients may be ignored.

6. A slowly varying pressure change or signal is transmitted through the mixture with a definite critical or "sonic" velocity, as distinguished from the anamolous dispersion and attenuation of high frequency pressure waves.

Tangren started with an equation of state for two-phase fluid, applied the Newton's equation of motion and utilized Bernouli equation for an incompressible fluid to come up with a dimensionless form of the velocity equation as follows:

$$\rho_l \frac{v^2}{2p_i} = -f_{li}\left(\ln \frac{p_{us}}{p_i}\right)\left(1 - \frac{p_{us}}{p_i}\right) \tag{2.1}$$

where:

ρ_l = Liquid density, slugs/cu ft

p_i = Pressure under initial conditions, psf

f_{li} = Initial volumetric liquid-mixture ratio

p_{us} = Upstream Tubing pressure, psf

v = Liquid velocity, ft/sec

Tangren, et al., formulated the nozzle flow parameters referred to throat conditions using a dimensionless form Thus the local Mach number ratio, velocity relative to throat, pressure ratio, volume ratio, density ratio, and area ratio were calculated and the performance of the nozzle was determined. A significant contribution of the Tangren approach was to show that when gas bubbles are added to an incompressible liquid, the mixture becomes compressible, and above the critical flow velocity the medium becomes incapable of transmitting upstream pressure changes against the flow, in other words, compressible flow is occuring at the critical velocity is occuring. As long as the liquid phase is continuous, the assumption that the velocities of both phases are equal is very reasonable. However, by fixing arbitarily a volume ratio of 2.0 there is no gurantee that the liquid phase will remain as the continuous phase. The opposite may occure. In practice, a liquid continuous phase is observed when the volumetric gas-liquid ratio is lower than one. For values above one, the gas becomes the continuous phase, and the preceding approach is not applicable.

2.2 Gilbert Approach (2)

Gilbert in 1954 has developed an empirical correlation using regularly reported individual well production data. He related tubing outlet pressure, gas-liquid ratio, gross liquid rate and bean size. By his empirical formula, approximate solutions for any one of the four variables when the other three are known, can be made. Gilbert used 268 data points from California's Ten Section Field to come up with his correlation. The choke sizes for his data range from 6 to 18 sixty fourth of an inch. It was assumed in his correlation that the actual mixture velocities through the bean exceed the speed of sound, for which condition the downstream, or flow line, pressure has no effect upon the tubing outlet pressure (i.e., pressure on the upstream side of the bean). Thus the formula applies only if the line pressure is less than 55 % of the tubing pressure. Field operators usually try to avoid operating below that range because fluctuations of the line pressure affect the well operation. Gilbert equation is as follows:

$$p_{us} = \frac{435\, R_p^{0.546}\, q_l}{S^{1.89}} \qquad (2.2)$$

where:

p_{us} = Flowing wellhead pressure, psia

R_p = Producing gas-liquid ratio, MSCF/STB

q_l = Gross liquid flow rate, STBD

S = Choke size, 64th of an in.

2.3 Ros Analysis (3,4,5)

Ros in 1960 presented a theoretical analysis on the mechanism of simultaneous flow of gas and liquid through a restriction at acoustic velocity. This analysis resulted in development of an equation relating mass flow of gas and liquid, restriction size and upstream pressure. The equation was based on well known energy balance equation:

$$\int\{g\,dh - V\,dp - d(\frac{1}{2}v^2) - dW\} = 0 \qquad (2.3)$$

where:

$g\,dh$ = Potential energy term

$V\,dP$ = Expansion energy term

$d(\frac{1}{2}v^2)$ = Kinetic energy term

dW = Irreversible energy term

and the following assumptions:

1. Gas is the continuous phase with the liquid phase dispersed in the gas phase.

2. The throat velocity is uniform; that is, the velocities of the phases are the same.

3. The gas expands polytropically.

4. The potential energy term $\int g\,dh$, and part of the irreversible energy term $\int dW$, namely surface energy and wall friction, can be ignored.

5. The remaining part of the irreversible energy term (slip losses) can be determined.

Ros also showed that the critical velocity corresponds to an approximate ratio of downstream/upstream pressures of .55 for a constant specific heat value, k. He plotted the critical ratio as a function of gas oil ratio as shown in figure 1.

2.3.1 Poetmann and Beck Adaptation (6)

To make Ros equation and analysis available to oil field personnel, Poetmann and Beck in 1963 converted the equation to oil field units and reduced it to a graphical form (Figure 2). In the construction of their chart, Borden and Rzasa (19) correlation was used for oil gravities of 20, 30, and 40 API. Figure 2 shows their developed chart for the 30 API gravity oil. Their final expression is as follows:

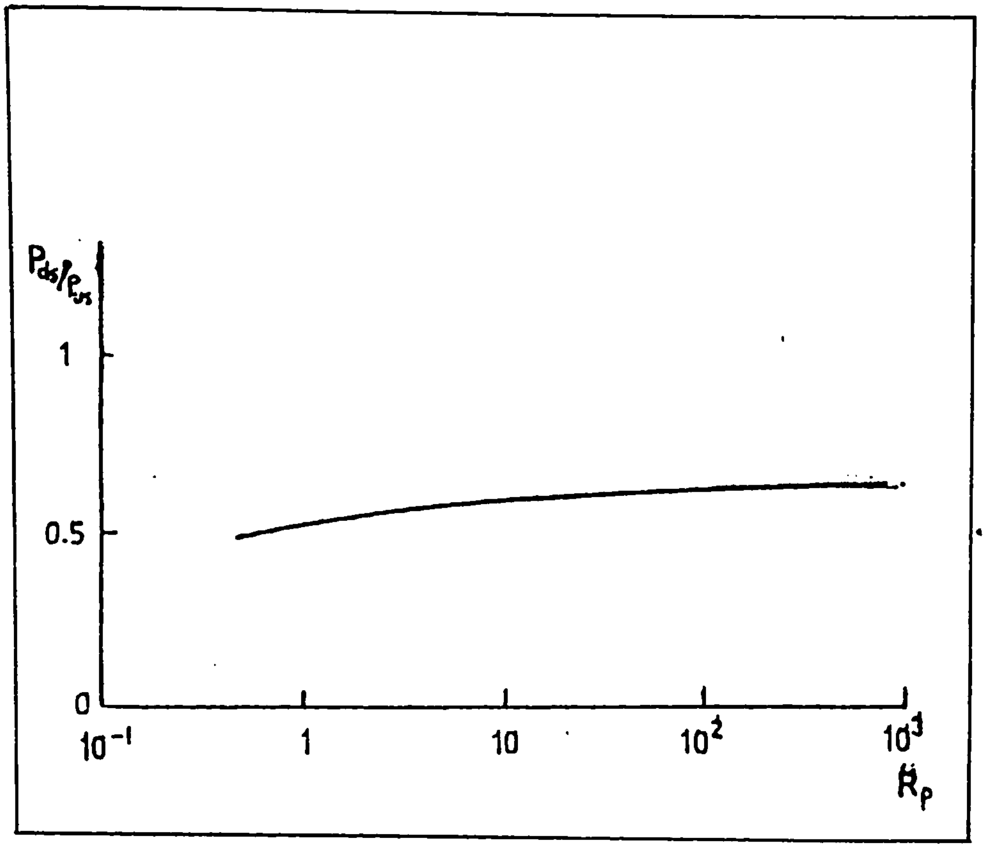

Figure 1: critical pressure ratio against volumetric gas liquid ratio for a constant n value (after ROS).

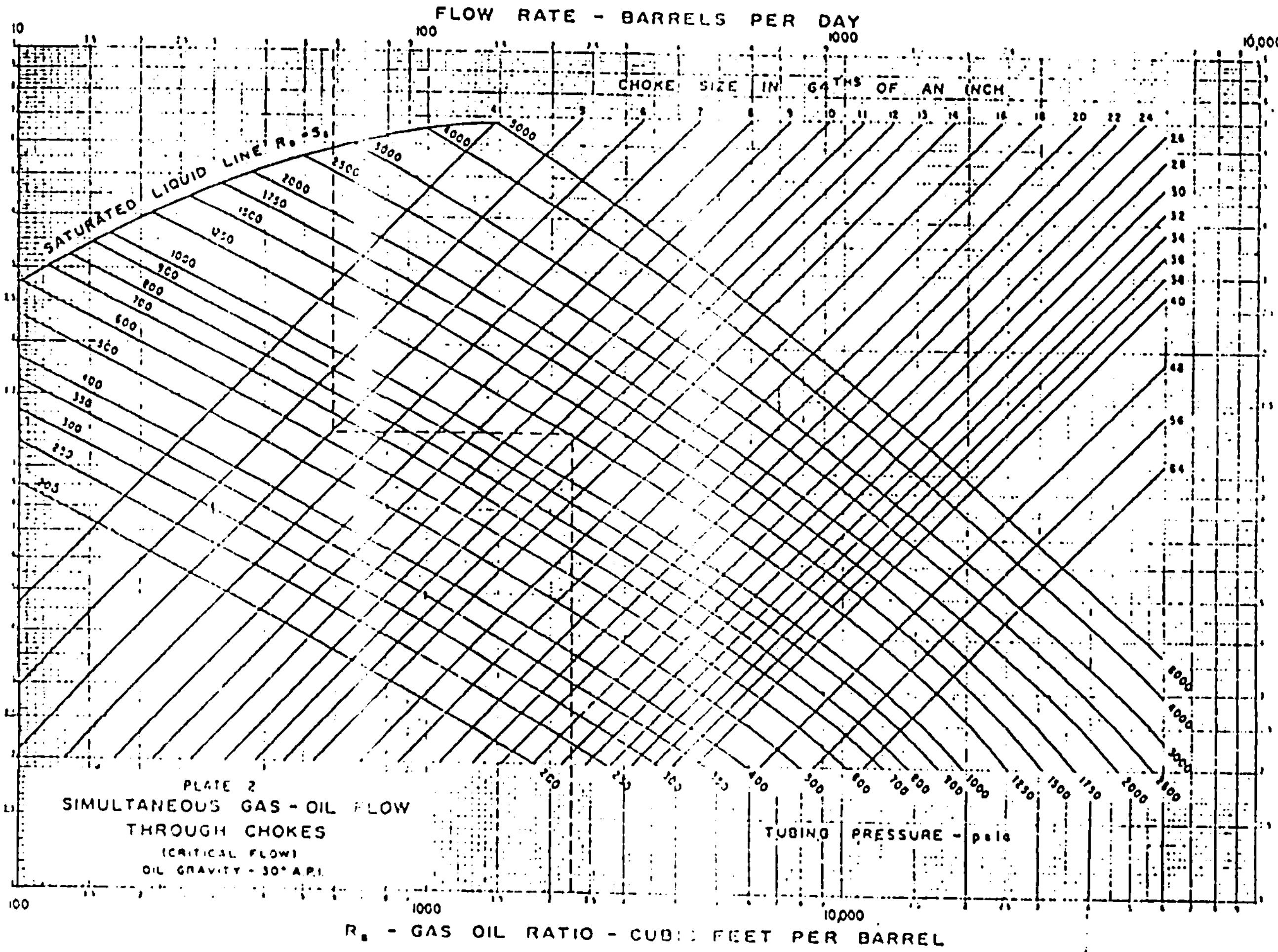

Figure 2: Choke performance chart (After Poettmann and Beck)

$$q_o = \frac{86{,}400 C_D A_c}{5.61\rho_{Ls} + .0765\gamma_g R_p} \sqrt{\left(\frac{9273.6 P_{ws}}{V_l(1 + 0.5x_l)}\right)} \left[\frac{.4513\sqrt{R + 0.7660}}{R + 0.5663}\right] \tag{2.4}$$

where

$$R = \frac{B_g(R_p - R_s)}{B_o} \tag{2.5}$$

$$x_l = 1/(1 + R\frac{\rho_g}{\rho_l}) \tag{2.6}$$

$$V_l = x_l/\rho_l \tag{2.7}$$

and,

q_o = Oil flow rate, STBD

C_D = Discharge coefficient

A_c = Cross sectional choke area, sq. ft

T = Tubing temperature (R)

z = Compressibility of gas at tubing pressure and 85 F

ρ_{Ls} = Density of crude in lb/cu ft at 60 F. and 14.7 psia

γ_g = Relative gas density at 60 F and 14.7 psia (Air = 1)

R_p = Producing gas-oil ratio in SCF/STB

p_{ws} = Tubing wellhead pressure , psia

V_l = Volume of liquid per unit mass of the mixture, cu ft/lb

x_l = Mass fraction of liquid in the mixture

R_s = Solution gas oil ratio at tubing pressure and 85 F

B_o = Formation volume factor of crude at tubing pressure and 85 F

ρ_l = Density of crude at tubing pressure and 85 F lb/cu ft

ρ_g = Density of gas at tubing pressure and 85 F, lb/cu ft

Poetmann and Beck concluded that variations in gas gravity on ultimate results are very small and could be neglected. Their charts are not valid if there is water production with oil, they reported. On their paper, the equation was tested with 108 field points resulting in 6.5 average percent relative error and 26.4% standard deviation. The range of the data tested was the following:

Oil gravity	21 - 56.3	API
GOR	175 - 18600	SCF/STB
Presuure	168 - 4374	psi
Choke size	4.2 - 28	1/64th inch
Production rate	10.5 - 1299	STBD

2.4 Sheldon and Schuder Approach (7)

Many valve manufacturers size valves by simply adding the standard sizing coefficients of gas and liquid, C_vs , which are determined for 100% liquid and 100% gas. The C_v is defined as follows:

$$C_{vl} = q_l \frac{\gamma_l}{\Delta p} \qquad \textit{for liquid} \qquad (2.8)$$

$$C_{vg} = q_l \frac{R_p}{p_{us}} \frac{\gamma_g T}{520} \qquad \textit{for gas} \qquad (2.9)$$

For a subcritical flow

$$C_{vg} = \frac{q_g}{(2.32\,\Delta p)^{.4425} \times \dfrac{\gamma_g T}{520\, p_{us}^{1.5575}}} \qquad (2.10)$$

where

q_l	=	Liquid flow rate in gallons per minute
γ_l	=	Liquid relative density (Water=1)
q_g	=	Gas flow rate, SCF/ hour
γ_g	=	Relative gas density (Air=1)
Δp	=	Pressure differential across choke, psi
p_{us}	=	Upstream wellhead pressure, psia
T	=	Temperature at choke, R
R_p	=	Producing gas oil ratio, SCF/STB

Although this method satisfies the energy equation; However, field experience has indicated that the valve chosen in this manner was undersized because of the losses introduced when the actual gas and liquid velocities tend to equalize at the orifice. A number of sizing rules have been proposed to minimize the error. In 1965, Sheldon and Schuder developed an application guide that minimizes the error by the use of correction curves based on liquid-gas volume ratio, pressure-drop ratio, and area ratio. Their study resulted in the following relationship:

$$C_{vr} = (C_{vl} + C_{vg})(1 + F_m M_a M_p) \qquad (2.11)$$

where:

C_{vr} = Corrected valve coefficient

C_{vl} = Valve coefficient of liquid in two phase flow

C_{vg} = Valve coefficient of gas in two phase flow

F_m = Mixture correction factor

M_a = Area ratio multiplier

M_p = Pressure drop ratio multiplier

Values for M_a, M_p and F_m correction factors can be obtained from figures 3-5. Sheldon and Schuder plotted the ratio of the corrected valve coefficient over the sum of the gas and liquid coefficients versus the gas volume ratio as shown on figure 6. The figure clearly indicates that as

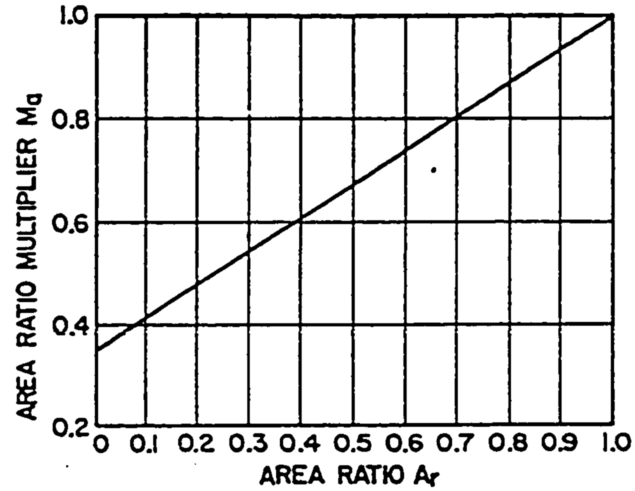

Figure 3: Area ratio multiplier
(After Sheldon and Schuder)

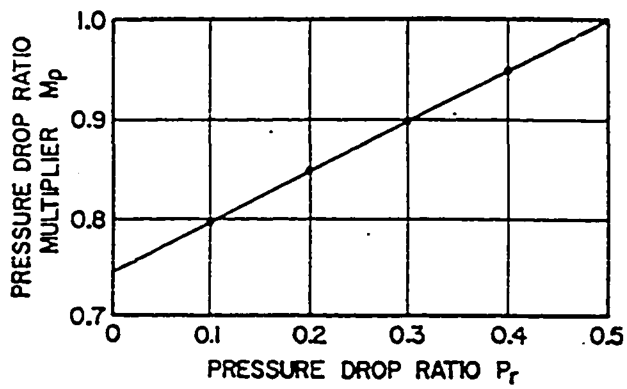

Figure 4: Pressure drop ratio multiplier
(After Sheldon and Schuder)

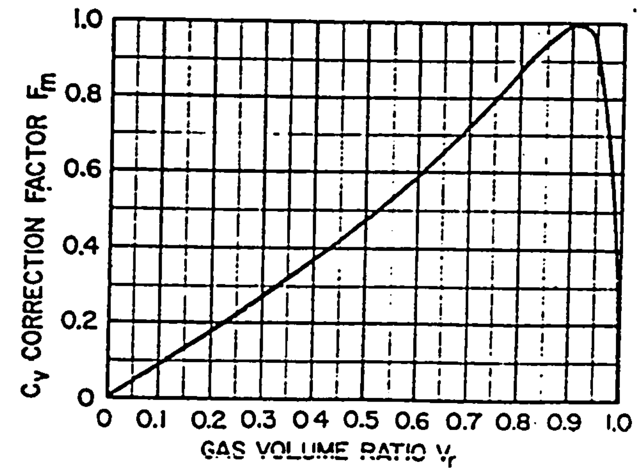

Figure 5: Mixture correction factor
(After Sheldon and Schuder)

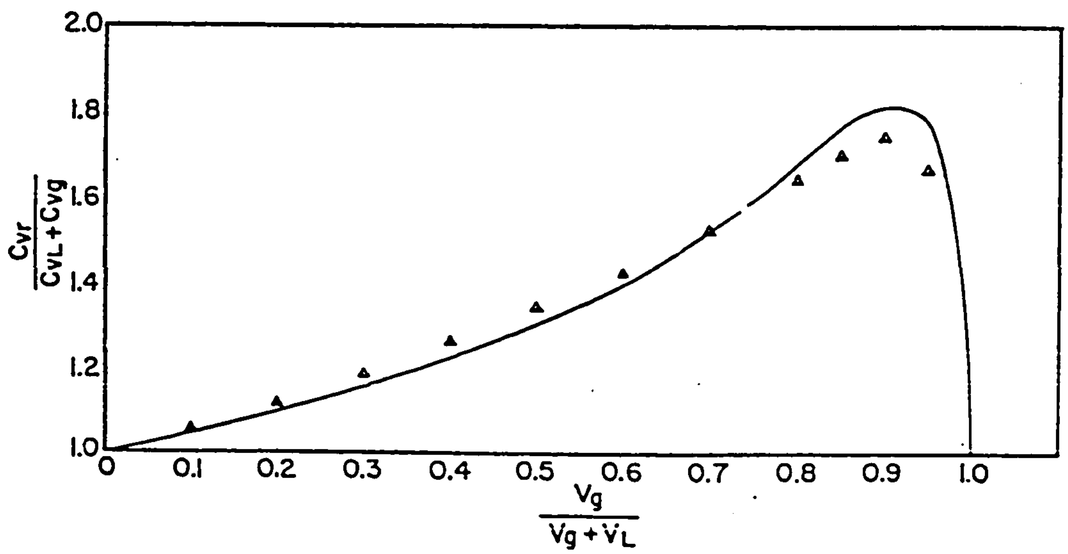

Figure 6: Correction factor for different volumetric gas ratio
(After Sheldon and Schuder)

the gas ratio increases, the need for the correction becomes more important.

It was found out that in the cases where the pressure drop exceeds the critical value, the C_v 's should be calculated using the gas critical pressure drop for both the gas and the liquid applying a pressure-ratio correction for $P_r = .5$. The effect of the gas/liquid density ratio was found to be negligible over the tested range and the fact was recognized that the correction (F_m) would decrease with density ratio approaching one. Once C_w which is the equivalent valve coefficient for 100% liquid flow is known, the corresponding choke size could be easily determined from tables provided by the valve manufacturers.

2.5 Omana Correlation (8)

In 1968, Omana used controlled field data taken at the facilities of Union Company of California's Tiger Lagon Field in Louisiana to check the existing correlations and develop his own. Field experiments were conducted with water and natural gas. He used dimensional analysis to obtain the following correlation

$$N_{ql} = .263 \ N_{\rho}^{-3.49} \ N_{pl}^{3.19} \ Q_d^{.657} \ N_d^{1.8} \qquad (2.12)$$

where the dimensionless parameters are defined as follows:

$$N_{ql} = 1.84\, q_l \left(\frac{\rho_l}{\sigma}\right)^{1.25} \tag{2.13}$$

$$N_p = \frac{\rho_g}{\rho_l} \tag{2.14}$$

$$N_{pl} = 1.74 \times 10^{-2} \frac{P}{(\rho_l \sigma)^{.5}} \tag{2.15}$$

$$Q_d = \frac{1}{1 + R_s} \tag{2.16}$$

$$N_d = .1574\, S \left(\frac{\rho_l}{\sigma}\right)^{.5} \tag{2.17}$$

and

σ = Liquid surface tension, dynes/cm

ρ_l = Liquid density, lb/ cu ft

Although his correlation gave good results with the data obtained in his experiment, his correlation is not widely accepted today for the following reasons:

1. Limitation of choke size (4 to 14/64 in.)

2. Limitation in flow rate (800 STBD maximum)

3. Limitation in pressure (400 to 1000 psig)

4. Use of water instead of oil or water-oil mixture in the field experiments.

2.6 Fortunati Analysis (9)

Fortunati in 1972 developed a new correlation for critical, subcritical multiphase flow and the boundary between these two regimes. He derived the following expressions:

$$p_{ds} = (q_l/A_c)\sqrt{(R_{si} - R_s)(\rho_o + \rho_g R_{si})\, P_i z T/T_i} \tag{2.18}$$

Under critical flow conditions, the above equation gives the possibility of knowing the oil flow rate passing through a choke with a cross-sectional area of A_c when the choke downstream pressure is p_{ds}.

For sub-critical flow, Fortunati based his correlation on curves that were previously published by Guzov and Medviediev. These curves give the velocity of the mixture in the chokes, for upstream / downstream pressure ratios, $\frac{p_{ds}}{p_{us}}$, from the critical value up to 1, and for different values of the parameter f_g, gas flow rate fraction of the mixture rate (Figure 7). Fortunati assumed no change in temperature at the choke and used liquid rate as a whole in the case of presence of some water cut. Liquid rate could be simply written as follows:

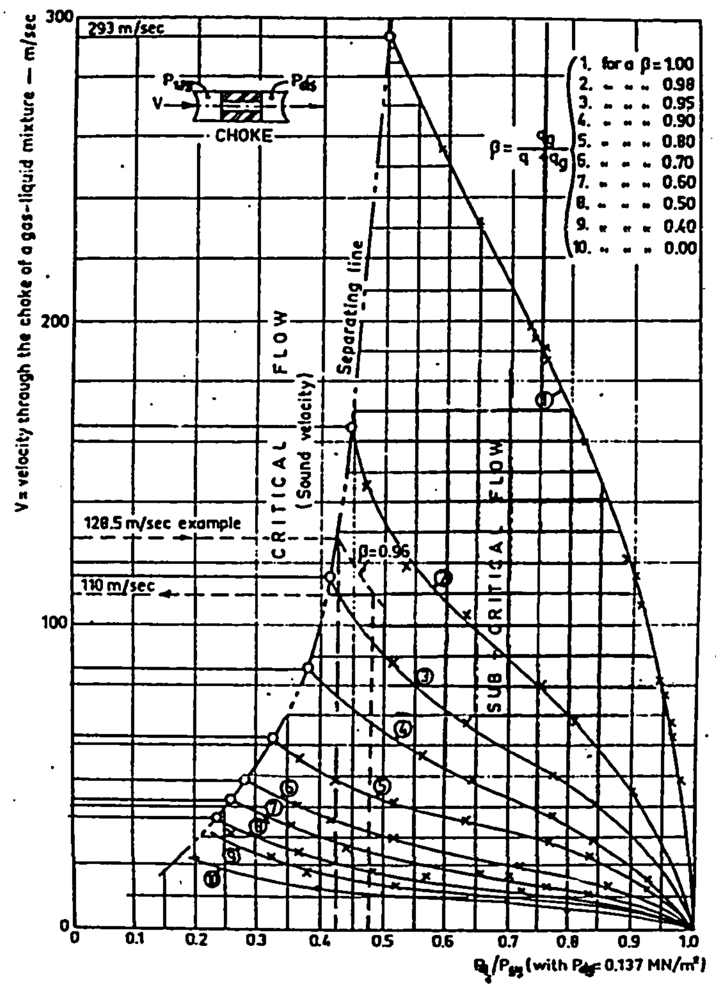

Figure 7: Velocity of gas-oil mixtures
(After Fortunati)

$$q_l = \frac{A_c(1 - f_g)}{B_o} C_v v_m (\sqrt{p_{ds}/p_{ds}^1}) \tag{2.19}$$

where:

A_c	=	choke cross-section area , m^2
C_v	=	Cumulative discharge coefficient
v_m	=	Mixture velocity read from figure 7, m/sec
f_g	=	Gas flow rate fraction of the mixture rate, $\frac{q_g}{(q_g + q_l)}$
q_l	=	Liquid flow rate, $m^3/$sec
p_{ds}^1	=	.137 $\times 10^6$ N/ m^2, choke downstream pressure used for plotting the experimental curves.
p_{ds}	=	Actual choke downstream pressure, 10^6 N/ m^2

The discharge coefficient, C_v, based on 250 cases was calculated to be 1.020 to 1.035. However, for other 150 cases, C_v was found to be 1.03 in agreement with Ros discharge coefficient. Fortunati commented that the discharge coefficient is a statistical consideration comparing the total daily rates to the oil measured in stock tanks. The fluid properties in his correlation were estimated at the downstream pressure. He stated that the model is valid as long as the downstream tubing pressure exceedes 1.5 atmospheres where the following two conditions are satisfied:

- Velocity through choke is greater than 10 m/sec

- Froude number, F_r, is greater than 600.

2.7 Ashford and Peirce Study (10)

Ashford and Peirce in 1975 developed a mathematical model relating dynamic orifice behavior in both critical and subcritical flow regimes. Orifice pressure drops and capacities were related to pertinent fluid properties and choke dimensions. Graphical correlations were also presented to predict the ultimate (critical) capacity of an orifice for any given set of dynamic conditions. To verify their model, a field test was designed and carried out in a flowing oil well. Both orifice pressure drops and fluid flow rates were measured in the well and the information was compared with analogous data predicted by the model. Their correlation was mainly intended for downhole safety valves. The correlation covered three phase flow and was basically an extension of Ros equation eliminating some of the assumptions made by Ros. The final form of the equation is as follows:

$$q_o = 3.51\, C_D\, S^2\, \alpha\, \beta \tag{2.20}$$

where

$$\alpha = (B_o + F_{wo})^{-1/2} \tag{2.21}$$

and

$$\beta = \frac{[(\frac{k}{k-1})Tz(R_p - R_s)(1 - \varepsilon^{\frac{k-1}{k}}) + 198.6p(1 - \varepsilon)]}{[198.6 + \frac{Tz}{p}(R_p - R_s)\varepsilon^{-1/k}]} \times \frac{(\gamma_o + 0.000217\gamma_g R_s + F_{wo}\gamma_w)^{1/2}}{(\gamma_o + 0.000217\gamma_g R_p + F_{wo}\gamma_w)} \qquad (2.22)$$

where

k = Specific heat ratio

ε = Choke downstream/upstream pressure ratio

γ_o = Relative oil density (Water = 1.0)

F_{WO} = Water oil ratio (WOR)

γ_w = Water density

Their range of data is as follows:

Pressure	1161 - 1230	psia
Choke size	14 - 20	1/64th inch
Condensate Flow Rate	261 - 596	STBD
Gas Oil Ratio	344 - 501	SCF/STB

2.8 Sachdeva et al. Analysis (11)

Sachdeva et al. in 1984 studied two-phase flow through chokes, including both critical and subcritical flow and the boundary between them. Data were gathered for air-water and air-kerosene flow through choke sizes of 16 to 32 sixty-fourth of an inch (The use of kerosene and water was mainly to cover the approximate range of fluid densities encountered in the field). A new theoretical model for predicting flow rates and the critical-subcritical flow boundary was tested against these data. The final form of their equation is as follows:

$$G_{ds} = C_D \left[(2g_c)\, 144 p_{us}\, \rho_{mds}^2 \left\{ (1 - f_{gus}) \frac{(1-\varepsilon)}{\rho_l} + \frac{f_{gus}\, k}{k-1} \left(\frac{1}{\rho_{gus}} - \frac{\varepsilon}{\rho_{gds}} \right) \right\} \right]^{.5} \quad (2.23)$$

where

$$G_{ds} = \frac{m_{gds} + m_{lds}}{A_c} \quad (2.24)$$

$$\frac{1}{\rho_{gds}} = \frac{1}{\rho_{gds}} \varepsilon^{\frac{-1}{k}} \quad (2.25)$$

and

$$\frac{1}{\rho_{mds}} = f_g \frac{1}{\rho_{gds}} \varepsilon^{\frac{-1}{k}} + (1 - f_g) \frac{1}{\rho_l} \quad (2.26)$$

and,

G_{ds}	=	Mass flux at downstream of choke, lbm/cu ft/sec
G_{us}	=	Mass flux at upstream of choke, lbm/cu ft/sec
g_c	=	Gravity constant
ρ_{mds}	=	Mixture density at downstream lb/cu ft
f_{gus}	=	Gas volumetric fraction upstream of choke, lbm/cu ft/sec
ρ_{gds}	=	Gas density downstream of choke, lbm/ cu ft
ρ_{gus}	=	Gas density upstream of choke, lbm/ cu ft
m_{gds}	=	Gas mass Flow rate downstream of choke, lbm/sec
m_{gus}	=	Gas mass Flow rate upstream of choke, lbm/sec
m_{lds}	=	Liquid mass Flow rate downstream of choke lbm/sec
ρ_l	=	Liquid density, lbm/cu ft
A_c	=	Choke throat area, ft^2

Their data summary is as follows:

Maximum upstream pressure	1230	psia
Maximum Gas flow rate	136.6	MSCFD
Maximum Liquid flow rate	1340	STBD
Choke size	16 - 32	1/64th inch

2.9 Osman and Dokla Correlation (12)

In 1988, Osman and Dokla used 87 production test data from a gas condensate reservoir located in the Middle East to develop empirical correlations that describe the behavior of gas condensate flow through chokes. Four forms of correlations were checked against data. One of the forms is to correlate choke upstream pressure with liquid production rate, gas liquid ratio and choke size. The second form is developed by using gas production rate instead of liquid rate in the previous form. The other two forms are developed by using the pressure drop across the choke instead of upstream pressure. Figure 8 shows one of their correlations presented in its graphical form. Five error parameters were used to check the accuracy of the different forms. They concluded that as far as their data is concerned, any one of the four forms is expected to give reasonable values and can be used; however, the critical error analysis indicates that the forms with pressure drops gave slightly less error. Following are the two equations with the oil rate term:

$$p_{us} = \frac{829.7(R_p)^{0.4344} q_l}{S^{1.8478}} \tag{2.27}$$

$$\Delta p = \frac{310.01(R_p)^{0.5919} q_l}{S^{1.86}} \tag{2.28}$$

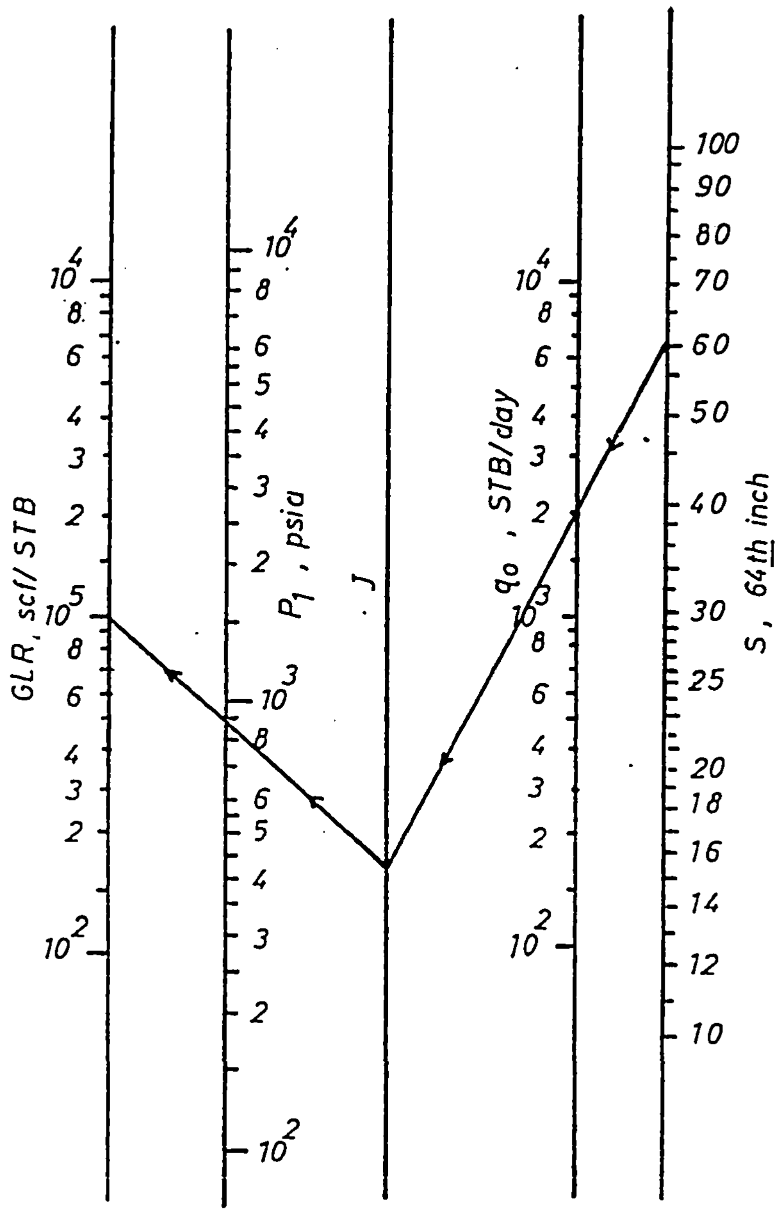

Figure 8: Correlation chart for choke upstream pressure
Example: for S = 60, q_o = 2000 bal/day and
GLR = 1000 scf/STB, P_1 = 860 psia
(After Osman and Dokla)

where:

P_{us}	=	Upstream tubing pressure, psia
Δp	=	Pressure drop across the choke, psi
q_l	=	Liquid flow rate, STBD
q_g	=	Gas flow rate, MSCFD
R_p	=	Producing gas-liquid ratio, MSCF/STB
S	=	bean size, 64th of an in.

Osman and Dokla stated that the above formula application should be limited to the following range of data which was used in their study:

Pressure	2950 - 5200	psia
Well head temperature	40 - 98.9	C
Choke size	28 - 72	1/64th inch
Condensate Flow Rate	10.5 - 1299	STBD
Water Flow Rate	0.0 - 1002.6	BPD
Gas Flow Rate	3.9 - 101	million SCFD

2.9 Surbey et al. Correlation (13)

In 1989, Surbey, Kelkar and Brill took a different approach in analyzing critical-flow data. Their experimental data was divided into two groups, subcritical and critical flow, on the basis of a comparison of the downstream to upstream pressure ratio to literature correlations (Wallis, Fortunati and Ashford et al.). When all the three correlations predicted a point to be in the critical region, that point was assumed to be critical. All other tests were grouped in the subcritical region. The over all range of experimental data is as follows:

Pressure	85 - 950	psig
Well head temperature	48 - 132	F
Liquid Flow Rate	450 - 3550	STBD
Gas Flow Rate	.40 - 2.50	million SCFD
Gas/liquid ratio	140 - 5200	SCF/STB

After evaluating his test data with the correlations from literature, Surbey et al. came with an equation similar to Gilbert form but with different constants as follows:

$$p_{us} = \frac{.2797 R_p^{.3955} q_l^{.5917}}{A_c^{.4664}} \tag{2.29}$$

Where

A_c = Cross-sectional area of the choke in squared inches.

Surbey et al. stated that their formula is restrictive to motor operated valve, MOV, type chokes. In their comparisons to other correlations, they concluded that Omana correlation had the least average absolute error and gave the least standard deviation.

For subcritical flow, they proposed an iterative procedure that converges when the calculated throat pressure is equal to the vapor pressure. This convergence gave a better estimate of the pressure drop (upstream/downstream) that causes the critical velocity. The following equations were used in their iterative program:

$$v_{tp} = ((\rho_l f_l + \rho_g f_g)(\frac{f_l}{\rho_l v_l^2} + \frac{f_g}{\rho_g v_g^2}))^{-1/2} \tag{2.30}$$

and

$$q_m = v_{tp} A_c \rho_m \tag{2.31}$$

where:

v_{tp}	=	Critical velocity, ft/sec
ρ	=	density, lbm/ cu ft
f	=	No-slip holdup volume fraction
v_l	=	Liquid choking velocity, ft/sec
v_g	=	Gas sonic velocity, ft/sec
q_m	=	Mixture flow rate, STBD
A_c	=	Choke cross-sectional area, square inches
ρ_m	=	mixture density, lbm/cu ft

CHAPTER 3

CHAPTER 3

DATA ACQUISITION

In this study data from different fields in the Middle East were collected to develop an empirical correlation that covers wide ranges of flow rates and choke sizes. The reported production test data includes oil and gas flow rates, choke sizes, downstream and upstream wellhead tubing pressures.

3.1 Data Description

3.1.1 Flow Rates

The gas and oil flow rates are measured at test traps on the gas oil separation plants or in portable separators designed for well testing. Although these separators are operated at different pressures and temperatures for different fields; However, the final reported data, which are used in this study, were all calibrated to surface conditions by field engineers utilizing the knowledge of PVT properties of the fluids and previous production tests done in the laboratory or in the field to describe the behavior of the fluid when changed from the separator conditions to the standard atmospheric conditions.

3.1.2 Tubing Wellhead Pressure

In most of these tests, the wellhead pressures are measured with mechanical spring gauges with an accuracy of $\pm$ 20 psi. In addition to the gauge accuracy, these gauges, occasionally, get stuck resulting in wrong readings; However, this seldom happens because these gauges are frequently checked and calibrated.

3.1.3 Choke Size

Adjustable chokes are the dominant chokes used in the tested wells. In many fields, sand production causes tear and wear of the chokes giving a nonrepresentative value of the actual choke opening. This too is not usually happening because if the sand cuts the choke, other surface equipment will be also affected, therefore, immediate action is always taken to remedy the problem mostly by either calibrating the choke or replacing it.

3.2 Production Test Data

Three thousand nine hundred and thirty one tests from 10 fields were collected and reorganized in a readable format for the purpose of this study. Table 1 shows the ranges of the data collected.

3.3 Fluid Properties Data

The production test data were collected from different fields; accordingly, corresponding PVT data for the respective fields were gathered. Table 2 shows the ranges of these fluid properties for the tested wells.

The table shows clearly the big diversity of the PVT properties studied. All these PVT properties were obtained from laboratory analysis of many of either downhole or/and surface fluid samples collected. PVT properties of each individual field were taken to be the average properties of the samples obtained from that field.

3.4 Data Screening

Since this correlation deals with multiphase flow, a criteria was applied to identify and remove single phase flow tests. The upstream wellhead pressures were cross checked with the fluid bubble-point pressure.

Should the upstream tubing pressure fall higher than the bubble point, indicating a single phase flow, this test data was dropped. A total of 52 tests were removed due to the single phase flow.

The main purpose of this study is to develop a correlation to be used in choke design, therefore this study deals primarily with data under the critical flow pattern. Supcritical flow data should not be included. The critical flow pattern occurs when the downstream tubing pressure is less than 55% of the upstream tubing pressure as was used by Gilbert (2), studied by Ros (3) and followed by many later researchers. Three hundred and twenty five tests that do not meet this criteria were dropped.

The final number of well tests that were used in the development of the new correlation was 3554 points.

TABLE 1: Production Data Summary

Description	Minimum	Maximum
Oil Flow Rate, STBD	172	33847
Gas Oil Ratio, SCF/STB	12	5026
Upstream Tubing Pressure, psia	97	1880
Downstream Tubing Pressure, psia	10	980
Choke Size, 64th of inch	16	160

TABLE 2: PVT Data Summary

Description	Minimum	Maximum
API Gravity	27	40
Oil Viscosity at rsvr. cond., cp	.29	4.6
Formation Volume Factor, RB/STB	1.16	1.60
Bubble Point Pressure, psia	300	3136
Reservoir Temperature, F	160	240
Gas Relative Density (Air=1)	.50	.91

CHAPTER 4

CHAPTER 4

REGRESSION THEORY AND ANALYSIS

Correlation refers to the degree of association between one variable and another or several others. Regression deals with the nature of the relation between these variables. In evaluating the degree of regression, all the error or imprecision is assumed to be in the measurement of one variable called the "dependent", while the other variables are assumed to be precisely known. These precise variables are called the "independent" variables.

4.1 Regression Theory (15)

The basic concept of regression analysis is to produce a linear or nonlinear combination of independent variables that will correlate as closely as possible with the dependent variable.

4.1.1 Linear Multiple Regression

Consider a set of observations of size n_d on which the properties $y, x_1, x_2, x_3............x_n$ are measured. The x's and y are the independent and dependent variables, respectively. The linear regression equation will then be written as follows:

$$y = a_0 + a_1x_1 + a_2x_2 + \ldots\ldots + a_nx_n \tag{4.1}$$

which represents a hyperplane in (n+1) dimensional space. Equation (4.1) can be written for any observation point i as:

$$y_i = a_0 + a_1x_{i1} + a_2x_{i2} + \ldots\ldots + a_nx_{in} \ ; \ i = 1, n_d \tag{4.2}$$

The n_d equations for the n_d experimental measurements can be expressed in matrix form as:

$$\begin{vmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1n} \\ 1 & x_{21} & x_{22} & \cdots & x_{2n} \\ 1 & x_{31} & x_{32} & \cdots & x_{3n} \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ 1 & x_{n_d1} & x_{n_d2} & \cdots & x_{n_dn} \end{vmatrix} \begin{vmatrix} a_0 \\ a_1 \\ a_2 \\ . \\ . \\ . \\ a_n \end{vmatrix} = \begin{vmatrix} y_1 \\ y_2 \\ y_3 \\ . \\ . \\ . \\ y_{nd} \end{vmatrix} \tag{4.3}$$

or in simpler form

$$X\vec{a} = \vec{y} \tag{4.4}$$

where

$X = n_d \times (n + 1)$ matrix

$\vec{a} = (n + 1)$ vector, and

$\vec{y} = n_d$ vector, and

n = total number of independent variables

Therefore, the objective is to solve for the vector $\vec{a}$ for which $X\vec{a}$ is as close as possible to vector y since the exact solution cannot be found. Such a vector is the least-squares solution. The unique least-square solution to this system presented in equation (4.4) is :

$$\dot{a} = (X^T X)^{-1} X^T \vec{y} \tag{4.5}$$

where $\dot{a}$ is the least-square solution to the system $X\vec{a} = \vec{y}$ and X^T is the transpose of the matrix X.

4.1.2 Nonlinear Multiple Regression

Although the choke size correlation is a nonlinear equation; However, it can be modified slightly to give a form of a multiple linear equation. The equation is as follows:

$$S = a'_0 \, q^{a1} \, p^{a2} \, \gamma^{a3} \tag{4.6}$$

which can be written as

$$\log S = a'_0 + a_1 \log(q) + a_2 \log(p) + a_3 \log(\gamma) \tag{4.7}$$

and therefore,

$$y = a_0 + a_1 x_1 + a_2 x_2 + \ldots\ldots + a_n x_n \tag{4.8}$$

where:

$y = \log\ (S)$

$a_0 = \log\ (a'_0)$

$x_1 = \log\ (q)$

$x_2 = \log\ (p)$

and,

$x_3 = \log\ (\gamma)$

Equation (4.8) can be solved by the method of linear multiple regression, as outlined earlier.

4.2 Error Analysis

Statistical and graphical error analysis were used to check the accuracy and performance of the correlation developed in this study.

4.2.1 Statistical Error Analysis

The accuracy of the correlation relative to the actual value is determined by using various statistical means. The criteria used in this study were average percent relative error, average percent absolute relative error, minimum/maximum absolute percent relative error, the root mean square error, standard deviation, the correlation coefficient, and the T-test.

4.2.1.1 Average Percent Relative Error

It is defined as:

$$E_r = \left(\frac{1}{n_d}\right)\sum_{i=1}^{n_d} E_i \tag{4.9}$$

E_i is the relative deviation in percent of an estimated value from a measured value and is defined by:

$$E_i = \{\frac{(x_{exp} - x_{est})}{x_{exp}}\}.100, \qquad i = 1, 2, ...n_d \tag{4.10}$$

where x_{exp} and x_{est} represent the experimental and estimated values respectively. E_r is an indication of the relative deviation in percent from the experimental values. The lower the value of E_r , the more equally distributed are the errors between positive and negative values.

4.2.1.2 Average Absolute Percent Relative Error

It is defined as:

$$E_a = (\frac{1}{n_d})\sum_{i=1}^{n_d} |E_i| \tag{4.11}$$

and indicates the relative absolute deviation, in percent, from the experimental values. A lower value implies a better correlation.

4.2.1.3 Minimum/Maximum Absolute Percent Relative Error

After calculating the absolute percent relative error for each data point, $|E_i|$, i = 1, 2, ... n_d , both the minimum and maximum values are scanned to determine the range of error for each correlation:

$$E_{min} = \min_{i=1}^{n_d} |E_i| \tag{4.12}$$

$$E_{max} = \max_{i=1}^{n_d} |E_i| \tag{4.13}$$

Accuracy of a correlation can be examined by maximum absolute percent relative error. The lower the value of maximum absolute percent relative error, the higher is the accuracy of the correlation.

4.2.1.4 Standard Deviation

Standard deviation of the errors, s, is a reflection of the dispersion of errors around the mean and a measure of the quality of the fit. It is expressed as the positive square root of the variance s^2

$$s^2 = \frac{1}{n_d - 1} \sum_{i=1}^{n_d} (E_i - E_r)^2 \tag{4.14}$$

A lower value of standard deviation means a smaller degree of scatter and a better quality of fit.

4.2.1.5 The Root Mean Square Error

It is another criteria to test the closeness of correlation prediction to the measured values and is defined as follows:

$$E_{rms} = \sqrt{\Sigma(E_i)^2/n} \tag{4.17}$$

The low value of E_{rms} indicates a good correlation. As the value goes higher, a worse fit is obtained.

4.2.1.6 The Correlation Coefficient

The correlation coefficient, r, represents the degree of success in reducing the standard deviation by regression analysis. The other term is coefficient of determination which is simply the square of the correlation coefficient and defined by:

$$r^2 = 1 - \sum_{i=1}^{n_d}(x_{exp} - x_{est})_i^2 \,/ \sum_{i=1}^{n_d}(x_{exp} - \bar{x})_i^2 \tag{4.15}$$

where

$$\bar{x} = (\frac{1}{n_d})\sum_{i=1}^{n_d}(x_{exp})_i \tag{4.16}$$

The correlation coefficient lies between 0 and 1. A value of 1 indicates a perfect correlation whereas a value of 0 implies no correlation at all among the given independent variables. The larger the value of r, the greater is the reduction in the sum of squares of errors, and the stronger is the relationship between the independent variable and the dependent ones.

4.2.1.7 T-Distribution Test (16)

The T-statistic test is an application of null hypothesis where each parameter in the model being tested is assumed to be zero. It is defined as :

$$t_j = \frac{b_j - \beta_j}{s_{bj}} \tag{4.17}$$

Where,

b_j is the estimated coefficient of each independent variable

β_j is the value under H_o for which it is most difficult to reject the null hypothesis (i,e $\beta_j = 0$), and ,

s_{bj} is the estimated standard error for each variable.

The estimated standard error for each of the independent variables is as follows:

$$s_{bj} = \frac{s}{\sqrt{\Sigma x_j^2(1 - r_j^2)}} \tag{4.18}$$

Where

$$s = \sqrt{\Sigma(Y_i - \dot{Y}_i)^2/(n - m - 1)} \tag{4.19}$$

$x_j = X_j - \overline{X}_j$ and ,

r_j^2 is the coefficient of determination between the tested variable, treated as dependent, and the rest of the independent variables. r_j^2 is defined as :

$$r_j^2 = 1 - \sum_{i=1}^{nd}(X_{j\,exp} - X_{j\,est})_i^2 \,/\, \sum_{i=1}^{nd}(X_{j\,exp} - \overline{X}_j)_i^2$$

4.2.2 Graphical Error Analysis

Graphical means help in visualizing the accuracy of a correlation. Four graphical analysis techniques were used and presented below.

4.2.2.1 Crossplot

A crossplot is a plot of one variable obtained by two different means. A 45 degree straight line is drawn to reflect the perfect correlation line on which the estimated and measured are equal. Then, The estimated values are plotted versus their corresponding measured values to form the crossplot. The closer the plotted data points to the perfect 45 line, the better the correlation is.

4.2.2.2 Incremental Range Analysis

In this analysis, the data are grouped within specific ranges of the dependent variable. The statistical results of each group predicted by the correlation can show the strengths and weaknesses of the correlation besides the cumulative results of the correlation. Values are plotted versus their corresponding dependent variable ranges. This analysis shows the consistency of the correlation and tests the validity of it in different ranges.

4.2.2.3 Error Distribution Analysis

This analysis shows error distribution histograms with overlaid normal-distribution curve of the correlation. It graphically shows the range of the error and at which the peak occurs indicating the adequacy of the prediction and the level of the correlation overestimation or underestimation.

4.2.2.4 Error Elimination Analysis

As a subset of the error distribution analysis, this analysis gives a graphical representation of the effect of eliminating data of the highest deviation. The number of points is reduced by 5% increments of the total number of data in order of the highest error to the lowest. If the correlation gives better prediction by eliminating small number of data, the confidence will be higher in such a correlation and indicates that error magnitudes could have been high only due to some few data points. This test was made for absolute relative error, the coefficient of determination and the root mean square error.

4.2.3 Sensitivity Analysis

Sensitivity analysis is made to determine the overall sensitivity to each of the independent variables. Each of the independent variables will be plotted against the dependent variable while the others are held constant. The plot will show the influence of the variable in different ranges. The influence of the variable to the correlation could be significant in a specific range without much of influence in the other ranges. Also the degree of influence will be indicated by the slope of the plotted points. If the absolute slope is low, indicating a slight change in the dependent variable value as a result of a big change in the independent, the effect of that variable in the final correlation is small. A horizontal line with zero slope indicates a negligible effect.

CHAPTER 5

CHAPTER 5

DEVELOPMENT OF THE CORRELATION

Non-linear multiple least square regression analysis was used to develop the correlation of choke size. Firstly, independent variables were determined by studying their relationships with the dependent variables. Secondly a model was chosen that best correlates against the field data. The least square regression coefficients thus obtained were fixed one after another to the nearest rounded or fraction values and the final correlation was formulated.

5.1 Selection of the Independent Variables

The first step in developing the correlation was to gather all the pertinent variables that may influence the fluid flow mechanisms in chokes. The variables considered in developing this correlation were as follows: choke size, mixture flow rate, gas flow rate, oil flow rate, upstream tubing pressure, downstream tubing pressure, liquid density, gas density, mixture relative density, liquid viscosity, surface tension, gas oil ratio, tubing temperature, the gas compressibilty factor, and the oil formation volume factor.

Through inspection of the existing correlations, the common variables that were of significant influence in determining the choke size in all of them were the oil flow rate, the upstream wellhead tubing pressure, and the gas oil ratio. Some other variables were introduced in some of the correlations like the liquid surface tension in Omana's correlation (8) or the mixture densities in Ros and Fortunati correlations (3,9). The specific heat was another variable that was introduced in Ashford, Sachdeva and Surbey correlations (10,11,13), however, because this property is not a field measured value and the lack of strong correlations in predicting the specific heat from different properties, and because this study was targeted for practical application in the oil industry, the specific heat was not introduced neither studied.

5.2 Choke Size Correlation

Non linear multiple regression showed the choke size to be a function of oil flow rate, upstream wellhead tubing pressure, and the mixture relative density.

$$S = f(q_o, p_{us}, \gamma_m) \tag{5.1}$$

The aforementioned independent variables were in agreement with the empirical correlations from literature (2,12,13) as far as the oil rate and the upstream pressure. The gas oil ratio that was used in these correlations was replaced in the new correlation by the mixture relative density. The mixture density, in general, was always an important variable in the formulation of the correlations that were based on theoritical background (3,9). That, in addition to the performed statistical analysis, strongly supports the inclusion of the relative mixture density in this study.

Several models were tested to reach the final form of the correlation. Firstly, the effect of each parameter was studied individually followed by the combined effect of the parameters. The model so obtained was then modified depending on the contribution of each parameter. This procedure was continued until the final model was reached.

The regression coefficients were determined by the least square method. The final values of the coefficients were chosen by fixing one coefficient to the nearest rounded or fraction value while the remaining other coefficients were determined by regression. The next coefficient was then fixed and the regression was carried out to determine the remaining coefficients. This process was repeated until all constants were determined. Statistical analysis was carried out at each step to avoid any instabilities. The final correlation is as follows:

$$S = 20.696 \frac{q_o^{.483} \gamma_m^{.707}}{p_{us}^{.474}} \tag{5.2}$$

where,

$$\gamma_m = \gamma_o + 2.18 \times 10^{-4} R_p \gamma_g \tag{5.3}$$

and,

S	=	Choke Size, 64th of inch
q_o	=	Oil flow rate, STBD
p_{us}	=	Upstream wellhead pressure, psia
γ_o	=	Oil relative density (water = 1)
γ_g	=	Gas relative density (air = 1)
R_p	=	Producing gas oil ratio, SCF/STB
γ_m	=	Mixture Relative Density (water = 1)

5.3 Statistical Analysis of The New Correlation

The statistical analysis of this correlation includes the T-statistics test, error distribution analysis, error elimination analysis, and finally sensitivity analysis of the independent variables.

5.3.1 T-statistics

Values of the T-statistcs for all coefficients were significantly different from zero at the 99 percent confidence level, that is, there is 99 percent certainty that the explanatory variables are meaningful predictors of choke size. The T-statistics are presented in table 3.

5.3.2 Error Elimination Analysis

Data from the field showing the greatest deviation are discarded incrementally. The absolute percent relative error drops from 8.4% to 6.7% after 10% of the data is deleted. After 50% of data is deleted, considering 1784 data point, the absolute percent relative error becomes 3.41%. Table 4 summarizes the results of error elimination for absolute percent relative error, the standard deviation, the coefficient of determination and the root mean square error. Figures 3 through 7 are graphical illustrations of this analysis.

5.3.3 Error Distribution Analysis

Figure 13 is the error distribution of the correlation. It clearly indicates a normal distribution with a mean of zero error, in other terms, the error is distributed evenly on both sides of the zero.

5.3.4 Sensitivity Analysis

The sensitivity of the variables that contribute to the final correlation was tested as shown in figures 14-17. Each variable was tested while holding the remaining variables constant at minimum, average, and maximum values producing three curves in each plot.

Figure 14 shows that the effect of the oil flow rate is strong in all the choke ranges; However, the greatest effect is observed at low choke sizes. Higher slope and relativley high values are observed when the other variables are at maximum values (the wellhead pressure and the mixture relative density).

For the tubing pressure, its effect on the correlation is drastically reduced when the pressure is at high values indicating no need for precise measurement of wellhead pressure in the high ranges for the practical use of this correlation (figure 15).

The mixture relative density plays an important rule in the correlation in all the choke size ranges equally as indicated by its consistent slope (figure 16).

Overall, mostly all aforementioned variables affect the correlation when they are in relatively small ranges. As their values become higher, may be breaking a "critical barrier", their influence decreases.

TABLE 3 :T-statistics for regression coefficients (eq. 5.2)

Independent variable	T-value
Intercept	42.80
log (oil flow rate, q_o)	167.98
log (Tubing pressure, p_{ut})	-57.41
log (Mixture relative density, γ_m)	21.33

TABLE 4 :Error Elimination Statistical analysis

% of data	No. of observations	E_a	s^2	r^2	E_{rms}
100	3554	8.372	113.296	0.909	10.777
95	3377	7.401	79.847	0.936	9.142
90	3200	6.710	62.295	0.950	8.114
85	3023	6.168	51.668	0.957	7.386
80	2846	5.689	43.398	0.964	6.766
75	2669	5.247	36.783	0.969	6.209
70	2492	4.850	31.289	0.974	5.721
65	2315	4.471	26.479	0.978	5.262
60	2138	4.102	22.204	0.982	4.815
55	1961	3.743	18.389	0.985	4.374
50	1784	3.414	15.456	0.988	3.995

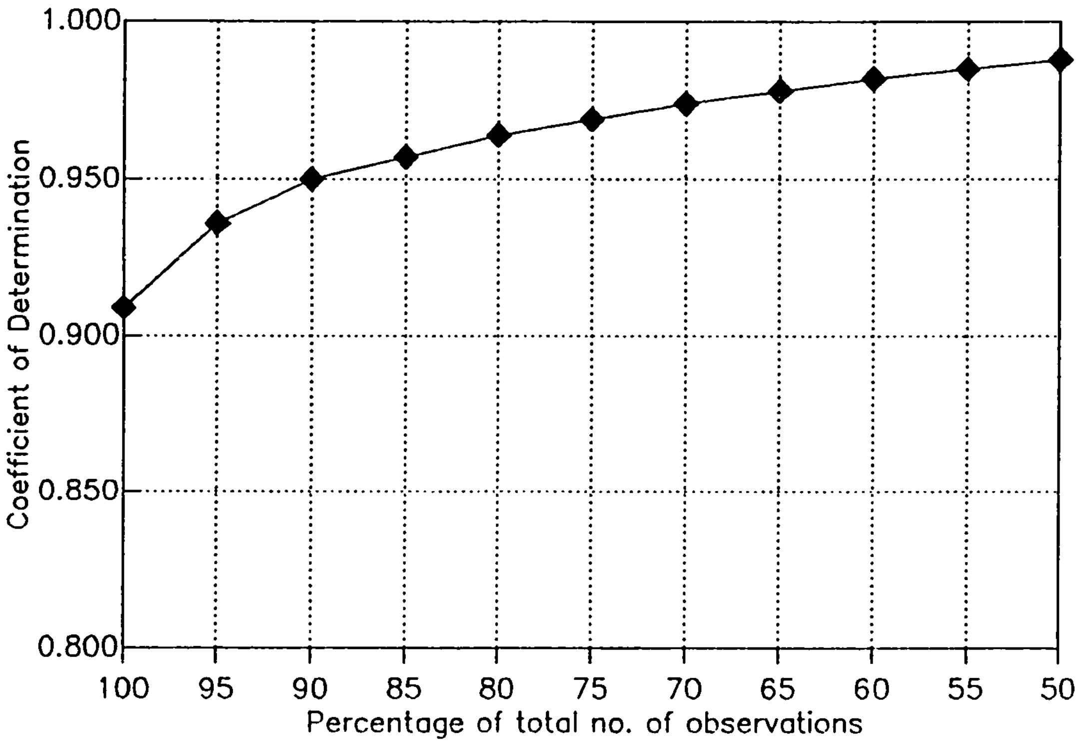

Fig 9: Error Elimination Analysis (Coefficient of Determination)

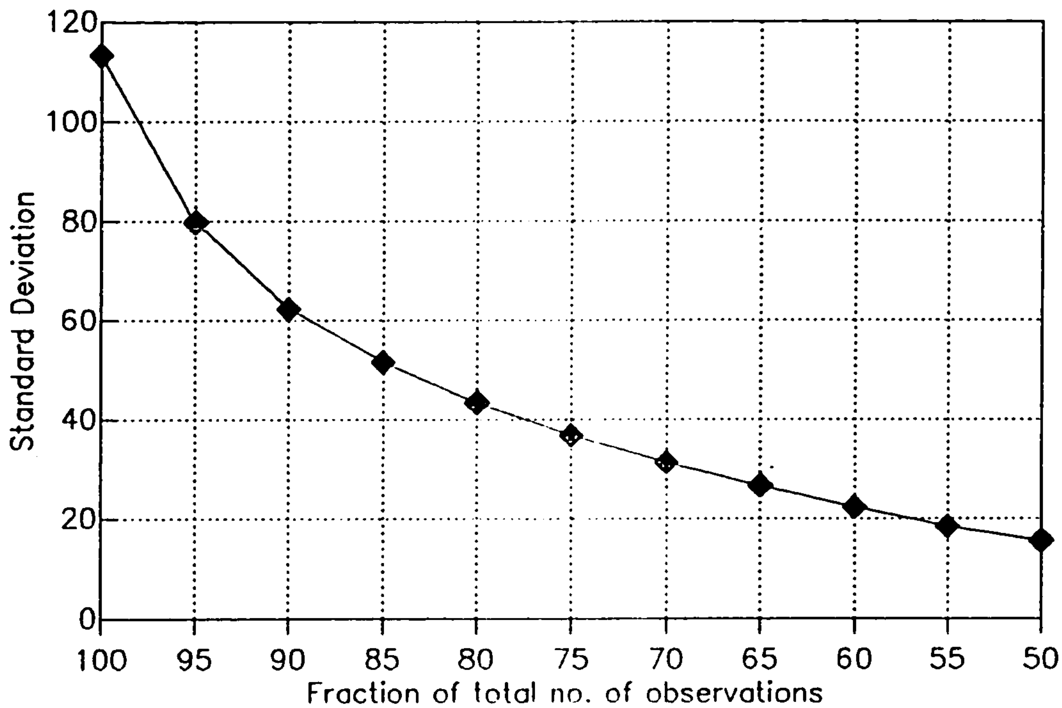

Fig 10: Error Elimination Analysis (Standard Deviation)

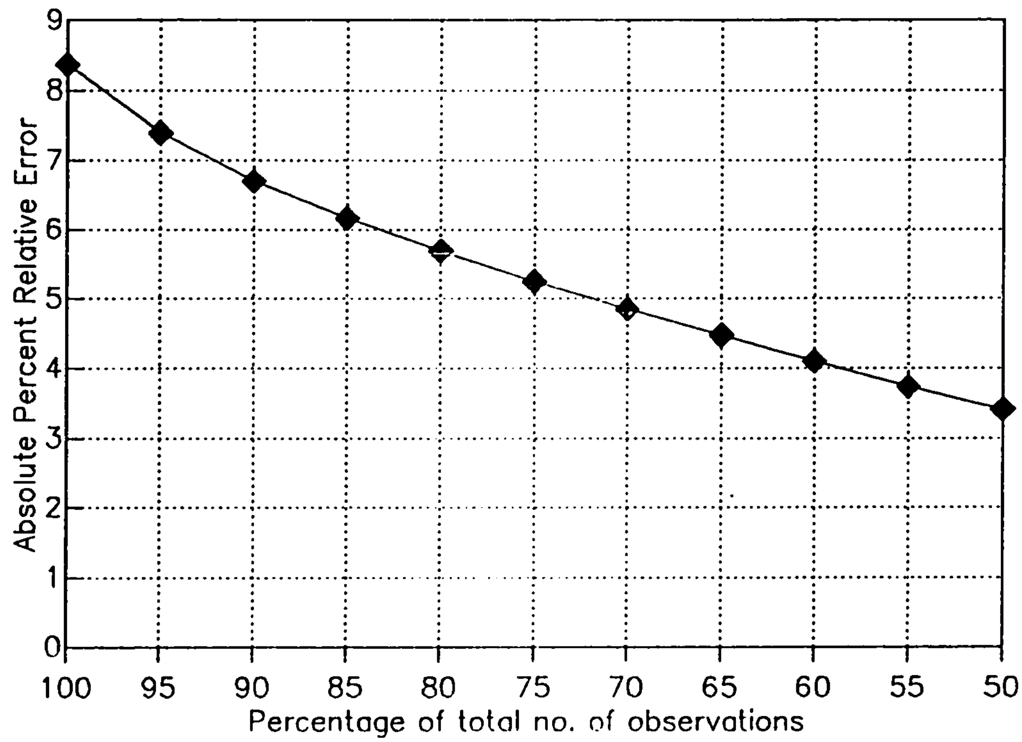

Fig 11: Error Elimination Analysis (Absolute percent Relative Error)

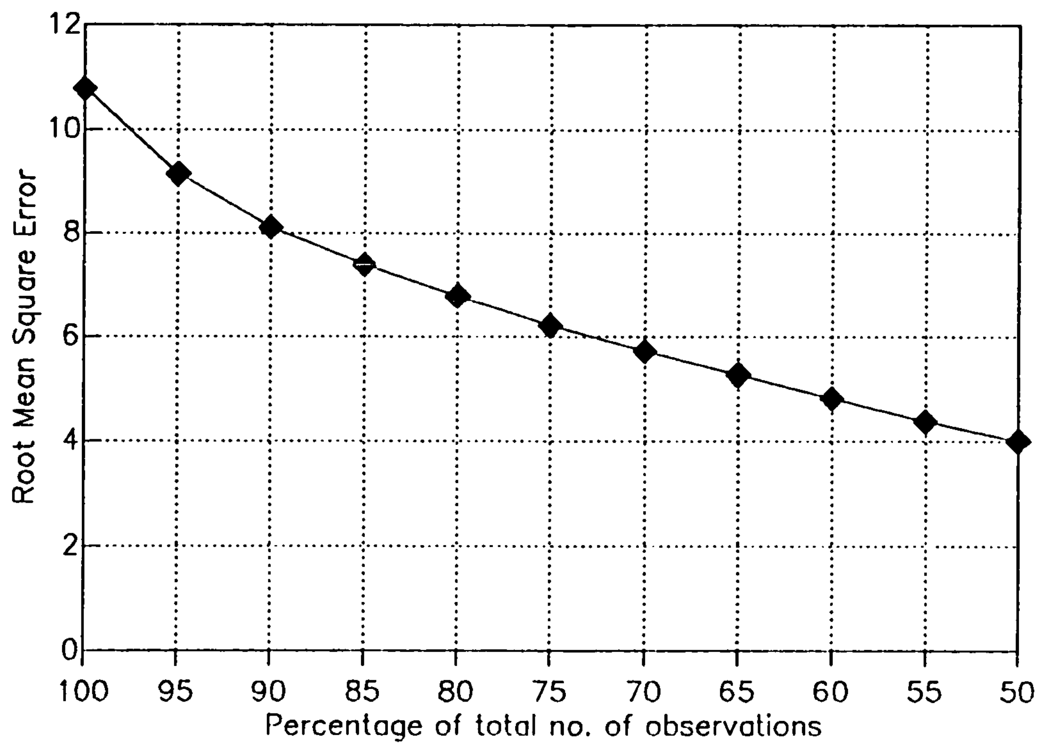

Fig 12: Error Elimination Analysis (Root Mean Square Error)

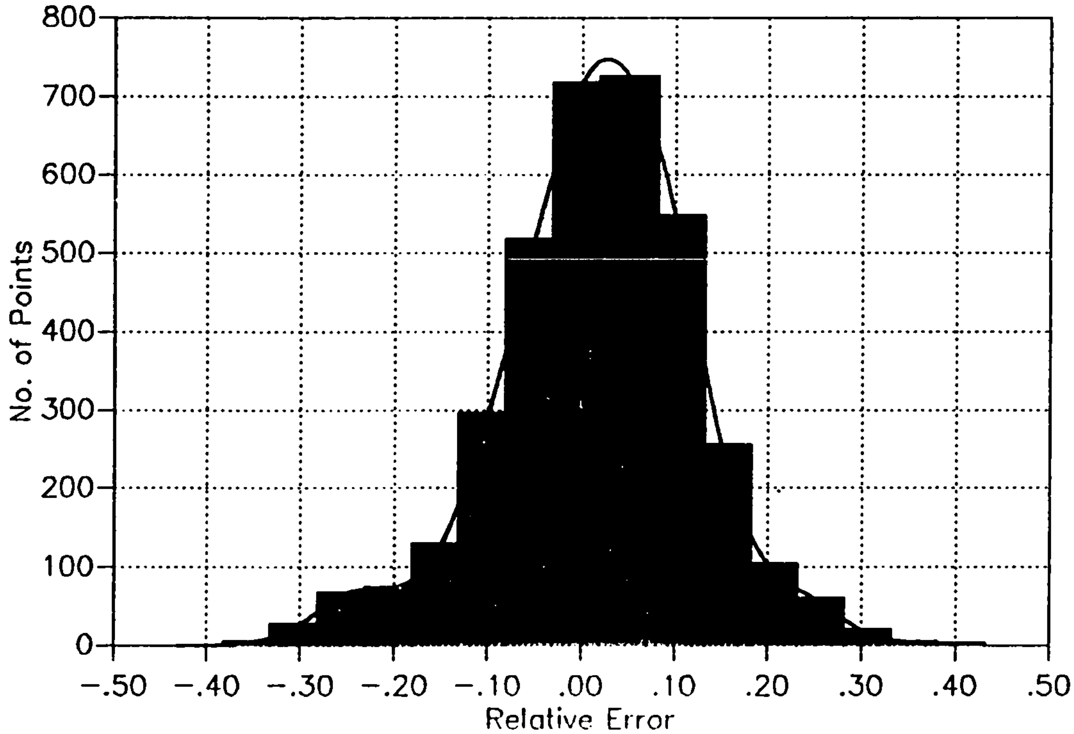

Fig 13: Error Distribution Plot

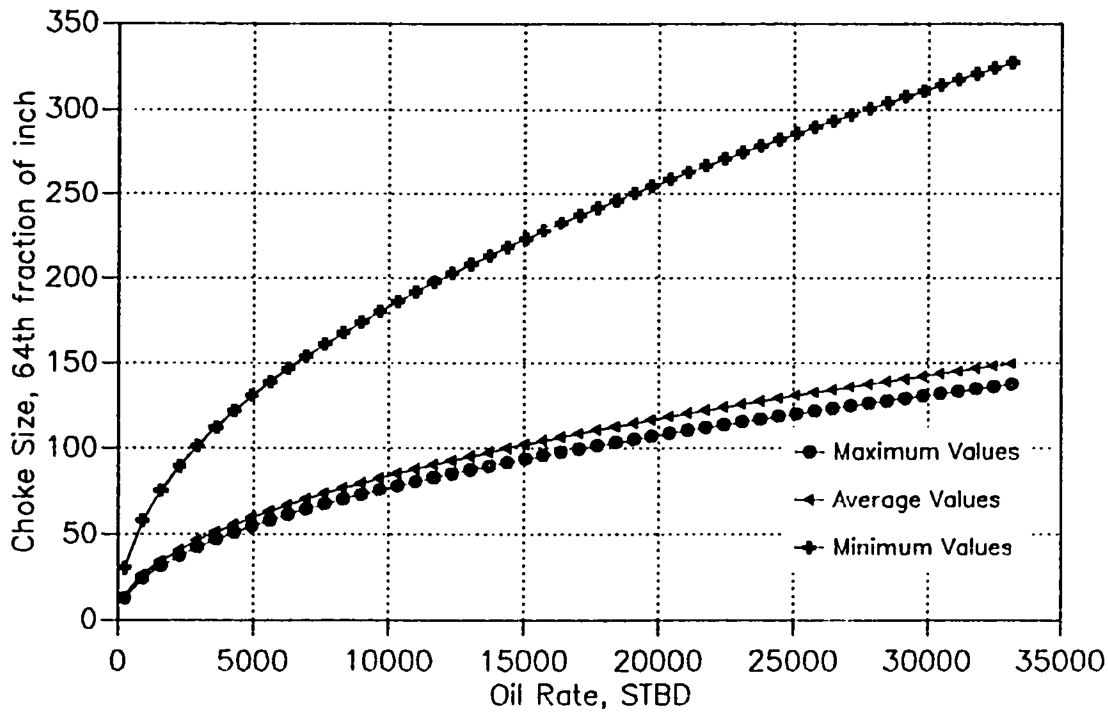

Fig 14: Sensitivity Analysis of Flow Rate

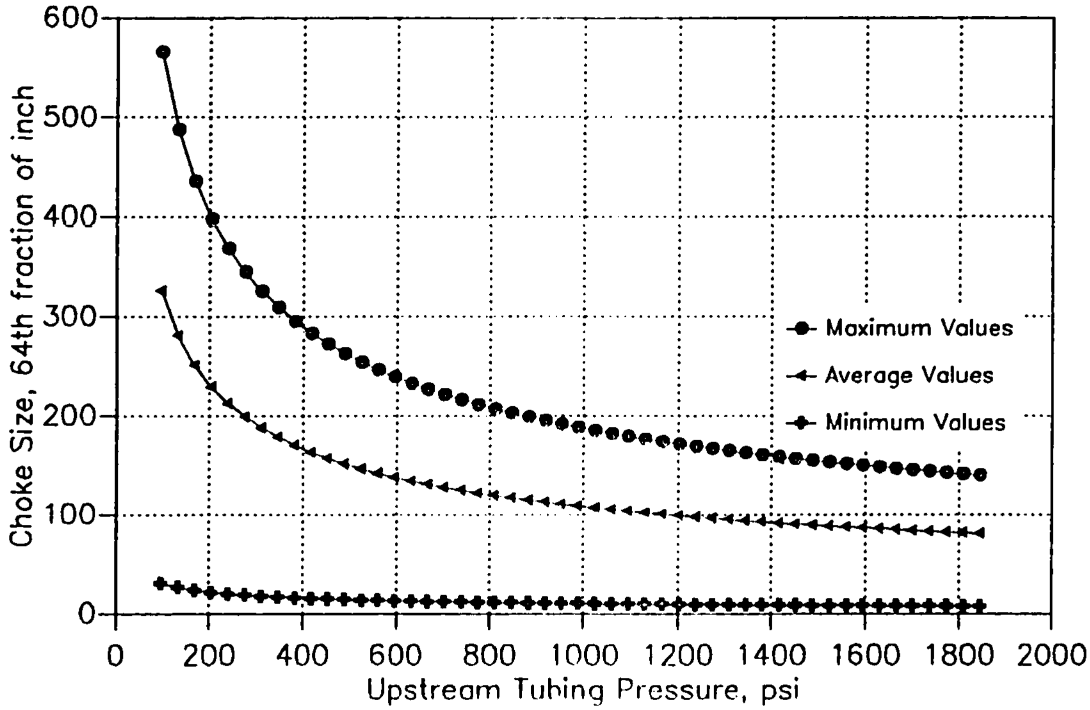

Fig 15: Sensitivity Analysis of Tubing Pressure

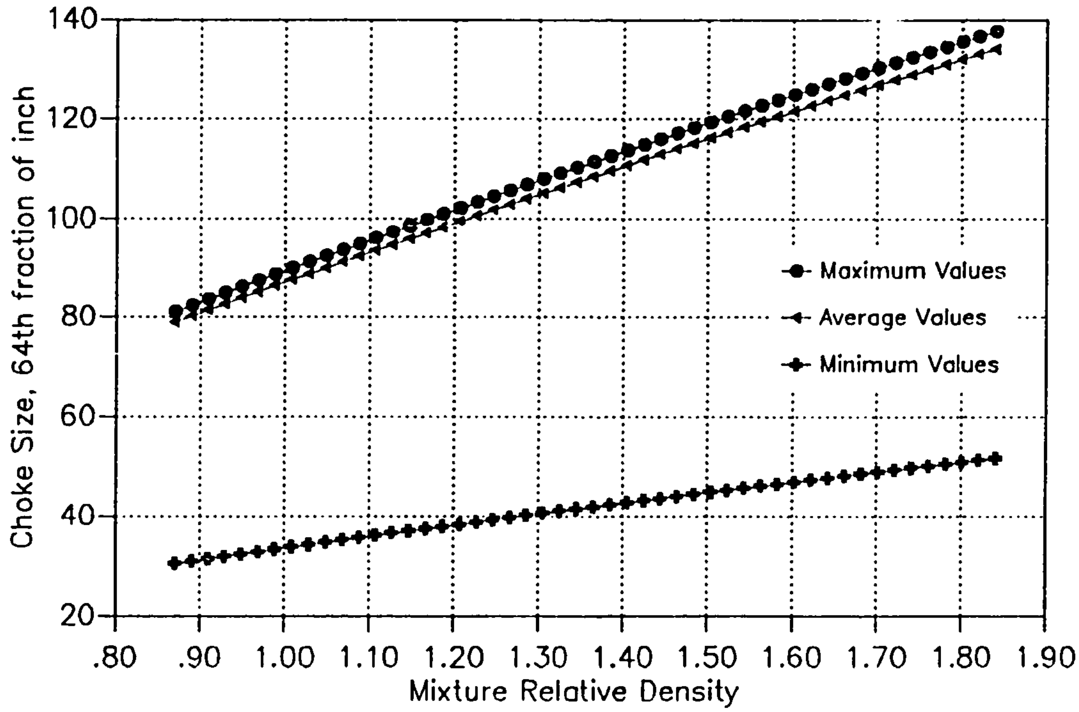

Fig. 16: Sensitivity Analysis of Mixture Relative Density

CHAPTER 6

CHAPTER 6

EVALUATION

OF

THE EXISTING CORRELATIONS

Literature dealing with multiphase flow through chokes have been discussed in the preceding chapters. Field data utilized to develop the new correlation was used for the purpose of testing the existing correlations. The means to compare the correlations are both statistical and graphical analyses means.

6.1 Statsitical Analysis

Five error parameters were used to evaluate the existing correlations and compare them with this study correlation. The five parameters are the average relative error, the absolute relative error, the minimum and maximum relative errors and finally the root mean square relative error. The analysis was made for choke size prediction and supplemented by the prediction of flow rates. Table 5 and 8 summarize the results of these statistical analyses.

6.1.1 Gilbert Corrclation

Gilbert formula was used to predict choke size as a function of gross liquid rate, the gas-liquid ratio, and the upstream wellhead tubing pressure. The production rate was also calculated to check for the applicability of the formula to predict production rates. Both the choke size and liquid rate predictions showed average absolute relative errors of 13% and 23% respectively.

6.1.2 Ros Correlation / Poetmann and Beck Adaptation

Ros's analysis was theoretically based on the assumption of heterogeneous system with a high gas oil ratio, where gas is the continuous phase. The formula adapted by Boetmann and Beck was tested with field data resulting in 9% and 23% average absolute relative errors for choke size and flow rate. The gas compressibility factor in this formula was obtained by Dranchuk and Abou-Kassem correlation (17).

6.1.3 Omana Correlation

In order to test the field data with Omana correlation, oil surface tension was approximated using Baker's correlation (18) which relates the surface tension to API oil gravity as follows:

$$\sigma_{68} = 39 - .2571\gamma_{API}$$

where the surface temperature is 68 F.

The average absolute relative errors for choke size and flow rate are 165% and 81% respectively.

6.1.4 Fortunati Correlation

The Fortunati formula was based on theoretical background as previously mentioned. In his derivation, fluid properties were estimated at the downstream pressure. But because of the lack of confidence in the reported downstream pressure, the upstream pressure was used to estimate the properties. The equation resulted in average absolute relative errors for choke size and flow rate of 24% and 71% respectively.

6.1.5 Osman and Dokla Correlation

Osman and Dokla formula is similar to Gilbert's formula with different constants. For sake of completeness, their formula was tested to the field data resulting in 77% and 64% average absolute relative errors for choke size and flow rate.

6.1.6 Surby et al. Correlation

Surby formula is also similar to Gilbert form. The only difference, in addition to the change of constants, is the use of the flow rate raised to a power. The equation resulted in average absolute relative errors for choke size and flow rate of 18% and 39% respectively.

6.1.7 The New Correlation

The new correlation developed in this study has shown a consistently better prediction than the other correlations. The absolute relative errors were 8% and 18%. Since the correlation was mainly established for choke design, more emphasis was made on the prediction of choke size, however, the prediction of flow rate also shows better results than the other formulas.

6.2 Graphical Analysis

Two graphical means were used for comparison and evaluation of the correlations, crossplots and incremental analysis.

6.2.1 Crossplots

The crossplots of estimated versus observed values of both the choke size and the oil flow rates are shown in figures 17-24 and 27-33 respectively.

The crossplot of this study indicates a very close scatter around the perfect line, the 45 line, in comparison of all the existing correlations. Although Gilbert correlation shows close to the perfect line yet the scatter is distributed over a bigger range indicating higher deviation. Ros and Fortunati in their correlation have a good scatter yet not on the perfect line which indicates the need for a fudge factor to minimize the error and bring their correlation closer to the perfect line. Ros correlation is the best predictor after the new correlation. Omana correlation shows two scatters, one approximately on the perfect line and the other group is predicting very high values. Such scatter indicates that an important parameter to the correlation was not included.

6.2.2 Incremental Analysis

The data was distributed by ranges of choke sizes. The absolute percent relative error and root mean square error for each range were plotted versus their corresponding choke size ranges in figures 25,26,34 and 35.

The incremental analysis indicates that this study predicted consistently better in all the ranges. The closest was Gilbert correlation in all the ranges consistently. Surbey correlation gave very good estimate similar to Gilbert in the 64-96 choke size range. His deviation increases as the choke size goes further from that range. Omana correlation, despite being established based on small choke size data, showed the highest deviation in the small choke size range decreasing as the size goes higher yet in all the ranges, Omana correlation showed the highest error.

6.3 Summary

It is observed that Omana correlation gave the highest deviation from measured values while Ros, one of the oldest correlations tested, gave the closest prediction relative to the others. Gilbert and Fortunati prediction accuracy could be improved if calibrated with a fudge factor.

In general, all the existing correlations gave a relatively high deviations starting from a 13% average absolute relative error. Due to the observed inconsistency and the high relative error of the existing correlations, the new correlation was attempted. The new correlation gave better prediction consistently for all five error parameters as demonstrated by the error analyses in this chapter.

Table 5: Statistical Accuracy for Choke Size Correlations

Correlation	E_r	E_a	E_{min}	E_{max}	E_{rms}
Gilbert	-8.63	13.16	0.02	80.00	16.46
Ros	6.50	9.39	0.01	52.60	12.40
Omana	-163.29	165.23	0.06	570.98	189.08
Fortunati	-17.47	24.01	0.01	154.89	30.36
Osman	-77.16	77.36	0.48	192.49	80.31
Surbey	2.57	17.98	0.00	196.47	25.06
This study	-1.69	8.37	0.00	43.87	10.78

Table 6: Statistical Accuracy for Choke Size Correlations For Different Choke Size Ranges (Absolute Percent Relative Error)

	Choke size Ranges in 64th of inch (No. of points)				
Correlation	16-32 (271)	32-64 (1391)	64-96 (1079)	96-128 (607)	128-160 (206)
Gilbert	11.033	12.909	14.873	11.935	12.221
Ros	7.819	8.668	8.143	11.684	12.323
Omana	253.692	208.044	145.057	93.584	76.056
Fortunati	49.296	32.468	26.024	19.398	17.541
Osman	64.420	77.535	81.894	75.724	74.265
Surbey	30.730	16.966	13.275	19.357	28.439
This study	7.660	8.609	7.949	8.497	10.248

Table 7: Statistical Accuracy for Choke Size Correlations For Different Choke Size Ranges (Root Mean Square Error)

	Choke size Ranges in 64th of inch (No. of points)				
Correlation	16-32 (271)	32-64 (1391)	64-96 (1079)	96-128 (607)	128-160 (206)
Gilbert	13.868	16.067	17.759	16.198	15.880
Ros	11.975	10.497	11.554	14.308	15.215
Omana	274.403	222.741	162.751	109.713	91.101
Fortunati	54.500	37.323	32.792	26.286	25.035
Osman	67.023	79.823	84.806	79.412	78.195
Surbey	32.701	21.715	19.514	30.276	39.341
This study	11.436	10.062	10.394	10.732	13.072

Table 8: Statistical Accuracy For Oil Flow Rate Prediction

Correlation	E_r	E_a	E_{min}	E_{max}	E_{rms}
Gilbert	8.79	23.35	0.031	1244.35	44.49
Ros	-25.25	28.26	0.01	286.45	41.57
Omana	38.52	81.10	0.040	3944.88	130.89
Fortunati	69.11	71.33	1.76	551.32	74.01
Osman	63.32	63.94	0.889	272.77	64.77
Surbey	-21.95	38.54	0.006	3990.88	113.26
This study	-0.38	17.85	0.00	230.61	25.60

Table 9: Statistical Accuracy for Flow Rate Prediction For Different Choke Size Ranges (Absolute Percent Relative Error)

	Choke size Ranges in 64th of inch (No. of points)				
Correlation	16-32 (271)	32-64 (1391)	64-96 (1079)	96-128 (607)	128-160 (206)
Gilbert	21.495	24.505	24.774	19.742	21.292
Ros	28.238	21.303	26.285	41.397	47.324
Omana	88.219	90.310	78.571	71.745	101.395
Fortunati	82.088	75.923	69.724	61.684	63.209
Osman	58.694	64.465	65.433	63.043	62.218
Surbey	92.484	49.041	21.786	22.803	30.652
This study	16.595	16.279	16.877	19.584	27.090

Table 10: Statistical Accuracy for Flow Rate Prediction For Different Choke Size Ranges (Root Mean Square Error)

	Choke size Ranges in 64th of inch (No. of points)				
Correlation	16-32 (271)	32-64 (1391)	64-96 (1079)	96-128 (607)	128-160 (206)
Gilbert	40.239	58.056	35.494	25.490	27.362
Ros	40.555	32.109	41.906	51.869	59.967
Omana	90.821	114.976	121.728	76.597	117.790
Fortunati	83.366	79.072	71.743	63.718	65.028
Osman	59.595	65.370	66.117	63.704	63.061
Surbey	137.683	163.803	46.194	30.244	35.981
This study	29.330	21.561	25.107	25.397	38.977

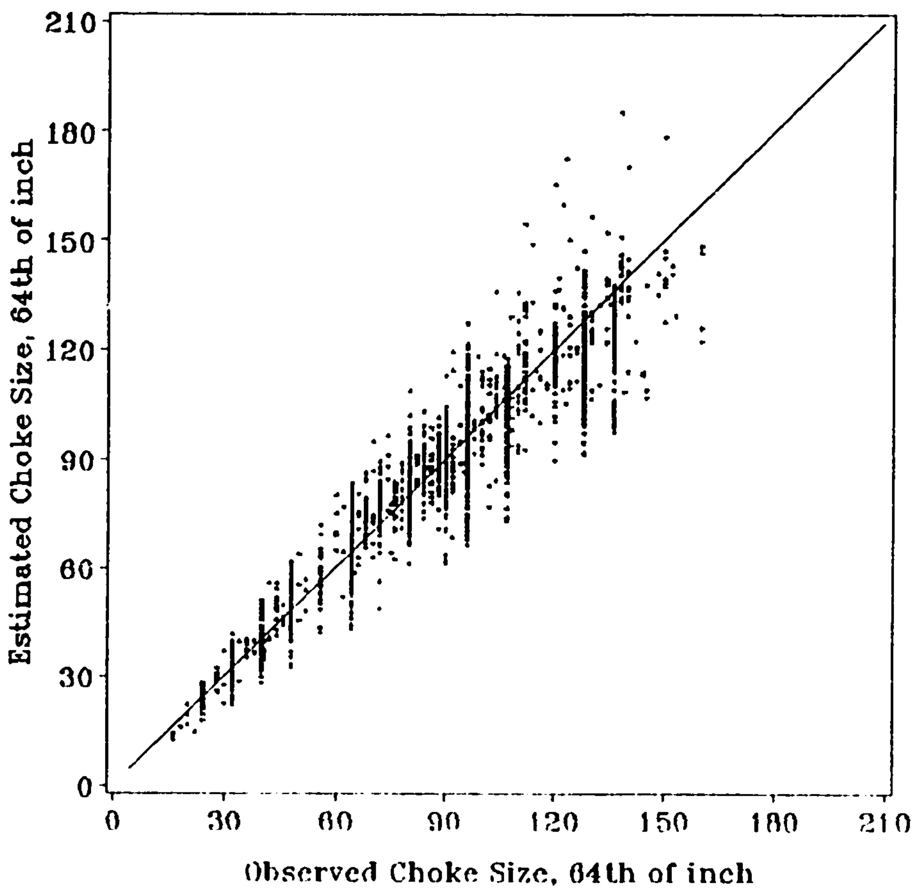

Fig.17: A Crossplot of The New Correlation for Choke Size

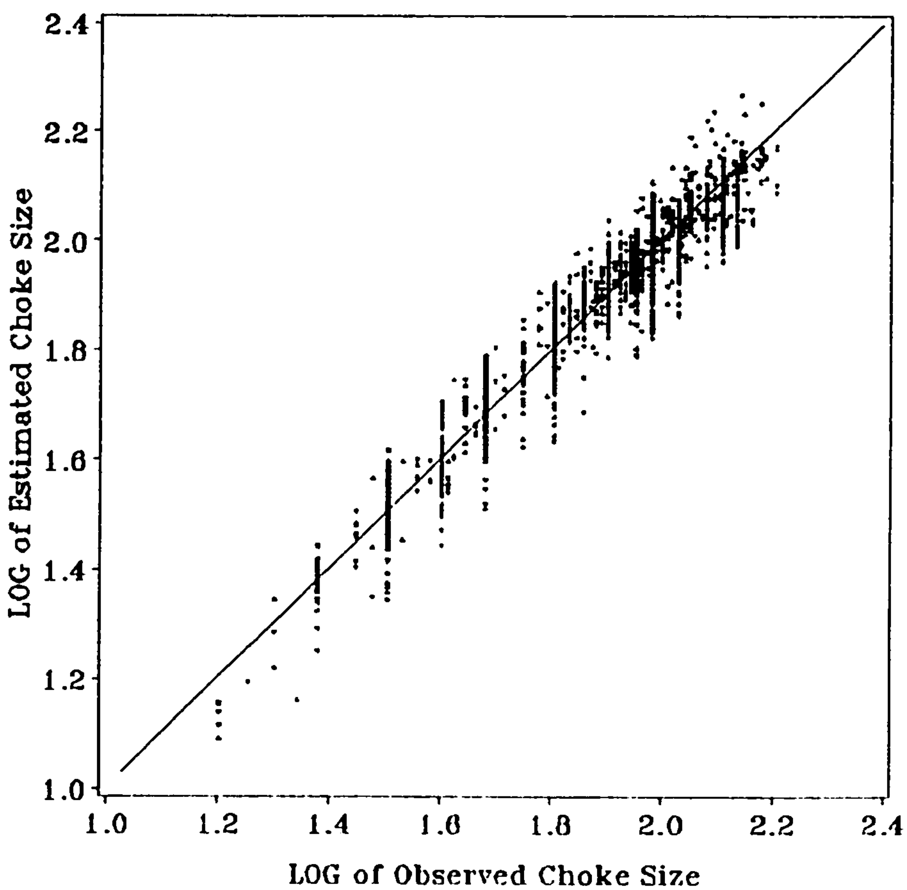

Fig.18: A Crossplot of The New Correlation for Choke Size

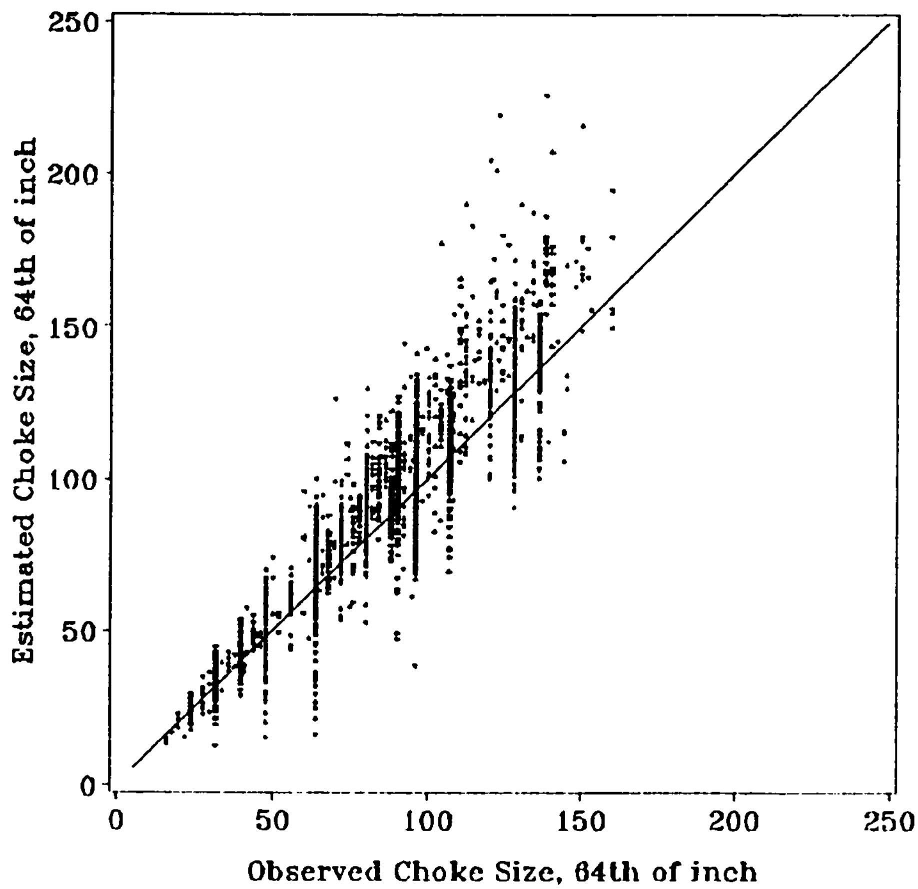

Fig.19: A Crossplot of Gilbert Correlation for Choke Size

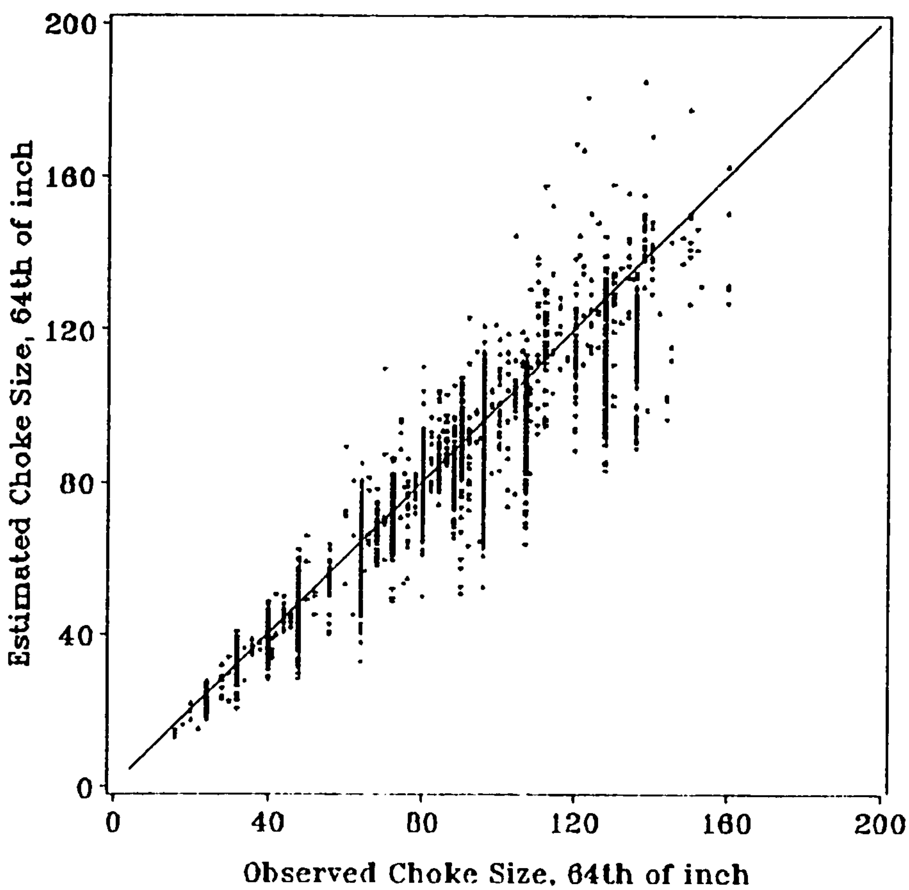

Fig.20: A Crossplot of Ros Correlation for Choke Size

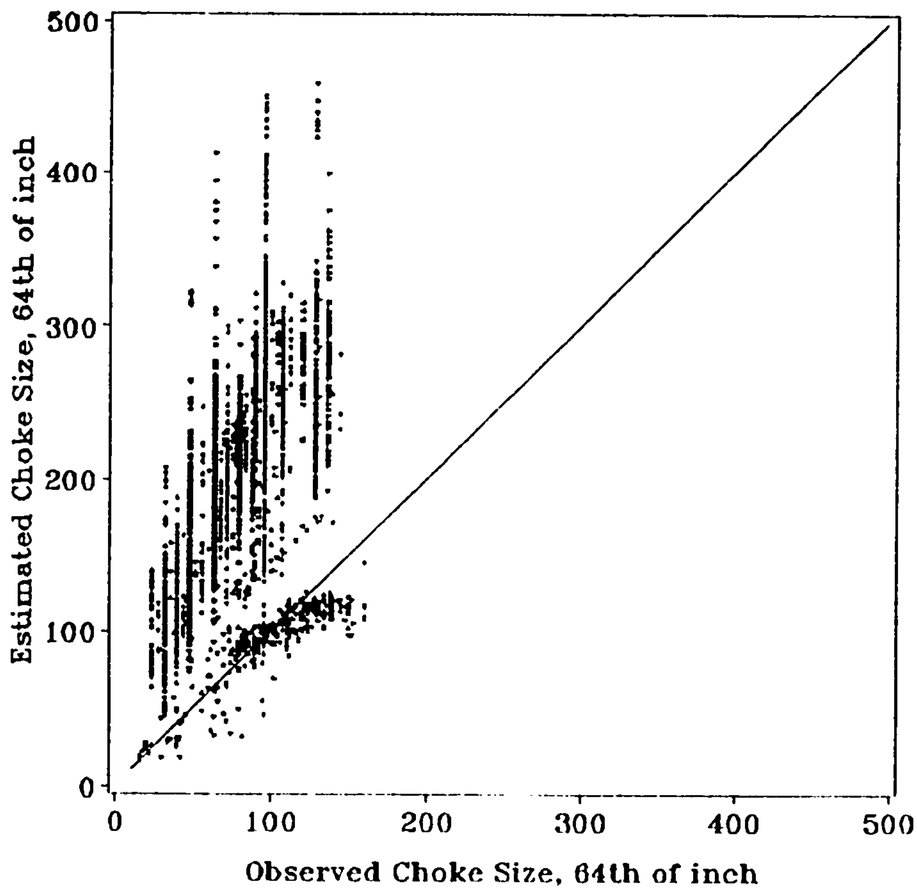

Fig.21: A Crossplot of Omana Correlation for Choke Size

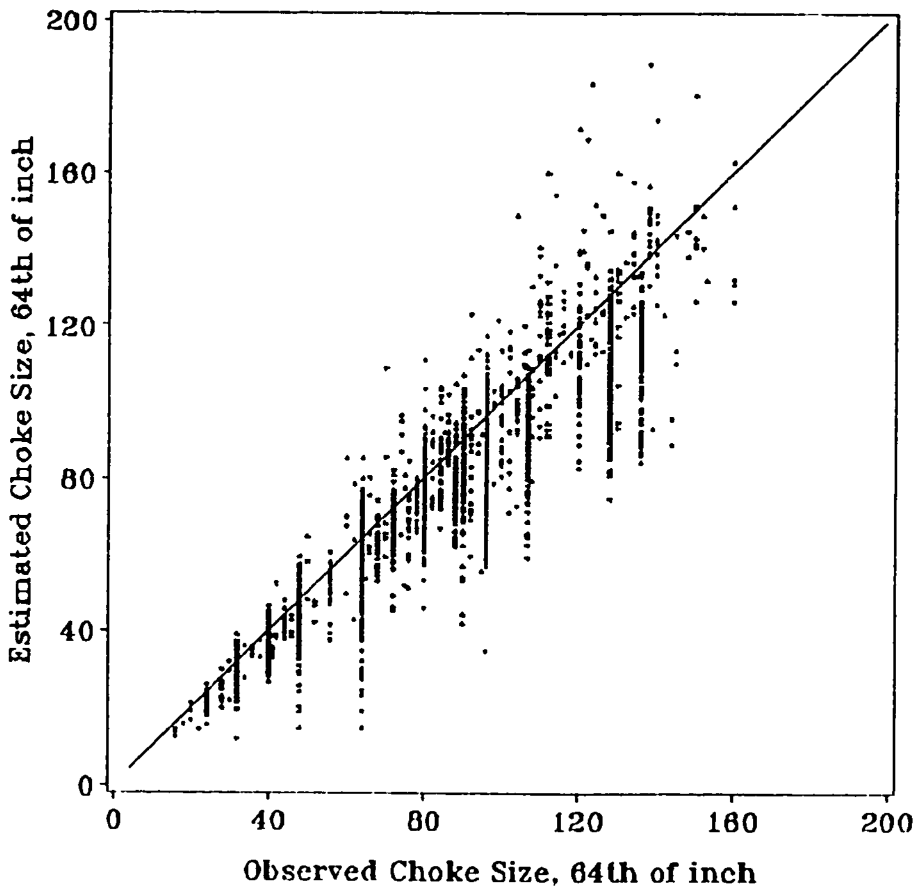

Fig.22: A Crossplot of Fortunati Correlation for Choke Size

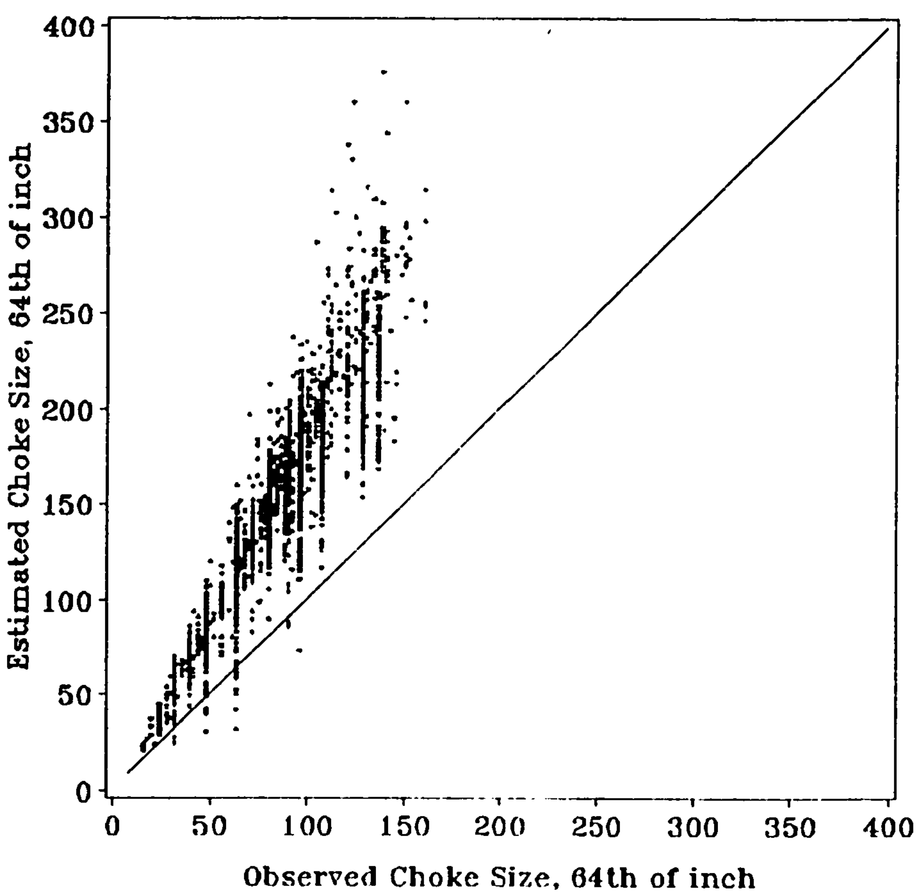

Fig.23: A crossplot of Osman & Dokla Correlation for Choke Size

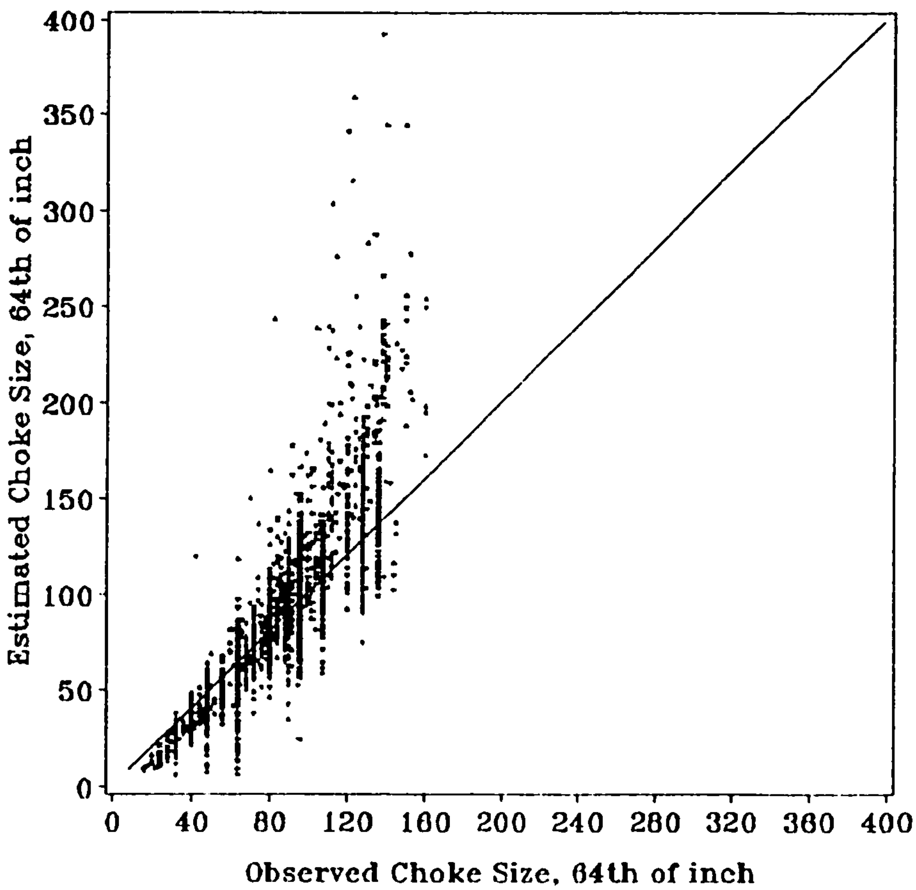

Fig.24: A Crossplot of Surbey Correlation for Choke Size

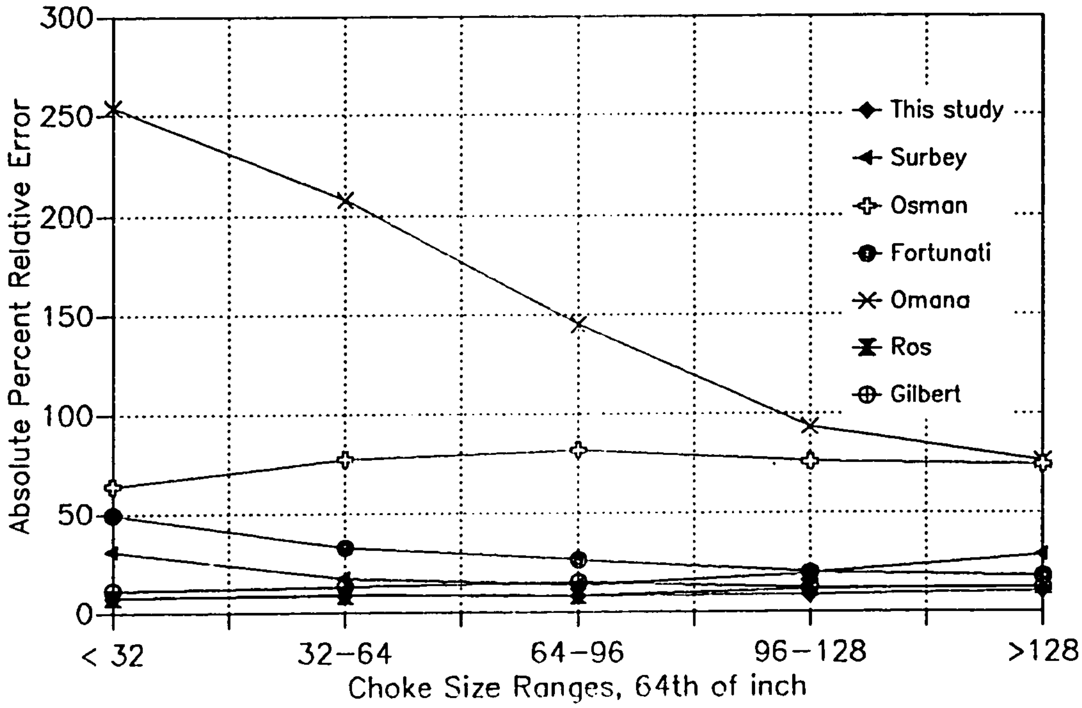

Fig.25: Accuracy of Correlations for Different Choke Size Ranges (Absolute Percent Relative Error)

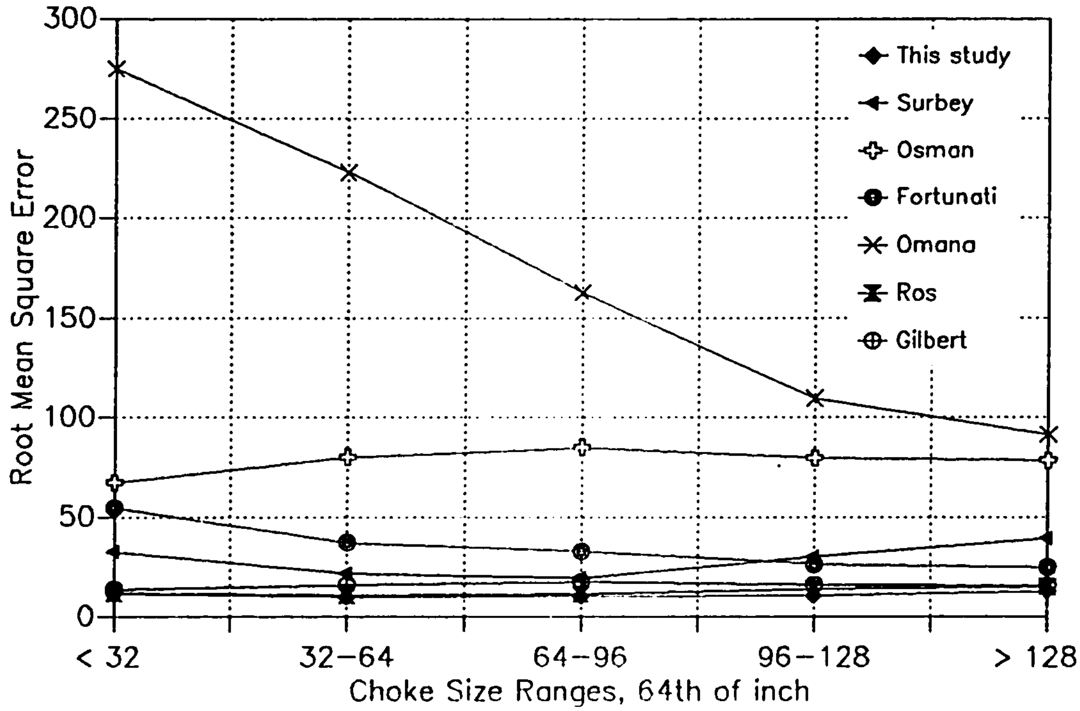

Fig. 26: Statistical Accuracy of Choke Size for Different Ranges of Choke Sizes (Root Mean Square Error)

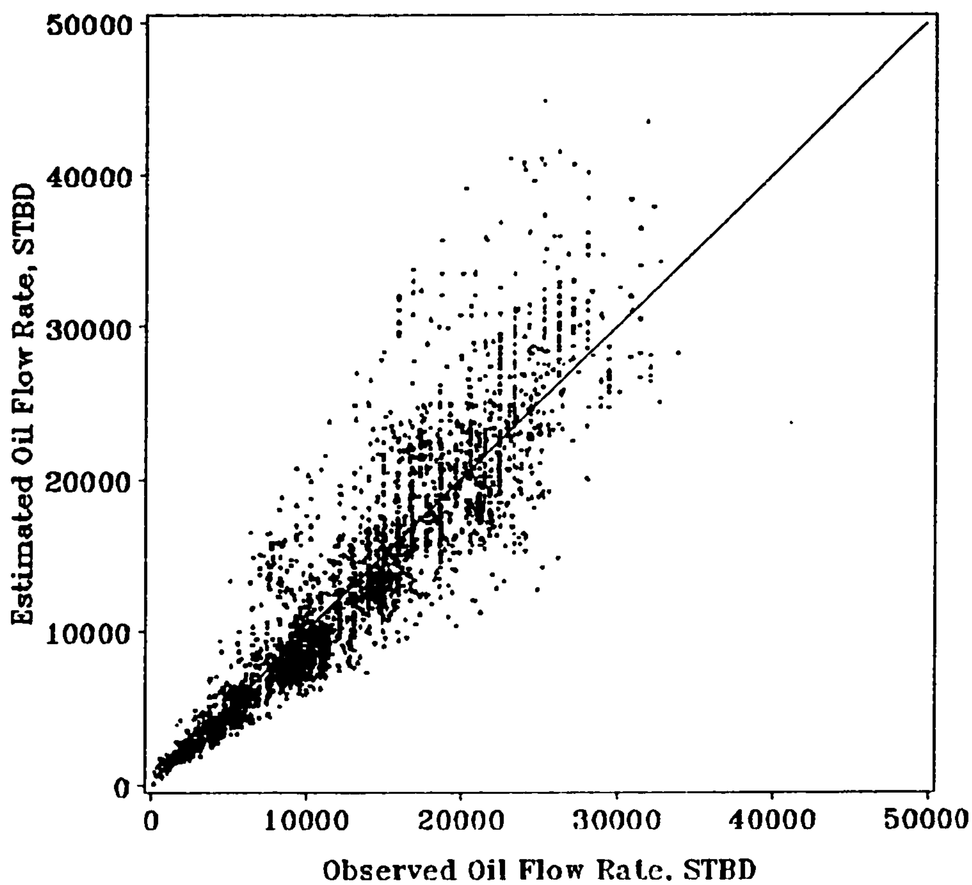

Fig.27: A Crossplot of The New Correlation for Oil Flow Rate

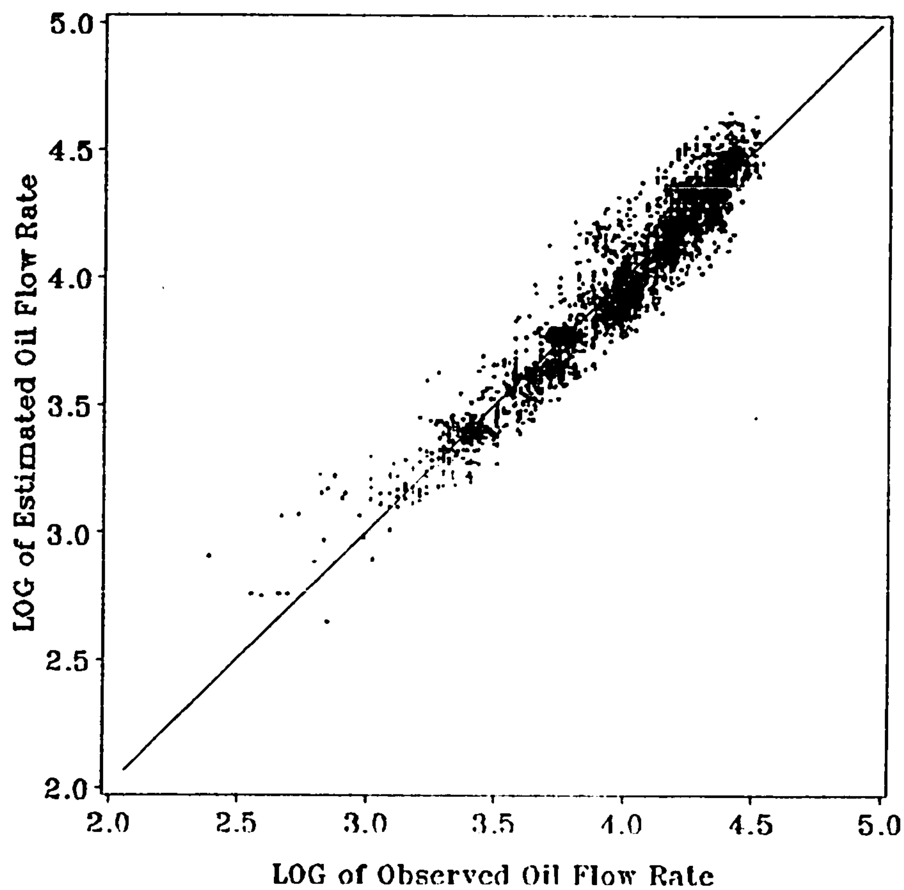

Fig.28: A Crossplot of The New Correlation for Oil Flow Rate

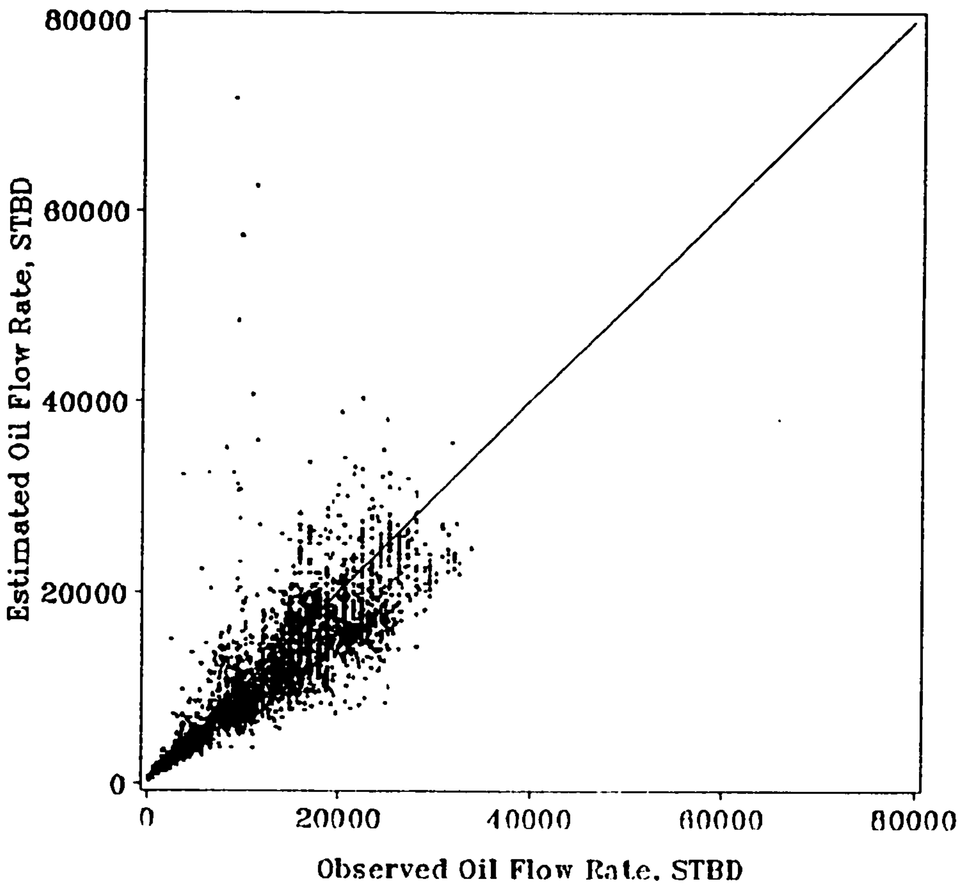

Fig.29: A Crossplot of Gilbert Correlation for Oil Flow Rate

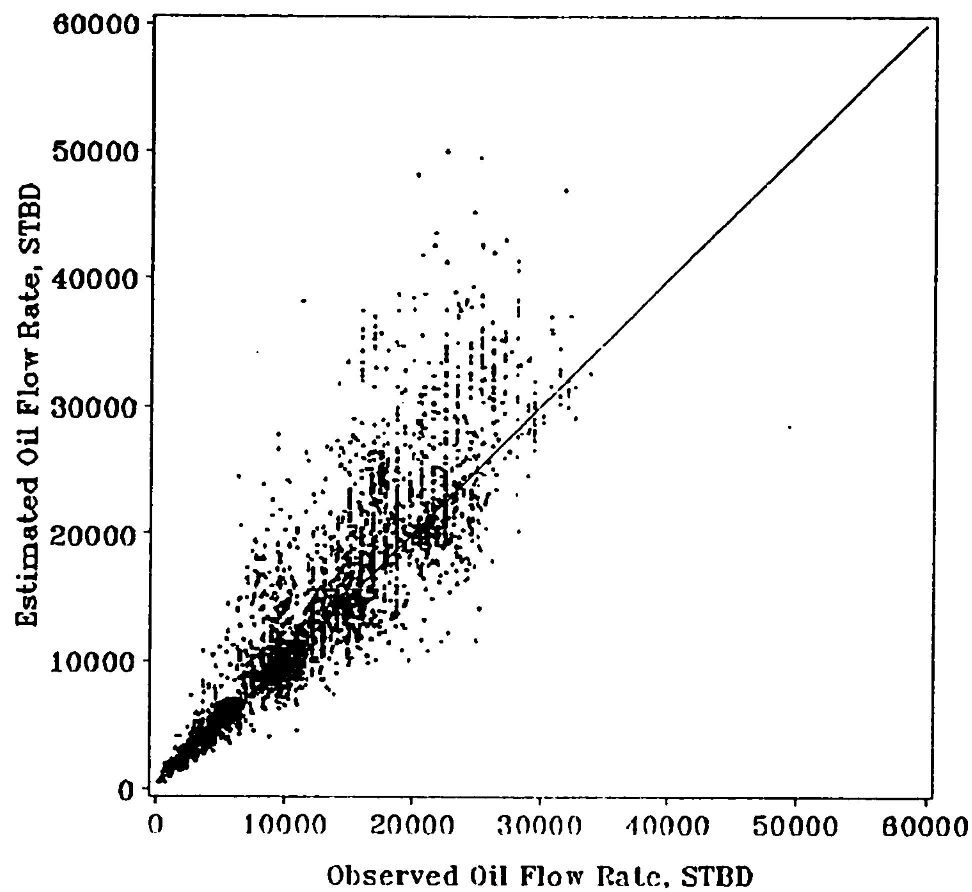

Fig.30: A Crossplot of Ros Correlation for Oil Flow Rate

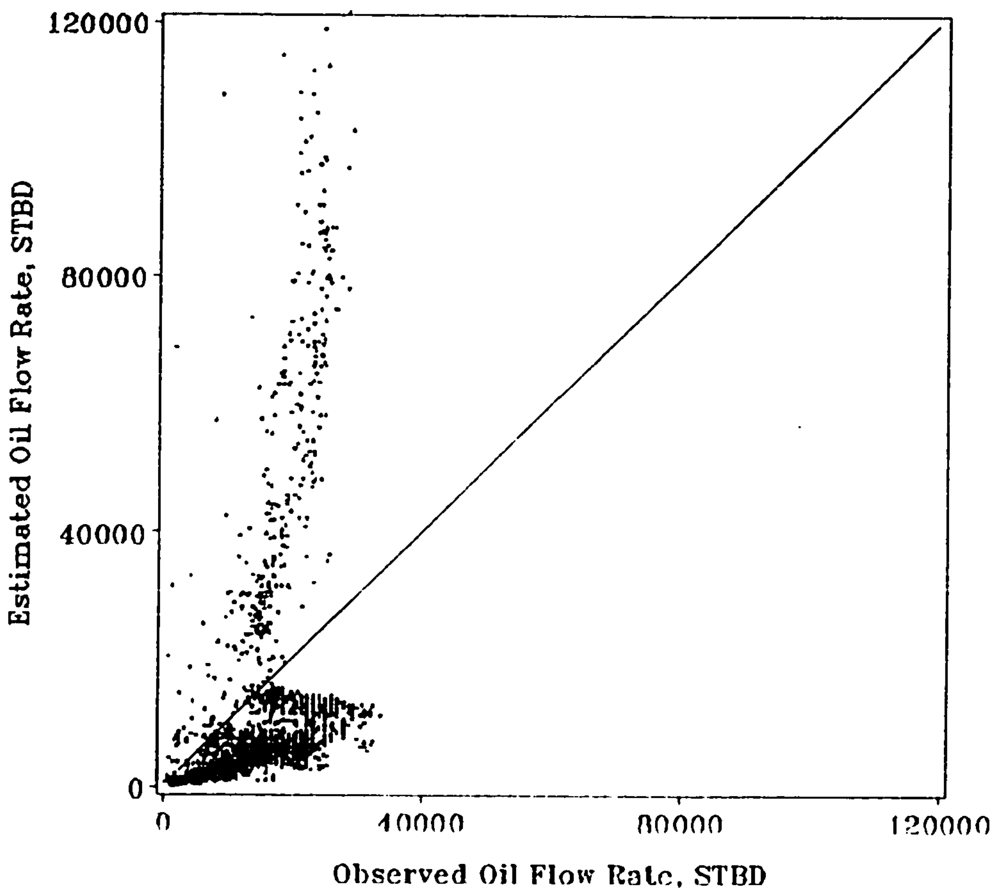

Fig.31: A Crossplot of Omana Correlation for Oil Flow Rate

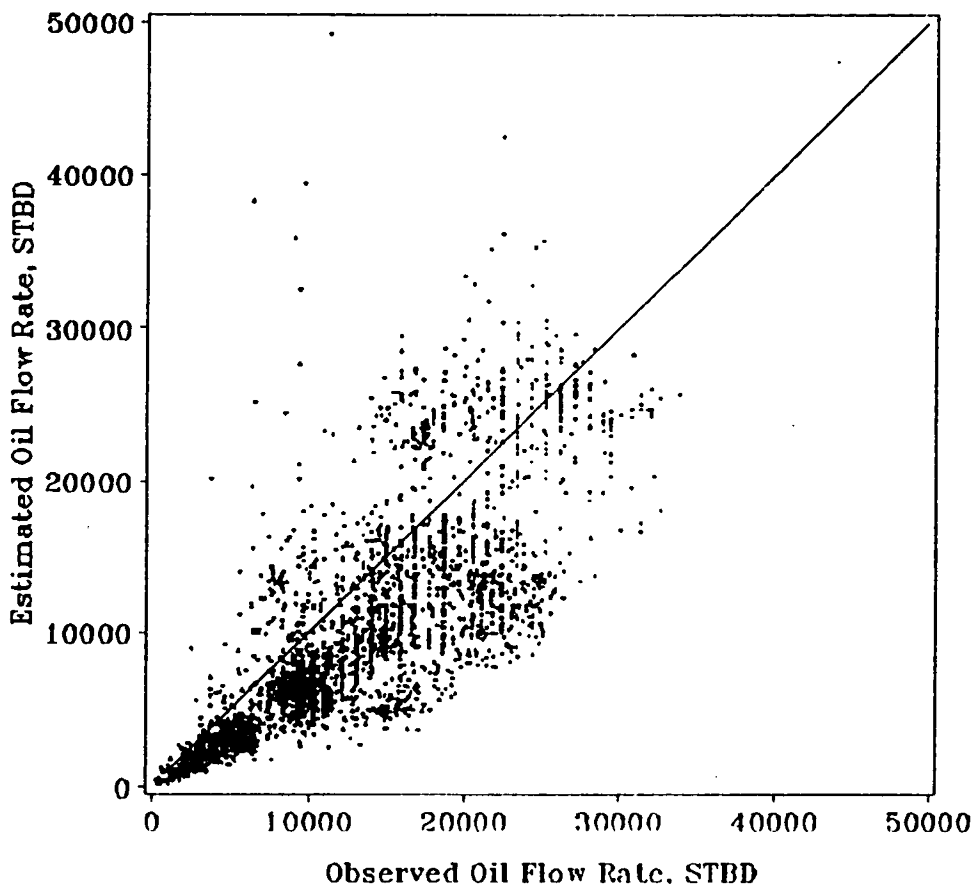

Fig.32: A Crossplot of Fortunati Correlation for Oil Flow Rate

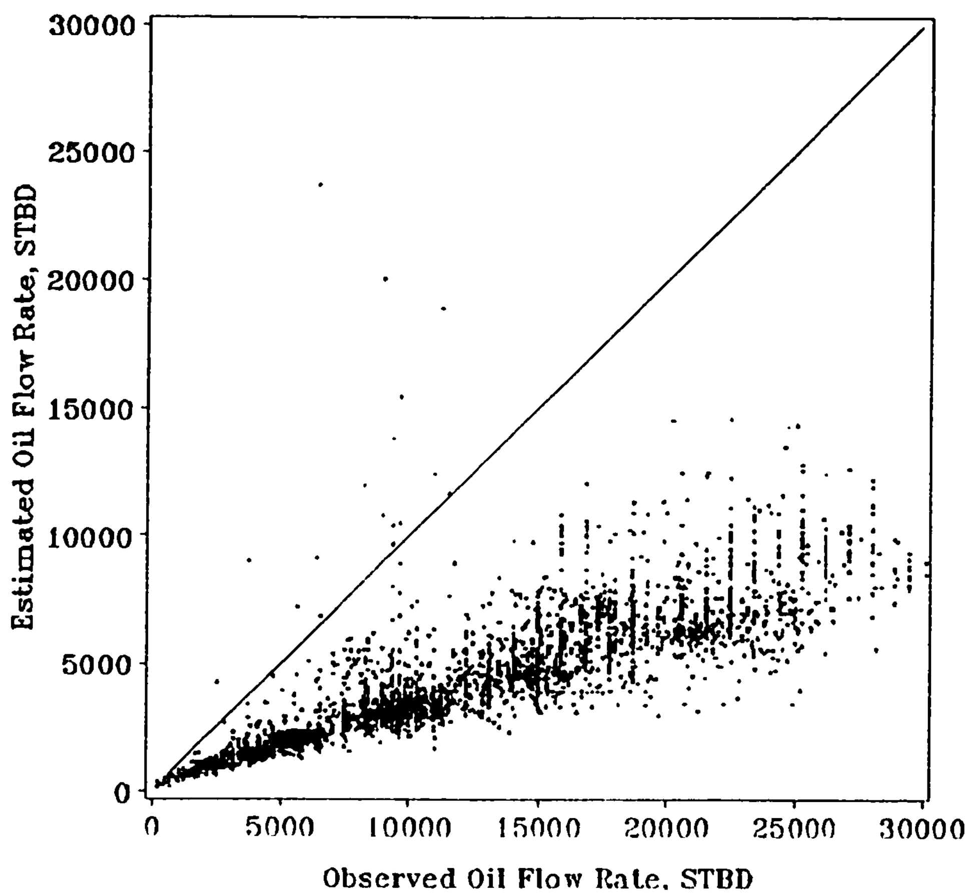

Fig.33: A Crossplot of Osman & Dokla Correlation for Oil Flow Rate

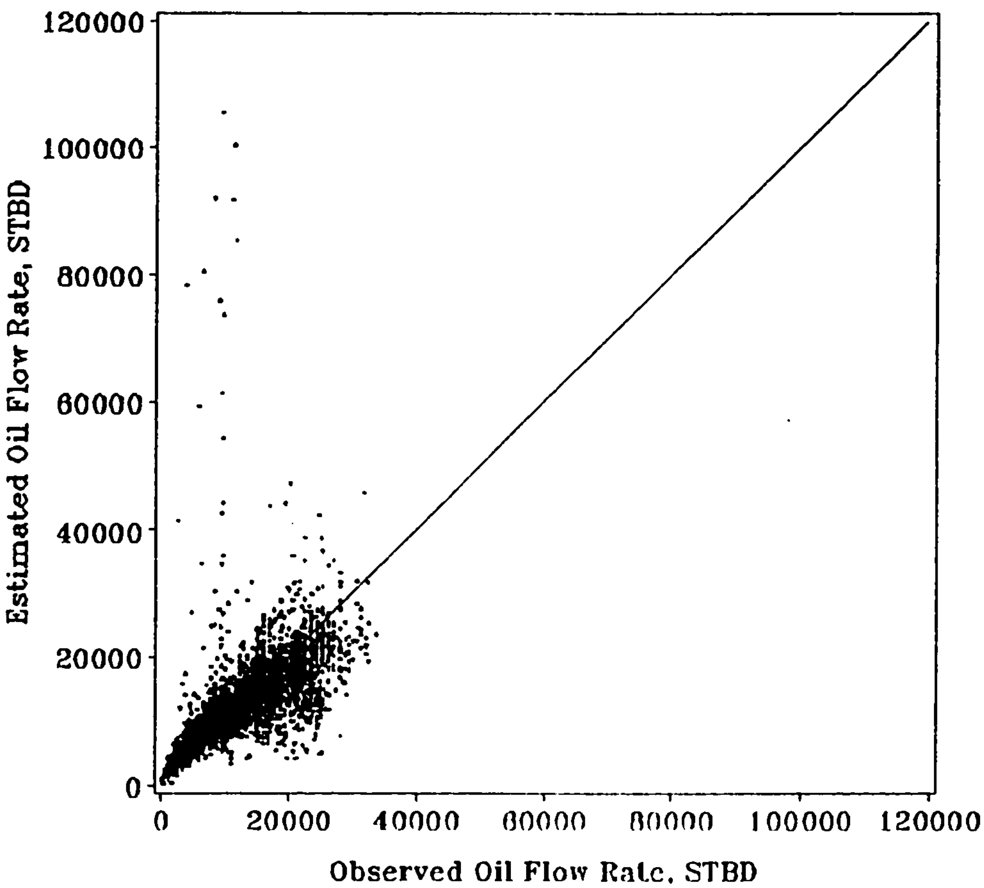

Fig.34: A Crossplot of Surbey Correlation for Oil Flow Rate

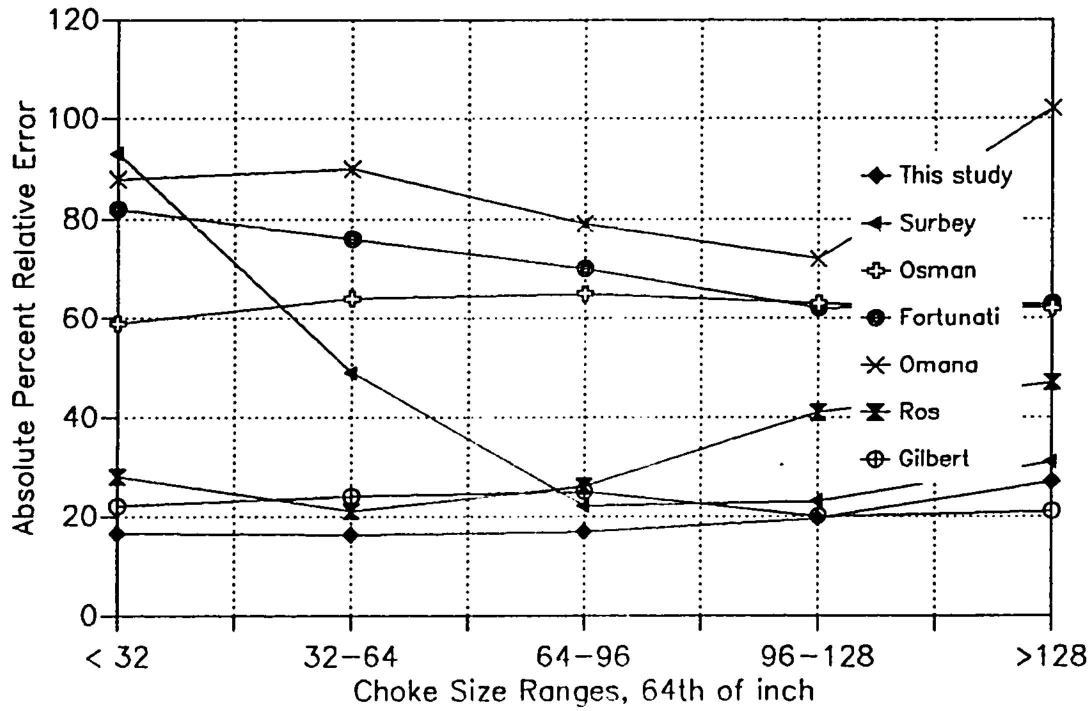

Fig. 35: Accuracy of Correlations for Flow Rate Prediction for Different Choke Size Ranges(Absolute Percent Relative Error)

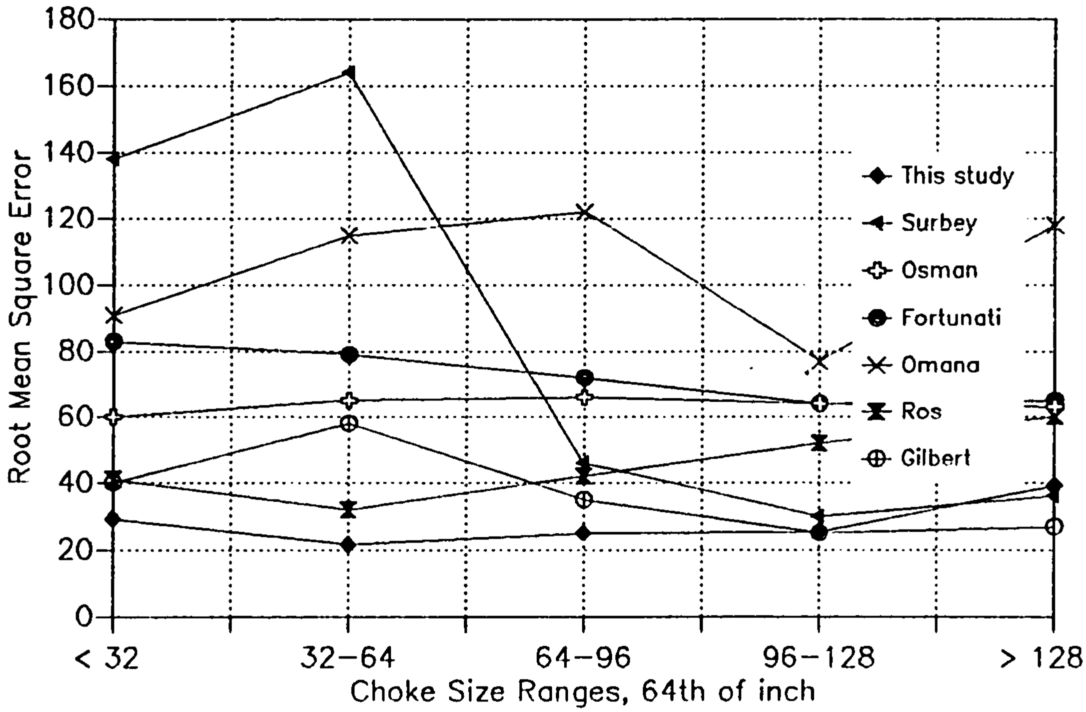

Fig. 36: Statistical Accuracy of Flow Rate Prediction for Different Choke Sizes (Root Mean Square Error)

CHAPTER 7

CHAPTER 7

VALIDATION

OF

THE NEW CORRELATION

In order to validate the new correlation, examine its applicability, and compare it to the existing ones, a further step was made when all the correlations were tested with data that was not used to develop the new correlation. Three groups of test data were used. First data group was published by Poetmann and Beck in 1963 (6). the second is recently published by Majeed-Ghassan (20) and the third is data from the Middle East fields for tests conducted very recently, therefore, they were not used in the new correlation.

7.1 Poetmann and Beck Test Data

The data is tabulated in table-11. Except for Fortunati correlation which requires the downstream wellhead pressure that is not available, all the other correlations were tested with this data. Although Gilbert predicted the best; However, the new correlation competes very well with Gilbert having very close statistical error. Table-12 summarizes these statistical errors.

7.2 Majeed-Ghassan Data

Table-13 shows Majeed-Ghassan data that was recently published. The new correlation outperforms all the correlations for this test data. Table-14 shows a summary of the statistical analysis performed.

7.3 Middle East data

The new correlation was developed for data from the Middle East and its best performance is expected to predict Middle East well performance. Three hundred data points were collected from production tests that were not included in the new correlation development were tested. Confirmed by table-15, the correlation is outperforming the other correlations.

Table 11: Production Test Data (after Poetmann & Beck)

API Gravity	P_{ut}	R_s	Choke Size	Flow Rate
36	7000	545	28	200
35	5000	385	21	130
42.0	987	715	12	227.7
42.4	881	1265	10	313.4
43	934	1215	12	312.5
42.2	798	665	12	254.8
42.4	840	965	12	342
43	950	915	10	325
43	736	715	12	240
50.6	3205	4374	12	760
51.3	3570	3965	12	575
51	3165	3887	12	644.4
48	1875	815	22	480
48.6	1988	565	14	206.6
50.8	1950	2440	8	260
49.8	1910	2740	9	255.7
47.5	1910	1391	16	544.5
49.4	937	265	20	215.3
50.1	544	815	24	1008
51	690	615	14	293
49.7	361	365	20	512
43.4	3002	2315	14	431
36.4	900	490	13	268
50.1	3150	915	14	236
32	465	515	12	306
32	175	565	14	343
32	556	515	16	557
32	657	315	40	1299
32	317	365	20	267
32	387	375	16	302
32	407	265	20	387
24	390	240	24	355
21	12800	965	23	475
30.2	184	178	21.8	405
30.2	217	172	18.4	313
30.5	132	175	21	500
30.5	221	287	21.8	398
30.8	6310	400	16	56
32.6	869	400	16	208
32.7	585	302	16	216
33	552	680	6	32
33	572	335	8	61
33	791	175	11	43
33	978	355	11	64
33	986	415	8	53
33	1057	210	8	31
33	1112	495	9	83
33	1235	635	8	51
33	1246	665	9	57
33	1660	715	7	49
33	1713	685	9	51
33	1852	175	11	56
33.1	900	325	16	224
34	464	1115	6	74
34	332	255	11	91

Continue- Production Test Data (after Poetmann & Beck)

API Gravity	P_{us}	R_s	Choke Size	Flow Rate
34	883	215	11	92
34	985	395	11	84
34	1020	435	10	88
34	1170	395	10	92
34	1382	615	10	93
34	1646	625	9	92
34	1818	655	8	79
35	413	815	4.6	52
35	556	865	4.2	37
35	580	865	4.2	40
35	8565	2015	4.6	24
36	494	840	4.4	27
36	514	815	4.4	39
36	535	915	4.6	46
36	584	890	4.8	38
36	586	865	4.4	43
36	586	865	4.6	51
36	586	915	4.8	59
36	614	995	4.2	38
36	629	1015	4.4	33
36	5814	2065	4.6	30
36	13759	2315	5.9	28
36.4	822	515	14	193
37	2835	265	16	49
37	3210	240	14	48
38.1	950	215	16	183
39.3	1100	315	18	180
42	1048	595	14	202
47	2020	715	10	94
47	4708	1445	12	148
47.1	4155	1465	8	60
47.2	4850	1515	7	50
47.5	2778	1215	14	232
48	925	565	10	102
48	2790	905	8	53
48	3807	895	8	50
48.1	1290	1315	8	115
48.1	1298	1415	9	144
48.4	1381	655	14	212
48.8	1209	819	14	280
49	526	585	14	207
56.3	196	576	12	513
44.4	2250	1264	6	60
49	2879	1025	13	164
33	1629	700	9	85.4
37	836	1200	4.8	59.3
35	8886	825	5.3	10.5
35	5705	2050	4.75	31.2
40	18594	1240	9	32.4
40	16923	1200	9	22.5
40	3361	700	7	39.2
31	188	168	10	67.0
35	2054	220	11	40.4

Table 12: Statistical Accuracy For Choke Size (Poetmann & Beck Data)

Correlation	E_r	E_a	E_{min}	E_{max}	E_{rms}
Gilbert	-3.83	12.08	0.05	55.34	16.22
Ros	-10.81	14.88	0.11	114.17	20.97
Omana	25.88	35.79	1.59	79.95	39.51
Osman	-54.22	54.31	4.59	115.80	60.34
Surbey	21.62	35.98	0.09	106.03	41.49
This study	1.14	14.47	0.19	57.96	17.84

Table 13: Production Test Data (after Majeed-Ghassan)

API Gravity	R_s	p_{us}	Choke Size	Flow Rate	Specific Gas Gravity
34	129.14	11383	32	834.8	.89
34	129.14	11328	32	830	.89
34	129.14	11707	24	300.52	.89
34	129.14	11273	24	302.11	.89
34	129.14	11935	20	214.66	.89
34	129.14	10342	48	1033.55	.89
34	129.14	10342	28	397.52	.89
40	142.5	8618.5	22	206.71	.75
40	142.5	12066	14	113.05	.75
30	129.14	11031.6	16	127.21	.70
20	178.13	4826.3	14	41.34	.80
20	178.13	5516	14	46.11	.80
20	178.13	6205.3	14	52.47	.80
20	178.13	6895	14	57.24	.80
20	178.13	8618.5	14	63.6	.80
20	178.13	10342	14	79.5	.80
20	178.13	12066	14	85.86	.80
40	106.88	1379	30	68.37	.68
40	106.88	1724	30	90.63	.68
40	106.88	2068.4	30	109.72	.68
40	106.88	2413.2	30	122.44	.68
40	106.88	2758	30	149.47	.68
40	106.88	3447.4	30	174.91	.68
40	106.88	4137	30	219.43	.68
40	106.88	4826.3	30	267.13	.68
40	106.88	5516	30	302.15	.68
40	106.88	6205.3	30	325.97	.68
40	106.88	6895	30	357.77	.68
40	106.88	8618.5	30	477.02	.68
40	106.88	10342	30	556.53	.68

Table 14: Statistical Accuracy For Choke size (Majeed-ghassan data)

Correlation	E_r	E_a	E_{min}	E_{max}	E_{rms}
Gilbert	-7.83	8.51	0.02	30.73	11.19
Ros	-8.74	8.77	0.41	28.34	10.79
Omana	-79.96	102.53	3.26	346.54	142.32
Fortunati	1.14	4.66	0.08	12.36	6.24
Osman	-68.06	68.06	43.61	105.87	69.64
Surbey	22.43	32.43	0.51	55.54	35.68
This study	-2.13	3.81	0.12	20.51	6.08

Table 15: Statistical Accuracy For Choke Size (Middle East Data)

Correlation	E_r	E_a	E_{min}	E_{max}	E_{rms}
Gilbert	-6.71	9.06	0.08	47.78	11.40
Ros	4.22	6.21	0.11	27.99	7.87
Omana	-170.28	170.60	2.17	447.13	183.02
Fortunati	-21.20	22.46	0.08	109.56	27.02
Osman	-74.17	74.17	34.71	130.62	75.68
Surbey	9.61	16.58	0.01	83.87	20.67
This study	-2.92	5.54	0.02	15.03	6.47

CHAPTER 8

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CONCLUSIONS

1. An empirical correlation to determine the optimum choke size for design purposes has been developed.

2. Statistical and graphical error analyses show that the new correlation is better than the existing correlations for both the data used to develop the new correlation and the data used to validate it.

3. The effect of mixture density, which was not included by previous empirical correlations, has been introduced in this study as an important parameter that should always be considered in the study of fluid flow mechanism in chokes.

4. The correlation could also be used to predict oil flow rate in the cases where measurment of flow rates is not handy or difficult to perform.

5. The correlation is applicable to wide ranges of flow conditions. High rates such as the ones in Saudi Arabia are within the correlaion range.

APPENDIX

SAMPLE CALCULATION

A choke need to be designed for a new well. The new well is expected to produce a maximum oil rate of 10000 STBD. The expected gas oil ratio in this field is 250 SCF/STB while the tubing wellhead pressure is in the range of 300 - 580 psia. The gas specific gravity is .9 while the oil API gravity is 30.

SOLUTION:

$$\gamma_o = \frac{141.5}{131.5 + API} = \frac{141.5}{131.5 + 30} = .876$$

$$\gamma_m = \gamma_o + 2.18 \times 10^{-4} R_s \gamma_g = .876 + 2.18 \times 10^{-4} \times 250 \times .9 = .925$$

$$S = 20.696 \times \frac{q_o^{.483} \times \gamma_m^{.707}}{R_p^{.474}} = 20.696 \times \frac{10000^{.483} \times .925^{.707}}{580^{.474}} = 82$$

The optimum choke size to handle this situation is 82/64 of an inch.

NOMENCLATURE

Symbol	Description
p	Tubing wellhead pressure, psia
R	Gas oil ratio, SCF/STB
v	Velocity, ft/sec
ρ	Density, lb/cu ft
f	Volumetric fraction
S	Choke size, 64th fraction of inch
q	Flow rate, STBD
C_D	Discharge coeffecient, dimensionless
A_c	Choke cross-sectional area, square inches
γ_l	Liquid relative density (Water = 1)
γ_g	Gas relative density (Air = 1)
g_c	Gravitional constant
B	Formation volume factor
x	Mass fraction of liquid
y	Mass fraction of gas
T	Tubing temperature, Rankin

Symbol	Description
z	Gas compressibility factor
C_v	Valve coefficient
Δp	Pressure drop across the choke, psi
F_m	Mixture correction factor, dimensionless
M_a	Area ratio multiplier
M_p	Pressure drop ratio multiplier
σ	Liquid surface tension, dynes/cm
F_{WO}	Water oil ratio (WOR)
k	Specific heat ratio
ε	Downstream / upstream tubing pressure ratio
m	Mass flow rate, lbm/sec
v_{tp}	Critical velocity, ft/sec
G	Mass flow rate, $lbm/ft^2/$ sec
E_a	Average absolute percent relative error
E_i	Relative deviation
E_r	Average percent relative error
E_{min}	Minimum absolute percent relative error
E_{max}	Maximum absolute percent relative error
E_{rms}	Root mean square error
n_d	Number of data points
n	Number of variables

Symbol	Description
s^2	Standard deviation
r^2	Coeffeicent of determination
r	Correlation Coeffeicent

Subscripts

g	Gas
o	Oil
w	Water
l	Liquid (oil + water)
m	Mixture (liquid + gas)
i	Initial
us	Upstream
ds	Downstream
s	Solution
p	Producing
c	Choke, critical

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