Heat Transfer Mechanisms Governing Laser Metal Interactions

by

Muhammad Sami

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

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DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the Requirements for the Degree of

MASTER OF SCIENCE

In

MECHANICAL ENGINEERING

June, 1993
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King Fahd University of Petroleum and Minerals (Saudi Arabia), 1993
HEAT TRANSFER MECHANISMS
GOVERNING
LASER METAL INTERACTIONS

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JUNE 1993
This thesis, written by

MUHAMMAD SAMI

under the direction of his thesis committee, and approved by all the members, has been presented to and accepted by the Dean, College of Graduate Studies, in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

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Date: 19-7-93
Dedicated to

My Parents,

Brothers and Sisters
Acknowledgment

In the name of Allah, Most Gracious, Most Merciful. Read in the name of thy Lord and Cherisher, Who created. Created man from a { leech-like } clot. Read and thy Lord is Most Bountiful. He Who taught { the use of } the pen. Taught man that which he knew not. Nay, but man doth transgress all bounds. In that he looketh upon himself as self-sufficient. verily, to thy Lord is the return { of all }.

(The Holy Quran, Surah 96)

First and foremost, all praise to the Almighty Allah Who gave me the courage and patience to carry out this work. I am happy to have had a chance to glorify His name in the sincerest way through this small accomplishment and ask Him to accept my efforts. May He guide us and the whole humanity to the right path (Aameen).

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Abstract

Name: Muhammad Sami

Title: Heat Transfer Mechanisms Governing Laser Metal Interactions

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The thesis describes the modelling of laser heating of metals for conduction as well as convection cases. Application of Electron Kinetic Theory in laser heating process, including heat conduction, melting and evaporation, is carried out. The energy equation developed using this theory is numerically solved for the appropriate boundary conditions. This makes possible to compare the results obtained from the classical work (Fourier Theory), previously studied, with the Kinetic theory results.

The energy equation resulting from the Electron Kinetic Theory is solved numerically using the explicit finite difference method. The stability problem is of concern and by careful choice of the space and time increments is controlled effectively. In the numerical scheme, forward differences are used for the derivatives.

In the final part of the work, the various laser pulses are considered and introduced in the energy equation. To achieve these various pulse profiles, a mathematical function describing the actual laser pulse is employed. Consequently the results obtained for continuous and pulse heating are compared. The effect of the laser pulse shape on the temperature profile is also studied and different pulse shapes are compared from the point of view of laser machining. This may provide useful information to laser designer to design more efficient laser beams required for laser machining.

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خلاصة الرسالة

الاسم: محمد سميع
عنوان الرسالة: مكتنات النقل الحراري المحكمة بتفاعلات المعادن بالليزر.
tخصص: هندسة ميكانيكا
تاريخ الشهادة: يونيو 1993م

الرسالة تصف نماذج تتسخين المعادن بالليزر في حالة التوصيل والانحلام. يتم تطبيق نظرية الاكترون الحركية في عملية التتسخين بالليزر، والمشكلة على التتسخين بالتصلب، وإضافة نتائج التحدي والتبخر. تم حل معادلة الطاقة، والتي طورت باستخدام النظرية الميكانيكية، عدداً لحالات حدود مناسبة. تلك الجيل من الممكن مقارنة النتائج المحصلة والمحددة من النظرية الكلاسيكية (نظرية فورير)، مع نتائج النظرية الحركية.

وتم حل معادلة الطاقة، التي يتم الحصول عليها من نظرية الاكترون الحركية، عندما باستخدام طريقة الفروق للحاسة ضمنية الانتشارية الصغر. إن مشكلة الاستقرار لهذة أهمية، ويوفر
التاريخ برامج اختيار درجات الحرارة والوقت، يتم التحكم بها بشكل فعال. في الإجراء الرقمي يتم استخدام الفروق المتقدمة لإيجاد الإدراك.

في الجزء الأخير من الرسالة يتم استخدام عدد نماذج للليزر التي تدخل في معادلة الطاقة. لتحقق هذه الأشكال المختلفة في النماذج يتم استخدام نمذجة البلازما الحقيقية للليزر عبر طريقة التحليل الرياضي. نتيجة لذلك يتم مقارنة النتائج التي يتم الحصول عليها لحالات التتسخين الحراري المتواصل والتبخر، وقد دراست تأثير 신اس البيزور على التوزيع الحراري، وعدد أشكال من النماذج يتم مقاولتها من وجهة نظر المكثنة بالليزر. هذا قد يزود معلومات هامة لتصميم الليزر حتى يتم
أشعة حرارية ذات كفاءة أعلى مطلوبة للمكثنة بالليزر.

درجة الماجستير في العلوم
جامعة الملك فهد للبترول والمعادن
الظهران - المملكة العربية السعودية
يونيو 1993م
Nomenclature

\( \alpha \)  
Thermal diffusivity  \( (m^2/sec) \)

\( \rho \)  
Density  \( (kg/m^3) \)

\( C_p \)  
Specific heat  \( (J/kg.K) \)

\( K \)  
Thermal conductivity  \( (W/m.K) \)

\( \delta \)  
Absorption coefficient  \( (1/m) \)

\( \lambda \)  
Mean free path of electrons  \( (m) \)

\( \bar{V} \)  
Electron mean velocity  \( (m/sec) \)

\( N \)  
Electron number density  \( (1/m^3) \)

\( k_B \)  
Boltzmann constant  \( (1.38 \times 10^{-23} \ J/K) \)

\( h \)  
Planck's constant  \( (6.626 \times 10^{-34} \ J.s) \)

\( T(s,t) \)  
Electron temperature  \( (K) \)

\( T(x,t) \)  
Phonon temperature  \( (K) \)

\( E_{s,t} \)  
Electron energy  \( (J) \)

\( E_{\tau,t} \)  
Phonon energy  \( (J) \)

\( I_o \)  
Laser power intensity  \( (W/m^2) \)

\( dx \)  
Spatial increment  \( (m) \)

\( dt \)  
Time increment  \( (sec) \)

\( m \)  
Mass of an electron  \( (9.12 \times 10^{-31} \ kg) \)

\( c \)  
Intensity scale factor
Chapter 1

Introduction

1.1 History of Laser Machining

Laser machining belongs to the large family of material removing or machining processes. It provides a feasible production method for the materials hard-to-machine and involves special applications such as micromachining, while it has limitations in some applications in terms of material removal rate and surface quality when compared with traditional machining methods. Nevertheless interest is growing in the use of lasers in welding, soldering, surface modification, marking, cutting, hole drilling and scribing. For example in the automotive industry in USA, CO\textsubscript{2} lasers in 5 to 9 kW power range have been placed in production for welding and flexible cutting of metal sheets[20]. This is due to the fact that laser beam possesses temporal and spatial energy properties which are different than the diffuse light.
The effectiveness of laser machining depends upon the thermal properties, and to some extent the optical properties rather than the mechanical properties of the material subjected to machining. Therefore, materials which exhibit a high degree of brittleness or hardness and have favorable thermal properties such as low thermal diffusivity and conductivity are particularly well suited for laser machining. Some of the advantages of using lasers in material processing are:

- No mechanical contact between the workpiece and the tool.

- Machining of parts which are situated in normally difficult to reach areas.

- Working in a variety of atmospheres.

- Minimal heat affected zones after machining.

- High speed processing.

- Easy to automate.

- High up-time.

- Minimal operator skill is required.

- Precise machining is possible.

Lack of tool contact eliminates the tool wear and makes favorable in applications where the tool force on the workpiece creates a problem. This also eliminates mechanically induced material damage and machine vibration. Since a laser beam is
coherent, it can be piped around a work area using mirrors and conduits and focused inside of cylinders or other hollow parts. With Nd-Glass or Nd-YAG lasers the beam can be transmitted through a transparent media such as through fiber or plastic cables before focusing on to the part. The laser beam used in material processing is not markedly attenuated in air and virtually any non reactant gas can be used as a shield gas, examples are N₂, He, Ar, and CO₂ [20]. A very high percentage of the energy absorbed by a part from a laser beam performs the function intended and very little portion appears as waste heat. Consequently very small heat affected zones (HAZ) are resulted. This implies minimal induced chemical change and reduced damage due to thermal cracking and distortion. In many applications, the laser can perform the operation faster than conventional or other exotic techniques. Because of the lack of tool contact and the ease with which a laser beam can be delivered to the workpiece, the laser is one of the most easily automated machine tool. Industrial lasers, in spite of the relatively short time they have existed, are highly reliable. Uptime for CO₂ lasers in heat treating and welding applications of 90 to 95% are routinely reported and results for other types of lasers should be comparable. The skills and training required for laser operators is minimal. Lasers are really very simple to operate once a system has been properly set-up. The following is a list of disadvantages that may apply in most cases:

- High initial cost
• Routine maintenance

• Skilled maintenance people

• Safety

Laser systems are expensive. The cost (in U.S. dollars) for a carbon dioxide laser runs from 150 $ per watt for a 500 watt laser to less than 100 $ per watt for multi kilowatt laser. Operating costs, including maintenance and consumables will run from 3 to 10 $ per hour or higher[20]. Lasers do require routine maintenance such as tube replacement, cleaning of optics or electrodes in some CO₂ lasers. This maintenance must be carried out carefully by properly trained and well supervised technicians. There are special safety considerations associated with lasers. Aside from the serious danger of electrocution from the high voltage power supplies used in these lasers, there is a severe risk of injury (particularly to the eyes) as a result of exposure to the direct, reflected or scattered radiation.

1.1.1 Applications

In early 1970's lasers were recognized as excellent tools for trimming resistor and capacitor hybrid networks. Repetitively pulsed Nd-YAG lasers, with pulses of short duration, trim thin film and thick film resistor materials[2]. Laser power can be controlled through a feedback control. Scribing of IC wafers is another early electronic application. Another material removal application is wire stripping of certain
tough types of insulation like kapton insulation which is used in aerospace applications. One of the most exotic material removal application involves making precision holes in cigarette filter-tip paper. Because most nonmetals absorb energy from the 10.6-μm wavelength CO₂ laser beam, a host of organic and inorganic material applications has opened up, from acrylic, polyethylenes, wood, rubber, and wool to diamonds. Hole drilling in plastics and rubber materials is one of the fastest growing applications. Both of these materials correspond well to CO₂ laser energy. Examples in plastics include hole drilling in aerosol valves, contact lenses, pipes, tubing and wire insulations. Rubber examples include hole drilling in baby bottle nipples and rubber seals. Plastic materials have been welded with CO₂ laser beams. Laser treatment of metals is by far the largest segment of industrial-laser applications. Two of the more popular metal treatment applications are welding and surface hardening, with steel and cast iron the most common metals treated. Other metals include aluminum, magnesium, titanium and many other metallic alloys. At the General Motors Saginaw steering gear division in Saginaw, Michigan, a photon sources a 1 kW CO₂ laser system has been hardening the power steering housings for all GM automobiles in the last four years[2]. The company is also using a GTE Sylvania 5 kW CO₂ laser to harden rail road engine cylinder liners at its electromotive engine facility in Lagrange-III. Seam welding of electronic components and packages is another growing application. At Medtronic Inc, a major manufacturer of heart pacemakers, circumferential seals of the pacemakers are welded using both CO₂ and
Nd-YAG lasers.

Another popular seam welding application is that of batteries, like the lithium ones being welded by a Nd-YAG laser at Honeywell, Minneapolis. Thousand of lead acid batteries used by the Bell's system power reserve grid are being seam welded at the gould industrial battery division using computer-controlled CO$_2$ laser system. One of the most common aerospace applications of laser is the drilling by ruby, Nd-YAG and Nd-Glass lasers of very fine intricate holes in aircraft turbine blades.

For products that are either too fragile or too small or that cannot be marked fast enough by conventional stamping, engraving, printing and scribing methods, the laser is a solution. Hundreds of marking, engraving and lettering applications for tool steels, automotive parts, aircraft wire insulations, and the like are springing up. Marking universal product codes on food containers is one of the newest applications.

The most obvious application of low power laser is alignment. These alignments could be spindle alignment or angular alignment. Gauging of work pieces is another application. Further more laser scanners have been highly successful for inspecting machined surfaces for defect such as rust, scratch, and pits. Lasers are also used in holographic interferometry to produce what is called hologram.
The first laser application in medicine was the removal of clusters of growth, called papillomas, on the vocal chords of a two years old boy. This was accomplished at Stanford University in 1963. In 1968 argon laser was used for the first time in delicate eye surgery at the Colombia hospital. Since then the argon lasers have become a common tool in eye surgery and other medical applications primarily due to its ability to stop bleeding. It is also used to operate inner ear, allocation almost as hard to reach through conventional surgery as the retina. The speed and precision of CO₂ laser are particularly beneficial where delicacy and quickness are vital, for example neurosurgery. Laser surgery cuts operating time by up to 50% eliminating much of the need for clamps, sponges and scalpel incision with resulting leading problems. Laser diagnosis is another area that looks promising. The list of medical uses for this versatile tool goes on and on.

Despite its multitude of constructive peace time uses, the lasers potentials as weapon caught the imagination of the news media and the military from the day of its inception. Space based laser weapons are currently the major focus of U.S. research. The army, navy and airforce all have programs under way to demonstrate the lethality of lasers against realistic air, ground and sea targets. The army uses lasers as range finders allowing accuracy never before possible. Navy uses laser detection system for targets like submarines. The airforce relies heavily on laser-assisted systems for the pin point accuracy needed in dog fights and air defense systems. In short, high
power lasers have given a new dimension to the state-of-the-art weapon system.

1.1.2 Lasers for Machining

The dominant lasers in material processing are the CO₂, Nd-YAG and Nd-Glass lasers with CO₂ lasers accounting for the largest percentage of sales. In this section, these three types of lasers and their output characteristics will be briefly described.

- Nd-YAG lasers

  This laser employs Neodymium (Nd) as the lasing material doped in a YAG crystal. Crystal size for a typical Nd-YAG laser ranges from 5-10 mm in diameter to 6-15 cm in length. Excitation is by one or two krypton or xenon lamps. The wavelength is 1.06 μm, which is close enough to the visible spectrum that conventional optics can be used for laser and windows. However, cavity mirrors must be of the dielectric enhanced refraction type to achieve sufficient reflectance. Nd-YAG lasers can be continuously operated from a few watts to several hundred watts but in most applications, pulsed operation is preferred. For low power application, such as marking or scribing (for example thin or thick film resistor trimming), q-switching is used to achieve peak powers of kilo watts at kilo hertz rate. For high power applications such as cutting of hermetic seam welding, electronic pulsing of the lamps is utilized to produce 5 to 10 pulses per seconds at tens of joule of energy per pulse and pulse lengths of 0.1 to 10 micro second depending on the application.
• Nd-Glass Laser

The Nd-Glass laser utilizes neodymium as the lasing, but the host interior is glass. Due to the poor thermal characteristics of the glass this laser is used chiefly for spot welding and hole piercing with pulse rate of 1 pulse per second and 30 to 50 joule per pulse. However more than one laser can be operated from the same power supply. Glass laser rods may be circular or rectangular cylinders. Rectangular cross section rods are frequently used for spot welding applications.

• CO₂ Laser

CO₂ lasers come in a variety of power ranges, sizes and designs. All use the molecular vibrations of CO₂ as a lasing mechanism. Generally, a mixture of CO₂, nitrogen and helium are employed, the nitrogen is active in the excitation process and helium acts as an internal heat sink. Some times a small amount of oxygen is added to reduce contamination from CO and carbon. CO₂ lasers emit radiation at 10.6 micrometer wavelength which is quite far into the infrared region. This causes problems with respect to the reflectance in processing metals such as copper, silver and gold, but alternatively these metals can be used as mirror materials internally or externally. Some small sealed off carbon dioxide lasers, such as the waveguide types, use RF excitation, but most use DC electrical discharge excitation. The three major designs used in industrial
processing applications are the slow axial gas flow with axial discharge, the fast axial gas flow with axial discharge and the fast transverse gas flow with transverse discharge. The slow flow design relies on thermal conduction for cooling of the gases and consequently the cross section area of the discharge region is limited. Roughly 50 to 70 watts of power per meter tube length can be obtained from this type of design. Beam quality is generally good with near Gaussian output being attainable because of the long narrow bore tube. The fast flow (about 60 meter per second) designs use convection cooling so that much larger discharge cross sections can be achieved. Hence the power per unit length is much higher, 600 watts per meter for fast axial flow and 2500 watts per meter for transverse fast flow. In fast transverse flow designs the beam is folded back and forth through the discharge region several times. Unstable resonator configurations are frequently employed for multi kilowatt CO₂ lasers to eliminate transmissive optics. The efficiency of these CO₂ lasers is approximately 10% (total input power divided into useful output power). This is about three times the efficiency of the other two industrial lasers described above.

In general laser machining produces parts with higher dimensional accuracy and surface quality than those produced with conventional processes, and with higher removal rates. Materials that can be machined by lasers include metals, ceramics, plastics, wood, glass and rubber. Laser drilling can produce holes as small as a few
microns in diameter in workpieces at a rate of 1 ms/hole. It is used in industry for producing holes in turbine blades, combustion chambers, and aerosol nozzles, among other applications[7]. Laser cutting is used to produce intricate two dimensional shapes in workpieces made out of materials such as sheet metal and paper upto 15 mm thick with high cutting speeds[7]. Laser scribing has been used to create channels in ceramic substrates for cooling and identification labels in finished parts. Finally three dimensional laser machining has been used in a research effort to implement turning, milling and threading operations. In many cases, lasers offer a way to perform drilling, cutting or shaping operations where conventional machining processes fail. In other cases, it offers an alternative method which improves machining speed, surface quality and/or dimensional accuracy.

1.2 Basics of Laser Machining

Laser machining can be divided into one, two or three dimensional processes by differentiating the kinematics of the erosion front during beam-material interaction. All laser machining processes exhibit common characteristics such as molten layer formation, possible plasma formation and beam reflection from the erosion front. Since the laser beam is directional heat source it can be viewed as a one dimensional line source with a line thickness equal to the focussed beam diameter. In the case of the one dimensional process (drilling), the laser beam is stationary relative to the
work piece. The erosion front, located at the surface of the drilled hole, propagates in the direction of the line source in order to remove the material. In the case of the two dimensional process (cutting), the laser beam is in relative motion with respect to the work piece. Material removal occurs by moving the line source in a direction perpendicular to the laser beam axis, thereby forming a two dimensional surface. The erosion front is located at the leading edge of the line source. For three dimensional machining, two or more laser beams are used, and each beam forms a surface through relative motion with the work piece. The erosion front for each surface is found at the leading edge of each laser beam. When the surfaces intersect, the three dimensional volume bounded by the surfaces is removed.

Three major issues in any laser machining process occur. These are:

- material removal.
- dimensional accuracy.
- surface quality.

The material removal rate is governed in each case by the propagation speed of the eroded front. In laser drilling (a one-dimensional process), material removal rate is determined by the speed that the eroded front moves in the beam direction. In laser cutting (a two dimensional process), the scanning velocity determines the rate at which the two dimensional surface increases in the work piece. In three dimensional
laser machining, two dimensional surfaces, produced by two laser beams, define a three dimensional volume of material to be removed. The speed at which these two surfaces propagate determines the time required to remove a given volume of material.

Dimensional accuracy is determined particularly by the hole geometry for laser drilling, kerf geometry for laser cutting, and groove shape for three dimensional machining.

Surface quality for all laser machining processes can be related to the factors such as surface roughness, dross formation, and heat affected zone[7].

1.2.1 Drilling (One-Dimensional Laser Machining)

Laser drilling involves a stationary laser beam which uses its high power density to melt and subsequently vaporizing the material from the workpiece. In principle, laser drilling is governed by an energy balance between the irradiating energy from the laser beam and the conduction and convection heat into and from the workpiece, i.e. the energy losses to the environment, and the energy required for phase change in the workpiece. The incident beam energy has a spatial intensity distribution, generally a Gaussian distribution. The radius of the beam is usually specified as the distance between the beam center and a point where the intensity
is reduced from its maximum value by a factor of $e^2$ as shown in figure 1.1. This industrial laser beam has an average diameter of around 250 $\mu m$ in the TEM$_{\infty}$ mode.

The average diameter of the drilled hole may be greater than the beam diameter due to various heat loss effects. These losses, primarily conduction to the interior of the workpiece and losses to the environment, divert beam energy away from the actual hole drilling process. These losses are due to a number of physical phenomena that occur during the laser drilling. These phenomena include

- If the material removal process involves melting, the molten material may accumulate along the side and bottom of the hole, causing laser beam energy to be lost in two ways. First, energy may be expended to superheat the
accumulated molten material in the hole above the melting point. Second, in percussion drilling where a sequence of pulses is used, the molten material may resolidify between successive pulses, therefore a portion of the beam energy is used to remelt the resolidified material during each pulse[7].

- Plasma formation may occur when material is vaporized. This absorbs the incoming beam energy and increases its temperature. In some cases, the heated plasma acts as a secondary heat source which improves the drilling process[7]. However, the directionality of the plasma is difficult to control, and this causes dimensional accuracy problems.

- The absorption of laser beam energy depends on both the wavelength of the radiation and the spectral absorptivity characteristics of the material processed. Some metals such as aluminum and copper exhibit high reflectivity for CO₂ laser radiation so Nd-YAG lasers are more effective[7]. Also for metals and ceramics, the presence of a molten layer changes the absorptivity value. Furthermore, the absorptivity of a surface depends on its orientation with respect to beam direction. For deep holes, multiple beam reflections may occur along the wall of the hole, thereby decreasing the availability of beam energy for material removal[7].

- The use of a gas jet during laser drilling may aid in cooling the erosion front through convection heat transfer. In situations where a high pressure gas jet
is used, a supersonic gas flow is formed and the thermal dissipation to the jet may become significant. Hence, more beam energy is required to maintain the melting/vaporization temperature at the erosion front[7].

Laser drilling has several advantages over mechanical methods:

- Due to the thermal nature of the laser drilling process, holes can be formed on materials which are difficult to machine with conventional methods, such as ceramics, hardened metals and composites.

- High accuracies and precise small diameters can be achieved. Depending on the focusing lens used, hole diameters between 0.018 mm and 1.3 mm are achievable[7].

- High drilling rates can be achieved in a production environment by using a pulsed beam source. By coordinating workpiece motions with the pulse period, drilling rates above 100 holes per second can be achieved. The controlling of process variables allows rapid changes in hole diameter and hole shapes to be made in-process, eliminating the need for tool changes[7].

- The laser allows holes to be drilled at high angles of incidence to the surface (upto 80 deg). Shallow angle drilling is difficult to achieve mechanically due to tool deflection.

However, in some cases, there are limitations:
- Holes with stepped diameters can not be drilled by a laser beam.

- Due to instabilities in the laser drilling process, depth control in blind hole drilling is difficult. However, continuous monitoring of beam mode and beam power regulation can provide substantial benefits when incorporated into the laser drilling system in the form of a controller.

- For deep holes, the effects of beam divergence may become important. This can be compensated for by using a longer focal length lens or by continuously moving the focal point from the workpiece surface to a point at the workpiece interior.

A variation on laser drilling, among others is the use of a reactive gas to enhance the process. In this case, chemical reactions between the workpiece and the reactive gas become important for the secondary material removal mechanism.

### 1.2.2 Cutting (Two-Dimensional Laser Machining)

In the laser through-cutting process, a kerf is created through relative motion between the beam and the workpiece. The physical mechanisms are similar to those for drilling. However, due to the relative beam/workpiece movement, the erosion front formed in front of the laser beam and the temperature field in the workpiece are stationary with respect to coordinate system moving with the laser beam; therefore laser cutting can be considered as a steady state thermal process. Since the work-
piece thickness is equal to the depth of cut, heat conduction occurs in the plane of
the workpiece surface. The width of the cut is determined largely by the laser beam.
The temperature inside the workpiece is dependent on the distance to the erosion
front[7]. The accumulated molten material at the erosion front can be expelled out
from the bottom of the kerf with the aid of a coaxial gas jet.

The laser may be used for cutting by two different techniques. In laser fusion
cutting, the material is melted by the radiation energy of the laser, the melt being
blown away by a gas. Alternatively, the material may be eroded by vaporization.
An inert or slow reacting shielding gas is required for combustible materials and for
oxide-free cutting of metals. In laser oxygen cutting a metallic material is heated
to its ignition temperature by the radiation energy of the laser and combusted in
a stream of oxygen. A fluid slag is produced which is ejected from the kerf by
the oxygen jet. The exothermic reaction provides additional energy and the cutting
speed can be substantially increased. In addition, the oxidation of the metal surface
in the presence of the hot focussed laser spot provides greater energy absorption of
the laser radiation by the metal.

Laser cutting has several advantages:

- For most industrial materials with workpiece thicknesses upto 10 mm, laser
cutting produces a significantly higher material removal rate than the mechan-
ational cutting or shearing.

- **Kerf widths are narrower than those achievable with mechanical cutting resulting in less material wastage.**

- **Shapes can be cut from curved workpieces. Conventional systems can cut flat workpieces effectively. Lasers can be applied to trimming operations to remove flash and burrs from curved parts.**

- **For cutting of fibrous material such as wood, paper, or composites, the laser beam vaporizes the volume of material to be removed thereby eliminating the residue and debris which remains after mechanical cutting. This eliminates the need for solid waste collection and disposal and reduces the health hazard in the work environment.**

The disadvantages of laser cutting are:

- **Laser cutting effectiveness decreases with increase in workpiece thickness.**

- **It produces a tapered kerf shape, compared to the straight vertical kerf walls achievable with conventional methods. The taper is a result of the beam divergence and becomes more pronounced as the workpiece thickness increases. This taper can be reduced by focal point adjustment of the laser beam to the interior instead of on the workpiece surface.**
1.2.3 Laser Welding (Two Dimensional Laser Machining)

Laser welding is especially useful when it is essential to limit the size of the heat affected zone (for instance, for metal-semiconductor welding), to reduce the roughness of the welded surface and to eliminate mechanical effects. Lasers are generally used for welding multilayer materials in which there are discontinuities in thermal properties at the interfaces where the layers come into contact. Generally, thin plates are joined by direct welding of the surfaces or by coating the surfaces with low melting point metal and subsequent welding. Application of a low melting point metal reduces the effect of the instability of thermal resistance at the contact. This coating must be sufficiently thick to eliminate the adverse effect of the variation in the gap width on the welding process. In addition, the coat thickness must be limited to prevent critical dependence of the welding process on the variation in the power intensity. The quality of welds can be considerably improved by preparation of the contact surfaces.

There are basically two types of laser welding, Continuous Wave(CW) and Pulsed. Seam welding is done both by CW CO₂ and over lapping pulses with Nd-YAG lasers[21]. Spot welding is done by CO₂, Nd-YAG, Nd-Glass and to a lesser extent, ruby lasers.
Continuous Laser Welding

At present continuous seam welding is done only with CO$_2$ lasers, usually of 500 W or higher power level. Theoretical approaches that give reasonable agreement for conduction limited welding are simple energy balance, the uniform irradiance model, and numerical finite difference or finite element approaches. The biggest problem with calculations in this regime is the uncertainty in the reflectance of the molten metal which may approach that of the solid metal.

Cover gases play an important role in laser welding as well. However, some unique problems are associated with cover gases and their method of application in laser welding and therefore the method of application varies with the material being welded and the quality of the weld required. The most frequently used cover gases are He, Ar, and nitrogen, each having certain advantages over the others. The cover gas is applied by means of a tube aimed directly at the weld location if bead oxidation is not a serious problem. A strong cross flow of air when welding materials that spatter vigorously like rimmed steel or high carbon steel is required. The flow is placed far above to avoid molten bead disturbance and dilution of the cover gas.

Pulsed Laser Welding

Pulsed laser welding comprises microwelding but more frequently applies to spot welding of larger parts. Pulsed laser welding is done with Nd-YAG or Nd-Glass.
lasers. Nd-Glass lasers are well suited for low repetition rate spot welding whereas Nd-YAG lasers are used for high speed spot welding or seam welding. Excellent hermetic welds are made by overlapping pulse welding with Nd-YAG lasers.

Basically the setup for welding with Nd-YAG or Nd-Glass lasers does not differ greatly from CO₂ laser welding. Cover gases are used and cross flow may be required to protect the optics.

The main advantages of laser welding are:

- Minimum heat input which results in very little distortion.

- Small heat affected zone because of the rapid cooling.

- Narrow, generally good looking weld bead achieved.

- High strength welds.

- Easily automated process.

- Welding some metals which are difficult to weld by other techniques, especially dissimilar metals.

- Welding in difficult-to-reach areas.

- Faster than other techniques.
The disadvantages of laser welding are:

- Extremely hard bead in hardenable materials; cold cracking and hot cracking may cause mechanical problems due to rapid heating and cooling.

- High capital investment (though it may be more than offset by improved part) quality, high up-time and low operating cost.

1.2.4 Turning/Milling (Three-Dimensional Laser Machining)

This method uses two intersecting laser beams to remove a volume of material. Unlike laser cutting, each beam creates a blind groove in the workpiece through single or multiple passes. A volume of material is removed when the two grooves intersect. The material removal rate is related to the depth of each groove. The groove depths define the boundary of the volume of material removed, and the scanning velocity defines the rate at which this volume is formed on the workpiece. The surface quality is related to the heat affected zone and surface roughness at the groove surface during beam-material interaction. Dimensional accuracy is related particularly to the taper angle for each of the two grooves.

Similar to laser grooving, laser scribing creates a blind groove on the surface of a workpiece. However, in laser scribing, the ratio of groove depth to groove width
is close to one and the groove depths are typically very small.

Three dimensional laser machining has the following advantages:

- **It can perform turning, threading and milling operations on difficult-to-machine materials.**

- **Laser can be used to scribe or mark permanent identification patterns on metallic or ceramic parts. Laser marked ID’s can withstand greater amounts of wear than those marked with other methods.**

- **Since lasers can be focussed to a small spot, they are ideal for micromachining applications to repair defective integrated circuit components which would otherwise be scrapped.**

The disadvantages are:

- **These processes result in molten material accumulation at the erosion front.** However unlike in laser cutting, a coaxial gas jet is not effective for ejecting molten material due to the presence of the groove bottom. The use of an off-axial jet can minimize the molten layer.

- **The groove depth can fluctuate due to disturbances caused by laser beam changes, mechanical vibrations, material impurities, and gas jet fluctuations.** Unevenness in groove depth can decrease surface quality and the mechanical
strength of the finished part. Consistency in groove depth can be maintained
by using a closed loop control scheme.

1.3 Scope of the Present Work

The application of laser as a machine tool requires knowledge of the interaction
between the laser beam and the workpiece. Such interactions are highly complex
involving initial heating, melting, evaporation, subsequent ejection of liquid glob-
ules and formation of plasma, and in addition, the effects of plasma and vapor on
the incident beam before it reaches the workpiece. Since all these effects are tem-
perature dependent, an analysis of laser-workpiece interaction must begin with the
determination of temperature distribution within the workpiece.

The basic law governing heat conduction is the well-known Fourier's heat conduction
law. The law, however is based on empirical observation and is not derived from
other physical principles [5]. In the analysis of Fourier Theory, the heat transfer
medium is assumed to be homogeneous and the heat flux through a given plane is
also considered as being a function of the spatial temperature distribution at that
plane. This depends on the assumption that the temperature gradient remains con-
stant between two successive and closely spaced planes. The distance between these
two planes is finite, so errors occur when higher order terms, which were neglected,
become important at high intensities. Thus, the absorption depth of laser irradiation is of the order of the mean free path of electron which is comparable with the interatomic spacing, so that over the scale of distances require to examine this problem, the material can no longer be considered as being homogeneous continuum. The heat flux through a given plane depends on the electron energy distribution through the material.

For these reasons, a new mathematical model for laser heating applications is needed. Yilbas[40] discussed the validity of the application of Fourier Theory in laser heating of metals and showed that the heat conduction mechanism must be examined on a microscopic level thereby indicating the need of Electron Kinetic Theory analysis. The model developed using this theory describes the transport of energy by electrons.

An extensive literature survey reveals that considerable work has been done but most of the workers have used the Fourier theory in their analysis. Using Fourier Theory, Yilbas and Apalak[41] considered the steady state evaporation in one dimension and obtained closed form solutions for temperature field. Relationships between the absorbed power intensity and the temperature and the recession velocity of the surface for different target materials were obtained. They also developed a time unsteady model using the Fourier Theory. Barillot[4] employed implicit Finite Difference Method (FDM) to simulate the transient heating and ablation of a
two dimensional axisymmetric target by a laser beam. He also employed the coordinate transformation scheme of Landau to immobilize the complex moving boundary. Bashenko et al.[29] investigated the effect of concentrated heating with CO₂ laser on the variation of the structure and properties of the surface layer of workpieces of low machineability materials subjected to cutting. Using Finite Element Method (FEM) and experiments, he modelled the process of laser assisted machining and classified the nature of laser heating required in machining materials various grades. Similar analyses for tool steels are done by Brover et. al.[35] using metallographic x-ray diffraction analysis. Chen and Subrata[6] developed a mathematical model for the temperature distribution in bone due to laser irradiation in order to estimate the adequacy of the laser heating of bone and minimize damage to bone tissues. They found the temperature dependence on the density, specific heat and thermal conductivity of the osseous tissue. Abraham and Halley[1] calculated the heat flow in an absorbing film which is exposed to Gaussian laser beam. They analysed the effects of film thickness, thermal conductivity and forced cooling on temperature profiles. Temperature distribution measurements for a Mg-silicate ceramic exposed to Continuous Wave (CW) laser are also done by Jeanloz and Heinz[15]. Similar studies on sheets of aluminum alloys are reported by Mikheev et. al.[22]. They experimentally determined the nature of radiation energy distribution in the heating spot and the values of the effective absorption coefficients. Using a mathematical model, temperature fields were calculated taking into account the real values of the
parameters and compared with experimentally determined temperature distribution. Mehrabian et. al.\cite{24} considered the rapid melting and solidification of the surface layer of a semi infinite solid subjected to step function stationary uniform in Gaussian heat flux distribution over a circular region. They carried out some preliminary experiments on aluminum-4.5\% copper specimen. The mathematical details of their work can be found in \cite{25}. Details of similar analysis with moving source are quoted in \cite{26}. Melvin\cite{19} found steady state temperature rise induced by a laser beam with a Gaussian radial exponential distribution and absorption. Using non-linear conduction equation, he found an integral representation of the spatial distribution of temperature rise. Surko et. al.\cite{32} studied the dynamics of the melting of the surface of a semiconductor during annealing with pulsed laser. They included the temperature dependence of thermal and optical properties. Surface temperature as a function of time was also calculated. Gnanamuthu et. al.\cite{18} obtained analytical solution for temperature distribution in semi infinite uniform medium exposed to a uniformly intense rectangular shaped moving laser beam. Similar analyses were done by Tan, Ho and Tsu\cite{34} using a Gaussian laser beam. In their analyses certain approximations such as linear heat flow and negligible latent heat effects were used. Newstein et. al.\cite{12} solved the Boltzman equation numerically to describe the transition layer next to an evaporating surface. Roth and Cantello\cite{30} evaluated the laser hardening of a turbine blade using both a mathematical model based on Fourier Analysis and experiment. Kawasumi\cite{16} analytically determined the tem-
perature distribution inside a hardening material for the case of a rectangular heat source.

On the other hand, very few researchers have used the Electron Kinetic theory in their analysis. In 1986, Yilbas[38] presented a new model for the conduction mechanism in laser heating of metals. Using quanto-mechanical approach, he obtained solutions within an accuracy greater than 90 percent. Harrington[13] has examined the case employing so many unnecessary assumptions such as he assumed without necessity an exponential temperature distribution at the surface and this causes the solution to fail by over-constraint.

Being aware of the extent of validity of the Fourier theory in laser heating applications, we therefore would refer to the Electron Kinetic Theory to develop a mathematical model for laser heating applications.

The fundamental understanding of the laser-solid interaction is still in a developing stage. The physics behind the problem is complex and microscopic analyses of the interaction must be carried out in order to fully understand the phenomenon. In the present work, this basic knowledge applied to laser heating of metals shall be developed using the Electron Kinetic Theory approach.

In the proposed work, application of Electron Kinetic Theory in laser heating, melt-
ing and evaporation will be carried out. The energy equation developed using this theory will then be solved for appropriate boundary conditions. This will make the classical work (Fourier Theory), previously done, possible to compare with the Kinetic Theory work. The solution shall also predict the velocity of surface front which is developed due to laser beam.

The energy equation resulting from the Electron Kinetic Theory analysis shall be solved numerically using the well known explicit finite difference method. The stability problem is of concern and by careful choice of the space and time increments can be controlled effectively. In the numerical scheme, forward differences will be used for the first derivatives.

In the final part of the work, laser pulse will be introduced in the energy equation. To achieve this, a mathematical function describing real laser pulse will be employed. Consequently continuous and pulse heating will be compared. In addition, attempt will be made to predict the pulse repetition rate required for similar continuous laser beam processing. The effect of the shape of the laser pulse on the temperature distribution will also be studied and different pulse shapes will be compared from the point of view of laser machining. This will help the laser designer to design more efficient laser beams required for laser machining.
Chapter 2

Mathematical Modelling of Laser Heating Mechanism

2.1 Electron Kinetic Theory Approach

In the analysis of the laser machining mechanism which have been carried out, to date the central feature has been the use of the conduction equation of Fourier to describe the way in which photon energy from the laser beam is transferred to the molecular lattice of the target material. It is this energy transfer mechanism which defines the laser-material interaction process. It is of the greatest importance in the analysis of laser machining of materials to accurately specify the surface temperature of the irradiated material since material removal rates are closely related to evaporation, which is essentially a surface phenomenon. It was shown that the Fourier
equation is only valid within strictly limits and is not valid in the analysis of heat transfer process which occurs with laser irradiated materials\([40]\). In the analysis of the classical Fourier heat conduction theory, the heat transfer flux through a given plane is also considered as being a function of the spatial temperature gradient at that plane. This depends on the assumption that the temperature gradient remains constant between two successive and closely spaced planes. The distances between these planes is finite. So errors occur when higher order terms which are neglected become important at high intensities. Thus the absorption depth of laser radiation is of the order of the mean free path of electrons, which is comparable to the inter-atomic spacing, so over the scale of distances required to examine this problem the material can no longer be considered as being a homogeneous continuum. The heat flux through a given plane depends on the electron energy distribution through the material.

For these reasons a new model is needed (for laser heating applications). The basis in examining the problem using a kinetic theory to describe the transport of energy by electrons. Some useful assumptions are made for simplicity. Steady state space charge is assumed, in which exactly the same number of electrons are emitted from the material as are returned from the space charge; energy losses due to thermionic emission can thereby be neglected. Energy transfer processes occur due to electron-phonon collisions; immediately after the collision the electrons may change their
directions, but the electron flux remains constant in any direction. During the collision some fraction of the excess electron energy is transferred to the phonons. In the solution, local equilibrium is assumed to prevail at $t = 0$ when material is at a uniform temperature. Wilson[36] showed that the interaction of the electrons with the lattice vibrations can be assumed to be weak at room temperatures without an applied external field. Another assumption is that the mean free path of electrons is not affected by the temperature change. Deby and Peierls[17] have showed that the mean free path is proportional to $\frac{1}{T}$ at temperature higher than the Deby temperature, but for metals the effect of temperature on the mean free path is small.

In the analysis, the electron-electron collision was assumed to occur in such a way that before and after the collision, the colliding electron has the same kinetic energy, and consequently the same momentum, i.e. collisions are assumed to be elastic[38]. Harrington[13] has examined the case although his analysis contains a significant error. Furthermore, he assumed without necessity, an exponential temperature distribution at the surface which causes the solution to fail by overconstraint. Riley[28] has attempted to introduce a semianalytical solution, using the Laplace transform, but he faced mathematical difficulties. Bakewell[3] modified this model by assuming thermal equilibrium during the collisions, which is that the energy transfer process has not taken place until the temperature equilibrium is achieved between electrons and phonons. This assumption can no longer be considered when the time required
for the collision is compared with the electron relaxation time (i.e. electron decay time)\[9].

2.2 Analysis of Heating and Conduction Processes at a Free Surface According to the Kinetic Theory

To examine the heat conduction mechanism, a free electron model is considered in the quantum field. Free electrons have certain velocities and move freely in the surface region. Electrons in the bulk can change their energies with respect to the Fermi distribution law. Molecules are situated in the lattice, having phonon energies associated with their vibrational motion. When the lattice vibration is considered, the displacement of an atom at the lattice point \( s \) can be expressed as,

\[
A_s e^{(g^3 - w_s t)}
\]

where \( A_s \) is the amplitude of the motion. At temperature \( T \) the average energy associated with this mode is given by Plank's formula\[9].

\[
E = \frac{\hbar w_s}{e^{(\frac{\hbar w_s}{k_B T})} - 1}
\]

The vibrational mode at the temperature \( T \) corresponds to the low portion of the
The energy of \( N \) molecules in one direction due to lattice vibration is,

\[ E_z = N_z k_B T \]

which can be described as the phonon energy.

For solids, the heat capacity is meant to be the heat capacity at constant volume, which is more fundamental than the heat capacity at constant pressure. The heat capacity at constant volume is defined as,

\[ C_v = T \left( \frac{\partial s}{\partial T} \right)_v = \left( \frac{\partial E}{\partial T} \right)_v \]

where \( s \), \( E \) and \( T \) are entropy, internal energy and temperature of the substance respectively.

After considering Plank's formula, it can be written as[38],

\[ N k_B = \rho C_v \]

The thermal resistance of substance in the quantum field is of interest where the electronic thermal conductivity is concerned. In general the thermal conductivity is usually defined with respect to steady state heat transfer through a homogeneous medium in a random process. Thus the thermal conductivity in one direction can be concluded as being[37].

\[ K = \frac{N \bar{v}_z k_B \lambda}{6} \]
Figure 2.1: Electron movement after considering mirror image at the surface $(X=0)$

It has been shown that for temperatures higher than the Deby temperature, thermal conductivity can be assumed to be constant[17]. In the analysis, a difficulty arises due to the reflected electrons from the surface. However, this is overcome by introducing a mirror image at the surface which is given in figure 2.1. In figure 2.2, the number of electrons leaving the section I with area $A$ in a time $dt$ in $x$ direction can be written as,

$$N_x A \bar{V}_x dt$$

After consideration of figure 2.1, the probability of an electron travelling a distance $x$ without making a collision is seen to be,

$$exp\left(-\frac{x}{\lambda}\right)$$
Figure 2.2: Electron movement in the metal (one dimensional case)

where $x \ll \lambda$.

The probability of an electron having just collided in $dx$ is,

$$\frac{dx}{\lambda}$$

With reference to figure 2.2, electrons may travel by either of the two paths "II", "I" or "III", "I". The probability of electrons which last collided in $dx$, travelling (II-III-I) is,

$$\exp \left[ -\frac{|x+s|}{\lambda} \right] \cdot \frac{ds \cdot dx}{\lambda \cdot \lambda}$$

In this argument, electrons which may escape from the surface are not considered.

Hence the total collision probability of electrons can be written as

$$\int_{-\infty}^{\infty} \exp \left[ -\frac{|x-s|}{\lambda} \right] \cdot \frac{ds \cdot dx}{\lambda \cdot \lambda}$$
The negative bound of the integral is due to a mirror image at the surface because of reflected electrons from the free surface.

The net transfer of energy during the electron-phonon collision through the entire body can be written as,

\[ \Delta E_{x,t} = \int_{-\infty}^{\infty} \exp \left[ -\frac{|x-s|}{\lambda} \right] \cdot \frac{ds}{\lambda} \cdot \frac{dx}{\lambda} \cdot f(E_{s,t} - E_{x,t}) \]

where \( E_{s,t} \) and \( E_{x,t} \) are the energy of electrons and energy of atoms at a considered region respectively. The parameter \( f \) used in the above equation represents a fraction of the electron energy and shall be discussed shortly.

Let us now consider a collimated beam of perfectly monochromatic radiation of unit cross sectional area passing through an absorbing medium. We assume for simplicity that there is only one relevant electron transition, which occurs between any two energy levels. Then the change in irradiance of the beam as a function of distance is given by

\[ \Delta I(x) = I(x + \Delta x) - I(x) \]

For a homogeneous medium \( \Delta I(x) \) is proportional both to the distance travelled \( \Delta x \) and \( I(x) \). That is

\[ \Delta I(x) = -\delta \cdot I(x) \cdot \Delta x \]

where the constant of proportionality, \( \delta \) is the absorption coefficient. The negative
sign indicates the reduction in beam irradiance due to absorption as \( \delta \) is a positive quantity. Writing this expression as a differential equation we have

\[
\frac{dI}{dx} = -\delta I(x)
\]

Integrating this equation gives

\[
I = I_o \exp(-\delta x)
\]

Where \( I_o \) is the peak intensity of incident irradiance.

Hence, using the above formula for the intensity of the laser beam, the rate of applied external energy (irradiated energy) at \( dx \) during the time interval \( dt \) can be given as,

\[
\Delta E_{x,t}|_{abs} = I_o \delta e^{-\delta x} A dt dx
\]

The total energy increase in the material at \( dx \) during the time \( dt \) is,

\[
N.A(E_{x,t+dt} - E_{x,t}).dx = \Delta E_{x,t} + \Delta E_{x,t}|_{abs}
\]

The total energy transfer equation can therefore be written as,

\[
N.A(E_{x,t+dt} - E_{x,t}).dx = I_o \delta \exp(-\delta x) A dt dx + \\
\int_{-\infty}^{\infty} \frac{N_v}{6\lambda^2} \exp(-\frac{|x-s|}{\lambda}).f(E_{x,t} - E_{x,t}).ds A dt dx
\]

(2.1)
However it is also suggested that the rate of transfer of energy between the electrons and the molecules will be determined only by the difference in temperature of the electrons and the lattice lattice vibrations [23]. After arranging the total energy transfer equation, the lattice temperature gradients can be written as,

\[
\frac{dT(x,t)}{dt} = \frac{I_o \delta}{\rho C_p} \exp(-\delta x) - \frac{K_f}{2 \lambda^2 \rho C_p} T(x,t) + \frac{K_f}{4 \lambda^3 \rho C_p} \left[ \int_0^\infty \exp\left(-\frac{|x-s|}{\lambda}\right) T(s,t) ds + \int_s^\infty \exp\left(-\frac{|x-s|}{\lambda}\right) T(s,t) ds \right] \tag{2.2}
\]

During the electron-phonon collisions, some fraction \( f \) of the electron excess energy is transferred to the phonon.

For any inelastic collision, conservation of energy in any section may be written as

\[
\text{Electron energy entering the section} = \text{Electron energy leaving the section} + \text{Energy transfer to phonons in the section.}
\]

This gives,

\[
f = \frac{\text{(electron energy)}_{\text{in}} - \text{(electron energy)}_{\text{out}}}{\text{(excess electron energy)}_{\text{in}}}
\]
or,

\[ f = \frac{(E_{el})_{in} - (E_{el})_{out}}{(E_{el})_{in} - E_{phonon}} \]

providing that \( 0 \leq f \leq 1 \) from energy conservation, where,

\[ (E_{el})_{excess} = (E_{el})_{in} - E_{phonon} \]

and \( E_{phonon} \) = mean energy of phonon

The effective value of \( f \) over a region large enough to allow many collisions approaches unity and this corresponds to the attachment of thermal equilibrium. However, in the case considered, electrons absorb energy from the radiation field and as a result equilibrium can not be achieved. In the case of a single collision, \( f \) depends only upon the masses of the colliding particles according to:

\[ f = \frac{2Mm}{(M + m)^2} \]

where \( M \) and \( m \) are the masses of a molecule and an electron respectively. Substituting appropriate values shows that the value of \( f \) is of the order of \( 10^{-4} \).

Harrington[14] suggested that the effective \( f \) value is of the order of \( 10^{-4} \), which agrees with Kittel[17]. Dekker[9] showed that the most energetic phonon is only 0.01 eV assuming \( c=10^5 \) cm/sec (velocity of sound in the solid), but electrons near the Fermi level have energies of several eV; hence when such electrons are scattered, only a small fraction of their energy can be given during an electron-phonon collision. In
the light of the above arguments, the values of $f$ in the present analysis is taken as $10^{-4}$ and assumed as constant over successive collisions. The heat conduction equation (2.2) describing the kinetic theory model is in the form of an integro-differential equation, which does not yield to analytical methods, so finite difference methods must be used. The solution can be attempted by implicit and explicit methods. Errors arising from these approximations are truncation errors and the degree to which approximate solution approaches the exact solution is termed the convergence. The fourth power of the spatial step size and the terms of the order of the square of the time increments are neglected, which gives a negligible error when the convergence is considered[8].

In order to include evaporation in the above conduction equation, we need to know the variation of latent heat of evaporation and surface velocity with surface temperature. This is discussed next.

### 2.3 Evaporation of the Surface

The liquid surface layer formed during a laser pulse, moves into the metal at a rate determined by the quantity of vapor expelled. As the temperature of the liquid molecules is increased so the additional energy needed to free them from the binding forces decreases. The latent heat of vaporization of liquid therefore decreases with
temperature, until, at the critical temperature and above it remains zero. It has been taken as an elliptic function of temperature which appears adequate for the accuracy sought[41].

$$L(T) = L_o \left[1 - \left(\frac{T}{T_c}\right)^2\right]^{\frac{1}{2}}$$

where $L_o$ is the latent heat of vaporization at absolute zero. The rate of change of latent heat with temperature can be expressed as[41].

$$\frac{dL}{dT} = \frac{L}{T} + (C_{p_2} - C_{p_1}) - \frac{L}{(v_2 - v_1)} \left[\left(\frac{\partial v_2}{\partial T}\right)_p - \left(\frac{\partial v_1}{\partial T}\right)_p\right]$$

where $C_{p_1}$ and $C_{p_2}$ are specific heats at constant pressure. $v_1$ and $v_2$ are specific volumes, and 1,2 subscripts denote liquid and vapor states respectively.

Although integration of the latent heat over the temperature range 0 to $T_c$ is impossible, it can be used to show that little inaccuracy is involved in taking the latent heat as the latent heat at absolute zero. By putting,

$$v_2 \gg v_1 \quad \text{and} \quad \left(\frac{\partial v_2}{\partial T}\right)_p \gg \left(\frac{\partial v_1}{\partial T}\right)_p$$

i.e. the specific volume of gas is much greater than the condensed liquid and its rate of change with temperature, at constant pressure, is correspondingly greater. Thus,

$$\frac{dL}{dT} = \frac{L}{T} + (C_{p_2} - C_{p_1}) - \frac{L}{v_2} \left(\frac{\partial v_2}{\partial T}\right)_p$$
Applying the perfect gas law,

\[ v_2 = \frac{R.T}{p} \]

and differentiating with respect to \( T \) so that

\[ \frac{\partial v_2}{\partial T} = \frac{R}{p} \]

we finally get,

\[ \frac{dL}{dT} = \frac{L}{T} + (C_p - C_v) - \frac{L}{T} = \Delta C_p \]

\( C_p \) is extremely small for temperatures up to the standard temperatures [33] and so little error will result in taking \( L_o \) as latent heat at standard temperatures.

According to Maxwell's law, the velocity distribution of molecules is[11],

\[ f(V_z) \, dV_z = \left[ \frac{m}{2\pi k_B T} \right]^{\frac{3}{2}} \exp \left[ -\frac{mV_z^2}{2\pi k_B T} \right] \, dV_z \]

where \( V_z \) is the velocity in the direction normal to the surface, \( T \) is the temperature of the solid, liquid or gas and \( m \) is the mass of the atom.

\[ f(V_z) \, dV_z = \frac{\text{number of atoms with velocity } V_z \text{ to } V_z + dV_z \text{ per unit volume}}{\text{number of atoms per unit volume}} \]
Only those molecules whose velocity is greater than that given by,

\[ \frac{1}{2} m V_{\text{min}}^2 = L(T) \]

where \( V_{\text{min}} \) lies in the z-direction, will escape from the retaining potential. If \( n \) is the number of atoms per unit volume then the number of atoms with velocities \( V_z \) to \( V_z + dV_z \) per unit volume is \( n f(V_z) dV_z \) and the number of atoms with these velocities passing a unit area per unit time is \( n f(V_z) dV_z \). All these atoms for which \( V_z > V_{\text{min}} \) do not return to their equilibrium position and are evaporated. If \( G \) is the number of atoms evaporated per unit time per unit area then,

\[
G = \int_{V_{\text{min}}}^{\infty} n f(V_z) V_z dV_z
\]

\[
\Rightarrow G = n \left[ \frac{m}{2\pi k_B T} \right]^{\frac{1}{2}} \int_{V_{\text{min}}}^{\infty} \exp \left[ \frac{-m V_z^2}{2\pi k_B T} \right] V_z dV_z
\]

\[
\Rightarrow G = n \left[ \frac{m}{2\pi k_B T} \right]^{\frac{1}{2}} \left[ \exp \left[ \frac{-m V_{\text{min}}^2}{2\pi k_B T} \right] - \exp \left[ \frac{-L(T)}{k_B T} \right] \right]
\]

If the atoms are equally spaced within the lattice, a surface layer would consist of \( n_{\text{s}}^2 \) atoms with an evaporation time of \( \frac{\pi}{G} \). The average velocity of the surface is
therefore,

\[ V_s = \frac{1}{n^3} \cdot \frac{G}{n^3} = \left[ \frac{k_b T_s}{2 \pi m} \right]^{\frac{1}{2}} \exp \left[ -\frac{L(T_s)}{k_b T_s} \right] \]

Anisimov et. al.[10] erroneously derived velocities of liquid surface based on the Deby frequency of the metal and a constant latent heat of vaporization. Such data yielded surface velocities comparable to those of Ready[27] but suggested a minimum velocity for low powers and a maximum for high powers.

2.3.1 Moving Front

The conservation of energy equation:

\[ I_s = \rho V_s (L(T) + C_p T_s) \]

and the velocity of evaporating surface:

\[ V_s = \left[ \frac{k_b T_s}{2 \pi m} \right]^{\frac{1}{2}} \exp \left[ -\frac{L(T_s)}{k_b T_s} \right] \]

are simultaneous equations in \( V_s \) and \( T_s \). They constitute conditions under which the surface will recede at a constant rate. This is true even for temperatures below the standard boiling temperature provided the evaporation takes place in vacuum.

Results obtained for the variation of power intensity and surface velocity with surface temperature for aluminum, copper, iron and tungsten[41] show the evaporation
course rising sharply from low surface temperatures up to the critical point when,

\[ V_s = \left[ \frac{k_B T_s}{2 \pi m} \right]^{\frac{1}{2}} \]

The energy equation, plotted for various values of power density, falls from left to right. The parametric solution is found at the intersection of the two curves. The energy equation at the critical point beyond is [41],

\[ V_s = \left[ \frac{I_s}{\rho C_p T_s} \right] \]

The presence of atmosphere affects the results obtained for \( V_s \) below the melting temperature such that they are not accurate for temperatures below fusion. The threshold for damage as predicted by these results is lower than that found in practice mainly because the evaporating velocity is calculated for molecular escape into a vacuum [37].

Beyond the critical temperature, the solution takes the analytic form,

\[ I = \rho C_p \left[ \frac{k_B}{2 \pi m} \right]^{\frac{1}{2}} T_s^{\frac{3}{2}} \]

and the moving front becomes vapor-solid. The latent heat of fusion is now greater than the latent heat of vaporization and so considerable error may be introduced by its omission. Another factor governing the accuracy of this region is the extrapolation of normal material constants, in particular the specific heat and thermal capacity of superheated solids. It is unwise therefore, to analyze material ejection
for temperatures above the critical temperature until such information is available.

The rate at which the evaporation front is moving is determined by the rate of heat transfer per unit volume used in vaporization. This heat transfer rate is given by

$$\rho C_p V_s \frac{\partial T}{\partial x}$$

Incorporating this convection term in the main equation (2.2), we get

$$\frac{dT(x,t)}{dt} = \frac{I_o \delta}{\rho C_p} \exp(-\delta x) - f \frac{K}{2 \lambda^2} T(x,t) + \frac{K f}{4 \lambda^3} \left[ \int_{0}^{\infty} \exp\left(-\frac{|x+s|}{\lambda}\right) T(s,t) \, ds + \int_{0}^{x} \exp\left(-\frac{|x-s|}{\lambda}\right) T(s,t) \, ds + \int_{x}^{\infty} \exp\left(-\frac{|x-s|}{\lambda}\right) T(s,t) \, ds \right] + V_s \frac{\partial T}{\partial x} \quad (2.3)$$

The above equation describes the kinetic theory model for laser conduction and convection heating and the solution of this equation is discussed in chapter 3.
Chapter 3

Numerical Approach and Solution

3.1 Introduction

The energy equation, developed in the previous chapter, describes the Kinetic Theory model and is in the form of an integro-differential equation. This equation does not yield to analytic solution due to the mathematical complexities involved and hence some other method of solution is required. As an alternative method, a numerical method may be employed. Numerical and analog techniques are available which are able to handle almost any problem of any degree of complexity. The detail and accuracy of the answer obtained depends mainly upon the proper selection of nodes, spatial and time increments. Hence, in developing a numerical scheme, a primary consideration is the trade-off between model detail and computational effort. The various numerical methods all yield numerical values for the dependent
variable at selected discrete points within the body being considered and only at discrete time intervals.

3.2 Numerical Method

Several techniques of numerical analysis exist. Among them, the most commonly used technique is the Finite Difference Approximation due to its simplicity and accuracy. Before proceeding to the finite difference method, it is appropriate to define certain properties of numerical solutions which determine the level of accuracy and details which can be achieved. These properties include[5]:

- Stability

- Consistency, and

- Convergence

Stability of a numerical solution describes whether or not the dependent variable is bounded. For transient analysis, the dependent variable is unstable if the solution oscillates with an amplitude that increases with time.

Consistency is a property based on the errors resulting from truncation of numbers during computation. A discretized equation is consistent if the error between the numerical and analytical solutions approach zero as the time and spatial in-
crements approach zero. Truncation errors primarily occur during application of
taylor's expansion of the terms in the differential equation[5]. In the numerical
methods, only the first few terms of Taylor’s expansion are used.

The third property of a numerical method is the degree of convergence of the so-
lution. A numerical solution is said to be convergent if the discretization error
approaches zero as the mesh is refined. In other words, the exact solution is ap-
proached numerically through mesh refinement by either increasing the number of
spatial increments or the time increments. For finite difference solutions, the Lax
theorem[31] states that the convergence is ensured if and only if the solution is both
stable and consistent.

3.3 Finite Difference Approximations

In this method, the differential equation to be solved is replaced by its equivalent
form in finite differences over the whole domain of interest. These finite difference
equations are algebraic in nature and can be solved without much difficulty. De-
pending upon the finite difference representation for the first derivatives, the finite
difference methods are divided into two classes:

1. Explicit Formulation

2. Implicit Formulation
3.3.1 Explicit Formulation

When the first derivatives are expressed in forward difference form, one can get the explicit formulation. The energy equation as resulted using the Kinetic Theory analysis is,

$$\frac{dT(x,t)}{dt} = \frac{I_o \delta}{\rho C_p} \exp(-\delta x) - f \frac{K}{2\lambda^2} T(x,t) + \frac{K f}{4\lambda^3} \left[ \int_0^\infty \exp\left(-\frac{|x+s|}{\lambda}\right) T(s,t) ds + \int_s^x \exp\left(-\frac{|x-s|}{\lambda}\right) T(s,t) ds \right] + V\left(\frac{\partial T}{\partial x}\right)$$

The first derivatives in the above equation in terms of forward differences are given as

$$\frac{\partial T}{\partial x} = \frac{T_{i+1,j} - T_{i,j}}{\Delta x}$$

where the index $i$ corresponds to the spatial coordinate $x$, while the index $j$ is for the time coordinate $t$. $\Delta x$ is the increment in $x$ direction. Similarly the time derivative is given as

$$\frac{\partial T}{\partial t} = \frac{T_{i,j+1} - T_{i,j}}{\Delta t}$$

where $\Delta t$ is the increment in time.

Using these difference expressions, the energy equation, may be written as,
\[ T(x, t + \Delta t) = \frac{I_\sigma \delta \Delta t}{\rho C_p} \exp(-\delta x) + \left[1 - \frac{\sigma K \Delta t}{2\lambda^2 \rho C_p}\right] T(x, t) + \frac{\sigma K \Delta t}{4\lambda^3 \rho C_p} \int_0^\infty \exp\left(-\frac{|x + s|}{\lambda}\right) T(s, t) ds + \int_0^x \exp\left(-\frac{|x - s|}{\lambda}\right) T(s, t) ds \]

\[ V \frac{\Delta t}{\Delta x} [T(x + \Delta x, t) - T(x, t)] \]  

(3.1)

This is the explicit finite difference representation of the energy equation.

3.3.2 Evaluation of the Integrals

The integrals involved in the energy equation are computed using the trapezoidal method. According to this method, a function \( f \) is computed from an initial point 1 to a final point \( n \) using the value of \( f \) at discrete points spaced equally apart. Mathematically,

\[ \text{Integral} = \frac{h}{2} [f_1 + 2f_2 + 2f_3 + \cdots + f_n] \]

where \( f_1, f_2, \cdots, f_n \) are the values of function at points 1, 2, \( \cdots \), \( n \) and \( h \) is the spatial increment between successive points.
Hence if we denote the integrals by

\[
G_1 = \int_0^\infty \exp\left(-\frac{|x+s|}{\lambda}\right)T(s,t)\,ds
\]

\[
G_2 = \int_0^x \exp\left(-\frac{|x-s|}{\lambda}\right)T(s,t)\,ds
\]

\[
G_3 = \int_x^\infty \exp\left(-\frac{|x-s|}{\lambda}\right)T(s,t)\,ds
\]

then the trapezoidal approximation to one of these integrals, \(G_1\), can be written as

\[
G_1 = \frac{\Delta x}{2} \left[ \exp\left(-\frac{|x+s_1|}{\lambda}\right)T(s_0,t) + 2\exp\left(-\frac{|x+s_1|}{\lambda}\right)T(s_1,t) + \cdots + \right.
\]

\[
\exp\left(-\frac{|x+s_{n-1}|}{\lambda}\right)T(s_{n-1},t)) + \exp\left(-\frac{|x+s_n|}{\lambda}\right)T(s_n,t) \right]
\]

(3.2)

where,

\[
s_1 = \Delta x
\]

\[
s_2 = 2\Delta x
\]

\[:\]

\[
s_n = n\Delta x
\]

Now considering the exponential term as

\[
a_j = \exp\left[-\frac{|x+s_j|}{\lambda}\right]
\]

then the approximation for \(G_1\) can be written as
\[ G_1 = \frac{\Delta x}{2} \left[ a_{1}T(\Delta x, t) + 2\sum_{j=2}^{n} a_{j}T(j \Delta x, t) + a_{n+1}T((n+1) \Delta x, t) \right] \]

The expressions for \( G_2 \) and \( G_3 \) also involve the exponentials, so in order to generalize, the exponential terms are redefined as:

\[ A_7(j, m) = \sum_{j=1}^{n+1} \sum_{m=1}^{n+1} \exp\left(-\frac{|j \Delta x - m \Delta x|}{\lambda}\right) \]

and

\[ A_8(j, m) = \sum_{j=1}^{n+1} \sum_{m=1}^{n+1} \exp\left(-\frac{|j \Delta x + m \Delta x|}{\lambda}\right) \]

Therefore, the three integrals yield in a final form as

\[ G_1 = \frac{\Delta x}{2} \left[ A_6(1, 1).T(1, t) + 2\sum_{r=2}^{n} A_6(j, r).T(r, t) + A_6(j, n+1).T(n+1, t) \right] \]

\[ G_2 = \frac{\Delta x}{2} \left[ A_7(1, 1).T(1, t) + 2\sum_{r=1}^{j-1} A_7(j, r).T(r, t) + A_7(j, j).T(j, t) \right] \]
\[ G_3 = \frac{\Delta x}{2} \left[ A_T(j,j) \cdot T(j,t) + 2 \sum_{r=j+1}^n A_T(j,r) \cdot T(r,t) + A_T(j,n+1) \cdot T(n+1,t) \right] \]

As it can be observed, the explicit representation gives the future temperature at \( x \) in terms of the current temperatures at \( x \) and its surrounding node. Hence knowing only the initial temperature for all the nodes, we can calculate the individual nodal temperatures for the next time step. However, the explicit formulation faces the problem of stability, and careful choice of the time increment is required. The upper limit of the allowable time increment for the conduction case only is given by

\[ \frac{f K \, \Delta t}{2 \lambda^2 \rho \, C_p} \leq 1 \]

so that the second term on the right hand side of the energy equation remains positive.

In this case, the velocity of the moving front, \( V \), is set to zero i.e. (no convection). When the convection term is also included in the analysis, we can rewrite the finite difference equation as,

\[
T(x,t+\Delta t) = \frac{I_s \delta \Delta t}{\rho \cdot C_p} \cdot \exp(-\delta x) + \left[ 1 - \left( \frac{f K \, \Delta t}{2 \lambda^2 \rho C_p} + V \frac{\Delta t}{\Delta x} \right) \right] \cdot T(x,t) +
\]
\[ + \frac{f K \Delta t}{4 \lambda^3 p C_p} \left[ \int_0^\infty \exp \left( -\frac{|x+s|}{\lambda} \right) T(s,t) ds \right] + \int_0^x \exp \left( -\frac{|x-s|}{\lambda} \right) T(s,t) ds + \]

\[ + \int_x^\infty \exp \left( -\frac{|x-s|}{\lambda} \right) T(s,t) ds \right] + V \frac{\Delta t}{\Delta x} T(x + \Delta x, t) \]  

(3.3)

and from there we get the stability criteria as,

\[
\left[ \frac{f K \Delta t}{2 \lambda^2 p C_p} + V \frac{\Delta t}{\Delta x} \right] \leq 1
\]

so that the second term on the right hand side of the energy equation does not yield negative.

If we let

\[ A = \frac{I_o \delta \Delta t}{p C_p} \]
\[ B = 1 - \left( \frac{fK \Delta t}{2\lambda^2 \rho C_p} + V \frac{\Delta t}{\Delta x} \right) \]

\[ C = \frac{fK \Delta t}{4\lambda^3 \rho C_p} \]

\[ D = V \frac{\Delta t}{\Delta x} \]

and \( G_1, G_2 \) and \( G_3 \) are already defining the integrals, then equation (3.3) can be written as,

\[ T(x, t + \Delta t) = A.\exp(-\delta x) + B.T(x, t) + C.(G_1 + G_2 + G_3) + D.T(x + \Delta x, t) \] (3.4)

and from there we get the stability criteria as,

\[ B \geq 0 \]

The explicit scheme can also be written in vector form as,

\[ \overline{T}(x, t + dt). \begin{bmatrix} B \end{bmatrix} = A. \begin{bmatrix} B \end{bmatrix} + \overline{T}(x, t).[X].S \]

where

\[ A = \frac{I_0 \delta}{\rho C_p}, \]

\[ \begin{bmatrix} B \end{bmatrix} = \exp(-\delta x) \ \text{vector} \]

\[ S = 1 - \frac{f \alpha dt}{2\lambda^2} \]

and \([X]\) is the matrix containing the integral terms involved in the equation.
3.3.3 Implicit Formulation

When the first derivatives are expressed in backward difference form, implicit formulation is obtained. These expressions are,

$$\frac{\partial T}{\partial x} = \frac{T_{i,j} - T_{i-1,j}}{\Delta x}$$

and for time derivative we have,

$$\frac{\partial T}{\partial t} = \frac{T_{i,j} - T_{i,j-1}}{\Delta t}$$

Using these expressions for $\frac{\partial T}{\partial x}$ and $\frac{\partial T}{\partial t}$, the energy equation becomes

$$T(x,t - \Delta t) = -\frac{I_c \delta \Delta t}{\rho C_p} \exp(-\delta x) \left[1 - \frac{f.K \Delta t}{2\lambda^2 \rho C_p}\right] T(x,t) -$$

$$\frac{f.K \Delta t}{4\lambda^2 \rho C_p} \left[\int_0^\infty \exp\left(-\frac{|x+s|}{\lambda}\right) T(s,t).ds + \int_0^x \exp\left(-\frac{|x-s|}{\lambda}\right) T(s,t).ds + \right.$$

$$\left.\int_x^\infty \exp\left(-\frac{|x-s|}{\lambda}\right) T(s,t).ds\right] - V \frac{\Delta t}{\Delta x} [T(x,t) - T(x - \Delta x,t)]$$

(3.5)

which may be written as

$$T(x,t - \Delta t) = -\frac{I_c \delta \Delta t}{\rho C_p} \exp(-\delta x) - (1 - \frac{f.K \Delta t}{2\lambda^2 \rho C_p} + V \frac{\Delta t}{\Delta x}) T(x,t)$$
\[
\frac{f.K \Delta t}{4\lambda^2 \rho C_p} \left[ \int_0^\infty \exp\left(-\frac{|x+s|}{\lambda}\right)T(s,t)\,ds + \int_0^x \exp\left(-\frac{|x-s|}{\lambda}\right)T(s,t)\,ds + \int_x^\infty \exp\left(-\frac{|x-s|}{\lambda}\right)T(s,t)\,ds \right] + V \frac{\Delta t}{\Delta x} T(x - \Delta x, t) \tag{3.6}
\]

If we let
\[
F = -\frac{I_0 \Delta t}{\rho C_p}
\]
\[
P = -(1 - \frac{f.K \Delta t}{2\lambda^2 \rho C_p} + V \frac{\Delta t}{\Delta x})
\]
\[
N = -\frac{f.K \Delta t}{4\lambda^2 \rho C_p}
\]
\[
M = V \frac{\Delta t}{\Delta x}
\]

and \(G_1, G_2\) and \(G_3\) are already defining the integrals, then equation (3.6) can be written as,
\[
T(x, t - \Delta t) = F \cdot \exp(-\delta x) + P \cdot T(x, t) + N \cdot (G_1 + G_2 + G_3) + M \cdot T(x - \Delta x, t) \tag{3.7}
\]

As can be observed, the future temperature at \(x\) in the implicit formulation always follows the thermodynamic sense no matter what the value of \(\Delta t\) is chosen. Hence
implicit formulation is always stable. In this scheme, initial temperature distribution is assumed to be known for each nodal point in the field. A set of simultaneous linear equations determines the temperature distribution at the end of a chosen time interval. The solution is obtained either by relaxation or matrix inversion.

The implicit form may be written, in matrix form, as

\[ T(x,t) \cdot [X] = A \left[ \overrightarrow{B} \right] + T(x,t - dt) \cdot \left[ \overrightarrow{B} \right] \]

where

\[ A = \frac{I_0.\delta}{\rho.C_p} \]

\[ \left[ \overrightarrow{B} \right] = \exp(-\delta.x) \ \text{vector} \]

and \([X]\) is the matrix containing the integral terms involved in the equation.

Solutions can be obtained for all nodal points with a given time increment by computing the above set of simultaneous linear equations.

This method has the disadvantage of requiring lengthy calculation of each time step and in requiring large storage when the number of nodes becomes large.
3.3.4 Explicit versus Implicit Formulation

A rather obvious advantage of the explicit representation over the implicit is the fact that the forward difference equation gives the future temperature of a single node in terms of current temperatures of that node and its neighbors. It is thereby possible to calculate the temperatures for a given time increment node by node. In the implicit, current nodal temperatures are expressed in terms of their initial values and the current values of their neighbors. Therefore, for one time step to the next, a set of equations must be solved.

3.4 Method of Solution

In the present work, explicit formulation was employed. Stability criteria, which is based on both conduction and convection processes, was considered and the time increment was calculated accordingly. The stability criteria was satisfied using a fixed spatial increment equal to one tenth of λ, \( \left( \frac{1}{10} \right) \). For this value of the spatial increment, which was calculated as \( 10^{-8} \) m, the requisite time increment was found to be \( 7.99 \times 10^{-10} \). This value was used in the conduction case only. For the convection case (evaporation), the time increment was further reduced by one tenth of the original value thereby giving more insight into the evaporation process. However, this increased the computation time and caused dynamic storage problems. The problem was solved by using the temporary dynamic storage available at the
mainframe which of course is not a permanent solution.

The computer program developed consists of a MAIN program and four subroutines. The MAIN program contains the logic of calculations. The subroutines, INT1, INT2, and INT3 perform the integration part of the energy equation while the fourth subroutine, VET, is called when the temperature of the node exceeds the melting point. The velocity of the moving front and the latent heat of vaporization is calculated accordingly in this subroutine. This velocity is used to calculate the temperature field after the melting point i.e. when the convection process starts. The flow chart of the computer program is given in figure 3.1. The listing of the program is also attached as Appendix A.

For each node in the x-direction, the numerical representation of the integrals resulted using a trapezoidal approximation, are evaluated to determine the future temperature at that node. Similarly, all the nodes are calculated for the future temperature at corresponding nodes. Once, this sweep in x-direction is complete, the next sweep starts for the next time step and so on until the whole time domain is traversed for all the nodes. The numerical value of the spatial node limit was set at 13 nodes, while the time dimension was set at 3000. At times, need arose to change these values and the respective changes were made keeping in view the limitations of storage. The real time calculations for the 13 x 3000 grid size always causes problem
of virtual memory being exceeded and an extra 2 mega bites of dynamic memory was always used. The spatial limit of 13 was set considering the fact that there were no changes afterwards in the spatial direction. The time grid of 3000 nodes was employed in order to accommodate micro-second analysis with a 0.8 nano-second time increment.
MAIN

Input laser/material properties, define the grid and initial and boundary conditions.

Evaluation of the integrals

T > T_m

Calculate new temperatures

Print T

t > t_m

STOP

END

SUBROUTINES

INT1

INT2

INT3

VET

Figure 3.1: Flow Chart of the Computer Program
Chapter 4

Results and Discussions

Fig. 4.2 shows the surface temperature profiles for four different types of laser beam input power. The Continuous Wave (CW) laser beam is time independent and has a constant power of $1 \times 10^8 \text{ W/m}^2$. The other three beams/pulses have power intensities which vary linearly with time in the initial stage and then become steady to reach a constant power intensity of $1 \times 10^9 \text{ W/m}^2$. These pulses have different slopes as shown in fig. 4.1. As observed from fig. 4.2, the surface temperature profile due to the CW laser heating starts parabolic and continues until a heating time of 0.2 $\mu$ second. This corresponds to the beginning of the melting process. The slope of the temperature curve reduces considerably from this point on up to evaporation temperatures. This is due to the phase change process.
Figure 4.1: Intensity profiles of CW laser beam and linearly varying laser pulses A, B and C.
The first pulse (pulse A) may be described mathematically in the form of:

\[ I(t) = 2 \times 10^{15} t \quad 0.0 \leq t \leq 0.5 \mu\text{sec}. \]

\[ I(t) = 1 \times 10^9 \quad 0.5 \leq t \leq 2.0 \mu\text{sec}. \]

In this case, the temperature profile is exponential in nature. Heating starts at a rate much less than that for CW heating and then increases exponential until the melting point is reached. After the melting point, the curve shows similar variation with time as described for CW heating. The second and third pulses (pulses B and C), can be described mathematically in the form:

**Pulse B:**

\[ I(t) = 1 \times 10^{15} t \quad 0 \leq t \leq 1.0 \mu\text{sec}. \]

\[ I(t) = 1 \times 10^9 \quad 1.0 \leq t \leq 2.0 \mu\text{sec}. \]

**Pulse C:**

\[ I(t) = 0.5 \times 10^{15} t \quad 0 \leq t \leq 2.0 \mu\text{sec}. \]

Due to different slopes, the rate of heating for the second pulse B is higher than that corresponds to the third pulse C but it is lower than that obtained for the first pulse A. This is due to the difference in rise time of the three pulses. Melting point obtained by the third pulse C is around 0.9 \( \mu\text{sec} \) as compared with 0.6 \( \mu\text{sec} \) for the second pulse B, 0.48 \( \mu\text{sec} \) for pulse A and 0.2\( \mu\text{sec} \) for the CW heating. This may indicate the effect of the pulse shape on the surface temperature rise.
Figure 4.2: Surface temperature profiles due to Continuous Wave (CW) and linearly varying laser pulses A, B and C.
Fig. 4.3 shows the temperature profiles inside the material at different heating times when the material is exposed to CW laser heating. The temperature decreases smoothly before the melting point is reached at low heating times but it gives a higher gradient ($\frac{dT}{dx}$) near the surface than that corresponds to further inside the material. Beyond the melting temperature, the temperature profile changes and temperature becomes higher than the surface temperature just beneath the surface. The curve resembles a small parabola near the surface and then decreases nearly exponentially with increasing distance inside the material. The reason for this behavior is that the convection effect due to melting is dominant at the surface resulting in a negative temperature gradient developed at the surface. This indicates that temperature underneath the surface reaches the melting temperature and further increase in heating causes liquid metal to reach higher temperature underneath the surface than that corresponds to the surface. In this case, super heating dominates the conduction resulting in non-steady equilibrium process developed underneath the workpiece. The temperature of the focusing area of the target material increases to its vaporization temperature when heating progresses. Vaporization of the front surface occurs which in turn removes some energy from the surface region. This results in cooling of the surface and consequently causing the maximum temperature to lie some where below the surface. If this temperature is high enough to cause boiling of the material at depths below the surface, very high pressures are resulted. This pressure increase derives the boiled material into its liquid phase. Therefore,
once the pressure inside this liquid phase reaches the destruction pressure of the surface, then it explodes removing material from the interior to outside[39].

If the recoil pressure is less than the saturated vapor pressure, then the liquid metal is superheated in which case nucleation will occur giving vapor bubbles within the bulk of the liquid. Explosion of these vapor bubbles will result in the ejection of liquid from the crater[39].
Figure 4.3: Temperature profiles inside the material due to CW laser beam.
Figures 4.4 to 4.6 show the variation of temperature with distance inside the material for the three pulses A, B, and C respectively. It is clear from the figures that as the time increases, the temperature increases steadily and after the melting point, a rise in temperature just underneath the surface is observed as similar to CW heating and melting profiles. However, the temperature obtained are much less than that for CW heating. Pulses having long rise time result in increased temperature rise time to reach the melting temperature and vice versa. This may also be seen clearly from figures 4.7 to 4.10 where the temperature profiles due to the four laser beam/pulses CW, A, B, and C are compared at different heating times. Figure 4.7 shows a good comparison at time $t=0.143 \mu\text{sec}$. CW beam causes a temperature of 1320° C compared with 200° C for pulse A, 108° C for pulse B and 80° C for pulse C. The temperature profile due to CW beam has a sharp slope near the surface because of the high intensity and less penetration time.
Figure 4.4: Temperature profiles inside the material due to Pulse A.
Figure 4.5: Temperature profiles inside the material due to Pulse B.
Figure 4.6: Temperature profiles inside the material due to Pulse C.
Similarly as heating proceeds with time, one can observe the same effect on a large scale as it is clear from figures 4.8 to 4.10. Another notable point is observed in fig.4.10 where the temperature profile caused by CW radiation shows an exponential decay with the distance inside the material. This is due to the fact that after the melting, the process of sensible heating towards evaporation starts giving rise to considerable temperature increase in a very short time.
Figure 4.7: Temperature profiles inside the material due to CW laser beam and linearly varying laser pulses A, B and C at a time of $t=0.143 \ \mu$sec.
Figure 4.8: Temperature profiles inside the material due to CW laser beam and linearly varying laser pulses A, B and C at a time of $t=0.639 \, \mu\text{sec}$. 
Figure 4.9: Temperature profiles inside the material due to CW laser beam and linearly varying laser pulses A, B and C at a time of t = 1.27 μsec.
Figure 4.10: Temperature profiles inside the material due to CW laser beam and linearly varying laser pulses A, B and C at time $t=2.236 \ \mu\text{sec.}$
After analyzing the continuous beam (CW) with the linearly varying pulses A, B, and C during interaction with the surface, the effect of a whole laser pulse containing both heating and cooling cycles may be established.

Pulse #1 has a power intensity varying linearly with time for the time interval of $0 \leq t \leq 0.8 \mu$ sec. For $0 \leq t \leq 0.5 \mu$ sec, the intensity increases from zero and reaches a maximum value of $1 \times 10^{10}$ W/m$^2$ and for $0.5 \leq t \leq 0.8 \mu$ sec, it decreases rapidly to a minimum value of $1.273 \times 10^9$ W/m$^2$. The remaining pulse $0.8 \leq t \leq 15.8 \mu$ sec has a power intensity which is parabolic with time having a maximum value of $3.59 \times 10^9$ W/m$^2$ obtained at $7.5 \mu$ sec and a minimum value of zero at $15.8 \mu$ sec which corresponds to the end of the pulse.

In order to introduce this pulse shape in the computer program, some mathematical expressions describing this pulse shape are required. For the time interval of $0 \leq t \leq 0.8 \mu$ sec, the pulse intensity variation is linear and the expressions for this time interval are given as:

$$ I(t) = 2 \times 10^{16} \ t \quad 0 \leq t \leq 0.5 \mu\text{sec} $$

$$ I(t) = 10^{10} - 2.909 \times 10^{16} (t - 0.5 \times 10^{-6}) \ t \quad 0.5 \leq t \leq 0.8 \mu\text{sec} $$
Figure 4.11: Intensity variation with time for Pulse #1.
For the time interval of $0.8 \leq t \leq 15.8 \mu$ sec, the expression for the parabolic variation of the intensity with time for pulse #1 is obtained using the Statistical Analysis System (SAS) package available on the mainframe system. After iterations, the following expression was obtained for the pulse shape:

$$I(t) = (7.356 + 7.374 t - 0.498 t^2) \times 10^8 \quad 0.8 \leq t \leq 15.8 \mu sec$$

The above three expressions are fed in the computer program and the effect of the complete pulse shape is analyzed. Fig. 4.12 shows the surface temperature profiles resulted due to pulse #1. Due to a very small rise time of the pulse, rapid heating occurs and a temperature of more than $1650^\circ C$ is obtained in $0.9 \mu$ sec. The first part of the cooling cycle of the pulse varies from $0.5 \mu$ sec to $0.8 \mu$ sec. In this case rapid reduction in the surface temperature occurs to about $1500^\circ C$ at around $1 \mu$ sec. From $0.8 \mu$ sec, exponential heating restarts due to the positive intensity gradient of the pulse and a temperature of $2900^\circ C$ is obtained at $12.5 \mu$ sec. After this heating process, cooling of the surface is observed. This is due to the cooling cycle of the pulse. The surface temperature decreases exponentially in the beginning of this cooling cycle and later on decreases smoothly to reach the ambient temperature.
Figure 4.12: Typical surface heating and cooling due to Pulse # 1
4.1 **Effect of the shape of the pulse**

In order to see the effect of the pulse shape on the temperature distribution, pulse# 1 was modified in such a way as to keep the total energy contents of the modified pulse equal to the total energy contents of the pulse# 1. This was achieved by measuring the area under the curve plotted between intensity and time axis for pulse# 1 and the modified pulse.

Four different modifications were made resulting in four different pulses. Each of these pulses will be described next.

4.1.1 **pulse# 2**

Fig.4.13 shows the intensity variations with time for pulse# 2. Area under the curve of fig.4.13 was found to be equal to $3.741 \times 10^4$ J/m$^2$ whereas from fig.4.11, the area for pulse# 1 was calculated as $3.738 \times 10^4$ J/m$^2$. Hence a negligible error of 0.08% is obtained. The data for pulse# 2 was fed in the regression program using the SAS package (as mentioned before) and the resulting mathematical expressions were fed in the main computer program for analysis. These mathematical expressions are given as:

\[
I(t) = 2 \times 10^{16} t \quad 0 \leq t \leq 0.5 \mu sec
\]

\[
I(t) = 10^{10} - 2.909 \times 10^{16}(t - 0.5 \times 10^{-6}) \quad 0.5 \leq t \leq 0.8 \mu sec
\]

\[
I(t) = (3.123 - 0.71 t + 0.16 t^2) \times 10^9 \quad 0.8 \leq t \leq 4 \mu sec
\]
$$I(t) = (0.774 + 0.694 \, t - 0.047 \, t^2) \times 10^9 \quad 4 \leq t \leq 15.8 \mu sec$$
Figure 4.13: Intensity variation with time for Pulse #2.
4.1.2 pulse\# 3

Fig. 4.14 shows the intensity variations with time for pulse\# 3. Area under the curve of fig. 4.14 was found to be equal to $3.8 \times 10^4$ J/m$^2$ whereas from fig. 4.11, the area for pulse\# 1 was calculated as $3.738 \times 10^4$ J/m$^2$. Hence a negligible error of 1.08% is obtained. The data for pulse\# 3 was fed in the regression program using the SAS package (as mentioned before) and the resulting mathematical expressions were fed in the main computer program for analysis. These mathematical expressions are given as:

\[
I(t) = 2 \times 10^{16} t \quad \quad 0 \leq t \leq 0.5 \mu\text{sec}
\]

\[
I(t) = 10^{10} - 2.909 \times 10^{16} (t - 0.5 \times 10^{-6}) t \quad \quad 0.5 \leq t \leq 0.73 \mu\text{sec}
\]

\[
I(t) = (4.427 - 0.399 t + 0.021 t^2) \times 10^9 \quad \quad 0.73 \leq t \leq 11.4 \mu\text{sec}
\]

\[
I(t) = (1 \times 10^{-9} + 0.794 t - 0.050 t^2) \times 10^9 \quad \quad 11.4 \leq t \leq 15.8 \mu\text{sec}
\]
Figure 4.14: Intensity variation with time for Pulse #3.
4.1.3 pulse# 4

Fig.4.15 shows the intensity variations with time for pulse# 4. Area under the curve of fig.4.15 was found to be equal to $3.7386 \times 10^4 \text{ J/m}^2$ whereas from fig.4.11, the area for pulse# 1 was calculated as $3.738 \times 10^4 \text{ J/m}^2$. Hence a negligible error of 0.01% is obtained. This pulse in fact consists of three linear variations of intensity with time. These variations can easily be described using straight line equations.

The mathematical expressions are given as:

$$I(t) = 2 \times 10^{16} t \quad \quad 0 \leq t \leq 0.5 \mu\text{sec}$$

$$I(t) = 10^9 - 2.909 \times 10^{16}(t - 0.5 \times 10^{-6}) t \quad 0.5 \leq t \leq 0.7 \mu\text{sec}$$

$$I(t) = \left[ 5 - \frac{5 \times 10^6}{15.1}(t - 7 \times 10^{-7}) \right] 10^9 \quad 0.7 \leq t \leq 15.8 \mu\text{sec}$$

4.1.4 pulse# 5

Fig.4.16 shows the intensity variations with time for pulse# 5 which is a square pulse. The intensity remains constant and then suddenly becomes zero at the end of the pulse. The area under the curve for this pulse is exactly the same as that obtained for pulse# 1 if a constant power intensity value of $2.3659 \times 10^8 \text{ W/m}^2$ is chosen. This value was put in the computer program for analysis. The mathematical expressions are given as:

$$I = 2.3659 \times 10^8 \text{W/m}^2 \quad \quad 0 \leq t \leq 15.8 \mu\text{sec}$$

$$I = 0.0 \quad \quad t > 15.8 \mu\text{sec}$$
Figure 4.15: Intensity variation with time for Pulse #4.
Figure 4.16: Intensity variation with time for Pulse #5.
4.1.5 pulse# 6

Fig 4.17 shows the intensity variations with time for pulse# 6. Area under the curve of fig.4.17 was found to be equal to $1.5219 \times 10^4$ J/m² whereas from fig.4.11, the area for pulse# 1 was calculated as $3.738 \times 10^4$ J/m². This means that pulse# 6 has an energy content which is 40.7% of that contained by pulse# 1. The data for pulse# 6 was fed in the regression program using the SAS package (as discussed before) and the resulting mathematical expressions were fed in the main computer program for analysis. These mathematical expressions are given as:

$$I(t) = 2 \times 10^{16} t \quad 0 \leq t \leq 0.5 \mu sec$$

$$I(t) = 10^{16} - 2.909 \times 10^{16}(t - 0.5 \times 10^{-6}) t \quad 0.5 \leq t \leq 0.8 \mu sec$$

$$I(t) = \left[1.273 - 0.671 \times 10^6 (t - 0.8 \times 10^{-6})\right] 10^9 \quad 0.8 \leq t \leq 1.6 \mu sec$$

$$I = 7.356 \times 10^8 \quad 1.6 \leq t \leq 15.8 \mu sec$$

$$I = 0.0 \quad t > 15.8 \mu sec$$

This pulse was introduced into the program in order to see the effect of the scale factor on temperature profiles. This pulse is not included in the comparison made between temperature profiles due to pulse # 1 to pulse # 5.
Figure 4.17: Intensity variation with time for Pulse #6.
When studying the pulse heating, five pulses (pulse#1 to Pulse#5) are introduced. These pulses contain the same amount of energy but having different shapes, i.e. different rise times. These pulse shapes are fed into the computer program in order to compare their outputs.

The intensity of the pulse should be within acceptable limits providing that the reaching of the critical temperature is avoided. This may be achieved by introducing the scaling factor for I(t). In the present study, a scale factor of 0.08 is employed to fulfill this requirement.

Fig.4.18 shows a comparison between the surface temperature profiles which are the result of different pulse shapes. It is clear from the figure that the pulse#4 causes the surface temperature to reach values higher than those corresponding to the remaining four pulses. The lowest temperature value is resulted due to pulse#5. Pulses #1 and 2 develop about the same temperature profile whereas in pulse#3 heating rate is higher than that of pulses #1 and 2. This is due to the fact that pulse#4 has an intensity variation in which the larger part of the total energy contents occurs in the beginning of the pulse. This causes intense heating at the pulse beginning. In contrast, pulse#5, which is basically a square pulse, has evenly distributed total energy contents. This results in slow heating mechanism and hence low temperature profiles are obtained despite the fact that total energy of both the
pulses #4 and #5 is the same. The same effect is observed, although at a smaller scale, when the effects of pulses #1 and 2 on heating mechanism are compared. The energy contained in the beginning of pulse #2 is slightly higher than that contained by the same portion of pulse #1. This results in high heat input giving rise to high temperature rise corresponding to pulse #2. Similarly pulse #3 has higher energy contents in the beginning of the pulse when compared with pulse #2 resulting in higher surface temperature than that results from pulse #2. After about 12 \( \mu \) sec of heating time, an exponential reduction occurs in surface temperature. This may indicate the start of the cooling cycle.
Figure 4.18: Typical surface heating and cooling profiles due to Pulse # 1 to Pulse # 5 for the whole pulse period of 15.8 μsec.
In order to analyze initial heating and cooling cycles, which occur in the time interval of $0 \leq t \leq 0.5 \mu$ sec, surface temperature profiles for the above time interval are plotted and shown in fig.4.19. This figure shows very clearly how the surface temperature varies (increase and consequent decrease) due to the pulse shape. The surface temperature profile due to pulse #$5$ shows the lowest temperature with a gradual increase in the surface temperature with time unlike the remaining pulses. This is due to the fact that pulse #$5$ is a square pulse with a constant power intensity resulting in no cooling cycle to occur. In contrast, all the remaining pulses have varying intensity profiles which develop both heating and cooling cycles and consequently resulting in surface temperature profiles different from that resulted due to pulse #$5$.

All the pulses #$1$, #$2$, #$3$ and #$4$ develop very similar temperature profiles for the time interval of $0 \leq t \leq 0.7 \mu$ sec. This is due to the similarity of the pulse shapes. However from this point onwards, the temperature profiles due to the four pulses mentioned start deviating. In the beginning of the heating process, the pulse #$5$ develops higher temperature than those developed by the remaining four pulses. This is due to the high intensity of pulse #$5$ in the beginning of the heating process. As the heating proceeds, the intensity of pulses #$1$, #$2$, #$3$ and #$4$ increases exponentially resulting in higher temperature than the one developed due to pulse #$5$.
Figure 4.19: Typical surface heating and cooling due to Pulse # 1 to Pulse # 5 for the time period of t=0-5 μsec.
Fig. 4.20 shows the later part of the surface temperature profiles for the time interval of $5 \leq t \leq 16 \mu$ sec. It is clear from the figure that surface temperature due to Pulse # 4 increases parabolically as soon as the boiling point of the material is reached at a time of $8 \mu$sec. This indicates the sensible heating after the evaporation process. It then starts dropping after a time of $13 \mu$sec due to the cooling cycle of the pulse. Surface temperature resulted due to Pulse # 5 increases linearly with small gradient and then starts decreasing at the end of the pulse due to the rapid decrease in the intensity at time of $t=15.8 \mu$sec. The surface temperature due to the remaining three pulses develop similar temperature profile after $11 \mu$ sec since the pulse shapes become identical. Fig. 4.21 shows the temperature profile inside the material resulting from the irradiation of pulse # 5 and for six different heating times. Since the intensity is constant with time for this pulse, there is always a gradual increase in temperature with time at every point inside the material. Initially the profile is fairly smooth but as the heating progresses becomes parabolic with maximum temperature occurring at the surface. However, the maximum temperature occurs just underneath the surface after the start of the melting process. This may be due to the same reason as was stated for continuous beam heating i.e. convection effects due to melting are dominant at the surface removing some energy from the surface region. This results in cooling of the surface and consequently maximum temperature lies somewhere below the surface.
Figure 4.20: Typical surface heating and cooling due to Pulse # 1 to Pulse # 5 for the time period of t=5-15.8 µsec.
Figure 4.21: Temperature profiles inside the material due to Pulse # 5 (Square Pulse).
Fig. 4.22 shows the temperature profiles inside the material due to pulse # 4. At low heating times, very similar behavior of the temperature profiles is observed as that observed for pulse # 5. However, at high heating times, that is near the end of the pulse, different temperature profiles are obtained for a time of $t=16.74 \mu \text{sec}$. At this instance of time, it can be seen that the temperature profile intersects the temperature profile corresponding to $t=11.97\mu \text{sec}$ at about $2\times10^{-8}\text{ m}$ below the surface. The intersection of profiles resulted at different time intervals is not observed in the case of pulse # 5. This may be due to the pulse shape. Similar behavior for the temperature profiles inside the material can be observed for the remaining three pulses # 3, 2 and 1. These temperature profiles for pulse # 3, 2 and 1 are shown in figures 4.23, 4.24 and 4.25 respectively.
Figure 4.22: Temperature profiles inside the material due to Pulse # 4.
Figure 4.23: Temperature profiles inside the material due to Pulse # 3.
Figure 4.24: Temperature profiles inside the material due to Pulse # 2.
Figure 4.25: Temperature profiles inside the material due to Pulse # 1.
A comparison of temperature profiles corresponding to the five pulses may be made when examining the fig.4.26. This figure clearly shows the effect of the pulse shape on temperature profiles developed inside the material at a heating time of 0.1438 μ sec. All the pulses # 1, 2, 3, and 4 result in the same temperature profile due to the similarity of the pulse shapes for the time interval of 0 \( \leq t \leq 0.1438 \) μ sec. However pulse # 5 develops a higher temperature gradient than those obtained for the remaining pulses. This is because of the relatively higher peak power intensity during the time interval of 0 \( \leq t \leq 0.1438 \) μ sec of pulse # 5.

Fig.4.26 shows temperature profiles before the melting process has started. The profiles corresponding to the melting process are shown in fig.4.27 at heating time of 0.63 μ sec. It is clear from the figure that pulse # 5 develops a lower temperature gradient than those obtained due to the remaining pulses. This temperature profile also shows that melting is initiated earlier for the pulses # 1, 2, 3 and 4 except for the pulse # 5. This is due to the pulse shape i.e. the power intensity increases exponentially which is not the case with pulse # 5.

Fig.4.28 shows the temperature profiles inside the material due to the five pulses at a heating time of 2.35 μ sec. All the pulses develop similar temperature profiles, but the magnitudes of these profiles differ. For example, at a distance of 3 \( \times 10^{-8} \) m below the surface, the temperature due to pulse # 5 is 650° C while pulse # 4
develops a temperature of 1070° C, hence a temperature difference of 420° C exists between the two pulses. The higher temperature due to pulse # 4 is again due to the pulse shape which contains large portion of energy in the beginning of the pulse. Similar arguments can be made in the case of temperature profiles due to pulse # 1, 2 and 3.
Figure 4.26: Temperature profiles inside the material due to Pulse #1 to #5 at a time of t=0.143 μsec.
Figure 4.27: Temperature profiles inside the material due to Pulse #1 to #5 at a time of t=0.638 μsec.
Figure 4.28: Temperature profiles inside the material due to Pulse # 1 to # 5 at a time of t=2.353 μsec.
Figures 4.29 to 4.31 show the temperature profiles due to the five pulses at times of 7.18 μsec, 11.97 μsec and 16.74 μsec respectively. Same behavior of the temperature profile is seen as that observed at 2.35 μsec, although at a larger scale.

In fig.4.29 the temperature profiles developed due to different pulses are shown. The profile corresponding to pulse # 3 lies below the temperature profile due to pulse # 2 at a time of 7.18 μsec. This is due to the fact that pulse # 3 has a power intensity less than that for pulse # 2 at 7.18 μsec. Another notable point in fig.4.29 is that the pulse # 1 has a power intensity which is greater than that of pulse # 3 at 7.18 μsec, but the temperature profile developed by pulse # 1 lies below the temperature profile due to pulse # 3. This behavior is opposite to the one observed while comparing pulse # 2 with pulse # 3. The reason for this behavior may be the energy contents of pulse # 1 in the beginning of the heating process which cause slow heating rate as compared to pulse # 3 and hence lower temperature result.

Fig.4.30 shows temperature profiles developed inside the material due to the different pulses. A big temperature difference is obtained between the temperature profiles due to pulse # 4 and the remaining pulses # 1, 2, 3 and 5 during the heating process. This is due to the sensible heating after the evaporation temperature is reached. A difference of 1000°C exists between temperature due to pulse # 4 and temperature due to pulse # 1, 2 and 3 at a distance of $3 \times 10^{-8}$ m below the
surface. Another noteworthy feature of fig.4.30 is the similarity in temperature profiles due to pulse # 1, 2 and 3. This is due to the similarity in the pulse shape after 11.97 μsec. Pulse # 5 shows lowest temperature profile and the distance between this temperature profile and the remaining temperature profiles is quite appreciable. However, when fig.4.31 is examined, it can be seen that the temperature profile due to pulse # 5 and temperature profiles due to Pulse # 1, 2 and 3 are closer to each other as compared with the temperature profiles at 11.97 μsec. This is due to the fact that the cooling cycle in pulses # 1, 2 and 3 results in decrease in temperature due to pulses # 1, 2 and 3. In contrast, there is no cooling cycle in pulse # 5 which results in a gradual increase in temperature and hence the temperature profiles come closer to each other.
Figure 4.29: Temperature profiles inside the material due to Pulse # 1 to # 5 at time $t=7.18 \mu$sec.
Figure 4.30: Temperature profiles inside the material due to Pulse # 1 to # 5 at time $t=11.97 \mu \text{sec.}$
Figure 4.31: Temperature profiles inside the material due to Pulse # 1 to # 5 at a time of $t=16.74 \mu$sec.
4.2 Effect of the Scale Factor

Fig. 4.32 shows surface temperature profiles developed due to pulse # 6. This figure shows the effect of the scale factor, which is used in the time dependent intensity profiles, on the surface temperature profiles. It is seen that increasing the scale factor results in rapid increase in the surface temperature. This is due to the fact that peak intensity increases with increasing scale factor. At a scale factor of 0.3, it can be seen that temperature as high as 8000° C are obtained at 8.5 \( \mu \) sec of heating time. As the scale factor is reduced to a value of 0.25, the peak temperature drops to 7400 ° C at a heating time of 17 \( \mu \) sec. Surface temperature decreases further when the scale factor is further reduced to 0.2. A peak temperature of 2250° C is obtained at 14 \( \mu \) sec. Similar behavior of the temperature profile is observed at a scale factor of 0.1. A rapid reduction in the surface temperature occurs when the heating cycle is completed. In this case, heat conduction dominates the heating which in turn cools the material.

In order to examine the heating process which occurs in the beginning of the pulse, surface temperature profiles are obtained for the time interval of \( 0 \leq t \leq 1 \mu \) sec and being shown in fig. 4.33.

At a scale factor of 0.1, the heating rate is small in the beginning of the pulse and the temperature profile becomes parabolic. The melting process starts at 0.4 \( \mu \) sec.
As the intensity is increased, by increasing the scale factor, surface temperature profiles start shifting towards left (fig. 4.33). At a scale factor of 0.2, the temperature profile is still parabolic and the melting starts at 0.28 \( \mu \) sec. Melting is achieved even earlier i.e. at a time of 0.25 \( \mu \) sec when the scale factor is increased to 0.25. Finally, at a scale factor of 0.3, melting of the surface starts at 0.23 \( \mu \) sec, which is 0.17 \( \mu \) sec earlier than that corresponding to a scale factor of 0.1. After the start of the melting process, it is seen that large values of scale factor result in large temperature gradients and vice versa. After about 0.8 \( \mu \) sec, there is a slight decrease in the surface temperature. This is due to the initial cooling effect of pulse \( \# 6 \) during the time interval of \( 0.5 \leq t \leq 0.8 \).
Figure 4.32: Typical surface heating and cooling due to Pulse # 6 for the whole pulse period of 15.8 μsec at different values of the scale factor.
Figure 4.33: Typical surface heating and cooling due to Pulse # 6 for the pulse period of 0-1 μsec at different values of the scale factor.
4.3 Pulse Integration

The pulse repetition rate should be defined in order to converge the process ( heating, melting and evaporation processes ) to continuous machining. This repetition rate may be calculated using the temperature and corresponding pulse intensity profiles.

The surface temperature of the target material reaches melting and evaporation values after the pulse heating progresses. However, temperature at the surface as well as inside the material remains high ( above the melting point ) for some time even after the pulse diminishes. This is due to the conduction effect occurring inside the material. Once the temperature drops to about melting point, introducing the new pulse increases the internal energy of the workpiece which in turn increases the temperature inside and at the surface of the workpiece. This process, introducing the laser pulse repeatedly whenever the temperature drops to the melting point, may be referred to as pulse integration. This technique provides quasi-steady type continuous heating.

The pulse repetition rate depends mainly on the type of laser beam ( solid state or gas ) and the workpiece material. A pulse repetition rate of 100 kHz is recommended for CO₂ laser pulses. Alternatively, this value may reduce to 100 Hz for Nd-YAG laser pulses.
Chapter 5

Conclusion

Heat transfer mechanisms during laser-metal interactions have been studied using the Electron Kinetic Theory Approach. The energy equation, which is in the form of an integro-differential equation, developed using this theory is solved numerically. The numerical scheme employed is the explicit finite difference approximation. The energy equation is discretized and solved for temperature distribution with time and distance inside the material. A computer program is developed to solve the energy equation for both conduction and convection heat transfer processes.

In the first part of the present study, effect of continuous wave (CW) laser beam on the heating mechanism is analyzed and compared with three types of linearly varying pulses. It is found that small rise time of the linearly varying pulses result in high surface temperature and consequently melting starts earlier with pulses having
small rise time than those corresponding to large rise time.

Temperature profiles developed inside the material due to pulses A, B, C and the CW laser beam are also compared at different heating times. It is concluded that, before the start of the melting process, the temperature profiles are parabolic with maximum temperature occurring at the surface. However, there is a considerable change in the temperature profiles after the melting process has started. In this case, maximum temperature lies somewhere below the surface and not at the surface. This is due to the effect of the melting process which remove some energy from the surface and results in decrease in the surface temperature.

Since laser pulses are being extensively employed in many laser machining applications, it is of greatest importance to examine the effect of complete laser pulses on the temperature distribution. Therefore, in the second part of this study, six different type of pulses are developed and introduced in the computer program for thermal analysis. One of these pulses, pulse #6, is used to determine the effect of scale factor, employed in the time dependent intensity profiles, on temperature distribution. Each of the remaining five pulses has same energy content but different shape. These pulses are examined and temperature profiles both at the surface as well as inside the material are obtained. It is noted that the pulse shape has a major effect on the heating mechanism. By putting more energy in the beginning
of the pulse, it is possible to develop high temperature within a small interval of
time at the start of the heating mechanism. Hence, by modifying the shape of the
laser pulse, more efficient laser pulses can be developed for processing the materi-
als. This will result in energy conservation and low manufacturing/production costs.

The effect of the scale factor on the temperature distribution is also analyzed and it
is found that for a given pulse shape, the temperature profiles are highly dependent
on the scale factor. Small change in the value of scale factor results in large variation
in the temperature profiles.

5.1 Recommendations for Future Work

- Studying the early plasma generation.

- Studying the effect of $f$ factor on heating mechanism.

- Study the temperature dependence of absorption coefficient.

- Laser Lab development
Appendix A

Listing of the Computer Program
C*******************************************************************************
C PROGRAM TO SOLVE THE CONDUCTION EQUATION OBTAINED AS A RESULT
C OF KINETIC THEORY ANALYSIS OF LASER HEATING OF METALS
C******************************************************************************

DIMENSION T(100,3000),A7(100,100),A8(100,100)
OPEN(1,FILE='laser2-2.data',STATUS='OLD')
OPEN(9,FILE='out9',STATUS='NEW')
OPEN(15,FILE='out15',STATUS='NEW')
OPEN(16,FILE='out16',STATUS='NEW')
OPEN(20,FILE='out20',STATUS='NEW')
OPEN(21,FILE='out21',STATUS='NEW')
OPEN(22,FILE='out22',STATUS='NEW')
OPEN(23,FILE='out23',STATUS='NEW')
OPEN(24,FILE='out24',STATUS='NEW')
OPEN(25,FILE='out25',STATUS='NEW')
OPEN(26,FILE='out26',STATUS='NEW')
READ(1,*) CP,DELTA,RO,CLAM,A10,F,BK,VT,C,VM,VL0,BKB,NX,NT,HT
1,STP,VTK,NNT
WRITE(15,201)
201 FORMAT(11X,'SURFACE HEATING DUE TO LASER IRRADIATION'/1X,70(1H-))
WRITE(15,210) CP,DELTA,RO,CLAM,A10
210 FORMAT(/1X,' CP',' DELTA ',' DENSITY ','
1',' LAMBDA ',' INTENSITY ',' 1/6X,F7.2,2X,E12.7,2X,F7.2,2(2X,E14.5))
WRITE(15,*)
WRITE(15,250) BK,F,VT,C,VM,VL0
WRITE(15,*)
WRITE(15,251) NX,NT
250 FORMAT(1X,'K(W/M-K) ',' F ',' TC(K) ',' TM(K) ','
1 L(J/EG)'/1X,2X,F5.1,2X,F7.5,2X,F8.2,2X,F8.2,2X,E14.5)
251 FORMAT(/1X,' NX ',' NT '/1X,2I4)
WRITE(15,2011)
2011 FORMAT(1X,70(1H-))
WRITE(15,2012)
2012 FORMAT(5X,'TIME',15X,'TEMPERATURE')
DT=7.99E-10
DX=CLAM/10

DO 73 J9=1,NX+1
DO 73 J8=1,NX+1
73 A7(J9,J8)=EXP(-ABS(J9*DX-J8*DX)/CLAM))
87 FORMAT(2X,I2,10(1X,E12.5))
51 FORMAT(2X,' CLAM=',E14.5,2X,'DX=',E14.5,2X,'NX=',I2)
DO 83 J9=1,NX+1
DO 83 J8=1,NX+1
83 A8(J9,J8)=EXP(-ABS(J9*DX+J8*DX)/CLAM))
52 FORMAT(2X, I2, 10(1X, E12.5))
   MT=0
   DO 1 J=1, NX+1
1   T(J,1)=20.0
     1, ', DT=', D21.13/90(')'), KSA=0
     MT=0
     ISAY=0
     ZT=0
     ZJ=0
     DO 99 IT=1, MTT
    ISAY=ISAY+1
     IF (ISAY.EQ.100) THEN
        WRITE(*,*) 'IT=', IT
        ISAY=0
     ELSE
    ENDIF
     DO 2 I=2, NT+1
     KSA=KSA+1
     MT=MT+1
     ZT=ZT+DT
     DO 3 J=1, NX+1
     IF (J.EQ.3) T(J-1, I)=SABIR
     ZJ=ZJ+DX
     CALL INT1(I, J, CLAM, DX, NX, NT, AL1, T, A8)
     CALL INT2(I, J, CLAM, DX, NX, NT, AL1, T, A7)
     CALL INT3(I, J, CLAM, DX, NX, NT, AL1, T, A7)
     AAL=(DT*AL1**DELT/(RO**CP))
     AA2=-DELT*J*DX
     AA3=EXP(AA2)
     AA=AAL*AA3
     BB1=(F*BR*DT)/(2*CLAM**2*RO**CP)
     CC=DT*BR*F/(4*CLAM**3*RO**CP)
     IF (J.EQ.NX) THEN
     DD3=T(J, I-1)
     ELSE
     DD3=T(J+1, I-1)
     ENDIF
     IF (T(J, I-1).GE.VTC) GOTO 133
     IF (T(J, I-1).GE.VM) THEN
     CALL VET(AL1, VL0, VTC, V.T, J, T, RO, CP, VL, ASF)
     BB2=V*(DT/DX)
     DD=V*(DT/DX)*DD3
     BB=(1-BB1-BB2)*T(J, I-1)
     ELSE

DT=7.99E-10
BB2=0
BB=(1-BB1-BB2)*T(J,I-1)
DD=0
ENDIF
BB3=1-BB1-BB2

T(J,I)=AA+BB+CC*(AL1+AL2+AL3)+DD

IF   (J.EQ.1.AND.ZT.LE.1E-06) THEN
    WRITE(15,111)ZT,T(J,I)
ELSE
    IF   (J.EQ.1.AND.I.GT.2950) THEN
        WRITE(16,111)ZT,T(J,I)
    ELSE
    ENDIF
ENDIF

111 FORMAT(2X,E14.6,5X,E14.6)

29 FORMAT(2X,'T(',E14.5,',',E14.5,')=','E14.6)
291 FORMAT(2X,E14.5,E14.5,E14.6)
27 FORMAT(1X,5('X,E12.5'))
3 CONTINUE
ZJ=0
2 CONTINUE
99 CONTINUE
133 STOP
END

******************************************************************************

SUBROUTINE INT1(I,J,CLAM,DX,NX,NT,AL1,T,A8)
DIMENSION T(100,3000),A8(100,100)

AL1=0
AL11=A8(J,1)**T(1,I-1)
AL12=A8(J,NX+1)**T(NX+1,I-1)
DO 5 I1=2,NX
5 ALL=ALL+2*A8(J,I1)**T(I1,I-1)
AL1=DX/2*(ALL1+ALL2+ALL)
RETURN
END

******************************************************************************

SUBROUTINE INT2(I,J,CLAM,DX,NX,NT,AL2,T,A7)
DIMENSION T(100,3000),A7(100,100)

AL2=0

IF   (J.EQ.1) THEN
    AL2=0
    GOTO 8
ELSE
ENDIF
ALL1=A7(J,1)*T(1,I-1)
ALL2=A7(J,J)*T(J,I-1)
DO 6 II=1,J-1
6 ALL=ALL+2*A7(J,II)*T(II,I-1)
   AL2=DX/2*(ALL1+ALL2+ALL)
8 RETURN
END

C**********************************************************************************************************
SUBROUTINE INT3(I,J,CLAM,DX,NX,NP,AL3,T,A7)
DIMENSION T(100,3000)
ALL=0
IF (J.EQ.NX+1) THEN
   AL3=0
   ALL2=0.
   GOTO 57
ELSE
ENDIF
ALL1=A7(J,J)*T(J,I-1)
ALL2=A7(J,NX+1)*T(NX+1,I-1)
DO 11 II=J+1,NX
11 ALL=ALL+2*A7(J,II)*T(II,I-1)
17 AL3=DX/2*(ALL1+ALL2+ALL)
57 RETURN
END

C**********************************************************************************************************
SUBROUTINE VET(AI0,VL0,VTC,V,T,J,I,RO,CP,VL,ASF)
DIMENSION T(100,3000)
VL=VL0*(1-(T(J,I-1)/VTC)**2)**(1./2.)
V=AI0/(RO*(VL+CP*T(J,I-1)))
RETURN
END
Bibliography


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