Optimum Process Targeting with Error and Error-Free Sampling Inspection Plan

BY

MOHAMMED AMJAD ALI SIDDIQUI

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This thesis, written by **MOHAMMED AMJAD ALI SIDDIQUI** under the direction of his Thesis Advisor and approved by his Thesis Committee, has been presented to and accepted by the Dean of Graduate Studies, in partial fulfillment of the requirements for the degree of **MASTER OF SCIENCE IN SYSTEMS ENGINEERING.**

Thesis Committee

5/10/2002

Dr. SALIH O. DUFFUAA (Chairman)

Dr. M. BEN – DAYA (Member)

Dr. U. M. AL – TURKI (Member)

Dr. M. DARWISH (Member)

Dr. N. RUHHAL (Member)

Department Chairman
(Dr. Umar M. Al-Turki)

Dean of Graduate Studies
(Dr. M. Osama Jannadi)

9/11/02
Date

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DEANSHIP OF GRADUATE STUDIES

Dedicated

to

My Dear Parents

Whose Prayers, Guidance and Inspiration led to this Accomplishment.

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All praises are for ALLAH(SWT), the most compassionate, the merciful. May peace and blessings be upon prophet Muhammed(PBUH), his family and his companions. I thank almighty ALLAH(SWT) for giving me the knowledge and patience to complete this work.

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THESIS ABSTRACT

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This thesis focuses on a class of targeting problem. The targeting problem is concerned with the selection of the optimal target mean for a production process. The literature in the area of process targeting focused more on targeting problem with the assumption that inspection is error-free. However, the inspection is error prone. Inspector commits type I and type II errors. These errors are expected to have tremendous impact on the optimal process target. In this thesis, three mathematical models are developed under acceptance sampling with and without error. The first model developed considers the effect of error in the sampling inspection plan on the optimal set point or mean of a filling operation. Sensitivity analysis is conducted to the above developed model to see the effect of error on the solution. The second model developed for the case of three-class targeting problem with quality sampling plan. The third model is the generalized model in which three-class targeting model is considered with inspection error in sampling inspection plan. Sensitivity analysis on errors and cost parameters are performed. Results show that type I and type II errors have tremendous impact on the optimal process parameters. The process mean is forced higher at higher values of type I error thus, reducing the expected profit and the process mean tends to decrease with the increase in the level of type II error. The thesis is concluded by suggesting a number of recommendations for future research.

Keywords: Process Targeting, Acceptance Sampling, Quality, Inspection Error.

Master of Science Degree

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خلاصة الأطروحة

الاسم : محمد أمجد علي صديقي
العنوان : " تحديد برامترات العمليات المثلى تحت تأثير خطط وأخطاء الفحص"
التخصص : هندسة النظم
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تدرس هذه الأطروحة مجموعة من مسائل تحديد البرامترات المثلى للعمليات الصناعية .الأدب المنشور في هذا المجال يرتكز على فرضية أن الفحص خالي من الخطأ. لكن الفحص عرضة للخطأ وغالباً ما يرتكب الفاحص أخطأ من النوع الأول وأخطـأ من النوع الثاني. ويتوقع أن تؤدي هذه الأخطاء إلى تأثير كبير على القيم المثلى لبراميترات العملية. تم في هذه الأطروحة تطوير ثلاثة نماذج رياضية تحت خطط الفحص مع وبدون الخطأ. النموذج الأول يعتبر أثر خطأ الفحص عـلى القـيمة الأفضل لمتوسط عملية التعبئة. أُجْري تحليل الحساسية على النموذج المتقدم لرؤية أثر الخطأ على الحلّ. الـنموذج الثاني طُورّ للحالة ذات ثلاثة تصانيف للمنتج مع وجود خطة جودة معتمدة على العينات. النموذج الثالث تعمـيم للـنموذج الثاني حيث تعتبر فيه ثلاثة تصانيف للمنتج مع وجود أخطاء الفحص. أجري تحليل الحساسية على الأخطـاء وعناصـر الـتكلفة. وأدت النتائج إلى أن أخطار الفحص لها تأثير كبير على القيم المثلى لبرامترات العملية. وخلصت الأطروحة باقتراح عدد من التوصيات للبحث المستقبلي.

درجة ماجستير في العلوم

جامعة الملك فهد للبترول والمعادن

الظهران – المملكة العربية السعودية

مايو ٢٠٠٢ م

Chapter 1

Introduction

1.1 Overview

In this chapter, a brief introduction to quality control problem is presented by defining different areas of quality control. The main area of concern in this thesis is the problem of process targeting. This area is introduced in this thesis by a basic model of process targeting. An introduction to inspection plans is presented in section 1.3, in which acceptance sampling and inspection error are discussed in detail. Acceptance sampling plan plays a vital role in the field of statistical quality control(SQC), and will be used in the thesis for the extension of several models in the literature. The problem under consideration is discussed in section 1.4, followed by the objectives of the thesis in section 1.5. The organization of the thesis is presented in section 1.6.

1.2 Quality Control

In any production process, regardless of how well designed or carefully maintained, a certain amount of inherent or natural variability will always exist. This natural variability is the cumulative effects of many small, essentially unavoidable causes. This variability in key quality characteristics usually arises from three sources: improperly adjusted machines, operator errors, or defective raw material.Thus, quality control plays a vital role in minimizing this variations and improving the overall characteristics of the product.

Quality control is a new way of thinking about and viewing management. Quality control can be defined as "A system of methods for the cost effective provision of goods or services whose quality is fit for the purchaser's requirements." The definition of quality has evolved with time. Initially it was defined as "Fitness for Use", then the definition modified by Juran given as "meeting specifications".

Statistical methods are essential and form the basis for quality control. For this reason, quality control is often called "statistical quality control". It provides a framework for obtaining the improvement in the quality of the products and services. Areas of quality control includes,

1. **Product Control**: involves the control of products at the source of production and through field service so that departures from the quality specification can be corrected before defective or non conforming items are shipped, this can be achieved through 100 percent inspection or sampling inspection etc.

2. **Statistical Process Control**: A major objective of statistical process control is to quickly detect the occurrence of the assignable causes of process shifts by the use of control charts or by the use of some other statistical tools. So that investigations of the process and corrective action may be undertaken before many non-conforming units are manufactured. This is usually achieved by control charts.

3. **Process Targeting**: An important aspect of SQC is the determination of the optimum values of the process parameters or machine settings. The general process-targeting problem is to find the optimal settings of the process mean and other process parameters to minimize total cost resulting from quality cost and cost of manufacturing.

4. **Quality Engineering**: Deals with setting values for the controllable design and process variables to minimize deviation from specific targets using design of experiments techniques. Taguchi presented a quadratic penalty for this deviation known as "Taguchi Quadratic Loss Function", in which the loss function concept for evaluating the quality level of a product by quantifying the deviation from the target value.

1.2.1 Process Targeting

An important problem in manufacturing involves determining the optimal mean or target value for the process. The process mean affects both the production cost and the chance of producing non-conforming items. Therefore, the general "Process Targeting" problem is to find the optimal settings of the process mean and other process parameters to minimize expected total cost or to maximize expected profit resulting from quality cost, cost of material, cost of manufacturing and processing etc. Consequently, the decision on setting a process mean should be based on the trade-off among material cost, payoff of conforming items, and the costs incurred due to non-conforming items. For example, suppose that there is a lower specification

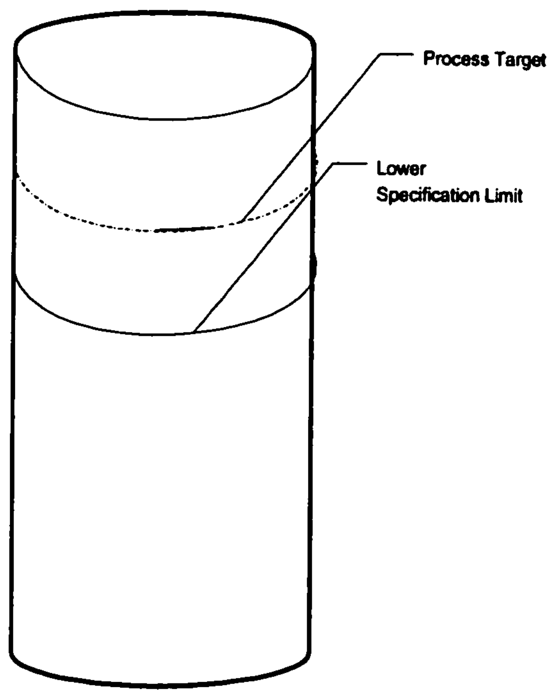

Figure 1.1: Process Targeting

limit (L) on the weight of a package. In many manufacturing process, each product is weighed using an automatic weighing machine. If the package weighs less than the lower specification limit, it is automatically rejected. We wish to determine the optimal target value($> L$).

Hunter and Kartha [2] have given an interesting formulation of this problem under the assumption that the process quality characteristic is normally distributed. In an industrial process in which items are produced continuously, suppose there is a lower specification limit (L) for a quality characteristic is fixed, as shown in figure[1.1], such that items with measured value less than L are re-processed or sold at a secondary market. Thus, the customer is compensated for poor quality but does not pay extra for excessive quality. A target value $T = L + \delta$ is selected so that the net income for the process is maximized. A net income function consists of the income from the accepted items, the give-away cost of the material in excess of the lower specification, and income from rejected items.

Notations:

x = the observed value of the quality characteristic (weight or volume),

L = the lower specification limit,

$T = L + \delta$ is the target value,

σ^2 = variance of the process,

a = the net selling price of an accepted item,

r = the net selling price of a rejected item, after re-processing($r < a$),

g = the cost of excess quality per unit measure for an accepted item.

Suppose that out of n items manufactured, n_a are accepted and n_r are rejected.

Then the net income per item given by Hunter and Kartha [2] is,

$$I = \begin{cases} a - g(x - L) & \text{if } x \geq L \\ r & \text{if } x < L \end{cases}$$

The expected net income derived as follows

$$E(I) = a\int_L^\infty f(x)dx - g\int_L^\infty (x - L)f(x)dx + r\int_{-\infty}^L f(x)dx \qquad (1.1)$$

Where

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(x - T)^2}{2\sigma^2}\right)$$

The first term on the right-hand side is the income from the accepted items, the second term is the give-away cost, and the third term is the income from the rejected items.

Let

$$\phi(t) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-t^2}{2}\right)$$

and

$$\Phi(x) = \int_{-\infty}^x \phi(t)dt.$$

Thus, equation(1.1) can be written as,

$$E(I) = a\Phi\left(\frac{\delta}{\sigma}\right) - g\int_L^\infty (x - L)f(x)dx + r\Phi\left(\frac{-\delta}{\sigma}\right)$$

Where δ is the average amount of product which is "given away"; i.e, it is the distance above the lower specification limit L to the target value T at which the product must be operated. For specified values of a, r, g and σ. The optimal value of δ is the solution to

$$\frac{dE(I)}{d\delta} = \frac{a}{\sigma}\phi\left(\frac{\delta}{\sigma}\right) - g\Phi\left(\frac{\delta}{\sigma}\right) - \frac{r}{\sigma}\phi\left(\frac{-\delta}{\sigma}\right) = 0 \tag{1.2}$$

$$\frac{dE(I)}{d\delta} = \frac{a-r}{\sigma}\phi\left(\frac{\delta}{\sigma}\right) - g\Phi\left(\frac{\delta}{\sigma}\right) = 0 \tag{1.3}$$

This can be solved by a trial and error using normal tables. Nelson(1978) provided a nomograph to determine the value of δ.

1.3 Inspection

Inspection for the purpose of deciding on the acceptability of material is carried on at many points in the manufacturing cycle, for example, when products are received from a vendor or from another department or when the products are shipped to customers. There are several methods on deciding whether to accept or reject the product. These are :

- **No inspection at all:** This obviously involves a great risk of accepting a product that is defective.
- **One hundred percent inspection:** Inspect every product received. But, as it is well known, 100 percent inspection is not 100 percent effective in removing defects. In addition to the fact that 100 percent inspection is often less effective than sampling . It has other drawbacks

  - 100 percent inspection is expensive.
  - It obscures the actual risk involved, because the margin of error is not known.
  - Because the margin of error is not known, the information provided by 100 percent inspection is relatively useless in improving the production process.
  - 100 percent inspection cannot be used for destructive testing.
  - scheduling delays.

- **Spot checking :** This is a compromise between no inspection and 100 percent inspection, but it means that many lots are accepted with no check on them.

1.3.1 Acceptance Sampling

Acceptance sampling is the process of evaluating a portion of the product in a lot for the purpose of accepting or rejecting the entire lot as either confirming or non confirming to a quality specification.

The main advantage of acceptance sampling is economy: the lower cost of inspecting since only parts of the lot results in overall cost reduction. In addition to this major advantage there are others.

- There is less damage to the product.
- The lot is disposed off in shorter time so that shop scheduling, inventory turns, and delivery are improved.

- It is usually less expensive because there is less inspection.

- It is applicable to destructive testing.

- Fewer personnel are involved in inspection activities.

- It often reduces the amount of inspection error.

It may be noted here that no sampling plan is perfect; there is always a chance that the sample may not always contain the same proportion of defectives items as the lot. On the basis of the sample, there are risks of accepting "bad" lots and rejecting "good" lots.

In earlier studies, on process targeting problem acceptance sampling and errors were not incorporated. It was assumed error free inspection. In this thesis, an attempt is made to extend existing models in the area of process targeting by incorporating sampling inspection plan and inspection error.

1.3.2 Inspection Error

In case of inspection(either 100 percent or sampling plan) is used for classification of product and quality control, there is always a chance of inspection error. There are two types of errors, type I and type II error. Type I error is classifying a non-defective item as defective and type II error is classifying a defective item as non-defective.

Considering two types of errors, type I and type II errors in the sampling inspection plan in which the observed number of defectives is the number of defectives as it appears to the inspector.

The expected observed number of defectives in the sample, given by Bennett et. al. [3] is

$$x_e = (n - x)e_1 + (1 - e_2)x. \tag{1.4}$$

where n = sample size,

x = actual number of defectives in the sample,

e_1, e_2 = probability of type I and type II error, respectively.

In the case of error free inspection, the decision to accept the lot is based on the actual number of defectives in the sample, i.e the lot is accepted if $x \leq d_0$, where d_0 is the allowable number of defectives in the sample.

However, in the presence of error, the decision to accept the lot is based on observed number of defectives in the sample i.e $x_e \leq d_0$. This is expected to have impact on decision made in process targeting models.

In this Thesis, an attempt is made to study the effect of inspection error in the sampling inspection plan in process targeting and extend the existing process targeting models in this area.

1.4 Statement of the Problem

In this thesis, a container filling process is considered defined by a random variable, which represents either the quantity or weight of material in an individual container. The product has a lower specification limit on its quality characteristics as shown in figure[1.2]. A container is defined as non-conforming, if it is filled less than the LSL. Sampling Plan is used in which a sample from a lot is drawn and evaluated. The

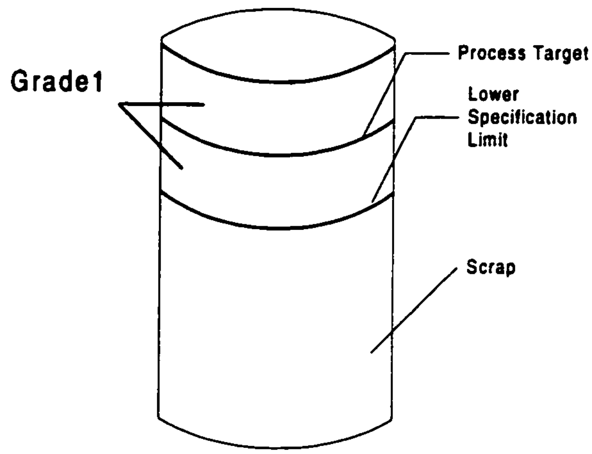

Figure 1.2: Two-Class Targeting Problem

decision to accept or reject the lot is based on the sample, i.e if the number of non conforming products is greater than the acceptance number then the lot is rejected otherwise accepted.

Considering the situation in which type I and type II errors enter into the inspection process. The number of defectives appear to the inspector is not the same as the actual number. However, in the presence of error, the decision to accept the lot is based on the observed number of non-conforming items in the sample.

In this thesis, a mathematical model will be developed to maximize the expected profit by finding the optimal mean or target value considering the effect of inspection error, two cases are considered for this model,

- the case of destructive testing and,

• the case of non-destructive testing.

Also sensitivity analysis will be conducted to the models developed to study the effect of changes in model parameters on its solutions.

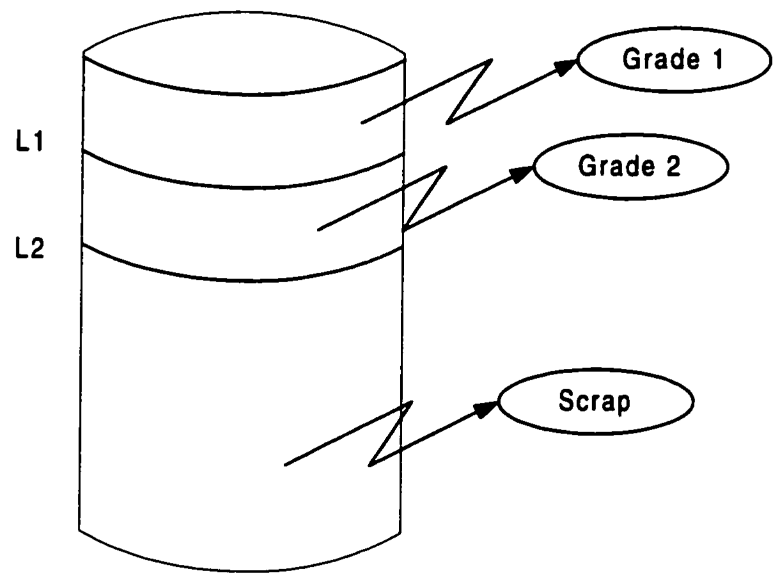

Figure 1.3: Multi-Class Targeting Problem

A model in the literature has been developed by Lee et.al [4] for three-class screening problem as shown in the figure[1.3]. The aim of the model is to maximize the expected profit by finding the optimal target value for the three-class screening problem. In this thesis, the model in [4] is extended by relaxing the assumption of 100 percent inspection by considering sampling inspection plan. Then, the developed model is extended by incorporating inspection error in the sampling inspection plan, and sensitivity analysis is performed to the developed model.

1.5 Objective of the Thesis

The objective of this thesis is to extend the existing process targeting models by incorporating sampling inspection plans under error and error free inspection. Specifically, the objectives are to:

1. Develop a process targeting model for single filling operations considering inspection error in the sampling plan.

2. Conduct sensitivity analysis to study the effect of error on the model developed in objective 1.

3. Develop a three-class targeting model using sampling inspection plan under error free inspection.

4. Extend the model developed in objective 3 by considering inspection error and to conduct sensitivity analysis to study the effect of error on its solution.

1.6 Thesis Organization

The thesis is organized as follows : Literature review in the area of process targeting is presented in the next chapter with emphasis on the models of Boucher and Jafari [1], Lee and Jang [4]. These models constitute basis for the models developed in this thesis. The process targeting model that incorporates inspection error together with sensitivity analysis to investigate the effect of inspection error is presented in chapter 3. A multi-class targeting problem using sampling inspection plan is presented in chapter 4. The multi-class targeting model that incorporates inspection error in sampling plan is presented in chapter 5 and sensitivity analysis on the error is also conducted in the same chapter. Finally, conclusions and recommendations for future research are outlined in chapter 6.

Chapter 2

Literature Review and Problem Definition

2.1 Introduction

The purpose of this chapter is to present the literature in the area of process targeting with emphasis on models extended in this thesis. Section 2.2, presents the literature on process targeting. The description of the model developed by Jafari and Boucher [1] (referred to as Model I) is presented in section 2.3. The model developed by Lee and Jang [4] (referred to as Model II) is presented in section 2.4. These models are used for the extension in this thesis.

2.2 Literature Review

In this section, the Literature in the area of process targeting is presented.

SPRINGER [5] considered a manufacturing situation where upper and lower specification limits are both present and the performance variable follows a gamma distribution. The per-item cost associated with non conforming items above the upper specification limit (over filled items) can be different from those below the lower specification limit (under filled items). However, these costs are assumed to be constant. The process mean that minimizes the total cost associated with non conforming items is obtained.

BETTES [6] studied a similar situation with a given lower specification limit and an arbitrary upper limit. Under filled and over filled items are re-processed at a fixed cost. The optimal process mean and upper specification limit are determined simultaneously.

BENNETT, CASE and SCHMIDT [3] considered the effects of inspection error on a cost based quality control system. The system examined is of single sampling plan involving several cost components. Both type I and type II errors are considered.

CASE and BENNETT [7] studied the economic effects of measurement errors on variable acceptance sampling.

HUNTER and KARTHA [2] discussed the situation where under filled items could be sold at a reduced price and a penalty (give-away cost) is incurred by conforming items with excess quality. They derived a procedure for calculating the optimal process mean.

NELSON [8] provided approximate solutions to the problem defined by Hunter and Kartha [2]. A four cycle arithmetic graph is provided for determining the target value.

NELSON [9] gave a nomograph for setting process mean to minimize scrap cost by assuming the distribution of the relevant characteristic of the individual item as normal.

BISGAARD, HUNTER and PALLENSEN [10] modified Hunter and Kartha's model by assuming that the selling price of non conforming items is a linear function of the process mean.

CARLSSON [11] discussed a more general sales situation where the selling prices of the conforming and non conforming items are linear function of excess (give-away) quality and "deficit in quality", respectively.

GOLHAR [12] assumed that only the regular market (fixed selling price) is available for the conforming items and that the under filled items are reprocessed and sold in a regular market. The process is assumed to have known variance.

TANG and SCHNEIDER [13] discussed how to determine screening limits when inspection error is present, and they investigated the economic effects of inspection imprecision on a screening procedure. It is assumed that the rejected items are reworked, and two rework conditions were considered. In the first situation the rejected items can be reworked so that the performance variable is exactly equal to the target value. In the second situation rework is based on the first inspection result; therefore, the value of the performance variable of the reworked items may not be exactly equal to the target value.

TANG and SCHNEIDER [14] discussed a method of determining the optimal inspection precision level based on the trade off of inspection cost and the cost incurred by inspection errors.

DANZIGER and PAPP [15] considers extension of the basic methodology of sampling plan to multiple criteria by defining tests for each criterion is discussed such that passage of all tests will lead to acceptance of, a given total fraction non conforming with specified risk.

VIDAL [16] provided a simple graphical solution for the problem stated in Bisgaard, Hunter and Pallensen[10].

GOLHAR and POLLOCK [17] extended the model by Golhar [12] to include the upper limit to reduce the cost associated with excess quality by re-processing the items above this limit. This model reduces to the model presented in Golhar's as upper limit tends to infinity. An implicit assumption in this model is that the process has an unlimited capacity that can be used to re-process items above the upper limit.

GOLHAR [18] provided a computer program for the above model.

CARLSSON [19] discussed a situation in which the lots produced by the production process are subjected to lot-by-lot acceptance sampling by variables.

SCHMIDT and PFEIFER [20] investigated the effects on cost savings from variance reduction in a single level canning problem and an approximate simple relationship between percentage reduction in standard deviation and the cost reduction was presented.

TANG and SCHNEIDER [21] showed that when inspection error is present, the observed value of the performance variable can be treated as a correlated variable. Consequently, all the results associated with using correlated variables in screening are applicable to the inspection error situation.

FOUNTAIN and CHOU [22] determined the minimum sample size for two-sided b-content tolerance intervals when the population size is finite.

R.SCHMIDT and P.PFEIFER [23] extended the model of D.Golhar [12] by considering the situation where the process capacity is fixed. In this work, a two level process control scheme is considered to determine both process mean and the upper control limit.

BOUCHER and JAFARI [1] extended this line of research by introducing sampling plan as opposed to 100 percent inspection, where the rejection criteria is based on the number of non conforming units in the sample.In this model, they examined the effect of single sampling plan on the optimal set point of a filling operation. There are two conditions examined,

(1) when sampling results in destructive testing and

(2) when the testing is non destructive.

ARCELUS and RAHIM [24] provided joint optimal settings for variable and attribute target means.

MELLOY [25] considered products that are subject to regulatory auditing (compliance tests) scheme. The performance variable is the weight of the package, which is determined by the weights of the product and the tare (boxes). The process mean and two-sided screening limits are used to minimize the "give-away" product weight, subject to an acceptable level of risk of failing the compliance tests.

CARLSSON [26] presents a model to determine the optimal two-dimensional process level and the optimal expected net income per lot. The model is based on the assumption that the joint distribution of quality characteristics follows a bi-variate normal distribution with known covariance.

TANG and LO [27] developed a model for jointly determining the optimal process mean and screening limits when a correlated variable is used in inspection. Since a correlated variable is not perfectly correlated with the quality characteristic, acceptance cost may be incurred by accepting non conforming items for shipment.

DO SUN BAI and MIN KOO LEE [28] presented the problem of selecting the process mean and the cutoff value of a correlated variable for a filling process in which inspection is based on the correlated variable rather than the process mean itself.

GEORGE TAGARAS [29] analyze an economic model for the selection of acceptance sampling by variables under the assumption of quadratic quality costs.

ARCELUS and RAHIM [30] presented a model for simultaneously selecting the optimal target means for both the variable and attribute quality characteristic. Optimality conditions are derived and a computational algorithm is given.

K.S.AL-SULTAN [31] extended the model of Boucher and Jafari [1] for two machines in series where a sampling plan is used. An algorithm for finding optimal machine parameters for the two machines in series case, with sampling inspection at each machine is given.

SHAUL P.LADANY [32] assumed the process in which oversized items and undersized items are repaired at different costs. The objective is to maximize the profit.

MIHALKO and GOLHAR [33] addressed the problem of estimating the expected profit for automatic filling operation when the standard deviation of the filling process is unknown. A method for the determination of the confidence interval for the optimal process setting for the case of unknown variance is proposed.

LIU, TANG and CHUN [34] considered the case of a filling process with limited capacity constraint. The optimal process parameters to be determined are process mean and upper specification limit.

ARCELUS and RAHIM [35] developed several models that controls both conformance to specification and uniformity of production. The models are proposed to minimize the functional interval width, with and without predetermining the value of the process coefficient of variation and to penalize for the deviation from the target value.

LEE and JANG [4] introduced the case of the three-class screening. In this paper, it is assumed that the products are sold in two different markets with different price structures. Two models were presented, in the first case; the objective is to find the optimal mean when inspection is based on the same quality characteristic. While in the second model it is assumed that the inspection is based on a correlated variable.

PULAK and AL-SULTAN [36] in this paper, a FORTRAN based computer program is presented for nine different process targeting models.

AL-SULTAN and AL-FAWZAN [37] extended the model of M.A.Rahim and P.K.Banerjee's(1988)model. This paper assumed a process with random linear drift and is assumed to have both lower and upper specification limits. The objective is to find the optimal initial mean and cycle length. Variance of the process is assumed to be known and constant.

AL-SULTAN and PULAK [38] presented a model for finding the optimal mean of a filling process under rectifying inspection. The effect of variance reduction is also considered for the case.

AL-SULTAN and AL-FAWZAN [39] studied the model of M.A.Rahim and P.K.Banerjee (1988) i.e. systems with linear drift for the case of variance reduction and optimal initial process mean and cycle time is given.

CAIN and JANSSEN [40] presented the model where is asymmetric across the target. a linear cost below lower specification limit and a quadratic cost above specification limit are assumed.

POLLOCK and GOLHAR [41] in this paper the canning process with constant demand and capacity constraint for the production process is considered. The model also assumes penalty for producing nonconforming items.

PFEIFER [42] provided a general canning problem model consisting of a piecewise linear profit function. This paper give a simple relationship between two competing objective functions for the canning problem, expected profit per fill-attempt and expected profit per can to be filled.

HONG and ELSAYED [43] extended the Golhar's [12] model for the case with normally distributed measurement error. They developed a model for determining jointly the optimum process mean and the cutoff value on the observed characteristic when measurement error is present.

RAHIM and AL-SULTAN [44] Consider the problem of simultaneously determining the optimal target mean and target variance.

MISIOREK and BARNETT [45] considered the problem of choosing the mean of a filling process for a number of model variations. The effects of change of the process variance on the optimal solution as well as on the expected profit are also discussed.

RAHIM, BHADURY and AL-SULTAN [46] Addressed the problem of selecting the most economical target mean and variance for a continuous production process, three approaches are suggested for the economic selection of a target variance integrated with the target mean.

LEE and AL-SAYED [47] Addressed the problem of determining the optimum process mean and screening limits of a product quality under a two-stage screening procedures.

As this thesis, extends the work done by Boucher and Jafari [1] and Lee and Jang [4], their models are presented in this section referred to as model I and model II, respectively.

2.3 Model I

The aim of the model is to find the optimum target value for a single filling process with quality sampling plan with the assumption that the inspection is error free. Two cases are examined.

- when sampling results in destructive testing and,
- when sampling results in non-destructive testing.

This model considers a filling process defined by a random variable "X", which represents the quantity of material in an individual container. A lower specification limit exists for X. Sampling plan is introduced in which a product is produced daily in lots of size "N". So, from each lot a sample of size n is drawn and evaluated. A container is classified as a "non-conforming" if it is filled less than the lower specification limit i.e, $(X < L)$. If D is the number of non-conforming items found in the sample and d_0 is the allowable number of non-conforming units,then the lot is accepted and sold at a price of A, if $D \leq d_0$ and the lot is rejected and sold at a reduced price of $(A - P)$ per product.

Notations

A = Selling price of the acceptable product,

P = Penalty or price reduction incurred per product,

c = Cost of excess quality per accepted item,

μ = Process Mean or Target.

Model Formulation

Thus, The expected revenue for a lot of size N, given by boucher and jafari [1].

$$E[R|D] = \begin{cases} AN & \text{if } D \leq d_0 \\ (A-P)N & \text{if } D > d_0 \end{cases}$$

Therefore, The marginal profit function is

$$E[\pi(\mu)] = ANPr.(D \leq d_0) + (A-P)NPr.(D > d_0) - Nc\mu$$

i.e

$$\frac{E[\pi(\mu)]}{N} = A - PPr.(D > d_0) - c\mu \qquad (2.1)$$

where $Nc\mu$ is the cost of processing for the lot N, given the set point μ.

The expected revenue function for destructive testing is

$$E[R|D] = \begin{cases} A(N-n) & \text{if } D \leq d_0 \\ (A-P)(N-n) & \text{if } D > d_0 \end{cases}$$

And the expected profit becomes,

$$E[\pi(\mu)] = A(N-n)Pr.(D \leq d_0) + (A-P)(N-n)Pr.(D > d_0) - Nc\mu$$

$$E[\pi(\mu)] = A(N-n) - P(N-n)Pr.(D > d_0) - Nc\mu$$

2.4 Model II

In this model [4], the product out of the production process is classified into three grades based on product specifications (that might be the weight or volume of the can). Considering can filling process, grade one is the product that has the net weight $\geq L_1$. The second grade product is the product that has a net weight between L_2 and L_1. The third grade is scrap, which has net weight $\leq L_2$.

The aim of this model is to find the optimal process mean, to maximize the profit resulting from grade 1 and grade 2.

Notations

Y = Performance variable representing the quality characteristic,

L_1 = Specification limit on Y for grade 1,

L_2 = Specification limit on Y for grade 2,

a_1 = Selling Price for grade 1,

a_2 = Selling Price for grade 2,

r = reduced price for scrap,

$c_0 + cy$ = Production cost per item,

c_p = Inspection cost per item.

Assumptions

- 100 percent inspection is considered.
- The inspection is assumed to be error free.
- A single item can be sold in two different markets with different cost(or)profit

structures.

- The quality characteristic is assumed to be normally distributed with process mean μ_y and known variance σ_y^2. The pdf of y is f(y).
- $a_1 > a_2 > r$.
- $L_1 > L_2$.
- The production cost per item is $c_0 + cy$.

Model Formulation

The profit function per item is given by

$$P_Y = \begin{cases} a_1 - (c_0 + cy + c_p) & \text{if } Y \geq L_1 \\ a_2 - (c_0 + cy + c_p) & \text{if } L_2 \leq Y < L_1 \\ r - (c_0 + cy + c_p) & \text{if } Y < L_2 \end{cases}$$

Then the expected profit per unit given by Lee and Tang [4]

$$E[P_y] = \int_{L_1}^{\infty} [a_1 - (c_0 + cy + c_p)] f(y)dy + \int_{L_2}^{L_1} [a_2 - (c_0 + cy + c_p)] f(y)dy + \int_{-\infty}^{L_2} [r - (c_0 + cy + c_p)] f(y)dy \quad (2.2)$$

This equation can be written as,

$$\begin{aligned} E[P_y] = [a_1 - c_0 - c_p] \int_{L_1}^{\infty} f(y)dy + [a_2 - c_0 - c_p] \int_{L_2}^{L_1} f(y)dy + [r - c_0 - c_p] \int_{-\infty}^{L_2} f(y)dy \quad (2.3) \\ -c \int_{L_1}^{\infty} yf(y)dy - c \int_{L_2}^{L_1} yf(y)dy - c \int_{-\infty}^{L_1} yf(y)dy \end{aligned}$$

This further can be written as,

$$E[P_y] = a_1 \int_{L_1}^{\infty} f(y)dy + a_2 \int_{L_2}^{L_1} f(y)dy + r \int_{-\infty}^{L_2} f(y)dy - (c_0 + c_p) - c\mu \quad (2.4)$$

Where $f(y)$ $N(\mu_y, \sigma_y^2)$

Let $z = \frac{L_1 - \mu_y}{\sigma_y} = \Gamma_1$

and $z = \frac{L_2 - \mu_y}{\sigma_y} = \Gamma_2$ Where $\phi(z)$ and $\Phi(z)$ are the pdf and cdf of the standard normal distribution,

$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{(\frac{-z^2}{2})}$

using the relationship above we can rewrite the equation of expected profit,

$E[P_y] = a_1[\Phi(-\Gamma_1)] + a_2[\Phi(\Gamma_1)] + (r - a_2)[\Phi(\Gamma_2)] - c_0 - c_p - c(L_1 - \Gamma_1\sigma_y)$

2.5 Conclusion

In this chapter, the literature in the area of process targeting is reviewed. The models of Boucher and Jafari [1], Lee and Jang [4] are presented to provide background for the work in this thesis. The next chapter extends model I to the case where inspection error is present.

Chapter 3

Process Targeting with Inspection Error in Sampling Plan

3.1 Introduction

The purpose of this chapter is to extend model I developed by Boucher and Jafari [1] by incorporating inspection error in the sampling inspection plan. Then to conduct sensitivity analysis to the developed model to study the effect of model parameters on its optimal solutions.

Classically, sampling inspection plans have assumed that the inspection process is perfect, with no errors in judgments being made by the inspector. This assumption is, of course, false. In reality an inspector whether human or machine subject to making two types of errors, these errors are

(1) Type I error: Classifying a non-defective item as defective

(2) Type II error: Classifying a defective item as non-defective.

Thus, inspection error may cause considerable loss due to misclassification of the product. The loss could be of replacement and warranty costs, loss of goodwill, or loss of profit by selling a higher-grade product as lower grade product due to misclassification.

The objective of this model is to maximize the expected profit by finding the optimum target value for single filling operations with quality sampling plans by considering the effect of inspection error. The following two cases are examined.

(1) When sampling results in destructive testing and

(2) When the sampling results in non-destructive testing.

This chapter is organized as follows, Model Development is presented in section 3.2, in which notations, model assumptions statement of the problem, and model formulation are discussed. Solution and Analysis of the model are discussed in section 3.3, results are shown in section 3.4, then sensitivity analysis is conducted in section 3.5.

3.2 Model Development

In this section, a Process Targeting model is developed incorporating inspection error in the sampling plan. At first, necessary notations are presented, followed by model assumptions, then the statement of the problem is provided.

3.2.1 Notations

The following are the notations which are adopted in this chapter

X	=	Quality characteristic of the product,
A	=	Selling price of the accepted product,
P	=	Penalty or price reduction incurred per product,
c	=	Ingredient cost per unit weight,
μ	=	The expected value of X,
σ^2	=	Process variance,
L	=	lower specification limit for the product attribute,
n	=	sample size for sampling,
q	=	lot fraction defective i.e $Pr[X < L] = \Phi(-z)$,
d_0	=	allowable number of non-conforming units in the sample,
x	=	actual number of defectives in the sample,
e_1	=	type I error,
e_2	=	type II error.
x_e	=	The apparent number of defectives in the sample,

3.2.2 Model Assumptions

1. The inspection process is assumed to be error prone.
2. The quality characteristic X is assumed to be normally distributed with mean μ and variance σ^2.
3. The variance of the process is assumed to be known and fixed.
4. Sampling plan is used for quality control.
5. Selling price is greater than the penalty incurred i.e,A>P.

3.2.3 Statement of the Problem

Consider a can filling process where the content of the container is represented by a random variable X, a lower specification limit exists for the quality characteristic (X>L). Sampling plan is used in which a product is produced daily in lots of size N and from each lot a sample of size n is drawn and inspected. A container is defined as non-conforming, if it is filled less than the lower specification limit (X<L). If x is the actual number of nonconforming units in the sample and d_0 is the allowable number of non-conforming units, the decision to accept the lot is based on actual number of non-conforming in the sample, i.e the lot is accepted if $x \leq d_0$. Considering the situation in which type I and type II errors is a part of the inspection process, the number of defectives appears to the inspector is not the same as the actual number of defectives in the sample. Thus, the apparent number of defectives are the number of defectives as it appears to the inspector. The expected apparent number of defectives in the sample, x_e given by [3] is

$$x_e = (n - x)e_1 + (1 - e_2)x$$

However, in the presence of error, the decision to accept the lot is based on the apparent number of defectives x_e. This is expected to have an impact on the decisions made in process targeting model. Containers from a production lot that is accepted are sold at a price A per container, while containers from an unacceptable lots incur a penalty of P per container, i.e they bring in incremental revenue of $(A - P)$.

3.2.4 Model Formulation

Non-Destructive Testing

Thus, The expected revenue for a lot of size N, given x_e.

$$E[R|x_e] = \begin{cases} AN & \text{if } x_e \leq d_0 \\ (A-P)N & \text{if } x_e > d_0 \end{cases}$$

For the set point of μ, the expected value of the marginal cost per lot is $Nc\mu$, where μ is the expected value of X.

Therefore, The marginal profit function is

$$E[\pi(\mu)] = ANP(x_e \leq d_0) + (A-P)NP(x_e > d_0) - Nc\mu$$

i.e

$$\frac{E[\pi(\mu)]}{N} = A - PP(x_e > d_0) - c\mu \tag{3.1}$$

Where $E[\pi(\mu)]$ is the expected profit when the set point μ is fixed at a specific value, looking at this equation one can say that the expected profit is not a function of the lot size and it is only defined in terms of the decision variable μ which affects both the probability of rejection of a lot and the cost of material per lot.

Destructive Testing

It is often the case that examining the weight of the contents of a package will result in destroying the product, Then, The expected revenue becomes

$$E[R|x_e] = \begin{cases} A(N-n) & \text{if } x_e \leq d_0 \\ (A-P)(N-n) & \text{if } x_e > d_0 \end{cases}$$

And the expected profit becomes,

$$\begin{aligned} E[\pi(\mu)] &= A(N-n)P(x_e \leq d_0) + (A-P)(N-n)P(x_e > d_0) - Nc\mu \\ E[\pi(\mu)] &= A(N-n) - P(N-n)P(x_e > d_0) - Nc\mu \end{aligned}$$

It should be noted that the expected profit for nondestructive testing is not directly comparable cost wise to destructive testing because there are different cost associated with the sampling method used. In nondestructive testing the container is salvageable, whereas it is rendered useless in destructive testing.

3.3 Solution and Analysis

We want to maximize $\frac{E[\pi(\mu)]}{N}$ defined above. A necessary condition for optimality is that the partial derivative with respect to μ vanishes at the target value μ.

i.e $\frac{\partial}{\partial\mu}\frac{E[\pi(\mu)]}{N} = 0$,

Therefore,

$$\frac{\partial}{\partial\mu}\frac{E[\pi(\mu)]}{N} = -P\frac{\partial}{\partial\mu}P(x_e > d_0) - c \tag{3.2}$$

Where

$$P(x_e > d_0) = 1 - P(x_e \leq d_0)$$

The probability of an item being defective q is independent of other items and the lot is assumed to be too large then x is said to follow binomial distribution.

since,

$$q_e = q(1-e_2) + (1-q)e_1$$

$$q = P(X \leq L) = \Phi(-z)$$

where

$$z = \frac{\mu - L}{\sigma}$$

Therefore, x_e is said to follow binomial distribution with sample size n and lot fraction defective q_e. i.e

$$x_e = \dot{o}(q_e, n)$$

Therefore,

$$P(x_e \leq d_0) = \sum_{i=0}^{d_0} \binom{n}{i} q_e^i (1-q_e)^{(n-i)}$$

Then,

$$\begin{aligned} \frac{\partial}{\partial\mu} P(x_e > d_0) &= -\frac{\partial}{\partial\mu} P(x_e \leq d_0) \\ &= -\frac{\partial q_e}{\partial\mu}\frac{\partial}{\partial q_e} P(x_e \leq d_0) \end{aligned} \tag{3.3}$$

we know that,

$$\begin{aligned} q_e &= q(1-e_2) + (1-q)e_1 \\ &= (1-e_1-e_2)q + e_1 \end{aligned}$$

Therefore,

$$\frac{\partial q_e}{\partial \mu} = -\frac{(1-e_1-e_2)}{\sigma}\phi\left(\frac{L-\mu}{\sigma}\right) \tag{3.4}$$

$$\begin{aligned}
\frac{\partial}{\partial q_e}P(x_e \le d_0) &= \frac{\partial}{\partial q_e}\sum_{i=0}^{d_0}\binom{n}{i}q_e^i(1-q_e)^{(n-i)} \\
&= \sum_{i=0}^{d_0}\binom{n}{i}\left[iq_e^{(i-1)}(1-q_e)^{(n-i)} - q_e^i(n-i)(1-q_e)^{(n-i-1)}\right] \\
&= -n(1-q_e)^{(n-1)} + \left[n(1-q_e)^{(n-1)} - n(n-1)q_e(1-q_e)^{(n-2)}\right] \\
&+ \left[n(n-1)q_e(1-q_e)^{(n-2)} - \frac{n(n-1)(n-2)}{2}q_e^2(1-q_e)^{(n-3)}\right] \\
&+ \ldots + \left[\frac{n!}{(n-d_0)!d_0!}nq_e^{(d0-1)}(1-q_e)^{(n-d_0)} - \frac{n!}{(n-d_0)!d_0!}(n-d_0)q_e^{d_0}(1-q_e)^{(n-d_0-1)}\right] \\
&= -\frac{n!}{(n-d_0)!d_0!}(n-d_0)q_e^{d_0}(1-q_e)^{(n-d_0-1)}
\end{aligned} \tag{3.5}$$

Substituting equations (3.4) and (3.5) in (3.3), we get

$$\frac{\partial}{\partial \mu}P(x_e > d_0) = -\frac{(1-e_1-e_2)}{\sigma}\phi\left(\frac{L-\mu}{\sigma}\right)\frac{n!}{(n-d_0)!d_0!}(n-d_0)q_e^{d_0}(1-q_e)^{(n-d_0-1)} \tag{3.6}$$

Substituting, equation (3.6) in (3.2) and equating it to zero, we get

$$P\frac{(1-e_1-e_2)}{\sigma}\phi\left(\frac{L-\mu}{\sigma}\right)\frac{n!}{(n-d_0-1)!d_0!}q_e^{d_0}(1-q_e)^{(n-d_0-1)} = c \tag{3.7}$$

The optimal value of μ can be obtained from this equation by numerically solving this equation.

Similar calculations are carried out to find the optimal condition for optimality for destructive testing, we arrive at the result shown below,

$$P\frac{(1-e_1-e_2)}{\sigma}\phi\left(\frac{L-\mu}{\sigma}\right)\frac{n!}{(n-d_0-1)!d_0!}q_e^{d_0}(1-q_e)^{(n-d_0-1)} = \left(\frac{N}{N-n}\right)c \tag{3.8}$$

The only change noted in the optimality condition for the two cases i.e for destructive testing and nondestructive testing is the term $\left(\frac{N}{N-n}\right)$ on the right hand side of the equation.

3.4 Results

In this section, an illustrative example for destructive and non-destructive testing are presented. Since, it is difficult to find a closed form solution, numerical search such as Golden Section method is used, and IMSL subroutines of ANORDF are used to evaluate the standard uni-variate normal distribution function. Computer code is written in Fortran 90, the program is run on a pentium III computer with 64 MB ram (see appendix B for the code).

Using the same values as given in the model developed by Boucher and Jafari [1] and the level of errors from the model of Bennett et. al. [3], the optimal process mean and expected profit for different sample size, and for different values of d_0 are computed. The expected profit obtained by considering the effect of inspection error is evaluated and compared with that of the profit without error. The values A, P, c, σ, e_1, e_2, L are given as,

$A = 67.5$, $P = 30.5$, $c = 55$, $\sigma = 0.00563$, $e_1 = 0.01$, $e_2 = 0.03$, $L = 1$.

The table below shows the values of the optimal process mean and expected profit for the can filling problem with and with out inspection error. The profit decreases with the when inspection error is incorporated in the model.

For n=10

	without error		with error	
d_0	μ	EP	μ	EP
1	1.014	11.68	1.0153	11.4370
2	1.0111	11.84	1.0117	11.7865

Table 3.1: Expected Profit and Mean with and without Error for n=10

For n=20

	without error		with error	
d_0	μ	EP	μ	EP
1	1.0154	11.60	1.0175	10.9328
2	1.0129	11.75	1.0142	11.6108

Table 3.2: Expected Profit and Mean with and without Error for n=20

For Destructive Testing

For n=10

	without error		with error	
d_0	μ	EP	μ	EP
1	1.01381	4.93	1.01514	4.7096
2	1.01097	5.097	1.01159	5.0437

Table 3.3: Expected Profit and Mean for Destructive testing, for n=10

	without error		with error	
d_0	μ	EP	μ	EP
1	1.01529	-1.889	1.01734	-2.45
2	1.01275	-1.74028	1.01407	-1.8664

Table 3.4: Expected Profit and Mean for Destructive testing, for n=20

The point to be noted here is that, the solution is independent of the lot size "N", whatever may be the lot size the value of the Expected Profit and Optimal Mean μ are not affected. As we can see from the table above, for the same value of d_0, as the sample size increases the set point also increases because for any value q, the OC curve of a sampling plan with higher "n"will show less probability of acceptance. In simple terms, sampling more units in a lot ensures the detection of the non-conformity and rejecting the lot. That's why the producer increases the set point. As we can see from the table above, as the sample size increases the mean is increased and if the mean is increased then the Expected Profit will decrease. Moreover, as the allowable number of defectives d_0 is increased then the optimum set point falls because, for the same "n" raising the value of d_0 has the effect of enlarging the area of acceptance under the OC curve, i.e the probability of acceptance increases with the increase in the value of acceptance number d_0.

As can be seen from the tables above, the expected profit for destructive testing is less than the expected profit for non destructive testing, but with the increase in the lot size N the expected profit for destructive testing increases, but if the lot size is infinite then the expected profit for both destructive and non destructive testing are equal.

3.4.1 Hyper-geometric distribution

Using the same values as given in Boucher and Jafari model [1], the expected profit and optimal mean are computed considering hypergeometric distribution. The graph shows the expected profit with and without error for binomial distribution and expected profit with and without error for hypergeometric distribution at different levels of the lot size N. The results shows that when the lot size is small the hypergeometric distribution gives the best results but as the lot size increases the expected profit for hypergeometric distribution is almost the same as the expected profit for binomial.

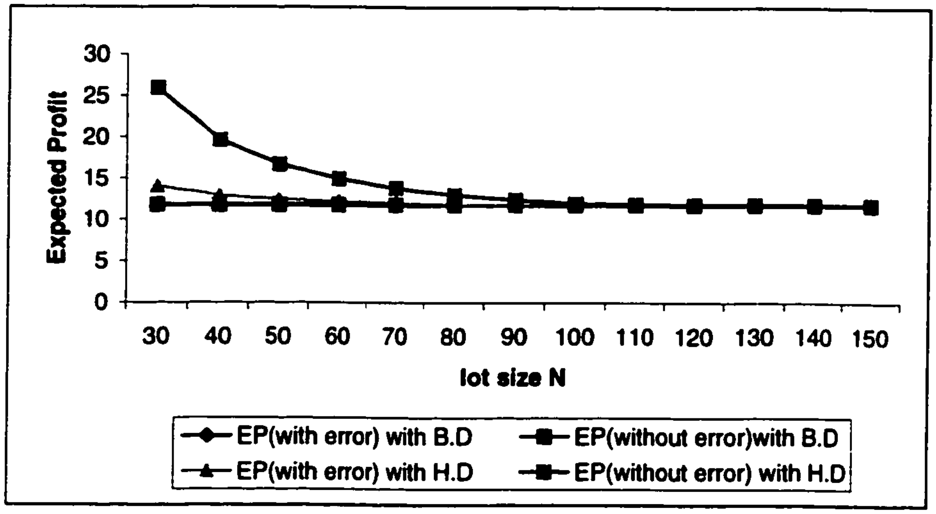

Figure 3.1: Expected Profit vs Lot Size for hypergeometric distribution

3.5 Sensitivity Analysis

The effect of type I and type II errors on expected profit and the process mean is investigated in the sensitivity analysis. Two Sampling Plans are considered with sample size n = 10 and 20, and with same acceptance number $d_0 = 2$. Table 3.5 and 3.6, shows the optimal mean and expected profit at different levels of type I and type II error. The levels considered for type I and type II errors are taken from the paper by Bennett [3]

Change in profit is defined as the difference between expected profit without error and expected profit with error. Thus, percentage change in profit is given by,

$$\text{percentage change in profit} = \frac{EP(\text{without error}) - EP(\text{with error})}{EP(\text{without error})} * 100 \quad (3.9)$$

The point to be noted in the tables shown below is that, the first few values of the change in profit is negative. This is expected because when type I error is nullified and increasing the value of type II error, we are classifying more number of defective items as good and selling them at a higher price. So, the profit with error is expected to be more than the profit without error. This is a limitation in the model it does explicitly considers the negative effect of type II error.

Expected profit without error = 11.84

percentage change in profit = $\frac{EP(withouterror)-EP(witherror)}{EP(withouterror)} * 100$

(e_1, e_2)	μ	EP	Percentage change in profit
(0,0.01)	1.01106	11.844	-0.033
(0,0.03)	1.01101	11.847	-0.059
(0,0.05)	1.01096	11.849	-0.076
(0,0.1)	1.01082	11.857	-0.143
(0,0.15)	1.01067	11.864	-0.202
(0.01,0)	1.01177	11.782	0.489
(0.01,0.01)	1.01175	11.783	0.481
(0.01,0.03)	1.01169	11.786	0.456
(0.01,0.05)	1.01164	11.789	0.4307
(0.01,0.10)	1.0115	11.796	0.371
(0.01,0.15)	1.01136	11.803	0.312
(0.03,0.0)	1.0132	11.594	2.077
(0.03,0.01)	1.01318	11.595	2.069
(0.03,0.03)	1.01313	11.598	2.0439
(0.03,0.05)	1.01311	11.6	2.0270
(0.03,0.1)	1.013	11.607	1.967
(0.03,0.15)	1.0128	11.613	1.917
(0.05,0)	1.01444	11.253	4.95
(0.05,0.01)	1.01441	11.254	4.949
(0.05,0.03)	1.01437	11.256	4.932
(0.05,0.05)	1.01435	11.259	4.907
(0.05,0.1)	1.01424	11.265	4.856
(0.05,0.15)	1.01407	11.271	4.805
(0.1,0.0)	1.01615	9.372	20.844
(0.1,0.01)	1.01615	9.373	20.836
(0.1,0.03)	1.0161	9.375	20.819
(0.1,0.05)	1.01604	9.377	20.802
(0.1,0.1)	1.01593	9.383	20.751
(0.1,0.15)	1.01582	9.39	20.692
(0.15,0.0)	1.0168	5.995	49.366
(0.15,0.01)	1.01677	5.997	49.349
(0.15,0.03)	1.01677	5.999	49.332
(0.15,0.05)	1.01672	6.001	49.315
(0.15,0.1)	1.0166	6.007	49.265
(0.15,0.15)	1.01644	6.014	49.206

Table 3.5: Expected Profit and Optimal Mean at different values of Error for n = 10 ;$d_0 = 2$

Expected profit without error = 11.75

(e_1, e_2)	μ	EP	Percentage change in profit
(0,0.01)	1.01284	11.751	-0.0085
(0,0.03)	1.01279	11.754	-0.0340
(0,0.05)	1.012	11.756	-0.051
(0,0.1)	1.01262	11.763	-0.11064
(0,0.15)	1.01249	11.769	-0.161
(0.01,0)	1.01424	11.607	1.217
(0.01,0.01)	1.01422	11.608	1.208
(0.01,0.03)	1.01417	11.61	1.191
(0.01,0.05)	1.01413	11.613	1.165
(0.01,0.10)	1.01401	11.619	1.114
(0.01,0.15)	1.0139	11.625	1.06383
(0.03,0.0)	1.01652	10.856	7.608
(0.03,0.01)	1.0165	10.857	7.6
(0.03,0.03)	1.01646	10.859	7.582
(0.03,0.05)	1.01642	10.861	7.565957
(0.03,0.1)	1.01631	10.866	7.523
(0.03,0.15)	1.0162	10.872	7.472
(0.05,0)	1.01761	9.137	22.238
(0.05,0.01)	1.01759	9.138	22.229
(0.05,0.03)	1.01756	9.14	22.212
(0.05,0.05)	1.01752	9.142	22.195
(0.05,0.1)	1.01741	9.147	22.153
(0.05,0.15)	1.0173	9.152	22.110
(0.1,0.0)	1.01831	1.549	86.817
(0.1,0.01)	1.01829	1.55	86.80851
(0.1,0.03)	1.01825	1.552	86.791
(0.1,0.05)	1.01821	1.554	86.774
(0.1,0.1)	1.01811	1.559	86.731
(0.1,0.15)	1.01799	1.565	86.680
(0.15,0.0)	1.01792	-6.72	157.191
(0.15,0.01)	1.0179	-6.726	157.242
(0.15,0.03)	1.01786	-6.724	157.22
(0.15,0.05)	1.01781	-6.722	157.208
(0.15,0.1)	1.01777	-6.716	157.157
(0.15,0.15)	1.0175	-6.71	157.106

Table 3.6: Expected Profit and Optimal Mean at different values of Error for n = 20 ;$d_0 = 2$

3.5.1 Effect of Type I Error on Expected Profit

In this section, expected profit at different levels of type I error are shown in Figures 3.2 to 3.7, each figure shows, Expected profit versus type I error at fixed levels of e_2, containing 2 plots each with different sampling plan with sample size n and acceptance number d_0. At a given level of type II error, the expected profit decreases with the increase in the value of type I error, this is expected because a good item is classified as defective when type I error is committed, and it is sold at a reduced price. Thus, reducing the expected profit.

Expected Profit decreases sharply as type I error tends to 0.1. As the sample size increases the expected profit is reduced because the sampling plan with a higher sample size will show a smaller probability of acceptance.

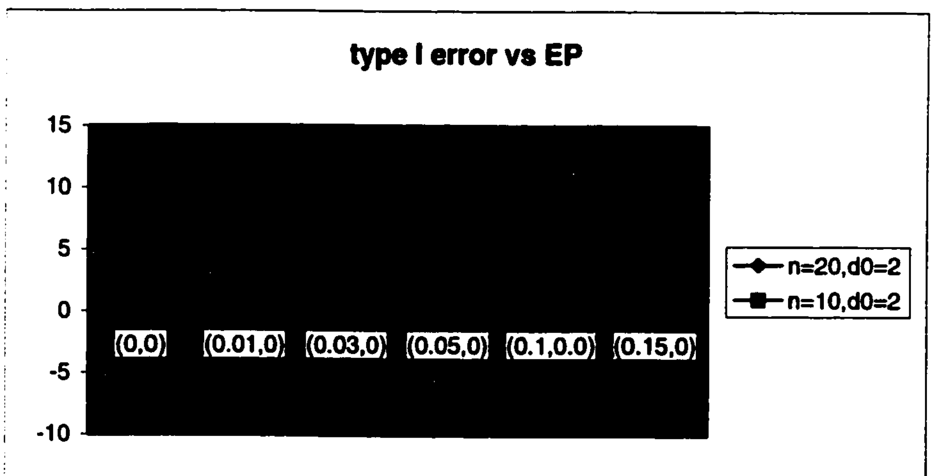

Figure 3.2: Expected Profit versus e_1 at $e_2 = 0$

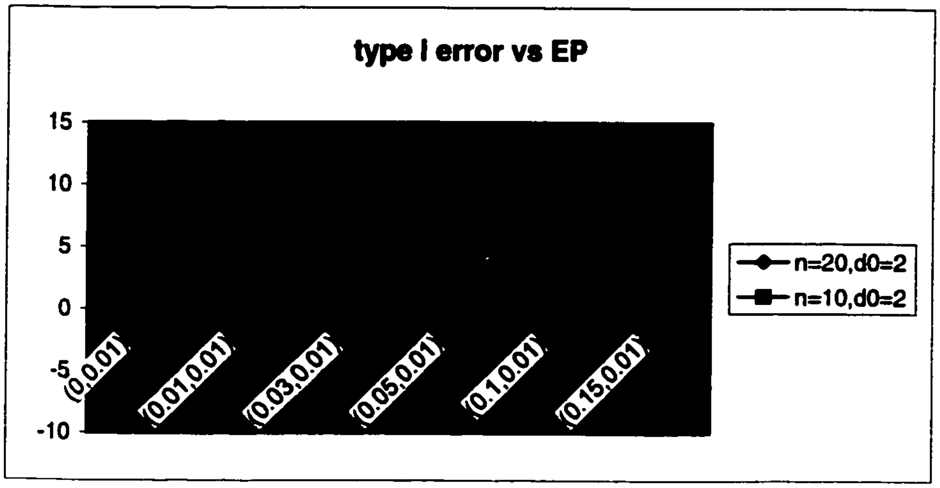

Figure 3.3: Expected Profit versus e_1 at $e_2 = 0.01$

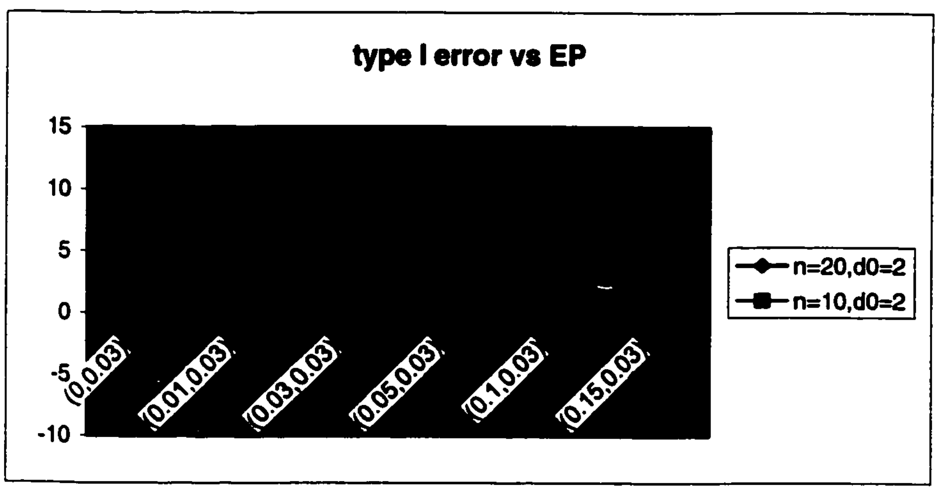

Figure 3.4: Expected Profit versus e_1 at $e_2 = 0.03$

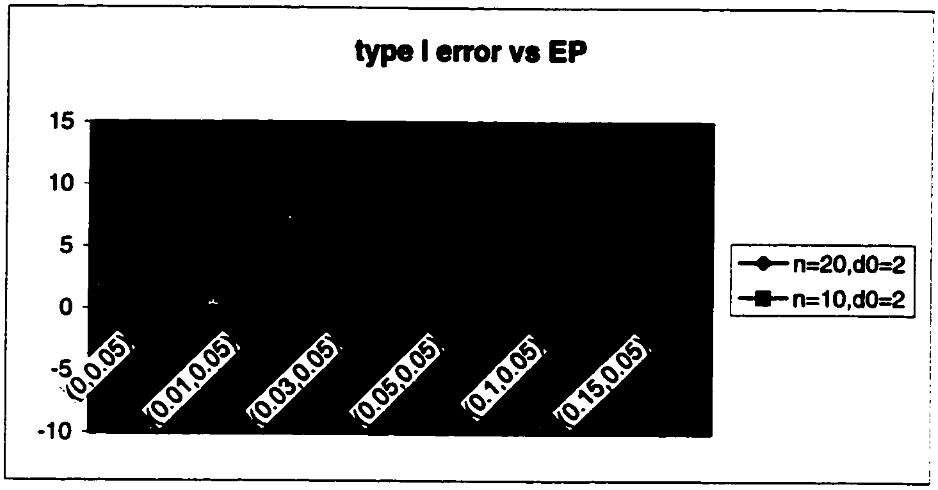

Figure 3.5: Expected Profit versus e_1 at $e_2 = 0.05$

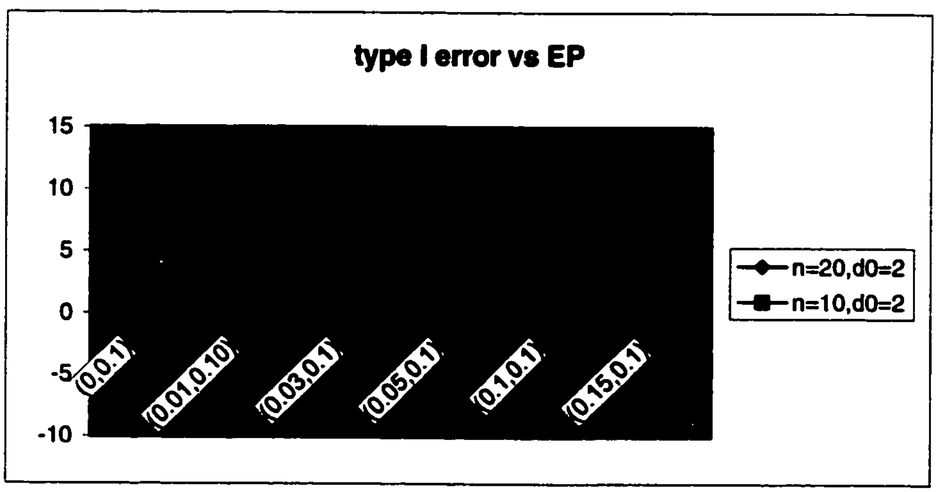

Figure 3.6: Expected Profit versus e_1 at $e_2 = 0.1$

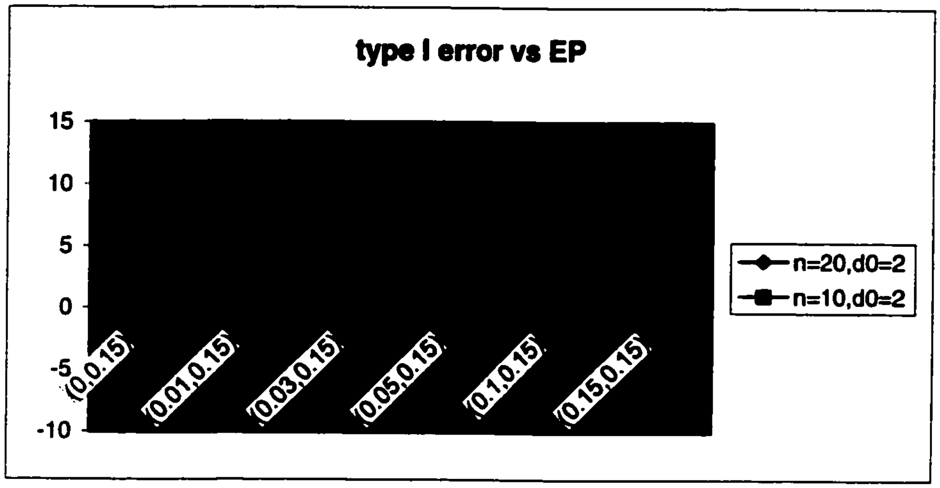

Figure 3.7: Expected Profit versus e_1 at $e_2 = 0.15$

3.5.2 Effect of Type II Error on Expected Profit

The Expected Profit at different levels of type II error are shown in this section. As shown in the figures 3.8 to 3.13 taking different values of e_1, for the same Sampling plan expected profit increases with the increase in the level of type II error, this is expected because increasing the level of type II error implies that we are classifying more number of defective items as good and selling it at a higher price.Thus, Expected Profit is assumed to be directly proportional to type II error.

Each figure shows Expected profit versus type II error at different levels of e_1 ; containing 2 plots each with different sampling plan with sample size n and acceptance number d_0.As the sample size increases the Expected Profit is reduced because the sampling plan with a higher sample size will show a smaller probability of acceptance for the same acceptance number d_0.Simply,Sampling more units in a lot increases the chances of detecting the non-conformity and rejecting the lot. As seen from figures 3.2 to 3.7, type I error reduces the value of EP. Thus, type I error has more impact on Expected Profit than type II error.

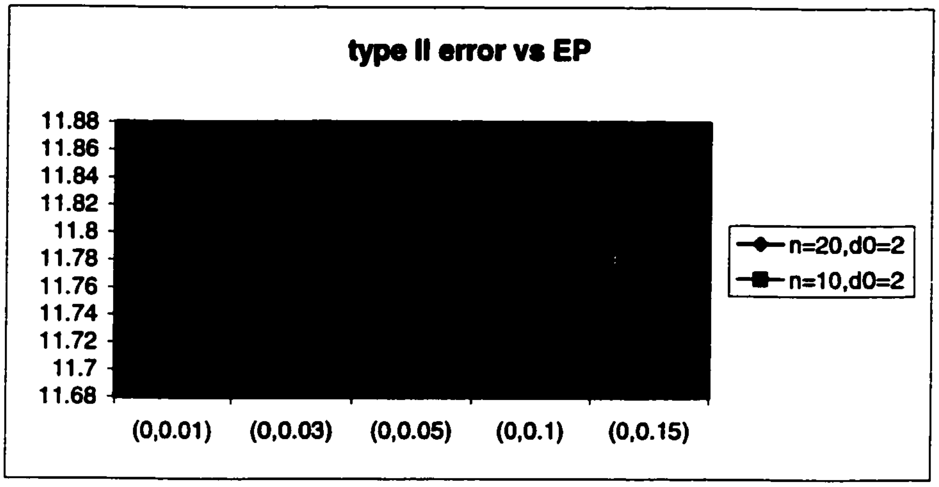

Figure 3.8: Expected Profit versus e_2 at $e_1 = 0$

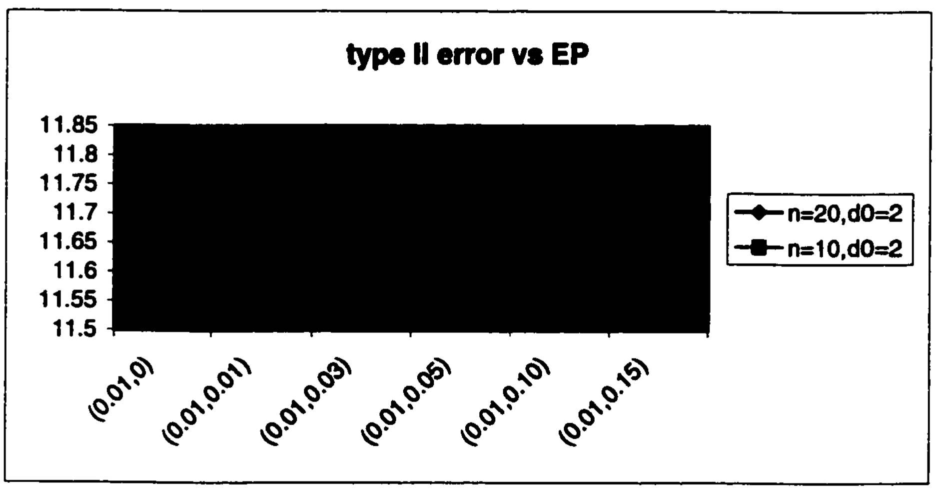

Figure 3.9: Expected Profit versus e_2 at $e_1 = 0.01$

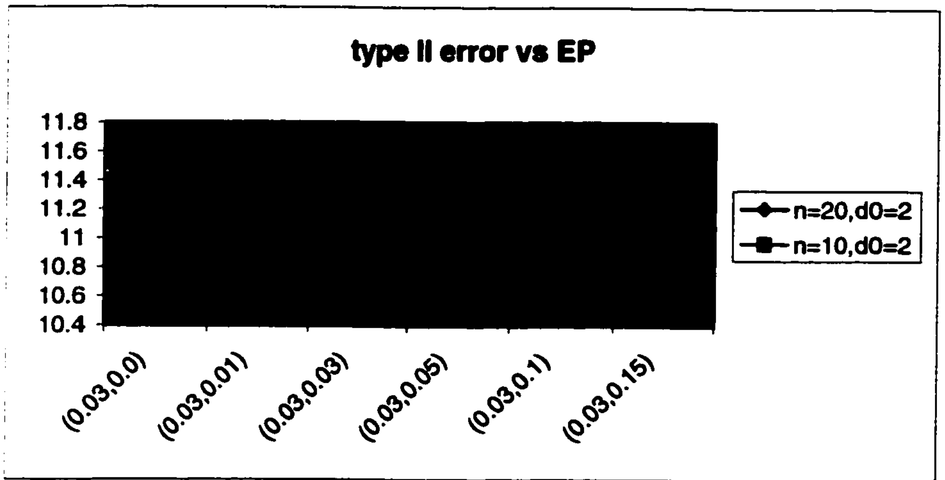

Figure 3.10: Expected Profit versus e_2 at $e_1 = 0.03$

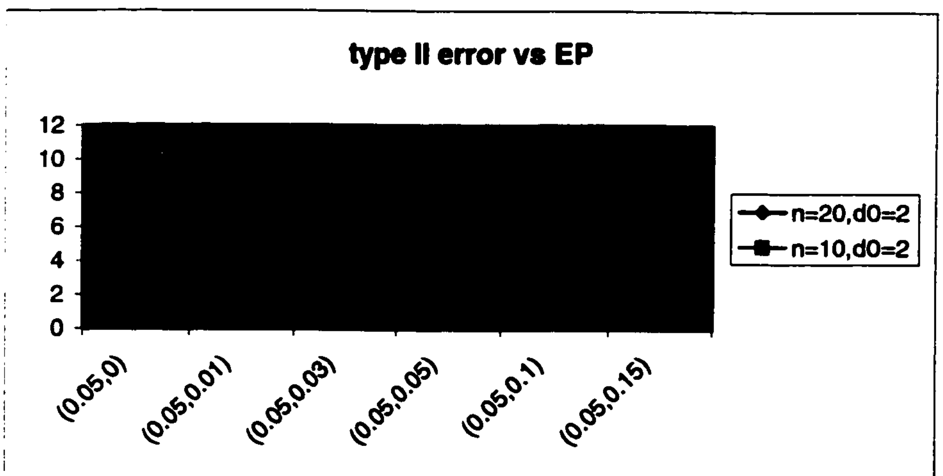

Figure 3.11: Expected Profit versus e_2 at $e_1 = 0.05$

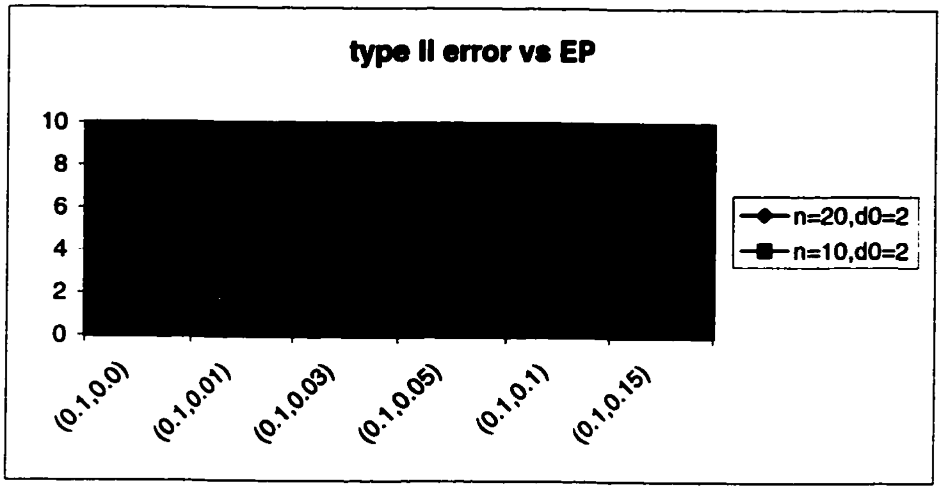

Figure 3.12: Expected Profit versus e_2 at $e_1 = 0.1$

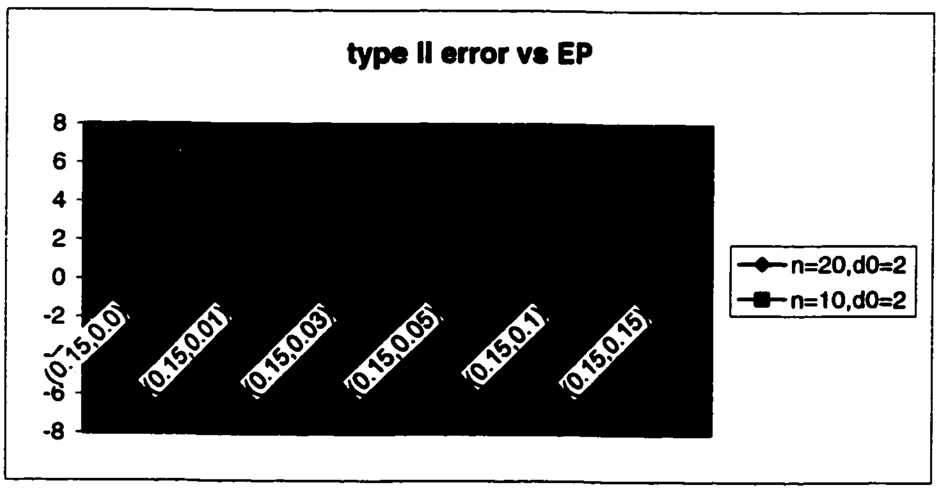

Figure 3.13: Expected Profit versus e_2 at $e_1 = 0.15$

3.5.3 Effect of type I and type II error on Optimal Mean

The effect of type I error on optimal mean are shown in figure 3.16 and 3.17, at a given level of type II error the mean tends to increase drastically with the increase in type I error, this shows that the process mean is forced higher at higher values of e_1.

In figure 3.14 and 3.15, keeping e_1 as constant, optimal mean versus type II error is plotted. the graphs shows that the mean tends to decrease with the increase in the value of e_2.

The graphs show two points with different sampling plan with sample size n and acceptance number d_0 . It can be seen that as the sample size increases the mean of the process is increased because for any value of probability of failure q_e, the OC curve of a sampling plan with higher n will show a lower probability of acceptance, this forces the producer to increase the set point.

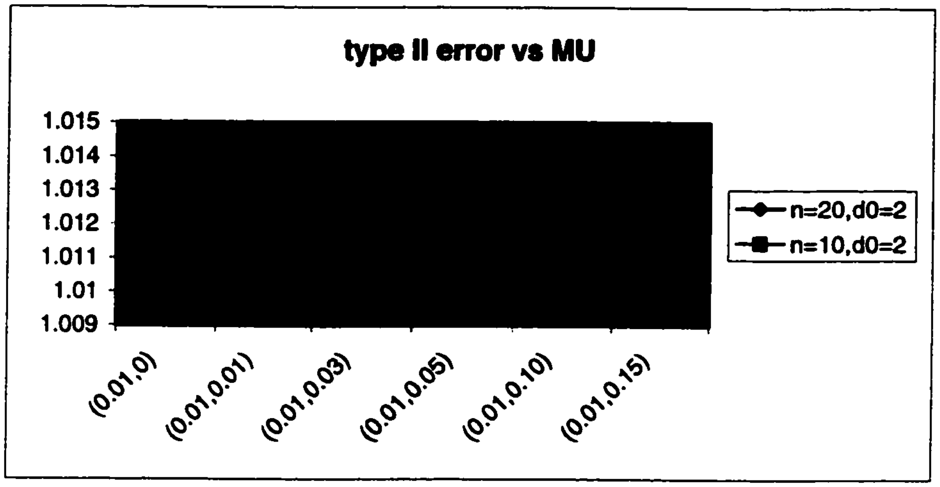

Figure 3.14: μ versus e_2 at $e_1 = 0.01$

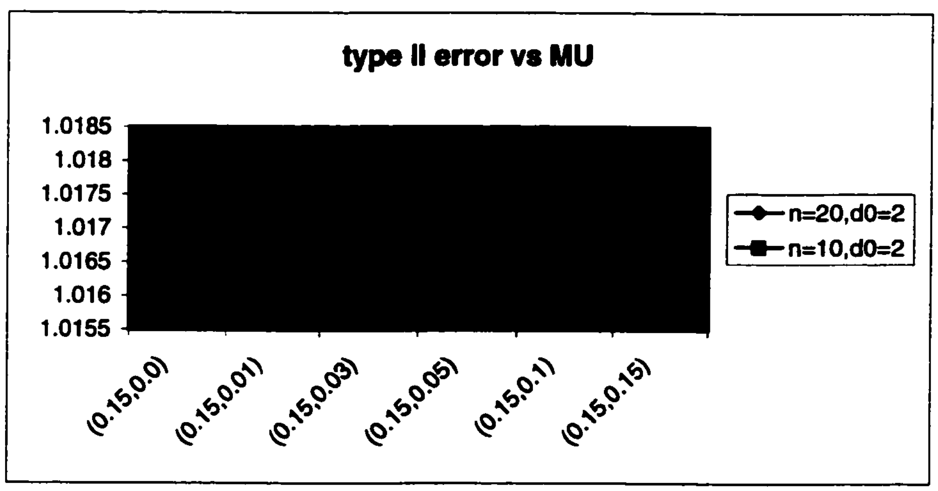

Figure 3.15: μ versus e_2 at $e_1 = 0.15$

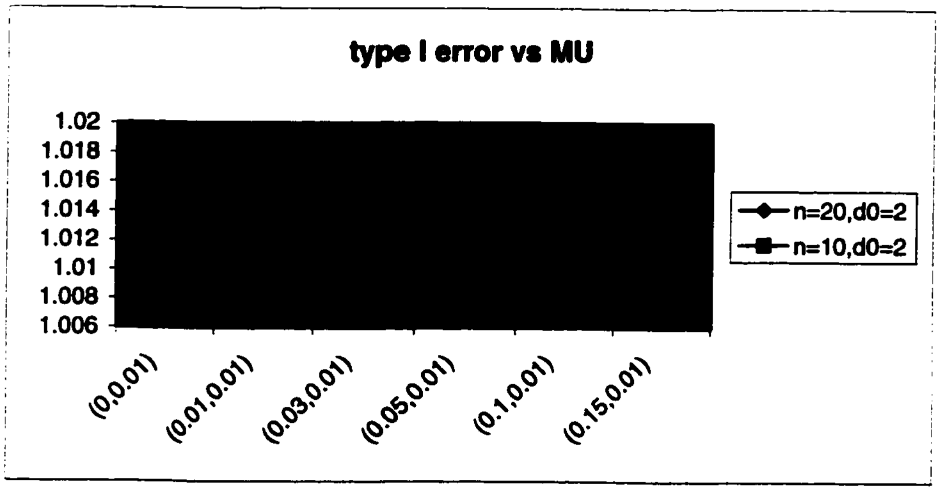

Figure 3.16: μ versus e_1 at $e_2 = 0.01$

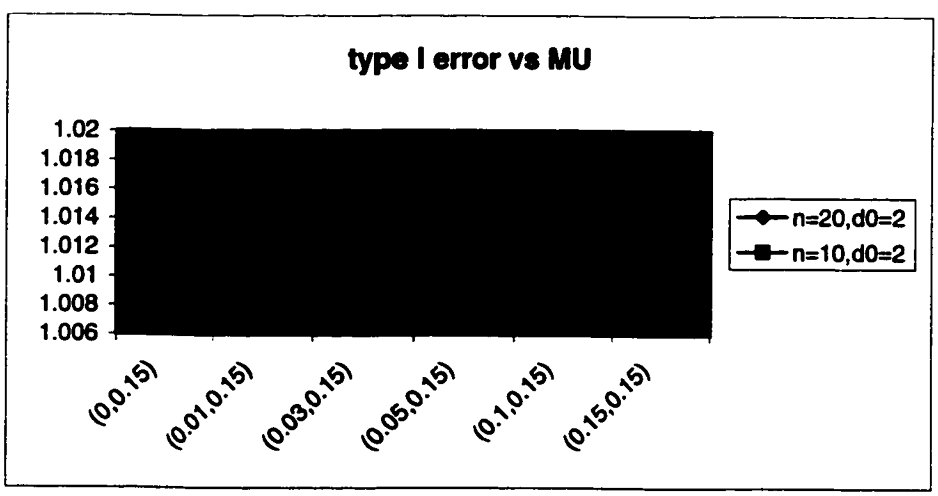

Figure 3.17: μ versus e_1 at $e_2 = 0.15$

3.5.4 Effect of Cost parameters on Expected Profit

The effect of different cost parameters on expected profit are studied in this section, by taking the partial derivative of the profit function w.r.t. the selling price A, P and processing cost c. Thus, the partial derivatives of equation 3.1 w.r.t different parameters are as follows:

$$
\begin{aligned}
\frac{\partial(\frac{E[\pi(\mu)]}{N})}{\partial A} &= 1 \\
\frac{\partial(\frac{E[\pi(\mu)]}{N})}{\partial P} &= -P(x_e > d_0) \\
\frac{\partial(\frac{E[\pi(\mu)]}{N})}{\partial c} &= -\mu
\end{aligned}
$$

The partial derivative w.r.t selling price A is constant, whereas the partial derivative w.r.t P the price reduction for rejected items is equal to the negative of the probability of rejection, and the rate of change of profit w.r.t cost c is the negative of the mean of the process.

The rate of change of profit w.r.t selling prices at different sample sizes are plotted against the acceptance number d_0. Figure 3.18, shows that the rate of change of profit w.r.t A is constant for all levels of the acceptance number d_0. As seen from the figure 3.19, for the same acceptance number, as the sample size increases the probability of acceptance decreases, because sampling more units in a lot have higher chances of detecting the defect and rejecting the lot. So, the probability of acceptance decreases with the increase in the sample size and hence the probability of rejection increases. The rate of change of expected profit w.r.t P is the negative of the probability of rejection. Therefore, for the same acceptance number, the rate of change of expected profit w.r.t P decreases because of the negative sign. But for the same level of sample size, the rate of change of profit w.r.t P increases with the increase in the level of acceptance number.

The rate of change of expected profit w.r.t c vs. d_0 is shown in figure. 3.20, the figure shows 4 plots with different sample size, the rate of change of expected profit is the negative of the mean of the process. With the increase in the acceptance number, the mean of the process decreases because raising the value of d_0 has the effect of enlarging the area of acceptance under the OC curve corresponding to the sampling plan. i.e. allowing more nonconforming units to occur gives the producer more latitude in producing nonconforming units, but as seen in the figure the rate of change of expected profit w.r.t. c increases because of the negative sign associated with the mean. The rate of change of expected profit w.r.t c increases with the increase in the acceptance number.

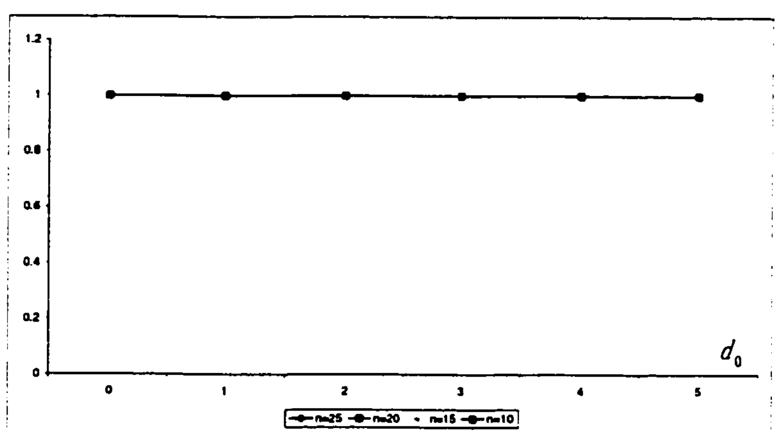

Figure 3.18: Rate of change of Expected Profit w.r.t A vs d_0

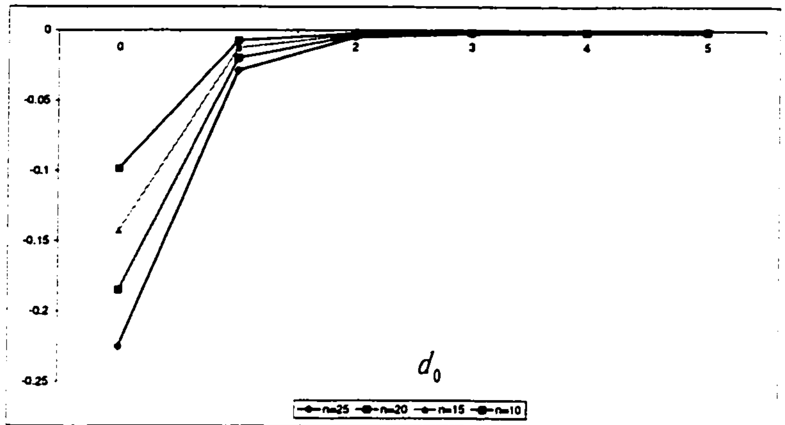

Figure 3.19: Rate of change of Expected Profit w.r.t P vs d_0

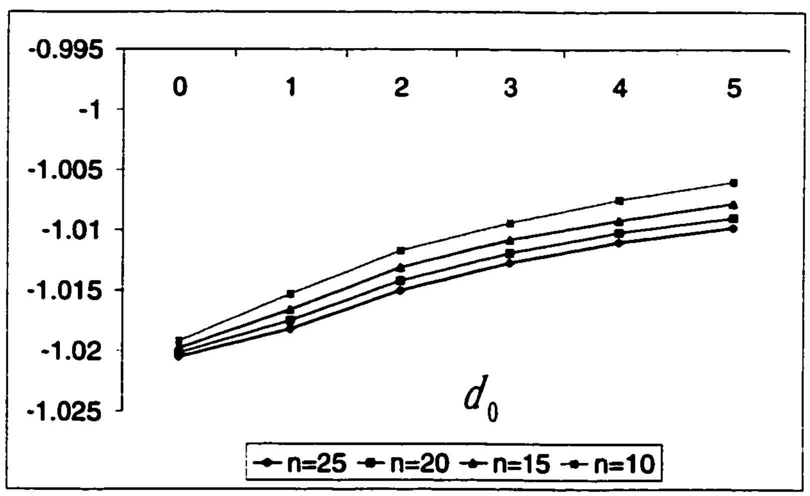

Figure 3.20: Rate of change of Expected Profit w.r.t c vs d_0

3.6 Conclusion

In this chapter a model for process targeting is developed by incorporating the effect of type I and type II error on the sampling inspection plan. The results showed that the expected profit increases with the increase in the level of type II error and decreases with the increase in the level of type I error. Sensitivity analysis is also conducted to study the impact of error on the solutions. It is concluded that type I and type II errors has a tremendous impact on the process optimal parameters.

Chapter 4

Multi-Class Process Targeting with Quality Sampling Plans

4.1 Introduction

The purpose of this chapter is to develop a multi-class targeting model using Sampling Inspection Plan. A container filling process is considered, in which a product out of the production process, is sold in one of two markets with different profit structures or scrapped, suppose the quality characteristic X has a lower specification limit L, the quality characteristic may be the weight or volume of the container. Sampling Inspection is used in this model instead of 100 percent inspection because of less inspection, less handling of the product, lower inspection error etc.

In this model lot disposition is done by sampling i.e, a sample is drawn from a lot, to classify the lot into three classes(grades) based on the acceptance number of the sample, i.e, grade 1 is the product in the lot with number of defectives less than the acceptance number for grade 1 d_1, the grade 2 product in the lot is the product that has acceptance number greater than d_1 and less than d_2 and the third class is scrap, number of defective greater than d_2.

The objective of this model is to maximize the expected profit by finding the optimum target value for multi-class targeting problem with quality sampling plans. Two cases are examined here.

(1) When sampling results in destructive testing and

(2) When the sampling results in non-destructive testing.

This chapter is organized as follows, model development is presented in sec.4.2, in this section notations, model assumptions and statement of the model, and model formulation are presented. Solution and Analysis of the problem is discussed in sec.4.3,and the results are shown in sec.4.4.

4.2 Model Development

The model developed in this chapter is presented in this section. The section contains, model notations, assumptions, problem statement and model formulation.

4.2.1 Notations

The following are the symbols which are used in this chapter,

a_1 = Selling price for grade 1 item,
a_2 = Selling price for grade 2 item,
r = Salvage value when an item is scrap,
c = Ingredient cost per unit weight,
X = Weight or volume of the product and is a random variable,
μ = The expected value of X or process mean,
σ^2 = Process variance,
L = Lower specification limit for the product attribute,
n = Sample size,
q = Probability of failure i.e $P[X < L] = \Phi(\frac{L-\mu}{\sigma})$,
d_1 = Allowable number of non-conforming units in the sample for grade1,
d_2 = Allowable number of non-conforming units in the sample for grade2,
x = Number of non-conforming units in the sample.

4.2.2 Model Assumptions

1. The inspection process is assumed to be error free.
2. The quality characteristic X is assumed to be normally distributed with mean μ and variance σ^2.
3. The variance of the process is assumed to be known and fixed.
4. Sampling plan is used for ensuring product quality.
5. The probability of an item being defective is independent of other items in the lot.
6. The selling price for grade 1 is greater than selling price for grade 2 is grade than the price for scrap, i.e $a_1 > a_2 > r$.
7. The acceptance number for grade 2 is greater than the acceptance number for grade 1, i.e $d_2 > d_1$

4.2.3 Statement of the model

Consider a can filling process or turning a metal bar. The quality characteristic X, for the can filling process is the weight of the material or the diameter of the turned metal bar, X is assumed normally distributed with mean μ and known variance σ^2. Products are produced in lots, sampling plan is used for ensuring product quality. A container is classified as a "non-conforming" if it is filled less than the lower specification limit (X<L). The lot disposition is based on the number of defective x in the sample. If the number of defective in the sample is less than or equal to acceptance number for grade 1, the lot is considered as grade 1. If the number of defective is greater than d_1 and less than or equal to d_2 it is considered as grade 2 product. If the number of defectives are greater than d_2, then the product is scrap. Containers from a production lot that is accepted for grade 1 are sold at a price "a_1" per container, whereas containers acceptable for grade 2 are sold at a price a_2 and salvage value r per container, is obtained if the lot is scrap.

4.2.4 Model Formulation

Non-Destructive Testing

Thus, The expected revenue for a lot of size N, given x.

$$E[R|x] = \begin{cases} a_1 N & \text{if } x \leq d_1 \\ a_2 N & \text{if } d_1 < x \leq d_2 \\ rN & \text{if } x > d_2 \end{cases}$$

For the set point of μ, the expected value of the marginal cost per lot is $Nc\mu$, where μ is the expected value of X.

Therefore, The marginal profit function is

$$E[\pi(\mu)] = a_1NP(x \le d_1) + a_2NP(d_1 < x \le d_2) + rNP(x > d_2) - Nc\mu$$

i.e

$$\frac{E[\pi(\mu)]}{N} = a_1P(x \le d_1) + a_2P(d_1 < x \le d_2) + rP(x > d_2) - c\mu \qquad (4.1)$$

Where $E[\pi(\mu)]$ is the expected profit when the set point μ is fixed at a specific value, the process mean may be set higher to reduce the cost incurred by producing defective items. Using a higher process mean, however, results in a higher production cost. Therefore, a process mean is to be selected so that the expected profit per item is maximized.

Destructive Testing

It is often the case that examining the weight of the contents of a package will result in destroying the product,

Then, The expected revenue becomes

$$E[R|x] = \begin{cases} a_1(N-n) & \text{if } x \le d_1 \\ a_2(N-n) & \text{if } d_1 < x \le d_2 \\ r(N-n) & \text{if } x > d_2 \end{cases}$$

And the expected profit becomes,

$$E[\pi(\mu)] = a_1(N-n)P(x \le d_1) + a_2(N-n)P(d_1 < x \le d_2) + r(N-n)P(x > d_2) - Nc\mu \quad (4.2)$$

It should be noted that the expected profit for nondestructive testing is not directly comparable cost wise to destructive testing because there are different cost associated with the sampling method used. In nondestructive testing the container is salvageable, whereas it is rendered useless in destructive testing.

4.3 Solution and Analysis

We want to maximize $\frac{E[\pi(\mu)]}{N}$ defined above. A necessary condition for optimality is that the partial derivative with respect to μ vanishes at the target value μ.
i.e $\frac{\partial}{\partial\mu}\frac{E[\pi(\mu)]}{N} = 0$,
Therefore,

$$\frac{\partial}{\partial\mu}\frac{E[\pi(\mu)]}{N} = a_1\frac{\partial}{\partial\mu}P(x \leq d_1) + a_2\frac{\partial}{\partial\mu}P(d_1 < x \leq d_2) + r\frac{\partial}{\partial\mu}P(x > d_2) - c = 0 \tag{4.3}$$

we know that x is binomially distributed with sample size n and probability of failure q. Therefore,

$$P(x \leq d_1) = \sum_{i=0}^{d_1}\binom{n}{i}q^i(1-q)^{(n-i)}$$

Where

$$q = P(X \leq L) = \Phi(-z)$$

and

$$z = \frac{\mu - L}{\sigma}$$

Then,

$$\frac{\partial q}{\partial\mu} = -\frac{1}{\sigma}\phi\left(\frac{L-\mu}{\sigma}\right)$$

therefore,

$$\frac{\partial}{\partial \mu}P(x \leq d_1) = \frac{\partial q}{\partial \mu}\frac{\partial}{\partial q}P(x \leq d_1)$$

$$\begin{aligned}
\frac{\partial}{\partial q}P(x \leq d_1) &= \frac{\partial}{\partial q}\sum_{i=0}^{d_1}\binom{n}{i}q^i(1-q)^{(n-i)} \\
&= \sum_{i=0}^{d_1}\binom{n}{i}\left[iq^{(i-1)}(1-q)^{(n-i)} - q^i(n-i)(1-q)^{(n-i-1)}\right] \\
&= -n(1-q)^{(n-1)} + \left[n(1-q)^{(n-1)} - n(n-1)q(1-q)^{(n-2)}\right] \\
&+ \left[n(n-1)q(1-q)^{(n-2)} - \frac{n(n-1)(n-2)}{2}q^2(1-q)^{(n-3)}\right] + \ldots \\
&+ \left[\frac{n!}{(n-d_1)!d_1!}d_1q^{(d_1-1)}(1-q)^{(n-d_1)} - \frac{n!}{(n-d_1)!d_1!}(n-d_1)q^{d_1}(1-q)^{(n-d_1-1)}\right] \\
&= -\frac{n!}{(n-d_1-1)!d_1!}q^{d_1}(1-q)^{(n-d_1-1)}
\end{aligned}$$

We can write the above equation as,

$$\frac{\partial}{\partial q}P(x \leq d_1) = -\frac{n!}{(n-d_1-1)!d_1!}q^{d_1}(1-q)^{(n-d_1-1)}$$

Therefore,

$$\frac{\partial}{\partial \mu}P(x \leq d_1) = \frac{\partial q}{\partial \mu}\left(\frac{\partial}{\partial q}P[x \leq d_1]\right)$$

i.e,

$$\frac{\partial}{\partial \mu}P(x \leq d_1) = \frac{1}{\sigma}\phi(z)\frac{n!}{(n-d_1-1)!d_1!}q^{d_1}(1-q)^{(n-d_1-1)} \tag{4.4}$$

Similar calculations are carried out to get the differential of$P(x \geq d_2)$ with respect to μ

$$\frac{\partial}{\partial \mu}P(x > d_2) = \frac{\partial}{\partial \mu}(1 - P[x \leq d_2])$$

i.e,

$$\frac{\partial}{\partial \mu}P(x > d_2) = -\frac{1}{\sigma}\phi(z)\frac{n!}{(n-d_2-1)!d_2!}q^{d_2}(1-q)^{(n-d_2-1)} \tag{4.5}$$

similarly,

$$\frac{\partial}{\partial \mu}P(d_1 < x \leq d_2) = \frac{\partial q}{\partial \mu}\left(\frac{\partial}{\partial q}P(d_1 < x \leq d_2)\right)$$

Therefore,

$$\begin{aligned}
\frac{\partial}{\partial q}P(d_1 < x \leq d_2) &= \frac{\partial}{\partial q}\sum_{i=d_1+1}^{d_2}\binom{n}{i}q^i(1-q)^{(n-i)} \\
&= \sum_{i=d_1+1}^{d_2}\binom{n}{i}\left[iq^{(i-1)}(1-q)^{(n-i)} - q^i(n-i)(1-q)^{(n-i-1)}\right] \\
&= \left[\begin{array}{c} \left(\frac{n!}{(n-d_1-1)!(d_1+1)!}(d_1+1)q^{d_1}(1-q)^{(n-d_1-1)}\right) \\ -\left(\frac{n!}{(n-d_1-1)!(d_1+1)!}(n-d_1-1)q^{d_1+1}(1-q)^{(n-d_1-2)}\right) \end{array}\right] \\
&+ \left[\begin{array}{c} \left(\frac{n!}{(n-d_1-2)!(d_1+2)!}(d_1+2)q^{d_1+1}(1-q)^{(n-d_1-2)}\right) \\ -\left(\frac{n!}{(n-d_1-2)!(d_1+2)!}(n-d_1-2)q^{d_1+2}(1-q)^{(n-d_1-3)}\right) \end{array}\right] \\
&+ \ldots + \left[\begin{array}{c} \left(\frac{n!}{(n-d_2+1)!(d_2-1)!}(d_2-1)q^{d_2-2}(1-q)^{(n-d_2+1)}\right) \\ -\left(\frac{n!}{(n-d_2+1)!(d_2-1)!}(n-d_2+1)q^{d_2-1}(1-q)^{(n-d_2)}\right) \end{array}\right] \\
&+ \left[\begin{array}{c} \left(\frac{n!}{(n-d_2)!d_2!}d_2q^{d_2-1}(1-q)^{(n-d_2)}\right) \\ -\left(\frac{n!}{(n-d_2)!d_2!}(n-d_2)q^{d_2}(1-q)^{(n-d_2-1)}\right) \end{array}\right]
\end{aligned}$$

That is equal to,

$$\frac{\partial}{\partial q}P(d_1 < x \leq d_2) = \frac{n!}{(n-d_1-1)!d_1!}q^{d_1}(1-q)^{(n-d_1-1)} - \frac{n!}{(n-d_2-1)!d_2!}q^{d_2}(1-q)^{(n-d_2-1)}$$

Therefore,

$$\frac{\partial}{\partial \mu}P(d_1 < x \leq d_2) = -\frac{1}{\sigma}\phi(z)\left[\frac{n!}{(n-d_1-1)!d_1!}q^{d_1}(1-q)^{(n-d_1-1)} - \frac{n!}{(n-d_2-1)!d_2!}q^{d_2}(1-q)^{(n-d_2-1)}\right] \tag{4.6}$$

Substituting equations(4.4),(4.5) and (4.6) in (4.3), and equating it to zero, we get

$$\frac{\phi(z)}{\sigma}\left[\frac{n!}{(n-d_1-1)!d_1!}q^{d_1}(1-q)^{(n-d_1-1)}(a_1-a_2)+\frac{n!}{(n-d_2-1)!d_2!}q^{d_2}(1-q)^{(n-d_2-1)}(a_2-r)\right]=c \tag{4.7}$$

The optimal value of μ can be obtained from this equation by numerically solving this equation.

Similar calculations are carried out to find the optimal condition for optimality for destructive testing, we arrive at the result shown below,

$$\begin{aligned}&\frac{\phi(z)}{\sigma}\left[\frac{n!}{(n-d_1-1)!d_1!}q^{d_1}(1-q)^{(n-d_1-1)}(a_1-a_2)+\frac{n!}{(n-d_2-1)!d_2!}q^{d_2}(1-q)^{(n-d_2-1)}(a_2-r)\right]\\ &= \left(\frac{N}{N-n}\right)c\end{aligned} \tag{4.8}$$

The only change noted in the optimality condition for the two cases i.e for destructive testing and non destructive testing is the term $\left(\frac{N}{N-n}\right)$ on the right hand side of the equation.

4.4 Results

In this section, an illustrative example for destructive and non-destructive testing are presented. Since, closed form solution cannot be obtained to this problem. A line search such as golden section method is used to get the solutions. computer code is written in Fortran 90, the program is run on a pentium III computer with 64 MB RAM (see appendix C for the code).

Using the same numerical values of the parameters as given in the model developed by Boucher and Jafari [1], the optimal process mean and expected profit for different sample size, and for different values of d_1 and d_2 are evaluated. Given $a_1 = 67.5$, $a_2 = 37$, $r = 10$,$c = 55$, $\sigma = 0.00563$, $L = 1$.

The table below shows the values of the optimal process mean and expected profit for the Multi-Class Process Targeting problem for sampling plans.

	$d_1 = 0$		$d_1 = 1$		$d_1 = 2$		$d_1 = 3$	
d_2	μ	EP	μ	EP	μ	EP	μ	EP
1	1.0206	11.289	-	-	-	-	-	-
2	1.0206	11.289	1.0155	11.5989	-	-	-	-
3	1.0206	11.289	1.0154	11.5989	1.0129	11.749	-	-
4	1.0206	11.289	1.0154	11.5989	1.0129	11.7505	1.0112	11.8494

Table 4.1: Expected Profit and Mean for n=20 using model I values

Using the numerical values of the parameters given by Lee and Jang [4], the optimal process mean and expected profit for different sample size, and for different values of d_1 and d_2 are obtained.

	$d_1 = 0$		$d_1 = 1$		$d_1 = 2$		$d_1 = 3$	
d_2	μ	EP	μ	EP	μ	EP	μ	EP
1	1.0195	11.346	-	-	-	-	-	-
2	1.0195	11.346	1.014	11.6741	-	-	-	-
3	1.0195	11.346	1.014	11.6748	1.011	11.8415	-	-
4	1.0195	11.346	1.014	11.6748	1.011	11.8415	1.0091	11.9582

Table 4.2: Expected Profit and Mean for n=10 using model I values

Given $a_1 = 5.5$, $a_2 = 5.1$, $r = 2.5$,$c = 0.01$, $\sigma = 1.25$, $L = 40$.

The table below shows the values of the optimal process mean and expected profit for the Multi-Class Process Targeting problem.

	$d_1 = 0$		$d_1 = 1$		$d_1 = 2$		$d_1 = 3$	
d_2	μ	EP	μ	EP	μ	EP	μ	EP
1	44.1758	5.0548	-	-	-	-	-	-
2	44.1606	5.0549	43.2239	5.0657	-	-	-	-
3	44.1604	5.0549	43.1809	5.0661	42.729	5.0711	-	-
4	44.1604	5.0549	43.1809	5.0661	42.6595	5.0717	42.3796	5.0748

Table 4.3: Expected Profit and Mean for n=20 using model II values

	$d_1 = 0$		$d_1 = 1$		$d_1 = 2$		$d_1 = 3$	
d_2	μ	EP	μ	EP	μ	EP	μ	EP
1	43.9105	5.0573	-	-	-	-	-	-
2	43.8847	5.0574	42.8734	5.0689	-	-	-	-
3	43.8865	5.0574	42.8149	5.0693	42.3141	5.075	-	-
4	43.8864	5.0574	42.8131	5.0694	42.2322	5.0756	41.9044	5.0794

Table 4.4: Expected Profit and Mean for n=10 using model II values

For Destructive Testing Using the same values of model I, the optimal process mean and Expected Profit for different sampling plans are shown for destructive testing.

For n=20,N=100;

	$d_1=0$		$d_1=1$		$d_1=2$		$d_1=3$	
d_2	μ	EP	μ	EP	μ	EP	μ	EP
1	1.0202	-2.1923	-	-	-	-	-	-
2	1.0202	-2.1922	1.0152	-1.8905	-	-	-	-
3	1.0202	-2.1922	1.0152	-1.8895	1.0127	-1.7420	-	-
4	1.0202	-2.1922	1.0152	-1.8895	1.0127	-1.7424	1.011	-1.6427

Table 4.5: Expected Profit and Mean for n=20 for destructive testing using model I values

For n=10,N=100;

	$d_1=0$		$d_1=1$		$d_1=2$		$d_1=3$	
d_2	μ	EP	μ	EP	μ	EP	μ	EP
1	1.0193	4.6051	-	-	-	-	-	-
2	1.0193	4.6052	1.0139	4.9299	-	-	-	-
3	1.0193	11.346	1.0139	4.9308	1.0111	5.0963	-	-
4	1.0193	11.346	1.0139	4.9308	1.0111	5.098	1.0090	5.2124

Table 4.6: Expected Profit and Mean for n=10 for destructive testing using model I values

Using the same values as given in model II, the optimal process mean and Expected Profit for different sample size, and for different values of d_1 and d_2 are obtained for destructive testing.

For n=20,N=100;

	$d_1=0$		$d_1=1$		$d_1=2$		$d_1=3$	
d_2	μ	EP	μ	EP	μ	EP	μ	EP
1	44.0918	3.9556	-	-	-	-	-	-
2	44.0837	3.9557	43.1814	3.9661	-	-	-	-
3	44.0837	3.9557	43.1112	3.9666	42.6937	3.9714	-	-
4	44.0835	3.9557	43.1112	3.9666	42.6209	3.9721	42.3550	3.9751

Table 4.7: Expected Profit and Mean for n=20 for destructive testing using model II values

For n=10,N=100;

	$d_1 = 0$		$d_1 = 1$		$d_1 = 2$		$d_1 = 3$	
d_2	μ	EP	μ	EP	μ	EP	μ	EP
1	43.8632	4.5077	-	-	-	-	-	-
2	43.8398	4.5078	42.8552	4.5192	-	-	-	-
3	43.8399	4.5078	42.7836	4.5196	42.2965	4.5252	-	-
4	43.8399	4.5078	42.7835	4.5196	42.2147	4.5258	41.8836	4.5296

Table 4.8: Expected Profit and Mean for n=10 for destructive testing using model II values

Looking at equation 4.3, one can say that, the solution is independent of the lot size N. The value of the expected profit and optimal mean μ are not effected for any value of the lot size,this is only applied for non-destructive testing, but for destructive testing the value of the expected profit and the optimal mean depends on the lot size as can be seen from the table.

As the lot size is increased the value of the expected profit also increases, but the expected profit for destructive testing is always less than expected profit for non-destructive testing, for any number of units in the lot. As can be seen from the table above, for the same value of d_2, as the sample size increases the set point also increases because for any value q, the OC curve of a sampling plan with higher "n"will show less probability of acceptance. In simple terms, sampling more units in a lot ensures of detecting the non-conformity and rejecting the lot. That's why the producer increases the set point.

As we can see as the sample size increases the set point is increased and if the set point is increased then the Expected Profit will decrease.

4.4.1 Effect of d_1 on Expected Profit

The expected Profit at different levels of acceptance number for different sample size are shown in figures 4.1 and 4.2.

For the same sampling plan, the expected profit increases with the increase in the level of acceptance number d_1 for grade 1, because for the same sample size the probability of acceptance increases with the increase in the level of acceptance number d_1.

As the sample size is increased, the expected profit decreases because sampling plan with a higher sample size will show a lower probability of acceptance.

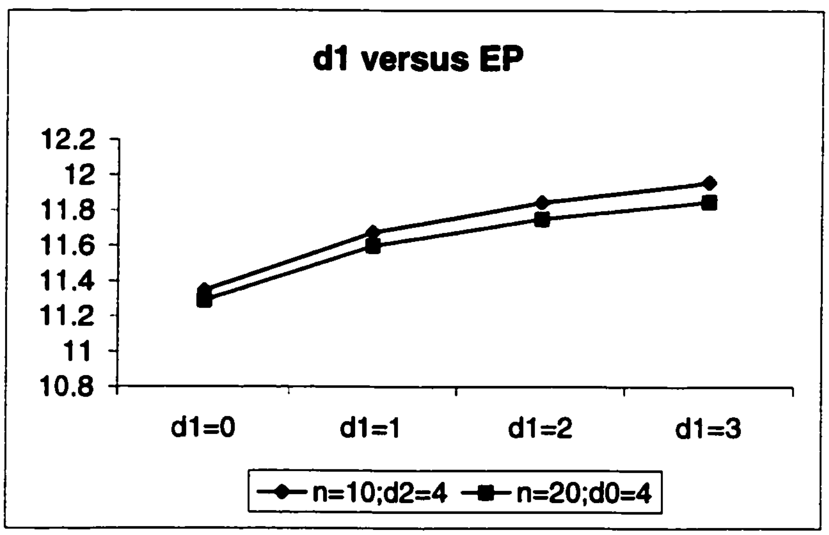

Figure 4.1: Expected Profit versus d_1 at $d_2 = 4$

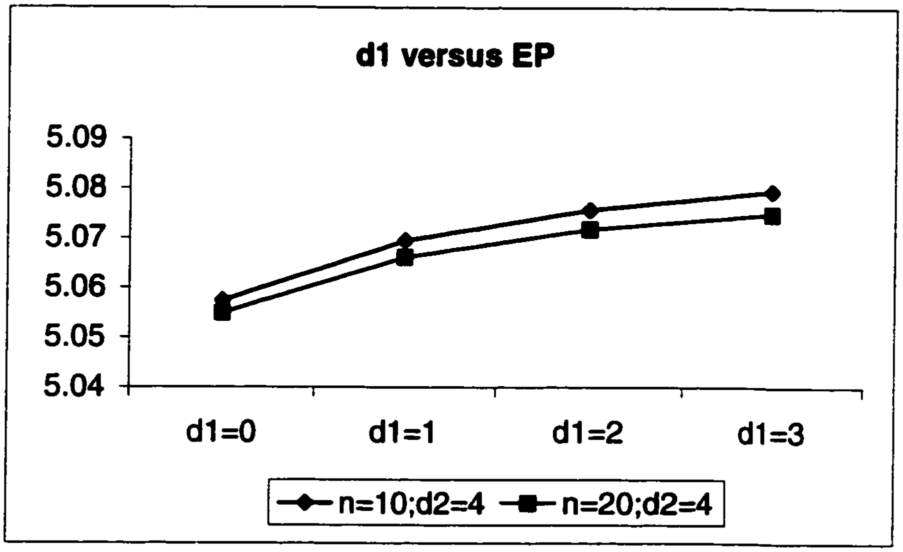

Figure 4.2: Expected Profit versus d_1 at $d_2 = 4$

4.4.2 Effect of d_1 on μ

The effect of acceptance number d_1 on optimal mean is shown in figures 4.3 and 4.4, the graphs shows that the mean tends to decrease with the increase in the value of acceptance for grade 1 d_1 .

As the sample size increases for the same value of acceptance numbers d_1 and d_2; the mean of the process is increased, because for any value of lot fraction defective q, the OC curve of the sampling plan with higher n will show a lower probability of acceptance.

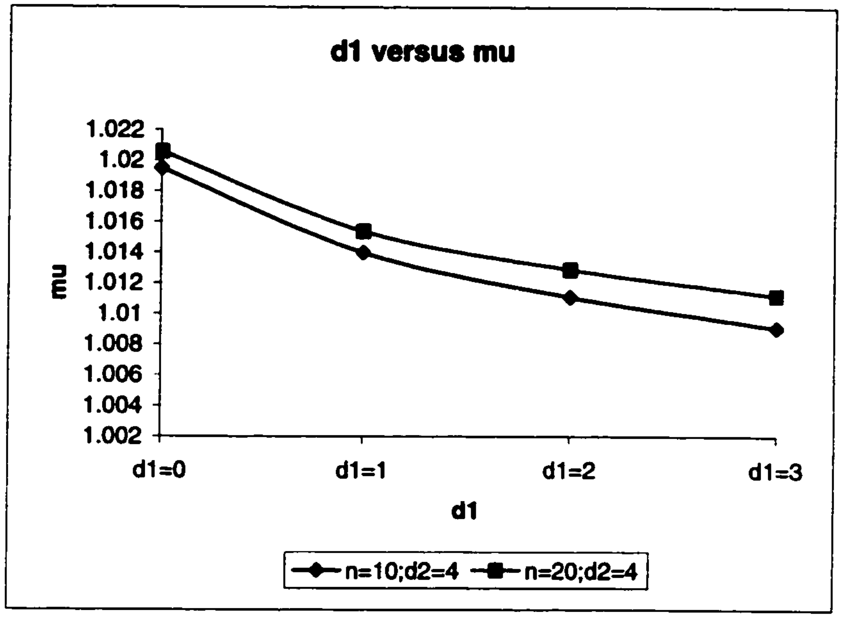

Figure 4.3: μ versus d_1 at $d_2 = 4$

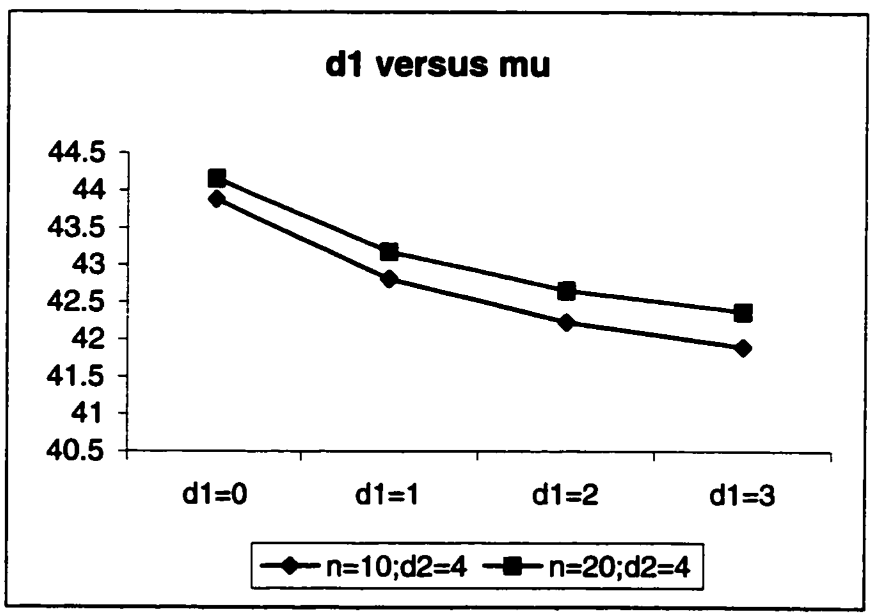

Figure 4.4: μ versus d_1 at $d_2 = 4$

4.4.3 Effect of Cost parameters on Expected Profit

The effect of different cost parameters on expected profit are studied in this section, by taking the partial derivative of the profit function with respect to. the selling price a_1, a_2, r and processing cost c. The rate of change of expected profit are computed in this section. thus, the partial derivatives of equation 4.1 with respect to these parameters are as follows:

$$\begin{aligned}
\frac{\partial(\frac{E[\pi(\mu)]}{N})}{\partial a_1} &= P(x \leq d_1) \\
\frac{\partial(\frac{E[\pi(\mu)]}{N})}{\partial a_2} &= P(d_1 < x \leq d_2) \\
\frac{\partial(\frac{E[\pi(\mu)]}{N})}{\partial r} &= P(x > d_2) \\
\frac{\partial(\frac{E[\pi(\mu)]}{N})}{\partial c} &= -\mu
\end{aligned}$$

The partial derivative with respect to selling price a_1 is the probability of the lot accepted for grade 1, and the partial derivative with respect to a_2 is equal to the probability of acceptance for grade 2, and the probability that the lot is scrap is the rate of change of expected profit with respect to selling price r. whereas, the rate of change of profit with respect to cost c is the negative of the mean of the process. The rate of change of profit at different sample sizes are plotted against the acceptance number d_1 and d_2. Figure 4.5 shows that the rate of change of profit with respect to a_1 is increases with the increase in the acceptance number d_1.

Fig.4.6 shows the rate of change of expected profit with respect to a_2 vs acceptance number for grade 1 d_1, the rate decreases with the increase in the acceptance number for grade 1. Fig 4.7, shows the rate of change of expected profit with respect to r vs

d_2, the rate of change decreases with the acceptance number d_2.

The rate of change of expected profit with respect to c vs. d_0 is shown in figure. 4.8, the figure shows 4 plots with different sample size, the rate of change of expected profit is the negative of the mean of the process. With the increase in the acceptance number, the mean of the process decreases because raising the value of d_0 has the effect of enlarging the area of acceptance under the OC curve corresponding to the sampling plan. i.e. allowing more nonconforming units to occur gives the producer more latitude in producing nonconforming units, but as seen in the figure the rate of change of expected profit with respect to. c increases because of the negative sign associated with the mean. The rate of change of expected profit with respect to c increases with the increase in the acceptance number.

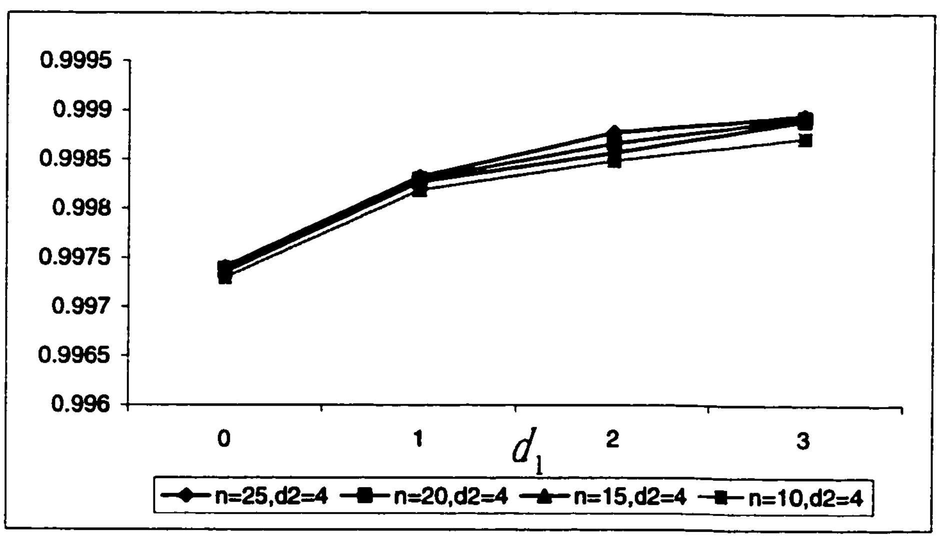

Figure 4.5: Rate of change of Expected Profit with respect to a_1 vs d_1

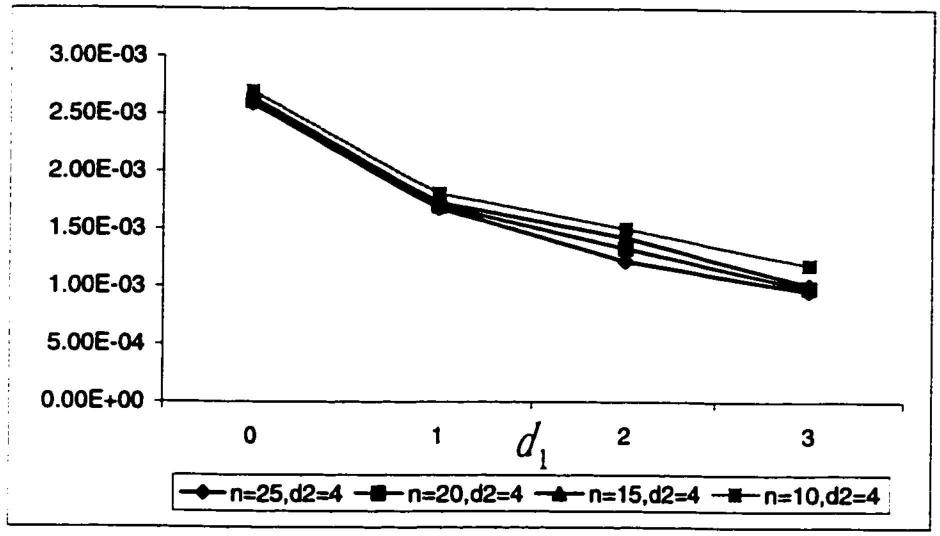

Figure 4.6: Rate of change of Expected Profit with respect to a_2 vs d_1

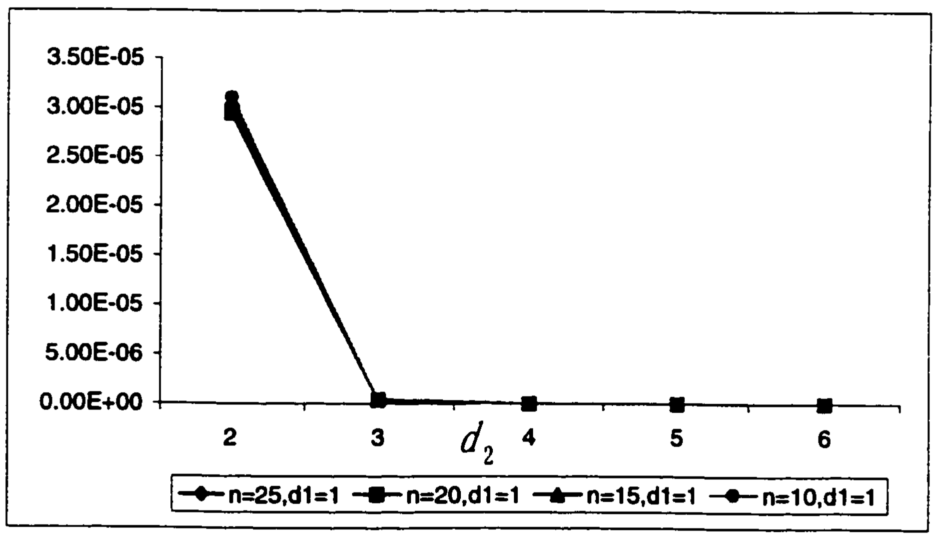

Figure 4.7: Rate of change of Expected Profit with respect to r vs d_2

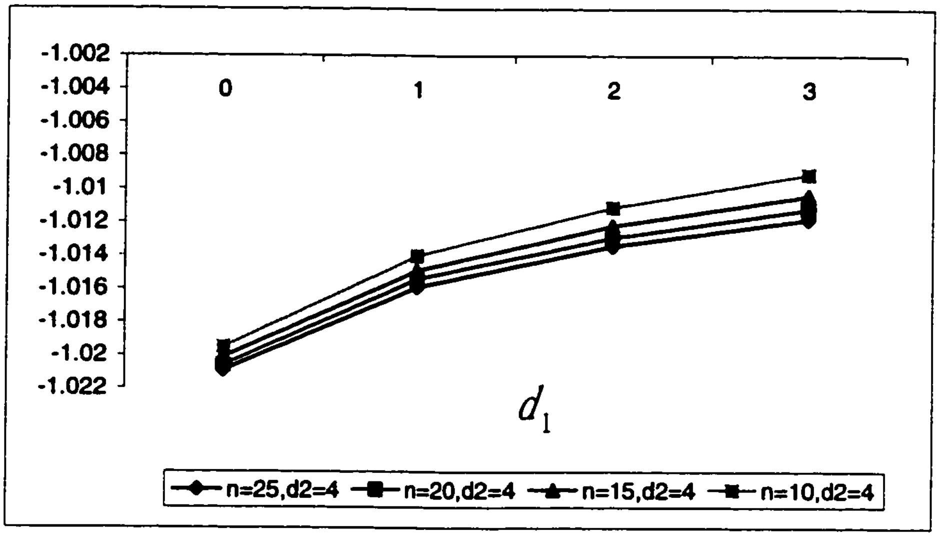

Figure 4.8: Rate of change of Expected Profit with respect to c vs d_1

4.5 Conclusion

In this chapter, a three class process targeting model is developed with quality sampling plans. Two cases are examined for sampling i.e, with without destructive testing. The effect of cost parameters on expected profit and optimal mean are also examined.

Chapter 5

Multi-Class Process Targeting with Quality Sampling Plans and Inspection Error

5.1 Introduction

The purpose of this chapter is to develop a model for selecting the most economical target mean for multi-class targeting problem when inspection error is present. then conduct sensitivity analysis to investigate the effect of the inspection error and model parameters on the result of the model.

The model is developed for a container filling process is considered where the weight and volume of the container is represented by a random variable X, that follows normal distribution with mean μ and variance σ^2. A fixed lower specification limit exists for the quality characteristic X of interest, The lot is classified into three classes using a sampling plan. The classes are grade 1, grade 2, and scrap grade 1 and grade 2 are sold in two different markets with different price structure.

The product is produced in lots and an inspection plan is used for classifying the product as follows, a sample of size n is drawn from the lot and based on the number of defective in the sample the lot is classified as grade 1, grade 2 or scrap. If the number of defectives is less than or equal to d_1, the lot is considered grade 1. However, if the number of defectives is more than d_1 and less or equal to d_2 the lot is considered grade 2. In case the number of defective more than d_2, the lot is sold as scrap.

In this model Inspection error is incorporated in the sampling plan in which an inspector commits two types of errors i.e

(1) Classify a good item as bad, i.e Type I error and

(2) Classify a bad item as good, i.e type II error

Thus, type I and type II error may cause considerable loss to the society. These losses could be the loss due to replacement and warranty, loss of goodwill or loss of profit by selling a higher grade product as a lower grade and selling them at a lower price.

The objective of this model is to maximize the expected profit by finding the optimum target value for multi-class targeting problem with inspection error in the sampling plan. Two cases are examined here.

(1) When sampling results in destructive testing and

(2) When the sampling results in non-destructive testing.

This chapter is organized as follows, model development is presented in section5.2, in which notations, model assumptions and statement of the problem are discussed. solution and analysis of the problem is discussed in sec.5.3, the model results are given in sec.5.4, then sensitivity analysis is provided in section 5.5.

5.2 Model Development

In this section, a multi-class process targeting model is developed considering sampling inspection plan and inspection error. At first, model assumptions, followed by, necessary notations are presented, then the statement of the problem is provided.

5.2.1 Model Assumptions

1. The inspection process is assumed to be error prone.
2. The quality characteristic X is assumed to be normally distributed with mean μ and variance σ^2.
3. The variance of the process is assumed to be known and fixed.
4. Sampling plan is used for quality control in this model.
5. $a_1 > a_2 > r$.
6. $d_2 > d_1$.

5.2.2 Notations

The following are the notations that are used in this chapter

a_1 = Selling price for grade 1,
a_2 = Selling price for grade 2,
r = Salvage value when an item is scrap,
c = Processing cost per item,
X = Weight or volume of the product and is a random variable,
μ = The expected value of X or process mean,
σ^2 = Process variance setting,
L = Lower specification limit for the product attribute,
n = Sample size in the inspection plan,
q = Lot fraction defective i.e $P[X < L] = \Phi(\frac{L-\mu}{\sigma})$,
d_1 = Allowable number of non-conforming units in the sample for grade1,
d_2 = Allowable number of non-conforming units in the sample for grade2,
e_1 = Probability of type I error,
e_2 = Probability of type II error,
x = Number of non-conforming items in the sample, x is a random variable,
x_e = Observed number of non-conforming units in the sample.

5.2.3 Statement of the Problem

Consider a can filling process or turning a metal bar defined by a quality characteristic X, which represents the quantity of material in an individual container or the mean diameter of the turned metal bar and it is assumed to be normally distributed with mean μ and variance σ^2. Sampling Plan is used in this model i.e, a product is produced daily in lots of size N and from each lot a sample of size n is drawn and evaluated. A container is defined as a "non-conforming" if it is filled less than the lower specification limit (X<L). If x is the number of non-conforming units in the sample, and follows binomial distribution and d_2 is the allowable number of non-conforming units, the decision to accept the lot is based on number of

non-conforming in the sample i.e the lot is accepted if $x \leq d_2$ otherwise scrapped. Considering the situation in which type I and type II error occurs in the inspection. The observed number of defectives as it appears to the inspector is not the same as the actual number of defective items in the sample. The expected observed number of defectives in the sample given by Bennett [3] is

$$x_e = (n - x)e_1 + (1 - e_2)x$$

let $P = P(X \leq L)$, the probability of a item being non-conforming is independent of the other item in the lot. the lot fraction defective under error is

$$P_e = (1 - P)e_1 + (1 - e_2)P$$

Therefore, the decision to accept the lot is based on the observed number of defectives in the sample i.e the lot is accepted if $x_e \leq d_2$. Once the lot is accepted, it is considered for grades i.e,if the observed number of non-conforming units are less than d_1 then the lot is accepted for grade 1 or else accepted for grade 2. Containers from a production lot that is accepted for grade 1 are sold at a price "a_1" per container, whereas containers acceptable for grade 2 are sold at a price a_2 and salvage value r per container, is obtained if the lot is scrap.

Non-Destructive Testing

Thus, The expected revenue for a lot of size N, given x_e.

$$E[R/x_e] = \begin{cases} a_1N & \text{if } x_e \leq d_1 \\ a_2N & \text{if } d_1 < x_e \leq d_2 \\ rN & \text{if } x_e > d_2 \end{cases}$$

For the set point of μ, the expected value of the marginal cost per lot is $Nc\mu$, where μ is the expected value of X.

Therefore, The marginal profit function is

$$E[\pi(\mu)] = a_1NP(x_e \leq d_1) + a_2NP(d_1 < x_e \leq d_2) + rNP(x_e > d_2) - Nc\mu$$

i.e

$$\frac{E[\pi(\mu)]}{N} = a_1P(x_e \leq d_1) + a_2P(d_1 < x_e \leq d_2) + rP(x_e > d_2) - c\mu \tag{5.1}$$

Where $E[\pi(\mu)]$ is the expected profit when the set point μ is fixed at a specific value, The first term in the expected profit function is the profit from accepted items, the second term is the profit from class 2 items, the third term is the profit obtained from scrap and $Nc\mu$ is the cost of processing for the set point μ. The process mean may be set higher to reduce the cost incurred by producing defective items. Using a higher process mean, however, increases the probability of producing grade 1 and hence obtaining higher sales and increases the cost. Therefore, the need to select an optimal mean is to obtain the highest profit exists.

Destructive Testing

It is often the case that examining the weight of the contents of a package will result in destroying the product,

Then, The expected revenue becomes

$$E[R/x_e] = \begin{cases} a_1(N-n) & \text{if } x_e \le d_1 \\ a_2(N-n) & \text{if } d_1 < x_e \le d_2 \\ r(N-n) & \text{if } x_e > d_2 \end{cases}$$

And the expected profit becomes,

$$E[\pi(\mu)] = a_1(N-n)P(x_e \le d_1) + a_2(N-n)P(d_1 < x_e \le d_2) + r(N-n)P(x_e > d_2) - Nc\mu \tag{5.2}$$

It should be noted that the expected profit for non destructive testing is not directly comparable cost wise to destructive testing because there are different cost associated with the sampling method used. In non destructive testing the container is salvageable, whereas it is rendered useless in destructive testing.

5.3 Solution and Analysis

The objective is to maximize $\frac{E[\pi(\mu)]}{N}$ defined above. A necessary condition for optimality is that the partial derivative with respect to μ vanishes at the target value μ. which is $\frac{\partial}{\partial\mu}\frac{E[\pi(\mu)]}{N} = 0$,

Therefore,

$$\frac{\partial}{\partial\mu}\frac{E[\pi(\mu)]}{N} = a_1\frac{\partial}{\partial\mu}P(x_e \le d_1) + a_2\frac{\partial}{\partial\mu}P(d_1 < x_e \le d_2) + r\frac{\partial}{\partial\mu}P(x_e > d_2) - c = 0 \tag{5.3}$$

we know that x_e is binomially distributed with sample size n and lot fraction defective q. Therefore,

$$P(x_e \leq d_1) = \sum_{i=0}^{d_1} \binom{n}{i} q_e^i (1 - q_e)^{(n-i)}$$

Where,

$$q_e = q(1 - e_2) + (1 - q)e_1$$

and

$$q = P(X \leq L) = \Phi(\frac{L - \mu}{\sigma})$$

and

$$z = \frac{\mu - L}{\sigma}$$

Then,

$$\frac{\partial q_e}{\partial \mu} = -\frac{(1 - e_1 - e_2)}{\sigma} \phi \left(\frac{L - \mu}{\sigma} \right)$$

therefore,

$$\frac{\partial}{\partial \mu} P(x_e \leq d_1) = \frac{\partial q_e}{\partial \mu} \frac{\partial}{\partial q_e} P(x_e \leq d_1)$$

$$
\begin{aligned}
\frac{\partial}{\partial q_e} P(x_e \leq d_1) &= \frac{\partial}{\partial q_e} \sum_{i=0}^{d_1} \binom{n}{i} q_e^i (1-q_e)^{(n-i)} \\
&= \sum_{i=0}^{d_1} \binom{n}{i} \left[i q_e^{(i-1)} (1-q_e)^{(n-i)} - q_e^i (n-i)(1-q_e)^{(n-i-1)} \right] \\
&= -n(1-q_e)^{(n-1)} + \left[n(1-q_e)^{(n-1)} - n(n-1) q_e (1-q_e)^{(n-2)} \right] \\
&+ \left[n(n-1) q_e (1-q_e)^{(n-2)} - \frac{n(n-1)(n-2)}{2} q_e^2 (1-q_e)^{(n-3)} \right] + \ldots \\
&+ \left[\frac{n!}{(n-d_1)! d_1!} d_1 q_e^{(d_1-1)} (1-q_e)^{(n-d_1)} - \frac{n!}{(n-d_1)! d_1!} (n-d_1) q_e^{d_1} (1-q_e)^{(n-a} \right. \\
&= -\frac{n!}{(n-d_1-1)! d_1!} q_e^{d_1} (1-q_e)^{(n-d_1-1)}
\end{aligned}
$$

We can write the above equation as,

$$
\frac{\partial}{\partial q_e} P(x_e \leq d_1) = -\frac{n!}{(n-d_1-1)! d_1!} q_e^{d_1} (1-q_e)^{(n-d_1-1)}
$$

Therefore,

$$
\frac{\partial}{\partial \mu} P(x_e \leq d_1) = \frac{\partial q_e}{\partial \mu} \left(\frac{\partial}{\partial q_e} P[x_e \leq d_1] \right)
$$

i.e,

$$
\frac{\partial}{\partial \mu} P(x_e \leq d_1) = \frac{(1-e_1-e_2)}{\sigma} \phi(z) \frac{n!}{(n-d_1-1)! d_1!} q_e^{d_1} (1-q_e)^{(n-d_1-1)} \qquad (5.4)
$$

Similar calculations are carried out to get the differential of$P(x_e \geq d_2)$ w.r.t μ

$$
\frac{\partial}{\partial \mu} P(x_e \geq d_2) = \frac{\partial}{\partial \mu} (1 - P[x_e \leq d_2])
$$

i.e,

$$
\frac{\partial}{\partial \mu} P(x_e \geq d_2) = -\frac{(1-e_1-e_2}{\sigma} \phi(z) \frac{n!}{(n-d_2-1)! d_2!} q_e^{d_2} (1-q_e)^{(n-d_2-1)} \qquad (5.5)
$$

similarly,

$$\frac{\partial}{\partial \mu}P(d_1 < x_e \geq d_2) = \frac{\partial q_e}{\partial \mu}\left(\frac{\partial}{\partial q_e}P(d_1 < x_e \geq d_2)\right)$$

Therefore,

$$\begin{aligned}
\frac{\partial}{\partial q_e}P(d_1 < x_e \leq d_2) &= \frac{\partial}{\partial q}\sum_{i=d_1+1}^{d_2}\binom{n}{i}q_e^i(1-q_e)^{(n-i)} \\
&= \sum_{i=d_1+1}^{d_2}\binom{n}{i}\left[iq_e^{(i-1)}(1-q_e)^{(n-i)} - q_e^i(n-i)(1-q_e)^{(n-i-1)}\right] \\
&= \begin{bmatrix} \left(\frac{n!}{(n-d_1-1)!(d_1+1)!}(d_1+1)q_e^{d_1}(1-q_e)^{(n-d_1-1)}\right) \\ -\left(\frac{n!}{(n-d_1-1)!(d_1+1)!}(n-d_1-1)q_e^{d_1+1}(1-q_e)^{(n-d_1-2)}\right) \end{bmatrix} \\
&+ \begin{bmatrix} \left(\frac{n!}{(n-d_1-2)!(d_1+2)!}(d_1+2)q_e^{d_1+1}(1-q_e)^{(n-d_1-2)}\right) \\ -\left(\frac{n!}{(n-d_1-2)!(d_1+2)!}(n-d_1-2)q_e^{d_1+2}(1-q_e)^{(n-d_1-3)}\right) \end{bmatrix} \\
&+ \ldots + \begin{bmatrix} \left(\frac{n!}{(n-d_2+1)!(d_2-1)!}(d_2-1)q_e^{d_2-2}(1-q_e)^{(n-d_2+1)}\right) \\ -\left(\frac{n!}{(n-d_2+1)!(d_2-1)!}(n-d_2+1)q_e^{d_2-1}(1-q_e)^{(n-d_2)}\right) \end{bmatrix} \\
&+ \begin{bmatrix} \left(\frac{n!}{(n-d_2)!d_2!}d_2q_e^{d_2-1}(1-q_e)^{(n-d_2)}\right) \\ -\left(\frac{n!}{(n-d_2)!d_2!}(n-d_2)q_e^{d_2}(1-q_e)^{(n-d_2-1)}\right) \end{bmatrix}
\end{aligned}$$

That is equal to,

$$\frac{\partial}{\partial q}P(d_1 < x_e \leq d_2) = \frac{n!}{(n-d_1-1)!d_1!}q_e^{d_1}(1-q_e)^{(n-d_1-1)} - \frac{n!}{(n-d_2-1)!d_2!}q_e^{d_2}(1-q_e)^{(n-d_2-1)}$$

Therefore,

$$\frac{\partial}{\partial \mu}P(d_1 < x_e \leq d_2) = -\frac{(1-e_1-e_2)}{\sigma}\phi(z)\left[\frac{n!}{(n-d_1-1)!d_1!}q_e^{d_1}(1-q_e)^{(n-d_1-1)} - \frac{n!}{(n-d_2-1)!d_2!}q_e^{d_2}(1-q_e)^{(n-d_2-1)}\right] \tag{5.6}$$

Substituting equations(5.4),(5.5) and (5.6) in (5.3), and equating it to zero, we get

$$\frac{(1-e_1-e_2)\phi(z)}{\sigma}\left[\frac{n!}{(n-d_1-1)!d_1!}q_e^{d_1}(1-q_e)^{(n-d_1-1)}(a_1-a_2) + \frac{n!}{(n-d_2-1)!d_2!}q_e^{d_2}(1-q_e)^{(n-d_2-1)}(a_2-r)\right] = c \tag{5.7}$$

The optimal value of μ can be obtained from this equation by numerically solving this equation.
Similar calculations are carried out to find the optimal condition for optimality for destructive testing, we arrive at the result shown below,

$$\frac{(1-e_1-e_2)\phi(z)}{\sigma}\left[\frac{n!}{(n-d_1-1)!d_1!}q_e^{d_1}(1-q_e)^{(n-d_1-1)}(a_1-r)-\frac{n!}{(n-d_2-1)!d_2!}q_e^{d_2}(1-q_e)^{(n-d_2-1)(a_2-r)}\right]=\left(\frac{N}{N-n}\right)c \tag{5.8}$$

The only change noted in the optimality condition for the two cases i.e for destructive testing and non destructive testing is the term $(\frac{N}{N-n})$ on the right hand side of the equation.

5.4 Results

An example for destructive and non-destructive testing for the targeting problem with two types of error in the sampling inspection plan are presented in this section. Since, it is difficult to get the closed form solution to this problem. A line search such as golden section method is used to get the solutions. Computer code is written in Fortran 90, the program is run on a pentium III computer with 64 MB RAM (see appendix D for the code).

Using the same numerical values of parameters as given in the model developed by Boucher and Jafari [1], and the same error levels as given in th model of Bennett et.al. [3], the optimal process mean and expected profit for different sample size, and for different values of d_1 and d_2 are evaluated.

Given $a_1 = 67.5$, $a_2 = 37$, $r = 10$,$c = 55$, $\sigma = 0.00563$, $L = 1$,$e_1 = 0.01$ and $e_2 = 0.03$.

The table below shows the values of the optimal process mean and expected profit for the multi-class process targeting problem for sampling plans with inspection error.

	$d_1 = 0$		$d_1 = 1$		$d_1 = 2$		$d_1 = 3$	
d_2	μ	EP	μ	EP	μ	EP	μ	EP
1	1.0204	5.2868	-	-	-	-	-	-
2	1.0202	5.7263	1.0177	10.8985	-	-	-	-
3	1.0202	5.7534	1.0176	10.9312	1.0143	11.6047	-	-
4	1.0202	5.7546	1.0176	10.9327	1.0142	11.6105	1.0119	11.7891

Table 5.1: Expected Profit and Mean for n=20

Comparing the tables above, w.r.t sample size and acceptance number

	$d_1 = 0$		$d_1 = 1$		$d_1 = 2$		$d_1 = 3$	
d_2	μ	EP	μ	EP	μ	EP	μ	EP
1	1.0194	8.3192	-	-	-	-	-	-
2	1.0192	8.4377	1.0154	11.4302	-	-	-	-
3	1.0192	8.4410	1.053	11.4369	1.0118	11.7836	-	-
4	1.0192	8.4410	1.053	11.4370	1.0117	11.7865	1.0094	11.9335

Table 5.2: Expected Profit and Mean for n=10

The mean of the process increases with the increase in the level of the sample size for the fixed value of d_1 and d_2, because the OC curve shows lower probability of acceptance for higher n in the sampling plan, i.e sampling more units in a lot shows higher chances of detecting the defect and rejecting the lot. That's why the producer increases the mean.

If the mean is increased the chance of producing the defectives is reduced at the expense of cost of manufacturing. Thus,the expected profit decreases with the increase in process mean.

Using the same values as given in the model developed by Lee and Jang [4], the optimal process mean and expected profit for different sample size, and for different values of d_1 and d_2 are obtained.

Given $a_1 = 5.5$, $a_2 = 5.1$, $r = 2.5$, $c = 0.01$, $\sigma = 1.25$, $L = 40$, $e_1 = 0.01$ and $e_2 = 0.03$.

The table below shows the values of the optimal process mean and expected profit for the multi-class process targeting problem.

	$d_1 = 0$		$d_1 = 1$		$d_1 = 2$		$d_1 = 3$	
d_2	μ	EP	μ	EP	μ	EP	μ	EP
1	44.3781	4.9363	-	-	-	-	-	-
2	44.1174	4.9799	43.6694	5.0504	-	-	-	-
3	44.0750	4.9828	43.4738	5.0546	42.9770	5.0669	-	-
4	44.0750	4.9829	43.4569	5.0548	42.8422	5.0682	42.5165	5.0728

Table 5.3: Expected Profit and Mean for n=20

	$d_1 = 0$		$d_1 = 1$		$d_1 = 2$		$d_1 = 3$	
d_2	μ	EP	μ	EP	μ	EP	μ	EP
1	44.0317	5.0068	-	-	-	-	-	-
2	43.8479	5.0192	43.1078	5.0634	-	-	-	-
3	43.8320	5.0196	42.9840	5.0647	42.4200	5.0733	-	-
4	43.8320	5.0196	42.9781	5.0648	42.3200	5.0742	41.9516	5.0786

Table 5.4: Expected Profit and Mean for n=10

For Destructive Testing Using the same values of model I, the optimal process mean and Expected Profit for different sampling plans are shown for destructive testing.

For n=20, N=100;

	$d_1 = 0$		$d_1 = 1$		$d_1 = 2$		$d_1 = 3$	
d_2	μ	EP	μ	EP	μ	EP	μ	EP
1	1.0201	-6.9935	-	-	-	-	-	-
2	1.0199	-6.6393	1.0173	-2.4735	-	-	-	-
3	1.0199	-6.6174	1.0172	-2.4459	1.0140	-1.8718	-	-
4	1.0199	-6.6164	1.0172	-2.4446	1.0139	-1.8661	1.0118	-1.6989

Table 5.5: Expected Profit and Mean for n=20 for destructive testing

For n=10, N=100;

	$d_1=0$		$d_1=1$		$d_1=2$		$d_1=3$	
d_2	μ	EP	μ	EP	μ	EP	μ	EP
1	1.0192	1.8813	-	-	-	-	-	-
2	1.0191	1.9885	1.0152	4.7031	-	-	-	-
3	1.0191	1.9915	1.0151	4.7095	1.0116	5.0408	-	-
4	1.0191	1.9916	1.0151	4.7097	1.0116	5.0437	1.0093	5.1886

Table 5.6: Expected Profit and Mean for n=10 for destructive testing

Using the same values as given in model II, the optimal process mean and Expected Profit for different sample sizes, and for different values of d_1 and d_2 are obtained for destructive testing.

For n=20, N=100;

	$d_1=0$		$d_1=1$		$d_1=2$		$d_1=3$	
d_2	μ	EP	μ	EP	μ	EP	μ	EP
1	44.3038	3.8603	-	-	-	-	-	-
2	44.0327	3.8958	43.5832	3.9531	-	-	-	-
3	43.9831	3.8982	43.4087	3.9568	42.9208	3.9676	-	-
4	43.9831	3.8983	43.3794	3.9510	42.7929	3.9689	42.4924	3.9732

Table 5.7: Expected Profit and Mean for n=20 for destructive testing

For n=10, N=100;

	$d_1=0$		$d_1=1$		$d_1=2$		$d_1=3$	
d_2	μ	EP	μ	EP	μ	EP	μ	EP
1	43.9831	4.4621	-	-	-	-	-	-
2	43.8017	4.4735	43.0808	4.5139	-	-	-	-
3	43.7860	4.4739	42.9505	4.5153	42.3964	4.5236	-	-
4	43.7860	4.4739	42.9505	4.5153	42.2915	4.5245	41.9307	4.5288

Table 5.8: Expected Profit and Mean for n=10 for destructive testing

As the lot size is increasing the value of the Expected Profit also increases, but the Expected Profit for destructive testing is always less than Expected Profit for non-destructive testing, for any number of units in the lot.

5.5 Sensitivity Analysis

The effect of type I and type II errors on expected profit and the process mean is investigated in the sensitivity analysis. Three Sampling Plans are considered with sample size n = 10,20 and 30, and with same acceptance numbers $d_1 = 3$ and $d_2 = 4$. Table 5.9, 5.10 and 5.11, shows the Optimal Mean and Expected Profit at different levels of type I and type II error.

Change in Profit is defined as the difference between Expected Profit without error and Expected Profit with error. Thus, percentage change in profit is given by,

$$\text{percentage change in profit} = \frac{EP(\text{without error}) - EP(\text{with error})}{EP(\text{without error})}$$

The point to be noted in the tables shown below is that, the first few values of the Change in profit is negative, this is expected because when type I error is nullified and increasing the value of type II error, we are classifying more number of defective items as good and selling it at a higher price. So, the Profit with error is expected to be more than the Profit without error.

5.5.1 Effect of Type I Error on Expected Profit

In this section, Expected Profit at different levels of type I error are shown. Figures 5.1 to 5.6, shows that at a given level of type II error, the Expected Profit decreases

Expected profit without error = 11.9582

percentage change in profit = $\frac{EP(withouterror) - EP(witherror)}{EP(withouterror)} * 100$

(e_1, e_2)	μ	EP	Percentage change in profit
(0,0)	1.0091	11.9582	0.00
(0,0.01)	1.0091	11.9597	-0.012
(0,0.03)	1.009	11.9628	-0.038
(0,0.05)	1.009	11.966	-0.0652
(0,0.1)	1.0088	11.9744	-0.135
(0,0.15)	1.0086	11.9833	-0.209
(0.01,0)	1.0095	11.929	0.244
(0.01,0.01)	1.0095	11.9305	0.231
(0.01,0.03)	1.0094	11.9335	0.206
(0.01,0.05)	1.0094	11.9367	0.179
(0.01,0.10)	1.0092	11.9449	0.111
(0.01,0.15)	1.009	11.9536	0.038
(0.03,0.0)	1.0104	11.8561	0.853
(0.03,0.01)	1.0104	11.8575	0.842
(0.03,0.03)	1.0103	11.8605	0.817
(0.03,0.05)	1.0103	11.8634	0.792
(0.03,0.1)	1.0101	11.8713	0.726
(0.03,0.15)	1.0099	11.8797	0.656
(0.05,0)	1.0114	11.7531	1.715
(0.05,0.01)	1.0114	11.7545	1.703
(0.05,0.03)	1.0113	11.7573	1.680
(0.05,0.05)	1.0113	11.7602	1.655
(0.05,0.1)	1.0111	11.7676	1.593
(0.05,0.15)	1.0109	11.7757	1.526
(0.1,0.0)	1.0139	11.1984	6.353
(0.1,0.01)	1.0139	11.1997	6.342
(0.1,0.03)	1.0138	11.2022	6.322
(0.1,0.05)	1.0138	11.2048	6.3002
(0.1,0.1)	1.0136	11.2116	6.243
(0.1,0.15)	1.0135	11.2189	6.182
(0.15,0.0)	1.0156	9.7541	18.431
(0.15,0.01)	1.0155	9.7553	18.421
(0.15,0.03)	1.0155	9.7577	18.401
(0.15,0.05)	1.0155	9.7602	18.380
(0.15,0.1)	1.0153	9.7668	18.325
(0.15,0.15)	1.0152	9.7739	18.266

Table 5.9: Expected Profit and Optimal Mean at different values of Error for n = 10 ;$d_1 = 3$;$d_2 = 4$

Expected profit without error = 11.8494

(e_1, e_2)	μ	EP	Percentage change in profit
(0,0)	1.0112	11.8494	0
(0,0.01)	1.0112	11.8507	-0.0109
(0,0.03)	1.0111	11.8533	-0.0329
(0,0.05)	1.0111	11.8562	-0.0573
(0,0.1)	1.0109	11.8634	-0.118
(0,0.15)	1.0108	11.871	-0.182
(0.01,0)	1.012	11.7854	0.540
(0.01,0.01)	1.012	11.7866	0.529
(0.01,0.03)	1.012	11.7892	0.508
(0.01,0.05)	1.0119	11.7918	0.486
(0.01,0.10)	1.0118	11.7987	0.427
(0.01,0.15)	1.0116	11.806	0.366
(0.03,0.0)	1.014	11.5532	2.499
(0.03,0.01)	1.014	11.5545	2.488
(0.03,0.03)	1.0139	11.5567	2.470
(0.03,0.05)	1.0139	11.5592	2.449
(0.03,0.1)	1.0138	11.5654	2.396
(0.03,0.15)	1.0136	11.5719	2.341
(0.05,0)	1.0159	10.9796	7.340
(0.05,0.01)	1.0158	10.9807	7.331
(0.05,0.03)	1.0158	10.9828	7.313
(0.05,0.05)	1.0157	10.985	7.294
(0.05,0.1)	1.0156	10.9907	7.246
(0.05,0.15)	1.0155	10.9969	7.194
(0.1,0.0)	1.0181	6.1937	47.729
(0.1,0.01)	1.0181	6.1947	47.721
(0.1,0.03)	1.018	6.1968	47.703
(0.1,0.05)	1.018	6.1988	47.686
(0.1,0.1)	1.0179	6.2043	47.640
(0.1,0.15)	1.0177	6.21	47.592
(0.15,0.0)	1.0188	-3.9585	133.406
(0.15,0.01)	1.0187	-3.9575	133.398
(0.15,0.03)	1.0187	-3.9554	133.380
(0.15,0.05)	1.0187	-3.9533	133.362
(0.15,0.1)	1.0186	-3.9477	133.315
(0.15,0.15)	1.0184	-3.9417	133.264

Table 5.10: Expected Profit and Optimal Mean at different values of Error for n = 20 ;$d_1 = 3$;$d_2 = 4$

Expected profit without error = 11.8494

(e_1, e_2)	μ	EP	Percentage change in profit
(0,0)	1.0122	11.795	0
(0,0.01)	1.0112	11.7962	-0.010173802
(0,0.03)	1.0122	11.7988	-0.032
(0,0.05)	1.0121	11.8014	-0.0542
(0,0.1)	1.012	11.808	-0.110
(0,0.15)	1.0118	11.8152	-0.171
(0.01,0)	1.0135	11.69	0.890
(0.01,0.01)	1.0135	11.6911	0.880
(0.01,0.03)	1.0134	11.6936	0.859
(0.01,0.05)	1.0134	11.696	0.839
(0.01,0.10)	1.0133	11.7022	0.786
(0.01,0.15)	1.0131	11.7089	0.729
(0.03,0.0)	1.0164	11.0957	5.928
(0.03,0.01)	1.0163	11.0966	5.921
(0.03,0.03)	1.0164	11.0987	5.903
(0.03,0.05)	1.0163	11.1008	5.885
(0.03,0.1)	1.0162	11.1062	5.839
(0.03,0.15)	1.016	11.112	5.790
(0.05,0)	1.0182	9.1388	22.519
(0.05,0.01)	1.0181	9.1399	22.510
(0.05,0.03)	1.0181	9.1418	22.494
(0.05,0.05)	1.018	9.1438	22.477
(0.05,0.1)	1.018	9.1489	22.434
(0.05,0.15)	1.0178	9.1543	22.388
(0.1,0.0)	1.0195	-4.149	135.175
(0.1,0.01)	1.0195	-4.148	135.167
(0.1,0.03)	1.0195	-4.1462	135.152
(0.1,0.05)	1.0195	-4.1443	135.136
(0.1,0.1)	1.0194	-4.1392	135.092
(0.1,0.15)	1.0192	-4.1337	135.046
(0.15,0.0)	1.0193	-22.176	288.011
(0.15,0.01)	1.0194	-22.1751	288.004
(0.15,0.03)	1.0193	-22.173	287.986
(0.15,0.05)	1.0192	-22.1709	287.968
(0.15,0.1)	1.0192	-22.1655	287.922
(0.15,0.15)	1.019	-22.1596	287.872

Table 5.11: Expected Profit and Optimal Mean at different values of Error for n = 30 ;$d_1 = 3$;$d_2 = 4$

with the increase in the value of type I error, this is expected because a good item is classified as defective when type I error is committed, and it is sold at a reduced price. Thus, reducing the expected profit.

Expected Profit reduces sharply as type I error tends to 0.1. Each figure shows, Expected profit versus type I error at different levels of e_2 ; containing 3 plots each with different sampling plan with sample size n and acceptance number d_1 and d_2. As the sample size increases the Expected Profit is reduced because the sampling plan with a higher sample size will show a smaller probability of acceptance.

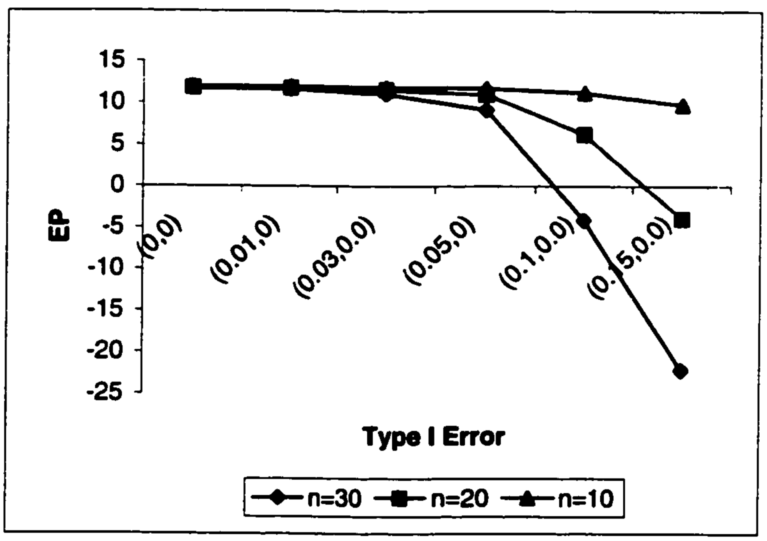

Figure 5.1: Expected Profit versus e_1 at $e_2 = 0$

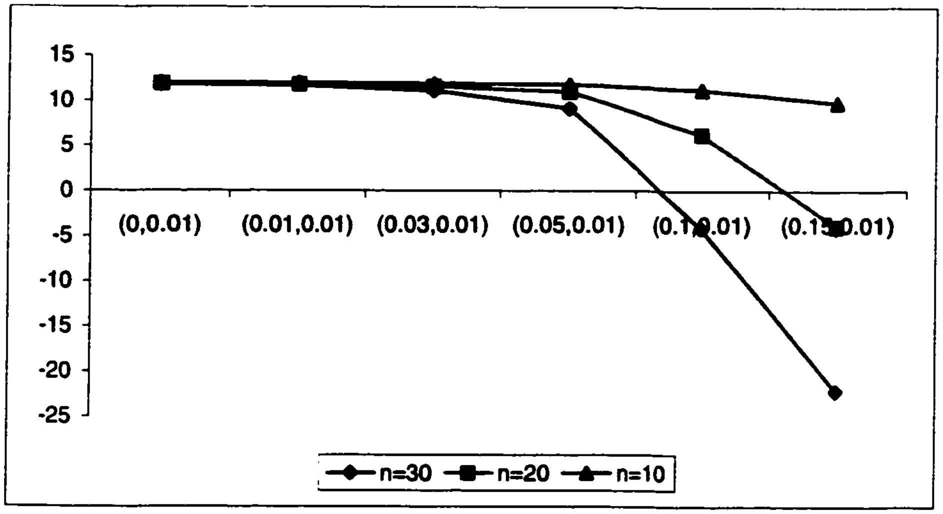

Figure 5.2: Expected Profit versus e_1 at $e_2 = 0.01$

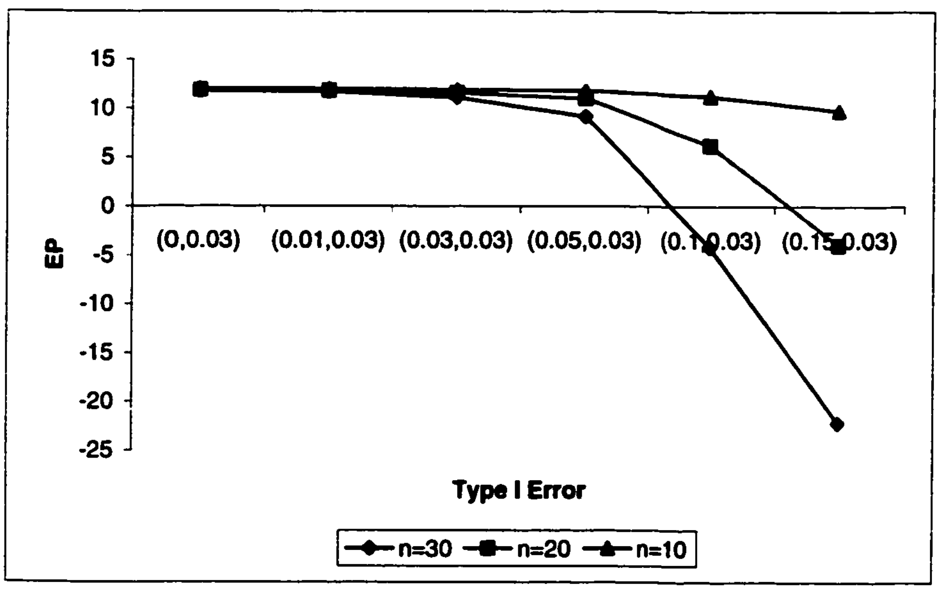

Figure 5.3: Expected Profit versus e_1 at $e_2 = 0.03$

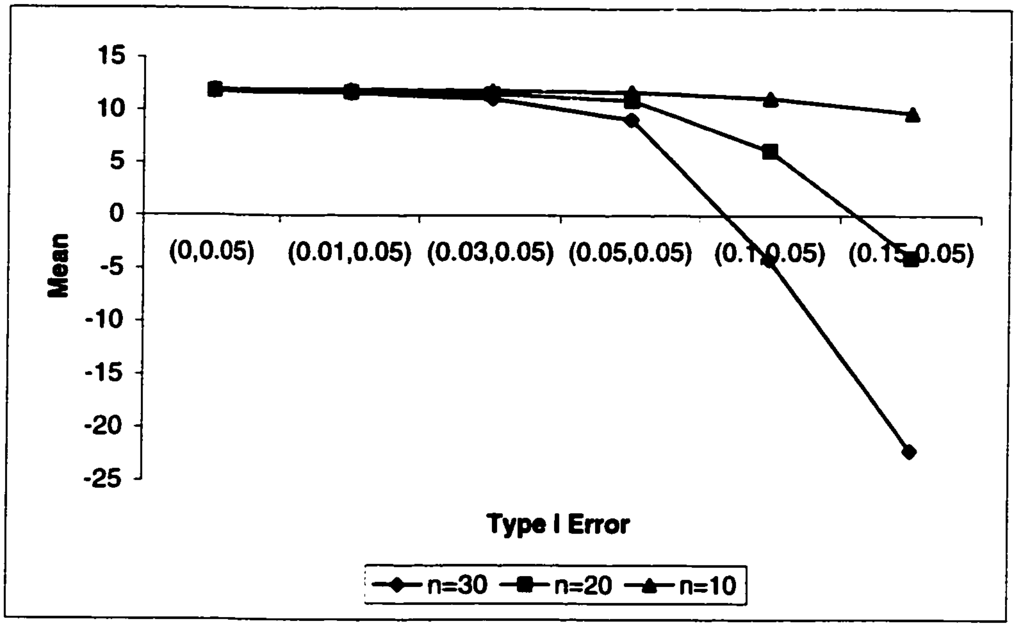

Figure 5.4: Expected Profit versus e_1 at $e_2 = 0.05$

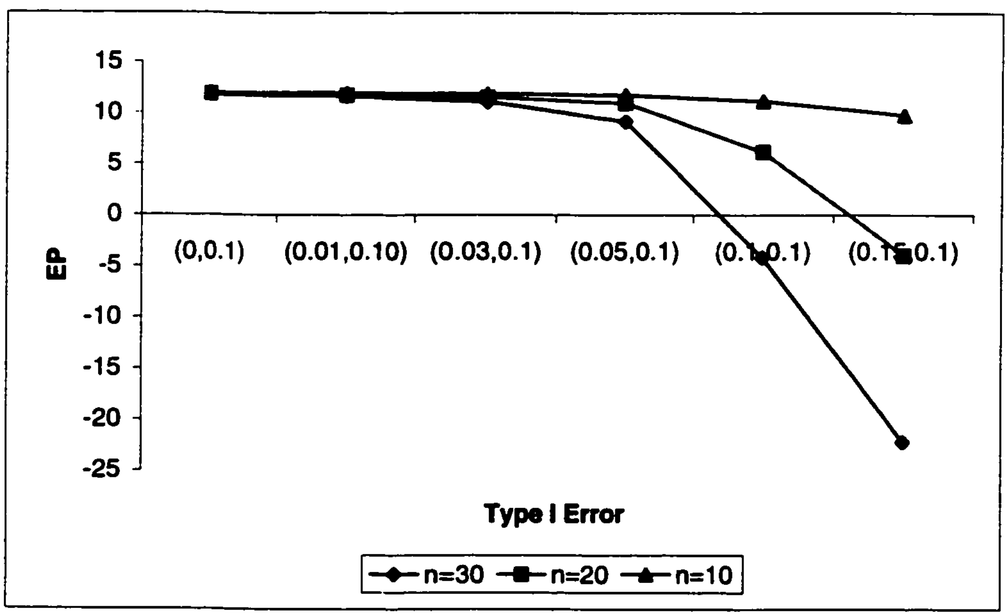

Figure 5.5: Expected Profit versus e_1 at $e_2 = 0.1$

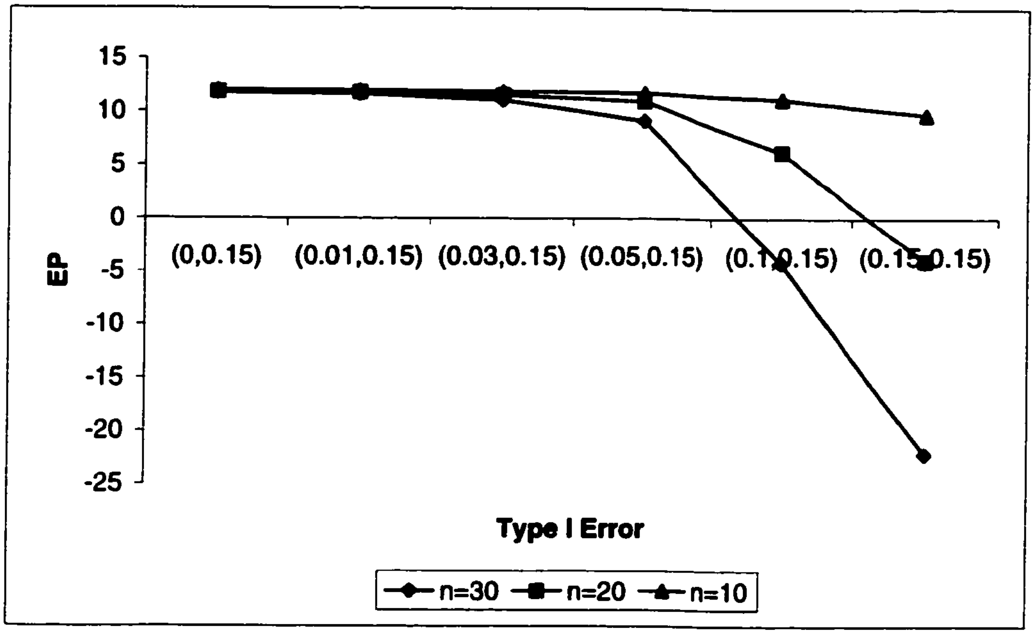

Figure 5.6: Expected Profit versus e_1 at $e_2 = 0.15$

5.5.2 Effect of Type II Error on Expected Profit

The Expected Profit at different levels of type II error are shown in this section. As shown in the figures 5.7 to 5.12 taking different values of e_1, for the same Sampling plan expected profit increases with the increase in the level of type II error, this is expected because increasing the level of type II error implies that we are classifying more number of defective items as good and selling it at a higher price. Thus, Expected Profit is assumed to be directly proportional to type II error.

Each figure shows Expected profit versus type II error at different levels of e_1 ; containing 3 plots each with different sampling plan with sample size n and acceptance number d_1 and d_2. As the sample size increases the Expected Profit is reduced because the sampling plan with a higher sample size will show a smaller probability of acceptance for the same acceptance number d_1 and d_2. Simply,Sampling more units in a lot increases the chances of detecting the non-conformity and rejecting the lot. As seen from figures 5.1 to 5.6, type I error reduces the value of EP. Thus, type I error has more impact on Expected Profit than type II error.

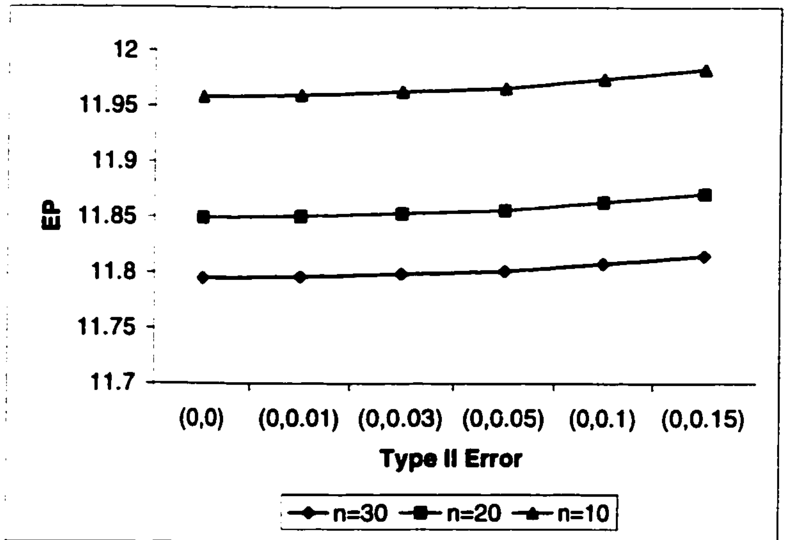

Figure 5.7: Expected Profit versus e_2 at $e_1 = 0$

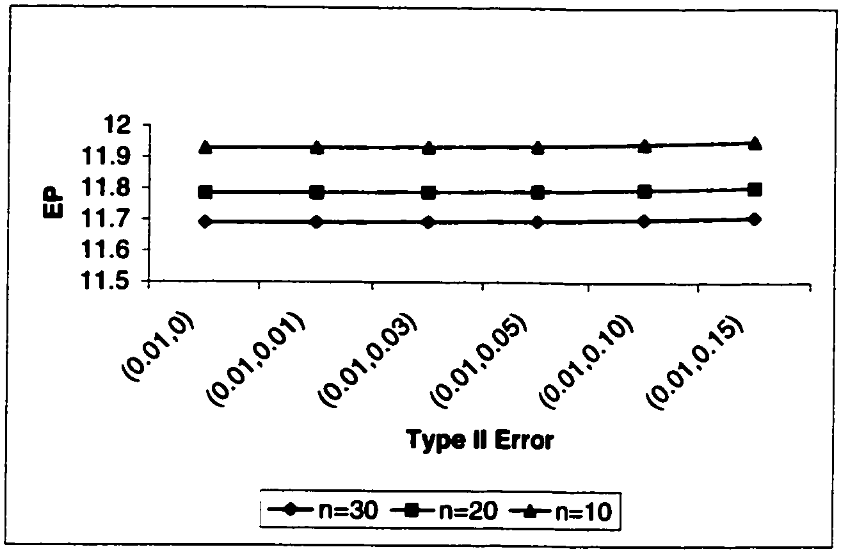

Figure 5.8: Expected Profit versus e_2 at $e_1 = 0.01$

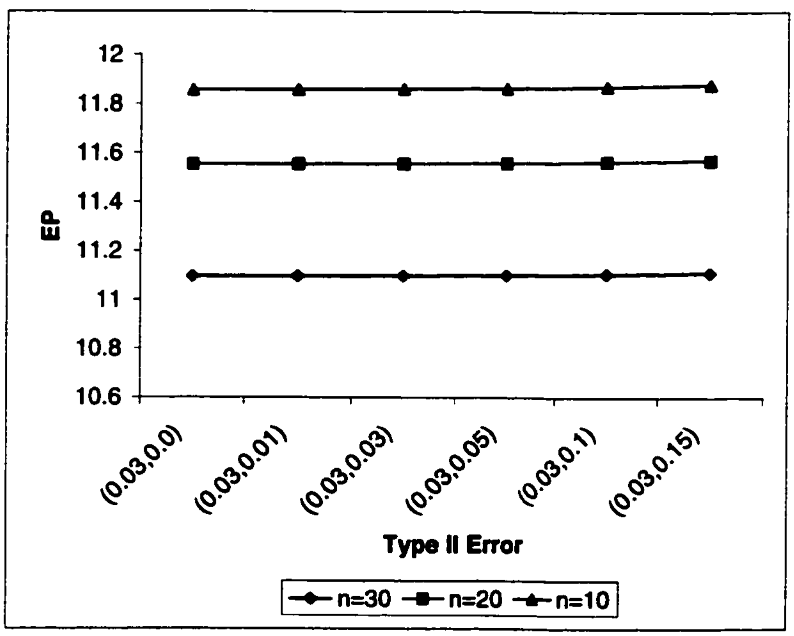

Figure 5.9: Expected Profit versus e_2 at $e_1 = 0.03$

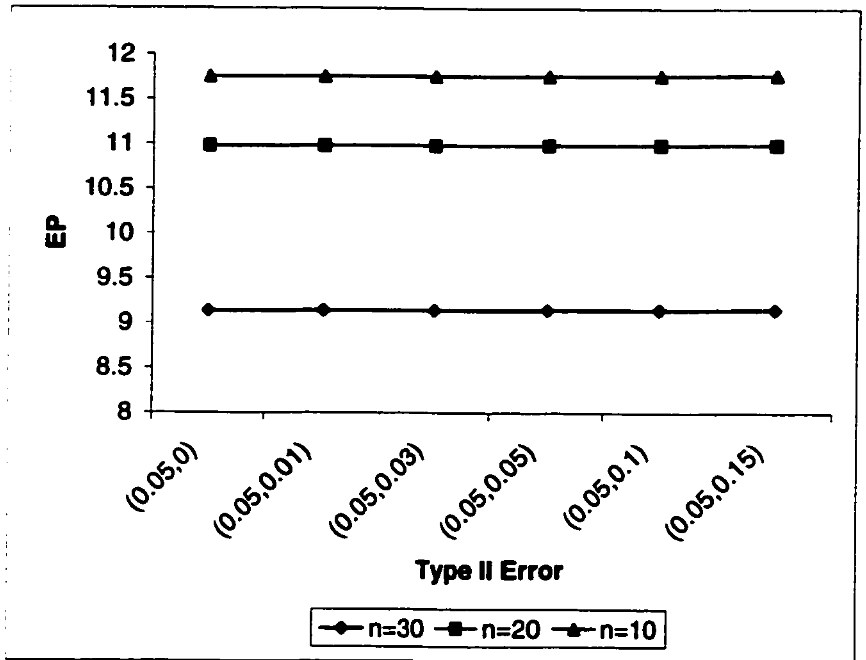

Figure 5.10: Expected Profit versus e_2 at $e_1 = 0.05$

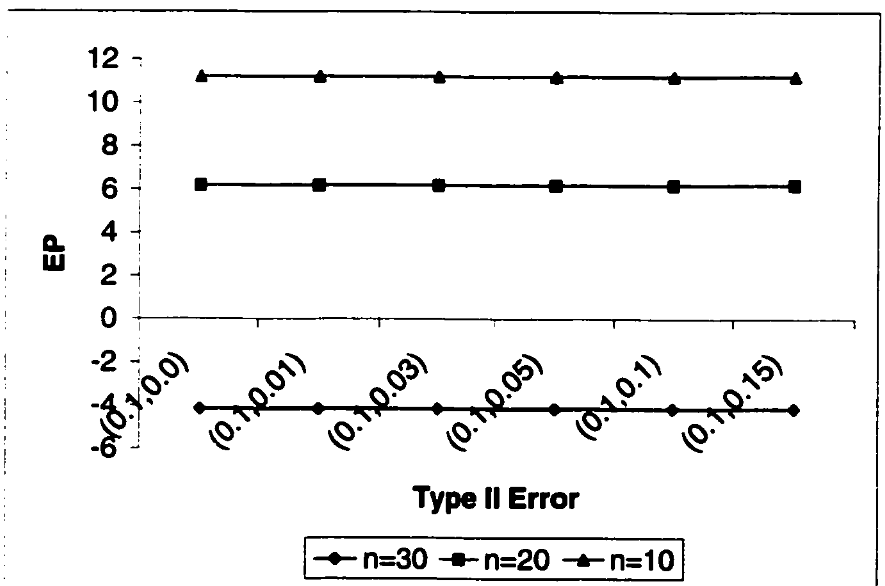

Figure 5.11: Expected Profit versus e_2 at $e_1 = 0.1$

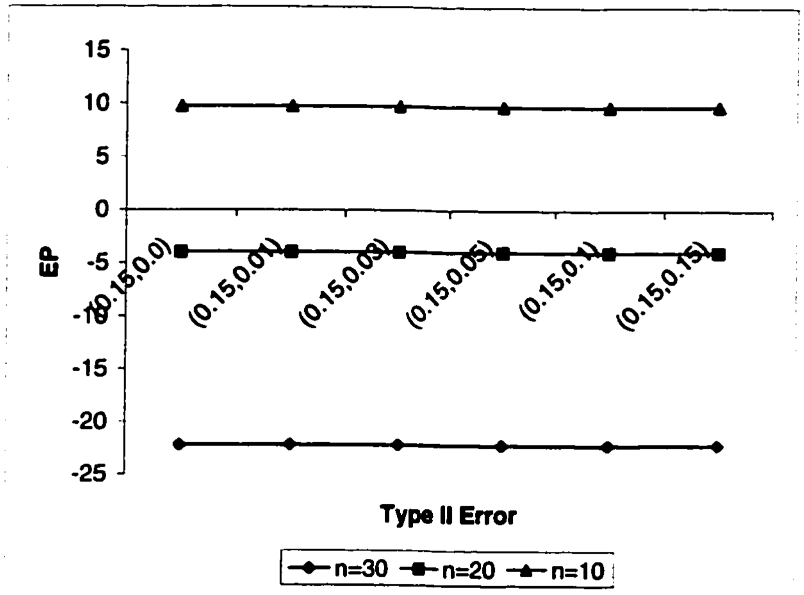

Figure 5.12: Expected Profit versus e_2 at $e_1 = 0.15$

5.5.3 Effect of type I and type II error on Optimal Mean

The effect of type I error on Optimal Mean are shown in figure 5.13 and 5.14, at a given level of type II error the mean tends to increase drastically with the increase in type I error, this shows that the Process mean is forced higher at higher values of e_1.

in figure 5.15 and 5.16, keeping e_1 as constant , Optimal Mean versus type II error is plotted. the graphs shows that the mean tends to decrease with the increase in the value of e_2.

The graphs show two points with different sampling plan with sample size n and acceptance number d_1 and d_2. It can be seen that as the sample size increases the Mean of the process is increased because for any value of probability of failure q_e, the OC curve of a sampling plan with higher n will show a lower probability of acceptance, this forces the producer to increase the set point.

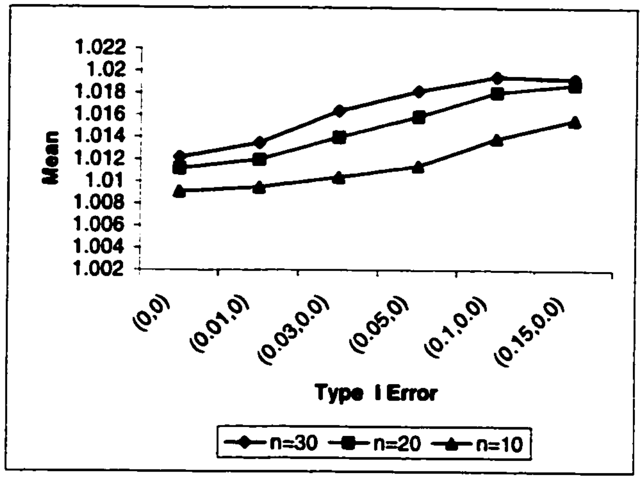

Figure 5.13: μ versus e_1 at $e_2 = 0$

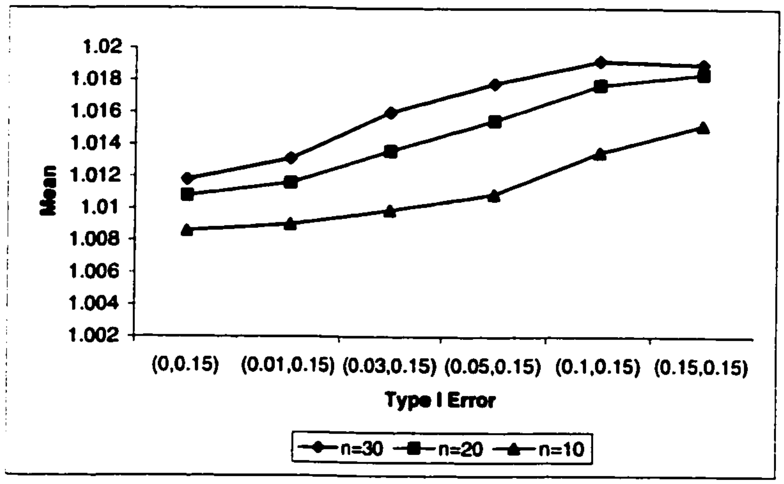

Figure 5.14: μ versus e_1 at $e_2 = 0.15$

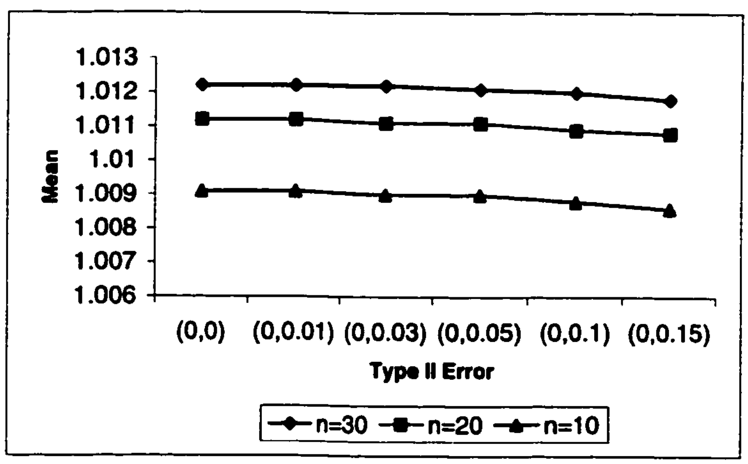

Figure 5.15: μ versus e_2 at $e_1 = 0$

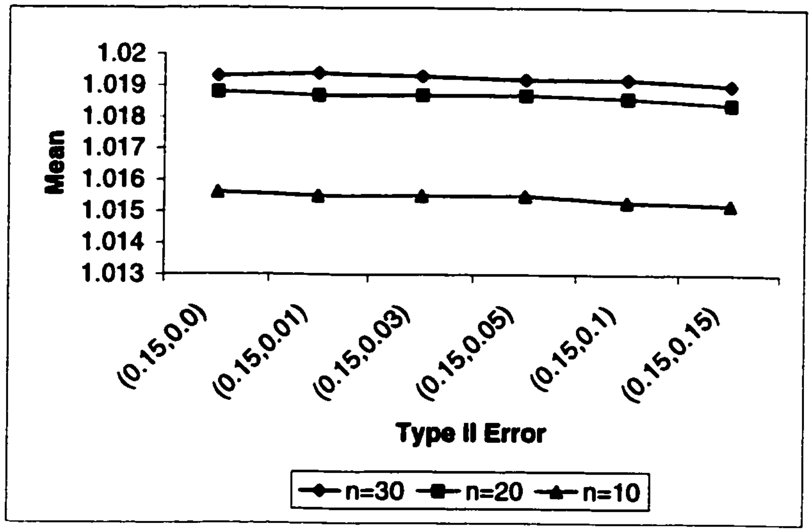

Figure 5.16: μ versus e_2 at $e_1 = 0.15$

5.5.4 Effect of Cost parameters on Expected Profit

The effect of different cost parameters on expected profit are studied in this section, by taking the partial derivative of the profit function w.r.t. the selling price a_1, a_2, r and processing cost c. thus, the partial derivatives of equation 5.1 w.r.t these parameters are as follows:

$$\begin{aligned}
\frac{\partial(\frac{E[\pi(\mu)]}{N})}{\partial a_1} &= P(x_e \leq d_1) \\
\frac{\partial(\frac{E[\pi(\mu)]}{N})}{\partial a_2} &= P(d_1 < x_e \leq d_2) \\
\frac{\partial(\frac{E[\pi(\mu)]}{N})}{\partial r} &= P(x_e > d_2) \\
\frac{\partial(\frac{E[\pi(\mu)]}{N})}{\partial c} &= -\mu
\end{aligned}$$

The partial derivative w.r.t selling price a_1 is the probability of the lot accepted for grade 1, and the partial derivative w.r.t a_2 is equal to the probability of acceptance for grade 2, and the probability that the lot is scrap is the rate of change of expected profit w.r.t selling price r. whereas, the rate of change of profit w.r.t cost c is the negative of the mean of the process.

The partial derivative w.r.t selling price a_1 increases with the increase in the acceptance number, because with the increase in the acceptance number, the probability of acceptance increases. For the same acceptance number, with the increase in the sample size the rate decreases because the probability of acceptance decreases with the increase in the sample size as shown in figure 5.17. Fig. 5.18 shows the rate of change of expected profit w.r.t a_2 vs d_1, the rate of change decreases with the increase in the acceptance number for grade 1, because the product is acceptable for grade 2. For the same acceptance number with the increase in the sample size, the rate of change increases because probability of acceptance decreases with the sample size so the probability of rejection increases. similar result is obtain for the case of the change in expected profit w.r.t r vs d_2.

The rate of change of expected profit w.r.t c vs. d_1 is shown in figure. 5.20, the figure shows 4 plots with different sample size, the rate of change of expected profit is the negative of the mean of the process. With the increase in the acceptance number, the mean of the process decreases because raising the value of d_1 has the effect of enlarging the area of acceptance under the OC curve corresponding to the sampling plan. i.e. allowing more nonconforming units to occur gives the producer more latitude in producing nonconforming units, but as seen in the figure the rate of change of expected profit w.r.t. c increases because of the negative sign associated with the mean. The rate of change of expected profit w.r.t c increases with the increase in the acceptance number.

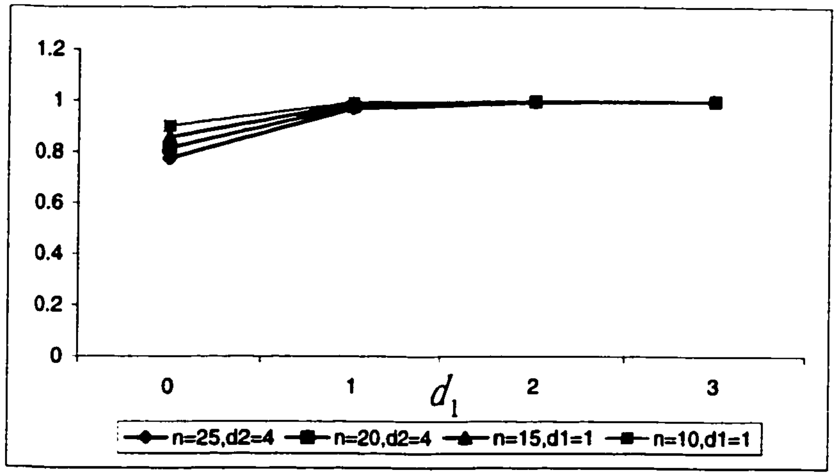

Figure 5.17: Rate of change of Expected Profit w.r.t a_1 vs d_1

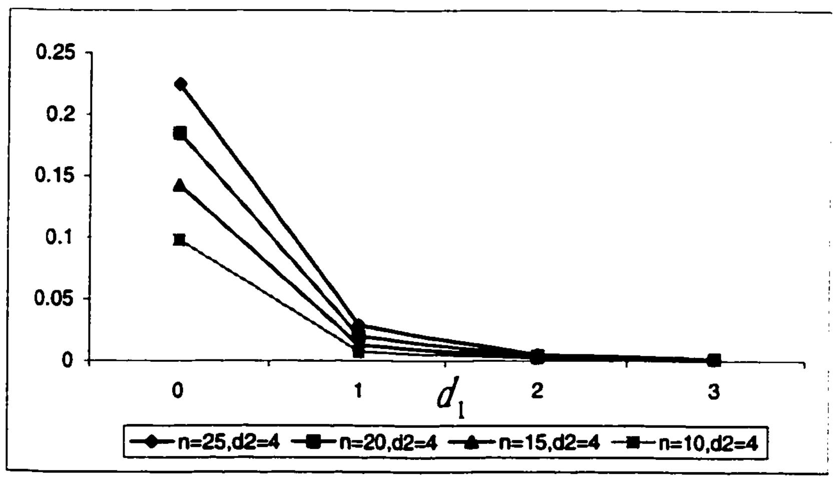

Figure 5.18: Rate of change of Expected Profit w.r.t a_2 vs d_1

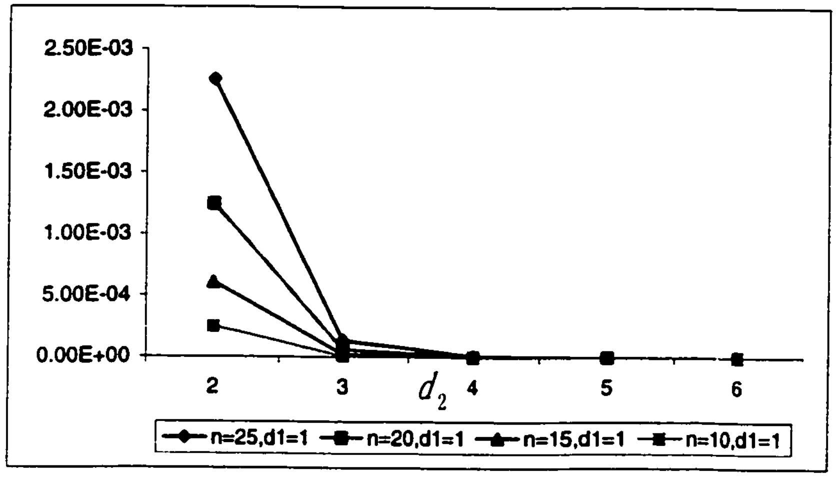

Figure 5.19: Rate of change of Expected Profit w.r.t r vs d_2

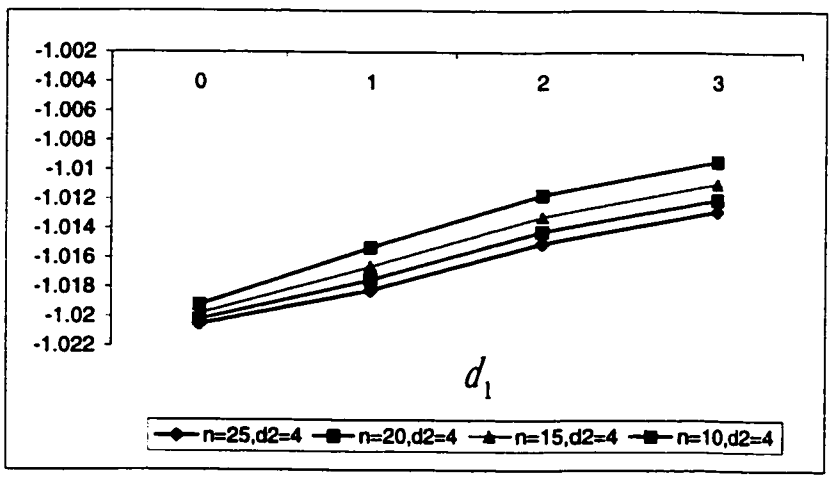

Figure 5.20: Rate of change of Expected Profit w.r.t c vs d_1

5.6 Conclusion

A three-class targeting model is developed in this chapter by incorporating inspection error in the sampling plan. Two cases are examined for inspection i.e with and without destructive testing. Sensitivity analysis is conducted to study the effect of type I and type II errors on expected profit and optimal mean and the effect of cost parameters on expected profit is also considered in this chapter.

Chapter 6

Conclusion and Future Research

6.1 Summary

In this thesis, relevant literature in the area of process targeting is reviewed, three different models of process targeting with and without inspection error in the sampling plan are developed.

Chapter 1 provides a brief introduction to quality control and state the problem under consideration in this thesis. Literature in the area of process targeting is reviewed along with Boucher and Jafari [1], Lee and Jang [4] models are presented in chapter 2. A process targeting model for a single filling operation considering inspection error in the sampling plan is developed in chapter 3. This model extends the work done by Boucher and Jafari [1] to the case where the inspection process is assumed to be error prone. Sensitivity analysis to the developed model is conducted to study the effect of model parameters on its solution. A three-class targeting problem under acceptance sampling is developed and sensitivity analysis with respect to cost parameters are presented in chapter 4. A three-class targeting model considering acceptance sampling and inspection error is developed and sensitivity analysis of the developed model is conducted with respect to error and cost parameters in chapter 5.

Thus, the work done in this thesis can be summarized as follows:

- Three models of process targeting are developed.
- In the first extension, two types of inspection errors are introduced in the model of Boucher and Jafari [1].
- In the second extension, Sampling inspection is introduced instead of 100 percent inspection in the model developed by Lee and Jang [4].
- In the third extension, the model developed in chapter 4 is extended to the case where inspection error is present.

6.2 Recommendations for Future Research

The research work in this thesis can be extended to the following areas.

- The three class targeting model could be extended to the case under asymmetric cost.
- The three class targeting problem could be extended to the case of joint economic selection of optimum mean and variance, by relaxing the assumption that the variance is known and fixed in this thesis.
- This work can be extended to the case of two machines in series with different process mean, by incorporating inspection error.
- Process targeting models can be developed considering inspection error for a single filling operations with rectifying inspection.
- The developed models can be extended to the case where the process deteriorates with time.
- A targeting model can be developed by relaxing the assumption that the sampling plan is known, The problem will be casted as the joint determination of the optimal process targets and inspection plan parameters.
- The models of Boucher and Jafari [1], the extension in this thesis and the three class targeting models can be extended by considering different cost function such as Taguchi quadratic loss function, or nonlinear cost function.

- The model of Boucher and Jafari [1] and the extension in this thesis could be extended to reflect the effect of type I and type II errors in the objective function.

Appendix A

Computer Code for Jafari model [1]

```
USE MSIMSL
     implicit none
     real:: z,p,s2,t2,phi,a1,Ans,\sigma,mu,ep,a2,c
     integer:: n,d0,j4,m,flag
     open(unit=6, file='amjay.txt',status='unknown',access='append')
     z = 0.2
     Ans=0.0
     flag=1
     do while (flag /= 0)
     z=z+0.001
     p = ANORDF(-z)
!    print*,"p",p
     sigma=0.00563
```

```
      m=100
      a2=30.5
      a1=67.5
      c=55
      d2=2
      n=20
      s2=0
      t2=0
!     print*,"st2",st2
      do 80 j4=d0+1,n
      s2=(fac(n)/(fac(j4)*fac(n-j4)))*((p**j4)*(1-p)**(n-j4))
      t2=t2+s2
80    continue
      print*,t2
      phi=0.3989672*exp(-z**2/2)
      Ans=(phi/sigma)*a1*((fac(n)/(fac(n-1-d0)*fac(d0)))*(P**d0*(1-P)**(n-1-d0)))
      mu=z*sigma+1
      ep=(a1)-(a2*t2)-(c*mu)
      print*,z
      print*,Ans
      print*,mu
      print*,ep
      if ((Ans  <=  56) .and. (Ans  > 54)) flag=0
      enddo
      end program
```

Appendix B

Computer Code for Objective I of the Thesis

```
program main
implicit none
real::a,b,lemda,meu,anew,bnew,lemdanew,meunew, alpha,f_lemda,f_meu
double precision ::epsilon
  integer::k,i
  a=1.0
  b=2.0
  alpha=0.618
  epsilon=0.0001
  lemda=a+(1-alpha)*(b-a)
  meu=a+alpha*(b-a)
  k=1
 do while( abs(b-a) >= epsilon)
```

```
        call get_objval(meu,f_meu)
    call get_objval(lemda,f_lemda)
    if(f_lemda<f_meu) then
        anew=lemda
        bnew=b
        lemdanew=meu
        meunew=anew+alpha*(bnew-anew)
    else
        anew=a
        bnew=meu
        meunew=lemda
        lemdanew=anew+(1-alpha)*(bnew-anew)
    endif
    k=k+1
    a=anew
    b=bnew
    lemda=lemdanew
    meu=meunew
   print 6,a,b,lemda,meu,f_lemda,f_meu
 6 format(tr2,f8.4,tr2,f8.4,tr2,f8.4,tr2,f8.4,tr2,f8.4,tr2,f8.4)
enddo
end program main

subroutine get_objval(x,f)
use MSIMSL
 implicit none
```

```
  real,intent(in)::x
  real,intent(out)::f
  real::z,l,sigma,a1,a2,c,p,s2,t2
  integer::j1,n,d0
  sigma=0.00563
  a1=67.5
  a2=30.5
  l=1.0
  c=55
  n=20
  d0=0
  s2=0
  t2=0
  z=(x-l)/sigma
  p=ANORDF(-z)
  do 80 j1=d0+1,n
  s2=(fac(n)/(fac(j1)*fac(n-j1)))*((p**j1)*(1-p)**(n-j1))
  t2=t2+s2
  80 continue
    f = a1-(a2*t2)-(c*x)
end subroutine get_objval
```

Appendix C

Computer Code for Objective III for the Thesis

```
    USE MSIMSL
    implicit none
    real:: z,p,s2,L,t2,s3,t3,t5,s5,phi,a1,Ans,sigma,mu,ep,a2,r,c
    integer:: n,d0,d1,j3,j4,j5,flag
! REAL
      ANORDF,A,P,N,c,c1,c2,l,sigma,e1,e2,n,d0,alpha,beeta,xe,Pe,i,j
      open(unit=6, file='amjay.txt',status='unknown',access='append')
    z = 0.1
    Ans=0.0
    flag=1
    do while (flag /= 0)
    z=z+0.001
    p= ANORDF(-z)
```

```
!    print*,"p",p
     sigma=0.00563
     a1=67.5, a2=30.5, r=10
     c=55
     d0=4
     d1=2
     n=20
     L=1
     s2=0
     t2=0
     s3=0
     t3=0
     t5=0
     s5=0
!    print*,"st2",st2
     do 80 j4=d0+1,n
     s2=(fac(n)/(fac(j4)*fac(n-j4)))*((p**j4)*(1-p)**(n-j4))
     t2=t2+s2
80   continue !print*,t2
 do 90 j3=0,d1
     s3=(fac(n)/(fac(j3)*fac(n-j3)))*((p**j3)*(1-p)**(n-j3))
     t3=t3+s3
90   continue !print*,t3
 do 100 j5=d1+1,d0
     s5=(fac(n)/(fac(j5)*fac(n-j5)))*((p**j5)*(1-p)**(n-j5))
     t5=t5+s5
```

```
100 continue !print*,t5
  phi=0.3989672*exp(-z**2/2)
  Ans=(phi/sigma)*((fac(n)/(fac(n-1-d1)*fac(d1)))*(P**d1*(1-P)**(n-1-d1)*(a2-r))-
  (fac(n)/(fac(n-1-d0)*fac(d0)))*(P**d0*(1-P)**(n-1-d0)*(a1-r)))
    mu=z*sigma+L
    ep=(a1*t3)+(a2*t5)+(r*t2)-(c*mu)


    print 4,z,Ans,mu,ep
    4 format(tr2,f8.4,tr2,f8.4,tr2,f8.4,tr2,f8.4)
    if ((Ans  <= 55.5) .and. (Ans  > 54.5)) flag=0
    enddo ! do while ((Ans .le. 5501) .and. (Ans .ge.5499))
    end program
```

Appendix D

Computer Code for Objective IV for the Thesis

```
program main
 implicit none
 real::a,b,lemda,meu,anew,bnew,lemdanew,meunew, &
  alpha,f_lemda,f_meu
    double precision ::epsilon
    integer::k
    a=1.0
    b=2.0
    alpha=0.618
    epsilon=0.00001
    lemda=a+(1-alpha)*(b-a)
    meu=a+alpha*(b-a)
    k=1
```

```
      do while( abs(b-a) >= epsilon)

          call get_objval(meu,f_meu)

          call get_objval(lemda,f_lemda)

          if(f_lemda<f_meu) then

              anew=lemda

              bnew=b

              lemdanew=meu

              meunew=anew+alpha*(bnew-anew)

          else

              anew=a

              bnew=meu

              meunew=lemda

              lemdanew=anew+(1-alpha)*(bnew-anew)

          endif

          k=k+1

          a=anew

          b=bnew

          lemda=lemdanew

          meu=meunew


          print 6,a,b,lemda,meu,f_lemda,f_meu

      6 format(tr2,f8.4,tr2,f8.4,tr2,f8.4,tr2,f8.4,tr2,f10.4,tr2,f10.4)

      enddo

  end program  main
```

```
subroutine get_objval(x,f)
 use MSIMSL
 implicit none
 real,intent(in)::x
 real,intent(out)::f
 real::z,l,sigma,a1,a2,c,p,s2,t2,pe,e,e2,s3,t3,s4,t4,r
  integer::j1,j2,j3,n,d2,d1,m
 sigma=0.00563
 a1=67.5
 a2=37
 r=10
 l=1.0
 c=55
 n=15
 d1=0
 d2=4
 s2=0
 t2=0
 s3=0
 t3=0
 s4=0
 t4=0
 e=0.0
 e2=0.0
 z=(x-l)/sigma
 p=ANORDF(-z)
```

```
pe=p*(1-e2)+(1-p)*e
do 80 j1=d2+1,n
s2=(fac(n)/(fac(j1)*fac(n-j1)))*((pe**j1)*(1-pe)**(n-j1))
t2=t2+s2
80 continue
do 90 j2=0,d1
s3=(fac(n)/(fac(j2)*fac(n-j2)))*((pe**j2)*(1-pe)**(n-j2))
t3=t3+s3
90 continue
do 70 j3=d1+1,d2
s4=(fac(n)/(fac(j3)*fac(n-j3)))*((pe**j3)*(1-pe)**(n-j3))
t4=t4+s4
70 continue
print *,t4
print *,t3
print *,t2

f=(a1*t3)+(a2*t4)+(r*t2)-(c*x)

end subroutine get_objval
```

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Vita

Mohammed Amjad Ali Siddiqui is a graduate student in Systems Engineering Department at King Fahd University of Petroleum and Minerals(KFUPM), Dhahran. He received his bachelor's degree in Mechanical Engineering in 1999 from Osmania University. He was associated with L.G.air-conditioners as a HVAC Engineer for a year. He joined the Department of Systems Engineering, King Fahd University of Petroleum and Minerals (KFUPM), as a Research Assistant in Fall 2000.