New Flavour Symmetry and NonLeptonic Decays of D and B Mesons

by

Farook Mohammad Fathi Al-Shamali

A Thesis Presented to the

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In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

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King Fahd University of Petroleum and Minerals (Saudi Arabia), 1992

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Ann Arbor, MI 48106

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COLLEGE OF GRADUATE STUDIES

This thesis, written by FAROOK MOHAMMAD FATHI AL-SHAMALI under the direction of his Thesis Advisor and approved by his Thesis Committee, has been presented to and accepted by the Dean of the College of Graduate Studies, in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE.

Thesis Committee

Thesis Advisor (Professor Riazuddin)

Member (Professor H. A. Mavromatis)

Member (Dr. H. Bahlouli)

Department Chairman

Dean, College of Graduate Studies

2-1-92

Date

إلي أبي و أمي

مع حبي وتقديري

To My Parents

With Love and Gratitude

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ABSTRACT

NAME: FAROOK MOHAMMAD FATHI AL-SHAMALI

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The form factors of the current matrix elements corresponding to the transitions $D \rightarrow K, K^*$ are available from two experimental measurements and two lattice calculations. These values have been used to study several nonleptonic D^0 decays that admit factorization and contain a strange meson (K, K^*) in the final state.

Furthermore, these form factors have been used to calculate the corresponding ones for $\overline{B}^0$ decays using flavour symmetry. This enabled us to study two factorizable B decays involving a strange meson.

Our results for the branching ratios provide a support for the factorization hypothesis. Also, flavour symmetry seems to be working very well.

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December 1992

الخلاصه

الإسم : فاروق محمد فتحي الشمالي

العنوان : تناظر جديد لنكهات الكوارك و التفكك غير الليبتوني للميزونات D و B

التخصص : فيزياء

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إن معاملات التركيب لعناصر مصفوفه التيار للتحول (D --> K, K^*)، متوفره في قياسات تجريبتين، و في حسابين مستقلين بطريقه المنظومه. وقد إستعملت هذه القيم لدراسه التفككات المختلفه غير الليبتونيه للميزون $D\dagger$ و التي تقبل التحليل و تحتوي على ميزون غريب (K, K^*) في الحاله النهائيه.

هذا و قد تم إستعمال هذه المعاملات لحساب المعاملات المقابله لتفككات الميزون B بإستخدام تناظرات نكهات الكوارك. وقد مكّننا هذا من دراسه تفككين للميزون B قابلين للتحليل و يحوي كلاً منهما ميزوناً غريباً.

إن نتائجنا لنسب التفرع تؤيد فرضيه التحليل ، وقد بدا أيضاً أن تناظرات نكهات الكوارك مناسبه للإستخدام في مثل هذه الحسابات.

درجه الماجستير في العلوم

جامعه الملك فهد للبترول والمعادن

الظهران ، المملكه العربيه السعوديه

رجب ، ١٤١٣هجريه

Chapter 1

Introduction

1.1 Classifications

Particles

A Few decades ago the only known particles were protons, neutrons and electrons. They were considered to be the elementary particles from which all matter (atoms) is built.

Soon after, the number of known particles started to increase. More particles were discovered in cosmic rays and then in high energy accelerators. So, it was not reasonable to consider all these particles to be elementary. And a way of organizing and classifying what was called the particles zoo became

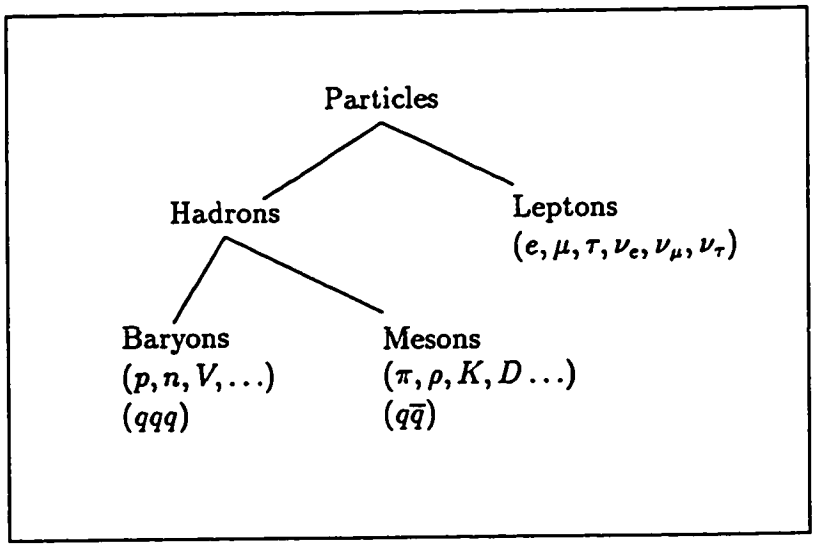

Figure 1.1: Particles classification.

necessary.

Now, particles are classified into two major groups: those which participate in strong interactions and called hadrons (from a Greek word for strong) and those which do not participate in strong interactions and called leptons. Furthermore, hadrons are divided into two subgroups: the first consists of baryons which have half-integral spins and obey Fermi statistics; the second consists of mesons which have integral spins and obey Bose statistics [24], see figure 1.1.

Among these categories, only leptons are elementary; whereas hadrons are believed to be composites of more fundamental particles called quarks.

Confidence in the quark theory is growing even though free quarks have never been observed and some people believe they will not be.

According to the quark model, quarks come in six flavours, up(u), down(d), strange(s), charm(c), bottom(b) and top(t). All baryons are composed of three quarks and antibaryons of three antiquarks. For example, a proton is composed of two up quarks and one down quark (uud) in the sense that this combination has the same total quantum numbers as those of the proton. On the other hand, all mesons are composed of quark-antiquark pairs. For example, π^+ is composed of an up quark and an antidown quark ($u\bar{d}$).

Mesons

Table 1.1 lists and summarizes some properties of a number of mesons that will show up through this study [25]. By looking at the table, we can make some distinctions between different groups of mesons. One group involves light mesons which are composed of up and down quarks only and have relatively low masses. Because of their low rest energies, they are the most abundant (easiest to produce in accelerators) and therefore the most thoroughly studied.

Other groups are strange mesons (contain an s quark), charmed mesons

Table 1.1: A List of mesons and their properties.

Particle	Quark Composition	$I(J^P)$	Mass (MeV)	Mean Life 10^{-13} (sec)
π^+, π^-	$u\bar{d}, \bar{u}d$	$1(0^-)$	139.5679 ± 0.0007	
π^0	$(u\bar{u} - d\bar{d})/\sqrt{2}$	$1(0^-)$	134.9743 ± 0.0008	
ρ^+, ρ^-	$u\bar{d}, \bar{u}d$	$1(1^-)$	768.1 ± 0.5	
ρ^0	$(u\bar{u} - d\bar{d})/\sqrt{2}$	$1(1^-)$	$\sim$	
A_1^+	$u\bar{d}$	$1(1^+)$	1260 ± 30	
K^+, K^-	$u\bar{s}, \bar{u}s$	$\frac{1}{2}(0^-)$	493.646 ± 0.009	
$K^0, \overline{K}^0$	$d\bar{s}, \bar{d}s$	$\frac{1}{2}(0^-)$	497.671 ± 0.031	
K^{*+}, K^{*-}	$u\bar{s}, \bar{u}s$	$\frac{1}{2}(1^-)$	891.59 ± 0.24	
$K^{*0}, \overline{K}^{*0}$	$d\bar{s}, \bar{d}s$	$\frac{1}{2}(1^-)$	896.10 ± 0.28	
ϕ	$s\bar{s}$	$0(1^-)$	1019.413 ± 0.008	
D^+, D^-	$c\bar{d}, \bar{c}d$	$\frac{1}{2}(0^-)$	1869.3 ± 0.5	10.66 ± 0.23
$D^0, \overline{D}^0$	$c\bar{u}, \bar{c}u$	$\frac{1}{2}(0^-)$	1864.5 ± 0.5	4.20 ± 0.08
D^{*+}, D^{*-}	$c\bar{d}, \bar{c}d$	$\frac{1}{2}(1^-)$	2010.1 ± 0.6	
$D^{*0}, \overline{D}^{*0}$	$c\bar{u}, \bar{c}u$	$\frac{1}{2}(1^-)$	2007.1 ± 1.4	
J/ψ	$c\bar{c}$	$0(1^-)$	3096.93 ± 0.09	
B^+, B^-	$u\bar{b}, \bar{u}b$	$\frac{1}{2}(0^-)$	5278.6 ± 2.0	(Average) 12.9 ± 0.5
$B^0, \overline{B}^0$	$d\bar{b}, \bar{d}b$	$\frac{1}{2}(0^-)$	5278.7 ± 2.1	$\tau_{B^+}/\tau_{B^0} = 0.93 \pm 0.16$

(contain a c quark) and bottom mesons (contain a b quark). Among these, charmed and bottom mesons are the heaviest. This is why they are called heavy mesons and c and b quarks are called heavy flavors.

Finally, it is believed that top quarks are the heaviest of all flavors. But, till now no topped particle has been discovered.

Decays

In high energy accelerators heavy mesons are produced. Shortly, and under weak interactions, they decay into lighter mesons and/or leptons through many decay channels, see figure 1.2.

In particle physics we distinguish between three kinds of weak decays:

- Leptonic: which involve only leptons in the initial and final states (e.g. $\mu^+ \rightarrow e^+ \nu_e \overline{\nu}_\mu$).
- Semileptonic: which involve both leptons and hadrons in the initial and final states (e.g. $\pi^+ \rightarrow \mu^+ \overline{\nu}_\mu$).
- Nonleptonic: which involve only hadrons in the initial and final states (e.g. $K^0 \rightarrow \pi^+ \pi^-$).

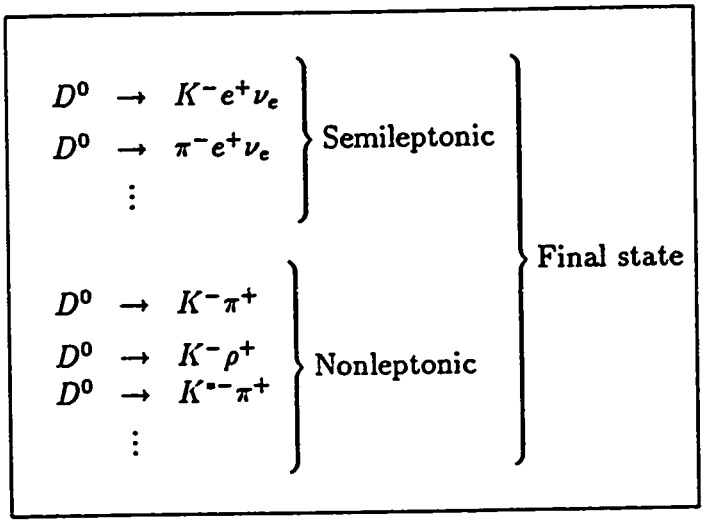

Figure 1.2: Some decay channels of D^0.

Now we can become more specific about the working area of this study. It will be concerned – as the title says – about nonleptonic decays of D and B mesons.

1.2 Flavour Symmetry

Recently, it was shown that hadrons containing a single heavy quark exhibit a new flavour-spin symmetry of QCD. This symmetry resulted from approximating the heavy quarks as essentially static sources of color in the Heavy Quark Effective Theory (HQET).

The $\overline{B}$ meson has the quantum number of a b quark ($B = 1$). However, it is inaccurate to picture it as a bound state of the heavy b quark and a light (u or d) antiquark. Rather, it should be thought of as a bound state of a b quark, a light antiquark, and a rather complicated superposition of gluons and additional light quark-antiquark pairs. Calculating the explicit form of this complicated superposition (known in the HQET trade as "brown muck") requires solution of non-perturbative dynamics [9].

So, according to HQET, the physics of heavy mesons is appropriately described by replacing the heavy quark by a static source of color. In this limit, the mass of the heavy quark becomes irrelevant to the dynamics. Quarks with different quantum numbers and large masses, like b and c quarks, form bound states in which the light degrees of freedom (the "brown muck") is in the same state. This means there is a symmetry which relates amplitudes involving states with $B = 1$ to states with $C = 1$, where B and C are bottom and charm quantum numbers, respectively.

An atomic physics analogy can be drawn which may clarify the ideas. In the analogy, electromagnetic interactions play the role of QCD, the nucleus that of the heavy quark, and the electrons and electromagnetic field that of the light degrees of freedom. The flavour symmetry, simply states that

different isotopes of the same element have the same chemical properties [9].

1.3 Transition Matrix Elements

For any transition, from an initial to a final state, we are usually concerned with evaluating the following matrix element:

$$\langle \text{ final state } |\mathcal{H}| \text{ initial state } \rangle.$$

Here, $\mathcal{H}$ is the interaction Hamiltonian which is responsible for the transformation.

In our work, we are studying nonleptonic weak decays of D and B mesons. The matrix elements (or decay amplitudes) for these decays are

$$\langle \text{ final } |\mathcal{H}^D_{eff}|D\rangle \text{ and } \langle \text{ final } |\mathcal{H}^B_{eff}|B\rangle.$$

If we use the effective dominant Hamiltonians and try to evaluate the above amplitudes, we find that some of them admit factorization [see below]. Examples are ($D^0 \to K^-\pi^+, D^0 \to K^{*-}\pi^+$ and $\overline{B}^0 \to K^0\psi$). After factorization we end up with current matrix elements of the form

$$\langle P(p')|V_\mu|M(p)\rangle,$$

$$\langle V(p')|J_\mu|M(p)\rangle,$$

$$\langle 0|A_\mu|P(p)\rangle$$

and

$$\langle 0|V_\mu|V(p')\rangle.$$

$|M(p)\rangle$ stands for $|D\rangle$ or $|B\rangle$ mesons, whereas $|P(p)\rangle$ and $|V(p')\rangle$ represent the relevant pseudoscalar and vector mesons respectively. For such matrix elements, we substitute the form factor decomposition defined in Chapter 2 which enables us to calculate the amplitude for each exclusive decay.

As an example, consider the decay channel $D^0 \to K^-\pi^+$. The effective Hamiltonian for D decays is

$$\mathcal{H}^D_{eff} = \frac{G_F}{\sqrt{2}} a_1 V^*_{sc} V_{ud} \bar{u}\gamma_\mu(1-\gamma_5) d\bar{s}\gamma^\mu(1-\gamma_5)c.$$

After factorization, the amplitude is written as

$$A(D^0 \to K^-\pi^+) = -\frac{G_F}{\sqrt{2}} a_1 V^*_{cs} V_{ud} \langle \pi^+|\bar{u}\gamma^\mu\gamma_5 d|0\rangle\langle K^-|\bar{s}\gamma_\mu c|D^0\rangle,$$

which contains the contraction of a vector and an axial vector current matrix element.

Chapter 2

Form Factors

Our ultimate goal in this work is to calculate the decay rates of a number of decay processes. This will be the subject of the coming chapter. Before that we have to make the necessary step of preparing the form factors to be used in these calculations.

As mentioned in the previous chapter, we are interested in the processes where we have a pseudoscalar D^0 or $\overline{B}^0$ mesons in the initial state and strange mesons (K or K^*) in the final state. So, we need to evaluate the current matrix elements that correspond to the transitions $D^0 \to K, K^*$ and $\overline{B}^0 \to K, K^*$. These will be specified once their form factors are known.

In practical life B mesons are heavier than D mesons ($m_{B^0}/m_{D^0} \approx 2.8$)

and therefore more difficult to study. This is why we have less information available for B decays than D decays.

In this chapter we make use of the quark constituent of these two mesons in order to relate the form factors of their current matrix elements. Since both of them consist of a heavy quark and a light quark, then it is a suitable situation where we can apply and test flavour symmetry. More precisely, we will use the data available on the form factors of the $D^0 \to K, K^*$ transitions to calculate the corresponding ones for the $\overline{B}^0 \to K, K^*$ transitions.

2.1 First Decomposition

First, let us present the form factor decomposition of the hadronic current matrix elements as used in [4]

$$\begin{aligned}
\langle K|\bar{s}\gamma_\mu c|D\rangle &= f_+(t)(p_D + p_K)_\mu + f_-(t)(p_D - p_K)_\mu, \\
\langle K^*|\bar{s}\gamma_\mu\gamma_5 c|D\rangle &= f(t)\varepsilon_\mu + a_+(t)(\varepsilon \cdot p_D)(p_D + p_{K^*})_\mu \\
&\quad + a_-(t)(\varepsilon \cdot p_D)(p_D - p_{K^*})_\mu, \qquad (2.1) \\
\langle K^*|\bar{s}\gamma_\mu c|D\rangle &= ig(t)\varepsilon_{\mu\rho\tau\sigma}\varepsilon^\rho(p_D + p_{K^*})^\tau(p_D - p_{K^*})^\sigma,
\end{aligned}$$

where

$$t = q^2 = (p_D - p_K)^2 \text{ or } (p_D - p_{K^*})^2.$$

Flavour symmetry relates heavy quarks of the same four-velocity v but different mass (and hence different four-momentum) [4,5]. From this symmetry, it follows that

$$\begin{aligned}
(f_+ + f_-)^{\overline{B}\to K} &= \left[\frac{m_c}{m_b}\right]^{1/2}\left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25}(f_+ + f_-)^{D\to K},\\
(f_+ - f_-)^{\overline{B}\to K} &= \left[\frac{m_b}{m_c}\right]^{1/2}\left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25}(f_+ - f_-)^{D\to K},\\
(a_+ + a_-)^{\overline{B}\to K^*} &= \left[\frac{m_c}{m_b}\right]^{3/2}\left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25}(a_+ + a_-)^{D\to K^*}, \qquad (2.2)\\
(a_+ - a_-)^{\overline{B}\to K^*} &= \left[\frac{m_c}{m_b}\right]^{1/2}\left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25}(a_+ - a_-)^{D\to K^*},\\
g^{\overline{B}\to K^*} &= \left[\frac{m_c}{m_b}\right]^{1/2}\left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25}g^{D\to K^*},\\
f^{\overline{B}\to K^*} &= \left[\frac{m_b}{m_c}\right]^{1/2}\left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25}f^{D\to K^*}.
\end{aligned}$$

In the following calculations, we take [5]

$$\alpha_s(m_b) = 0.189 \quad \text{and} \quad \alpha_s(m_c) = 0.29.$$

Also, for quark masses we take [1]

$$m_c = 1.7 \text{ GeV} \quad \text{and} \quad m_b = 4.9 \text{ GeV}.$$

In equations (2.2), form factors are evaluated at the same four velocity transfer $v \rightarrow v'$. Therefore, $D \rightarrow K, K^*$ form factors at $t_D = (m_D v - p')^2$ are related to the $\overline{B} \rightarrow K, K^*$ form factors at $t_B = (m_B v - p')^2$. So,

$$\begin{aligned} t_B &= (m_B v_\mu - p'_\mu)(m_B v^\mu - p'^\mu) \\ &= m_B^2 + m_X^2 - 2m_B(v \cdot p'), \end{aligned}$$

where X is either K or K^*. Similarily,

$$t_D = m_D^2 + m_X^2 - 2m_D(v \cdot p').$$

By eleminating v we get

$$t_B = m_B^2 + m_X^2 - m_B m_D - \frac{m_B}{m_D}(m_X^2 - t_D). \tag{2.3}$$

2.2 Second Decomposition

In our work we are interested in a different form factor decomposition of the matrix elements. It is this decomposition which was used to construct the model in [1,2], and to experimentally measure the form factors in the semileptonic decays of $D^0 \rightarrow K^- e^+ \nu_e$ and $D^+ \rightarrow \overline{K}^{*0} e^+ \nu_e$.

This decomposition is given below:

$$\langle K|\bar{s}\gamma_\mu c|D\rangle = \left(p_D + p_K - \frac{m_D^2 - m_K^2}{q^2} q\right)_\mu f_1(t) + \frac{m_D^2 - m_K^2}{q^2} q_\mu f_0(t),$$

$$\langle K^*|\bar{s}\gamma_\mu\gamma_5 c|D\rangle = -(m_D+m_{K^*})A_1(t)\varepsilon_\mu + \frac{A_2(t)}{m_D+m_{K^*}}(\varepsilon\cdot q)(p_D+p_{K^*})_\mu + \frac{(\varepsilon\cdot q)}{q^2}2m_{K^*}q_\mu[A_3(t)-A_0(t)], \quad (2.4)$$

$$\langle K^*|\bar{s}\gamma_\mu c|D\rangle = -i\frac{2V(t)}{m_D+m_{K^*}}\varepsilon_{\mu\rho\tau\sigma}\varepsilon^\rho p_D^\tau p_{K^*}^\sigma,$$

where $q = p_D - p_K(p_{K^*})$, and $A_3(t)$ is defined to be

$$A_3(t) = \frac{m_D+m_{K^*}}{2m_{K^*}}A_1(t) - \frac{m_D-m_{K^*}}{2m_{K^*}}A_2(t). \quad (2.5)$$

At this moment let us relate the form factors in this decomposition with those in (2.1). For example, from (2.1) and (2.4) we can write

$$\begin{aligned} & f_+(p_D+p_K)_\mu + f_-(p_D-p_K)_\mu \\ &= \left(p_D+p_K-\frac{m_D^2-m_K^2}{q^2}q\right)_\mu f_1(t) + \frac{m_D^2-m_K^2}{q^2}q_\mu f_0(t) \\ &= f_1(t)(p_D+p_K)_\mu + \frac{m_D^2-m_K^2}{q^2}(f_0(t)-f_1(t))(p_D-p_K)_\mu. \end{aligned} \quad (2.6)$$

So, we get

$$f_+(t) = f_1(t), \text{ and } f_-(t) = \frac{m_D^2-m_K^2}{t}[f_0(t)-f_1(t)]. \quad (2.7)$$

Similarily, we can also prove that

$$a_+(t) = \frac{A_2(t)}{m_D+m_{K^*}}, \; a_-(t) = \frac{-2m_{K^*}}{t}[A_0(t)-A_3(t)], \quad (2.8)$$

$$g(t) = \frac{V(t)}{m_D+m_{K^*}}, \text{ and } f(t) = -(m_D+m_{K^*})A_1(t), \quad (2.9)$$

where the superscripts $D \to K(K^*)$ of the form factors are omitted.

In our work, single pole dominance for form factors is assumed. So, the t dependance is approximated as

$$F(t) = F(0)\left[\frac{1}{1 - t/M^2}\right], \tag{2.10}$$

where M is the relevant pole mass value. Also, we have the constraints

$$f_0(0) = f_1(0) \text{ and } A_0(0) = A_3(0).$$

The values of pole masses used in the numerical estimates are displayed in table 2.1.

In [5] Tanimoto used double pole approximation for A_2 and V. Actually, we tried both ways and we found no reasonable justification for his assumption.

Before we continue, let us find the values of $f_-(t)$ and $a_-(t)$ at zero momentum transfer. From (2.7), (2.8), (2.10) and table 2.1 we can prove after some simple algebra that

$$f_-(0) = (m_D^2 - m_K^2) f_1(0)\left[\left(\frac{1}{M_{c\bar{s}}(0^+)}\right)^2 - \left(\frac{1}{M_{c\bar{s}}(1^-)}\right)^2\right] \tag{2.11}$$

$$a_-(0) = 2m_{K^*} A_3(0)\left[\left(\frac{1}{M_{c\bar{s}}(1^+)}\right)^2 - \left(\frac{1}{M_{c\bar{s}}(0^-)}\right)^2\right]. \tag{2.12}$$

Table 2.1: Values of pole masses in (GeV).

Form factor	J^P	$M_{c\bar{s}}(J^P)$	$M_{b\bar{s}}(J^P)$	$M_{b\bar{c}}(J^P)$
f_0	0^+	2.60	5.89	6.80
f_1	1^-	2.11	5.43	6.34
A_0	0^-	1.97	5.38	6.30
A_1	1^+	2.53	5.82	6.73
A_2	1^+	2.53	5.82	6.73
A_3	1^+	2.53	5.82	6.73
V	1^-	2.11	5.43	6.34

2.3 Numerical Calculations

To find needed form factors either you construct a model and use it to calculate their values, or you get them from experiment. Bauer, Stech and Wirble (BSW) worked on a model in which they expressed the form factors of the hadronic current in terms of relativistic bound state wave functions for which they took solutions of relativistic harmonic oscillator [1,2]. From their results we can get values for all needed form factors; see Appendix B. These values are shown in table 2.2.

Experimental measurements of the form factors did not become available until 1990, when the E691 Collaboration measured the three form factors

Table 2.2: Predictions of form factors at $t = 0$ from the BSW model.

Transition	$f_0(0) = f_1(0)$	$A_0(0) = A_3(0)$	$A_1(0)$	$A_2(0)$	$V(0)$
$D \to K$	0.760				
$D \to K^*$		0.731	0.887	1.15	1.28
$B \to K$	0.381				
$B \to K^*$		0.322	0.329	0.331	0.370

governing the decay $D^+ \to \overline{K}^{*0} e^+ \nu_e$ in the Fermilab photoproduction experiment E691 [15,5].

Early this year, another experimental measurement of the same form factors conducted by a different group became available. It was done also in the Fermilab by the E653 Collaboration by studying the $D^+ \to \overline{K}^{*0} \mu^+ \nu_\mu$ decay. In this experiment only the A_2/A_1 and V/A_1 ratios have been measured [16]. In our study we will use the values given by Pham in [17] where he used a previous measurment of the branching ratio of $D^+ \to \overline{K}^{*0} e^+ \nu_e$ to fix the value of $A_1(0)$ and calculate the values of $A_2(0)$ and $V(0)$. Also, he gives a little smaller value for the $D \to K$ form factor $f_1(0)$ which will be considered to belong to the E653 Collaboration.

In addition to the two experimental results two recent studies (BES and ELC) of $D \to K, K^*$ decays using what is called *lattice calculations* are also

Table 2.3: Values of form factors at zero momentum transfer as produced by E691 and E653 experiments and by lattice calculations.

Group	Transition	$f_0(0) = f_1(0)$	$A_1(0)$	$A_2(0)$	$V(0)$
E691 [15]	$D \to K$	0.79 ± 0.078			
(exp.)	$D \to K^*$		0.46 ± 0.05	0.0 ± 0.2	0.9 ± 0.3
E653 [16]	$D \to K$	0.71 ± 0.06			
(exp.)	$D \to K^*$		0.53 ± 0.08	0.43 ± 0.15	1.06 ± 0.3
ELC [14]	$D \to K$	0.58 ± 0.04			
(lattice)	$D \to K^*$		0.53 ± 0.03	0.19 ± 0.21	0.86 ± 0.10
BES [12,13]	$D \to K$	0.90 ± 0.04			
(lattice)	$D \to K^*$		0.83 ± 0.14	0.59 ± 0.14	1.43 ± 0.45

available [12,13]. See table 2.3 for a list of these form factor values.

Using the assumption of single pole dominance, the values of these form factors can be calculated for any t.

In this section our aim is to calculate the form factors for the transitions $\overline{B} \to K, K^*$ at any momentum transfer. This can be done in two ways. In the first way , we use the relations (2.7 - 2.9, 2.11, 2.12) and the flavour symmetry relations (2.2) to relate $(f_0^{\overline{B}\to K}(t_B), f_1^{\overline{B}\to K}(t_B), A_0^{\overline{B}\to K^*}(t_B), \ldots)$ with $(f_0^{D\to K}(t_D), f_1^{D\to K}(t_D), A_0^{D\to K}(t_D), \ldots)$. The dependence of t_B on t_D is given

in (2.3). This way we have the form factors for $\overline{B} \rightarrow K, K^*$ at any t assuming, of course, single pole dominance dependence for $D \rightarrow K, K^*$.

In the second way, we calculate the form factors for $\overline{B} \rightarrow K, K^*$ at a particular momentum transfer as mentioned above. Then we use single pole dominance for $\overline{B} \rightarrow K, K^*$ form factors to give us the full t dependence. A suitable momentum is $t_B = 17.6(16.6)$ GeV, that corresponds to $t_D = 0$.

The form factors defined by both methods using E691 experimental results are plotted in figures 2.1–2.7, and it is clear that they cross at $t = 17.6$ or 16.6 GeV as they should. In particular, the values of form factors for $\overline{B} \rightarrow K, K^*$ at zero momentum transfer produced by the first way are shown in tables 2.4 and 2.5, and those produced by the second way are shown in tables 2.6 and 2.7. For the details of the calculations see Appendix A.

It is the second way that was used in [5] and will be used in this work to evaluate the branching ratios of the two studied $\overline{B}$ decays. However, the differences between the results of both ways are small (smaller than their uncertainties) and they are noticed only in few form factors in the region of interest ($t = m_\psi^2 \approx 10$ GeV). So, there is no strong point in using both ways in the coming calculations.

Table 2.4: Flavour symmetry prediction of form factors for B decays using the first method.

Form factor	E691	E653
$f_0(0)$	0.444 ± 0.044	0.399 ± 0.034
$f_1(0)$	0.444 ± 0.044	0.399 ± 0.034
$A_0(0)$	0.612 ± 0.306	0.190 ± 0.286
$A_3(0)$	0.612 ± 0.306	0.190 ± 0.286
$A_1(0)$	0.202 ± 0.022	0.233 ± 0.035
$A_2(0)$	0.035 ± 0.101	0.250 ± 0.079
$V(0)$	0.568 ± 0.189	0.669 ± 0.189

Table 2.5: Flavour symmetry prediction of form factors for B decays using the first method.

Form factor	ELC	BES
$f_0(0)$	0.326 ± 0.022	0.506 ± 0.022
$f_1(0)$	0.326 ± 0.022	0.506 ± 0.022
$A_0(0)$	0.478 ± 0.292	0.397 ± 0.354
$A_3(0)$	0.478 ± 0.292	0.397 ± 0.354
$A_1(0)$	0.233 ± 0.013	0.364 ± 0.062
$A_2(0)$	0.133 ± 0.105	0.351 ± 0.079
$V(0)$	0.543 ± 0.063	0.903 ± 0.284

Table 2.6: Flavour symmetry prediction of form factors for B decays using the second method.

Form factor	E691	E653
$f_0(0)$	0.323 ± 0.032	0.290 ± 0.024
$f_1(0)$	0.453 ± 0.045	0.407 ± 0.034
$A_0(0)$	0.619 ± 0.220	0.384 ± 0.222
$A_3(0)$	0.471 ± 0.266	0.081 ± 0.243
$A_1(0)$	0.198 ± 0.022	0.228 ± 0.034
$A_2(0)$	0.086 ± 0.097	0.287 ± 0.080
$V(0)$	0.576 ± 0.192	0.679 ± 0.192

Table 2.7: Flavour symmetry prediction of form factors for B decays using the second method.

Form factor	ELC	BES
$f_0(0)$	0.237 ± 0.016	0.368 ± 0.016
$f_1(0)$	0.333 ± 0.023	0.516 ± 0.023
$A_0(0)$	0.568 ± 0.201	0.666 ± 0.295
$A_3(0)$	0.338 ± 0.256	0.216 ± 0.294
$A_1(0)$	0.228 ± 0.013	0.356 ± 0.060
$A_2(0)$	0.183 ± 0.097	0.413 ± 0.087
$V(0)$	0.551 ± 0.064	0.916 ± 0.288

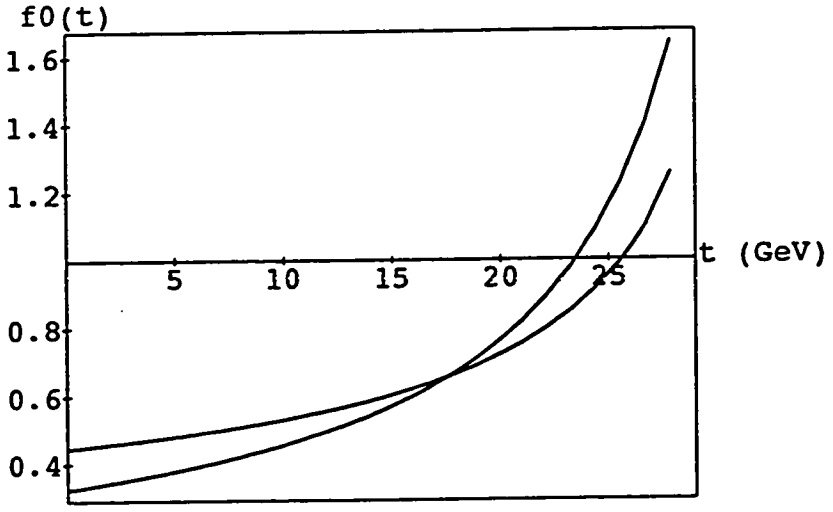

Figure 2.1: $f_0(t)$ for $\overline{B} \to K$ as generated by both methods using the results of E691 experiment.

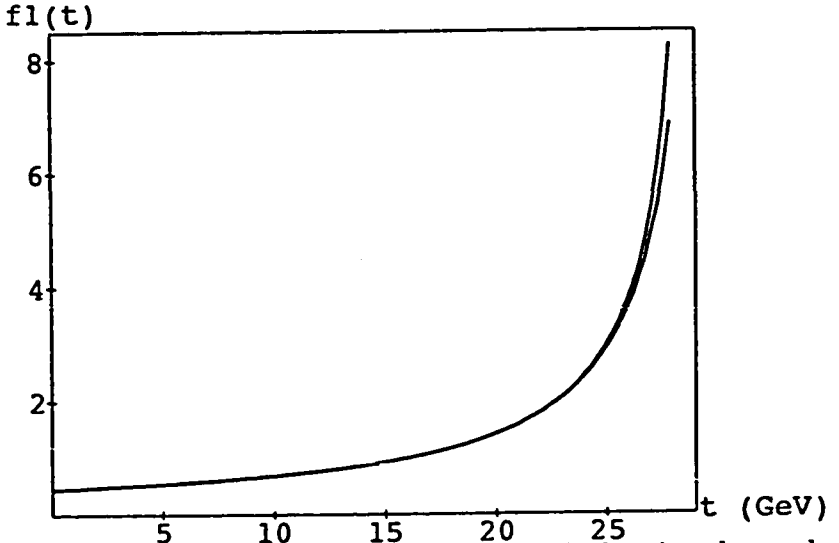

Figure 2.2: $f_1(t)$ for $\overline{B} \to K$ as generated by both methods using the results of E691 experiment.

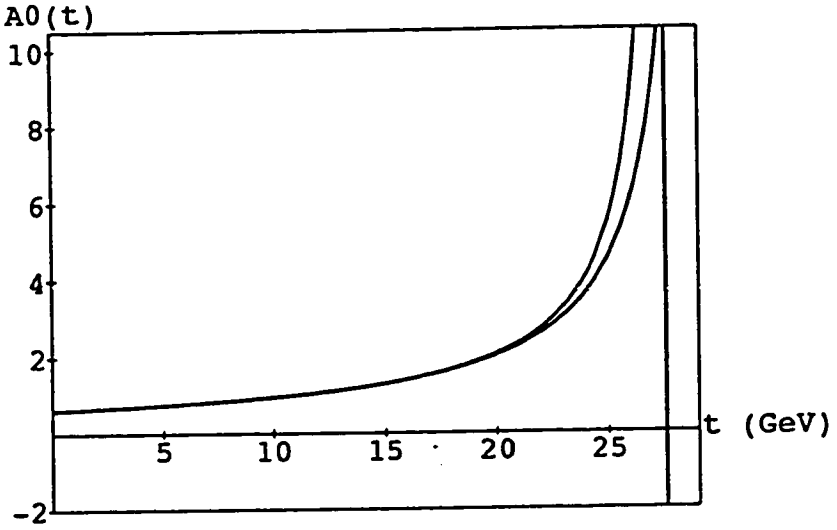

Figure 2.3: $A_0(t)$ for $\overline{B} \to K^*$ as generated by both methods using the results of E691 experiment.

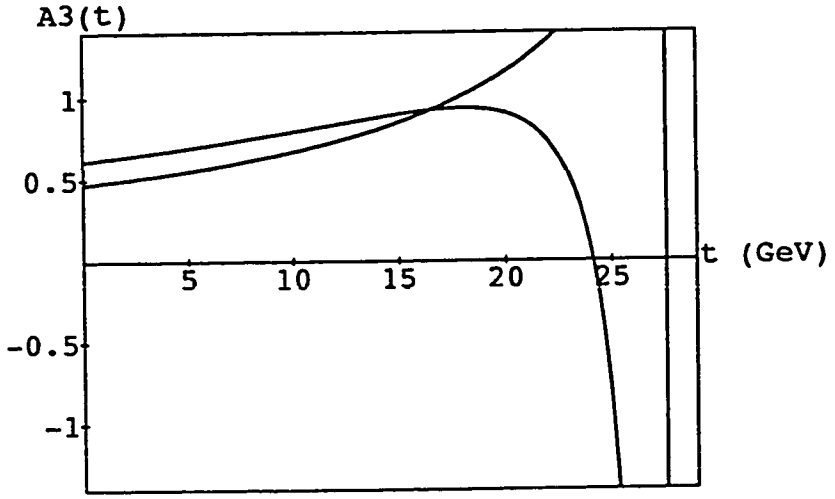

Figure 2.4: $A_3(t)$ for $\overline{B} \to K^*$ as generated by both methods using the results of E691 experiment.

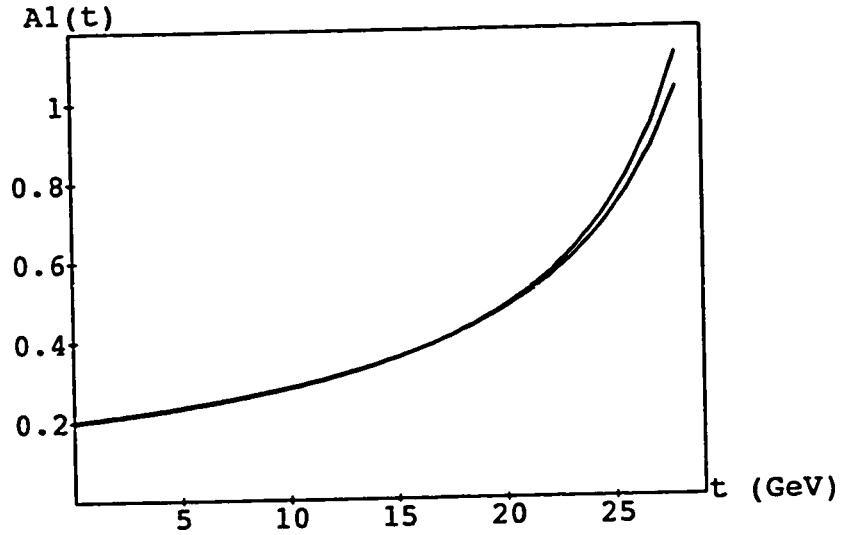

Figure 2.5: $A_1(t)$ for $\overline{B} \to K^*$ as generated by both methods using the results of E691 experiment.

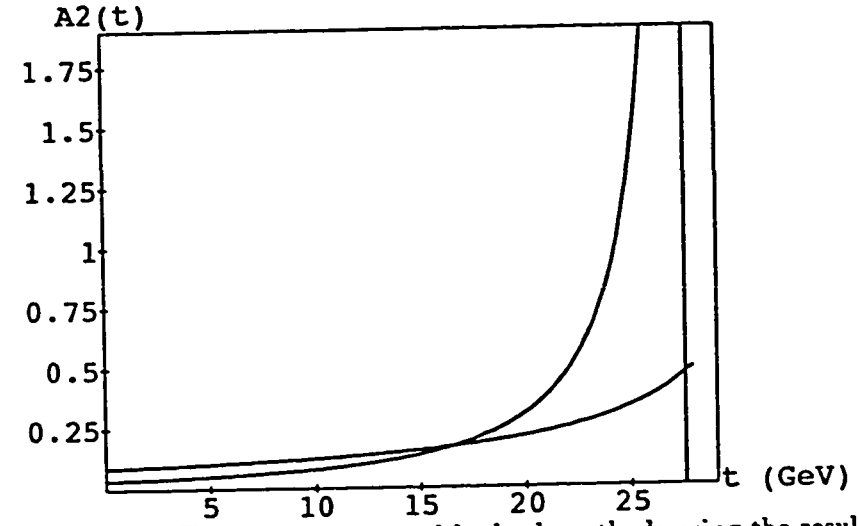

Figure 2.6: $A_2(t)$ for $\overline{B} \to K^*$ as generated by both methods using the results of E691 experiment.

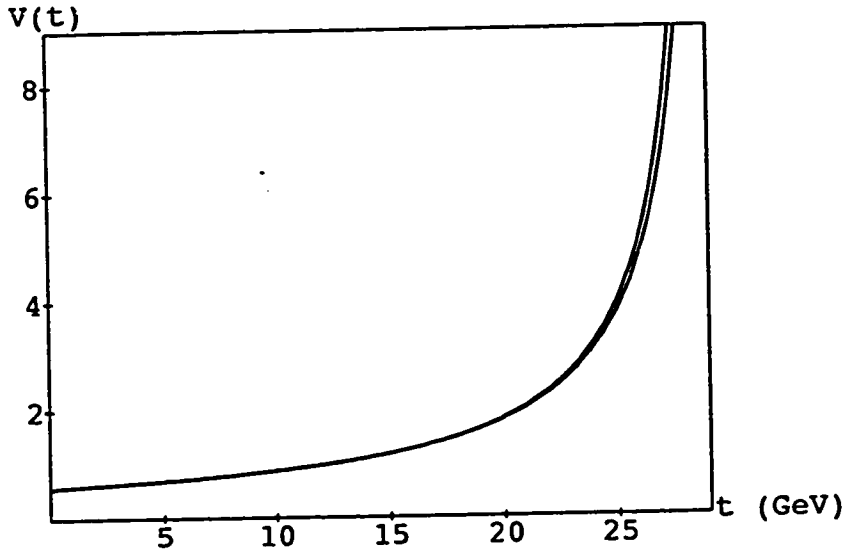

Figure 2.7: $V(t)$ for $\overline{B} \to K^*$ as generated by both methods using the results of E691 experiment.

Chapter 3

$1/N$ Expansion Method

In this chapter we will study nonleptonic decays of D and B mesons using form factors obtained in the previous chapter. For the study, we are going to use the $1/N$ expansion approach which leads to the factorization of the weak hadronic matrix elements and the neglect of final state interactions in the leading order.

3.1 Effective Hamiltonians

The effective dominant Hamiltonian for nonleptonic weak decays of D meson is given by:

$$\mathcal{H}^{D}_{eff} = \frac{G_F}{\sqrt{2}}\left(c_1 + \frac{c_2}{N_c}\right) V^{*}_{sc} V_{ud} \bar{u}\gamma_\mu(1-\gamma_5) d\bar{s}\gamma^\mu(1-\gamma_5)c. \qquad (3.1)$$

c_1 and c_2 are the Wilson coefficient functions which include short distance QCD corrections and depend only on the initial state. For the D meson, they have the values: $c_1 = 1.24$ and $c_2 = -0.47$.

The dominant part of the effective weak nonleptonic Hamiltonian for B meson is given by:

$$\mathcal{H}^{B}_{eff} = \frac{G_F}{\sqrt{2}}\left(c_2 + \frac{c_1}{N_c}\right) V^{*}_{sc} V_{bc} \bar{b}\gamma_\mu(1-\gamma_5) s\bar{c}\gamma^\mu(1-\gamma_5)c \qquad (3.2)$$

where the QCD coefficients have the values $c_1 = 1.1$ and $c_2 = -0.25$.

3.2 Decay Amplitudes

Let us start by presenting definitions of the pseudoscalar and vector couplings as:

$$\begin{aligned} \langle P(q)|\overline{Q}\gamma_\mu\gamma_5 Q|0\rangle &= f_P q_\mu, \\ \langle V(q,\varepsilon)|\overline{Q}\gamma_\mu Q|0\rangle &= g_V m_V \varepsilon_\mu. \end{aligned} \qquad (3.3)$$

3.2.1 $D \to PP$

We will study first the decay process $D^0 \to K^-\pi^+$. This is an example of $D \to PP$, where P represents a pseudoscalar meson.

By sandwiching $\mathcal{H}_{eff}^D$ between the initial and final states of this process and then applying factorization, the following form for the decay amplitude is obtained:

$$A(D^0 \to K^-\pi^+) = -\frac{G_F}{\sqrt{2}} a_1 V_{cs}^* V_{ud} \langle \pi^+ | \bar{u}\gamma^\mu\gamma_5 d|0\rangle \langle K^- | \bar{s}\gamma_\mu c | D^0\rangle, \qquad (3.4)$$

where

$$a_1 = \left(c_1 + \frac{c_2}{N_c}\right) = \left(1.24 - \frac{0.47}{N_c}\right).$$

By substitution from (2.1) and (3.3), we get:

$$A(D^0 \to K^-\pi^+) = -\frac{G_F}{\sqrt{2}} a_1 V_{cs}^* V_{ud} f_\pi (p_\pi)^\mu \times \left[f_+^{D-K}(t)(p_D + p_K)_\mu + f_-^{D-K}(t)(p_D - p_K)_\mu \right]. \qquad (3.5)$$

Remembering that

$$(p_\pi)_\mu = (p_D - p_K)_\mu$$

and

$$t = (p_D - p_K)^2 = m_\pi^2,$$

the decay amplitude becomes

$$\begin{aligned} & A(D^0 \to K^- \pi^+) \\ &= -\frac{G_F}{\sqrt{2}} a_1 V_{cs}^* V_{ud} f_\pi \left[(p_D^2 - p_K^2) f_+^{D\to K}(m_\pi^2) + (p_D - p_K)^2 f_-^{D\to K}(m_\pi^2) \right] \\ &= -\frac{G_F}{\sqrt{2}} a_1 V_{cs}^* V_{ud} f_\pi \left[(m_D^2 - m_K^2) f_+^{D\to K}(m_\pi^2) + m_\pi^2 f_-^{D\to K}(m_\pi^2) \right]. \end{aligned} \tag{3.6}$$

3.2.2 $D \to PV$

The second process we will study is $D^0 \to K^- \rho^+$. This is an example of $D \to PV$, where V represents a vector meson.

The amplitude for this decay after factorization is

$$\begin{aligned} & A(D^0 \to K^- \rho^+) \\ &= \frac{G_F}{\sqrt{2}} a_1 V_{cs}^* V_{ud} \langle \rho^+ | \bar{u} \gamma^\mu \gamma_5 d | 0 \rangle \langle K^- | \bar{s} \gamma_\mu c | D^0 \rangle \\ &= \frac{G_F}{\sqrt{2}} a_1 V_{cs}^* V_{ud} (g_\rho m_\rho \varepsilon^\mu) \\ &\quad \times \left[f_+^{D\to K}(t)(p_D + p_K)_\mu + f_-^{D\to K}(t)(p_D - p_K)_\mu \right]. \end{aligned} \tag{3.7}$$

Since ε_μ is the polarization vector for ρ^+ meson, then

$$t = (p_D - p_K)^2 = m_\rho^2,$$

$$\varepsilon^\mu \cdot (p_D + p_K)_\mu = \varepsilon^\mu \cdot (2p_D - p_\rho)_\mu = 2\varepsilon^\mu \cdot (p_D)_\mu = 2\varepsilon^\mu \cdot (p_K)_\mu,$$

and

$$\varepsilon^{\mu} \cdot (p_D - p_K)_{\mu} = \varepsilon^{\mu} \cdot (p_\rho)_{\mu} = 0. \tag{3.8}$$

Also, we should remember that the square of the amplitude $|A(D^0 \to K^- \rho^+)|$ is summed over all polarization vectors.

So, from (3.8) and (3.7), the square of the amplitude becomes,

$$|A(D^0 \to K^- \rho^+)|^2 = \left|\frac{G_F}{\sqrt{2}} a_1 V_{cs}^* V_{ud} g_\rho m_\rho\right|^2 \left|2 f_+^{D-K}(m_\rho^2)\right|^2 \sum_{r=1}^{3} |p_K \cdot \varepsilon|^2, \tag{3.9}$$

where the polarization index (r) has been omitted.

Let us evaluate the summation part of (3.9).

$$\begin{aligned}
\sum_{r=1}^{3} |p_K \cdot \varepsilon|^2 &= (p_K)_\mu (p_K)_\nu \sum_{r=1}^{3} \varepsilon^\mu \varepsilon^\nu \\
&= (p_K)_\mu (p_K)_\nu \left[-g^{\mu\nu} + \frac{(p_\rho)^\mu (p_\rho)^\nu}{m_\rho^2}\right] \\
&= -p_K^2 + \frac{(p_K \cdot p_\rho)^2}{m_\rho^2} \\
&= -p_K^2 + \frac{(p_D^2 - p_K^2 - p_\rho^2)^2}{4m_\rho^2} \\
&= \frac{(m_D^2 - m_K^2 - m_\rho^2)^2 - 4m_K^2 m_\rho^2}{4m_\rho^2}.
\end{aligned} \tag{3.10}$$

In order to put the amplitude in a more compact form we can use conservation of energy to write,

$$E_D = E_\rho + E_K$$

so,

$$m_D = (|\mathbf{p}|^2 + m_\rho^2)^{1/2} + (|\mathbf{p}|^2 + m_K^2)^{1/2}.$$

With little algebra we can show that

$$\frac{(m_D^2 - m_K^2 - m_\rho^2)^2 - 4m_K^2 m_\rho^2}{4m_\rho^2} = |\mathbf{p}|^2 \frac{m_D^2}{m_\rho^2},$$

where $\mathbf{p}$ is a three vector such that

$$|\mathbf{p}|^2 = |\mathbf{p}_\rho|^2 = |\mathbf{p}_K|^2.$$

So, now we can write,

$$\sum_{r=1}^{3} |p_K \cdot \varepsilon|^2 = |\mathbf{p}_\rho|^2 \frac{m_D^2}{m_\rho^2},$$

and

$$\sqrt{\sum_{r=1}^{3} |p_K \cdot \varepsilon|^2} = |\mathbf{p}_\rho| \frac{m_D}{m_\rho}. \tag{3.11}$$

By substitution of (3.11) in (3.9) we end up with the decay amplitude as,

$$A(D^0 \to K^- \rho^+) = \frac{G_F}{\sqrt{2}} a_1 V_{cs}^* V_{ud} m_D |\mathbf{p}_\rho| 2 g_\rho f_+^{D-K}(m_\rho^2), \tag{3.12}$$

where

$$|\mathbf{p}_\rho| = \left[\frac{(m_D^2 - m_K^2 - m_\rho^2)^2 - 4m_K^2 m_\rho^2}{4m_D^2} \right]^{1/2}. \tag{3.13}$$

3.2.3 $D \to VP$

The third process we will study is $D^0 \to K^{*-}\pi^+$. This is an example of $D \to VP$.

The amplitude for this process after factorization is

$$\begin{aligned}
&A(D^0 \to K^{*-}\pi^+) \\
&= -\frac{G_F}{\sqrt{2}} a_1 V_{cs}^* V_{ud} \langle \pi^+ | \bar{u}\gamma^\mu\gamma_5 d|0\rangle \langle K^{*-}|\bar{s}\gamma_\mu(1-\gamma_5)c|D^0\rangle \\
&= -\frac{G_F}{\sqrt{2}} a_1 V_{cs}^* V_{ud} f_\pi (p_\pi)^\mu \Big[ig^{D\to K^*}(t)\varepsilon_{\mu\rho\tau\sigma}\varepsilon^\rho (p_D + p_{K^*})^\tau (p_D - p_{K^*})^\sigma \\
&\quad - f^{D\to K^*}(t)\varepsilon_\mu - a_+^{D\to K^*}(t)(\varepsilon\cdot p_D)(p_D + p_{K^*})_\mu \\
&\quad - a_-^{D\to K^*}(t)(\varepsilon\cdot p_D)(p_D - p_{K^*})_\mu \Big] . \qquad (3.14)
\end{aligned}$$

The axial vector part when it is contracted with p_π will give us zero. Also, the square of the amplitude is summed over all polarization vectors. So,

$$\begin{aligned}
|A(D^0 \to K^{*-}\pi^+)|^2 &= \left(\frac{G_F}{\sqrt{2}} a_1 V_{cs}^* V_{ud} f_\pi\right)^2 \\
&\times \sum_{r=1}^{3} \Big| (p_\pi\cdot\varepsilon) f^{D\to K^*}(m_\pi^2) + (p_D\cdot\varepsilon)(p_D^2 - p_{K^*}^2) a_+^{D\to K^*}(m_\pi^2) \\
&+ (p_D\cdot\varepsilon)(p_D - p_{k^*})^2 a_-^{D\to K^*}(m_\pi^2) \Big|^2 . \qquad (3.15)
\end{aligned}$$

Since

$$(p_D\cdot\varepsilon) = (p_{k^*} + p_\pi)\cdot\varepsilon = (p_\pi\cdot\varepsilon),$$

we can also prove – as we did for the previous decay process – that

$$\sum_{r=1}^{3} |p_\pi \cdot \varepsilon|^2 = \frac{m_D^2}{m_{K^*}^2} |\mathrm{p}_{K^*}|^2.$$

By substitution back in (3.15) we get

$$|A(D^0 \to K^{*-}\pi^+)|^2 = \left(\frac{G_F}{\sqrt{2}} a_1 V_{cs}^* V_{ud} f_\pi\right)^2 \frac{m_D^2}{m_{K^*}^2} |\mathrm{p}_{K^*}|^2 \times \left[f^{D\to K^*}(m_\pi^2) + (m_D^2 - m_{K^*}^2) a_+^{D\to K^*}(m_\pi^2) + m_\pi^2 a_-^{D\to K^*}(m_\pi^2)\right]^2. \tag{3.16}$$

So, we can write the amplitude as,

$$A(D^0 \to K^{*-}\pi^+) = \frac{G_F}{\sqrt{2}} a_1 V_{cs}^* V_{ud} f_\pi \frac{m_D}{m_{K^*}} |\mathrm{p}_{K^*}| \times \left[f^{D\to K^*}(m_\pi^2) + (m_D^2 - m_{K^*}^2) a_+^{D\to K^*}(m_\pi^2) + m_\pi^2 a_-^{D\to K^*}(m_\pi^2)\right]. \tag{3.17}$$

3.2.4 $B \to VV$

The fourth process we will study is $\overline{B}^0 \to K^{*0}\psi$. This is an example of $B \to VV$.

The amplitude for this decay after factorization is

$$A(\overline{B}^0 \to K^{*0}\psi)$$

$$
\begin{aligned}
&= \frac{G_F}{\sqrt{2}} a_2 V_{cs}^* V_{bc} \langle \psi | \bar{c} \gamma^\mu c | 0 \rangle \langle K^{*0} | \bar{s} \gamma_\mu (1 - \gamma_5) b | \overline{B}^0 \rangle \\
&= \frac{G_F}{\sqrt{2}} a_2 V_{cs}^* V_{bc} (g_\psi m_\psi \varepsilon^\mu) \\
&\quad \times \Big[i g^{\overline{B} \to K^*}(t) \varepsilon_{\mu\rho\tau\sigma} \eta^\rho (p_B + p_{K^*})^\tau (p_B - p_{K^*})^\sigma \\
&\quad - f^{\overline{B} \to K^*}(t) \eta_\mu - a_+^{\overline{B} \to K^*}(t) (\eta \cdot p_B)(p_B + p_{K^*})_\mu \\
&\quad - a_-^{\overline{B} \to K^*}(t) (\eta \cdot p_B)(p_B - p_{K^*})_\mu \Big] ,
\end{aligned} \tag{3.18}
$$

where ε and η are the polarization vectors for ψ and K^* respectively, and

$$
a_2 = \left(c_2 + \frac{c_1}{N_c} \right) = \left(-0.25 + \frac{1.11}{N_c} \right) .
$$

By contracting the two currents we get

$$
\begin{aligned}
A(\overline{B}^0 \to K^{*0} \psi) &= \frac{G_F}{\sqrt{2}} a_2 V_{cs}^* V_{bc} g_\psi m_\psi \\
&\times [i g(t) \varepsilon^\mu \varepsilon_{\mu\rho\tau\sigma} \eta^\rho (p_B + p_{K^*})^\tau (p_\psi)^\sigma - f(t)(\varepsilon \cdot \eta) - 2 a_+(t)(\eta \cdot p_B)(\varepsilon \cdot p_B)] .
\end{aligned} \tag{3.19}
$$

As we did before, here also the square of the amplitude is summed over all polarization vectors. So,

$$
\begin{aligned}
|A(\overline{B}^0 \to K^{*0} \psi)|^2 &= \left(\frac{G_F}{\sqrt{2}} a_2 V_{cs}^* V_{bc} g_\psi m_\psi \right)^2 \\
&\times \sum_{r_\varepsilon r_\eta} [i g(t) \varepsilon^\mu \varepsilon_{\mu\rho\tau\sigma} \eta^\rho (p_B + p_{K^*})^\tau (p_\psi)^\sigma - f(t)(\varepsilon \cdot \eta) \\
&- 2 a_+(t)(\eta \cdot p_B)(\varepsilon \cdot p_B)]
\end{aligned}
$$

$$\times \left[-ig(t)\varepsilon^{*\mu}\varepsilon_{\mu'\rho'\tau'\sigma'}\eta^{*\rho'}(p_B+p_{K^*})^{\tau'}(p_\psi)^{\sigma'} - f(t)(\varepsilon^*\cdot\eta^*)\right.$$

$$\left.-2a_+(t)(\eta^*\cdot p_B)(\varepsilon^*\cdot p_B)\right]. \tag{3.20}$$

By expanding the amplitude above, we notice that terms containing vector-axial vector contractions vanish when we perform summations over all polarization vectors. So, we are left with the following five terms:

$$|A(\overline{B}^0\to K^{*0}\psi)|^2 = \left(\frac{G_F}{\sqrt{2}}a_2V_{cs}^*V_{bc}g_\psi m_\psi\right)^2$$

$$\sum\left[g^2(t)\varepsilon^\mu\varepsilon^{*\mu'}\varepsilon_{\mu\rho\tau\sigma}\varepsilon_{\mu'\rho'\tau'\sigma'}\eta^\rho\eta^{*\rho'}(p_B+p_{K^*})^\tau(p_B+p_{K^*})^{\tau'}p_\psi p_{\psi'}\right.$$

$$+ f^2(t)(\varepsilon\cdot\eta)(\varepsilon^*\cdot\eta^*)$$

$$+ a_+(t)f(t)(\varepsilon\cdot\eta)(\eta^*\cdot p_B)(\varepsilon^*\cdot p_B)$$

$$+ a_+(t)f(t)(\varepsilon^*\cdot\eta^*)(\eta\cdot p_B)(\varepsilon\cdot p_B)$$

$$\left.+ 4a_+^2(t)(\varepsilon\cdot p_B)(\varepsilon^*\cdot p_B)(\eta\cdot p_B)(\eta^*\cdot p_B)\right]. \tag{3.21}$$

Before we start summation over all indeces and all polarization vectors we perform the following substitution:

$$\varepsilon_{\mu\rho\tau\sigma}\varepsilon_{\mu'\rho'\tau'\sigma'} = -\begin{vmatrix} g_{\mu\mu'} & g_{\mu\rho'} & g_{\mu\tau'} & g_{\mu\sigma'} \\ g_{\rho\mu'} & g_{\rho\rho'} & g_{\rho\tau'} & g_{\rho\sigma'} \\ g_{\tau\mu'} & g_{\tau\rho'} & g_{\tau\tau'} & g_{\tau\sigma'} \\ g_{\sigma\mu'} & g_{\sigma\rho'} & g_{\sigma\tau'} & g_{\sigma\sigma'} \end{vmatrix}. \tag{3.22}$$

This determinant contains 24 terms. If we exclude terms that contain $g_{\mu\sigma'}$ or $g_{\sigma\mu'}$, then we end up with 14 terms only. The excluded terms contain the contraction of p_ψ and its polarization vector which vanishes. So, the square of the amplitude will be

$$
\begin{aligned}
|A(\overline{B}^0 \to K'^{*0}\psi)|^2 = & \left(\frac{G_F}{\sqrt{2}} a_2 V_{cs}^* V_{bc} g_\psi m_\psi\right)^2 \\
& \times \sum_{r_\xi r_\eta} \Big[-g^2(t) \Big\{ (p_\psi \cdot (p_B + p_{K^*}))^2 (\varepsilon \cdot \eta^*)(\varepsilon^* \cdot \eta) \\
& - (p_\psi \cdot (p_B + p_{K^*}))^2 (\varepsilon \cdot \varepsilon^*)(\eta \cdot \eta^*) \\
& + (p_\psi \cdot (p_B + p_{K^*}))(\varepsilon \cdot \varepsilon^*)(\eta \cdot p_B)(\eta^* \cdot p_B) \\
& - 2(p_\psi \cdot (p_B + p_{K^*}))(\varepsilon \cdot \eta^*)(\varepsilon^* \cdot p_B)(\eta \cdot p_B) \\
& + (p_\psi \cdot (p_B + p_{K^*}))(\varepsilon \cdot \varepsilon^*)(\eta \cdot p_B)(\eta^* \cdot p_B) \\
& - 2(p_\psi \cdot (p_B + p_{K^*}))(\varepsilon^* \cdot \eta)(\varepsilon \cdot p_B)(\eta^* \cdot p_B) \\
& + 4(\varepsilon \cdot p_B)(\varepsilon^* \cdot p_B)(\eta \cdot p_B)(\eta^* \cdot p_B) \\
& - (p_B + p_{K^*})^2 (\varepsilon \cdot \varepsilon^*)(\eta \cdot p_B)(\eta^* \cdot p_B) \\
& - p_\psi^2 (\varepsilon \cdot \varepsilon^*)(\eta \cdot p_B)(\eta^* \cdot p_B) \\
& + 2p_\psi^2 (\varepsilon \cdot \eta^*)(\varepsilon^* \cdot p_B)(\eta \cdot p_B) \\
& + 2p_\psi^2 (\varepsilon^* \cdot \eta)(\varepsilon \cdot p_B)(\eta^* \cdot p_B) \\
& - (p_B + p_{K^*})^2 p_\psi^2 (\varepsilon \cdot \eta^*)(\varepsilon^* \cdot \eta)
\end{aligned}
$$

$$
\begin{aligned}
&- 4p_\psi^2(\varepsilon\cdot p_B)(\varepsilon^*\cdot p_B)(\eta\cdot\eta^*) \\
&+ (p_B+p_{K^*})^2 p_\psi^2(\varepsilon\cdot\varepsilon^*)(\eta\cdot\eta^*)\Big\} \\
&+ f^2(t)(\varepsilon\cdot\eta)(\varepsilon^*\cdot\eta^*) \\
&+ 2a_+(t)f(t)(\varepsilon\cdot\eta)(\varepsilon^*\cdot p_B)(\eta^*\cdot p_B) \\
&+ 2a_+(t)f(t)(\varepsilon^*\cdot\eta^*)(\varepsilon\cdot p_B)(\eta\cdot p_B) \\
&+ 4a_+^2(t)(\varepsilon\cdot p_B)(\varepsilon^*\cdot p_B)(\eta\cdot p_B)(\eta^*\cdot p_B)\Big]. \qquad (3.23)
\end{aligned}
$$

The final step to get the amplitude is to perform the summation over all polarization vectors. In doing so, we encounter expressions like

$$\sum(\varepsilon\cdot\varepsilon^*), \sum(\varepsilon\cdot\eta^*)(\varepsilon^*\cdot\eta), \ldots$$

which need to be evaluated. As an example, let us evaluate the first one in detail,

$$
\begin{aligned}
\sum_{r=1}^{3}(\varepsilon\cdot\varepsilon^*) &= \sum_{r=1}^{3}\varepsilon^\alpha\varepsilon_\alpha^* = g_{\alpha\beta}\sum_{r=1}^{3}\varepsilon^\alpha\varepsilon^{*\beta} \\
&= g_{\alpha\beta}\left(-g^{\alpha\beta}+\frac{(p_\psi)^\alpha(p_\psi)^\beta}{m_\psi^2}\right) \\
&= -4+\frac{p_\psi^2}{m_\psi^2} \\
&= -3. \qquad (3.24)
\end{aligned}
$$

Similarily, we can prove that

$$\sum(\eta\cdot\eta^*) = -3, \qquad (3.25)$$

$$\sum(\varepsilon\cdot\eta^*)(\varepsilon^*\cdot\eta) = \left[2+\frac{(p_\psi\cdot p_{K^*})^2}{m_\psi^2 m_{K^*}^2}\right], \tag{3.26}$$

$$\sum(\varepsilon\cdot p_B)(\varepsilon^*\cdot p_B) = \left[-m_B^2+\frac{(p_B\cdot p_\psi)^2}{m_\psi^2}\right], \tag{3.27}$$

$$\sum(\eta\cdot p_B)(\eta^*\cdot p_B) = \left[-m_B^2+\frac{(p_B\cdot p_{K^*})^2}{m_{K^*}^2}\right], \tag{3.28}$$

and

$$\sum(\varepsilon\cdot\eta^*)(\varepsilon^*\cdot p_B)(\eta\cdot p_B) = \left[m_B^2-\frac{(p_B\cdot p_{K^*})^2}{m_{K^*}^2}-\frac{(p_B\cdot p_\psi)^2}{m_\psi^2}+\frac{(p_B\cdot p_\psi)(p_B\cdot p_{K^*})(p_{K^*}\cdot p_\psi)}{m_{K^*}^2 m_\psi^2}\right]. \tag{3.29}$$

Also, from conservation of four momentum, we have

$$p_B = p_{K^*}+p_\psi.$$

So, by contracting p_B by itself we get

$$\begin{aligned} p_B^2 &= (p_{K^*}+p_\psi)\cdot(p_{K^*}+p_\psi) \\ &= p_{K^*}^2+p_\psi^2+2(p_{K^*}\cdot p_\psi) \end{aligned}$$

which leads to

$$(p_{K^*}\cdot p_\psi) = \left(\frac{m_B^2-m_{K^*}^2-m_\psi^2}{2}\right). \tag{3.30}$$

Similarily, we can prove that

$$(p_B\cdot p_\psi) = \left(\frac{m_B^2-m_{K^*}^2+m_\psi^2}{2}\right), \tag{3.31}$$

$$(p_B \cdot p_{K^*}) = \left(\frac{m_B^2 + m_{K^*}^2 - m_\psi^2}{2}\right), \tag{3.32}$$

$$p_\psi \cdot (p_B + p_{K^*}) = (m_B^2 - m_{K^*}^2), \tag{3.33}$$

and

$$(p_B + p_{K^*})^2 = (2m_B^2 + 2m_{K^*}^2 - m_\psi^2). \tag{3.34}$$

The substitution from (3.24–3.29) and (3.30–3.34) into (3.23) was done using *Mathematica*, See Appendix B. For the coefficients of $2g^2(t)$, the result was

$$\begin{aligned}(m_B - m_{K^*} - m_\psi)&(m_B + m_{K^*} - m_\psi)(m_B - m_{K^*} + m_\psi)(m_B + m_{K^*} + m_\psi) \\ &= (m_B^2 - m_{K^*}^2 - m_\psi^2)^2 - 4m_{K^*}^2 m_\psi^2 \\ &= 4m_B^2 |\mathrm{p}_{K^*}|^2, \end{aligned} \tag{3.35}$$

where p_{K^*} above, is a three momentum, see (3.13).

The final result for the square of the amplitude is now

$$\begin{aligned}|A(\overline{B}^0 \to K^{*0}\psi)|^2 &= \left(\frac{G_F}{\sqrt{2}} a_2 V_{cs}^* V_{bc} g_\psi m_\psi\right)^2 \\ &\times \left[8m_B^2 |\mathrm{p}_{K^*}|^2 g^2(t) + 2f^2(t) + \left(\frac{(m_B^2 - m_{K^*}^2 - m_\psi^2) f(t) + 4m_B^2 |\mathrm{p}_{K^*}|^2 a_+(t)}{2m_{K^*} m_\psi}\right)^2\right]\end{aligned} \tag{3.36}$$

where $t = m_\psi^2$.

3.2.5 Other Decays

Until now, we have disscussed four decays that cover the four types $D \to PP, D \to PV, D \to VP$, and $\overline{B} \to VV$. Any other decay of a D or a B particle that admit factorization is one of these types. So, its amplitude can be deduced from the corresponding one already derived by making suitable replacements.

The other decays that will be considered are:

- $D^0 \to K^- A_1^+$; it is of the $D^0 \to PV$ type. Its amplitude is deduced from $D^0 \to K^- \rho^+$ amplitude by replacing ρ^+ by A_1^+ everywhere in (3.12).

- $\overline{B}^0 \to \overline{K}^0 \psi$; this one is of the $\overline{B} \to PV$ type. Its amplitude is also deduced from $D^0 \to K^- \rho^+$ amplitude in (3.12). In addition to the $\rho^+ \to \psi$ and $D \to \overline{B}^0$ replacements we should not forget to replace $(V_{cs}^* V_{ud} a_1)$ by $(V_{cs}^* V_{bc} a_2)$, because it is a B meson decay.

- $D^0 \to K^{*-} \rho^+$; this is of the $D \to VV$ type. Its amplitude is deduced from $\overline{B}^0 \to \overline{K}^{*0} \psi$ amplitude in (3.36) by making suitable substitutions similar to the decays above.

3.3 Transition Probability

What we have been doing so far is just theoretical calculations. In order to check that we have the correct picture or at least something close to it, we should predict values of some observables (things that can be measured in the lab).

So, in the next section we will use the decay amplitudes to calculate the branching ratios of each of the decay channels discussed in the previous section. But now, we will derive the relation that allow us to do these calculations.

For the decay of a single particle to several final paricles (*e.g.* $D^0 \rightarrow K^-\pi^+, \mu^- \rightarrow e^-\overline{\nu}_e\nu_\mu, \ldots$) the differential transition probability per unit time is given as [20]

$$d\Gamma = (2\pi)^4\delta^4(p_f - p_i)\prod_k \frac{m_k}{E_k}\prod_l \frac{1}{2\omega_l}\prod_f \frac{d^3\mathrm{p}_f}{(2\pi)^3}|A_{fi}|^2, \qquad (3.37)$$

where E_k is the energy of the kth fermion and ω_l is the energy of the lth boson. In (3.37), $\prod_k$ is a product over all fermions, $\prod_l$ is a product over all bosons and $\prod_f$ is a product over all final particles.

In our work, we are interested in nonleptonic decays that has the form

$D \rightarrow ab$, where both a and b are mesons. So, for this case $d\Gamma$ becomes

$$d\Gamma = \frac{1}{8(2\pi)^2}\delta^4(p_a + p_b - p_D)\frac{1}{\omega_a\omega_b\omega_D}d^3|\mathrm{p}_a|d^3|\mathrm{p}_b||A(D \rightarrow ab)|^2. \quad (3.38)$$

Of course, the square of the amplitude $|A(D \rightarrow ab)|^2$ is summed over the polarizations of the final state particles.

Knowing that the energy of a particle forms the fourth component of its four momentum, we can write

$$d\Gamma = \frac{1}{8(2\pi)^2}\delta^3(\mathrm{p}_a+\mathrm{p}_b-\mathrm{p}_D)\frac{\delta(\omega_a + \omega_b - \omega_D)}{\omega_a\omega_b\omega_D}d^3\mathrm{p}_a d^3\mathrm{p}_b|A(D \rightarrow ab)|^2. \quad (3.39)$$

The energies of the particles are

$$\begin{aligned} \omega_D &= (|\mathrm{p}_D|^2 + m_D^2)^{1/2} = m_D, \\ \omega_a &= (|\mathrm{p}_a|^2 + m_a^2)^{1/2}, \\ \omega_b &= (|\mathrm{p}_b|^2 + m_b^2)^{1/2}, \end{aligned}$$

where $\mathrm{p}_D = 0$, because we are working in the center-of-mas rest frame. So,

$$d\Gamma = \frac{1}{8(2\pi)^2}\delta^3(\mathrm{p}_a + \mathrm{p}_b)\frac{\delta((|\mathrm{p}_a|^2 + m_a^2)^{1/2} + (|\mathrm{p}_b|^2 + m_b^2)^{1/2} - m_D)}{m_D(|\mathrm{p}_a|^2 + m_a^2)^{1/2}(|\mathrm{p}_b|^2 + m_b^2)^{1/2}} \times d^3\mathrm{p}_a d^3\mathrm{p}_b|A(D \rightarrow ab)|^2. \quad (3.40)$$

Now, if we perform integration over p_a we get

$$\Gamma = \frac{1}{8\pi m_D}\int_0^\infty \frac{\delta((|\mathrm{p}_b|^2 + m_a^2)^{1/2} + (|\mathrm{p}_b|^2 + m_b^2)^{1/2} - m_D)}{m_D(|\mathrm{p}_b|^2 + m_a^2)^{1/2}(|\mathrm{p}_b|^2 + m_b^2)^{1/2}} \times |\mathrm{p}_b|^2 d|\mathrm{p}_b||A(D \rightarrow ab)|^2. \quad (3.41)$$

By defining

$$q = (|\mathbf{p}_b|^2 + m_a^2)^{1/2} + (|\mathbf{p}_b|^2 + m_b^2)^{1/2}, \tag{3.42}$$

and substituting back in (3.41), we get

$$\begin{aligned}\Gamma &= \frac{1}{16\pi m_D}\int_{m_a+m_b}^{\infty}\frac{\delta(q-m_D)}{q^2} \\ &\quad\times\left[\left\{q^2-(m_a-m_b)^2\right\}\left\{q^2-(m_a+m_b)^2\right\}\right]^{1/2}dq|A(D\to ab)|^2. \\ &= \frac{1}{16\pi m_D}|A(D\to ab)|^2\left[\left\{1-(m_a/m_D-m_b/m_D)^2\right\}\right. \\ &\quad\left.\times\left\{1-(m_a/m_D+m_b/m_D)^2\right\}\right]^{1/2}. \end{aligned}\tag{3.43}$$

For the nonleptonic decays of B mesons, we just replace D by B everywhere in the previous equation.

3.4 Numerical Calculations

The branching ratio of a certain exclusive decay is defined to be its transition probability (Γ) multiplied by the mean life of the decaying particle. In this section, we will use what we have done so far in order to predict the branching ratios of several nonleptonic decays. In particular, we will need the form factors produced in Chapter 2, and the decay amplitudes derived in the previous sections of this chapter. Furthermore, we will need the masses of

Table 3.1: Some constants needed in the numerical calculations

Quantity	Symbol	Value
Fermi Coupling Constant	G_F	$1.16639 \times 10^{-5}\text{GeV}^{-2}$
Plank Constant, reduced	$\hbar$	$6.5S2122 \times 10^{-25}\text{GeV} \cdot \text{s}$
Kobayashi-Maskawa Matrix Elements	V_{ud}	0.9753
	V_{cs}	0.9743
	V_{cb}	0.043
Decay Constants	f_π	0.132 GeV
	g_ρ	0.20S GeV
	$g_{A_1^+}$	0.334 GeV
	g_ψ	0.3S2 GeV

the particles in table 1.1 and several other parameters and decay constants shown in table 3.1.

Our results for the two cases ($N_c = \infty$ and $N_c = 3$) are shown in tables 3.2 and 3.3. For the details of the calculations see Appendix D.

The results of other work are displayed in table 3.4. The work done by Tanimoto [5] is similar to ours in which he used E691 results for the form factors to calculate the branching ratios for the two cases ($N_c = \infty$ and $N_c = 3$). Also, he used flavour symmetry to calculate the form factors for the B

Table 3.2: Predicted branching ratios for D^0 and $\overline{B}^0$ decays with $N_c = \infty (N_c = 3$ in parentheses).

Process	E691 (%)	E653 (%)	Experimental (%)
$D^0 \to K^- \pi^+$	6.8 ± 1.3 (5.2 ± 1.0)	5.5 ± 0.9 (4.2 ± 0.7)	3.71 ± 0.25
$D^0 \to K^- \rho^+$	10.8 ± 2.1 (8.2 ± 1.6)	8.7 ± 1.5 (6.6 ± 1.1)	7.8 ± 1.1
$D^0 \to K^- A_1^+$	3.9 ± 0.8 (3.0 ± 0.6)	3.1 ± 0.5 (2.4 ± 0.4)	7.4 ± 1.3
$D^0 \to K^{*-} \pi^+$	3.1 ± 1.6 (2.3 ± 1.2)	2.1 ± 1.5 (1.6 ± 1.1)	4.6 ± 0.6
$D^0 \to K^{*-} \rho^+$	6.8 ± 2.2 (5.2 ± 1.7)	8.0 ± 3.1 (6.1 ± 2.4)	6.2 ± 2.5
$\overline{B}^0 \to \overline{K}^0 \psi$	0.073 ± 0.014 (0.017 ± 0.003)	0.059 ± 0.010 (0.014 ± 0.002)	0.065 ± 0.031
$\overline{B}^0 \to \overline{K}^{*0} \psi$	0.16 ± 0.10 (0.037 ± 0.022)	0.14 ± 0.10 (0.033 ± 0.020)	0.13 ± 0.04

Table 3.3: Predicted branching ratios for D^0 and $\overline{B}^0$ decays with $N_c = \infty (N_c = 3$ in parentheses).

Process	ELC (%)	BES (%)	BSW (%)
$D^0 \to K^- \pi^+$	3.7 ± 0.5 (2.8 ± 0.4)	8.8 ± 0.8 (6.7 ± 0.6)	6.3 (4.8)
$D^0 \to K^- \rho^+$	5.8 ± 0.8 (4.4 ± 0.6)	14.0 ± 1.2 (10.7 ± 0.9)	10.0 (7.6)
$D^0 \to K^- A_1^+$	2.1 ± 0.3 (1.6 ± 0.2)	5.0 ± 0.4 (3.8 ± 0.3)	3.6 (2.8)
$D^0 \to K^{*-} \pi^+$	3.1 ± 1.4 (2.4 ± 1.1)	5.6 ± 3.4 (4.3 ± 2.6)	3.2 (2.5)
$D^0 \to K^{*-} \rho^+$	8.4 ± 1.6 (6.4 ± 1.2)	19.5 ± 7.9 (14.9 ± 6.0)	19.4 (14.8)
$\overline{B}^0 \to \overline{K}^0 \psi$	0.039 ± 0.005 (0.009 ± 0.001)	0.095 ± 0.008 (0.022 ± 0.002)	0.052 (0.012)
$\overline{B}^0 \to \overline{K}^{*0} \psi$	0.16 ± 0.06 (0.036 ± 0.15)	0.32 ± 0.20 (0.075 ± 0.046)	0.22 (0.050)

decays.

In Pham work [17] the approach is somewhat different. For the first 3 decays, he related the branching ratios of the nonleptonic decays to known branching ratios of the semileptonic decays $D \rightarrow Ke\nu$ and $D \rightarrow K^*e\nu$ without the need to find the form factors. For the other two decays, he also related the branching ratios to those of semileptonic decays, but he also needed the values of the form factors. So, he used the results of both E691 and E653 experiments.

Kramer and Palmer followed a different approach. In their paper titled *Direct CP asymmetries in the decays $B \rightarrow VV$ from an effective weak Hamiltonian* [18] we found only one decay of interest to us.

Table 3.4: Branching ratios of D^0 and $\overline{B}^0$ decays as appeared in different papers.

Process	Tanimoto (%)	Pham (%)	Kramer Palmer
$D^0 \to K^-\pi^+$	$7.64 \pm 0.07(N_c = \infty)$ $(5.83 \pm 0.06)(N_c = 3)$	6.16 ± 0.72	
$D^0 \to K^-\rho^+$	$11.5 \pm 0.11(N_c = \infty)$ $(8.81 \pm 0.09)(N_c = 3)$	8.89 ± 1.04	
$D^0 \to K^- A_1^+$		1.98 ± 0.23	
$D^0 \to K^{*-}\pi^+$	$3.46 \pm 0.18(N_c = \infty)$ $(2.64 \pm 0.14)(N_c = 3)$	3.01 ± 1.18 (E691) 1.98 ± 0.77 (E653)	
$D^0 \to K^{*-}\rho^+$		5.67 ± 2.26 (E691) 5.74 ± 2.29 (E653)	
$\overline{B}^0 \to \overline{K}^0\psi$	$0.077 \pm 0.0008(N_c = \infty)$ $(0.018 \pm 0.0002)(N_c = 3)$		
$\overline{B}^0 \to \overline{K}^{*0}\psi$	$0.16 \pm 0.03(N_c = \infty)$ $(0.037 \pm 0.006)(N_c = \infty)$		0.241

Chapter 4

Discussion and Conclusions

4.1 Discussion of the Results

Form Factors

In Chapter 2, we presented the form factors available for $D \to K, K^*$ transitions from two experimental measurements, two lattice calculations and quark model (BSW). Also, we calculated the corresponding ones for $B \to K, K^*$ transitions by using flavour symmetry. By looking at those values we notice that there are some differences between them. For example, the form factors that describe the $D \to K$ transition ($f_0(0) = f_1(0)$) vary between 0.58 for the ELC group and 0.9 for the BES group.

For the $D \to K^*$ transitions, we notice that the values given by the first three groups (E691, E653 and ELC) almost agree for the values of $A_1(0)$ and $V(0)$. However, for the value of $A_2(0)$ there are clear differences. It seems that this particular form factor is more difficult to measure. Also, these values given for $A_2(0)$ are in big disagreement with that predicted by the BSW model.

The values given by the BES lattice calculations are higher than those given by the other three groups, even for the value of $f_0(0)$. It would be thus interesting to see which value of $A_2(0)$ would be supported by the non-leptonic D decays.

Branching Ratios

In Chapter 3, we calculated the branching ratios for several decay processes using the form factors prepared in Chapter 2. By taking a first general look at these predictions we find something that goes with experiment and not something that is irrelevant.

A closer look at these tables allow us to note the following points:

- For the process $D^0 \to K^- A_1^+$, all groups are giving small predictions for the branching ratio: less than half the experimental value for most

of them.

- For processes of the types $D, B \rightarrow K$, the E653 group gives branching ratios which are closer to experiment than those given by the E691 group. This could indicate that ($f_0(0) = 0.71$) is more appropriate than ($f_0(0) = 0.79$) given by the E691 group. However, for processes of the type $D, B \rightarrow K^*$ the opposite case is correct.

- The predictions of the lattice group BES are clearly very high.

- With the exception of $D^0 \rightarrow K^-\pi^+$ processes, the ELC lattice group gives predictions that are smaller than experiment for decays of the type $D, B \rightarrow K$, whereas for decays of the type $D, B \rightarrow K^*$ the agreement is good.

By looking at the predictions as a whole, we can say: for the decays where we have pseudoscalar (K) meson in the final state the E653 group seems to give the predictions that are closest to experiment. On the other hand, for the decays where vector (K^*) meson is in the final state, the E691 seems to be the best. However we should be careful about these statements because the errors are high and sometimes exceed the differences between the predictions of different groups.

4.2 Conclusions

In this work, we hoped to test two assumptions that were used in the calculations. The first one is the factorization hypothesis that enabled us to decompose the decay amplitudes into two current matrix elements which could then be evaluated. The second one is flavour symmetry.

Factorization

By looking at the branching ratios as predicted by the first three groups, we conclude that factorization, which is exact only in the large N_c (number of colours) limit, is not a bad assumption. This statement can be made sharper when we have more precisely measured form factors.

The low branching ratio for the decay $D^0 \to K^- A_1^+$, could mean that factorization is not enough for this particular process, assuming, of course, an accurate value for the decay constant of A_1^+ meson.

The high values predicted by BES as compared to the other three groups and to the experimental values allow us to doubt about the BES values for the form factors and not about factorization.

Flavour Symmetry

Our predictions of the branching ratios for the two B decays are in good agreement with experiment. Actually, it is better than expected, since the two assumptions (factorization and flavour symmetry) have been used in the calculations. So, we can say that flavour symmetry seems to be working very well. This statement can be made stronger and more general by studying more decays of B mesons using this new symmetry.

4.3 Summary

We studied the non-leptonic decays of D and B mesons involving K and K^* mesons with a view to test the factorization hypothesis, which is exact only in large N_c limit, and new flavour symmetry.

The form factors of the current matrix elements corresponding to the transitions $D \to K, K^*$ are available from two experimental measurements and two lattice calculations. These values have been used to study several nonleptonic D^0 decays that admit factorization and contain a strange meson (K, K^*) in the final state.

Furthermore, these form factors have been used to calculate the corre-

sponding ones for $\overline{B}^0$ decays using flavour symmetry. This enabled us to study two factorizable B decays involving strange mesons.

The comparison of these calculations with experiment shows that the factorization hypothesis is not bad and that flavour symmetry works well.

Appendices

Appendix A

All numerical calculations are done using *Mathematica*. This *Mathematica* note book is organized in 6 parts as follows:

First, we define the masses of B, D, K and (K* = K1) mesons.

In[1]:=

```
mB  = 5.2787;
mD  = 1.8645;
mK  = 0.493646;
mK1 = 0.89159;
```

Part 1: Form Factors Defintion (D meson)

In this part we define form factors for the processes D-->K,K* assuming single pole dominance. So, we start with the values of the form factors at t=0.

In[2]:=

```
fD0[0] = fD1[0] = f1;
AD1[0] = A1;
AD2[0] = A2;
 VD[0] = V;
```

Then, we calculate the values of A0(0) and A3(0) using (2.5).

In[3]:=

```
AD0[0] = AD3[0] =
(mD + mK1)/(2 mK1) AD1[0] - (mD - mK1)/(2 mK1) AD2[0]
```

Out[3]=

```
1.5456 A1 - 0.545604 A2
```

Now, the form factors as a function of t are defined below.

In[4]:=

```
fD0[t_] = fD0[0] /(1 - t/2.60^2);
fD1[t_] = fD1[0] /(1 - t/2.11^2);
AD0[t_] = AD0[0] /(1 - t/1.97^2);
AD1[t_] = AD1[0] /(1 - t/2.53^2);
AD2[t_] = AD2[0] /(1 - t/2.53^2);
AD3[t_] = AD3[0] /(1 - t/2.53^2);
 VD[t_] =  VD[0] /(1 - t/2.11^2);
```

Part 2: Transformation (D decay)

Here we relate form factors of the **First Decomposition** to those of the **Second Decomposition** using the transformations in (2.8), (2.9), and (2.10). Here fDn(t) = f_(t), fDp(t) = f+(t), ...

In[5]:=

```
fDn[0]  = (mD^2 - mK^2) fD1[0] (1/2.6^2 - 1/2.11^2);
aDn[0]  = 2 mK1 AD3[0] (1/2.53^2 - 1/1.97^2);
```

In[6]:=

```
fDp[t_] = fD1[t];
fDn[t_] = (mD^2 - mK^2)/t (fD0[t] - fD1[t]);
aDp[t_] = AD2[t] /(mD + mK1);
aDn[t_] = - 2 mK1/t (AD0[t] - AD3[t]);
 gD[t_] = VD[t] /(mD + mK1);
 fD[t_] = - (mD + mK1) AD1[t];
```

The values of form factors of the **First Decomposition** for D-->K,K* decays are evaluated below at zero momentum transfer (t=0).

In[7]:=

```
{fDp[0], fDn[0], aDp[0], aDn[0], fD[0], gD[0]} //
Simplify //TableForm
```

Out[7]//TableForm=

f1

-0.247895 f1

0.362833 A2

-0.279589 A1 + 0.098696 A2

-2.75609 A1

0.362833 V

Part 3: Flavour Symmetry (D-B mesons)

In[8]:=

```
alphas = (0.189/0.29)^(-6/25);
mc = 1.7;
mb = 4.9;
```

In[9]:=

```
c1 = c4 = c5 = (mc/mb)^(1/2) alphas;
c2 = c6 = (mb/mc)^(1/2) alphas;
c3 = (mc/mb)^(3/2) alphas;
```

From (2.3) we can define tD as a function of tB as follows:

In[10]:=

```
tD = mD^2 + mK^2 - mB mD - mD/mB (mK^2 - tB);
```

In[11]:=

```
tD1 = mD^2 + mK1^2 - mB mD - mD/mB (mK1^2 - tB);
```

Now we make use of flavor symmetry relations (2.2) to relate the form factors of D-->K,K* and B-->K,K* at the same velocity transfer.

In[12]:=

```
fBp[tB_] = 1/2 ((c1 + c2) fDp[tD]  + (c1 - c2) fDn[tD]);
fBn[tB_] = 1/2 ((c1 - c2) fDp[tD]  + (c1 + c2) fDn[tD]);
aBp[tB_] = 1/2 ((c3 + c4) aDp[tD1] + (c3 - c4) aDn[tD1]);
aBn[tB_] = 1/2 ((c3 - c4) aDp[tD1] + (c3 + c4) aDn[tD1]);
 gB[tB_] = c5 gD[tD1];
 fB[tB_] = c6 fD[tD1];
```

Part 4: Form Factors (B dcay)

In this part we give form factors of B-->K,K* as in the Second Decomposition.

In[13]:=

```
fB1[t_] = fBp[t];
fB0[t_] = t/(mB^2 - mK^2) fBn[t] + fBp[t];
AB1[t_] = -1/(mB + mK1) fB[t];
AB2[t_] = (mB + mK1) aBp[t];
AB3[t_] = (mB + mK1)/(2 mK1) AB1[t] -
                              (mB - mK1)/(2 mK1) AB2[t];
AB0[t_] = -t/(2 mK1) aBn[t] + AB3[t];
VB[t_]  = (mB + mK1) gB[t];
```

At the same velocity transfer, the momentum transfers tB that coresponds to zero momentum trasfers tD(tD1) for D decays are

In[14]:=

```
Solve[tD == 0, tB]
```

Out[14]=

```
{{tB -> 17.5763}}
```

In[15]:=

```
Solve[tD1 == 0, tB]
```

Out[15]=

```
{{tB -> 16.5669}}
```

In[16]:=

```
tBK = 17.5763;
tBK1 = 16.5669;
```

■ First Method

The form factors at zero momentum transfer (t=0) as calculated by the first method; see section 2.3.

In[17]:=

```
{fB0[0], fB1[0], AB0[0], AB3[0], AB1[0], AB2[0], VB[0]} //
Simplify //TableForm
```

Out[17]//TableForm=

```
0.562347 f1

0.562347 f1

1.33073 A1 - 1.19844 A2

1.33073 A1 - 1.19844 A2

0.439037 A1

0.0765991 A1 + 0.487118 A2

0.631443 V
```

■ Second Method

The form factors at zero momentum transfer (t=0) as calculated by thesecond method; see section 2.3.

In[18]:=

```
ffB0[0] = fB0[tBK] (1 - tBK/5.89^2) //Simplify
```

Out[18]=

```
0.408796 f1
```

In[19]:=

```
ffB1[0] = fB1[tBK] (1 - tBK/5.43^2) //Simplify
```

Out[19]=

```
0.573283 f1
```

In[20]:=

```
AAB0[0] = AB0[tBK1] (1 - tBK1/5.38^2) //Simplify
```

Out[20]=

```
1.34502 A1 - 0.764022 A2
```

In[21]:=
```
AAB3[0] = AB3[tBK1] (1 - tBK1/5.82^2) //Simplify
```
Out[21]=
```
1.02352 A1 - 1.07394 A2
```
In[22]:=
```
AAB1[0] = AB1[tBK1] (1 - tBK1/5.82^2) //Simplify
```
Out[22]=
```
0.429363 A1
```
In[23]:=
```
AAB2[0] = AB2[tBK1] (1 - tBK1/5.82^2) //Simplify
```
Out[23]=
```
0.187862 A1 + 0.436512 A2
```
In[24]:=
```
VVB[0] = VB[tBK1] (1 - tBK1/5.43^2) //Simplify
```
Out[24]=
```
0.640264 V
```

▩ Part 5: Numerical Vlaues of Form Factors

In this part we define the form factors for D-->K,K* obtained by the two experimental groups E691 and E653 and the two lattice groups ELC and BES. Then, we obtain the corresponding form factors for B-->K,K* using the two methods.

In[25]:=
```
error[x_] := Map[Abs, Simplify[x]]
```

■ E691 (experimental)

In[26]:=
```
E691 = {f1->0.79, A1->0.46, A2->0.0, V->0.9};
```
In[27]:=
```
errE691 = {f1->0.078, A1->0.05, A2->0.2, V->0.3};
```

□ First Method

In[28]:=

```
N[{fB0[0], fB1[0], AB0[0], AB3[0], AB1[0],
AB2[0], VB[0]} /.E691, 3]
```

Out[28]=

```
{0.444, 0.444, 0.612, 0.612, 0.202, 0.0352, 0.568}
```

In[29]:=

```
N[(error /@ {fB0[0], fB1[0], AB0[0], AB3[0], AB1[0],
AB2[0], VB[0]}) /.errE691, 3]
```

Out[29]=

```
{0.0439, 0.0439, 0.306, 0.306, 0.022, 0.101, 0.189}
```

□ Second method

In[30]:=

```
N[{ffB0[0], ffB1[0], AAB0[0], AAB3[0], AAB1[0],
AAB2[0], VVB[0]} /.E691, 3]
```

Out[30]=

```
{0.323, 0.453, 0.619, 0.471, 0.198, 0.0864, 0.576}
```

In[31]:=

```
N[(error /@ {ffB0[0], ffB1[0], AAB0[0], AAB3[0], AAB1[0],
AAB2[0], VVB[0]}) /.errE691, 3]
```

Out[31]=

```
{0.0319, 0.0447, 0.22, 0.266, 0.0215, 0.0967, 0.192}
```

■ E653 (experimental)

In[32]:=

```
E653 = {f1->0.71, A1->0.53, A2->0.43, V->1.06};
```

In[33]:=

```
errE653 = {f1->0.06, A1->0.08, A2->0.15, V->0.3};
```

□ First Method

In[34]:=

```
N[{fB0[0], fB1[0], AB0[0], AB3[0], AB1[0],
AB2[0], VB[0]} /.E653, 3]
```

Out[34]=

```
{0.399, 0.399, 0.19, 0.19, 0.233, 0.25, 0.669}
```

In[35]:=

```
N[(error /@ {fB0[0], fB1[0], AB0[0], AB3[0], AB1[0],
AB2[0], VB[0]}) /.errE653, 3]
```

Out[35]=

```
{0.0337, 0.0337, 0.286, 0.286, 0.0351, 0.0792, 0.189}
```

□ Second method

In[36]:=

```
N[{ffB0[0], ffB1[0], AAB0[0], AAB3[0], AAB1[0],
AAB2[0], VVB[0]} /.E653, 3]
```

Out[36]=

```
{0.29, 0.407, 0.384, 0.0807, 0.228, 0.287, 0.679}
```

In[37]:=

```
N[(error /@ {ffB0[0], ffB1[0], AAB0[0], AAB3[0], AAB1[0],
AAB2[0], VVB[0]}) /.errE653, 3]
```

Out[37]=

```
{0.0245, 0.0344, 0.222, 0.243, 0.0343, 0.0805, 0.192}
```

■ ELC (lattice)

In[38]:=

```
ELC = {f1->0.58, A1->0.53, A2->0.19, V->0.86};
```

In[39]:=

```
errELC = {f1->0.04, A1->0.03, A2->0.21, V->0.10};
```

□ First Method

In[40]:=

```
N[{fB0[0], fB1[0], AB0[0], AB3[0], AB1[0],
AB2[0], VB[0]} /.ELC, 3]
```

Out[40]=

```
{0.326, 0.326, 0.478, 0.478, 0.233, 0.133, 0.543}
```

In[41]:=

```
N[(error /@ {fB0[0], fB1[0], AB0[0], AB3[0], AB1[0],
AB2[0], VB[0]}) /.errELC, 3]
```

Out[41]=

```
{0.0225, 0.0225, 0.292, 0.292, 0.0132, 0.105, 0.0631}
```

□ Second method

In[42]:=

```
N[{ffB0[0], ffB1[0], AAB0[0], AAB3[0], AAB1[0],
AAB2[0], VVB[0]} /.ELC, 3]
```

Out[42]=

```
{0.237, 0.333, 0.568, 0.338, 0.228, 0.183, 0.551}
```

In[43]:=

```
N[(error /@ {ffB0[0], ffB1[0], AAB0[0], AAB3[0], AAB1[0],
AAB2[0], VVB[0]}) /.errELC, 3]
```

Out[43]=

```
{0.0164, 0.0229, 0.201, 0.256, 0.0129, 0.0973, 0.064}
```

■ BES (lattice)

In[44]:=

```
BES = {f1->0.90, A1->0.83, A2->0.59, V->1.43};
```

In[45]:=

```
errBES = {f1->0.04, A1->0.14, A2->0.14, V->0.45};
```

□ First Method

In[46]:=

```
N[{fB0[0], fB1[0], AB0[0], AB3[0], AB1[0],
AB2[0], VB[0]} /.BES, 3]
```

Out[46]=

```
{0.506, 0.506, 0.397, 0.397, 0.364, 0.351, 0.903}
```

In[47]:=

```
N[(error /@ {fB0[0], fB1[0], AB0[0], AB3[0], AB1[0],
AB2[0], VB[0]}) /.errBES, 3]
```

Out[47]=

```
{0.0225, 0.0225, 0.354, 0.354, 0.0615, 0.0789, 0.284}
```

□ Second method

In[48]:=

```
N[{ffB0[0], ffB1[0], AAB0[0], AAB3[0], AAB1[0],
AAB2[0], VVB[0]} /.BES, 3]
```

Out[48]=

```
{0.368, 0.516, 0.666, 0.216, 0.356, 0.413, 0.916}
```

In[49]:=

```
N[(error /@ {ffB0[0], ffB1[0], AAB0[0], AAB3[0], AAB1[0],
AAB2[0], VVB[0]}) /.errBES, 3]
```

Out[49]=

```
{0.0164, 0.0229, 0.295, 0.294, 0.0601, 0.0874, 0.288}
```

Part 6

In this part we compare form factors of B-->K,K* obtained in two ways:

1) Those which were completely obtained from the D-->K,K* form factors using flavour symmetry (fB0, fB1, AB0, . . .).

2) Those which were defined using simple pole dominance and the value of form factors at a particular momentum transfer (ffB0, ffB1, AAB0, . . .).

In[50]:=

```
ffB0[t_] = ffB0[0] /(1 - t/5.89^2);
ffB1[t_] = ffB1[0] /(1 - t/5.43^2);
AAB0[t_] = AAB0[0] /(1 - t/5.38^2);
AAB1[t_] = AAB1[0] /(1 - t/5.82^2);
AAB2[t_] = AAB2[0] /(1 - t/5.82^2);
AAB3[t_] = AAB3[0] /(1 - t/5.82^2);
VVB[t_]  =  VVB[0] /(1 - t/5.43^2);
```

In[51]:=

```
Plot[{fB0[t]/.E691, ffB0[t]/.E691}, {t, 0, mB^2},
PlotRange -> {{0, 29}, Automatic},
Framed -> True,
AxesLabel -> {"t (GeV)", "f0(t)"},
AspectRatio -> 0.75]
```

Appendix B

In this Appendix we calculate the predictions of the BSW model [1, 2] for the transitions D-->K, B-->K, D-->K* and B-->K*.

In[1]:=

```
<<IntegralTables.m
```

Out[1]=

```
Integrator`
```

In[2]:=

```
w = 0.4;
```

In[3]:=

```
m[u] = m[d] = 0.35;
m[s] = 0.55; m[c] = 1.7; m[b] = 4.9;
```

In[4]:=

```
m[K]   = 0.497671;
m[K1]  = 0.89610;
m[D]   = 1.8645;
m[B]   = 5.2787;
```

In[5]:=

```
L[Nm_, i_, q1_, q2_] :=
Nm Sqrt[x (1 - x)] Exp[- p^2/(2 w^2)] *
Exp[- m[i]^2/(2 w^2) *
(x - 1/2 - (m[q1]^2 - m[q2]^2)/(2 m[i]^2))^2]
```

In[6]:=

```
normalize[i_, q1_, q2_] :=
(aa = Integrate[L[n, i, q1, q2]^2,
{x, 0, 1}, {p, 0, Infinity}];
bb = Solve[aa == 1, n]; n /. bb[[1]])
```

D --> K

f0 = f1

In[7]:=

```
f0 = Integrate[   L[normalize[D, c, u], D, c, u]*
                  L[normalize[K, s, u], K, s, u],
                  {x, 0, 1}, {p, 0, Infinity}] //N
```

Out[7]=

```
0.760523
```

B --> K

f0 = f1

In[8]:=

```
f0 = Integrate[   L[normalize[B, b, u], B, b, u]*
                  L[normalize[K, s, u], K, s, u],
                  {x, 0, 1}, {p, 0, Infinity}] //N
```

Out[8]=

```
0.380694
```

D --> K*

hA0 = hA3

In[9]:=

```
A0 = Integrate[   L[normalize[D, c, u], D, c, u]*
                  L[normalize[K1, s, u], K1, s, u],
                  {x, 0, 1}, {p, 0, Infinity}] //N
```

Out[9]=

```
0.730952
```

hv and hA1

In[10]:=

```
j = Integrate[    L[normalize[D, c, u], D, c, u]*
                  L[normalize[K1, s, u], K1, s, u]/x,
                  {x, 0, 1}, {p, 0, Infinity}] //N
```

Out[10]=

```
1.07622
```

In[11]:=

```
Av = (m[c] - m[s])/(m[D] - m[K1]) j
```

Out[11]=

```
1.27803
```

In[12]:=

```
A1 = (m[c] + m[s])/(m[D] + m[K1]) j
```

Out[12]=

```
0.877159
```

A2

In[13]:=

```
A2 = (m[D] + m[K1])/(m[D] - m[K1]) A1 -
                              2 m[K1]/(m[D] - m[K1]) A0
```

Out[13]=

```
1.14774
```

B --> K*

hA0 = hA3

In[14]:=

```
A0 = Integrate[   L[normalize[B, b, u], B, b, u]*
                  L[normalize[K1, s, u], K1, s, u],
                  {x, 0, 1}, {p, 0, Infinity}] //N
```

Out[14]=

```
0.3216
```

hv and hA1

In[15]:=

```
j = Integrate[    L[normalize[B, b, u], B, b, u]*
                  L[normalize[K1, s, u], K1, s, u]/x,
                  {x, 0, 1}, {p, 0, Infinity}] //N
```

Out[15]=

```
0.372305
```

In[16]:=

```
Av = (m[b] - m[s])/(m[B] - m[K1]) j
```

Out[16]=

```
0.369536
```

In[17]:=

```
A1 = (m[b] + m[s])/(m[B] + m[K1]) j
```

Out[17]=

```
0.328604
```

A2

In[18]:=

```
A2 = (m[B] + m[K1])/(m[B] - m[K1]) A1 -
                              2 m[K1]/(m[B] - m[K1]) A0
```

Out[18]=

```
0.331468
```

Appendix C

In here we substitute from (3.24 - 3.29) and (3.30 - 3.34) into (2.23) in order to evaluate the amplitude for B -> K* ψ.

In[1]:=

```
pKY = (mB^2 - mK^2 - mY^2)/2;
pBY = (mB^2 - mK^2 + mY^2)/2;
pBK = (mB^2 + mK^2 - mY^2)/2;
pYBK = mB^2 - mK^2;
pBK2 = 2 mB^2 + 2 mK^2 - mY^2;
```

In[2]:=

```
f1 = -3;
f2 = 2 + pKY^2 /(mY^2 mK^2);
f3 = -mB^2 + pBK^2 /mK^2;
f4 = -mB^2 + pBY^2 /mY^2;
f5 = mB^2 - pBK^2 /mK^2 - pBY^2 /mY^2 +
                              pKY pBY pBK /(mY^2 mK^2);
```

In[3]:=

```
a1 = -(pYBK^2 f2 - pYBK^2 f1^2 + pYBK f1 f3 -
2 pYBK f5 + pYBK f1 f3 - 2 pYBK f5 + 4 f3 f4 -
pBK2 f1 f3 - mY^2 f1 f3 + 2 mY^2 f5 + 2 mY^2 f5 -
pBK2 mY^2 f2 - 4 mY^2 f1 f4 + pBK2 mY^2  f1^2);
```

In[4]:=

```
A1 = Simplify[a1]
```

Out[4]=

```
2 (mB - mK - mY) (mB + mK - mY) (mB - mK + mY)

   (mB + mK + mY)
```

In[5]:=

```
A = g[t]^2 A1 + f[t]^2 Simplify[f2] +
4 ap[t] f[t] Simplify[f5] + 4 ap[t]^2 Simplify[f3 f4]
```

Out[5]=

$$
\begin{aligned}
&((mB - mK - mY)^2 (mB + mK - mY)^2 (mB - mK + mY)^2 \\
&\quad (mB + mK + mY)^2 ap[t]^2) / (4 mK^2 mY^2) + \\
&((mB - mK - mY) (mB + mK - mY) (mB - mK + mY) \\
&\quad (mB + mK + mY) (mB^2 - mK^2 - mY^2) ap[t] f[t]) / \\
&\left(2 mK^2 mY^2 + \left(2 + \frac{(mB^2 - mK^2 - mY^2)^2}{4 mK^2 mY^2}\right) f[t]^2 + \right. \\
&\left. 2 (mB - mK - mY) (mB + mK - mY) (mB - mK + mY) \right. \\
&\left. \quad (mB + mK + mY)^2 g[t] \right)
\end{aligned}
$$

Appendix D

All numerical calculations in this Appendix are done using *Mathematica* . The *Mathematica* note book is organized in 5 parts. The first 4 contain all the definitions that *Mathematica* need, while the last part contains the commands for specific calculations.

Part 1 : Constants

In this part we define all the constants needed in the calculations.

In[1]:=

```
GF = 1.16639 10^(-5);
hh = 6.582122 10^(-25);
```

Above, hh = h/(2π) (the reduced Plank constant).

In[2]:=

```
Vud = 0.9753;
Vcs = 0.9743;
Vbc = 0.043;
```

In[3]:=

```
a1[Nc_] =  1.24 - 0.47/Nc;
a2[Nc_] = -0.25 + 1.11/Nc;
```

In[4]:=

```
m[pi]  = 0.1395679;
m[rho] = 0.7681;
m[a1n]  = 1.260;
m[psi] = 3.09693;
m[K]   = 0.497671; m[Kn]  = 0.493646;
m[K1]  = 0.89610;  m[K1n] = 0.89159;
m[D]   = 1.8645;   m[Dn]  = 1.8693;
m[B]   = 5.2787;
```

Above, are the masses of the needed mesons. Here, K1 = K* and the small n in Kn, K1n and Dn indicates the negative charge of these mesons.

In[5]:=

```
T[D]  = 4.20 10^(-13);
T[B]  = 12.9 10^(-13);
```

Above, the mean lifes of D and B mesons are defined.

In[6]:=

```
DC[pi]  = 0.132;
DC[rho] = 0.208;
DC[psi] = 0.382;
DC[aln] = 0.334;
```

Above, the decay constant of π, ρ and ψ mesons are defined.

Part 2 : Form Factors

In this part we define the form factors for D-->K(K*) and B-->K(K*).

In[7]:=

```
f0[D, 0] = f1[D, 0] = f1;
A1[D, 0] = A1;
A2[D, 0] = A2;
 V[D, 0] = V;
```

In[8]:=

```
A0[D, 0] = A3[D, 0] =
(m[D] + m[K1])/(2 m[K1]) A1 - (m[D] - m[K1])/(2 m[K1]) A2;
```

In[9]:=

```
f0[D, t_] = f0[D, 0] /(1 - t/2.60^2);
f1[D, t_] = f1[D, 0] /(1 - t/2.11^2);
A0[D, t_] = A0[D, 0] /(1 - t/1.97^2);
A1[D, t_] = A1[D, 0] /(1 - t/2.53^2);
A2[D, t_] = A2[D, 0] /(1 - t/2.53^2);
A3[D, t_] = A3[D, 0] /(1 - t/2.53^2);
 V[D, t_] =  V[D, 0] /(1 - t/2.11^2);
```

Above, are the form factors for D-->K (K*).

In[10]:=

```
f0[B, 0] = 0.408796 f1;
f1[B, 0] = 0.573283 f1;
A0[B, 0] = 1.34502 A1 - 0.764022 A2;
A1[B, 0] = 0.429363 A1;
A2[B, 0] = 0.187862 A1 + 0.436512 A2;
A3[B, 0] = 1.02352 A1 - 1.07394 A2;
 V[B, 0] = 0.640264 V;
```

In[11]:=

```
f0[B, t_] = f0[B, 0] /(1 - t/5.89^2);
f1[B, t_] = f1[B, 0] /(1 - t/5.43^2);
A0[B, t_] = A0[B, 0] /(1 - t/5.38^2);
A1[B, t_] = A1[B, 0] /(1 - t/5.82^2);
A2[B, t_] = A2[B, 0] /(1 - t/5.82^2);
A3[B, t_] = A3[B, 0] /(1 - t/5.82^2);
 V[B, t_] =  V[B, 0] /(1 - t/5.43^2);
```

Above, are the form factors for B-->K(K*). In here, we used the values of form factors at zero momentum produced in [[Table 2.2]].

In[12]:=

```
fn[B, P_, 0]  := (m[B]^2 - m[P]^2) f1[B, 0] *
                                    (1/5.89^2 - 1/5.43^2);
fn[D, P_, 0]  := (m[D]^2 - m[P]^2) f1[D, 0] *
                                    (1/2.6^2 - 1/2.11^2);
an[B, V1_, 0] := 2 m[V1] A3[B, 0] *
                                    (1/5.82^2 - 1/5.38^2);
an[D, V1_, 0] := 2 m[V1] A3[D, 0] *
                                    (1/2.53^2 - 1/1.97^2);
```

In[13]:=

```
fp[M_, P_, t_]  := f1[M, t]
fn[M_, P_, t_]  := (m[M]^2 - m[P]^2)/t *
                                    (f0[M, t] - f1[M, t])
ap[M_, V1_, t_] := A2[M, t]/(m[M] + m[V1])
an[M_, V1_, t_] := -2 m[V1]/t (A0[M, t] - A3[M, t])
 g[M_, V1_, t_] := V[M, t]/(m[M] + m[V1])
 f[M_, V1_, t_] := -(m[M] + m[V1]) A1[M, t]
```

Above, are the form factors as in second decomposition.

Part 3 : Decay amplitudes

In[14]:=

```
factor[B,  Nc_] = GF/Sqrt[2.] Vbc Vcs a2[Nc];
factor[D_, Nc_] = GF/Sqrt[2.] Vcs Vud a1[Nc];
```

In[15]:=

```
p[M_, M1_, M2_] := Sqrt[(m[M]^2 - m[M1]^2 - m[M2]^2)^2 -
4 m[M1]^2 m[M2]^2]/(2 m[M]);
```

M --> PP

In[16]:=

```
Amplitude[M_, Pk_, P2_, PP, Nc_] :=
(- factor[M, Nc] DC[P2] *
((m[M]^2 - m[Pk]^2) fp[M, Pk, m[P2]^2] +
m[P2]^2 fn[M, Pk, m[P2]^2]))^2
```

Above, is the square of the amplitude for decays of the type (B or D)--> PP.

M --> PV

In[17]:=

```
Amplitude[M_, Pk_, V_, PV, Nc_] :=
(factor[M, Nc] m[M] p[M, Pk, V] 2 DC[V] fp[M, Pk, m[V]^2])^2
```

Above, is the square of the amplitude for decays of the type (B or D)--> PV.

M --> VP

In[18]:=

```
Amplitude[M_, Vk_, P_, VP, Nc_] :=
(factor[M, Nc] DC[P] m[M]/m[Vk] p[M, Vk, P] *
(f[M, Vk, m[P]^2] + (m[M]^2 - m[Vk]^2) ap[M, Vk, m[P]^2] +
m[P]^2 an[M, Vk, m[P]^2]))^2
```

Above, is the square of the amplitude for decays of the type (B or D)--> VP.

■ M --> VV

In[19]:=

```
Amplitude[M_, Vk_, V2_, VV, Nc_] :=
(factor[M, Nc] DC[V2] m[V2])^2 *
(8 m[M]^2 p[M, Vk, V2]^2 g[M, Vk, m[V2]^2]^2 +
2 f[M, Vk, m[V2]^2]^2 +
((m[M]^2 - m[Vk]^2 - m[V2]^2) f[M, Vk, m[V2]^2] +
4 m[M]^2 p[M, Vk, V2]^2 ap[M, Vk, m[V2]^2])^2 /
(4 m[Vk]^2 m[V2]^2));
```

Above, is the square of the amplitude for decays of the type (B or D)--> VV.

■ Part 4 : Branching ratios definitions

In this part we define the transition probability per unit time (Γ = gamma) which leads to the branching ratios.

In[20]:=

```
gamma[M_, M1_, M2_, type_, Nc_] :=
1/(16 Pi m[M]) Amplitude[M, M1, M2, type, Nc] *
phi[m[M1]/m[M], m[M2]/m[M]] /hh
```

where phi[x, y] is the phase space correction factor defined below as

In[21]:=

```
phi[x_, y_] := Sqrt[(1 - (x + y)^2) (1 - (x - y)^2)]
```

Notice above that we devided Γ by hh. This is to allow it to have the appropriate units.

In order to get the branching ratio we just multiply Γ by the mean life time of the decaying particle.

In[22]:=

```
BranchingRatio[M_, M1_, M2_, type_, Nc_] :=
gamma[M, M1, M2, type, Nc] T[M] 100
```

We multiplied above by 100 in order to get the answer in %.

Part 5

In this part the branching ratios of several decays are evaluated. Before that we define the values of the form factors for D-->K,K* at zero momentum transfer and their uncertainities. They are organized as {E691, E653, ELC, BES, BSW} respectively.

In[23]:=

```
nf1 = {0.79, 0.71, 0.58, 0.90, 0.760};
nA1 = {0.46, 0.53, 0.53, 0.83, 0.877};
nA2 = {0.0,  0.43, 0.19, 0.59, 1.147};
nV  = {0.9,  1.06, 0.86, 1.43, 1.278};
```

In[24]:=

```
errf1 = {0.078, 0.06, 0.04, 0.04, 0};
errA1 = {0.05,  0.08, 0.03, 0.14, 0};
errA2 = {0.2,   0.15, 0.21, 0.14, 0};
errV  = {0.3,   0.30, 0.10, 0.45, 0};
```

D --> Kn π

N = Infinity

In[25]:=

```
BR[f1_] = BranchingRatio[D, Kn, pi, PP, Infinity] //N
```

Out[25]=

$$10.8744\ f1^2$$

In[26]:=

```
BR[nf1]
```

Out[26]=

```
{6.7867, 5.48178, 3.65814, 8.80825, 6.28104}
```

In[27]:=

```
error = 10.8744 (2 nf1 errf1)
```

Out[27]=

```
{1.34016, 0.926499, 0.504572, 0.782957, 0}
```

□ N= 3

In[28]:=

```
BR[f1_] = BranchingRatio[D, Kn, pi, PP, 3] //N
```

Out[28]=

$$8.30014\ f1^2$$

In[29]:=

```
BR[nf1]
```

Out[29]=

```
{5.18012, 4.1841, 2.79217, 6.72311, 4.79416}
```

In[30]:=

```
error = 8.30014 (2 nf1 errf1)
```

Out[30]=

```
{1.02291, 0.707172, 0.385126, 0.59761, 0}
```

■ D --> Kn ρ

□ N = Infinity

In[31]:=

```
BR[f1_] = BranchingRatio[D, Kn, rho, PV, Infinity] //N
```

Out[31]=

$$17.2526\ f1^2$$

In[32]:=

```
BR[nf1]
```

Out[32]=

```
{10.7674, 8.69705, 5.80378, 13.9746, 9.96512}
```

In[33]:=

```
error = 17.2526 (2 nf1 errf1)
```

Out[33]=

```
{2.12621, 1.46992, 0.800521, 1.24219, 0}
```

□ N = 3

In[34]:=

```
BR[f1_] = BranchingRatio[D, Kn, rho, PV, 3] //N
```

Out[34]=

$$13.1685\ f1^2$$

In[35]:=

```
BR[nf1]
```

Out[35]=

```
{8.21845, 6.63824, 4.42988, 10.6665, 7.60612}
```

In[36]:=

```
error = 13.1685 (2 nf1 errf1)
```

Out[36]=

```
{1.62289, 1.12196, 0.611018, 0.948132, 0}
```

■ D --> Kn a1n

□ N = Infinity

In[37]:=

```
BR[f1_] = BranchingRatio[D, Kn, a1n, PV, Infinity] //N
```

Out[37]=

$$6.21736\ f1^2$$

In[38]:=

```
BR[nf1]
```

Out[38]=

```
{3.88025, 3.13417, 2.09152, 5.03606, 3.59114}
```

In[39]:=

```
error = 6.21736 (2 nf1 errf1)
```

Out[39]=

```
{0.766227, 0.529719, 0.288486, 0.44765, 0}
```

□ N = 3

In[40]:=

```
BR[f1_] = BranchingRatio[D, Kn, a1n, PV, 3] //N
```

Out[40]=

$$4.74555\ f1^2$$

In[41]:=

```
BR[nf1]
```

Out[41]=

```
{2.9617, 2.39223, 1.5964, 3.8439, 2.74103}
```

In[42]:=

```
error = 4.74555 (2 nf1 errf1)
```

Out[42]=

```
{0.584842, 0.404321, 0.220194, 0.34168, 0}
```

■ D --> Kn* π

In[43]:=

```
redpower[a_ x_^n_] := a (Expand[x])^n
```

□ N = Infinity

In[44]:=

```
BR[A1_, A2_] =
BranchingRatio[D, K1n, pi, VP, Infinity] //redpower//N
```

Out[44]=

$$1.8868\ (-2.76997\ A1 + 0.977799\ A2)^2$$

In[45]:=

```
BR[nA1, nA2]
```

Out[45]=

```
{3.06333, 2.07083, 3.10248, 5.59605, 3.22674}
```

In[46]:=

```
error = 1.8868 2 (-2.76997 nA1 + 0.977799 nA2) *
(2.76997 errA1 + 0.977799 errA2) //Abs
```

Out[46]=

```
{1.60624, 1.45589, 1.39572, 3.40984, 0}
```

□ N = 3

In[47]:=

```
BR[A1_, A2_] =
BranchingRatio[D, K1n, pi, VP, 3] //redpower//N
```

Out[47]=

$$1.44015\ (-2.76997\ A1 + 0.977799\ A2)^2$$

In[48]:=

```
BR[nA1, nA2]
```

Out[48]=

```
{2.33816, 1.58061, 2.36804, 4.27133, 2.46289}
```

In[49]:=

```
error = 1.44015 2 (-2.76997 nA1 + 0.977799 nA2) *
(2.76997 errA1 + 0.977799 errA2) //Abs
```

Out[49]=

```
{1.22601, 1.11124, 1.06532, 2.60265, 0}
```

■ D --> Kn* ρ

In[50]:=

```
errordef[aa_ A1^2 + bb_ A1 A2 + cc_ A2^2 + dd_ V^2] :=
Abs[2 aa nA1 + bb nA2] errA1 +
Abs[2 cc nA2 + bb nA1] errA2 + Abs[2 dd] nV errV
```

□ N = Infinity

In[51]:=

```
BR[A1_, A2_, V_] =
BranchingRatio[D, K1n, rho, VV, Infinity] //Expand//N
```

Out[51]=

$$29.792\ A1^2 - 5.04352\ A1\ A2 + 0.396548\ A2^2 + 0.652246\ V^2$$

In[52]:=

```
BR[nA1, nA2, nV]
```

Out[52]=

```
{6.8323, 8.02534, 8.3574, 19.5257, 19.4275}
```

In[53]:=

```
errordef[BR[A1, A2, V]]
```

Out[53]=

```
{2.18665, 3.1175, 1.56052, 7.86705, 0}
```

□ N = 3

In[54]:=

```
BR[A1_, A2_, V_] =
BranchingRatio[D, K1n, rho, VV, 3] //Expand//N
```

Out[54]=

$$22.7395\ A1^2 - 3.84959\ A1\ A2 + 0.302675\ A2^2 + 0.497843\ V^2$$

In[55]:=

```
BR[nA1, nA2, nV]
```

Out[55]=

```
{5.21492, 6.12554, 6.37899, 14.9035, 14.8285}
```

In[56]:=

```
errordef[BR[A1, A2, V]]
```

Out[56]=

```
{1.66901, 2.37951, 1.19111, 6.00472, 0}
```

■ B --> K ψ

□ N = Infinity

In[57]:=

```
BR[f1_] = BranchingRatio[B, K, psi, PV, Infinity] //N
```

Out[57]=

$$0.116741\ f1^2$$

In[58]:=

```
BR[nf1]
```

Out[58]=

```
{0.0728579, 0.058849, 0.0392716, 0.09456, 0.0674294}
```

In[59]:=

```
error = 0.116741 (2 nf1 errf1)
```

Out[59]=

```
{0.0143872, 0.00994633, 0.00541678, 0.00840535, 0}
```

□ N = 3

In[60]:=

```
BR[f1_] = BranchingRatio[B, K, psi, PV, 3] //N
```

Out[60]=

$$0.0268971\ f1^2$$

In[61]:=

```
BR[nf1]
```

Out[61]=

```
{0.0167865, 0.0135588, 0.00904817, 0.0217866, 0.0155357}
```

In[62]:=

```
error = 0.0268971 (2 nf1 errf1)
```

Out[62]=

```
{0.0033148, 0.00229163, 0.00124803, 0.00193659, 0}
```

■ B --> K* ψ

□ N = Infinity

In[63]:=

```
BR[A1_, A2_, V_] =
BranchingRatio[B, K1, psi, VV, Infinity] //Expand//N
```

Out[63]=

$$0.543113\ A1^2 - 0.426772\ A1\ A2 + 0.109022\ A2^2 + 0.0594436\ V^2$$

In[64]:=

```
BR[nA1, nA2, nV]
```

Out[64]=

```
{0.163072, 0.142248, 0.157485, 0.324667, 0.228945}
```

In[65]:=

```
errordef[BR[A1, A2, V]]
```

Out[65]=

```
{0.0963458, 0.0890457, 0.0638624, 0.199052, 0}
```

□ N = 3

In[66]:=

```
BR[A1_, A2_, V_] =
BranchingRatio[B, K1, psi, VV, 3] //Simplify//N
```

Out[66]=

$$0.125133\ A1^2 - 0.0983282\ A1\ A2 + 0.0251187\ A2^2 + 0.0136958\ V^2$$

In[67]:=

```
BR[nA1, nA2, nV]
```

Out[67]=

```
{0.0375718, 0.032774, 0.0362845, 0.0748033, 0.0527488}
```

In[68]:=

```
errordef[BR[A1, A2, V]]
```

Out[68]=

```
{0.0221981, 0.0205161, 0.0147139, 0.0458617, 0}
```

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