Estimation and Filtering of Harmonics

by

Shaikh Uvais Ahmad Qidwai

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the Requirements for the Degree of

MASTER OF SCIENCE

In

ELECTRICAL ENGINEERING

June, 1997
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This thesis, written by Shaikh Uvais Ahmad Qidwai, under the direction of his Thesis Advisor and approved by his Thesis Committee, has been presented to and accepted by the Dean of the College of Graduate Studies, in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in ELECTRICAL ENGINEERING.

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Dedicated to my parents
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Many thanks to Allah Subhanahu wa Ta’la, who guided me and enabled me to complete this work successfully. I pray for His Mercy and Guidance in all countenance of life.

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ABSTRACT

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Title: Estimation and Filtering of Harmonics.
Major Field: Electrical Engineering.
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The extensive use of modern nonlinear power electronic devices and nonlinear loads has made the problem of harmonic distortion more challenging to researchers and utility engineers. Estimation of these serious harmonic interferences is necessary for the analysis and design of modern equipment and for the system security. The need for the effective elimination of such unwanted frequency components becomes essential in power system planning, analysis, and operation.

The work proposed in this thesis is two folds. In the first part, different frequency components of a distorted signal have been estimated. Different estimation algorithms were used and tested over standard signals with different frequency components present. The robustness of the algorithms was tested in the context of additive noise, for different noise levels, corresponding to different signal to noise ratios (SNR).

In the second part of the thesis, two new filtering schemes for harmonic elimination are proposed. These techniques have been studied for different types of practical signals reported in the literature. The robustness of the schemes has been tested in context with the additive noise of different SNRs. Apart from the simulation of these techniques, the prototype implementations on a small voltage scale are also presented and evaluated based upon the criteria of robustness, accuracy, and reliability.
خلاصة الرسالة

اسم الطالب: شيخ أوس أحمد قدواني
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إن الاستخدام المتزايد لأجهزة الطاقة الإلكترونية الغير خطية الحديثة وكذلك الأعمال غير خطية جعلت مشكلة التشوه التوافقية معضلة للباحثين والمختصين. إن تقييم هذه التوافقيات الدقيقة مهم في تحليل وتصميم الأجهزة الحديثة. وكذلك مهم حماية النظام المولد للطاقة الكهربائية. فالأخطار لطريقة شاملة لتقييم مثل هذه الترددات الغير مرغوبة أصبحت حاجة ماسة في نظم الطاقة الكهربائية في مراحل التخطيط والتحليل والتشغيل.

العمل المعرض في هذا البحث يتكون من جزئين رئيسيين. في الجزء الأول، تم تقدير الأجزاء الترددية المختلفة في الإشارة المشوهة. وقد استخدمت خوارزميات مختلفة لهذا التقدير وقد استعمل هذا التقدير باستخدام إشارات معتمدة بوجود أجزاء ترددية مختلفة. وكذلك أظهرت هذه الخوارزميات استخدام تشويش مضاف تضمنيات عدة وهذه المستويات تقابل نسب مختلفة من نسبة قوة الإشارة إلى قوة التشويش.

وفي الجزء الثاني تم عرض طريقتين جديدتين لترشيح تصلح لتقييم التوافقيات. تم دراسة واعتبار هاتين الطريقتين باستخدام أنواع عدة من الإشارات العملية المستخدمة في الأجهزة. وكذلك اعتبرت مئات هاتين الطريقتين باستخدام تشويش مضاف بنسب مختلفة من نسبة قوة الإشارة إلى قوة التشويش. وكجزء من محاكاة هذه الطريقة، تطبيقات التصميم التجريبي على الضغوط المخفضة قد تقدم وقيمتي على أساس المثا والدقة الإعتمادية.

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CHAPTER 1

INTRODUCTION

Harmonics are defined as the frequencies which are integral multiples of a fundamental frequency. This chapter introduces the problem of harmonics in electrical power systems. Various sources of harmonics are mentioned and the adverse effects produced by these harmonics are briefly discussed. The problem for the work undertaken in this thesis is stated with the motivation that there is a strong need to overcome the problems in the system created by the harmonics.

1.1. Harmonics.

Electrical power systems are designed to operate with the pure sine wave. This is due to the fact that single frequency sine wave is the purest form of wave that can be used. It does not change its shape even by crossing attenuators, differentiators or integrators. However, the presence of certain nonlinear loads, like electronic switching devices, arc furnaces, ballast tubes, and other nonlinear devices, causes the shape of pure sine wave to change [1,2,3,4]. The distortion produced is considered as a superposition of the fundamental frequency in the system with other harmonics frequencies.

The study of power system harmonics is an area of much concern among the power and control engineers. Such studies are used to provide the quantified information about the type and level of voltage and current distortion in the system. Much of the research is focused on the estimation and the elimination of the system harmonics, and it is an
outcome of this that from time to time new methods for both estimation, modeling, and elimination are developed.

1.2. Sources of Harmonics.

Harmonics are injected in the system through many different sources [1,2,3,5,6]. Some of these are as follows:

1. Transformer magnetizing currents, saturated cores and hysteresis.

2. Network non linearity from loads such as voltage controllers, frequency converters and similar power electronic devices.

3. A change in the equipment design philosophy. In the past, equipment design tended to be underrated or over-designed. Now, in order to be competitive, power devices and equipments are more critically designed and in case of iron-core devices, their operating points are more into non linear regions. Operating in these regions results in sharp increase in harmonics.

4. Interconnection of wind and solar power converters with distribution systems.

5. Tooth ripples in the voltage waveform of rotating machines.


7. Flux distortion in the machines due to sudden load changes.


9. HVDC power conversion and transmission.

10. Static VAR compensators.
11. The potential use of direct energy conversion devices, such as magneto hydrodynamics, storage batteries, and fuel cells, that require DC/AC power converters.

12. Cycloconverters used for low speed, high torque machines.

13. Pulse burst modulated heating elements for large furnaces.

14. Ballast driven fluorescent tubes and other such illumination equipment. The prime function of the ballast is to prevent current runaway and to operate the lamp at its correct electrical characteristics. While the use of ballast solves one big problem, i.e., of lamp ignition and operation, it produces two very harmful effects. Firstly, it lowers the power factor of the system and secondly, the magnetizing currents of the core of ballast produces the non-linearities in the system which appears as harmonics in the system.

1.3. Effects of harmonics.

Modern technology has made possible the rectification and chopping of the regular waveform in the power lines in order to make the power drives and other such applications. These are although more versatile than the past methods, but these can also have detrimental effects on the quality of the AC system specially when they comprise a significant portion of the system [2,5, 7-11].

These effects are generally called the "Pollution of line". The main effect is the generation of harmonics in the power lines. With the introduction of harmonics, several
problems are created in the system. The degree to which harmonics can be tolerated is determined by the susceptibility of the load (or power source) to them. The least susceptible are equipments like furnaces, where the main function is heating. Although, harmonics would contribute to the heating signal, yet these can produce the thermal stresses over the dielectric. The most susceptible loads are categorized in communication and data processing equipments, where the design and functioning relies over the sinusoidal nature of the electric signal present. An intermediate load is that of motors which are mostly tolerant to the harmonics. How different components of an electrical system are affected with harmonics, is given briefly as follows:

In rotating machines, harmonics produces additional heating due to the iron and copper losses at the harmonic frequencies, produces audible noise, and additional mechanical oscillations. These effects reduces the efficiency and life of the machinery. In transformers, production of audible noise, parasitic heating, core saturation, and insulation stresses are the main effects produced by harmonics. Similar problems of voltage and thermal stresses also exist in the cables.

Harmonic currents overload the capacitor banks, can produce voltage stresses because of resonance with the system causing shortened capacitor life, even rupture during the critical resonance condition.

Power electronic equipment is susceptible to misoperation caused by harmonic distortion because of changes in the properties of waveform. Other types of electronic equipment
can be affected by transmission of AC supply harmonics through the equipment power supply or by magnetic coupling of harmonics into the equipment components.

Metering and instrumentation are affected by the harmonic components, particularly if harmonic resonant conditions exist.

As with other types of equipment, harmonic currents can increase heating and losses in the switchgear, thereby reducing the steady state current carrying capability of the contacts and reducing the life of the insulation. Similarly, fuses suffer derating due to harmonic heating during normal operation and relays tend to operate slower and/or with higher pickup values, rather than faster operation and lower pickup value.

The presence of harmonic currents or voltages in circuitry associated with power conversion apparatus can produce magnetic and electric fields that will impair the satisfactory performance of communication system that, by virtue of their proximity and susceptibility, can be disturbed.

Static power converters generally are perpetrators in that they generate harmonics as an outcome of their operation. In some situations, the converters can themselves be affected by the harmonics, either self generated or from a similar unit usually in parallel. This include the thermal stresses in the associated circuits, abnormal control, commutation irregularity, and uncharacteristic harmonics generation.

1.4. Problem Statement

The work done in this thesis is composed of two parts:
1. Harmonic Estimation.

2. Harmonic Filtering.

1.4.1 Harmonic Estimation.

For a given distorted signal, composed of a fundamental frequency along with \( n \) harmonics buried in stochastic noise, the problem is to estimate the desired frequency components in the signal. This estimation refers to estimating the amplitude and phase of the concerned frequency component. The noise could be white Gaussian or Uniform, with zero mean.

In this problem, the distorted signal is first modeled as a linear combination of harmonic terms along with the noise, and is then converted to the standard form used in the estimation algorithms, called Normal equation. The algorithms used in this context are based upon the well known least square criterion. These are, least square estimator (LS), weighted least square estimator (WLS), recursive least square estimator information form (RLS-I), recursive least square estimator covariance form (RLS-C), extended least square estimator - 1 (ELS-1), and extended least square estimator - 2 (ELS-2). Some of these algorithms are recursive and can therefore be used online for harmonic estimation. The above algorithm are implemented and tested using distorted voltage signal from six pulse rectifier for different signal to noise ratios (SNR).
1.4.2 Harmonic Mitigation.

The second part of problem deals with the development of two new techniques for harmonic elimination. These scheme are based upon the comparison-reinjection principle. The main purpose is to devise such techniques that can compensate for the shortcomings in the currently existing methods, like detuning of filters, problems of drift due to thermal stresses, parametric deviation due to noise.

Other important feature for these schemes is their adaptive structure that can be used to eliminate different kinds of frequency components. This include non harmonic frequency components, sub synchronous frequency components, and high power noise. Specially. when non harmonic frequencies exist in the system, almost all of the current techniques either fail completely or degrade heavily in terms of their performance.

The two areas appears to be quite segregated from each other. However, they can be considered as complimentary to each other. By using estimation technique, a better understanding of the characteristics of the signal can be obtained. This information can be utilized for a better strategy of harmonic elimination.
CHAPTER 2

LITERATURE SURVEY

This chapter presents an extensive literature survey, carried out in connection with estimation and filtering of power system harmonics. Research papers have been found from almost all of the different periodical sources related to this subject. However, the IEEE Transactions on Power Delivery, IEEE Transactions on Industry applications, and IEEE Transactions on Power systems contain a large number of papers in this area. Bibliographies of such papers have also been reported in the literature as an important finding. [12,13]. Although, it is very difficult to classify the amount of work found in this field, it is presented here as two broad classes acclamatory to the two directions of work as stated in section 1.4.

2.1. Harmonic Estimation.

Harmonic estimation studies are focused to utilize the powerful theories from the fields of optimization, control, and signal processing. Based on these theories, estimation algorithms have been developed to estimate certain parameters from the signal in order to enhance the power of measurement in the system and to get the maximum information about the system.

2.1.1. Fourier Transform Method.

The Fourier algorithm for the estimation of power system harmonics is probably the first alternative, researcher took as a tool to identify the multiple sinusoids. The fundamental waveform of voltage (or current) in an electrical system may be defined as,

\[ y(t) = X_s \sin(\omega t) + X_c \cos(\omega t) \]  \hspace{1cm} (2.1)
The parameter \( X_a \) and \( X_b \) can be calculated from the \( N \) sampled values. Any sampled waveform \( f(t) \) can be approximated by a function \( f^*(c, t) \) where parameter \( c \) is adjusted to minimize the square error

\[
E^2(c) = \int_0^T \left[ f(t) - f^*(c, t) \right]^2 dt
\]  

(2.2)

Generally the approximation function \( f^*(c, t) \) can be described as polynomial.

\[
f^*(c, t) = \sum_{m=0}^{M-1} C_m \phi_m(t)
\]  

(2.3)

If we have \( N \) sampled values \( f_n \) of the function \( f(t) \), a total square error is given by,

\[
E^2 = \sum_{n=0}^{N-1} \left[ f_n - \sum_{m=0}^{M-1} C_m \phi_{mn} \right]^2
\]  

(2.4)

where \( \phi_{mn} = \phi_m(t_n) \) is the value of the function \( \phi_m \) at time \( t_m \), and \( [C_m] \) is a set of parameters which should be evaluated. The best estimate is specified as the estimate which minimized the mean square error of each signal component simultaneously. It requires that,

\[
\frac{\partial E^2}{\partial C_k} = -2 \sum_{n=0}^{N-1} \phi_{kn} \left[ f_n - \sum_{m=0}^{M-1} C_m \phi_{mn} \right] = 0
\]  

(2.5)

or

\[
\sum_{n=0}^{N-1} \sum_{m=0}^{M-1} C_m \phi_{mn} \phi_{kn} = \sum_{n=0}^{N-1} f_n \phi_{kn}, \quad k = 0, 1, \cdots, M - 1
\]  

(2.6)

Assuming the basic signal waveform as describe above, we have,

\[
C_1 = X_a \quad ; \quad \phi_{1n} = \sin(n\omega T)
\]

\[
C_2 = X_b \quad ; \quad \phi_{2n} = \cos(n\omega T)
\]  

(2.7)
where $T$ is the sampling time and $\omega$ is the fundamental frequency. Hence the solution for $X_a$ and $X_b$ are given as follows,

\[
X_a = \frac{1}{AC - B^2} \sum_{n=0}^{N-1} f_n \left[ C \sin(n\omega T) - B \cos(n\omega T) \right] \\
X_b = \frac{1}{AC - B^2} \sum_{n=0}^{N-1} f_n \left[ -B \sin(n\omega T) + A \cos(n\omega T) \right] \quad (2.8)
\]

where

\[
A = \sum_{n=0}^{N-1} \sin^2(n\omega T) \\
B = \sum_{n=0}^{N-1} \sin(n\omega T) \cos(n\omega T) \\
C = \sum_{n=0}^{N-1} \cos^2(n\omega T) \quad (2.9)
\]

This is the case when there is only fundamental present along with the additive noise. This is also the case when we are interested in estimating the fundamental only and not the other harmonics, which are treated as noise in the signal along with the additive noise.

Such formulations have been reported by many authors [14-21]. Apart from the classical algorithm, many extensions and improvements have been published.

Lobos [14], in 1989 compared FFT, Kalman Filters and the Walsh methods to estimate the basic waveform signals. It was discovered that the Fourier algorithm rejects the higher harmonics very efficiently. The Kalman method was found to be good for the non harmonic higher frequencies, while Walsh algorithm, although very simple, can not be used with a higher fidelity specially for odd harmonics. The same author, in 1991 presented a modification
for multiple frequency estimation and strategy to use the FFT algorithm recursively as given below [15].

\[
X_{an} = X_{an-1} \cos(\omega T) + X_{bn-1} \sin(\omega T) + \frac{2}{N} \sin(\omega T) (f_n - f_{n-N}) \\
X_{bn} = X_{bn-1} \cos(\omega T) - X_{bn-1} \sin(\omega T) + \frac{2}{N} \cos(\omega T) (f_n - f_{n-N})
\]  

(2.10)

Andria [16], and other co-workers, in 1992 compared the FFT algorithm with Extended Kalman Filter (EKF). Their findings were: using zoomed and windowed FFT give rise to very accurate off line measurements of the harmonic components and THD factor. Instead EKF technique can be very useful for real time monitoring of these quantities.

Zbigniew and Chwaleba [17], modified the FFT algorithm using the interpolation method to reduce the numerical short comings due to quantization errors in the acquisition units. Another improvement was from Moo and Chang [18,19] in 1995, when they developed a strategy to identify the group harmonic distortion. Their proposed strategy was successful in reducing the errors due to spectral leakage in FFT algorithm. In the same year, Nyarko and Stromsmoe [20] used trapezoidal compensating windows in the FFT algorithm to minimize the long term leakage effects, thus improving harmonic measurement accuracy.

During 1996, Chicharo and Xi [21], presented an improvement for recursive DFT algorithms by modifying the input data in such a way so that the sample sequence become an ideal sequence which is synchronized with the signal subjected to sampling. They showed a decrease in errors due to leakage in DFT algorithm and truncation problems in sampling wattmeters.
2.1.2. Kalman Filtering

Kalman filtering has been used for a number of systems in order to estimate different states or parameters in a system. For the distorted signal \( x(t) \), samples \( X(k) \) are obtained and can be used to model the signal as a state equation.

\[
X(k) = A(k)X(k - 1) + W(k - 1)
\]  
(2.11)

where \( X(k) \) = the \( n \times 1 \) process state vector, \( A(k) = n \times n \) transition matrix, and \( W(k) \) = the \( n \times 1 \) noise vector.

This signal is fed into the measurement instrument with the parameter \( C(k) \) and is distorted by additional measurement noise \( V(k) \). The model for measurement vector is given by,

\[
F(k) = C(k)X(k) + V(k)
\]  
(2.12)

where \( F(k) = m \times 1 \) measurement data vector, \( C(k) = m \times n \) matrix describing the connection between the measurement and the state vector and \( V(k) = \) measurement noise vector.

The measurement noise covariance matrix is given by,

\[
R(k) = E\{V(k)V(k)^T\}
\]  
(2.13)

The Kalman filter processes the measurement results to obtain a minimum error estimate of the state vector \( X(k) \). The general recursive Kalman filter equation is described by,

\[
\hat{X}(k) = A(k)\hat{X}(k - 1) + K(k)\{F(k) - C(k)A(k)\hat{X}(k - 1)\}
\]  
(2.14)
where \( \hat{X}(k) \) is the estimate of the signal \( X(k) \) at the time \( t_k \), which simultaneously minimizes the least square error for each signal component. The solution to the estimation equation is given by the equation for Kalman filter gain \( K(k) \) in the form of \( n \times m \) matrix,

\[
K(k) = P(k)C(k)^T R(k)^{-1}
\]

(2.15)

where the error covariance matrix \( P(k) \) is given by,

\[
P(k)^{-1} = P(k-1)^{-1} + C(k)^T R(k)^{-1} C(k)
\]

(2.16)

This method was used by Lobos [14] in 1989 for fundamental harmonic estimation in a power signal and has found it to be very efficient, and much superior than the Fourier algorithm.

Dash and Sharaf [22], in 1988 used Kalman filtering technique for estimation of harmonics in the power system by using simple, linear and robust algorithm to estimate the magnitude of the known harmonics embedded in the signal along with stochastic noise. The model considered by them is as follows,

\[
z_k = c_k^0 + \sum_{l=1}^{N} \left[ c_k^l \cos\left(\frac{2\pi kl}{2N+1}\right) + s_k^l \sin\left(\frac{2\pi kl}{2N+1}\right) \right] + v_k
\]

(2.17)

where \( z_k \) is the measurement vector, \( v_k \) is the additive noise, and \( c_k^l \) and \( s_k^l \) are slowly time varying functions, and are the required parameters to be estimated.

The time invariant model for this signal is given by,

\[
\begin{align*}
X_{k+1} &= FX_k \\
z_k &= H^T X_k + v_k
\end{align*}
\]

(2.18)

where
\[ F = \begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
\cos 2\theta & \sin 2\theta \\
-\sin 2\theta & \cos 2\theta \\
\vdots & \vdots \\
\cos N\theta & \sin N\theta \\
-\sin N\theta & \cos N\theta
\end{bmatrix} \]

\[ X_k = \begin{bmatrix}
c^k \cos k\theta + s^k \sin k\theta \\
-c^k \sin k\theta + s^k \cos k\theta \\
c^2 \cos 2k\theta + s^2 \sin 2k\theta \\
-c^2 \sin 2k\theta + s^2 \cos 2k\theta \\
\vdots \\
c^N \cos Nk\theta + s^N \sin Nk\theta \\
-c^N \sin Nk\theta + s^N \cos Nk\theta
\end{bmatrix} \]

\[ H^T = \begin{bmatrix} 1 & 0 & 1 & \cdots & 1 & 0 & 1 \end{bmatrix} \]

and

\[ \theta = \frac{2\pi}{2N + 1} \]

Using the least error squared performance criterion approach, the estimator is given by,

\[ X_{k+1} = \left[F - K_k H^T\right] X_k + K_k z_k \quad (2.19) \]

where the Kalman filter gain \( K_k \) is obtained as follows,

\[ K_k = P_k H \left[H^T P_k H + R\right]^{-1} \quad (2.20) \]

and the covariance \( P_k \) is given by,

\[ P_{k+1} = \left[I - K_k H^T\right]^{-1} P_k \quad (2.21) \]
Their results show a better noise rejection and estimation compared to FFT algorithms. They also presented an estimation model for time varying multiple harmonic estimation case. A similar study was made by Ma and Girgis [23] in 1996. They showed that the Kalman filter can also be used to solve the problems in harmonic source identification and the optimal dynamic estimates of harmonic injections and their locations. Unlike Dash, who used magnitudes as states, they have used harmonic injections as the state variables.

2.1.3. Neural Networks.

Neural networks have been used in many areas of science and engineering. Many researchers have applied the same in the estimation of sinusoids. Osowski [24] in 1992, used optimization neural networks under quadratic objective function with global minimum, for the estimation of harmonic components in the power system. In the same year, Mori and other co-workers [25], used the back propagation learning technique for feed forward neural networks, in order to estimate the voltage harmonics in power systems. They used three layered neural network, and have found it comparable to standard adaptive estimation algorithms. A similar work has been done by Zrida and other co-workers [26] in 1995. They have used single neuron and online adaptive learning of the network along with the estimation.

2.1.4. Maximum Likelihood Method.

Among the classical methods of estimation of frequencies and their respective amplitudes and phases, maximum likelihood estimation algorithms have been used by many researchers in the
area. The distorted signal is modeled as a complex waveform, having \( N \) data points corresponding to a DC level and \( M \) complex sinusoid embedded in circular (real and imaginary parts of noise are uncorrelated) Gaussian noise,

\[
y[k] = x[k] + w[k], \quad k = 0, \ldots, N - 1
\]
\[
x[k] = \sum_{m=0}^{M} B_m e^{j \omega_m^k}, \quad B_m = b_m e^{j \theta_m}, \quad m = 0, \ldots, M
\]
\[
\theta = 2\pi f T_s, \quad N \geq 2M
\]

where \( f \) is the fundamental frequency and \( T_s \) is the sampling time.

The maximum likelihood principle is rather simple and its results are widely used by the theory of statistical estimation. In this case, the number \( M \) of harmonic components must be known, and the amplitudes and phases should not vary with time and should not be considered as deterministic.

The maximum likelihood (ML) estimator \( \hat{B} \) for the parameter vector \( B \) is one that maximizes the probability density function of the signal or its function, called the loglikelihood function. Since the additive noise is complex Gaussian circular, the loglikelihood function for the above discussed model can be given by,

\[
L(B,Y) = -N \text{Log} [\pi] - \text{Log} (\text{det}(C)) - (Y - EB)^H C^{-1}(Y - EB)
\]

\[
\text{with} \quad [C]_{ij} = R[w_i - j] \quad \text{for} \quad i, j = 1, \ldots, N
\]

where \( E \) is a matrix that represents the position of frequencies on the unit circle, \( H \) represents the conjugate transpose operation, \( C \) is the covariance matrix of noise, and \( R \) represents the
autocorrelation function. Furthermore, since $C$ does not depends upon the parameter vector $B$, the objective function reduces to the minimization of a simplified criterion $J(B)$ over $B$,

$$J(B) = (Y - EB)^H C^{-1} (Y - EB)$$

$$\Rightarrow \hat{B} = \left(E^H C^{-1} E\right)^{-1} E^H C^{-1} Y = H(\theta)Y$$

(2.24)

This maximum likelihood estimator has many desirable properties, like unbiasedness, which means that its averaged value is equal to the true unknown value, and it is efficient, which means that its covariance matrix is equal to the lower possible bound (called the Cramer-Rao bounds) for any estimate. The above two properties can be written as follows,

$$E[\hat{B}] = B \quad \text{and} \quad \text{Var}(\hat{B}) = \left(E^H C^{-1} E\right)^{-1} = \text{Cramer-Rao Bound}$$

(2.25)

In general, if an estimator exists that is unbiased and reaches the Cramer-Rao bound, then it is a maximum likelihood estimator. Consequently, the statistical performance of all other estimators are lower than the performance of maximum likelihood estimator.

Rife and co-workers [27,28] in 1974 and 1976, probably did one of the earliest work in this area, when they estimated single as well as multiple tones in a multi-tone signal using finite number of noisy discrete-time observations. They have developed the appropriate Cramer Rao bounds and the related properties for the maximum likelihood algorithm used in their study. They have also developed other algorithms that are not as complex as the maximum likelihood algorithms but can yield results close to those found with maximum likelihood algorithm.
The algorithm given in equation 2.8 was formulated by Gourad and others [29] in 1994. Their specific concern was power system harmonics and they have shown better results for moderately stationary conditions in the signal.

Kay [30], in 1989 proposed a new frequency estimator for a single complex sinusoid in complex white Gaussian noise. The algorithm is more computationally effective. However, there are some stringent assumptions like differenced phase data to avoid unwarping of phase, etc. Further improvements in maximum likelihood algorithms kept on coming. Chicharo and Wang, in 1993 and 1994 [31,32], proposed a new recursive algorithm to resolve the problem of power system harmonic signal retrieval. Their algorithm was based upon the Approximate maximum likelihood structure. The scheme was an adaptive Infinite Impulse Response (IIR) Line Enhancer (LE) comb filter.

Another recursive maximum likelihood estimator was proposed by White [33] in 1993. His proposed algorithm was based upon the recursive ML algorithm for incomplete data, and the parameters to be estimated were period and the complex amplitude of the harmonics present.

2.1.5. Singular Value Decomposition.

Another technique used in the estimation domain is Singular value decomposition (SVD). Osowski [34], in 1994 proposed an algorithm to estimate the harmonic amplitudes and phases. His algorithm was used off-line in the work and has found it to work well under moderately high SNRs. Sreenivas and co-workers [35] in 1992, used the same formulation in conjunction
with the statistical properties of zero crossing intervals. Their proposed method worked well for the low frequency spectrum and in the presence of low SNR noise.

2.1.6. Least Square Method.

A very popular method for parameter estimation is Least Square (LS) algorithm. The first part of the work done in this thesis, i.e., estimation of power system harmonics, is based on various formulations of LS algorithms. The details related to the algorithms are discussed in chapter 3. Sachdev and co-workers [36] used LS algorithm to estimate the line frequency in the power system. They utilized the digitized samples from the network and implemented the LS curve fitting technique to estimate the frequency. Narendra and Chandrasekhariah [37], in 1993, proposed a matrix method for using a least square criterion to approximate the given signal with that of the assumed Fourier series representation. Emphasis is given for waveforms in the transient regions like relaying operations etc.

Terzija and co-workers [38] in 1995, proposed a new self tuning digital signal processing algorithm for local system frequency deviation measurement. The algorithm utilized the non-recursive LS along with an updating mechanism that renders the improvements in the algorithm like measurement range, immunity to noise and accuracy.
2.1.7. Least Absolute Value Estimator.

This method is based upon the minimization of sum of the absolute values of residuals. The LAV has been known for a long time, but the algorithm that uses this criterion are not very old. In this method, the objective function $J$ is given by:

$$ J(x) = \sum_{i=1}^{m} |Z_i - \sum_{j=1}^{n} A_{ij} \theta_j| $$

which can be written as

$$ J(x) = \sum_{i=1}^{m} |r_i| $$

(2.26)

where $r_i$ is the residual (error). The problem can be solved numerically by using Simplex method from the linear programming techniques.

The least absolute value (LAV) algorithm is another algorithm to optimally track the parameters of power system harmonics. Soliman and others [39] in 1995, presented this algorithm in an application related to power system harmonic estimation. They have used the following system model,

$$ X(k+1) = \phi(k) X(k) + \Gamma(k) u(k) $$

(2.27)

where $X(k)$ is the $n*1$ state vector of estimates at step $k$, $\phi(k)$ is the $n*n$ state transition matrix, $\Gamma(k)$ is the $n*s$ control matrix, and $u(k)$ is the $s*1$ control vector at step $k$. The initial condition $X(0)$ is a Gaussian random vector with the following statistics.
\[ E[X(0)] = \bar{X}(0) \]
\[ E[(X(0) - \bar{X}(0))[X(0) - \bar{X}(0)]^T] = M(0) \tag{2.29} \]

The input \( u(k) \) is a Gaussian random vector sequence with the following properties,
\[ E[u(0)] = u(0) \]
\[ E[(u(k) - \bar{u}(k))[u(k) - \bar{u}(k)]^T] = V(k)\delta_{ij} \tag{2.30} \]

where \( \delta_{ij} \) is the Kronecker delta.

The measurement model for the process is given by,
\[ z(k) = H(k) x(k) + v(k) \tag{2.31} \]

where \( z(k) \) is the m*1 vector of measurements, \( H(k) \) is the m*n matrix giving the ideal noiseless connection between the measurement and the state vector, and \( v(k) \) is the m*1 measurement error, assumed to be a white sequence with known covariance.

The other covariances needed in the algorithm are given as follows,
\[ M(k) = \phi(k) P(k-1)\phi^T(k-1) + \Gamma(k-1)\nu(k-1)\Gamma^T(k-1) \]
\[ P(k) = [(I - K(k)H(k))M(k)[I - K(k)H(k)]^T + K(k)R(k)K^T(k)] \tag{2.32} \]

Here, \( K(k) \) is the filter gain at step \( k \) and is given by,
\[ K(k) = [H(k) + R(k)CM^{-1}(k)]^{-1} \]
\[ C = Le^T \tag{2.33} \]

with \( L \) and \( e \) being m*1 and n*1 unity column vectors, respectively. The estimation algorithm is given by,
\[
\hat{X}(k) = \bar{X}(k) + K(k)[Z(k) - H(k)\bar{X}(k)]
\]
where
\[
\bar{X}(k) = \phi(k - 1)\bar{X}(k - 1) + \Gamma(k - 1)u(k - 1)
\]

(2.34)

It is assumed that the model parameters: $\phi, H, \Gamma, R, M(0)$ and $V$ are known.

Najjar and Heydt [40] made an interesting study about prediction of harmonic bus voltages and line currents, in 1990. Their proposed scheme utilized the quadratic LS and non quadratic LAV criteria and used SVD on a feasible application in the large scale system. Kim and other co-workers [41], used modified Kay estimator to propose a new single frequency estimator in 1996. Their algorithm produced lower threshold than Kay estimator and at the same time, it was computationally efficient. The threshold was shown to be $20 \log_{10} K$ dB less than the Kay estimator. However, unlike their related method. Djuric and Begovic [42] proposed methods that utilized the auto regression as underlying model and determined the frequency directly, without estimating its parameters separately. However, their simulation results were focused for signals in very weak noise.

2.1.8. Discussion

Many techniques for harmonic estimation are known in the literature, however, the most popular methods for this purpose have been discussed in section 2.2. These techniques have produced good results under certain conditions, but all of them have limitations. This subsection discusses the conclusion from this and it is summarized as follows.
1. Fourier algorithms are although the very fundamental algorithms, but they are constrained by many properties like Aliasing, spectral leakage, and picket fence effect.

2. The noise rejection properties of Fourier algorithms are not very good, and the algorithm works well only in low noise signals.

3. Fourier algorithms are computationally very extensive and requires more time than other techniques.

4. Kalman filtering, although much superior than Fourier techniques, but it requires a very careful problem presentation.

5. The computational burden in Kalman algorithm is very large, although much less than the Fourier techniques.

6. Compared to Fourier, it is easy to use Kalman algorithm for Online estimation.

7. The most serious problem with Kalman filtering is its sensitivity to high noise. This requires pre filtering to reduce the noise content.

8. The least absolute value (LAV) algorithm is not very old and is quite promising with respect to its noise rejection properties even in moderately high noise levels.

9. However, LAV requires a higher number of samples for accurate estimation.

10. The best estimator, theoretically, is Maximum likelihood (ML) estimation algorithm. The most desirable properties of unbiasedness, efficiency, and consistence are found in this estimator. It is also used as standard for all other estimator, because the performance of this estimator to superior to all other estimators. It has been used in some applications, but its
use is quite problem dependent and computationally extensive. Further, the time invariance condition is a constraint in practical implementation. In fact, the estimator fails for even slight frequency deviation. However, under the conditions of uncorrelated, zero mean, white, Gaussian noise, the LS estimator behaves quite closely to the ML estimator, and hence the estimate acquires the properties of efficiency, unbiasedness, and consistency.

The harmonic estimation which is carried out in this thesis, is based upon the least square (LS) algorithms. The theory behind these algorithms is although very old, but the use of LS algorithms to power system harmonic estimation is quite new. In fact, new properties and improvements in the basic algorithm are now finding the LS algorithm and its extension to be a good alternative to other techniques. Therefore, in this work, it is intended to use LS to overcome the shortcomings from the other algorithms, specially insensitivity to noise and frequency deviation and computational efficiency.

2.2. **Harmonic Elimination.**

The second classification is related to the measures that are usually taken to eliminate the hazardous, unwanted signals. Different techniques are found in the literature.

The most popular method to reduce harmonic distortion is to use harmonic filters in the system. There are basically two types of filters. (i) Passive, where the filter components are passive elements such as resistor, inductor, and capacitor, and (ii) active, where the filter has controlled current or voltage source. Among the passive filters, there are still two classes. (a) Series filter, that are placed in series with the load flow direction and stops the higher
harmonics from traveling further in the network. and (b) Shunt filters, which are placed in parallel with the load and gives an easy path to higher harmonics, thus preventing them from entering into the system. The primary objective of a filter is to reduce the amplitude of one or more frequency components in the signal. Lot of work has been and is being done on the improvements and efficiency enhancement of these filters.

Figure 2.1 shows a typical electrical network. Usually, the shunt filters $S$ are used in the system to makeup for the distortion. Specially, compound filters like SC, that has many tuning stages to filter out different frequencies like $a$ or $b$ and one or more high pass stages like $c$.

Series filters $SF$, on the other hand, are used in very few applications where the load is light and distortion is mostly due to the voltage harmonics.

Elham and co-workers [43], in 1993, presented a study in which they have shown how to design appropriate filters based upon the noisy data available in the system. They also have presented the steady-state and harmonic filters designed for real world systems. A similar study was made by Merhaj and Nichols [44] in 1994. Their findings revealed some important properties of filters used in the off-shore systems. Specially, the resonant behavior A more generalized approach to design filters according to the data available in the system was presented by Phipps [45] in 1995. His approach uses the transfer function based procedure in conjunction with the new IEEE-519 harmonic distortion limits. The philosophy which governs the design is based on a numerical/graphical iterative solution.
Peng and others [46], in 1990, proposed an arrangement to take advantage of both active and passive filters. They have used shunt passive filters along with small rated active series filters thus incorporating both practical and economical points of view. A similar study is also performed by Rastogi and his co-workers [47] in 1995.

However, people are now concentrating to find out the solution of filtering in terms of active filters. Enjeti and others [48], in 1994, proposed a new topology for the implementation of active power filters in order to cancel harmonic currents in the three phase four-wire system. This scheme has successfully reduced the undesirable frequency components in the neutral wire and drastically improved the electric system quality. Similar scheme for similar functions was proposed by Koozehkanani and others [49] and Lott etc. [50] during 1995.

Zhang and Asplund [51] in 1994 presented a practical active filter using latest digital signal processing (DSP) and pulse width modulation (PWM) techniques. In this scheme, the harmonic current is measured by a transducer and is fed to a DSP unit. This unit calculated the necessary control pulses to be given to the power stages where the harmonic components are re-injected in the system with opposite phases.

Another type of filtering technique is adaptive filtering in which the filter parameters are updated as per system changes. The adaptation structure is similar to the adaptive frequency cancellation algorithm, as discussed in previous section, however, for the realization, filter bank switching or line conditioning units are employed. This recent strategy is at the moment a good compromise for the existing system with respect to the solution present and the
Figure 2.1. A typical Electrical Network. HVT is a high voltage transformer, LVT is low voltage transformer, R = relay, M represent motor loads, HL = heating load, DL = distribution load, S is the shunt filter, SF is the series filter, and SC is the compound shunt filter.
economy involved. Ashton and Emanuel [52], in 1991 proposed a method for voltage harmonic minimization based on the use of line conditioners controlled by means of an adaptive system. It combines the features of measurement and estimation algorithms with the appropriate remedial actions.

A similar work was done by Bryan and Orchard [53] in 1996. However, they have used filter banks in conjunction with the stochastic gradient adaptive filtering algorithms, conforming to the mean square criterion. Fukuda and Sugawa [54] proposed a simple and improved DSP controlled active power filter. They have utilized an adaptive digital filter operating as series compensator for the active filter, thus improving its performance.

Some groups of researchers also viewed the harmonic problem from an entirely different prospective. Since the problem is created in the system by the controlled power electronic devices, people have tried to develop new controlling scheme that will still serve the same purpose but at reduced harmonic pollution levels. These schemes are usually named as Pulse Width Modulation (PWM) techniques.

One such scheme is based on adding a harmonic current to the rectangular waveform produced by converter circuits. Such schemes utilizes the triplen harmonics and an external triplen source. This principle, as shown in Figure 2.2 (a), can be used to reduce some of the harmonics at a given operating point [2].

For controlled rectifiers, the output ripple can be removed by injecting the equivalent AC ripple on the AC side. The principle of such DC ripple reinjection is shown in Figure 2.2(b).
Bowes and Clark [55] has presented the implementation and generation of different PWM based strategies for INMOS transputers as well as the design for microprocessor based implementation. Their scheme corresponds to the derivation of new switching equations and thus defining new switching angles in order to minimize the angle difference between the exact harmonic switching angles and those produced from the regular sampling process, which reduces the harmonic amplitudes.

An improvement in the performance of such PWM controllers was proposed by Zargari and others [56] in 1994. They proposed a systematic approach to design the input filter in an efficient way in order to satisfy the filtering requirements of the current-source PWM filters and hence improve the filter performance.

Sun and co-workers [57], in 1996, proposed an optimal PWM technique for harmonic reduction in single switch three phase boost rectifiers. They have developed a scheme to minimize the input current THD and maximize the converter power by injecting a sixth harmonic to the duty ratio signal. They have analyzed the closed loop system with a PI controller with low crossover frequency, and have found it quite successful in reducing the THD in the signal.

In 1993, Phipps and Nelson [58] proposed a practical design aspect for controlling power system harmonics through the use of transformer connections applied to a distributed set of
Figure 2.2. Harmonic elimination (a) by triplen reinjection, (b) DC ripple reinjection.
industrial loads. For six pulse converters, they have shown that by introducing a shift of 30° between the sets of such converters, significant reduction in harmonic amplitudes can be achieved, provided the relative current amplitudes and firing angles are fairly coincident.

Ned Mohan [59], in 1993, proposed an approach to achieve nearly sinusoidal line current rectification of 3-phase utility voltages by DC link current modulation. The proposed scheme was characterized with simple design and less disadvantages as found in passive filters.

Another very popular method for harmonic elimination is current injection. This corresponds to the injection of equal but anti-phased harmonic current in order to cancel them out. This method is characterized with elimination of a large number of harmonics applicable to a variety of loads [60]. The earliest works in this area were from Bird [61] and Amentani [62]. They have presented studies in which certain frequencies were injected into the converter to cancel the harmonic components.

Arillaga and co-workers [63,64], presented methods to determine a current waveshape which, when injected into six-pulse converter transformer secondary, increases the pulse number of a bridge. Lawrence and others [65], in 1996 proposed a similar methodology and has presented good results.

Kim and others [66], in 1994, proposed another method to improve power factor and to reduce the harmonics. The scheme comprise of a novel interconnection of a star-delta transformer between the AC and the DC side of the diode rectifier configuration. This interconnection, along with the 120° conduction period of diode produces a third harmonic
Figure 2.3. Flux compensation scheme by Sasaki [67].
current circulating between the AC and the DC side of the bridge and is responsible for the elimination of harmonics.

The flux compensation method was first proposed by Sasaki in 1971 [67]. The method corresponds to cancellation of harmonics in the line by making use of specially designed transformers in which the cancellation can take place through the magnetic flux changes. This is shown in Figure 2.3. A current transformer is used to detect the harmonic components coming from the nonlinear load and these are fed, through an amplifier, into the tertiary windings of a transformer in such a way as to cause cancellation of harmonic currents concerned. The main area of concern with this system involves the coupling of the output of the amplifier to the tertiary winding, in such a way that the fundamental current flow does not damage the amplifier. A quaternary winding and filter are used to reduce the fundamental current in the amplifier output.

Based on similar approach, Bhattacharya [68], in 1995, proposed synchronous frame flux based control method for parallel active filter applications. Although, the theory of flux based compensation is very old, not much of the work has been done in this area.

2.2.1. Discussion

Various harmonic elimination methods which have been briefly described in section 2.2. are being used in one place or the other. All of the methods have produced good results within certain constraints. However, as reported by different researchers, these methods also have
certain limitations of operation. This subsection discusses the pitfalls in the above mentioned techniques.

Filtering techniques are mostly used for by passing the higher frequency components. However, due to following problems in filters, their performance is degraded.

- Non-idealistic cutoff and attenuation characteristics of filters can vary with changing frequency components.
- Changes in filter parameters due to alteration in capacitor and inductor values which is a result of thermal and voltage overstresses in the filter.
- Reduced performance of filters for unexpected frequencies and higher noise levels.

The injection and other PWM based techniques are good only for certain specific configuration. These cannot be generalized as a solution for any other harmonic source.

The flux based techniques, as reported in the literature is limited due to technological constraints and are still under experimental stages.

Due to these problems, this work is intended to propose new schemes for harmonic elimination. The emphasis is on the use of technologically possible, yet general purpose solutions for harmonic elimination or reduction within the environment of uncertainties and high power noise.
CHAPTER 3

ESTIMATION OF SINUSOIDS

This chapter presents the estimation of power system harmonics present in a distorted voltage or current waveform. Different noise levels have been considered corresponding to different signal to noise ratios (SNR). The methods used here are different Least Square (LS) based algorithms. Simulation has been performed in MATLAB and PSPICE software. The results have been presented in the form of time plots, frequency plots and comparative tables.

3.1. Signal Model.

The general structure for any linear system embedded in additive noise is given by the following model,

\[ Z(k) = \Phi(k)\theta(k) + v(k) \] (3.1)

where,

\[ Z(k) \] is the noisy measurement,

\[ \Phi(k) \] is the system structure matrix.

\[ \theta(k) \] is the vector for unknown parameters that are to be estimated, and

\[ v(k) \] is the additive noise.

For the case of a voltage or current signal with fundamental frequency \( \omega_i \), the harmonics are the multiples of this frequency. Hence the equation 3.1 can be written as follows:

\[ Z(k) = A_1 \sin(\omega_1 k + \psi_1) + A_2 \sin(2\omega_1 k + \psi_2) + \cdots + v(k) \] (3.2)

where, \( A_i \) is the Amplitude of the \( i^{th} \) harmonic and \( \psi_i \) is its phase. Equation 3.2 is highly suitable to be used with the identification and estimation algorithms. For the estimation of the
fundamental sinusoid, it is required to consider rest of the distortion present in the signal.

either noise or other harmonics. as the additive noise. the new structure will become.

\[ Z(k) = A_i \sin(\omega_i k + \psi_i) + \mu(k) \]  \hspace{1cm} (3.3)

By simple manipulation, this equation can be modified for the purpose of estimation:

Since.

\[ A_i \sin(\omega_i k + \psi_i) = A_i \sin(\omega_i k) \cos(\psi_i) + A_i \sin(\psi_i) \cos(\omega_i k) \]  \hspace{1cm} (3.4)

Therefore, the new model can be written as:

\[ Z(k) = [\sin(\omega_i k \cos(\omega_i k)] \begin{bmatrix} \alpha \\ \beta \end{bmatrix} + \mu(k) \]  \hspace{1cm} (3.5)

or. in the standard form.

\[ Z(k) = \Phi(k) \theta + \mu(k) \]  \hspace{1cm} (3.6)

where. \( \alpha \) and \( \beta \) are the parameters to be estimated. and are given by.

\[ \alpha = A_i \cos \psi_i \]
\[ \beta = A_i \sin \psi_i \]  \hspace{1cm} (3.7)

The Actual required parameters are \( A_i \) and \( \psi_i \), which can be given in terms of \( \alpha \) and \( \beta \) as.

\[ A_i = \sqrt{\alpha^2 + \beta^2} \]
\[ \psi_i = \tan^{-1} \frac{\beta}{\alpha} \]  \hspace{1cm} (3.8)

The same model can be modified in order to estimate the amplitudes and phases of different harmonics in the signal. The modification is in the observation matrix \( \Phi \) and parameter matrix \( \theta \). In this case. the assumption. which was made earlier for considering the harmonics as a kind of the noise present in the signal, is now discarded. Each harmonic has its
representative term appearing in the matrix $\Phi$. However, it is assumed that one has the knowledge of the order of harmonics that are present. The idea can also be extended by assuming that certain harmonics are present in the signal, and if they are not there, the estimation algorithm has given estimates close to zero, unless if the noise is appreciably high. in which case, the assumed harmonics may actually present. The new equations are as follows:

$$
\Phi(k) = \begin{bmatrix}
\sin(\omega_{11}) \cos(\omega_{11}) & \sin(5\omega_{11}) \cos(5\omega_{11}) & \ldots & \sin(13\omega_{11}) \cos(13\omega_{11}) \\
\sin(\omega_{12}) \cos(\omega_{12}) & \sin(5\omega_{12}) \cos(5\omega_{12}) & \ldots & \sin(13\omega_{12}) \cos(13\omega_{12}) \\
\vdots & \vdots & \ddots & \vdots \\
\sin(\omega_{1n}) \cos(\omega_{1n}) & \sin(5\omega_{1n}) \cos(5\omega_{1n}) & \ldots & \sin(13\omega_{1n}) \cos(13\omega_{1n})
\end{bmatrix}
$$

and

$$\theta(k) = [\alpha_1 \beta_1 \alpha_5 \beta_5 \ldots \alpha_{13} \beta_{13}]^T$$

(3.9)

### 3.2. Algorithms used in Simulation.

The above models were simulated by six well known methods based on the least square criterion, namely, Least Square Estimator (LS), Weighted Least Square Estimator (WLSE), Recursive Least Square Estimator Information Form (RLS-I), Recursive Least Square Estimator Covariance Form (RLS-C), Extended Least Square Estimator-1 (ELS-1) and Extended Least Square Estimator-2 (ELS-2). Four different Forgetting factors were used, $\lambda = 1$, $\lambda = 0.98$, $\lambda = 0.95$, and $\lambda = 0.9$. These were used for faster convergence, and to study their effects on algorithm robustness. It was found that $\lambda < 0.95$ gave poor results. These algorithms are discussed as follows.
3.2.1 Least Square Estimator (LS)

For the linear model of equation 3.6, the estimation model is.

\[ \hat{Z}(k) = \phi(k)\hat{\theta}(k) \]  

(3.10)

The estimate for the required parameter vector \( \theta \) can be obtained by minimizing the objective function.

\[ J[\hat{\theta}(k)] = \bar{Z}^T(k)\bar{Z}(k) \]  

(3.11)

where \( \bar{Z}(k) \) represents the error in the estimated and the measured values. Differentiating with respect to \( \theta \) and then setting it to zero gives the required estimation algorithm.

\[ \hat{\theta}_{ls}(k) = \left[ \phi(k)\phi^T(k) \right]^{-1} \phi(k)Z(k) \]  

(3.12)

3.2.2 Weighted Least Square Estimator (WLS)

This estimator is given by

\[ \hat{\theta}(k) = \left[ \Phi^T W \Phi \right]^{-1} \Phi^T WZ \]  

(3.13)

The weighing matrix \( W \) is used to give more importance to some of the observations as compared to other. Usually, the most recent observations are given more importance and hence are weighted more heavily. The usual choice of this matrix is

\[ W(k) = \begin{bmatrix}
\mu^1 & 0 & \cdots & 0 \\
0 & \mu^2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \mu^k
\end{bmatrix} \]  

(3.14)
The choice of $\mu$ lies between 0 and 1, however, usually it is recommended to use $\mu$ close to 1.

In simulation, $\tilde{z}$ is used for $\mu$. The WLS is better estimator in terms of convergence. However, it requires, like the original LS algorithm, all the $k$ observation in a batch.

### 3.2.3 Extensions in Least Square Estimators.

To improve the performance in the Least Square Algorithm, many extensions and modifications are found in the literature [69-72]. The enhancement in performance which is gained through these modifications are faster convergence, less deviations, recursive calculations, recursive estimation, and better estimates. Specially, the feature for recursive calculation is attractive in connection with using the algorithms online. This work, utilizes the following recursive algorithms which are extensions to LS algorithm.

#### 3.2.3.1 Recursive Least Square Estimator - Information Form (RLS-I).

This comprise of three step calculation in each iteration.

\[
P^{-1}(k + 1) = P^{-1}(k) + \phi^T(k + 1)w(k + 1)\phi(k + 1)
\]

\[
K_w(k + 1) = P(k + 1)\phi^T(k + 1)w(k + 1)
\]

\[
\hat{\theta}(k + 1) = \hat{\theta}(k) + K_w(k + 1)[z(k + 1) - \phi(k + 1)\hat{\theta}(k)]
\]

(3.15)

These equations are initialized by taking some initial values for $\hat{\theta}(k)$ and $P$. For $P$, usually a rule of thumb is used which requires $P = \alpha I$. where $\alpha$ is a large number and $I$ is the identity matrix of order $n \times n$, where $n$ is the number of parameters to be estimated.

#### 3.2.3.2 Recursive Least Squares Estimator - Covariance Form (RLS-C).

This algorithm corresponds to the following computations:
\[
K_w(k + 1) = P(k)\phi^T(k + 1)\left[\phi(k + 1)P(k)\phi^T(k + 1) + \frac{1}{w(k + 1)}\right]^{-1}
\]
\[
P(k + 1) = \left[I - K_w(k + 1)\phi(k + 1)\right]P(k)
\]
\[
\hat{\theta}(k + 1) = \hat{\theta}(k) + K_w(k + 1)\left[z(k + 1) - \phi(k + 1)\hat{\theta}(k)\right]
\]

The choice of \( P \) remains the same as for Information form.

3.2.3.3 Extended Least Square Estimator -1 (ELS-1)

This algorithm is simpler to calculate and provides a method to eliminate the bias present in the methods based upon the actual Least Square algorithm. This technique rearranges the Observation matrix \( \Phi \) and formulates the problem as follows [72]:

\[
P^{-1}(k + 1) = P^{-1}(k) + \phi^T(k)\phi(k)
\]
\[
\hat{\theta}(k + 1) = \hat{\theta}(k) + P(k + 1)\phi^T(k)\left[z(k + 1) - \phi(k)\hat{\theta}(k)\right]
\]

3.2.3.4 Extended Least Square Estimator -2 (ELS-2)

Another modification in the extended least square can be found in the literature. This method is specially suitable for fast convergence [72]:

\[
P(k + 1) = P(k) - P(k)\phi^T(k)\phi(k)P(k)\left[w(k) + \phi(k)P(k)\phi^T(k)\right]^{-1}
\]
\[
K(k + 1) = P(k + 1)\phi^T(k + 1)
\]
\[
\hat{\theta}(k + 1) = \hat{\theta}(k) + K(k + 1)\left[z(k + 1) - \phi(k + 1)\hat{\theta}(k)\right]
\]

3.3. Test Signal

The test signal used for estimation in this work has been referred to by many authors in this area [22,73]. Figure 3.1 shows the sample system which comprise of a two bus three phase
Figure 3.1. Sample system, a two bus architecture with six pulse full wave bridge rectifier supplying the load.
system with a full wave six pulse bridge rectifier at the load bus. The test signal is a distorted voltage waveform at the terminals of the load bus for the system of Figure 3.1.

Table 3.1 shows the harmonics present in the system, and Figure 3.2 shows the resulting signal with its frequency spectrum.

The signal is used with two sets of noise, Gaussian and Uniform. The Signal to Noise Ratio (SNR) is calculated as follows:

$$SNR = 10 \log_{10} \frac{P_{\text{SIGNAL}}}{P_{\text{NOISE}}}$$  \hspace{1cm} (3.19)

To generate noise, following calculations were done:

3.3.1 Signal Power.

The power in a sinusoidal signal is given by:

$$P_{\text{SIGNAL}} = \frac{V_m^2}{2}$$ \hspace{1cm} (3.20)

When there are several sinusoids present, by using superposition, the total signal power can be calculated by:

$$P_{\text{SIGNAL}} = \frac{1}{2} [V_{m1}^2 + \ldots + V_{mn}^2]$$ \hspace{1cm} (3.21)

3.3.2 Uniform Noise

The MATLAB function to generate uniform noise is \texttt{rand}, and the command syntax is as follows:

$$Pn = \text{a} \ast (2 \ast \text{rand}(1,N) - 1) :$$  \hspace{1cm} (3.22)
<table>
<thead>
<tr>
<th>Harmonic Order</th>
<th>Amplitude (P.U)</th>
<th>Phase (Degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental (60Hz)</td>
<td>0.95</td>
<td>-2.02</td>
</tr>
<tr>
<td>5th (300 Hz)</td>
<td>0.09</td>
<td>82.1</td>
</tr>
<tr>
<td>7th (420 Hz)</td>
<td>0.043</td>
<td>7.9</td>
</tr>
<tr>
<td>11th (660 Hz)</td>
<td>0.03</td>
<td>-147.1</td>
</tr>
<tr>
<td>13th (780 Hz)</td>
<td>0.033</td>
<td>162.6</td>
</tr>
</tbody>
</table>

Table 3.1. Frequency content of the test signal, Voltage at the load bus.
Figure 3.2. Test signal (a) and its spectrum (b). Dotted line in (a) represents the fundamental present in the signal.
where $P_n$ is the noise vector of the desired power, $N$ is the number of noise samples needed. and $a$ is the amplitude multiplier which is based upon the power requirement for the noise. It is given by $\sqrt{3P_{\text{noise}}}$ \cite{72}. $P_{\text{noise}}$ is the power in noise and it is calculated from the required SNR relation.

Figure 5.3 shows the sample signals and their spectrums for different set of Uniform noise. with three different SNRs in dB, 0, 10, 20, while the no noise case is the one shown in Figure 5.2.

### 3.3.3 Gaussian Noise

For Gaussian noise generation, the power in the noise is given by

$$P_{\text{noise}} = \sigma^2 + \mu^2$$ \hfill (3.23)

where $P_{\text{noise}}$ is the noise power, $\sigma^2$ is the variance of noise and $\mu$ is the mean. For a zero mean case, power in the noise is equal to its variance and the generation of such noise can be done as follows:

$$\text{randn('seed',sum(100*clock))}$$

$$x = \text{randn}(1,N)$$

$$y = x \ast \text{sqrt}(P_{\text{noise}}/\text{cov}(x))$$ \hfill (3.24)

where $N$ is the number of noise samples in the signal. Figure 5.4 shows the sample signals and their spectrums for different set of Gaussian noise, with three different SNRs in dB, 0, 10, 20. Again, the no noise case is the one in Figure 5.2.
Figure 3.3. Sample signal and its spectrum in uniform noise, (a), (b) SNR = 20, (c), (d) SNR = 10, and (e), (f) SNR = 0 dB.
Figure 3.4. Sample signal and its spectrum in Gaussian Noise, (a), (b) SNR = 20, (c), (d) SNR = 10, and (e), (f) SNR = 0 dB.
Although the figures, 5.3 and 5.4, does not convey much information about the noise, and one could think of them as similar signal, but in reality, they are very much statistically different. An important thing to note, however, is the effect of noise on higher frequency spectrum and harmonics of lower amplitudes. For low SNRs, the noise seems to have annihilated the harmonics present. This is due to the fact that most of the noise components lie in the high frequency spectrum. Since the spectrum of noise is very wide, even very low SNR noise signals can not have appreciable magnitude at high frequencies. But, the higher frequencies present in the signal as harmonics also have comparable magnitudes, and therefore, get mingled with the noise. As the later results show, it becomes increasingly difficult to estimate higher frequency components in high noise.

3.4. Simulation Results

For estimation of harmonics, the above mentioned routines were written and tested in MATLAB. The simulation strategy is shown in Figure 5.5. Each simulation was repeated 50 times (hence 50 realization of the same setup) and the average was calculated. The simulation is also classified as per power of noise injected in the system. Four sets of such signals were generated in each realization, Without Noise, Noise with SNR = 20 dB (Moderate noise), Noise with SNR = 10 dB (Strong Noise), and Noise with SNR = 0 dB (High Power Noise).

As stated above, the average for each method is taken over 50 realizations, and the results for estimation of Amplitudes are given in Table 3.2 to 3.11.

The true values that are being estimated are the amplitude and phase of the fundamental frequency component, as given in Table 3.1.
Figure 3.5. Simulation flow chart.
### Table 3.2. Estimation results for Fundamental harmonic amplitude. True value 0.95 p.u. \( \lambda \) represents the forgetting factor.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>SNR</th>
<th>( \lambda )</th>
<th>LS</th>
<th>WLS</th>
<th>RLS-I</th>
<th>RLS-C</th>
<th>ELS-1</th>
<th>ELS-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 Hz (Gaussian)</td>
<td>1000</td>
<td>1</td>
<td>0.9500</td>
<td>0.9500</td>
<td>0.9500</td>
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<th>Frequency</th>
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<th>( \lambda )</th>
<th>LS</th>
<th>WLS</th>
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<th>RLS-C</th>
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<td>RLS-C</td>
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Table 3.3. Estimation results for Fifth harmonic amplitude. True value 0.09 p.u. $\lambda$ represents the forgetting factor.
## ESTIMATION ALGORITHMS

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Table 3.4. Estimation results for Seventh harmonic amplitude. True value 0.043 p.u. $\lambda$ represents the forgetting factor.
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Table 3.6. Estimation results for Thirteenth harmonic amplitude. True value 0.033 p.u. $\lambda$ represents the forgetting factor.
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Table 3.7. Estimation results for Fundamental harmonic Phase. True value -202°. $\lambda$ represents the forgetting factor.
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Table 3.8. Estimation results for Fifth Harmonic Phase. True value 82.1°. $\lambda$ represents the forgetting factor.
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Table 3.9. Estimation results for Seventh harmonic Phase. True value 7.9°. $\lambda$ represents the forgetting factor.
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Table 3.10. Estimation results for Eleventh harmonic Phase. True value $-147.1^\circ$. $\lambda$ represents the forgetting factor.
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Table 3.11. Estimation results for Thirteenth harmonic Phase. True value 162.6°. $\lambda$ represents the forgetting factor.
The sampling frequency is selected to be 1620 Hz, which is one fundamental guard band more than the Nyquist frequency for the test signal. Hence for one full period of fundamental, there are 27 samples per period giving a timing window of about 600 m seconds per sample.

The actual value for the amplitude of fundamental is 0.95 P.U. and that of phase is -2.02°. As can be seen from the results in these figures, the estimation is fairly close to the exact value for almost no noise case (simulated as an SNR of 1000 dB) by all of the algorithms used. Any discrepancy could be a result of numerical approximation and finite word length effects, especially for the calculation of phase. The maximum deviation in amplitude estimates is 0.002 P.U. corresponding to an error of 2%. Similarly for the case of phase, except for one estimate, the results are within 5% error.

The true value for the amplitude of fifth harmonic is 0.09 P.U. and the phase is 82.1°. In case of amplitude estimation, the maximum error for 1000, 20, and 10 dB SNRs is about 1.5%. While for 0 dB the error increases to 6.7%. The reason is adding of noise components at the same frequency with the fifth harmonic component to produce a higher amplitude harmonic. However, the estimation for phase is much closer to the actual producing an error of about 1%.

The true value for amplitude of 7th harmonic is 0.043 P.U., and that of phase is 7.9°. The maximum error in the estimation of amplitude is 8% and it is the case when SNR is 0 dB. Similarly, the error in phase is about 7.6%. However, larger deviation of values for different algorithms can be observed. In fact, the higher frequencies are more difficult to track in the presence of noise by the help of the algorithms used. However, the percentage error is well within the acceptable range.
For the $11^{th}$ harmonic, the true values are 0.03 P.U. and $-147.1^0$ for amplitude and phase respectively. The amplitude estimates show a maximum error of 6.5%. The error in the estimate of phase is approximately 4%.

The $13^{th}$ harmonic has true value for amplitude as 0.033 P.U and that for phase is $162.6^0$. The estimation error for amplitude estimates for most of the algorithms is about 7%, while the worst estimates have deviated about 12%. For the phase estimates, the estimation error is about 5%.

3.5. Discussion.

The LS based algorithms have been very successful in estimating the amplitudes and phases of sinusoids in the signal. The estimation algorithms have been found to have produced equally good results for both type of noise, Gaussian as well as Uniform, although the estimates in case of Gaussian noise are closer to the actual value than the estimates in uniform noise. It is well known in estimation literature that LS is optimal with respect to Gaussian noise.

The worst results are obtained for the case where SNR is 0 dB. This could be the worst case scheme, because usually such levels of noise does not occur. However, this shows the robustness of the estimation algorithms even in such hostile conditions.

The four recursive algorithms based on LS criterion, presented in this chapter, can be used for recursive estimation of amplitudes and phases of harmonics in on-line applications. The computing hardware would define the actual computation time required by each method to produce one estimate. However, it has been determined that RLS-C requires minimum time for its formulation. As can be seen from equation 3.16, the operation of matrix inversion is
modified to scalar division which results in faster calculation. ELS-I has \( n \times n \) inversion involved, where \( n \) is the number of parameters to be estimated, but has fewer operations other than this. It has been found to be 12% slower than RLS-C. Similarly, ELS-1 is 14% slower and RLS-I is about 30% slower than RLS-C.

The presence of forgetting factor affects the convergence properties of these algorithms. The factor \( w(k + 1) \) in equations 3.15, 3.16, and 3.18 is the same forgetting factor, \( \lambda \), as mentioned in section 3.2, with increasing power with respect to time difference of final time and the initial sampling instant. However, ELS-I is independent of this factor and has uncontrollable convergence structure. In this regard. RLS-C produces best results. RLS-I and RLS-2 have almost similar convergence characteristics. However, with improper selection of \( P \) matrix.

RLS-C algorithm has higher chances of blowup (i.e. diverging estimates).

The LS estimators are unbiased, efficient and consistent provided the following holds true.

- The noise is zero mean white.
- The covariance matrix \( R \) of this noise should be positive definite.
- These corresponds to \( R \) being diagonal matrix with each diagonal entry corresponding to constant variance \( \sigma^2 \), i.e., \( R(k) = \sigma^2 I \), where \( I \) is the identity matrix at sampling instant \( k \).
- \( \Phi \) and \( \nu \) should be uncorrelated.
- \( \Phi \) should be deterministic.
- Under such conditions, the well known estimators converges, i.e.,

\[
\hat{\theta}_{LS} = \hat{\theta}_{BLUE} = \hat{\theta}_{ML}
\]
However, for recursive calculations, other factors are also important. Consider the estimate update equation,

$$\hat{\theta}(t) = \hat{\theta}(t-1) + L(t)(z(t) - \phi(t)\hat{\theta}(t-1))$$

This can be written as,

$$\hat{\theta}(t) = (I - L(t)\phi(t))\hat{\theta}(t-1) + L(t)z(t)$$

If an error $\delta\hat{\theta}(t_o)$ is introduced at instant $t_o$, then,

$$\Delta\hat{\theta}(t) = \prod_{k=t_o}^{t} (I - L(k)\phi(k))\delta\hat{\theta}(t_o)$$

The error propagation properties are thus dependent on the matrix,

$$\phi^T(t, t_o) = \prod_{k=t_o}^{t} (I - L(k)\phi(k))$$

Now, suppose that the algorithm is fed with the correct $L(t)$ sequence, i.e.,

$$L(t) = P(t)\phi^T(t)$$

then we can have

$$\phi(t, t_o) = \lambda^{t-t_o} P(t)P^{-1}(t_o)$$

The flow diagram for an LS algorithm is shown if Figure 3.6. It can be deduced that as long as $P(t)$ remains uniformly bounded and $\lambda < 1$ we thus conclude that the effect of a single error
Figure 3.6. Signal flow in LS algorithms.
decays exponentially. In case $\lambda = 1$, we have $P(t)P^{-1}(t_0) \leq I$. Then we have stability. If, in addition, $P(t)$ tends to zero we have asymptotic, but not exponential stability [75].

A similar concern with the RLS techniques lies in the implementation of the algorithm in practical application. As reported by Malik and others [76]. The use of RLS identification in a real time environment does not produces similar results when used in simulations. The discrepancy appears due to the following factors.

- The system to be identified has been more or less simplified or idealized for simulation purpose.
- Sampling is considered as instantaneous in many simulation strategies.
- Unpredicted disturbances that can cause the stationarity of the system are usually not considered in simulation schemes.
- Samples are assumed to have given errorless values.

The remedies that can be undertaken for the practical consideration of RLS algorithms are as follows.

- Elimination of the effects of unpredicted disturbances.
- Appropriate selection of the RLS identification parameters, such as system model and sampling period.
- Increasing the parameter tracking speed, i.e. preventing the identifier from going to 'sleep'.
- Avoiding unstable system parameter identification, i.e. to avoid covariance matrix 'blow-up'.

CHAPTER 4

PROPOSED HARMONIC REDUCTION ELECTRONIC SCHEMES

This chapter presents the second part of the work stated in chapter 1. This implies to
techniques developed in order to eliminate or reduce the harmonic contents in a complex
signal. The basic scheme which is based on the comparison-reinjection technique. The
 technique can be thought of a combination of current-reinjection and flux compensation
schemes. Several variants of the main scheme are presented in this chapter. The scheme has
been simulated using MATLAB and PSPICE. Experimental verification is also carried out.

4.1. The Proposed Schemes

This chapter describes some of the techniques that were developed in this thesis and the
related work [77], in order to eliminate or reduce the harmonic levels in the network. A new
scheme is proposed in this work to efficiently eliminate harmonics. A block diagram of the
scheme is given in Figure 4.1. x(t) is the actual electrical signal which is distorted due to the
presence of harmonics and random noise, such that it is given by

\[ x(t) = A_1 \sin(\omega_0 t + \psi_1) + A_2 \sin(2\omega_0 t + \psi_2) + \cdots + A_n \sin(n\omega_0 t + \psi_n) + V_{\text{noise}} \]  

(4.1)

where \( A_1, A_2, \ldots, A_n \) are the respective magnitudes. and \( \psi_1, \psi_2, \ldots, \psi_n \) are the respective
phases of each of the \( n \) sine waves associated with the distorted signal, with \( A_1 \) being the
magnitude of fundamental frequency \( \omega_0 \) and \( \psi_1 \) is its phase. \( V_{\text{noise}} \) is the noise voltage. In this
scheme, the signal \( x(t) \) is fed to a comparator which compares it to \( r(t) \) which is the reference
Figure 4.1. Block Diagram for the Proposed Scheme.
signal of the same amplitude, frequency and phase as that of the fundamental. Therefore,

\[ r(t) = A_i \sin(\omega_i t + \psi_i) \]  

(4.2)

The output of this block is \( e(t) \) or the error signal. This signal is given by.

\[ e(t) = x(t) - r(t) \]

\[ = A_2 \sin(2\omega_0 t + \psi_2) + A_3 \sin(3\omega_0 t + \psi_3) + \cdots + A_n \sin(n\omega_0 t + \psi_n) + V_{\text{noise}} \]  

(4.3)

This signal is then fed to an inversion block. The output of this block is \( d(t) \) or the difference signal,

\[ d(t) = -\left\{ A_2 \sin(2\omega_0 t + \psi_2) + A_3 \sin(3\omega_0 t + \psi_3) + \cdots + A_n \sin(n\omega_0 t + \psi_n) + V_{\text{noise}} \right\} \]  

(4.4)

When this signal is added back to the original \( x(t) \), the final output \( y(t) \) is obtained as

\[ y(t) = x(t) + d(t) \]

\[ = A_i \sin(\omega_i t + \psi_i) + A_2 \sin(2\omega_0 t + \psi_2) + \cdots + A_n \sin(n\omega_0 t + \psi_n) + V_{\text{noise}} \]

\[ -\left\{ A_2 \sin(2\omega_0 t + \psi_2) + A_3 \sin(3\omega_0 t + \psi_3) + \cdots + A_n \sin(n\omega_0 t + \psi_n) \right\} - V_{\text{noise}} \]  

(4.5)

Hence the fundamental harmonic is in principle recovered at the output of this scheme as desired.

In order to implement this scheme, different methodologies are proposed. The preliminary simulations have been carried out for some of the schemes using PSPICE. Standard signals from different generators were used and summed together to form the distorted signal. The selection of these values is random. For this simulation, the phases are all considered as \( 0^\circ \).

This assumption may not be true all the time, however, the results are unaffected even by
considering the phases. This will be demonstrated in the experimental results. The selected value of each sinusoid is given in Table 4.1.

4.1.1 Injection of Noise.

Noise is injected as a Piece Wise Linear (PWL) voltage source in the simulation. Gaussian noise of the required SNR is first generated with MATLAB, and is then changed to the structure of a library with a PWL source to be used in PSPICE simulation. Selected values for noise injection apart from no noise case are noise with SNR = 30, noise with SNR = 10, and noise with SNR = 0, in the order of increasing power in noise. The different sample signals used in simulation are shown in Figure 4.2 along with the fundamental signal which is shown with dotted line. The spectrums for these sample signals are given in Figure 4.3.

As can be seen from these figures, the increasing power in noise appears in the form of significant frequency components in the signal. Specially, in case of SNR = 0 dB, which corresponds to a condition in which noise has equal amount of power as in the signal, the higher harmonics present in the signal originally, are now completely merged with the noise. Other frequency components, which are not harmonics, are also visible in the signal. Such frequency components are not the multiples of the fundamental frequency and are not characterized with a particular source of noise.

Another important feature with high noise is the presence of significant sub-harmonic frequencies, specially the DC level. These frequencies are the most difficult components to eliminate from the signal, not only due to their presence outside the spectrum for many low
<table>
<thead>
<tr>
<th>Harmonics</th>
<th>Frequency (Hz)</th>
<th>Amplitude (V)</th>
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</table>

Table 4.1. Contents of the distorted voltage test signal for simulation.
Figure 4.2. Sample signals used along with the fundamental signal. (a) Sample signal without noise, (b) sample signal with noise SNR = 30dB, (c) sample signal with noise SNR = 10dB, (d) sample signal with noise = 0 dB.
Figure 4.3. Sample signal spectrum. (a) Sample signal without noise. (b) Sample signal with noise SNR = 30dB. (c) Sample signal with noise SNR = 10dB. (d) Sample signal with noise = 0 dB.
pass filters, but also due to the significant power in these frequency components. The implementation of the main theory depends on the voltage levels, available components, and cost. Some of the schemes for the possible implementation are proposed next.

4.2. Standard Local Oscillator

In this scheme, the reference signal $r(t)$ is locally generated sinusoid of the required amplitude and frequency. The block diagram for this scheme is shown in Figure 4.4. The results of the simulations are shown in Figure 4.5.

The locally generated signal is of much smaller rating than the actual signal. Only the amplitude and phase of voltage are the same as those of the fundamental harmonic, the supply current is very small. This scheme requires that the reference signal and the main signal should be perfectly synchronized. This is a very stringent requirement. However a pure sine wave can be synchronized with a distorted one using the usual technique for parallel operation of generator with the main bus.

The results in Figure 4.5 shows that the scheme works perfectly well to recover the required fundamental sinusoid. The immunity to noise is an attractive feature here. The fundamental sinusoid is recovered, even from the worst signal, i.e., SNR = 0 dB. The actual implementation of this scheme has also been tested and is discussed later in this chapter. However, to have a perfectly synchronized reference is a very strict condition. In order to alleviate this stringency, other variants of the local oscillator scheme are also proposed next.
Figure 4.4. Block diagram for Local Oscillator Scheme.
Figure 4.5. Output of local oscillator scheme, (a) Sample signal without noise, (b) sample signal with noise SNR = 30dB, (c) sample signal with noise SNR = 10dB, (d) sample signal with noise = 0 dB.
which are based upon the extraction of information related to the fundamental harmonic in
the distorted signal.

4.3. Band pass filter as reference generator.

In this scheme, the local oscillator is replaced with the output signal of a band pass filter
which provides the necessary fundamental signal. The block diagram of this scheme is shown
in Figure 4.6.

Band Pass Filters (BPF) are used in the electrical networks but not very extensively like the
shunt filters. One reason being the losses involved in the BPF which are connected in series
(or partially in series), with the load and therefore, full load current passes through them. In
this scheme, the purpose of filter is not to pass the high current through it but to extract the
information about the required signal to be used later as reference signal for the comparison
with distorted signal.

The BPF design could be any of the available analog designs like Butterworth filter.
Chebychev filter, or Elliptic filter. The higher the order of the filter, the better is the response.
Any standard procedure for filter design can be used. This work is not concerned with the
optimal filter requirement and design, instead the intention is to use the prevalently available
components to get the maximum efficiency. However, filters that have been used in this work
are based upon the standard procedures like in [74].
Figure 4.6. Scheme to use BPF as reference generator.
4.3.1 Butterworth realization.

A Butterworth filter is one that has maximum flat response in the pass band. However, the
cutoff characteristics of this filter are not very good. One should use a higher order filter for
this purpose. To test the proposed scheme, a Sallen-Key standard filter design has been used
and simulated for 4th order filters. The simulation results are shown in Figure 4.7.

The problem associated with the phase shift of the fundamental frequency is quite
prominent. But, this can be removed by inserting proper delays or phase compensation
networks. The response for the first three cases, i.e. the sample signals with SNRs > 0, show
a much better response as compared to the last case of very strong noise. However, in all
cases, the unwanted harmonics as well as noise components have been eliminated or reduced.
Drift in the high noise case is due to the saturation effects in the comparators and other RC
components.

4.3.2 Chebychev realization.

A Band Pass Chebychev filter is one that allows some ripples in either pass band (type I) or
in stop and (type II), but the cut off characteristics of this filter are much superior than that
of the Butterworth filter. The results of simulation for 4th order Chebychev II filter are given
in Figure 4.8.

As can be seen from the results, there are some ripples in the stop band but the cut off
characteristics of the filter are comparable with the results that were obtained with the local
Figure 4.7. Output of 4th order Butterworth BPF scheme, (a) Sample signal without noise, (b) sample signal with noise SNR = 30dB, (c) sample signal with noise SNR = 10dB, (d) sample signal with noise = 0 dB.
Figure 4.8. Output of 4th order Chebychev BPF scheme, (a) Sample signal without noise, (b) sample signal with noise SNR = 30dB, (c) sample signal with noise SNR = 10dB, (d) sample signal with noise = 0 dB.
oscillator scheme. However, compared to Butterworth filter, these ripples are more prominent for the high noise case.

4.3.3 Elliptic filter realization

Elliptic filters are characterized by the presence of ripples in both pass band as well as stop band but the cutoff characteristics are much better than those of the other filters. The results of simulation for 4th order elliptic filter are given in Figure 4.9.

This arrangement has excellent cut off characteristics and lesser ripples both in stop band as well as the pass band. But, compared to Butterworth and Chebychev designs, the elliptic filter have been seen to have some problems with the offset DC levels in lower frequency band. This could be due to the inherent offsets in the op amp models or due to the saturation effects caused by improper biasing.

4.3.4 The COMBINED scheme.

The best solution in the light of above discussion is to use the local oscillator. However, the strict synchronization requirement proposes the use of Band Pass filter to extract the fundamental from the system. But, the response of practical filters can not be taken 100% replica of the fundamental frequency. Further, the introduction of noise would lead to some deviations as well. Also, the effect of delays and phase shifts are inherent with the usage of practical filters.
Figure 4.9. Output of 4th order Elliptic BPF scheme, (a) Sample signal without noise, (b) sample signal with noise SNR = 30dB, (c) sample signal with noise SNR = 10dB, (d) sample signal with noise = 0 dB.
However, the best features of the two schemes can be combined in another scheme. In this scheme, the local oscillator is still there, but the Band Pass filter is now used to determine the phase and amplitude of the fundamental frequency. In order to avoid any phase shift, and attenuation, the filter is further supplemented with the phase corrector and the amplifier blocks. Hence the output of this design would be a close replica of the fundamental. At this stage one can implement two types of schemes. The first one suggests that this output should now be injected in the system as the reference signal. But since there is one chance of not 100% replication of the fundamental, the second method would be to use this output for the adjustment of amplitude and phase of local generator, which is afterward injected as the reference signal. The block diagram of this scheme is given in Figure 4.10.

4.4. Implementation and testing of the schemes

In order to test the integrity of simulated results in the real world, the schemes were tested at lower voltage levels. The distorted signal was formed by summing three sine waves from different function generators. The detail of the input signal frequency content is given in Table 4.2. The resulting signal along with the constituent frequencies is shown in Figure 4.11. The other elements of the schemes discussed were implemented with op amps, resistors and capacitors.
Figure 4.10. Block diagram for Combined scheme. The dotted line represents one possibility of using the BPF output to tune the local oscillator/generator.
4.4.1 Local Oscillator Scheme.

The circuits used for the implementation of the basic scheme (Local Oscillator scheme) is shown in Figure 4.12. The OP-Amp. OP1 is used as an adder to add the signal from three different sources in order to generate the sample distorted signal $x(t)$. The OP2 is the comparator that compares $x(t)$ with the reference signal $r(t)$. Where $r(t)$ is taken from either the same function generator that generates the fundamental signal in the formation of $x(t)$ or another function generator which is synchronized with the fundamental generator. Since the comparison is complementary, the output is $d(t)$, the difference signal. Hence we need not to invert it again. This signal is fed to OP3 along with the distorted signal in order to cancel out the distortion. Hence the final output $y(t)$ is the pure sinusoid of fundamental frequency. The results of this test are given in Figure 4.13.

4.4.2 BPF Scheme.

In order to use Band Pass Filters (BPFs) in this scheme, a slight modification is made. As shown in Figure 4.14, the existing circuit of Figure 4.12 is now added with a new block of BPF. The same scheme could be used to implement Butterworth, Chebychev and elliptic filters. The reference signal $r(t)$ is not any locally generated signal but is extracted from the same signal $x(t)$. The extraction is done by the BPF used in the block shown in the Figure 4.14.
<table>
<thead>
<tr>
<th>Harmonics</th>
<th>Frequency (Hz)</th>
<th>Amplitude (V)</th>
<th>Phase (Deg.)</th>
</tr>
</thead>
<tbody>
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<td>60</td>
<td>20</td>
<td>-21.6°</td>
</tr>
<tr>
<td>3rd</td>
<td>180</td>
<td>10</td>
<td>64.8°</td>
</tr>
<tr>
<td>Arbitrary</td>
<td>400</td>
<td>12</td>
<td>-32.4°</td>
</tr>
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Table 4.2. Set of frequencies used to generate the distorted signal for experimental implementation.
Figure 4.11. Experimental test signals. (a) Fundamental signal, (b) High frequency arbitrary signal, (c) Third harmonic, and (d) Overall distorted signal.
Figure 4.12. Test circuit for local oscillator scheme. $R_A$, $R_B$, and $R_f$ are selected for appropriate amplification.
Figure 4.13. Results from the local oscillator scheme. (a) Actual fundamental signal, (b) Output of the scheme.
Figure 4.14. Test circuit for BPF scheme. $R_A$, $R_B$, and $R_f$ are selected for appropriate amplification.
The values used in the circuit are selected according to the op amp used, and the working voltage. The filter design is done by using the standard design procedures and by using the standard Sallen and Key circuits.

Since the noise used in the sample signal is not of low SNR, the experimental setup could be simplified by using the low pass filter configurations instead of band pass filters. If the scheme can work with low pass filter (LPF), then the performance will be better with the BPF. The schemes of section 4.3 are tested with 4th order Butterworth, 2nd order Chebychev, and 2nd order Elliptic filters. The results of these implementations are shown in Figures 4.16 to 4.18.

The actual amplitude values can be adjusted by using right values of components and further adjusting the biasing and offsets. As can be seen from the above figures (4.16 to 4.18), the fundamental is recovered to an acceptable degree of consistence. Since each LPF stage is a 2nd order stage, the result of each stage is a phase inversion. As the 4th order Butterworth response shows, there is no phase shift between the actual fundamental and the one that is obtained by the LPF scheme. However, the Chebychev filter shows a 180° shift while the elliptic filter response corresponds to a shift of about 90°. However, these shifts are known before hand and can be compensated for by suitable design.

4.5. Estimation of Amplitude and Phase of Fundamental

Another method which can alleviate the stringent requirements for the local oscillator scheme is to estimate the amplitude and phase of the fundamental frequency present in the polluted
Figure 4.15. (a) Original fundamental, (b) output with 4th order Butterworth filter.
Figure 4.16. (a) Original fundamental, (b) output with 2\textsuperscript{nd} order Chebychev filter.
Figure 4.17. (a) Original fundamental, (b) output with 2\textsuperscript{nd} order Elliptic filter.
Figure 4.18. Replacing local oscillator with the estimates
signal and use it as the reference signal in the local oscillator scheme, by either adjusting parameters of local oscillator, or filter, or even generating the sinusoid using power electronic components. This is shown in Figure 4.19.

Hence, one can utilize the methods discussed in chapter 3 to estimate the amplitude and phase of fundamental harmonic. This provides a pertinence between the two seemingly different segregated areas of this work. First, the estimation can provide a better knowledge for the voltage signal present. Then, this information can be used to provide recursive updates to eliminate the distortion using the techniques discussed in this chapter. An estimation of upto 5% error can be easily achieved by using the recursive calculations, given in chapter 3. These estimates can be used as excitation signal or synchronizing signal for local oscillator scheme or BPF scheme.

The same test signal which was used in chapter 3 as the sample signal for estimation of amplitude and phase, is used here. The difference is the use of some of the estimates as obtained by the estimation algorithms in chapter 3, as reference signal in the main scheme, by replacing the local oscillator with the estimate of the sinusoid. As stated above, this estimate can be used for this purpose by either tuning the local oscillator, or tuning filter parameters, or even controlling the inverter circuits to produce the required sinusoid. Hence figure 4.1 is modified, and simulated using SIMULINK, as shown in Figure 4.20.

The whole structure was simulated with the previously used sample signals, i.e. the four types of noise. The results of simulation are shown in Figure 4.21 and 4.22.
The results show an agreement with the previous results, and the fundamental is retrieved with some error depending upon the percentage of error in the estimate. If the estimation and reinjection process is carried out on-line then the error in the estimate will eventually vanish, and the result would be the required sinusoid.

The results include three types of estimated reference signals. One with the true estimate i.e. $0.95 \angle -2.02^\circ$, while other two signals are those having $\pm 5\%$ error in the estimate of both amplitude and phase. This corresponds to $0.9975 \angle -2.121^\circ$ and $0.9025 \angle -1.919^\circ$. But, with the basic scheme, the results can be seen to be representing the actual fundamental sinusoid.

4.6. Discussion of Results

The different schemes proposed in this chapter have produced satisfactory results in eliminating the harmonic pollution in the power signal. In general, the proposed schemes are simple in design. They require commonly used components in the system. The elimination methodology can be used for any harmonic environment. The design does not involve calculation for specific frequency components, therefore, both characteristic as well as uncharacteristic harmonics can be eliminated. The robustness to noise is a powerful feature of the schemes.

Out of many variants discussed in foregoing sections, the selection of a particular method would depend on the requirement and feasibility. The three type of filter schemes, for instance, produces almost similar results. However, Butterworth filter can not be used where critical frequencies exist close to the fundamental frequency. This is due to the poor cutoff
characteristics of the filter. Chebychev filter, although, has better cutoff, but is more complicated in design than Butterworth filter. It also allows ripples in the stop band, which could be a major concern if appreciable components of noise are present. Elliptic filter is the most expansive of the three filters, but has the most desirable properties. It can produce the same performance with a much lower order filter as would be with other filters.

Similarly, the use of estimation techniques for providing the necessary information about fundamental signal is also dependent on the nature of application. If there are frequent drifts in fundamental frequency and higher noise levels, then recursive estimation algorithms can be used for adaptively tracking the amplitude and phase of fundamental.

However, the techniques discussed in this chapter, are constrained by the availability of required hardware. Operational Amplifiers are not available for more than 100 volts at present. Hence the above discussed schemes can be used for such applications like elimination of noise in communication lines and telephone lines.

Since Op-Amps. Have been used here for merely comparison and summing two signals, this can also be done by using equivalent circuits to that of the Op-Amp. This can be achieved by utilizing the differential amplifier configurations. This requires two to five transistors based on different loading performances. The bipolar junction transistors are at present available in access of 1.2 kV, while JFETs and MOSFETs are available for values of upto 5kV. This can make the use of the techniques discussed in this chapter possible to be implemented for distribution voltages.
Figure 4.19. SIMULINK file for testing estimated values with the basic scheme.
Figure 4. 20. Results of SIMULINK simulation, for estimated $r(t)$. (a) Sample signal $\text{SNR} = 1000 \text{ dB}$, (b) Output, (c) Sample signal $\text{SNR} = 20 \text{ dB}$, (d) Output.
Figure 4.21. Results of SIMULINK simulation, for estimated r(t). (a) Sample signal SNR = 10 dB, (b) Output, (c) Sample signal SNR = 0 dB, (d) Output.
CHAPTER 5

PROPOSED FLUX COMPENSATION BASED HARMONIC FILTERING

This chapter presents another technique which has been proposed to eliminate the harmonics from power system networks. The basic idea of this scheme is still comparison-reinjection, as presented in chapter 4. However, the implementation methodology is different. This chapter presents the mathematical formulation and the simulation as well as practical implementation of the scheme on the laboratory scale.

5.1. Flux Compensation based scheme.

This scheme utilizes the interactions of sinusoids within the transformer core. The main scheme is shown in Figure 5.1. The multi-winding transformer receives the distorted signal \( x(t) \) in one of its isolated primary winding, and the reference signal \( r(t) \) in a second primary winding. Both of these primary input signals interact within the core of the transformer with opposing flux, as shown by the dot conventional notation of transformers.

The result of this interaction is the negative of the distortion present in the distorted signal \( x(t) \), and this signal has been called \( d(t) \) as discussed earlier with the basic scheme, in chapter 4. This anti-phase distortion can now be injected back into the main line via series potential transformers with low loss properties, or with specially designed current transformers.

5.2. Mathematical Analysis.

The circuit shown in Figure 5.1 can be redrawn for the analysis purpose as shown in Figure 5.2. This circuit can be routinely analyzed by using the Kirchoff’s equations. For the purpose of analysis, the main line voltage \( x(t) \) is shown in two places, as two separate sources. But
Figure 5.1. Basic scheme, implemented via flux compensation strategy.
Figure 5.2. Modified circuit for analysis.
since the current will divide, as per Figure 5.1. the two resulting loops have been assigned different currents. Hence for the four loops, we obtain the following set of equations.

\[
\begin{align*}
  j\omega(L_4I_1 - MI_2 - MI_3) &= X \\
  j\omega(MI_1 + L_2I_2 - MI_3) &= R \\
  j\omega(MI_1 - MI_2 + L_3I_3 + L_4I_1 + MI_4) &= 0 \\
  j\omega(- NI_1 + L_4I_4) &= X - Y
\end{align*}
\]  
\( (5.1) \)

Letting,

\[
  k = \frac{1}{j\omega}
\]  
\( (5.2) \)

We can have,

\[
  I_4 = \frac{kX - kY + NI_1}{L_4}
\]  
\( (5.3) \)

Substituting this value of \( I_4 \) in equation 5.1 for \( I_3 \), we get,

\[
  I_3 = \frac{A(Y - X) - MI_1 + MI_2}{B}
\]  
\( (5.4) \)

where,

\[
  A = \frac{Nk}{L_3} \quad \text{and} \quad B = L_3 + L_4 + \frac{N^2}{L_4}
\]

Putting this in equation 5.1 for \( I_2 \), we get,

\[
  I_2 = \frac{KR + EX - EX - CI_1}{D}
\]  
\( (5.5) \)

where.
\[ C = M + \frac{M^2}{B} \]
\[ D = L_2 - \frac{M^2}{B} \quad \text{and} \]
\[ E = \frac{MA}{B} \]

Substituting equations 5.3 and 5.4 in 5.1 for \( I_1 \), we get the required equation for \( I_1 \) in terms of \( X, R, \) and \( Y \) as follows.

\[ I_1 = \frac{G}{F}R - \frac{H}{F}X + \frac{H}{F}Y \]  \hspace{1cm} (5.6)

where,

\[ B = L_1 + L_2 + \frac{N^2}{L_4} \]
\[ F = L_1 + \frac{MC}{D} + \frac{M^2}{B} + \frac{M^2C}{BD} \]
\[ G = \frac{MK}{D} \quad \text{and} \]
\[ H = \frac{ME}{D} \]

Considering the ideal transformers in the system, the currents are lagging by 90\(^\circ\). Although this is true only for ideal transformer, the real transformers would correspond to some other value of shift, based upon their design. Hence, \( I_1 = X \angle -90^\circ = jX \), and we get,

\[ Y = X + j \frac{F}{H}X - \frac{G}{H}R \]  \hspace{1cm} (5.7)

Equation 5.7 is a modified interpretation of equation 4.5. In words, this equation can be stated as the output \( Y \) is obtained when the difference of the scaled distorted signal and the scaled reference signal is re-injected in the original distorted signal \( X \). The scaling factors and
the shift operators are obtained as part of the transformer design and the values of different elements present in the design.

Similar to the simulations of section 4.3, the flux compensation based scheme is also simulated with a sample distorted signal. The signal is given in Table 5.1. The resulting sample signal and its corresponding spectrums are shown in Figures 5.3 and 5.4.

As before, the simulation is carried out for four set of sample signals, without noise. SNR = 30 dB, SNR = 10 dB, and SNR = 0 dB, respectively. As can be seen from the figures, the presence of noise hides many frequency components, specially for those that does not have appreciable amplitude or are at higher frequencies.

5.2.1 Local Oscillator Scheme.

In this simulation, the reference signal is a locally generated sinusoid, which has the same amplitude and phase as that of the fundamental component present in the distorted signal. Again, the rating of this signal is not as much as that of the actual signal and is used only to compare the distorted signal.

The results of simulation for this arrangement are given in Figure 5.5. It can be seen from the figure that the fundamental sinusoid has been retrieved in all cases of noise pollution with almost equal fidelity. This is a very strong feature of this scheme. However, in case of practical transformers, the harmonics may not be eliminated completely, rather they can be reduced. The reason being the non linearities and saturation problems with the transformers.
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<th>Harmonic</th>
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<th>Amplitude (V)</th>
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<td>17&lt;sup&gt;th&lt;/sup&gt;</td>
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</table>

Table 5.1. Test signal components used for simulation.
Figure 5.3. Sample signals used in simulation. (a) Without noise, (b) SNR = 30 dB, (c) SNR = 10 dB, (d) SNR = 0 dB.
Figure 5.4. Sample signal spectrums used in simulation, (a) Without noise, (b) $\text{SNR} = 30 \, \text{dB}$, (c) $\text{SNR} = 10 \, \text{dB}$, (d) $\text{SNR} = 0 \, \text{dB}$. 
Figure 5.5. Results of simulation with Local Oscillator scheme. (a) without noise. (b) SNR = 30 dB. (c) SNR = 10 dB, (d) SNR = 0 dB.
5.2.2 Band Pass Filter (BPF) for Reference Input.

As have been mentioned earlier, the only problem with the local oscillator scheme is the stringent synchronization requirements. This was alleviated earlier in section 6.3 with BPFs. Here again, the flux compensation scheme is supplemented with this modification, by extracting the required reference signal from the main distorted signal with the help of BPFs. Similar to section 6.3, 4th order Butterworth, Chebychev and Elliptic filter have been used for this modification. The results of simulation for these schemes are given in Figures 5.6, 5.7, and 5.8.

The results are not only consistent among each other but also with the results obtained for the op amp based schemes as shown earlier.

The no noise case and the noise SNR = 30 dB cases are almost identical. However, elliptic filter is seemed to be the one which had given the best results in these categories. Butterworth filter has clean curves but the cut off is not quite sharp. Chebychev filter, on the other hand has good cut off but some ripple in the stop band.

For the case of SNR = 10 dB, the Chebychev and Elliptic filters have very good performance. The Butterworth filter has some offsets in the stop band due to the cut off properties. However, for 0 dB case, Elliptic filter has more ripples in the stop band than others. Butterworth filter response in almost similar to its other three responses, while the ripples are also increased for the Chebychev filter.
Figure 5.6. Results of simulation with 4th order Butterworth BPF, (a) without noise, (b) SNR = 30 dB, (c) SNR = 10 dB, (d) SNR = 0 dB.
Figure 5.7. Results of simulation with 4th order Chebychev BPF. (a) without noise. (b) SNR = 30 dB. (c) SNR = 10 dB. (d) SNR = 0 dB.
Figure 5.8. Results of simulation with 4th order Elliptic BPF, (a) without noise, (b) SNR = 30 dB, (c) SNR = 10 dB, (d) SNR = 0 dB.
Overall, the performance of the scheme with the BPFs as reference signal generator are quite satisfactory, and the ordinary components have produced results that are acceptable as well as quite robust to the noise.

5.3. Experimental Results.

The flux based scheme was tested in the laboratory at low voltage levels, using the available transformers. Due to losses in the transformers, and the weak input signals, the output amplitude did not correspond to the actual fundamental voltage, but the shape of the waveform was improved, giving the confidence that the scheme can be used practically provided a correct design is used.

For laboratory testing, the distorted signal was constructed with a fundamental and a time varying, variable frequency signal, added together within a multi-winding auto transformer. The variable distortion signal is shown in Figure 5.9. This signal served two purposes.

- It provided variable SNR throughout the experiment.
- It provided different harmonics, both characteristic as well as non-characteristic.

As a result, the complex signal which was obtained by mixing just two signals was enough to be used as a good sample signal which has all the desired problems as discussed earlier.

The results obtained from this scheme shows highly reduced distortion. Two results are shown in Figures 5.10 and 5.11, corresponding to two different noise excitation powers. As can be seen from the figures, the resultant signal has slightly reduced amplitude, but the distortion in the original signal has been reduced to an appreciable extent.
Figure 5.9. Variable frequency signal which was added with the fundamental to make the distorted signal.
Figure 5.10. Distorted signal (top) with low power noise excitation, and the output from the scheme (bottom).
Figure 5.11. Distorted signal (top) with high power noise excitation, and the output from the scheme (bottom).
CHAPTER 6

CONCLUSION AND RECOMMENDATIONS

6.1. Conclusion.

This thesis has contributed to two complimentary areas related to power system harmonics, i.e., estimation of amplitudes and phases of different frequency components in a distorted voltage or current signal, and filtering of these unwanted frequencies. Although, the emphasis has been on the power system harmonics, the algorithms and techniques developed in this work can be used in any situation which has similar problems and structure.

For the first part the estimation algorithms based on the well known Least Square criterion and their usage for the estimation of different frequency components in a voltage signal are presented. Most of the estimates are within 3 to 4% error. Maximum deviation is within 9 to 10%. The results obtained from the recursive estimation are fairly close to the estimates obtained by the batch algorithms. This finding enables the use of recursive algorithms to be implemented on-line and provide estimates with time-varying tracking capability.

The estimates for fundamental with single frequency estimation algorithm are closer to the true values, compared to the estimates with multiple frequency estimation algorithm. The reason lies in the calculation of SNR. With multiple frequency estimation, the signal power is the sum of the powers in all harmonics, and hence the corresponding noise power is large. However, with single frequency estimation, the signal power is only due to the fundamental frequency. Hence part of the noise power is shared by harmonics and overall stochastic power in the spectrum decreases. This produces better estimates.
The estimates thus obtained, provide the necessary information about the signal frequency contents. Based on this information, important decisions for system component designs, operation and protection can be taken.

Due to optimality of LS based algorithms with respect to Gaussian noise, the estimates with this kind of noise distribution were found to be closer to the true values as compared to those in uniform noise.

For the second part two new schemes to efficiently eliminate the harmonics from a distorted voltage signal in electric power network are presented. The techniques are characterized with their ability to cancel out harmonics of different nature. Robustness to time varying, and variable power noise is an attractive feature of these schemes. Specially the noise mitigation capability can also be used in other areas, like in telephone and other communication lines.

The performance of the schemes is constrained, however, with the design of the related components. This include the design of high voltage based differential amplifiers and high saturation threshold transformers. The design of injection transformer is also very critical. It should be capable to withstand the load current and the related thermal and potential stresses, and at the same time accommodate the injecting signal $d(t)$ from the previous stages of the schemes. The practical implementation at low voltages, builds confidence in the scheme and it can be expected that the scheme would work equally well for high voltages by proper design.

6.2. Recommendations.

In the light of this work many recommendations and suggestion have emerged.
• It is recommended as an essential part of design and planning, that the harmonic analysis section must be included in order to:
  
a) Estimate the type of harmonics present in the system, and the distortion they are producing, or expected to produce.

b) Have a better designing tool to efficiently design equipment for maximum performance.

c) Determine the resonance frequency in the system to ensure that it is not coincident with any harmonic frequency.

• The estimation algorithm must be implemented over the real time data to build a database related to system harmonic information.

• The manufacturers must strictly follow the standard harmonic limits. The imposing of more strict legislation is highly needed and recommended.

• It is recommended to change the design philosophy, where every equipment is being utilized exhaustively, in order to drive more output for the same cost. This tends to saturate things and produces the environment for harmonics to flourish.

• In harmonic environment, the thermal stress on the transformer is proportional to the square of the harmonic frequency. It is, therefore, recommended to either derate the transformer or to use K factor transformers.

• Harmonic filters must be installed in the system and their values and other design features must be based upon the harmonic estimates obtained from the analysis of the system along with future worst case design strategy.
• It is recommended to use active and adaptive filters (as discussed in this work), in order to filter out characteristic as well as uncharacteristic harmonics.

6.3. Future Work.

Further possible extensions and improvements in the presented work are stated as follows.

• Development of estimation algorithms using nonlinear signal models.

• Development of estimation algorithms based on maximum likelihood and Kalman filtering techniques with nonlinear signal models.

• Application of genetic algorithms for harmonic estimation.

• Application of recursive estimation techniques, developed in this work, in real time environment.

• Design for flux based high voltage harmonic filtering scheme.

• Implementation of op-amp based scheme using high voltage transistors.
APPENDIX

Following is the main program in MATLAB to estimate the amplitude and phase of all the assumed harmonics in a distorted signal. The sample signal is the same as discussed in chapter 5.

% Main Routine est16.m
% This program estimates the amplitude and phases of sinusoids as given by subroutines
% hgen1.m and hgen2.m.
% 
% % frs = Frequencies assumed to be present in the signal.
% % For any specific simulation, change the frequency contents in this vector and
% % correspondingly in hgen1 and hgen2.
% % ratios = Different Signal to Noise Ratios used.
% % Can be modified for any number of SNRs.
% % mts = Represents the six different estimation algorithms used.
% % Since only six methods were used, therefore the count in vector does not exceed 6.
% % ffs = Forgetting factors used in the algorithms.

% % %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% % Begin Initialization.
% % %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear all
frs = [ 60 300 420 660 780 ];
mts = [ 1 2 3 4 5 6 ];
ratios = [ 10000 20 10 0 ];
ffs = [ 1 0.98 0.95 0.9 ];
ts = [ 1 2 ];
c6 = 0 ;
itr = 20 ;

% % %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% % Begin Main Loop.
% % %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for c1 = 1 : length(frs)
disp(frs(c1))
for c2 = 1 : length(mts)
    for c3 = 1 : length(ratios)
        ...
    end
end
for c4 = 1 : length(ratios)
    for c5 = 1 : length(ffs)
        met = mts(c3) :
        snr = ratios(c4) :
        fs = frs(c1) :
        lambda = ffs(c5) :
        c6 = c6 + 1 :
        for i = 1 : itr
            if nts(c2) == 1
                [n.x.] = hgen1(snr) :
            else
                [n.x.] = hgen2(snr) :
            end
            v = x + n :
            if met == 1
                [a,b] = lse(v,t,fs) :
            elseif met == 2
                [a,b] = wlse(v,t,lambda,fs) :
            elseif met == 3
                [a,b] = rlsie(v,t,ns,lambda,fs) :
            elseif met == 4
                [a,b] = rlsce(v,t,ns,lambda,fs) :
            elseif met == 5
                [a,b] = xls(v,t,ns,lambda,fs) :
            elseif met == 6
                [a,b] = xls1(v,t,ns,lambda,fs) :
            end
            amp(i) = sqrt(a^2+b^2):
            ph(i) = atan2(b/a)*180/pi:
        end
        AMP(c6) = mean(amp) :
        PH(c6) = mean(ph) :
    end
end
end

% **********************************************************************
% Subroutine hgen1.m
% **********************************************************************

function [n.x.t] = hgen1(nmax) :
% The purpose of this function is to generate sample vectors
% to represent the sampled distorted signal with additive
% Uniform Noise.
% For actual calculation, refer to section 5.2.
% fs describes the frequency components to be included.
% am are the amplitudes for each frequency respectively.
% pm are the corresponding phases.
% ammax refers to the maximum voltage in the signal.

fs = [ 60 5 7 11 13 ];
am = [ 0.95 0.09 0.043 0.03 0.033 ];
pm = [ -2.02 82.1 7.9 212.9 162.6 ] * pi / 180 ;
ammax = 1 ;
f1 = 2*pi*60 ;
T = 1/60 ;
sk = 0.05/1620 ;
t = [ 0 : sk : T ] ;
l1 = length(fs) ;
x1 = zeros(l1,length(t)) ;
for n = 1:l1
    x1(n,:) = ammax * am(n) * sin((f1*fs(n)*t)+pm(n));
end
x = sum(x1) ;
ap = ammax^2 * am.^2 / 2 ;
px = sum(ap) ;
R = 10^ap(nmax/10) ;pn = px / R ;
a = sqrt(3*pn) ;
no = a * ( 2 * rand(1,length(x))-1 ) ;
return

% ****************************************
% Subroutine hgen2.m
% ****************************************

function [no,x,t] = hgen(nmax) ;

% The purpose of this function is to generate sample
% vectors to represent the sampled distorted signal in
% additive Gaussian noise.
% For actual calculation, refer to section 5.2.
% fs describes the frequency components to be included.
% am are the amplitudes for each frequency respectively.
% pm are the corresponding phases.
% ammax refers to the maximum voltage in the signal.

fs = [ 60.4/60 5 7 11 13 ];
am = [ 0.95 0.09 0.043 0.03 0.033 ];
pm = [ -2.02 82.1 7.9 212.9 162.6 ] * pi / 180 ;
ammax = 1 ;
f1 = 2*pi*60 ;
T = 1/60 ;
sk = 0.05/1620 ;
t = [ 0 : sk : T ] ;
l1 = length(fs) ;
x1 = zeros(l1,length(t)) ;
for n = 1:l1
    x1(n,:) = ammax * am(n) * sin((f1*fs(n)*t)+pm(n));
end
x = sum(x1) ;
ap = ammax^2 * am.^2 / 2 ;
px = sum(ap) ;
R = 10^(nmax/10) ;
pn = px / R ;
no = noise(length(x),0,pn) ;
return

% ******************************************************************************
% Subroutine noise.m
% ******************************************************************************

function x = noise(n,m,v)

% This function generates Gaussian
% noise of the required mean and variance
% and the required number of points.
% x = noise(n,m,v)
% n = number of required points.
% m = mean
% v = variance

randn('seed',sum(100*clock))
x = randn(1,n) ;
y = x * sqrt(v/cov(x)) ;
x = y+(m-mean(y)) ;
% Subroutine lse.m, the least square estimator.

function [a,b] = lse(v,t,fs):
    w = 2 * pi * fs;
    A = [ sin(w*t) : cos(w*t) ]';
    Y = [ v ];
    X = pinv(A) * Y;
    a = X(1);
    b = X(2);
    return
end

% Subroutine wlse.m, the weighted least square estimator.

function [a,b] = wlse(v,t,lemda,fs):
    wo = 2 * pi * fs;
    A = [ sin(wo*t) : cos(wo*t) ]';
    Y = [ v ];
    W = diag((1/lemda).^t);
    X = inv(A'*W*A) * A'*W*Y;
    a = X(1);
    b = X(2);
    return
end

% Subroutine rlsie.m, the recursive least square estimator information form.

function [a,b] = rlsie(v,t,ns,lemda,fs):
    N = length(t);
    n = 2;
    w = 2 * pi * fs;
    offset = ceil(N / ns);
    n1 = [ 1 : offset : N ]:
    t1 = t(n1):
vl = v(n1);
K = zeros(n,1);
P = 1000 * eye(n);
aa = sin(w*t1);
bb = cos(w*t1);
h = [aa' bb'];
Th = [0 0]';

for i = 1:length(vl)
    P = inv(inv(P) + h(i,:)' / lemda * h(i,:));
    K = P * h(i,:)' / lemda;
    Th = Th + K * (vl(i) - h(i,:)' * Th);
end

a = Th(1);
b = Th(2);

return
end

% Subroutine rlsce.m, the recursive least square estimator covariance form.
function [a,b] = rlsce(v,t,ns,lemda,fs);

N = length(t);
n = 2;
w = 2 * pi * fs;
offset = ceil(N / ns);
n1 = [1 : offset : N];
t1 = t(n1);
vl = v(n1);
K = zeros(n,1);
P = 1000 * eye(n);
aa = sin(w*t1);
bb = cos(w*t1);
h = [aa' bb'];
Th = [0 0]';
Nm = length(vl);
for i = 1: Nm

\[ K = P \ast h(i,:)' \ast \text{inv} \left( h(i,:) \ast P \ast h(i,:)' + \text{lemda}^2 (t(Nm)-t(i)) \right) ; \]
\[ P = \left( \text{eye}(n) - K \ast h(i,:) \right) \ast P \text{./ \text{lemda}^2 ((Nm)-t(i))} ; \]
\[ P = P*(\text{eye}(n)-h(i,:)'*h(i,:)*P/(\text{lemda}+h(i,:)*P*h(i,:)'))/\text{lemda} ; \]
\[ K = P \ast h(i,:) ; \]
\[ Th = Th + K \ast ( v1(i) - h(i,:) \ast Th ) ; \]

end

a = Th(1) ;
b = Th(2) ;

return

end

%*****************************************************************************************************************************************

% Subroutine xlse.m, the extended least square estimator type 1.
%*****************************************************************************************************************************************

function [a,b] = xlse(v,t,ns,lemda,fs) ;

N = length(t) ;
n = 2 ;
w = 2 * pi * fs ;
offset = ceil(N / ns) ;
n1 = [1 : offset : N] ;
t1 = t(n1) ;
v1 = v(n1) ;
K = zeros(n,1) ;
P = 1000 * eye(n) ;
aa = sin(w*t1) ;
bb = cos(w*t1) ;
h = [aa' bb'] ;
Th = [0 0]' ;

for i = 1:length(v1)

G = lemda + h(i,:)*P*h(i,:)';
P = P * (\text{eye}(n)-h(i,:)'*h(i,:)*P/G) ;
K = P * h(i,:)';

end
\[ Th = Th - K * ( vl(i) - h(i,:) * Th ) ; \]

eend

a = Th(1) ;
b = Th(2) ;

return

eend

% Subroutine xsel1.m, the extended least square estimator type 2.
% **********************************************************************
function [a,b] = xsel1(v,t,ns,lemda,fs) :

N = length(t) ;
n = 2 ;
w = 2 * pi * fs ;
offset = ceil(N / ns) ;
n1 = [ 1 : offset : N ] ;
t1 = t(n1) ;
v1 = v(n1) ;
K = zeros(n,1) ;
P = 1000 * eye(n) ;
aa = sin(w*t1) ;
bb = cos(w*t1) ;
h = [ aa' bb' ] ;
Th = [ 0 0 ] ;

for i = 1:length(v1)

P = inv( inv(P) + h(i,:) * h(i,:) ) ;
Th = Th + P * h(i,:) * ( vl(i) - h(i,:) * Th ) ;
end
a = Th(1) ;
b = Th(2) ;
return
end
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VITA

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