Effect of Quality on Lotsizing, Warranty and Inventory Control Decisions

by

SYED MUBASHER NOMAN

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This thesis, written by **SYED MUBASHER NOMAN** under the direction of his Thesis Advisor and approved by his Thesis Committee, has been presented to and accepted by the Dean of Graduate Studies, in partial fulfillment of the requirements for the degree of **MASTER OF SCIENCE IN SYSTEMS ENGINEERING.**

Thesis Committee

____
Dr. MOHAMMED BEN – DAYA (Chairman)

____
Dr. SALIH O. DUFFUAA (Member)

____
Dr. M. DARWISH (Member)

____
Department Chairman

____
Dean of Graduate Studies

____
Date

Dedicated

to

My Dear Parents

Whose Prayers, Guidance and Inspiration led to this

Accomplishment.

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All praises are for ALLAH(SWT), the most compassionate, the merciful. May peace and blessings be upon prophet Muhammed(PBUH), his family and his companions. I thank almighty ALLAH(SWT) for giving me the knowledge and patience to complete this work.

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THESIS ABSTRACT

Name: SYED MUBASHER NOMAN

Title: Effect of Quality on Lotsizing, Warranty and Inventory Control Decisions

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This thesis focuses on production, inventory, warranty and quality control problem from both producer's and consumer's point of view. The purpose of this thesis is twofold. First, models for the producers which integrate lot sizing, warranty and quality improvement through preventive maintenance are developed. Second, integrated inventory and quality assurance models for the consumer are developed.

In the first part, joint lot sizing and warranty models with and without preventive maintenance are developed for an imperfect production process. The process starts in in-control state then shifts at a random point in time to out-of-control state and starts producing more fraction of nonconforming items. The time to shift follows a general distribution. Effects of preventive maintenance on quality and warranty costs are investigated. In the second part, consumer's inventory and quality assurance decisions through inspection are integrated in a single model. Models for both deterministic and stochastic demand are considered. It is assumed that the lot fraction nonconforming is a random variable following a beta distribution. Constraints on consumer and producer's risks are included to generate fair plans for both parties.

Numerical examples are presented and sensitivity analysis is conducted to investigate different aspects and features of the proposed models. It is shown through numerical examples that the joint policies always result in a better solution then separate once.

Keywords: Inventory, Quality, Inspection, Warranty.

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خلاصة الأطروحة

الاســـــم : سيد مبشر نعمان
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تركز هذه الرسالة على مسألة الإنتاج والمخزون والضمان وضبط الجودة من وجهة نظر كلٍ من المنتج والمستهلك. لهذه الرسالة هدف مزدوج. أولاً تم تطوير نماذج للمنتج يتكامل فيها تحديد حجم كمية الإنتاج والضمان وتحسين الجودة عبر الصيانة الوقائية. ثانياً تم تطوير نماذج متكاملة للمخزون وضمان الجودة للمستهلك.

في الجزء الأول تم تطوير نماذج تحديد الكمية والجودة معاً عند وجود وعدم وجود الصيانة الوقائية لعملية إنتاج غير سليمة، حيث تبدأ العملية بحالة السيطرة ثم تنحرف في وقت عشوائي إلى حالة عدم السيطرة وتبدأ في إنتاج وحدات غير سليمة. هذا الوقت يتبع توزيعاً عاماً. تم التحقق من آثار الصيانة الوقائية على كلٍ من كلفة الجودة والضمان. في الجزء الثاني تمت مكاملة قرارات المخزون وضمان الجودة عبر الفحص في نموذج واحد للمستهلك. تم اعتبار نماذج كل من الطلب المحدد والطلب الاحتمالي. افترض أن نسبة الوحدات التالفة متغير عشوائي يتبع توزيع بيتا قيود الخاطر على المستهلك والمنتج ضمنت لإيجاد خطط منصفة لكل من الطرفين.

تم عرض أمثلة عددية وأجري تحليل الحساسية للتحقق من مختلف الجوانب والملامح للنماذج المقترحة، حيث اتضح من الأمثلة العددية أن السياسات الموحدة تعطي دائماً حلولاً أفضل تعطيه السياسات المنفصلة.

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Chapter 1

Introduction

1.1 Overview

In this chapter, a brief introduction to inventory and quality control problem is presented by defining different areas of inventory and quality. The main area of concern in this thesis is the determination of joint inventory, maintenance and quality policies. This area is introduced in this thesis by two basic models of inventory and warranty, and inventory and quality. An introduction to inspection plans is presented in section 1.4, in which acceptance sampling is discussed in detail. The problem under consideration is discussed in section 1.5, followed by the objectives of the thesis in section 1.6. The organization of the thesis is presented in section 1.7.

1.2 Plan and Strategy

Every firm has a plan or strategy. A business strategy is composed of several elements,

- The marketing strategy.
- The financial strategy.
- The production and operations strategy.

In this thesis the sum total effect of these strategies on a production system is discussed and effect of quality on lot sizing, warranty and inventory is studied. Before further discussion a brief definitions of the terms used, is given.

1.3 Terms and Terminology

1. **Inventory:** Inventory is defined as the raw material, semi-finished parts and assemblies, and finished goods that are in the production system at any point in time. Inventory serves as buffer between stages of production system and between the production system and customers. Inventory cost is the sum total of ordering cost or fixed cost (cost of replenishing inventory), holding cost (cost of carrying inventory), shortage cost (penalty on incurring shortage).

2. **Inventory policies:** Inventory policies refer to review and ordering discipline used in controlling the inventory. Depending upon when to order and how

much to order, most commonly used inventory policies are (i) Periodic review policy, (ii) Order up to 'R' policy, (iii) Continuous review policy, (iv) Fixed reorder quantity policy, (v) Base stock policy, etc.

3. **Warranty:** A warranty is a contractual agreement which requires the manufacturer to rectify all failures occurring within the warranty period. The rectification action can include repair, replacement or cash refund. Warranty is important for both consumers and manufacturers. It protects consumer against non-conforming or poor quality items that do not function satisfactorily . At the same time, it protects manufacturer against unreasonable claims from customer. It is a very popular tool in marketing.

4. **Inspection:** Inspection is further classified as process inspection and product inspection. (i) Process inspection shows whether the process is in good production state, producing good units within its natural variability. Process inspection is usually scheduled at the beginning of the cycle or at various intervals during the production period. (ii) Product inspection is done online (during production) and/or at the end, it is discussed in detail in the next section.

5. **Restoration:** Restoration is defined as bringing a production system back to its natural state. If after inspection, the system is found in a state where it is producing low quality items than it is desired/desinged to (out-of-control

state), repair work is done on the system and it is brought back to its natural state. Usually it is assumed this type of restoration gets system back to work without decreasing its chances to go back to out-of-control state.

6. **Preventive maintenance:** Preventive maintenance is the maintenance carried out at predetermined intervals or intended to minimize the probability of failure or the performance degradation of equipment.

1.4 Inspection

Inspection for the purpose of deciding on the acceptability of material is carried on at many points in the manufacturing cycle, for example, when products are received from a vendor or from another department or when the products are shipped to customers. There are several methods on deciding whether to accept or reject the product. These are :

- **No inspection at all:** This obviously involves a great risk of accepting a product that is defective.

- **One hundred percent inspection:** Inspect every product received. But, as it is well known, 100 percent inspection is not 100 percent effective in removing defects. In addition to the fact that 100 percent inspection is often less effective than sampling. It has other drawbacks:

  - 100 percent inspection is expensive.
  - It obscures the actual risk involved, because the margin of error is not known.
  - Because the margin of error is not known, the information provided by 100 percent inspection is relatively useless in improving the production process.
  - 100 percent inspection cannot be used for destructive testing.
  - scheduling delays.

- **Spot checking :** This is a compromise between no inspection and 100 percent inspection, but it means that many lots are accepted with no check on them.

1.4.1 Acceptance Sampling

Acceptance sampling is the process of evaluating a portion of the product in a lot for the purpose of accepting or rejecting the entire lot as either conforming or non conforming to a quality specification.

The main advantage of acceptance sampling is economy; the lower cost of inspecting. Since only a part of the lot is inspected, it results in overall cost reduction. In addition to this major advantage there are others.

- There is less damage to the product.

- The lot is disposed off in shorter time so that shop scheduling, inventory turns, and delivery are improved.
- It is usually less expensive because there is less inspection.
- It is applicable to destructive testing.
- Fewer personnel are involved in inspection activities.
- It often reduces the amount of inspection error.

It may be noted here that no sampling plan is perfect; there is always a chance that the sample may not always contain the same proportion of defectives items as the lot. On the basis of the sample, there are risks of accepting "bad" lots and rejecting "good" lots.

1.5 Joint Optimal Policy

Traditionally, an optimal production run length can be obtained by analyzing economic manufacturing quantity (EMQ) model. In traditional EMQ models, there is a basic assumption that the production process is perfect and stationary. Under this assumption, the production system does not deteriorate with time and continuously produces conforming items. However, this assumption is not true in practice. Many production systems continuously deteriorates due to usage, age, corrosion, fatigue and cumulative wear etc. Because of inevitable deterioration, the operating condition of a production system is usually classified into two states; in-control and out-of-control. The process starts in an in-control state and then may shift to the out-of-control state producing more non-conforming or substandard items than in in-control state. Quality of the items produced by an imperfect production process will have an effect on several decisions made by the producer and the customer. Here we are interested in the following two aspects of the decision making process.

1. For the producer, nonconforming products that reach the customer incur more post-sales service, especially when the product is sold with warranty. In this case it is important to have an integrated model that helps the producer make rational decision regarding the tradeoffs between warranty expenses and prevention costs aimed at improving the quality of product during production.

2. For the customer, nonconforming products affect his inventory policy. In this case it is important to have an integrated model that helps the customer make rational decision regarding both ordering and the quality assurance policy.

1.5.1 Warranty and EMQ

Warranty is being used as a marketing tool by many manufacturers. To reduce the warranty cost, one should shorten the production run length to allow frequent restorations, thereby retaining the system in in-control state to reduce the number of nonconforming items produced. However, a short production run length results in more setup cost and restoration cost. Therefore, there is a need to determine an optimal production run length such that the expected total cost per item is minimized. Most of the work in literature is concentrated on minimizing the cost incurred by the product before it is sold but not after. For products sold with warranty, the resulting post sale warranty cost is closely related to the quality of the items produced by a deteriorating production system. Hence it is important to incorporate the warranty cost in EMQ model to better evaluate the effect of quality and thereby justify quality improvement measures.

1.5.2 Quality Assurance and Inventory Control

Typically the operating policy for the inventory control system for a commodity is developed independent of operating policy for the quality control system for that commodity and vice versa. In many circumstances, these systems are dependent on one another. When a lot is manufactured or received, management may use some type of quality control system to decide whether the lot should be accepted and placed in inventory or rejected. If the lot is rejected, it cannot be used immediately to satisfy demand. The quality of incoming product and its probability of rejection becomes a source of uncertainty in determining the inventory control parameters. Specifically, a higher probability of rejection implies a longer expected production time or less quantity in lot. This may affect the lot size to be ordered. If the lot size is affected, the design of the quality control system has to be altered based on new lot size. Hence it is important to have a model that could determine optimal joint inventory and quality control policy.

The purpose of this thesis is twofold. First, there is a need to develop an integrated model for the producer which integrate lot sizing, warranty and quality improvement aspects. Quality improvement can be achieved by improving the performance of the production process through proper preventive maintenance activities. Second, an integrated inventory, quality assurance model for the customer will be helpful in formulating rational joint inventory control and quality assurance decisions.

1.6 Objective of the Thesis

The objective of this thesis is to extend the existing inventory models by incorporating warranty, quality and sampling inspection plans. Specifically, the objectives are to:

1. Develop an integrated EPQ, quality, warranty model for general distribution with intermediate inspection stages.

2. Extend the above developed Model to the case where Preventive maintenance is done and study the tradeoff between PM and warranty.

3. Integrate Inventory Control policy and Quality control Policy for EOQ, where both the sampling plan and optimal order quantity are decision variables and

4. Extend the model developed in objective III to the case of continuous review inventory policy.

1.7 Thesis Organization

The thesis is organized as follows : Literature review in the areas of deteriorating production system (DPS), deteriorating production system (DPS) with warranty and economic order quantity (EOQ) with inspection plan is presented in the next chapter with emphasis on the models of Yeh et. al. [1] and M. D. Hanna and J. M. Jobe [2]. These models constitute basis for the models developed in this thesis.

The generalized model that incorporates inventory, quality and warranty costs is presented in chapter 3. A general warranty inventory model with preventive maintenance is presented in chapter 4. The integrated inventory and inspection policy for EOQ model is presented in chapter 5 and integrated inventory and inspection policy for continuous review (Q,r) model is presented in chapter 6. Finally, conclusions and recommendations for future research are outlined in chapter 7.

Chapter 2

Literature Review and Problem Definition

2.1 Introduction

The purpose of this chapter is to present the literature in the area of deteriorating production systems (DPS), deteriorating production system (DPS) with warranty and economic order quantity (EOQ) with inspection plan. Section 2.2, presents the literature in this area. The description of the model developed by of Yeh et. al. [1] (referred to as Model I) and M. D. Hanna and J. M. Jobe [2] (referred to as Model II) is presented in section 2.3. These models are used for the extension in this thesis.

2.2 Literature Review

Ever since the EMQ (EOQ) model was presented many people came up with many ideas to make it closer to practical situations. Literature is full of efforts done in various directions with deterministic and stochastic parameters. Here we focus on deteriorating production systems and warranty and on EOQ and sampling plan.

2.2.1 Deteriorating production system (DPS)

Classical approaches to the problem of determining economic ordering quantity have always assumed implicitly that the items produced are of perfect quality. Product quality, however, is not always perfect, and usually is a function of state of production process. When the production process is in in-control condition, items produced may be of high or perfect quality. As time goes on, the process may deteriorate as a result of which the items produced contain some defectives or items that are of substandard quality. The relationship between production lot size and the quality of the product is thus significant. Here are key models in literature on deteriorating production systems. For an extensive review the reader is refered to Ben-Daya and Rahim [3]

Lee and Rosenblatt [4] for the first time relaxed the assumption of perfect stationary process, which is realistic. They gave a model which follows simple markov process; a production system starts in in-control state and then shifts to out of control state and will remain there until it is brought back to in-control state by restoration. They showed that the optimal production run length is less than that of traditional EMQ model.

E.L Porteus [5] gave the mathematical models linking product quality with lot size; with the concept of smaller lot size giving better quality.

Lee and Rosenblatt [6] gave the optimal production run length and maintenance schedule simultaneously for the same type of the system discussed above.

G. Keller and H. Noori [7] extended Porteus [5] for a situation where demand during lead time is probabilistic and presented a model for justification of investing in quality improvement.

Lee and Rosenblatt [8] extended their model of simultaneous determination of optimal production run length and maintenance schedule considering restoration cost a function of detection delay.

S.T.Tseng [9] gave the optimal preventive maintenance policy for deteriorating production system. Effort was also done in direction of finding optimal lot size to reduce rework cost of defective items.

H.C Kim and Y.Hong [10] and **Hariga and Ben-Daya**[11] gave an extension to EMQ model for failure prone machines for general lifetime distribution.

Most of the work mentioned earlier concentrated on minimizing the cost incurred by the product before it is sold but not after.

2.2.2 Deteriorating production system with warranty cost

In this section we review the literature that integrates lotsizing and warranty decisions.

A warranty of some type, is associated with almost all products, whether sold directly to customer or to a producer for assembly into a consumer product. The nature and the extent of warranty affect the sales, market share, cost and profits of many businesses. A warranty can be defined as an assurance from a seller to buyer that the product sold is guaranteed to perform satisfactorily up to a certain length of time (warranty period). Warranties are important for both sides, buyers need warranty to satisfy their need for assurance that the product will perform satisfactorily, seller uses warranty as a marketing tool. Sales are thus boosted by converting potential sales into actual sales. The knowledge of total expected warranty liability will help manufactures plan operations more effectively.

Here are key references that deals with the warranty and its application in inventory models.

A. Mitra and J. G. Patankar [12] gave goal programming model for a multi objective decision problem viz. warranty time, total warranty cost, achievement of specific market share, production quantity and lot size. Priorities are set for different goals and goal programming approach is used to solved the formulated problem

Djamaluddin et.al. [13] For the first time incorporated the warranty cost in the model, as this post sales warranty cost is directly dependent on the quality of the product. They presented a cost model to derive an optimal lot size, but without considering holding cost.

Yeh et.al [1] gave the model to derive optimal production run length for deteriorating production system whose products are sold with free minimal repair warranty and proved the existence of unique optimal production run length and also gave a closed form solution with an approximation for a system which is characterized by shift following exponential distribution.

R. Ganeshan, S. Kulkarni and T. Boone [14] looked the quality in Taguchi's perspective instead of traditional; binary decision. They gave a mathematical model so as to reduce the Taguchi's quality loss and inventory cost.

2.2.3 Economic order quantity and Inspection plan

Little work has been done on joint consideration of inventory control and quality control systems. Most of the previous work concentrates on the effect of defective items on ordering policies. In traditional inventory model it is assumed implicitly that the quantity received is of good quality and can be used to satisfy the demand. stockouts can occur only when demand exceeds the amount of inventory on hand. It is not however, the way stockout occur. Stockout may also be triggered, for instance, by unexpected presence of defective products in inventory. This assumption of perfect quality underlying most of the inventory models does not, in fact, appear to be very realistic because of extensive use of acceptance sampling by business and industry in accepting or rejecting a lot.

Here are key references which take quality assurance into consideration while designing inventory models.

Wei Shih [15] modified the EMQ model and a single-period inventory model by assuming that defective items are not repairable and cannot be used to fill demand.

Kalro and Gohil [16] considered a lot size model with backlogging where the number of items received may be different from the order quantity. The difference, caused by some uncontrollable factors, which is assumed to be a normal random variate.

Lee and Rosenblatt [17] derived optimal ordering quantities under two inspection policies. In the first policy, a lot is accepted without inspection, and is partially inspected before it is sold to consumers. as a result, some defective items could reach the market causing, loss of goodwill, repairing cost, or scrapping cost. In the second case it is assumed that all the items received have been inspected prior to purchase, and thus are defective free. Although no defective items reach consumer, this inspection policy requires a higher inspection cost and a higher unit purchase price.

M.H.Peters et.al. [18] integrated inventory and quality control policy, Considering rejection in lot to have an effect on lead time. They considered the time to rework on the rejected items as increased lead time. The uncertainty of the number of rejected items was translated as variable lead time and joint optimal policy was derived.

Kut C. So and C. S. Tang [19] gave classification of repair policies as repair all or repair none and developed the conditions for these to be optimal. they also showed that there exits a threshold batch size which governs the shift of repair policy.

M.D.Hanna and J.M. Jobe [2] included quality costs in the lot sizing decision for a fixed sampling plan for a EOQ model under a condition of non-replacement of defective items.

C. M. Wright and A. Mehrez [20] made a literature review of research that includes the relationship between quality and inventory

Cheung and Leung [21] extended M.H.Peters et.al. [18] for a case of two items. They had supply chain approach and considered supplier and consumer as a joint system.

C. H. Wang and S. H. Sheu [22] gave a inspection policy for a deteriorating production system. considering first few items to be of good quality and 100% inspection for the rest.

2.3 Description of Basic Models

In this section two basic models which are much closer to realistic situation are presented. These models try to integrate different industrial aspect ie. EPQ and warranty and EOQ and quality. These models will be used as reference to establish more realistic extended integrated model.

2.3.1 Model I : EPQ and Warranty

Among the models discussed above, Yeh et.al. [1] model addressed the integration of EPQ and warranty for a deteriorating process. They considered not only pre-sales (manufacturing) cost but also post sales (warranty) cost. A model without consideration of post sales cost will give sub-optimal results and can even cause great damage to firm, both in profit and goodwill.

Yeh et.al. [1] Considered a production system producing a single product. It is assumed that demand and production rate are constant and the production rate exceeds the demand rate. No backlogging is permitted. The process starts in the in-control state. However, the process may shift to the out-of-control after time 'X'. It is assumed that the elapsed time, X, of the process in the in-control state follows exponential distribution with finite mean $1/\lambda$. Once the process shifts to the out-of-control state, it stays there till the end of production run. After the completion of a production run, the process is setup with a cost $k > 0$ and is inspected to reveal the state of the process. If the process is out-of- control, then it is brought back to in-control state with an additional restoration cost.

Assumptions and Notations

d	$=$	demand rate(units/unit time),
p	$=$	production rate,
k	$=$	setup cost,
h	$=$	inventory carrying cost \$/unit time,
C_m	$=$	production cost excluding setup,
t	$=$	production run length,
$C(t)$	$=$	expected total cost per unit produced,
$M(t)$	$=$	expected pre-sale cost per item,
N	$=$	number of non-conforming items in production run,
$E(N)$	$=$	expected number of non-conforming items produced,
w	$=$	length of warranty period,
$r_1(\tau)$	$=$	hazard function for conforming items,
$r_2(\tau)$	$=$	hazard function for non-conforming items,
C_r	$=$	minimal repair cost during warranty period,
WC	$=$	warranty cost per items,
r	$=$	Restoration cost.

$q(t)$ = fraction of non-conforming items,

θ_1 = probability of producing a non-conforming item in in-control state,

θ_2 = probability of producing a non-conforming item in out-of-control state.
It is assumed that the $r_1(\tau) < r_2(\tau)$ and $\theta_1 < \theta_2$

Model formulation

Manufacturing Costs: In addition to the setup cost, k/pt, and the production cost per item C_m, the following costs are incurred per item:

Inventory carrying cost:

$$E(HC) = \frac{(p-d)ht}{2d} \tag{2.1}$$

Restoration cost: The process is restored to the in-control state whenever it is found out-of-control during an inspection.Therefore the expected restoration cost is given by:

$$E(RC) = \frac{r(1-e^{-\lambda t})}{pt}$$

Total manufacturing(pre-sale)cost:

$$M(t) = C_m + \frac{(p-d)th}{2d} + \frac{k}{pt} + \frac{r(1-e^{-\lambda t})}{pt} \tag{2.2}$$

Number of non-conforming items:

$$N = \begin{cases} \theta_1 Pt & \text{for} X \geq t \\ \theta_1 PX + \theta_2 P(t-X) & \text{for } X < t \end{cases} \tag{2.3}$$

The expected value of $N$is given by

$$\begin{aligned} E(N) &= \int_t^\infty \theta_1 pt\lambda e^{-\lambda x} dx + \int_0^t [\theta_1 px + \theta_2 p(t-x)]\lambda e^{-\lambda x} dx \\ &= \theta_2 pt + p(\theta_1 - \theta_2)\frac{1-e^{-\lambda t}}{\lambda} \end{aligned} \tag{2.4}$$

Fraction defective:

$$q(t) = \frac{E(N)}{pt} = \theta_2 + (\theta_1 - \theta_2)\frac{1-e^{-\lambda t}}{\lambda t} \tag{2.5}$$

Since the hazard function of the conforming items is $r_1(\tau)$ and that of the non-conforming is $r_2(\tau)$, then the expected warranty cost is given by:

$$WC = C_r \left[(1-q)\int_0^w r_1(\tau)d\tau + q\int_0^w r_2(\tau)d\tau\right] \tag{2.6}$$

The expected total cost per item is made up of the manufacturing cost,the inventory carrying cost,the restoration cost,and the warranty cost.thus

$$C(t) = M(t) + W \tag{2.7}$$

$$C(t) = C_m + \frac{(p-d)th}{2d} + \frac{k}{pt} + \frac{r(1-e^{-\lambda t})}{pt} + C_r\left[(1-q)\int_0^w r_1(\tau)d\tau + q\int_0^w r_2(\tau)d\tau\right] \quad (2.8)$$

Approximating e^x to second degree and differentiating with respect to t and equating to zero we get,

$$\tilde{t^*} = \sqrt{\frac{2kd}{p[(p-d)h - A\lambda^2 d]}} \quad (2.9)$$

where

$$A = \frac{r}{p} - \frac{C_r(\theta_2 - \theta_1)}{\lambda}\left[\int_0^w r_2(\tau)d\tau - \int_0^w r_1(\tau)d\tau\right] \quad (2.10)$$

This model have several limitations as it covers only exponential case, whereas a model with weibull or general distribution could be helpful for many practical cases. Preventive maintenance can be applied to it and tradeoff between investment in preventive maintenance and warranty can be established.

2.3.2 Model II : Quality and Inventory Control

Among the models discussed in literature review of EOQ and inspection, Jobe and Hanna's [2] model is a good attempt to integrate inspection policy (sampling plan) and inventory policy. There are some errors in the model and major modeling flaw but still its a good basis for further discussions.

M.D.Hanna and J.M. Jobe [2] considered the inventory cost model including quality cost. The three alternatives with regard to inspection policy are no inspection, sampling inspection and 100% inspection. They combined Deming's [23] rule and the acceptance sampling plane outlined by Duncan [24] to have the best from above three . As per Deming's [23] policy, if the ratio of K_1 (cost of inspection) and K_2 (cost of accepting a bad item) is less than fraction of nonconforming (p), then 100% inspection is performed. If $K_1/K_2 > p$ then no inspection is done. As per Orsini [25], as attributed by Duncan [24], 100% inspection should be performed when $K_1/K_2 < 0.001$ and no inspection when $K_1/K_2 > 0.100$ and when K_1/K_2 is between these values sampling inspection should be done.

Combining these two policies they came up with the scheme as shown in Figure 1 and gave the cost models and optimal lot size for all three cases.

Their model contained some errors and does not consider the sampling plan as a decision variable

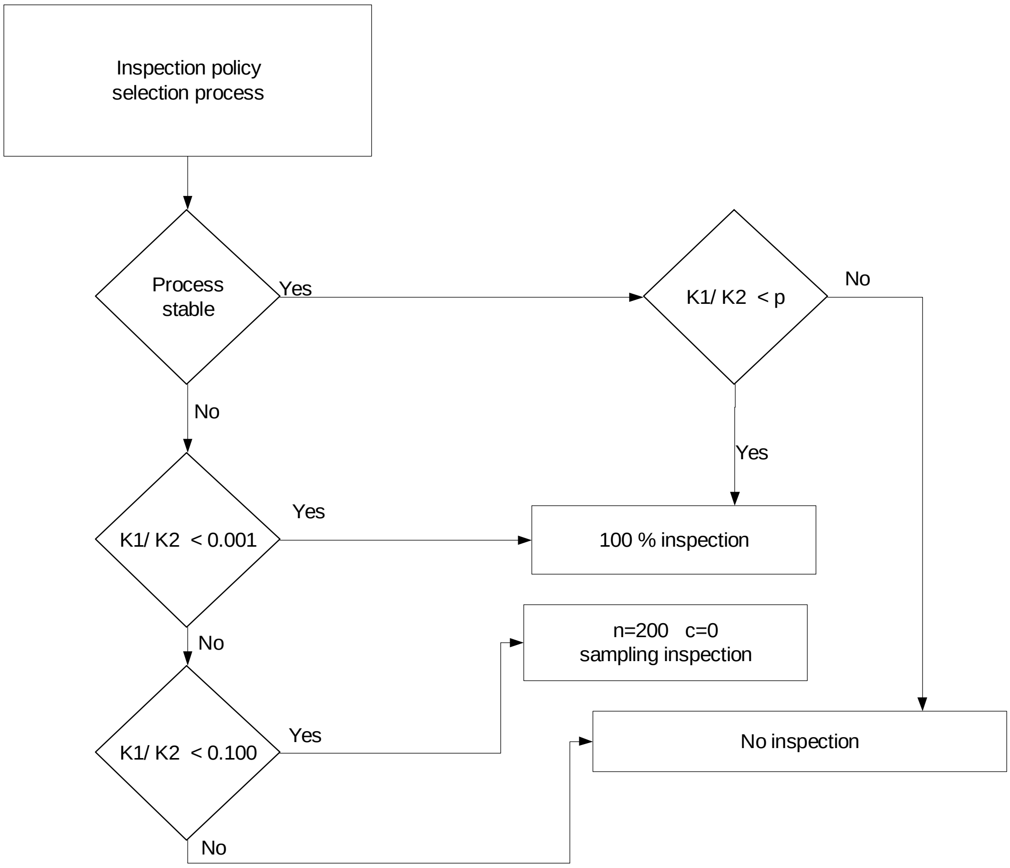

Figure 2.1: Sampling Scheme

Model formulation

In this section we present a corrected version of their model.

C_i = Quality cost per lot,

Q = lot size,

p = proportion of nonconforming in lot,

n = sample size for sampling inspection,

P_a = probability of acceptance of lot.

X = fixed ordering cost not including quality cost.

- No inspection

$$C_i = QpK_2 \tag{2.11}$$

$$TC = \frac{D}{Q}X + \frac{Q}{2}H + DpK_2 \tag{2.12}$$

- Sampling inspection

$$C_i = QpK_1 + (Q-n)pK_2P_a + (Q-n)(1-P_a)K_1 \tag{2.13}$$

$$TC = \frac{D}{Q}X + \frac{Q}{2}H + \frac{D}{Q}\left[nK_1 + (Q-n)pK_2P_a + (Q-n)(1-P_a)K_1\right] \tag{2.14}$$

- 100% inspection

$$C_i = QK_1 \tag{2.15}$$

$$TC = \frac{D}{Q}X + \frac{Q}{2}H + DK_1 \tag{2.16}$$

Optimal lot size is given by

$$Q^* = \sqrt{\frac{2DX}{H}} \tag{2.17}$$

for No inspection and 100% inspection

$$Q^* = \sqrt{\frac{2D(X - nP_a[pK_2 - K_1])}{H}} \tag{2.18}$$

for sampling inspection.

In addition to few mistakes, the inspection plan is also not optimal. Inventory and quality policies are derived by fixing a sampling plan which gives non optimal results. There is a serious mistake in the paper in the way stochastic fraction nonconforming is included in the model.

2.4 Conclusion

In this chapter, the literature in the area of deteriorating production system (DPS), deteriorating production system (DPS) with warranty and economic order quantity (EOQ) with inspection plan is reviewed. Models developed by of Yeh et. al. [1] (Model I) and M. D. Hanna and J. M. Jobe [2] (Model II) were presented to provide background for the work in this thesis. The next chapter extends model I to a case where shift follows general distribution.

Chapter 3

Optimal Production Run Length and Inspections Policy for the System Whose Products are Sold with Warranty

3.1 Introduction

The purpose of this chapter is to extend model I developed by Yeh et. al. [1] by incorporating intermediate process inspection and considering the time for the process to shift to follow general probability distribution.

Classically,an optimal production run length is obtained by analyzing economic manufacturing quantity (EMQ) model. It is Just tradoff between setup cost and holding cost. It has a basic assumption that the production process is perfect and stationary. Under this assumption, the production system does not deteriorate with time and continuously produces conforming items. However, this assumption is not true. Every production system continuously deteriorates. Because of inevitable deterioration, the operating condition of a production system is usually classified into two states; in-control and out-of-control.

1. In-Control state: The process is said to be in in-control state if it is working with its natural variability, producing items of quality for which it was designed or in other words giving out put under acceptable quality level.

2. Out-of-Control state: The process is said to be in out-of-control state if it is producing more nonconforming items then it was designed for.

The new model is able to answer following important research questions. The basic assumption of shift following exponential distribution has been relaxed to a general distribution.

- The effect of introducing the inspection in system which produces more bad quality items following a shift in the production process.

- The effect of introducing the inspection cost.

- The effects of introducing the inspection in the system on the quality of lot, there by on the warranty cost.

- The effects of introducing the inspection on the lot size and production run length for a general distribution.

The objective of this model is to maximize the expected profit by finding the optimum production run length and inspection schedule for the process discussed. This chapter is organized as follows, Model Development is presented in section 3.2, in which notations, model assumptions, statement of the problem, and model formulation are discussed. Solution and Analysis of the model are discussed in section 3.3, results are shown in section 3.4, then numerical examples are solved in section 3.5.

3.2 Model Development

In this section, an inventory, warranty model is developed incorporating intermediate process inspection. At first, necessary model assumptions are presented, followed by notations, then the statement of the problem is provided.

3.2.1 Model Assumptions

1. The process inspection is assumed to be error free.

2. The probability of producing a nonconforming unit in in-control state is less

than that in out-of-control state.

3. The hazard rate of nonconforming item is more than that of a conforming item.

4. Repair cost of both conforming and nonconforming items is same.

5. Units produced in in-control state can be differentiated from those produced in out-of-control state and can be sold at higher price.

6. The process restoration cost is a function of detection delay.

7. The scheduled inspection and restoration are instantaneous.

3.2.2 Notations

The following are the notations which are adopted in this chapter

3.2.3 Statement of the Problem

Consider a deteriorating production system that at any point in time can be classified into one of the two states; in-control or out-of-control. It is assumed that the elapsed time, t, of the process in in-control state follows a known general distribution. Once the process shifts to out-of-control state, it stays there until the process is inspected to reveal its state and restored back to in-control state if found out-of-control. System incurs a setup cost for every new cycle. It is assumed that

n	=	number of inspection intervals,
Q	=	production lot size,
d	=	demand rate(units/unit time),
p	=	production rate,
A	=	setup cost,
υ	=	selling price per items produced during the in-control state,
s	=	selling price per items produced during the out-of-control state,
h	=	inventory carrying cost \$/unit time,
C_p	=	production cost excluding setup,
π_1	=	probability of producing nonconforming item in in-control state,
π_2	=	probability of producing nonconforming item in out-of-control state,
t_j	=	production time at the end of the j^{th} inspection interval,
ETC	=	expected total cost per unit produced,
N_j	=	expected number of defective items produced in the j^{th} interval,
$E(N)$	=	expected number of items produced during the out-of-control period,
W	=	length of warranty period,
$r_1(t)$	=	hazard function for conforming items,
$r_2(t)$	=	hazard function for nonconforming items,
C_m	=	minimal repair cost during warranty period,
WC	=	warranty cost per items,
C_{r0}	=	fixed restoration cost,
C_{r1}	=	time dependent restoration cost,
ν	=	Inspection cost of the process.

all the items produced are operational and can be classified as being either conforming or nonconforming depending on whether its performance meets the product specifications or not. Due to manufacturing variability, an item is nonconforming with probability π_1 when the process is in-control and π_2 when the process is out-of-control, where $\pi_1 < \pi_2$. Since a nonconforming items can only be detected after a period of time in use, all the items produced are released for sale with a free minimal repair warranty, failure that occur within the warranty period (W) results in valid warranty claim and are rectified by minimal repairs instantaneously at no cost to the buyers. After minimal repair, the hazard rate of an item remains the same as that just before the failure. Every minimal repair incurs a cost of C_m to the manufacturer. However, it is assumed that due to few obvious characteristics the units produced in out-of-control state can be differentiated from those produced in in-control state and can be sold at less price s. A non differential case will be just a special case of this type, with the selling price of units produced in in-control state and in out-of-control being the same ($\upsilon = s$).

For the deteriorating production system discussed above, the expected total cost includes manufacturing cost, inventory holding cost, setup cost, inspection cost, restoration cost and warranty cost. Now our objective is to maximize the profit and get the corresponding total production run length t_n and inspection schedule $t_j,\ j = \{1, ..., n-1\}$.

3.2.4 Model Formulation

$$profit = Revenue - Costs$$

Revenue:

The profit per unit produced is given by:

$$REV = \frac{\upsilon(pt_n - E(N)) + sE(N)}{pt_n}$$

$$= \quad \upsilon - (\upsilon - s)\frac{E(N)}{pt_n} \tag{3.1}$$

Costs

In addition to the setup cost A, the production cost per item C_p, and inspection cost ν at the end of each inspection interval, the following costs are incurred per cycle:

Inventory carrying cost:

$$E(HC) = \frac{(p-d)t_n h}{2d} \tag{3.2}$$

Number of items during out-of-control period:

The expected number of items produced during the out-of-control period of the interval (t_{j-1}, t_j) is given by:

$$E(N_j) = pE(toc_j)$$

where $E(toc_j)$ is the time the machine is in out-of-control state before it is inspected and restored back to in-control state and is given by:

$$E(toc_j) \quad = \quad \frac{\int_{t_{j-1}}^{t_j}(t_j - t)f(t)dt}{\overline{F}(t_{j-1})}$$

$$= \frac{\left\{t_j[F(t_j) - F(t_{j-1})] - \int_{t_{j-1}}^{t_j} tf(t)dt\right\}}{\overline{F}(t_{j-1})} \tag{3.3}$$

therefore expected production in out-of-control state is given by:

$$E(N_j) = \frac{p\left\{t_j[F(t_j) - F(t_{j-1})] - \int_{t_{j-1}}^{t_j} tf(t)dt\right\}}{\overline{F}(t_{j-1})} \tag{3.4}$$

The total expected number of items produced during the out-of-control periods during a production cycle is given by:

$$E(N) = \sum_{j=1}^{n} E(N_j) = \sum_{j=1}^{n} \frac{p\left\{t_j[F(t_j) - F(t_{j-1})] - \int_{t_{j-1}}^{t_j} tf(t)dt\right\}}{\overline{F}(t_{j-1})} \tag{3.5}$$

Restoration cost:

The process is restored to the in-control state whenever it is found out-of-control during an inspection. Therefore the expected restoration cost during the interval(t_{j-1}, t_j) is given by:

$$E(RC_j) = \frac{\int_{t_{j-1}}^{t_j} [C_{r0} + C_{r1}(t_j - t)] f(t)dt}{\overline{F}(t_{j-1})}$$

hence the expected restoration cost per item is:

$$E(RC) \quad = \quad \sum_{j=1}^{n-1} \frac{C_{r0} \int_{t_{j-1}}^{t_j} f(t)dt}{pt_n \overline{F}(t_{j-1})} + \sum_{j=1}^{n-1} \frac{E(toc_j)}{pt_n}$$

$$\begin{aligned} &= \frac{C_{r0}}{pt_n}\left[n-1-\sum_{j=1}^{n-1}\frac{\overline{F}(t_j)}{\overline{F}(t_{j-1})}\right] \\ &+ \frac{C_{r1}}{pt_n}\sum_{j=1}^{n-1}\frac{\left\{t_j[F(t_j)-F(t_{j-1})]-\int_{t_{j-1}}^{t_j} tf(t)dt\right\}}{\overline{F}(t_{j-1})} \end{aligned} \tag{3.6}$$

Warranty cost:

Since items procured during in control periods will be nonconforming with probability π_1 and items produced during out of control periods will be non conforming with probability π_2, the total expected number of non conforming items per cycle is given by:

$$\begin{aligned} E(NC) &= \pi_1[pt_n - E(N)] + \pi_2 E(N) \\ &= (\pi_2 - \pi_1)E(N) + pt_n\pi_1 \end{aligned} \tag{3.7}$$

The fraction of nonconforming items per cycle is as follows.

$$q = \frac{E(NC)}{pt_n} = \frac{\sum_{j=1}^{n}\frac{p\left\{t_j[F(t_j)-F(t_{j-1})]-\int_{t_{j-1}}^{t_j} tf(t)dt\right\}}{\overline{F}(t_{j-1})}[\pi_2-\pi_1]+pt_n\pi_1}{pt_n} \tag{3.8}$$

Since the hazard function of the conforming items is $r_1(x)$ and that of the nonconforming is $r_2(x)$, the expected warranty cost is given by:

$$E(WC) = C_m \left[(1-q)\int_0^w r_1(x)dx + q\int_0^w r_2(x)dx\right] \tag{3.9}$$

The expected total cost per item is made up of manufacturing cost, inspection cost, inventory carrying cost , restoration cost and warranty cost. Thus

$$ETC = C_p + \frac{A+(n-1)\nu}{pt_n} + E(HC) + E(RC) + E(WC) \tag{3.10}$$

Where $E(HC)$, $E(RC)$ and $E(WC)$ are given by (3.2), (3.5)and (3.8), respectively.

So, we have

Therefore the expected profit per unit can be obtained as:

$$E(PR) = \upsilon - (\upsilon - s)\frac{E(N)}{pt_n} - ETC \tag{3.11}$$

$$\begin{aligned} E(PR) &= \upsilon - \frac{(\upsilon-s)E(N)}{pt_n} - C_p - \frac{A+(n-1)\nu}{pt_n} - \frac{(p-d)t_n h}{2d} \\ &- \frac{C_{r0}}{pt_n}\left[n-1-\sum_{j=1}^{n-1}\frac{\overline{F}(t_j)}{\overline{F}(t_{j-1})}\right] - \frac{C_{r1}}{pt_n}\sum_{j=1}^{n-1}E(toc_j) \\ &- C_m\left[\frac{(E(N)(\pi_2-\pi_1)+pt_n\pi_1)(R_2-R_1)}{pt_n} + R_1\right] \end{aligned}$$

Which can be simplified to.

$$
\begin{aligned}
E(PR) &= \upsilon - \frac{[\upsilon - s + C_m(\pi_2 - \pi_1)(R_2 - R_1)]}{t_n} \left\{ \sum_{j=1}^{n} \frac{t_j[F(t_j) - F(t_{j-1})] - \int_{t_{j-1}}^{t_j} tf(t)dt}{\overline{F}(t_{j-1})} \right\} \\
&- C_p - \frac{A + (n-1)\nu}{pt_n} - \frac{(p-d)t_n h}{2d} - \frac{C_{r0}}{pt_n}\left[n - 1 - \sum_{j=1}^{n-1} \frac{\overline{F}(t_j)}{\overline{F}(t_{j-1})} \right] \\
&- \frac{C_{r1}}{pt_n} \sum_{j=1}^{n-1} \frac{\left\{ t_j[F(t_j) - F(t_{j-1})] - \int_{t_{j-1}}^{t_j} tf(t)dt \right\}}{\overline{F}(t_{j-1})} - C_m[\pi_1(R_2 - R_1) + R_1] \quad (3.12)
\end{aligned}
$$

Where R_1 and R_2 are $\int_0^w r_1(x)dx$ and $\int_0^w r_2(x)dx$ respectively.

3.3 Solution and Analysis

We want to maximize $E(PR)$ defined above. The above model can be tailored for given distribution and can be simplified. Assumption such as equal integrated hazard rate throughout inspection intervals can help in solving the model. For the case of equal integrated hazard rate $\overline{F}(t_j) = [\overline{F}(t_1)]^j$ as given by Rahim and Benerje [26]. By this condition we can establish t_n, t_{n-1}, t_j, $F(t_n)$ and $F(t_{n-1})$ in terms of t_1.

Solution Algorithm

Step 1 : Set $n = 1$

Step 2 : Calculate optimal t_1 which maximizes profit $E(PR)_{(n,t_1^*)}$. Either by optimality condition (if possible) or by golden section method for given n.

Step 3 : If $n = 1$ set $E(PR)^* = E(PR)_{(n,t_1^*)}$

Step 4 : If $E(PR)_{(n,t_1^*)} \geq E(PR)^*$ then $n = n + 1$ go to step (2) else

Step 5 : $n^* = n - 1$ and $t_1^* = t_1$ corresponding to n^*.

3.4 Results

In this section, an illustrative example for exponential distribution and weibull distribution are presented. Since, it is difficult to find a closed form solution, approximate solution for exponential case and numerical solutions for weibull case are obtained. Golden section method is applied for t_1 and integer search is applied over n following the above algorithm. Computer code is written in Fortran, IMSL subroutines of GAMIC are used to evaluate gamma function. The program is run on pentium III computer with 256 MB ram (see appendix A for the code).

Examples with exponential and weibull are taken to show how the model can be easily tailored for these distributions. Closed form solution can be achieved with approximation of e^x to second degree for exponential case.

3.4.1 Exponential Case

$$\begin{aligned} f(t) &= \lambda e^{-\lambda t} \\ F(t) &= 1 - e^{-\lambda t} \\ \overline{F}(t) &= e^{-\lambda t} \end{aligned}$$

And for equal integrated hazard rate, we have

$$\begin{aligned} \overline{F}(t_j) &= [\overline{F}(t_1)]^j \\ e^{-\lambda t_j} &= [e^{-\lambda t_1}]^j \\ t_j &= j t_1 \end{aligned} \tag{3.13}$$

And

$$\int_{t_{j-1}}^{t_j} t f(t) dt = \int_{(j-1)t_1}^{jt_1} t \lambda e^{-\lambda t} dt = e^{-\lambda(j-1)t_1} \left[(j-1)t_1 + \frac{1}{\lambda} - e^{-\lambda t_1}(jt_1 + \frac{1}{\lambda}) \right] \tag{3.14}$$

The profit function for exponential distribution is given by

$$\begin{aligned} E(PR) &= \upsilon - \frac{[\upsilon - s + C_m(\pi_2 - \pi_1)(R_2 - R_1)]}{t_1} \left[t_1 - \frac{1}{\lambda} + \frac{e^{-\lambda t_1}}{\lambda} \right] \\ &- C_p - \frac{A + (n-1)\nu}{npt_1} - \frac{(p-d)nt_1 h}{2d} - \frac{C_{r0}(n-1)\left(1 - e^{-\lambda t_1}\right)}{npt_1} \\ &- \frac{C_{r1}(n-1)}{npt_1} \left[t_1 - \frac{1}{\lambda} + \frac{e^{-\lambda t_1}}{\lambda} \right] - C_m[\pi_1(R_2 - R_1) + R_1] \end{aligned} \tag{3.15}$$

by approximating $e^{-x} = 1 - x + \frac{x^2}{2!}$ profit function can be rewritten as

$$\begin{aligned} E(PR) &= \upsilon - [\upsilon - s + C_m(\pi_2 - \pi_1)(R_2 - R_1)]\frac{\lambda t_1}{2} \\ &- C_p - \frac{A + (n-1)\nu}{npt_1} - \frac{(p-d)nt_1 h}{2d} - \frac{C_{r0}(n-1)}{np}\left(\lambda - \frac{\lambda^2 t_1}{2}\right) \\ &- \frac{C_{r1}(n-1)}{np}\frac{\lambda t_1}{2} - C_m[\pi_1(R_2 - R_1) + R_1] \end{aligned} \tag{3.16}$$

For optimality taking partial derivative w.r.t t_1 and solving we get,

$$\widetilde{t_1^*} = \sqrt{\frac{2d(A + (n-1)\nu)}{ndp\lambda[\upsilon - s + C_m(\pi_2 - \pi_1)(R_2 - R_1)] + n^2 ph(p-d) - d\lambda(n-1)(\lambda C_{r0} - C_{r1})}} \tag{3.17}$$

and optimality condition for n^* is derived using $E(PR)_{n^*-1} \leq E(PR)_{n^*} \geq E(PR)_{n^*+1}$

$$n(n-1) \leq \frac{2d\left[(A-\nu) - t_1\left\{C_{r0}\left(\lambda - \frac{\lambda^2 t_1}{2}\right) + C_{r1}\frac{\lambda t_1}{2}\right\}\right]}{p(p-d)t_1^2 h} \leq n(n+1) \tag{3.18}$$

eq.(3.17)and eq.(3.18) can be solved iteratively for t_1^* and n^*.

3.4.2 Weibull Case

Where

$$f(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} e^{-\left(\frac{t}{\theta}\right)^{\beta}}$$

$$\begin{aligned} F(t) &= 1 - e^{-\left(\frac{t}{\theta}\right)^{\beta}} \\ \overline{F}(t) &= e^{-\left(\frac{t}{\theta}\right)^{\beta}} \end{aligned}$$

And for equal integrated hazard rate, we have

$$\begin{aligned} \overline{F}(t_j) &= [\overline{F}(t_1)]^j \\ e^{-\left(\frac{t_j}{\theta}\right)^{\beta}} &= [e^{-\left(\frac{t_1}{\theta}\right)^{\beta}}]^j \\ t_j &= j^{1/\beta} t_1 \end{aligned} \tag{3.19}$$

And

$$\begin{aligned} \int_{t_{j-1}}^{t_j} t f(t) dt &= \int_{t_{j-1}}^{t_j} t \frac{\beta}{\theta} \left(\frac{t}{\theta}\right)^{\beta-1} e^{-\left(\frac{t}{\theta}\right)^{\beta}} dt \\ &= \theta \int_{(j-1)\left(\frac{t_1}{\theta}\right)^{\beta}}^{j\left(\frac{t_1}{\theta}\right)^{\beta}} U^{1/\beta} e^{-U} dU \end{aligned}$$

This is similar to incomplete gamma function. A incomplete gamma function $\gamma(a, x) = \int_0^x t^{a-1} e^{-t} dt$, is bounded by complete gamma function $\Gamma(a) = \int_0^\infty t^x e^{-t} dt$ and can be calculated from numerical tables or by subroutines for numerical integration. Therefore the above expression can be written as

$$\theta\gamma\left(1 + 1/\beta, j\left(\frac{t_1}{\theta}\right)^{\beta}\right) - \theta\gamma\left(1 + 1/\beta, (j-1)\left(\frac{t_1}{\theta}\right)^{\beta}\right) \tag{3.20}$$

The profit function for weibull distribution is given by

$$\begin{aligned} E(PR) &= \upsilon - \frac{[\upsilon - s + C_m(\pi_2 - \pi_1)(R_2 - R_1)]}{n^{1/\beta} t_1} \left\{ \sum_{j=1}^{n} j^{1/\beta} t_1 (e^{\left(\frac{t_1}{\theta}\right)^{\beta}} - 1) - \theta \Psi(t_j) \right\} \\ &- C_p - \frac{A + (n-1)\nu}{p n^{1/\beta} t_1} - \frac{(p-d) n^{1/\beta} t_1 h}{2d} - \frac{C_{r0}(n-1)}{p n^{1/\beta} t_1} \left(1 - \frac{1}{e^{\left(\frac{t_1}{\theta}\right)^{\beta}}} \right) \\ &- C_{r1} \left\{ \sum_{j=1}^{n-1} j^{1/\beta} t_1 (e^{\left(\frac{t_1}{\theta}\right)^{\beta}} - 1) - \theta \Psi(t_j) \right\} - C_m[\pi_1(R_2 - R_1) + R_1] \quad (3.21) \end{aligned}$$

where

$$\Psi(t_j) = \gamma \left(1 + 1/\beta, j \left(\frac{t_1}{\theta} \right)^{\beta} \right) - \gamma \left(1 + 1/\beta, (j-1) \left(\frac{t_1}{\theta} \right)^{\beta} \right)$$

Our objective is to maximize $E(PR)$, it can be done by simple integer search on n and golden section on t_1.

3.5 Numerical example

Suppose that the life time distribution of both conforming and non conforming items are Weibull with hazard rate function $h_1(t) = \lambda_1^{\beta_1} \beta_1 t^{\beta_1 - 1}$ and $h_2(t) = \lambda_2^{\beta_2} \beta_2 t^{\beta_2 - 1}$, respectively. Assume that the shape parameters are $\beta_1 = \beta_2 = 2$ and the scale parameters are $\lambda_1 = 1/36$ and $\lambda_2 = 1/12$ and the warranty period $W = 6$. The remaining parameters are $p = 600$, $d = 400$, $A = 1000$, $\nu = 20$, $\upsilon = 20$, $s = 15$,

$\pi_1 = 0.15$, $\pi_2 = 0.65$, $C_m = 1$, $C_{r0} = 20$, $C_{r1} = 1$ and $h = 1$ For the case of exponential $\lambda = 1$ and for weibull $\theta = 1$.

Table 3.5 shows the optimal production run length, number of inspection intervals

	Type	t_1^*	n	t_n	Profit
exponential	Approximation	0.4187	3	1.2562	7.28359
	numeric solution	0.118838	21	2.4956	8.021774
Weibull					
β	2.0	0.3843	21	1.7624	7.7405
	2.5	0.4768	17	1.4805	7.6533
	3.0	0.5441	14	1.3121	7.6045
θ	0.1	0.0782	18	0.3331	0.1528
	0.4	0.2148	26	1.0947	6.3656
	0.8	0.3336	24	1.6365	7.5070
	1.0	0.3843	21	1.7624	7.7405

Table 3.1: Expected Profit, Inspection schedule and Production run length

and profit. Two cases, exponential and weibull are taken as examples and model is tailored for them. Table shows the comparison of approximate and numerical solution for exponential case. Difference in profit is within 10%. As expected the approximate solution costs higher i.e. profit with approximate solution is less when compared to numerical one.

For the case of weibull β and θ are varied and there effect is studied. With the increase in β, t_n and profit drop, as the shape factor is increases the spread of the distribution decreases, increasing the probability on failure. Thereby decreasing t_n. It could be seen that t_1 increases with β, it means model forces the system to work more in initial intervals were the chance of failure is less compared to that in later intervals. And with increase in θ, t_1, t_n and profit increases. The expected life of system increases thereby reducing the chances of failure. This model is developed to be a reference for more general models in chapter 4. More extensive sensitivity analysis will be performed in chapter 4.

3.6 Conclusions

A combined inventory warranty model is presented for general probability distribution. A computation methodology can be developed based on type of the distribution. However numerical techniques can always be used to solve such models.

Chapter 4

Optimal Production Run Length and PM Policy for the System Whose Products are Sold with Warranty

4.1 Introduction

The purpose of this chapter is to extend the model developed in chapter 3 by incorporating preventive maintenance with inspection and considering the shift to follow general probability distribution.

Prior proceeding further, the term preventive maintenance must be well defined.

Preventive maintenance is planned maintenance of plant and equipment that is designed to improve equipment life and avoid any unplanned maintenance activity. The effective age of the system is reduced and the performance of the system is restored between as good as new and as bad as old. It is intuitive that preventive maintenance could help a system to perform better. A model is developed incorporating preventive maintenance to maximize the expected profit by finding the optimum production run length and preventive maintenance schedule for the process.

This chapter is organized as follows, Model Development is presented in section 4.2, in which notations, model assumptions, statement of the problem, and model formulation are discussed. Solution and Analysis of the model are discussed in section 4.3, results are shown in section 4.4, then numerical examples are solved in section 4.5.

4.2 Model Development

The model developed in this chapter is presented in this section. The section contains model notations, assumptions, problem statement and model formulation.

4.2.1 Model Assumptions

1. The process inspection is assumed to be error free.

2. The probability of producing a nonconforming unit in in-control state is less than that in out-of-control state.

3. The hazard rate of nonconforming item is more than that of conforming item.

4. Repair cost of both conforming and nonconforming item is same.

5. Hazard rate of the products is dependent on the amount invested in PM.

6. Units produced in in-control state can be differentiated from those produced in out-of-control state and can be sold at higher price.

7. The process restoration cost is a function of detection delay.

8. The scheduled inspection, preventive maintenance and restoration are instantaneous.

4.2.2 Notations

The following are the notations which are adopted in this chapter.

n = number of intervals in production run,
Q = production lot size,
d = demand rate(units/unit time),
p = production rate,
A = setup including initial inspection cost,
v = selling price per items produced during the in-control state,
s = selling price per items produced during the out-of-control state,

h	=	inventory carrying cost \$/unit time,
C_p	=	production cost excluding setup,
π_1	=	probability of producing nonconforming item in in-control state,
π_2	=	probability of producing nonconforming item in out-of-control state,
y_j	=	age of the system before PM,
b_j	=	age reduction factor after j^{th} PM,
η	=	imperfectness factor,
w_j	=	$b_j y_j$ age of the system after PM,
h_j	=	production time interval at j^{th} PM,
H	=	$\sum_{j=1}^{n} h_j$ total production time,
ETC	=	expected total cost per unit produced,
$E(toc_j)$	=	expected production time in out-of-control in j^{th} interval,
N_j	=	expected number of defective items produced in the j^{th} interval,
$E(N)$	=	expected number of items produced during the out-of-control period,
W	=	length of warranty period,
$r_1(t)$	=	hazard function for conforming items,
$r_2(t)$	=	hazard function for nonconforming items,
C_m	=	minimal repair cost during warranty period,
WC	=	warranty cost per items,
C_{r0}	=	fixed restoration cost,
C_{r1}	=	time dependent restoration cost,
C_{pm}	=	Inspection and preventive maintenance cost of the process,
C_{pm}^0	=	Maximum PM cost resulting in perfect system.

4.2.3 Statement of the Problem

Consider a deteriorating production system that at any point in time can be classified into one of the two states; in-control or out-of-control. It is assumed that the elapsed time t, of the system in in-control state follows a known general distribution. Once the system shifts to out-of-control state, it stays there until the system is inspected to reveal its state and restored back to in-control state if found out-of-control, costing restoration cost which is function of detection delay. System incurs a setup cost for every new cycle. The system under goes preventive maintenance at regular intervals of length h_j. The system is rejuvenated and its effective age is reduced by factor b_j. The total production time is $\sum_{j=1}^{n} h_j$. It is assumed that all the items produced are operational and can be classified as being either conforming or nonconforming depending on whether its performance meets the product specifications or not. Due to manufacturing variability, an item is nonconforming with probability π_1 when the process is in-control and π_2 when the process is out-of-control, where $\pi_1 < \pi_2$. Since nonconforming items can only be detected after a period of time in use, all the items produced are released for sale with a free minimal repair warranty, failure that occur within the warranty period (W) results in valid warranty claim and are rectified by minimal repairs instantaneously at no cost to the buyers. After minimal repair, the hazard rate of an item remains the same as that just before the failure. Every minimal repair incurs a cost of C_m to the manufacturer. However, it is assumed that due to few obvious characteristics the units produced in out-of-control state can be differentiated from those produced in in-control state and can be sold at a lesser price s. A non differential case will just be a special case of this type, with the selling price of units produced in in-control state and in out-of-control being the same $(v = s)$.

For the deteriorating production system discussed above, the expected total cost incurred includes manufacturing cost, inventory holding cost, setup cost, inspection and preventive maintenance cost, restoration cost and warranty cost. Now our objective is to maximize the profit and get the corresponding total production run length t_n and inspection schedule $h_j,\ \ j\ =\ \{1, ..., n\}$.

4.2.4 Model Formulation

$$Profit = Revenue - Costs$$

Revenue:

The profit per unit produced is given by:

$$\begin{aligned} REV &= \frac{v(pH - E(N)) + sE(N)}{pH} \\ &= v - (v - s)\frac{E(N)}{pH} \qquad (4.1) \end{aligned}$$

4.2.5 Costs

In addition to the setup cost A, and the production cost per item C_p, and inspection cost C_{pm} at the end of each inspection interval,the following costs are incurred per cycle:

Inventory carrying cost:

$$E(HC) = \frac{(p - d)Hh}{2d} \qquad (4.2)$$

Number of items during out-of-control period:

The expected number of items produced during the out-of-control period of the interval h_j is given by:

$$E(N_j) = pE(toc_j)$$

where $E(toc_j)$ is the time the machine is in out-of-control state before it is inspected and restored back to in-control state and is given by:

$$\begin{aligned} E(toc_j) &= \frac{\int_{w_{j-1}}^{y_j} (y_j - t) f(t) dt}{\overline{F}(w_{j-1})} \\ &= \frac{\left\{ y_j [F(y_j) - F(w_{j-1})] - \int_{w_{j-1}}^{y_j} t f(t) dt \right\}}{\overline{F}(w_{j-1})} \end{aligned} \tag{4.3}$$

therefore expected production in out-of-control state is given by:

$$E(N_j) = \frac{p \left\{ y_j [F(y_j) - F(w_{j-1})] - \int_{w_{j-1}}^{y_j} t f(t) dt \right\}}{\overline{F}(w_{j-1})} \tag{4.4}$$

The total expected number of items produced during the out-of-control periods during a production cycle is given by:

$$E(N) = \sum_{j=1}^{n} E(N_j) = \sum_{j=1}^{n} \frac{p \left\{ y_j [F(y_j) - F(w_{j-1})] - \int_{w_{j-1}}^{y_j} t f(t) dt \right\}}{\overline{F}(w_{j-1})} \tag{4.5}$$

Restoration cost:

The process is restored to the in-control state whenever it is found out-of-control during an inspection. Therefore the expected restoration cost during the interval h_j is given by:

$$E(RC_j) = \frac{\int_{w_{j-1}}^{y_j}[C_{r0} + C_{r1}(y_j - t)]f(t)dt}{\overline{F}(w_{j-1})}$$

hence the expected restoration cost per item is:

$$\begin{aligned} E(RC) &= \sum_{j=1}^{n-1} \frac{C_{r0}\int_{w_{j-1}}^{y_j} f(t)dt}{pH\overline{F}(w_{j-1})} + \sum_{j=1}^{n-1} \frac{E(toc_j)}{pH} \\ &= \frac{C_{r0}}{pH}\left[n - 1 - \sum_{j=1}^{n-1} \frac{\overline{F}(y_j)}{\overline{F}(w_{j-1})}\right] \\ &+ \frac{C_{r1}}{pH}\sum_{j=1}^{n-1} \frac{\left\{y_j[F(y_j) - F(w_{j-1})] - \int_{w_{j-1}}^{y_j} tf(t)dt\right\}}{\overline{F}(w_{j-1})} \qquad (4.6) \end{aligned}$$

Warranty cost:

Since items procured during in control periods will be nonconforming with probability π_1 and items produced during out of control periods will be non conforming with probability π_2, the total expected number of non conforming items per cycle is given by:

$$E(NC) = \pi_1[pH - E(N)] + \pi_2 E(N)$$

$$= \quad (\pi_2 - \pi_1)E(N) + pH\pi_1 \tag{4.7}$$

The fraction of nonconforming items per cycle is as follows.

$$q = \frac{E(NC)}{pH} = \sum_{j=1}^{n} \frac{p\left\{y_j[F(y_j) - F(w_{j-1})] - \int_{w_{j-1}}^{y_j} tf(t)dt\right\}}{\overline{F}(w_{j-1})} \frac{[\pi_2 - \pi_1]}{pH} + \pi_1 \tag{4.8}$$

Since the hazard function of the conforming items is $r_1(x)$ and that of the nonconforming is $r_2(x)$, the expected warranty cost is given by:

$$E(WC) = C_m \left[(1-q)\int_0^w r_1(x)dx + q\int_0^w r_2(x)dx\right] \tag{4.9}$$

The expected total cost per item is made up of manufacturing cost, inspection cost, inventory carrying cost , restoration cost and warranty cost. Thus

$$ETC = C_p + \frac{A + (n-1)C_{pm}}{pH} + E(HC) + E(RC) + E(WC) \tag{4.10}$$

Where $E(HC)$, $E(RC)$ and $E(WC)$ are given by eq.(4.2), eq.(4.6)and eq.(4.9), respectively.

Therefore the expected profit per unit can be obtained as:

$$E(PR) = \upsilon - (\upsilon - s)\frac{E(N)}{pH} - ETC \tag{4.11}$$

$$\begin{aligned} E(PR) &= \upsilon - \frac{(\upsilon - s)E(N)}{pH} - C_p - \frac{A + (n-1)C_{pm}}{pH} - \frac{(p-d)Hh}{2d} \\ &- \frac{C_{r0}}{pH}\left[n - 1 - \sum_{j=1}^{n-1} \frac{\overline{F}(y_j)}{\overline{F}(w_{j-1})}\right] - \frac{C_{r1}}{pH}\sum_{j=1}^{n-1} E(toc_j) \\ &- C_m\left[\frac{(E(N)(\pi_2 - \pi_1) + pH\pi_1)(R_2 - R_1)}{pH} + R_1\right] \end{aligned}$$

Which can be simplified to

$$\begin{aligned} E(PR) &= \upsilon - \frac{[\upsilon - s + C_m(\pi_2 - \pi_1)(R_2 - R_1)]}{H}\left\{\sum_{j=1}^{n} \frac{y_j[F(y_j) - F(w_{j-1})] - \int_{w_{j-1}}^{y_j} tf(t)dt}{\overline{F}(w_{j-1})}\right\} \\ &- C_p - \frac{A + (n-1)C_{pm}}{pH} - \frac{(p-d)Hh}{2d} - \frac{C_{r0}}{pH}\left[n - 1 - \sum_{j=1}^{n-1} \frac{\overline{F}(y_j)}{\overline{F}(w_{j-1})}\right] \\ &- \frac{C_{r1}}{pH}\sum_{j=1}^{n-1} \frac{\left\{y_j[F(y_j) - F(w_{j-1})] - \int_{w_{j-1}}^{y_j} tf(t)dt\right\}}{\overline{F}(w_{j-1})} - C_m[\pi_1(R_2 - R_1) + R_1] \end{aligned} \tag{4.12}$$

Where R_1 and R_2 are $\int_0^w r_1(x)dx$ and $\int_0^w r_2(x)dx$ respectively.

4.3 Solution and Analysis

We want to maximize $E(PR)$ defined above. The above model can be tailored for given distribution and can be simplified. Assumption such as equal integrated hazard rate throughout inspection intervals can help in solving the model. For the case of equal integrated hazard rate $\overline{F}(y_j) = \overline{F}(w_{j-1})\overline{F}(h_1)$. By this condition we can establish H, w_{j-1}, y_j, $F(y_j)$ and $F(w_{n-1})$ in terms of h_1.

Solution Algorithm

Step 1 : Set $n = 1$

Step 2 : Calculate optimal h_1 which maximizes profit $E(PR)_{(n,h_1^*)}$. Either by equating derivative to zero (if possible) or by golden section method for given n.

Step 3 : If $n = 1$ set $E(PR)^* = E(PR)_{(n,h_1^*)}$

Step 4 : If $E(PR)_{(n,h_1^*)} \geq E(PR)^*$ then $n = n + 1$ go to step (2) else

Step 5 : $n^* = n - 1$ and $h_1^* = h_1$ corresponding to n^*.

4.4 Results

In this section, an illustrative example for the weibull distribution is presented. As it is difficult to find a closed form solution for weibull case, numerical solutions is obtained. Golden section method is applied to find h_1 and integer search is applied over n, following the above algorithm. Computer code is written in Fortran, IMSL subroutines of GAMIC are used to evaluate gamma function. The program is run on pentium III computer with 256 MB ram (see appendix B for the code).

4.4.1 Weibull Case

The weibull density function $f(t)$, distribution function $F(t)$ and reliability function $\overline{F}(t)$ are given below.

$$\begin{aligned} f(t) &= \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} e^{-\left(\frac{t}{\theta}\right)^{\beta}} \\ F(t) &= 1 - e^{-\left(\frac{t}{\theta}\right)^{\beta}} \\ \overline{F}(t) &= e^{-\left(\frac{t}{\theta}\right)^{\beta}} \end{aligned}$$

And for equal integrated hazard rate, we have

$$\begin{aligned} \overline{F}(y_j) &= \overline{F}(w_{j-1})\overline{F}(h_1) \\ e^{-\left(\frac{y_j}{\theta}\right)^{\beta}} &= e^{-\left(\frac{w_{j-1}}{\theta}\right)^{\beta}} e^{-\left(\frac{h_1}{\theta}\right)^{\beta}} \\ h_j &= [w_{j-1}^{\beta} + h_1^{\beta}]^{1/\beta} - w_{j-1} \qquad (4.13) \end{aligned}$$

$$\int_{w_{j-1}}^{y_j} t f(t) dt = \int_{w_{j-1}}^{y_j} t \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} e^{-\left(\frac{t}{\theta}\right)^{\beta}} dt$$

$$= \theta \int_{\left(\frac{w_{j-1}}{\theta}\right)^{\beta}}^{\left(\frac{y_j}{\theta}\right)^{\beta}} U^{1/\beta} e^{-U} dU$$

This is similar to incomplete gamma function. A incomplete gamma function $\gamma(a,x) = \int_0^x t^{a-1}e^{-t}dt$, is bounded by complete gamma function $\Gamma(a) = \int_0^\infty t^x e^{-t}dt$ and can be calculated from numerical tables or by subroutines for numerical integration. Therefore the above expression can be written as

$$\theta\gamma\left(1 + 1/\beta, \left(\frac{y_j}{\theta}\right)^{\beta}\right) - \theta\gamma\left(1 + 1/\beta, \left(\frac{w_{j-1}}{\theta}\right)^{\beta}\right) \tag{4.14}$$

The profit function for weibull distribution is given by

$$\begin{aligned} E(PR) &= \upsilon - \frac{[\upsilon - s + C_m(\pi_2 - \pi_1)(R_2 - R_1)]}{H} \left\{ \sum_{j=1}^{n} \frac{y_j \left(e^{-\left(\frac{w_{j-1}}{\theta}\right)^{\beta}} - e^{-\left(\frac{y_j}{\theta}\right)^{\beta}} \right) - \theta\Psi_j}{e^{-\left(\frac{w_{j-1}}{\theta}\right)^{\beta}}} \right\} \\ &- C_p - \frac{A + (n-1)C_{pm}}{pH} - \frac{(p-d)Hh}{2d} - \frac{C_{r0}}{pH}\left(n - 1 - \sum_{j=1}^{n-1} \frac{e^{-\left(\frac{y_j}{\theta}\right)^{\beta}}}{e^{-\left(\frac{w_{j-1}}{\theta}\right)^{\beta}}} \right) \\ &- \frac{C_{r1}}{pH} \left\{ \sum_{j=1}^{n} \frac{y_j \left(e^{-\left(\frac{w_{j-1}}{\theta}\right)^{\beta}} - e^{-\left(\frac{y_j}{\theta}\right)^{\beta}} \right) - \theta\Psi_j}{e^{-\left(\frac{w_{j-1}}{\theta}\right)^{\beta}}} \right\} - C_m[\pi_1(R_2 - R_1) + R_1] \end{aligned} \tag{4.15}$$

where

$$\Psi_j = \gamma\left(1 + 1/\beta, \left(\frac{y_j}{\theta}\right)^{\beta}\right) - \gamma\left(1 + 1/\beta, \left(\frac{w_{j-1}}{\theta}\right)^{\beta}\right)$$

Our objective is to maximize $E(PR)$, it can be done by simple integer search on n and golden section on h_1.

4.5 Numerical example

Suppose that the life time distribution of both conforming and non conforming items are Weibull with hazard rate function $r_1(t) = \lambda_1^{\beta_1}\beta_1 t^{\beta_1-1}$ and $r_2(t) = \lambda_2^{\beta_2}\beta_2 t^{\beta_2-1}$, respectively. Assume that the shape parameters are $\beta_1 = \beta_2 = 1.5 + 0.5\left(1 - \frac{C_{pm}}{C_{pm}^0}\right)$ and the scale parameters are $\lambda_1 = 1/6$ and $\lambda_2 = 1/2$ and the warranty period $W = 6$. The remaining parameters are $p = 600$, $d = 400$, $A = 1000$, $C_{pm} = 80$, $C_{pm}^0 = 100$, $\upsilon = 20$, $s = 15$, $\pi_1 = 0.15$, $\pi_2 = 0.65$, $C_m = 1$, $C_{r0} = 20$, $C_{r1} = 1$, $C_p = 10$ $h = 1$, $b_j = \left(1 - \eta^{j-1}\frac{C_{pm}}{C_{pm}^0}\right)$, $\beta = 2$ and $\theta = 1$.

Table 4.1 shows the effect of θ and thereby mean on various costs, profit, t_1, n and t_p. It can be seen for lower values of θ where t_p is much higher than mean, profits with PM is much higher than those without PM. This difference between the profits with and without PM goes on decreasing with increase in mean. For mean which is higher than t_p, it could be seen that the profit difference between with PM and without PM is not much. As intuitive, if the expected life of the system is higher than its run time, PM does not have much effect.

Table 4.2 shows the effect of minimal repair cost C_m on various costs, profit, t_1, n and t_p. It can be seen as C_m increases the profit decreases as expected. This is

θ	Mean	t_1	n	t_p	QC	RC	WC	Profit	
0.3	0.265	0.149	19	2.200	0.457	0.059	1.939	5.145	With PM
		0.157	16	0.626	0.739	0.191	2.791	2.662	W/O PM
0.5	0.443	0.206	14	2.289	0.316	0.029	1.871	5.724	With PM
		0.209	31	1.170	0.458	0.138	2.566	4.258	W/O PM
0.8	0.709	0.272	11	2.418	0.218	0.015	1.824	6.097	With PM
		0.288	27	1.490	0.325	0.070	2.460	5.074	W/O PM
1.0	0.886	0.322	9	2.364	0.194	0.011	1.812	6.235	With PM
		0.331	24	1.620	0.272	0.049	2.417	5.357	W/O PM
1.3	1.152	0.372	8	2.449	0.154	0.007	1.793	6.371	With PM
		0.390	21	1.790	0.218	0.032	2.374	5.622	W/O PM
1.5	1.329	0.416	7	2.412	0.144	0.006	1.788	6.435	With PM
		0.426	19	1.860	0.193	0.025	2.354	5.741	W/O PM
2.0	1.772	0.491	6	2.465	0.113	0.003	1.773	6.546	With PM
		0.503	16	2.010	0.149	0.015	2.319	5.937	W/O PM
3.0	2.658	0.688	4	2.363	0.096	0.002	1.766	6.669	With PM
		0.632	12	2.190	0.101	0.007	2.281	6.134	W/OPM

Table 4.1: Expected Profit, Inspection and PM schedule, Production run length, holding, PM, Restoration, Quality and Warranty cost per item for varying mean by changing θ.

C_m	t_1	n	t_p	QC	RC	WC	Profit	
1.0	0.322	9	2.364	0.194	0.011	1.812	6.235	With PM
	0.331	24	1.623	0.272	0.049	2.417	5.357	W/O PM
1.5	0.299	10	2.428	0.169	0.010	2.701	5.331	With PM
	0.312	26	1.592	0.240	0.048	3.588	4.155	W/O PM
2.0	0.291	10	2.364	0.160	0.010	3.593	4.432	With PM
	0.299	27	1.553	0.219	0.047	4.750	2.963	W/O PM
2.5	0.274	11	2.434	0.143	0.009	4.471	3.534	With PM
	0.287	28	1.517	0.201	0.046	5.901	1.779	W/O PM
3.0	0.268	11	2.383	0.137	0.009	5.357	2.641	With PM
	0.277	28	1.466	0.187	0.045	7.048	0.602	W/O PM

Table 4.2: Expected Profit, Inspection and PM schedule, Production run length, holding, PM, Restoration, Quality and Warranty cost per item for varying cost of minimal repair.

mainly due to increase in warranty cost, as it is directly related to C_m. We can't do any thing to it. So, the system forces itself to work more in in-control state by reducing the length of interval and increasing the number of PMs thereby reducing the chances of going out-of-control. Point to be noticed is the percentage difference in profit between the case with PM and that without PM . For higher values of C_m the percentage difference is more than for lower values. This implies that PM is very effective in such cases.

W	t_1	n	t_p	QC	RC	WC	Profit	
3.0	0.508	5	2.204	0.445	0.014	0.638	6.839	With PM
	0.369	22	1.728	0.341	0.051	0.618	7.188	W/O PM
4.0	0.498	5	2.161	0.429	0.013	1.002	6.472	With PM
	0.356	23	1.707	0.317	0.051	1.090	6.709	W/O PM
5.0	0.488	5	2.116	0.412	0.013	1.421	6.048	With PM
	0.343	24	1.682	0.293	0.050	1.690	6.098	W/O PM
6.0	0.477	5	2.068	0.395	0.013	1.889	5.573	With PM
	0.331	24	1.623	0.272	0.049	2.417	5.357	W/O PM
7.0	0.466	5	2.021	0.378	0.013	2.401	5.052	With PM
	0.318	25	1.589	0.249	0.048	3.265	4.487	W/O PM
8.0	0.455	5	1.975	0.362	0.012	2.955	4.488	With PM
	0.305	26	1.555	0.228	0.047	4.236	3.492	W/O PM
9.0	0.425	6	2.194	0.321	0.012	3.524	3.883	With PM
	0.293	27	1.522	0.210	0.046	5.327	2.370	W/O PM
10.0	0.416	6	2.146	0.308	0.012	4.150	3.244	With PM
	0.281	28	1.489	0.193	0.046	6.540	1.124	W/O PM

Table 4.3: Expected Profit, Inspection and PM schedule, Production run length, holding, PM, Restoration, Quality and Warranty cost per item for varying warranty period.

Table 4.3 shows the effect of warranty period W on various costs, profit, t_1, n and t_p for $C^0_{pm} = 300$ and $C_{pm} = 250$. It can be seen that profit goes down with increase in W as expected. Main reason here again is the warranty cost, it is directly related to W. To compensate for the increase in warranty cost, the system forces itself to work more in in-control by reducing the span of intervals and by increasing number of such intervals(PMs). By increasing number of PMs and shortening length of intervals the system tries to compensate the increase in warranty cost by reducing quality cost and restoration cost. Here again point to be noted is the profit difference for with and without PM. It goes on increasing with increase in W, implying PM to be highly effective and recommended in such cases.

C_{pm}	$t1$	n	$S+PM$	QC	RC	WC	Profit
20	0.244	28	1.073	0.136	0.021	2.155	6.015
40	0.275	17	1.135	0.164	0.016	2.035	6.046
60	0.299	12	1.158	0.183	0.013	1.920	6.127
80	0.322	9	1.155	0.194	0.011	1.812	6.235
100	0.344	7	1.130	0.195	0.009	1.711	6.364

Table 4.4: Expected Profit, Inspection and PM schedule, Production run length, holding, PM, Restoration, Quality and Warranty cost per item for varying investment in PM.

Table 4.4 shows the effect of investment in PM on various costs, profit, t_1 and n. It can be seen for higher ratio of C_{pm} to C_{pm}^0 the profit increases. For very little investment in PM the improvement factor is not much to compensate for the investment. The savings are mostly from warranty and restoration costs. For higher investment in PM the system tries to increase the span of production thereby reducing setup and PM cost per unit.

Table 4.5 shows the effect of perfect PM cost, when 80% of it is invested in PM. It can be seen for lower cost of perfect PM, savings can be seen for all involved costs. As the cost of perfect PM increases the savings are seen mainly in restoration and warranty costs. It can also be seen that it is profitable to perform PM when

C_{pm}^0	C_{pm}	$t1$	n	QC	RC	WC	Profit
100	80	0.322	9	0.194	0.0111	1.812	6.235
200	160	0.399	7	0.291	0.0128	1.859	5.847
300	240	0.476	5	0.401	0.0135	1.912	5.549
400	320	0.532	4	0.486	0.0136	1.953	5.301
500	400	0.590	3	0.578	0.0128	1.997	5.090
W/O PM		0.331	24	0.272	0.049	2.417	5.357

Table 4.5: Expected Profit, Inspection and PM schedule, Production run length, holding, PM, Restoration, Quality and Warranty cost per item for varying investment in PM.

$C_{pm}^0 < 400$, above this value the PM cost is higher than the benefits it offers. In this type of cases PM is not advisable.

4.6 Conclusion

In this chapter, a model that combines inventory warranty and preventive maintenance is presented for a case where the production process time to shift follows general distribution. It is seen that the model with PM has more profit when compared to model without PM in most of the cases. Special cases where the investment is too less to improve the system or for the case where the expected life of system is much higher than the total run time and for the case where warranty period is too short, PM is not recommended.

Chapter 5

Joint Optimal Ordering and Inspection Policy for Deterministic Demand

5.1 Introduction

The purpose of this chapter is to develop a model for joint economical ordering quantity and inspection plan.

The model is developed for a process where the units are inspected upon arrival, with both the quantity to be ordered and inspection policy being decision variables. As it was discussed before in chapters 1 and 2, that the inspection policy and order quantity must not be separately designed in order to optimize overall system performance.

In general there are three alternatives with regard to inspection policy. They are no inspection, sampling inspection and 100% inspection. Inspection policy to be applied to a lot is governed by combining Deming's [23] 'kp' rule, the acceptance sampling plan outlined by Ducan [24] and agreed consumer and producer risks. The basis of this scheme is discussed first.

If the fraction of nonconforming items, p, is in statistical control (known and fixed), the manufacturing process may be considered to be stable. Deming [23] has demonstrated that the most economical inspection alternative will be either 100% or zero inspection. Deming's 'kp' rule may be used to determine which of these two alternatives is best. This rule is based on the relation between the cost of inspection C_i and the internal failure cost for an item that fails during its performance C_d (cost of accepting a nonconforming item). If C_i/C_d is less than the average incoming quality (p), then 100% inspection is to be performed. If C_i/C_d is greater than p, then no inspection is an economical option. If $C_i/C_d = p$, then both options give same result and any one of them can be pursued. Deming also suggested that 100% inspection should be used when the fraction nonconforming is unpredictable, but within narrow range including C_i/C_d. As per Duncan [24] for C_i/C_d between 0.001 and 0.1 sampling inspection is better option. After determining the parameters of optimal sampling plan the sampling plan is checked for consumer and producer risk. If it gives risk within acceptable/agreed limits the sampling plan is accepted else a new sampling plan is derived satisfying the risk equations. If expected cost with this sampling plan is less than minimum of 100% inspection or no inspection, it is adopted or which ever gives low expected cost is adopted.

This chapter is organized as follows, model development is presented in section 5.2, in which notations, model assumptions, statement of the problem and model formulation are discussed. Solution and analysis of the problem is discussed in sec.5.3, the model results are given in sec.5.4.

5.2 Model Development

In this section, a joint inventory and quality model is developed considering practical situations of consumer and producer risk. At first, model assumptions, followed by, necessary notations, statement of the problem and then model formulation is provided.

5.2.1 Model Assumptions

1. Production rate of manufacturing system is always greater than demand rate.

2. Shortages are not allowed.

3. Incoming Quality is stable or follows known (beta) distribution.

4. Inspection is error free.

5. Inspection is instantaneous (no extra time).

6. Either defective items are all reworked and replaced or all discarded.

7. Risks are not considered if 100% or no inspection is done. They are considered only in sampling inspection.

5.2.2 Notations

The following are the notations that are used in this chapter

$$
\begin{aligned}
A &= \text{Fixed ordering cost,}\\
D &= \text{Demand rate(units/unit time),}\\
Q &= \text{Lot size,}\\
n &= \text{Sample size,}\\
c &= \text{Number of defective allowed in sample,}\\
C_i &= \text{Inspection cost,}\\
\bar{p} &= \text{Average quality of incoming lot,}\\
C_d &= \text{Cost of accepting a defective item,}\\
C_r &= \text{Cost of rework on defective item,}\\
h &= \text{Holding cost,}\\
P_a &= \text{Probability of acceptance,}\\
K &= \text{Expected cost,}\\
T &= \text{Expected cycle length,}\\
\alpha_r &= \text{Producer's risk,}\\
\beta_r &= \text{Consumer's risk.}
\end{aligned}
$$

5.2.3 Statement of the Problem

Consider a manufacturing system where units are inspected at customer's arrival window or at manufacturer's dispatch window. Inspection policy and quantity to be ordered both are decision variables. The aim is to find joint optimal inventory and inspection policy. Manufacturer and consumer agreed to have some protection by agreeing on producer's risk and consumer's risk. Incoming quality is stochastic in nature following a known beta distribution. If sampling inspection is adopted, a sampling plan is developed with the help of model developed, i.e. a sample size of 'n' units is taken from the lot and inspected. If this sample contains defective items less than or equal to the critical number 'c', whole lot is accepted or else the lot is subjected to 100% screening.

There are two scenarios here, the defectives observed during inspection and screening are either all reworked and replaced or all discarded. Both of these cases are discussed here as non-replacement case and replacement case.

5.2.4 Model Formulation

The distribution of number of lot defective X is given by:

$$g(X) = \int_0^1 \binom{Q}{X} p^X (1-p)^{Q-X} \xi(p) dp \tag{5.1}$$

where $\xi(p)$ follows beta distribution; given by:

$$\xi(p) = \frac{\Gamma(\theta+\phi)}{\Gamma(\theta)\Gamma(\phi)} p^{\theta-1}(1-p)^{\phi-1} \tag{5.2}$$

substituting eq.(5.2) in eq.(5.1) we get beta-binomial distribution for $g(X)$; given by:

$$g(X) = \binom{Q}{X} \frac{\Gamma(\theta+\phi)}{\Gamma(\theta)\Gamma(\phi)} \frac{\Gamma(\theta+X)\Gamma(Q+\phi-X)}{\Gamma(\theta+\phi+Q)} \tag{5.3}$$

The number of defectives x in sample of size n is described by conditional distribution $t(x|X)$, and the Joint distribution of X and x is given by:

$$p(X,x) = t(x|X)g(X)$$

And the marginal distribution of x is given by:

$$f_n(x) = \sum_X p(X,x)$$

The conditional distribution $t(x|X)$ is as sumed to be hypergeometric distribution as a result marginal distribution of X and x fall into a well known family of reproducible distribution [27] and therefore both $g(X)$ and $f_n(x)$ follows beta-binomial distribution with parameters (Q,θ,ϕ) and (n,θ,ϕ) respectively and their respective mean is given by:

$$E(X) = \frac{Q\theta}{\theta+\phi} = Q\bar{p} \tag{5.4}$$

$$E(x) = \frac{n\theta}{\theta + \phi} = n\bar{p} \tag{5.5}$$

Probability of acceptance is given by

$$P_a = \sum_{x=0}^{c} f_n(x)$$

The expected number of defective units in noninspected portion of lot, given x defectives in sample is given by $E[X-x|x]$ and $P_n(x)$ is the average fraction defective in the noninspected portion of the lot, given x defectives in sample of size n. Then

$$P_n(x) = \frac{E[X - x|x]}{(Q - n)} \tag{5.6}$$

This implies

$$E[X - x|x] = (Q - n)P_n(x) \tag{5.7}$$

It can be shown that

$$P_n(x) = \frac{\theta + x}{\theta + \phi + n} \tag{5.8}$$

Proof: Let the number of defectives in noninspected portion of lot be $U = X - x$ then eq.(5.6) can be rewritten as

$$P_n(x) = \frac{E[U|x]}{(Q-n)}$$

If $h(U|x)$ gives the distribution of defectives in noninspected portion of lot. Then

$$E[U|x] = \sum_{U=0}^{Q-n} U h(U|x)$$

And from Bayers theorem $h(U|x)$ is given by

$$h[U|x] f_n(x) = t(x|X) g(X)$$

therefore

$$h[U|x] = \frac{t(x|X) g(X)}{f_n(x)}$$

this implies

$$E[U|x] = \sum_{U=0}^{Q-n} U \frac{n!}{(n-x)!x!} \frac{(Q-n)!}{(Q-n-U)!U!} \frac{(Q-X)!X!}{Q!} \frac{g(X)}{f_n(x)}$$

after few rearrangements this could be written as

$$\begin{aligned} E[U|x] &= \frac{(Q-n)(x+1)}{n+1} \frac{\binom{n+1}{x+1}\binom{Q-(x+1)}{U-1} g(X)}{\binom{Q}{X} f_n(x)} \\ &= \frac{(Q-n)(x+1)}{n+1} \frac{f_{n+1}(x+1)}{f_n(x)} \end{aligned}$$

substituting this expression of $E[U/x]$ in $P_n(x) = \frac{E[U|x]}{(Q-n)}$ and using eq.(5.3) we have

$$P_n(x) = \frac{(x+1)}{n+1} \frac{\binom{n+1}{x+1} \frac{\Gamma(\theta+\phi)}{\Gamma(\theta)\Gamma(\phi)} \frac{\Gamma(\theta+x+1)\Gamma(n+1+\phi-(x+1))}{\Gamma(\theta+\phi+n)}}{\binom{n}{x} \frac{\Gamma(\theta+\phi)}{\Gamma(\theta)\Gamma(\phi)} \frac{\Gamma(\theta+x)\Gamma(Q+\phi-x)}{\Gamma(\theta+\phi+n)}}$$

Which could be simplified to give

$$P_n(x) = \frac{\theta + x}{\theta + \phi + n}$$

5.2.5 Model Formulation Case(1)

Case(1): Without replacement of non-conforming items.

- Deterministic p
  - No inspection

$$K = \frac{AD}{Q} + C_d D p + \frac{hQ}{2} \tag{5.9}$$

– 100% inspection

$$K = \frac{AD}{Q(1-p)} + \frac{C_i D}{1-p} + \frac{hQ(1-p)}{2} \tag{5.10}$$

– Sampling Inspection

$$K = \frac{D\{A - nX + Q[X + C_i]\}}{QY - npP_a} + \frac{h}{2}\left[QY - npP_a\right] \tag{5.11}$$

where

$X = P_a(pC_d - C_i)$

$Y = 1 - p + pP_a$

- Stochastic p

  – No inspection

$$K = \frac{AD}{Q} + C_d D\bar{p} + \frac{hQ}{2} \tag{5.12}$$

  – 100% inspection

$$K = \frac{AD}{Q(1-\bar{p})} + \frac{C_i D}{1-\bar{p}} + \frac{hQ(1-\bar{p})}{2} \tag{5.13}$$

  – Sampling Inspection

In this case the expected number of items that may slip through inspection process $E[X - x/x]$ incur an additional cost of C_d per unit.

Inspection and Quality cost if lot is accepted is given by

$$nC_i \sum_{x=0}^{c} \sum_{X} p(X, x) + C_d \sum_{x=0}^{c} \sum_{X} (X - x) p(X, x)$$

If the lot is rejected then remaining $Q - n$ items are also subjected to 100% inspection.

The expected (Inspection) cost in this case is given by

$$C_i Q \sum_{x=c+1}^{n} \sum_{X} p(X, x)$$

Combining both scenario the total setup, Inspection and Quality cost is given by

$$K_{i,q} = A + nC_i \sum_{x=0}^{c} f_n(x) + C_d(Q - n) \sum_{x=0}^{c} P_n(x) f_n(x) + C_i Q \sum_{x=0}^{n} f_n(x) - C_i Q \sum_{x=0}^{c} f_n(x)$$

since

$$\begin{aligned} P_n(x) &= \frac{E[X - x/x]}{Q - n} \\ &= \frac{1}{Q - n} \frac{\sum_X (X - x) p(X, x)}{f_n(x)} \\ \sum_X (X - x) p(X, x) &= (Q - n) P_n(x) f_n(x) \end{aligned}$$

$$K_{i,q} = A + QC_i + (Q-n)\sum_{x=0}^{c}\left[C_d P_n(x) - C_i\right] f_n(x) \tag{5.14}$$

Holding Cost:

Reduction in the total quantity due to defectives found in inspection(accepted lot) and while screening(rejected lot) is given by

$$\sum_{x=0}^{c} x \sum_{X} p(X,x) + \sum_{x=c+1}^{n} \sum_{X} X p(X,x)$$

Which could be simplified to

$$\begin{aligned} &= \sum_{x=0}^{c} x \sum_{X} p(X,x) + \sum_{x=0}^{n} \sum_{X} X p(X,x) - \sum_{x=0}^{c} \sum_{X} X p(X,x) \\ &= Q\bar{p} - \sum_{x=0}^{c} \sum_{X} (X-x) p(X,x) \end{aligned}$$

which can be simplified as

$$= Q\bar{p} - (Q-n)\sum_{x=0}^{c} P_n(x) f_n(x)$$

Therefore net inventory is

$$Q - \left\{ Q\bar{p} - (Q-n)\sum_{x=0}^{c} P_n(x) f_n(x) \right\} \tag{5.15}$$

And Holding cost is given by

$$K_h = \frac{h}{2D}\left[Q - \left\{Q\bar{p} - (Q-n)\sum_{x=0}^{c} P_n(x)f_n(x)\right\}\right]^2 \tag{5.16}$$

Cycle length is given by:

$$T = \frac{1}{D}\left[Q - \left\{Q\bar{p} - (Q-n)\sum_{x=0}^{c} P_n(x)f_n(x)\right\}\right] \tag{5.17}$$

Total cost per unit time is given by

$$\frac{A + QC_i + (Q-n)\sum_{x=0}^{c}\left[C_d P_n(x) - C_i\right] f_n(x) + \frac{h}{2D}\left[Q - \{Q\bar{p} - (Q-n)\sum_{x=0}^{c} P_n(x)f_n(x)\}\right]^2}{\left[Q - \{Q\bar{p} - (Q-n)\sum_{x=0}^{c} P_n(x)f_n(x)\}\right]/D}$$

which could be rearranged

$$K = \frac{D\{A + QC_i + (Q-n)\Psi(n,c)\}}{Q(1-\bar{p}) + (Q-n)\lambda(n,c)} + \frac{h}{2}\left[Q(1-\bar{p}) + (Q-n)\lambda(n,c)\right]$$

Which could be simplified to

$$K = \frac{D\{A - n\Psi(n,c) + Q[\Psi(n,c) + C_i]\}}{Q(1-\bar{p}+\lambda(n,c)) - n\lambda(n,c)} + \frac{h}{2}\left[Q(1-\bar{p}+\lambda(n,c)) - n\lambda(n,c)\right]$$

which can be rewritten as

$$K = \frac{D\{A - n\Psi(n,c) + Q[\Psi(n,c) + C_i]\}}{QZ - n\lambda(n,c)} + \frac{h}{2}\left[QZ - n\lambda(n,c)\right] \tag{5.18}$$

where

$\Psi(n,c) = \sum_{x=0}^{c}\left[C_d P_n(x) - C_i\right] f_n(x)$

$\lambda(n,c) = \sum_{x=0}^{c} P_n(x) f_n(x)$

$Z = 1 - \bar{p} + \lambda(n,c)$

5.2.6 Model Formulation Case(2)

Case(2): With replacement of non-conforming items.

In this case the main difference is of lot size after inspection. Here the defective items are reworked or replaced by other conforming items.

The Expected cost in this case is given by

- Deterministic p
  - No inspection

$$K = \frac{AD}{Q} + C_d D p + \frac{hQ}{2} \tag{5.19}$$

  - 100% inspection

$$K = \frac{AD}{Q} + C_i D + C_r D p + \frac{hQ}{2} \tag{5.20}$$

- Sampling Inspection

$$\begin{aligned} K &= \frac{D\{A - n[p(C_d - C_r) - C_i]P_a\}}{Q} \\ &+ \{[p(C_d - C_r) - C_i]P_a + C_i + pC_r\}D + \frac{hQ}{2} \end{aligned} \tag{5.21}$$

- Stochastic p
  - No inspection

$$K = \frac{AD}{Q} + C_d D\bar{p} + \frac{hQ}{2} \tag{5.22}$$

  - 100% inspection

$$K = \frac{AD}{Q} + C_i D + C_r D\bar{p} + \frac{hQ}{2} \tag{5.23}$$

  - Sampling Inspection:

$$\begin{aligned} K_c &= A + nC_i \sum_{x=0}^{c} \sum_{X} p(X,x) + C_r \sum_{x=0}^{c} \sum_{X} xp(X,x) + C_d \sum_{x=0}^{c} \sum_{X} (X-x)p(X,x) + \frac{hQ^2}{2D} \\ &+ QC_i \sum_{x=c+1}^{n} \sum_{X} p(X,x) + C_r \sum_{x=c+1}^{n} \sum_{X} Xp(X,x) \end{aligned}$$

Which could be rewritten as

$$K_c = A + nC_i \sum_{x=0}^{c} f_n(x) + QC_i \sum_{x=0}^{n} f_n(x) - QC_i \sum_{x=0}^{c} f_n(x) + C_r \sum_{x=0}^{c} \sum_{X} xp(X,x)$$

$$+ \quad C_r \sum_{x=0}^{n} \sum_{X} Xp(X,x) - C_r \sum_{x=0}^{c} \sum_{X} Xp(X,x) + C_d(Q-n) \sum_{x=0}^{c} P_n(x) f_n(x) + \frac{hQ^2}{2D}$$

Which could be further simplified to

$$\begin{aligned} K_c &= A + QC_i - (Q-n)C_i \sum_{x=0}^{c} f_n(x) - C_r \sum_{x=0}^{c} \sum_{X} (X-x)p(X,x) + C_r Q\bar{p} \\ &+ (Q-n) \sum_{x=0}^{c} C_d P_n(x) f_n(x) + \frac{hQ^2}{2D} \end{aligned}$$

Therefore total cost per unit time is given by

$$K = \frac{A + QC_i + C_r Q\bar{p} + \frac{hQ^2}{2D} + (Q-n) \sum_{x=0}^{c} \left[(C_d - C_r) P_n(x) - C_i \right] f_n(x)}{Q/D}$$

Which can be arranged as

$$K = \frac{D[A + (Q-n)\chi(n,c) + Q(C_i + C_r\bar{p})]}{Q} + \frac{hQ}{2}$$

Which can be written as

$$K = \frac{D[A - n\chi(n,c)]}{Q} + (\chi(n,c) + C_i + C_r\bar{p})D + \frac{hQ}{2} \tag{5.24}$$

Where

$$\chi(n,c) = \sum_{x=0}^{c} \left[(C_d - C_r) P_n(x) - C_i \right] f_n(x)$$

5.3 Solution and Analysis

The objective is to minimize 'K' defined above in eq.(5.18) and eq.(5.24). A necessary condition for optimality is that the partial derivative with respect to 'Q' is equal to zero. Integer search is made for optimal 'n' and 'c' according to algorithm 5.1

Differentiating equations 5.18 and 5.24 w.r.t 'Q' and equating to zero for given n, and c,

- For non replacement case deterministic p.
  - For 100% inspection

$$Q^* = \frac{1}{1-p}\sqrt{\frac{2DA}{h}} \tag{5.25}$$

  - For sampling inspection

$$Q^* = \frac{npP_a}{Y} + \sqrt{\frac{2npP_aD(X+C_i) + 2YD(A-nX)}{hY^3}} \tag{5.26}$$

  - For no inspection

$$Q^* = \sqrt{\frac{2DA}{h}} \tag{5.27}$$

- For non replacement case stochastic p.
  - For 100% inspection

$$Q^* = \frac{1}{1-\bar{p}}\sqrt{\frac{2DA}{h}} \tag{5.28}$$

- For sampling inspection

$$Q^* = \frac{n\lambda(n,c)}{Z} + \sqrt{\frac{2n\lambda(n,c)D(\Psi(n,c) + C_i) + 2ZD(A - n\Psi(n,c))}{hZ^3}} \tag{5.29}$$

- For no inspection

$$Q^* = \sqrt{\frac{2DA}{h}} \tag{5.30}$$

- For replacement case deterministic p
  - For case of 100% and no inspection

$$Q^* = \sqrt{\frac{2DA}{h}} \tag{5.31}$$

  - And for sampling inspection

$$Q^* = \sqrt{\frac{2D\{A - n[p(C_d - C_r) - C_i]P_a\}}{h}} \tag{5.32}$$

- For replacement case stochastic p.
  - For case of 100% and no inspection

$$Q^* = \sqrt{\frac{2DA}{h}} \tag{5.33}$$

– And for sampling inspection

$$Q^* = \sqrt{\frac{2D(A - n\chi(n,c))}{h}} \tag{5.34}$$

Following algorithm can be applied using respective equations for non-replacement and replacement case to get optimal 'n', 'c'.

Algorithm 5.1

$Step$ 0 : Set $c = 0$

$Step$ 1 : $n = c + 1$

$Step$ 2 : Calculate $K(n, c)$ and $Q(n, c)$ from eq.5.18 (or 5.24) and eq.5.29 (or 5.34)

$Step$ 3 : Find n^* such that $K(n^* - 1, c) \geq K(n^*, c) \leq K(n^* + 1, c)$

$Step$ 4 : If $K(n^*, c) \leq K(n^*, c - 1)$ then $c = c + 1$ go to step (1)

$Step$ 5 : Find c^* such that $K(n^*, c^* - 1) \geq K(n^*, c^*) \leq K(n^*, c^* + 1)$

A computer program is written in Fortran to follow above algorithm and the scheme for selection of sampling plan to get optimal inspection and inventory policy. Parameters of sampling plan 'n' and 'c', whenever need are calculated as per algorithm 5.1. The over all solution procedure is given by algorithm 5.2.

Algorithm 5.2

1. UL (upper limit) = min{Cost of no inspection, Cost of 100% inspection}

2. Optimal sampling plan is obtained by integer search over 'n', and 'c' according

to algorithm 5.1

3. If 'α_r' and 'β_r' from this sampling plan are less than the agreed risk, the sampling plan is accepted.

4. If either 'α_r' or 'β_r or both is/are more than the desired value, new sampling plan is calculated using the 'Risk equations'.

$$1 - \alpha_r = \sum_{d=0}^{c} \frac{n!}{d!(n-d)!} p_1^d (1-p_1)^{n-d} \qquad (5.35)$$

$$\beta_r = \sum_{d=0}^{c} \frac{n!}{d!(n-d)!} p_2^d (1-p_2)^{n-d} \qquad (5.36)$$

5. If the cost, using the sampling plan obtained from eq.(5.35) and eq.(5.36) is less than UL, the sampling plan is accepted.

6. Else 100% inspection or no inspection, which ever has low cost is adapted.

5.4 Results

An example for non replacement and replacement case for the inventory-quality problem are presented in this section. Since, it is difficult to get the closed form solution to this problem. Integer search method is used to get the solutions. Computer code is written in Fortran, the program is run on a pentium III computer with 256 MB RAM (see appendix C and D for the code).

Using the similar numerical values of parameters as given in the model developed by Hanna and Jobe [2], the optimal order quantity and sampling plan for different values of parameters are evaluated.

Given $A = 75$, $D = 50000$, $h = 5$, $C_i = 1.5$, $C_d = 24$, $C_r = 2$, $\alpha_r = 0.05$, $\beta_r = 0.15$, $p_1 = 0.01$ and $p_2 = 0.06$, considering incoming quality to follow beta distribution with parameter $\beta(5, 5)$.

The tables below shows the optimal order quantity and sampling scheme over different parameters. Examples of OC curves for the sampling plans obtained in the above numerical are given in following figures.

	W risk				W/O risk			
C_i	Q^*	K	n	c	Q^*	K	n	c
1	2449.49	106123.7	100 % Insp.		2449.49	106123.7	100 % Insp.	
1.2	2449.49	126123.6	223	5	2449.49	126123.6	223	5
1.3	2449.49	136123.5	201	5	2449.49	136123.5	201	5
1.4	2449.50	146123.3	190	6	2449.50	146123.3	190	6
1.5	2449.50	156123.1	181	6	2449.50	156123.1	181	6
1.8	2449.45	186123.7	100 % Insp.		2449.53	186121.9	151	6
1.9	2449.49	196123.0	77	2	2449.55	196121.2	142	6
2	2449.49	206122.6	77	2	2449.57	206120.2	134	6
7	1224.70	606123.0	No Inspection		1224.70	606123.0	No Inspection	

Table 5.1: Expected cost, Optimal Quantity and Sampling scheme for Non-replacement case for different values of C_i

Tables 5.1 shows the effect of C_i on joint inventory and quality policy for non-replacement cases . As intuitive, it is evident that increase in C_i reduces the sample size. It is seen that for $C_i = 1.8$ the inspection policy jumps to 100% inspection, the inspection plan with these values does not satisfy the risk constrains and the sampling plan given by the risk equations is more costly than the one given in the table. This is the reason for any such abnormality in any of the tables.

	W risk				W/O risk			
C_d	Q^*	K	n	c	Q^*	K	n	c
5	1244.70	131123.7	No Inspection		1244.70	131123.7	No Inspection	
10	2449.56	156118.8	77	2	2451.77	156035.4	149	18
15	2449.52	156121.3	77	2	2449.73	156114.5	150	10
20	2449.49	156123.7	100 % Insp.		2449.48	156122.0	166	7
24	2449.50	156123.1	181	6	2449.50	156123.1	181	6
25	2449.50	156123.3	179	5	2449.50	156123.3	179	5
27	2449.50	156123.4	196	5	2449.50	156123.4	196	5
30	2449.49	156123.7	100 % Insp.		2449.49	156123.7	126	1

Table 5.2: Expected cost, Optimal Quantity and Sampling scheme for Non-replacement case for different values of C_d

Tables 5.2 shows the effect of C_d on joint inventory and quality policy for non-replacement case. As intuitive, it is evident that increase in C_d makes the sampling plan more tight, from no inspection to 100% inspection. It is also evident from the tables that increase in C_i and C_d does not affect ordering quantity but with the change in inspection policy. The change in ordering quantity is seen when ever inspection policy changes from no inspection to sampling inspection or 100 % inspection.

Tables 5.3 shows the effect of A on joint inventory and quality policy. As expected, it is evident that increase in A has direct effect on the ordering quantity and thereby effects the sampling plan.

Table 5.4 shows the effect of h on joint inventory and quality policy. For these

A	Q^*	K	n	c
40	1788.85	154472.1	100% Inspection	
50	2000.01	154999.6	152	4
70	2366.4	155915.6	172	5
75	2449.5	156123.1	181	6
90	2683.3	156707.5	194	7
100	2828.45	157070.3	220	8
120	3098.4	157745.1	235	9
150	3464.13	158659.3	286	12

Table 5.3: Expected cost, Order quantity and Sampling scheme for Non-replacement case for different values of A

	W/O Replacement				W Replacement			
h	Q^*	K	n	c	Q^*	K	n	c
1	5477.27	152738.0	188	6	2738.64	127738.1	168	6
2	3873.01	153872.4	188	6	1936.51	128872.5	167	6
5	2449.5	156123.1	181	6	1224.75	131123.3	169	6
10	1732.06	158659.7	176	6	866.03	133659.8	171	6
12	1581.15	159486.3	169	6	790.57	134486.4	173	6
14	1463.86	160246.4	188	6	731.93	135246.6	172	6
16	1369.31	160953.9	182	6	684.66	135954.1	169	6
18	1291.00	161618.4	176	6	645.50	136618.6	172	6
20	1224.70	162246.9	173	6	612.37	137247.1	170	6

Table 5.4: Expected cost, Optimal Quantity and Sampling scheme for Non-replacement and Replacement case for different values of h

values of h, no sampling plan invokes risk constrains. It is just an coincidence and combination of parameters of incoming fraction non conforming (θ and ϕ), that the ordering quantity in replacement case is half to that of in non-replacement case. It can be seen that increase in h has effect of reducing ordering quantity, as expected. But the sampling plan is not much affected.

Table 5.5 shows the effect of D on joint inventory and quality policy for replacement and non-replacement cases. As intuitive, it is evident that increase in D

D	W/O Replacement				W Replacement			
	Q^*	K	n	c	Q^*	K	n	c
5000	774.60	16936.4	188	6	387.3	14436.46	164	6
25000	1732.06	79329.8	176	6	866.03	66829.92	171	6
35000	2049.40	110123.1	171	6	1024.7	92623.18	171	6
45000	2323.80	140808.9	187	6	1161.9	118309.1	171	6
50000	2449.50	156123.1	181	6	1224.75	131123.3	169	6
55000	2569.06	171422.0	173	6	1284.53	143922.1	168	6
65000	2792.87	201981.3	185	6	1396.44	169481.5	167	6
75000	3000.00	232499.1	187	6	1500.01	194999.3	171	6
85000	3193.77	262983.3	188	6	1596.89	220483.6	166	6

Table 5.5: Expected cost, Optimal Quantity and Sampling scheme for Non-replacement and Replacement case for different values of D

increases the order quantity. But again the sampling plan is not much affected.

C_i	W risk				W/O risk			
	Q^*	K	n	c	Q^*	K	n	c
1	1224.74	106123.7	100 % Insp.		1224.74	106123.7	162	0
1.2	1224.75	116123.6	221	6	1224.75	116123.6	221	6
1.4	1224.75	126123.4	184	6	1224.75	126123.4	184	6
1.5	1224.75	131123.3	169	6	1224.75	131123.3	169	6
1.8	1224.75	146123.3	77	2	1224.77	146122.4	137	6
12	1224.74	606123.8	No Inspection		1224.74	606123.8	No Inspection	

Table 5.6: Expected cost, Optimal Quantity and Sampling scheme for Replacement case for different values of C_i

Tables 5.6 shows the effect of C_i on joint inventory and quality policy for replacement case. As intuitive, it is evident that increase in C_i reduces the sample size. Whenever any risk constrain is violated we go back to inspection policy given by risk constrains, as in case with $C_i = 1$ and 1.8. The major difference in replacement case is the ordering quantity for any inspection policy, it nearly remains the same. As the non conforming units are replaced by good once.

C_d	W risk				W/O risk			
	Q^*	K	n	c	Q^*	K	n	c
4	1224.74	106123.7	No Inspection		1224.74	106123.7	No Inspection	
10	1224.78	131120.8	77	2	1228.47	130993.0	154	25
15*	1224.76	131122.1	77	2	1225	131114.0	146	12
20	1224.75	131123.3	77	2	1224.78	131122.2	158	8
24	1224.75	131123.3	169	6	1224.75	131123.3	169	6
25	1224.75	131123.4	180	6	1224.75	131123.4	180	6
30	1224.74	131123.7	100 % Insp.		1224.74	131123.7	100 % Insp.	

Table 5.7: Expected cost, Optimal Quantity and Sampling scheme for Replacement case for different values of C_d.

Tables 5.7 shows the effect of C_d on joint inventory and quality policy for replacement case. As intuitive, it is evident that increase in C_d makes the sampling plan more tight, from no inspection to 100% inspection. As mentioned earlier whenever any risk constraint is violated we go back to inspection plan given by risk constraints, this provide protection to both customer and producer. $*$ in table corresponds to OC curve shown in figure (5.4) for with and without risk constrains. It can be seen from the figure that the inspection plan without risk constrains is too lose. Even lot with 7.5% non conforming units have 80% chance of acceptance. Therefore inspection plan with risk constrains is used to give a good level of protection to customer.

Tables 5.8 shows the effect of A on joint inventory and quality policy. As expected, it is evident that increase in A has direct effect on the ordering quantity and thereby effects the sampling plan. $*$ in table corresponds to OC curve shown in figure (5.4). As it can be seen in figure that the sampling plan given without risk constraints is too tight on producer. For a lot with only 2% fraction non conforming

A	W risk Q^*	K	n	c	W/O risk Q^*	K	n	c
10*	447.21	137236.0	77	2	447.21	137236.0	68	0
30	774.59	128873.0	100 % Insp.		774.60	128872.8	113	2
50	1000.01	129999.7	137	4	1000.01	129999.7	137	4
70	1183.23	130915.6	167	6	1183.23	130915.6	167	6
75	1224.75	131123.3	169	6	1224.75	131123.3	169	6
90	1341.64	131708.2	100 % Insp.		1341.66	131707.7	199	8
120	1549.19	132746.0	100 % Insp.		1549.21	132745.3	230	10
150	1732.05	133660.3	100 % Insp.		1732.07	133659.5	270	13

Table 5.8: Expected cost, Optimal Quantity and Sampling scheme for Replacement case for different values of A.

the chance of acceptance is about only 20%. Therefore a sampling plan with risk constraints is proposed to have protection against too tight inspection. Table 5.9

C_r	W risk Q^*	K	n	c	W/O risk Q^*	K	n	c
1	1224.75	106123.4	178	6	1224.75	106123.4	178	6
2	1224.75	131123.3	169	6	1224.75	131123.3	169	6
3	1224.74	156124.7	100 % Insp.		1224.74	156123.1	172	7
4	1224.74	181123.7	100 % Insp.		1224.77	181122.9	166	7
5	1224.74	206123.6	77	2	1224.75	203122.7	158	7
15	1224.78	456121.1	77	2	1226.75	456052.1	151	21
22	1224.74	606123.8	No Inspection		1244.75	606123.8	No Inspection	

Table 5.9: Expected cost, Optimal Quantity and Sampling scheme for Replacement case for different values of C_r

shows the effect of C_r on joint inventory and quality policy for replacement case. As any other quality parameters for replacement case C_r does not have much effect on order quantity but its effect on sampling plan is evident.

C_i	Joint Policy				Separate Policy			
	Q^*	K	n	c	Q^*	K	n	c
Without Replacement								
1	1848.47	71650.9	172	6	1224.745	72185.8	126	4
1.5	1822.28	102678.6	77	2	1224.745	103199.2	77	2
6	1224.74	348980.9	No Inspection		1224.745	348980.9	No Inspection	
With Replacement								
1	1266.81	81501.96	77	2	1224.745	81505.57	77	2
1.5	1304.78	103521.5	77	2	1224.745	103534.6	77	2
8	1224.74	348980.9	No Inspection		1224.745	348980.9	No Inspection	

Table 5.10: Expected cost, Optimal Quantity and Sampling scheme showing joint policy is always better than the separate policy, for Replacement and Non-replacement case for different values of C_i.

Table 5.10 shows joint inventory and quality policy and a policy where quantity to be order is decided without consideration of inspection policy, once quantity is decided inspection policy is decided depending on it. It is intuitive that a joint policy will always be better than separate inventory and quality policy. Results shown in table for $\theta = 0.8$ and $\phi = 2.0$ (parameters of incoming quality) confirms joint policy to be superior to separate one.

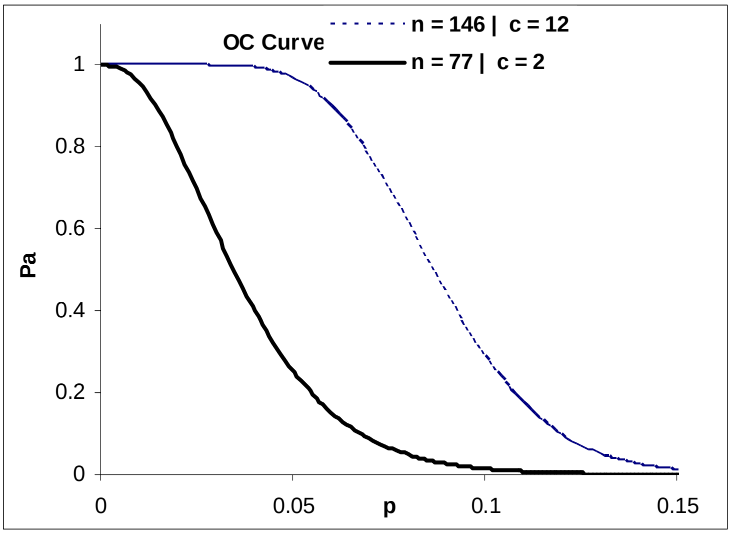

Figure 5.1: OC curve for with and without risk constrains

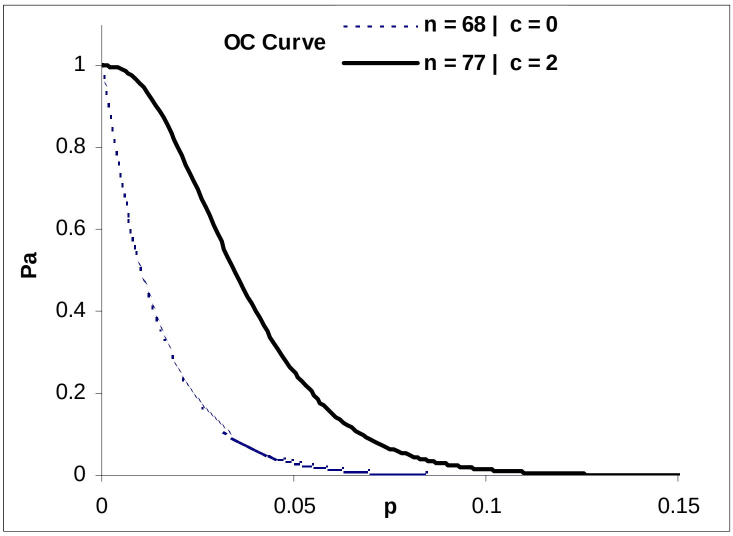

Figure 5.2: OC curve for with and without risk constrains

5.5 Conclusion

Joint inventory and inspection model is developed in this chapter by incorporating producer and consumer risk. Two cases are examined i.e with and without replacement of defective items found in inspection and in screening. The quantity in non-replacement model was found to be always higher to that in replacement model i.e. to compensate for the discarded units. It was also seen that a slight compromise in cost can give a inspection policy within the agreed limits.

Chapter 6

Joint Optimal Ordering and Inspection Policy for Stochastic Demand

6.1 Introduction

The purpose of this chapter is to develop a model for joint optimal ordering quantity, reorder point and inspection plan. This is followed by sensitivity analysis in order to investigate the effect of the inspection cost and model parameters on the results of the model.

The model is developed for a process where the units are inspected upon arrival, the quantity to be ordered, reorder point and inspection policy are decision variables.

In previous chapter ordering and inspection policy for deterministic demand were discussed. In this chapter the assumption of fixed deterministic demand is relaxed. The inspection policy, sampling scheme and rules under lying the sampling scheme are same as discussed in chapter 5.

This chapter is organized as follows, model development is presented in section 6.2, in which notations, model assumptions, statement of the problem and model formulation are discussed. Solution and analysis of the problem are discussed in sec.6.3, the results are given in sec.6.4.

6.2 Model Development

In this section, a joint optimal inventory and quality model for a case with stochastic demand is developed considering practical situations of consumer and producer risk. At first, model assumptions are discussed, followed by necessary notations, statement of the problem and then model formulation.

6.2.1 Model Assumptions

1. Demand during lead time follows known distribution

2. Shortages are backordered.

3. Incoming Quality is stable or follows known (beta) distribution.

4. Inspection is error free.

5. There costs no extra time for inspection.

6. Either defective items are all reworked and replaced or all discarded.

7. Risks are not considered if 100% or no inspection is done. They are considered only in sampling inspection.

6.2.2 Notations

The following are the notations that are used in this chapter

A	=	Fixed ordering cost,
D	=	Demand rate,
μ	=	Mean demand during lead time,
Q	=	Lot size,
r	=	Reorder point,
n	=	Sample size,
c	=	Number of defective allowed in sample,
C_i	=	Inspection cost,
$\bar{p}$	=	Average quality of incoming lot,
C_d	=	Cost of accepting a defective item,
C_r	=	Cost of rework on defective item,
h	=	Holding cost,
π	=	Shortage cost
P_a	=	Probability of acceptance,
K	=	Expected cost,
T	=	Expected cycle length,
α_r	=	Producer's risk,
β_r	=	Consumer's risk.

6.2.3 Statement of the Problem

Consider a manufacturing system where units are inspected at customer's arrival window or at manufacturer's dispatch window. Inspection policy, quantity to be ordered and when to be ordered are decision variables. Due to uncertainty in demand during lead time (time between order placed and received) there are chances of stockouts, if demand is underestimated and high holding costs if demand is overestimated. If shortages occur, they are backordered i.e. customers are not lost but their demand is satisfied with some extra penalty 'π' to the supplier. The aim is to find joint optimal inventory and inspection policy. Manufacturer and consumer agreed to have some protection by agreeing on producer's risk and consumer's risk. Incoming quality is stochastic in nature following a known beta distribution. If sampling inspection is adopted, a sampling plan is developed with the help of model developed. A sample size of 'n' units is taken from the lot and inspected, if this sample contains defectives less than or equal to the critical number 'c', whole lot is accepted else the lot is subjected to 100% screening.

There are two scenarios here, the defectives observed during inspection and screening are either all reworked and replaced or all discarded. Both of these cases are discussed here as non-replacement case and replacement case.

6.2.4 Model Formulation

The distribution of number of lot defective X is given by:

$$g(X) = \int_0^1 \binom{Q}{X} p^X (1-p)^{Q-X} \xi(p) dp \tag{6.1}$$

where $\xi(p)$ follows beta distribution; given by:

$$\xi(p) = \frac{\Gamma(\theta+\phi)}{\Gamma(\theta)\Gamma(\phi)} p^{\theta-1}(1-p)^{\phi-1} \tag{6.2}$$

substituting eq.(6.2) in eq.(6.1) we get beta-binomial distribution for $g(X)$; given by:

$$g(X) = \binom{Q}{X} \frac{\Gamma(\theta+\phi)}{\Gamma(\theta)\Gamma(\phi)} \frac{\Gamma(\theta+X)\Gamma(Q+\phi-X)}{\Gamma(\theta+\phi+Q)} \tag{6.3}$$

The number of defectives x in sample of size n is described by conditional distribution $t(x|X)$, and the Joint distribution of X and x is given by:

$$p(X,x) = t(x|X)g(X)$$

And the marginal distribution of x is given by:

$$f_n(x) = \sum_X p(X,x)$$

The conditional distribution $t(x|X)$ is as sumed to be hypergeometric distribution as a result marginal distribution of X and x fall into a well known family of reproducible distribution [27] and therefore both $g(X)$ and $f_n(x)$ follows beta-binomial distribution with parameters (Q, θ, ϕ) and (n, θ, ϕ) respectively and their respective mean is given by:

$$E(X) = \frac{Q\theta}{\theta + \phi} = Q\bar{p} \tag{6.4}$$

$$E(x) = \frac{n\theta}{\theta + \phi} = n\bar{p} \tag{6.5}$$

Probability of acceptance is given by

$$P_a = \sum_{x=0}^{c} f_n(x)$$

The expected number of defective units in noninspected portion of lot, given x defectives in sample is given by $E[X - x|x]$ and $P_n(x)$ is the average fraction defective in the noninspected portion of the lot, given x defectives in sample of size n. Then

$$P_n(x) = \frac{E[X - x|x]}{(Q - n)} \tag{6.6}$$

This implies

$$E[X - x|x] = (Q - n)P_n(x) \tag{6.7}$$

It can be shown that (proof in chapter 5)

$$P_n(x) = \frac{\theta + x}{\theta + \phi + n} \tag{6.8}$$

Average expected shortage is given by

$$\bar{S}(s) = \int_0^\infty S(s)f(s)ds = \int_r^\infty (s - r)f(s)ds \tag{6.9}$$

6.2.5 Model Formulation Case(1)

Case(1): Without replacement of non-conforming items.

- No inspection

$$K = \frac{D(A + \pi\bar{S}(s))}{Q} + h\left(\frac{Q}{2} + r - \mu\right) + C_d D\bar{p} \tag{6.10}$$

- 100% inspection

$$K = \frac{D(A + \pi \bar{S}(s))}{Q(1-\bar{p})} + h\left(\frac{Q(1-\bar{p})}{2} + r - \mu\right) + \frac{DC_i}{1-\bar{p}} \tag{6.11}$$

- Sampling Inspection:

In this case the expected number of items that may slip through inspection process $E[X - x/x]$ incur an additional cost of C_d per unit.

Inspection and Quality cost if lot is accepted is given by

$$nC_i \sum_{x=0}^{c} \sum_{X} p(X, x) + C_d \sum_{x=0}^{c} \sum_{X} (X - x) p(X, x)$$

If the lot is rejected then remaining $Q-n$ items are also subjected to 100% inspection. The expected (Inspection) cost in this case is given by

$$C_i Q \sum_{x=c+1}^{n} \sum_{X} p(X, x)$$

Combining both scenario the total setup, Inspection and Quality cost is given by

$$\begin{aligned} K_{i,q} &= A + \pi \bar{S}(s) + nC_i \sum_{x=0}^{c} f_n(x) + C_d(Q-n) \sum_{x=0}^{c} P_n(x) f_n(x) \\ &+ C_i Q \sum_{x=0}^{n} f_n(x) - C_i Q \sum_{x=0}^{c} f_n(x) \end{aligned}$$

since

$$\begin{aligned}
P_n(x) &= \frac{E[X - x/x]}{Q - n} \\
&= \frac{1}{Q - n} \frac{\sum_X (X - x) p(X, x)}{f_n(x)} \\
\sum_X (X - x) p(X, x) &= (Q - n) P_n(x) f_n(x)
\end{aligned}$$

$$K_{i,q} = A + \pi \bar{S}(s) + QC_i + (Q - n) \sum_{x=0}^{c} \left[C_d P_n(x) - C_i \right] f_n(x) \tag{6.12}$$

Holding Cost:

Reduction in the total quantity due to non conforming units found in inspection(accepted lot) and while screening(rejected lot) is given by

$$\sum_{x=0}^{c} x \sum_X p(X, x) + \sum_{x=c+1}^{n} \sum_X X p(X, x)$$

This could be simplified to

$$\begin{aligned}
&= \sum_{x=0}^{c} x \sum_X p(X, x) + \sum_{x=0}^{n} \sum_X X p(X, x) - \sum_{x=0}^{c} \sum_X X p(X, x) \\
&= Q\bar{p} - \sum_{x=0}^{c} \sum_X (X - x) p(X, x)
\end{aligned}$$

This can be simplified as

$$= Q\bar{p} - (Q-n)\sum_{x=0}^{c} P_n(x) f_n(x)$$

Therefore net inventory is

$$\hat{Q} = Q - \left\{ Q\bar{p} - (Q-n)\sum_{x=0}^{c} P_n(x) f_n(x) \right\} \tag{6.13}$$

And Holding cost is given by

$$K_h = h\left\{ \frac{\hat{Q}}{2} + r - \mu \right\} \frac{\hat{Q}}{D}$$

$$= h\left\{ \frac{Q - \{Q\bar{p} - (Q-n)\sum_{x=0}^{c} P_n(x) f_n(x)\}}{2} + r - \mu \right\} \frac{Q - \{Q\bar{p} - (Q-n)\sum_{x=0}^{c} P_n(x) f_n(x)\}}{D}$$

Cycle length is given by:

$$T = \frac{1}{D}\left[Q - \left\{ Q\bar{p} - (Q-n)\sum_{x=0}^{c} P_n(x) f_n(x) \right\} \right] \tag{6.14}$$

Total cost per unit time is given by

$$\frac{A + \pi\bar{S}(s) + QC_i + (Q-n)\sum_{x=0}^{c}\left[C_d P_n(x) - C_i\right] f_n(x) + \frac{h}{2D}[\hat{Q}^2 + 2(r-\mu)\hat{Q}]}{\left[Q - \{Q\bar{p} - (Q-n)\sum_{x=0}^{c} P_n(x) f_n(x)\}\right]/D}$$

This could be simplified to

$$\frac{2AD + 2D\pi\bar{S}(s) + 2DQC_i + 2D(Q-n)\Psi(n,c)}{2\left[Q(1-\bar{p}+\lambda(n,c)) - n\lambda(n,c)\right]}$$
$$+ \quad \frac{h\left\{[Q(1-\bar{p}+\lambda(n,c)) - n\lambda(n,c)]^2 + 2(r-\mu)[Q(1-\bar{p}+\lambda(n,c)) - n\lambda(n,c)]\right\}}{2\left[Q(1-\bar{p}+\lambda(n,c)) - n\lambda(n,c)\right]}$$

$$K = \frac{D\{A + \pi\bar{S}(s) + QC_i + (Q-n)\Psi(n,c)\}}{Q(1-\bar{p}) + (Q-n)\lambda(n,c)} + h\left[\frac{Q(1-\bar{p}) + (Q-n)\lambda(n,c)}{2} + r - \mu\right]$$

This can be rewritten as

$$K = \frac{D\{A + \pi\bar{S}(s) - n\Psi(n,c) + Q[\Psi(n,c) + C_i]\}}{QZ - n\lambda(n,c)} + h\left[\frac{QZ}{2} - n\lambda(n,c) + r - \mu\right] \tag{6.15}$$

where

$\Psi(n,c) = \sum_{x=0}^{c} \left[C_d P_n(x) - C_i\right] f_n(x)$

$\lambda(n,c) = \sum_{x=0}^{c} P_n(x) f_n(x)$

$Z = 1 - \bar{p} + \lambda(n,c)$

6.2.6 Model Formulation Case(2)

Case(2): With replacement of non-conforming items.

In this case the main difference is of lot size after inspection. Here the defective items are reworked or replaced by other conforming items.

The Expected cost in this case is given by

- No inspection

$$K = \frac{D(A + \pi\bar{S}(s))}{Q} + h\left(\frac{Q}{2} + r - \mu\right) + C_d D\bar{p} \tag{6.16}$$

- 100% inspection

$$K = \frac{D(A + \pi\bar{S}(s))}{Q} + h\left(\frac{Q}{2} + r - \mu\right) + DC_i + C_r D\bar{p} \tag{6.17}$$

- Sampling Inspection

$$\begin{aligned} K_c &= A + \pi\bar{S}(s) + nC_i \sum_{x=0}^{c} \sum_{X} p(X,x) + C_r \sum_{x=0}^{c} \sum_{X} xp(X,x) + C_d \sum_{x=0}^{c} \sum_{X} (X - x)p(X,x) \\ &+ h\left\{\frac{Q}{2} + r - \mu\right\} \frac{Q}{D} + QC_i \sum_{x=c+1}^{n} \sum_{X} p(X,x) + C_r \sum_{x=c+1}^{n} \sum_{X} Xp(X,x) \end{aligned}$$

Which could be rewritten as

$$\begin{aligned} K_c &= A + \pi\bar{S}(s) + nC_i \sum_{x=0}^{c} f_n(x) + QC_i \sum_{x=0}^{n} f_n(x) - QC_i \sum_{x=0}^{c} f_n(x) \\ &+ C_r \sum_{x=0}^{c} \sum_{X} xp(X,x) + C_r \sum_{x=0}^{n} \sum_{X} Xp(X,x) - C_r \sum_{x=0}^{c} \sum_{X} Xp(X,x) \\ &+ C_d(Q - n) \sum_{x=0}^{c} P_n(x) f_n(x) + \frac{hQ^2}{2D} + \frac{hQ(r - \mu)}{D} \end{aligned}$$

This could be further simplified to

$$\begin{aligned} K_c &= A + \pi\bar{S}(s) + QC_i - (Q-n)C_i\sum_{x=0}^{c} f_n(x) - C_r\sum_{x=0}^{c}\sum_{X}(X-x)p(X,x) + C_rQ\bar{p} \\ &+ (Q-n)\sum_{x=0}^{c} C_dP_n(x)f_n(x) + \frac{hQ^2}{2D} + \frac{hQ(r-\mu)}{D} \end{aligned}$$

Therefore total cost per unit time is given by

$$K = \frac{A + \pi\bar{S}(s) + QC_i + C_rQ\bar{p} + \frac{hQ^2}{2D} + \frac{hQ(r-\mu)}{D} + (Q-n)\sum_{x=0}^{c}\left[(C_d - C_r)P_n(x) - C_i\right]f_n(x)}{Q/D}$$

$$K = \frac{D[A + \pi\bar{S}(s) + Q(C_i + C_r\bar{p}) + (Q-n)\chi(n,c)]}{Q} + h\left[\frac{Q}{2} + r - \mu\right] \qquad (6.18)$$

This can be written as

$$K = \frac{D[A + \pi\bar{S}(s) - n\chi(n,c)]}{Q} + \left(C_i + C_r\bar{p} + \frac{h(r-\mu)}{D} + \chi(n,c)\right)D + \frac{hQ}{2} \qquad (6.19)$$

Where

$$\chi(n,c) = \sum_{x=0}^{c}\left[(C_d - C_r)P_n(x) - C_i\right]f_n(x)$$

6.3 Solution and Analysis

The objective is to minimize 'K' defined above. A necessary condition for optimality is that the partial derivative with respect to 'Q' and r are equal to zero. Integer search is made for optimal 'n' and 'c' according to algorithm 6.1

Partially differentiating equations 6.15 and 6.19 w.r.t 'Q'and r and equating to zero. For given n and c,

- For non replacement case.
  - For 100% inspection

$$Q^* = \frac{1}{1-\bar{p}}\sqrt{\frac{2D(A+\pi\bar{S}(s))}{h}} \tag{6.20}$$

and

$$\int_{r^*}^{\infty} f(x)dx = \frac{hQ^*(1-\bar{p})}{\pi D} \tag{6.21}$$

  - For sampling inspection

$$Q^* = \frac{n\lambda(n,c)}{Z} + \sqrt{\frac{2D[n\lambda(n,c)(\Psi(n,c)+C_i)+Z(A-n\Psi(n,c)+\pi\bar{S}(s))]}{hZ^3}} \tag{6.22}$$

and

$$\int_{r^*}^{\infty} f(x)dx = \frac{h\{Q^*(1-\bar{p}+\lambda(n,c)) - n\lambda(n,c)\}}{\pi D} \tag{6.23}$$

– For no inspection

$$Q^* = \sqrt{\frac{2D(A+\pi\bar{S}(s))}{h}} \tag{6.24}$$

and

$$\int_{r^*}^{\infty} f(x)dx = \frac{hQ^*}{\pi D} \tag{6.25}$$

- For replacement case.

  – For case of 100% and no inspection

$$Q^* = \sqrt{\frac{2D(A+\pi\bar{S}(s))}{h}} \tag{6.26}$$

and

$$\int_{r^*}^{\infty} f(x)dx = \frac{hQ^*}{\pi D} \tag{6.27}$$

– And for sampling inspection

$$Q^* = \sqrt{\frac{2D(A + \pi\bar{S}(s) - n\chi(n,c))}{h}} \tag{6.28}$$

and

$$\int_{r^*}^{\infty} f(x)dx = \frac{hQ^*}{\pi D} \tag{6.29}$$

Following algorithm can be applied using respective equations for non-replacement and replacement case to get optimal 'n', 'c'.

Algorithm 6.1

Step 0 : Set $c = 0$

Step 1 : $n = c + 1$

Step 2 : Calculate $K(n, c)$ and $Q(n, c)$ from eq.6.15 (or 6.19) and eq. 6.22 (or 6.28)

Step 3 : Find n^* such that $K(n^* - 1, c) \geq K(n^*, c) \leq K(n^* + 1, c)$

Step 4 : If $K(n^*, c) \leq K(n^*, c - 1)$ then $c = c + 1$ go to step (1)

Step 5 : Find c^* such that $K(n^*, c^* - 1) \geq K(n^*, c^*) \leq K(n^*, c^* + 1)$

A computer program is written in Fortran to follow above algorithm and get optimal inspection and inventory policy. Parameters of sampling plan 'n' and 'c', when ever need are calculated as per algorithm 6.1. The over all solution procedure is similar to one done in chapter 5 and is given by algorithm 6.2.

Algorithm 6.2

1. UL (upper limit) = min{Cost of no inspection, Cost of 100% inspection}

2. Optimal sampling plan is obtained by integer search over 'n', and 'c' according to algorithm 5.1

3. If 'α_r' and 'β_r' from this sampling plan are less than the agreed risk, the sampling plan is accepted.

4. If either 'α_r' or 'β_r or both is/are more than the desired value, new sampling plan is calculated using the 'Risk equations'.

$$1 - \alpha_r = \sum_{d=0}^{c} \frac{n!}{d!(n-d)!} p_1^d (1-p_1)^{n-d} \tag{6.30}$$

$$\beta_r = \sum_{d=0}^{c} \frac{n!}{d!(n-d)!} p_2^d (1-p_2)^{n-d} \tag{6.31}$$

5. If the cost, using the sampling plan obtained from eq.6.30 and eq.6.31 is less than UL, the sampling plan is accepted.

6. Else 100% inspection or no inspection, which ever has low cost is adopted.

6.4 Results

An example for non replacement and replacement case for the inventory-quality problem are presented in this section. Since, it is difficult to get the closed form solution to this problem. Integer search method is used to get the solutions. Computer code is written in Fortran, the program is run on a pentium III computer with 256 MB RAM (see appendix E and F for the code).

Using the similar numerical values of parameters as used in chapter 5, the optimal order quantity and sampling plan for different values of parameters are evaluated. Given $A = 75$, $D = 50000$, $h = 5$, $C_i = 1.5$, $C_r = 5$, $C_d = 30$, $\pi = 20$, $\alpha_r = 0.05$, $\beta_r = 0.1$, $p_1 = 0.01$, $p_2 = 0.06$, considering incoming quality to follow beta distribution with parameter $\beta(5, 5)$. Demand during lead time follows uniform distribution between 0 and 50 with mean 25, ie. $b = 50$ and $\mu = 25$.

The tables below shows the optimal order quantity, reorder point and sampling scheme over different parameters. Examples of OC curves for the sampling plans obtained in the above numerical are given in following figures.

C_i	W risk					W/O risk				
	Q^*	r^*	K	n	c	Q^*	r^*	K	n	c
0.9	2449.79	49.69	96149.2	100 % Insp.		2449.79	49.69	96149.2	100 % Insp.	
1.3	2449.80	49.69	136149.0	319	9	2449.80	49.69	136149.0	319	9
1.6	2449.82	49.69	166148.5	240	9	2449.82	49.69	166148.5	240	9
1.9	2449.78	49.69	196148.8	87	2	2449.86	49.69	196146.8	212	10
5	2449.99	49.69	506135.1	87	2	2475.84	49.68	502790.1	110	24
8	1224.78	49.84	756248.1	No Insp.		2742.96	49.52	713667.0	108	54
9	1224.78	49.84	756248.1	No Insp.		1224.78	49.84	756248.1	No Insp.	

Table 6.1: Expected cost, Optimal Quantity and Sampling scheme for Non-replacement case for different values of C_i

Table 6.1 shows the effect of C_i on joint inventory and quality policy for non-replacement cases . As intuitive, it is evident that increase in C_i reduces the sample size. The inspection plan goes from 100% inspection to no inspection. C_i does not have direct effect on ordering quantity but change in ordering quantity is seen when ever inspection policy changes from no inspection to sampling inspection or 100 % inspection. Reorder point is hardly affected.

C_d	W risk					W/O risk				
	Q^*	r^*	K	n	c	Q^*	r^*	K	n	c
5	1224.78	49.84	131248.2	No Insp.		1224.78	49.84	131248.2	No Insp.	
15	2449.82	49.69	156146.6	87	2	2450.59	49.69	156109.5	221	23
25	2449.78	49.69	156149.2	87	2	2449.85	49.69	156147.7	258	12
30	2449.81	49.69	156148.7	269	9	2449.81	49.69	156148.7	269	9
35	2449.80	49.69	156149.0	292	7	2449.80	49.69	156149.0	292	7
40*	2449.72	49.69	156153.2	87	2	2449.79	49.69	156149.2	177	0
45	2449.70	49.69	156154.5	87	2	2449.79	49.69	156149.2	245	1
55	2449.79	49.69	156149.2	100 % Insp.		2449.79	49.69	156149.2	100 % Insp.	

Table 6.2: Expected cost, Order quantity, Reorder point and Sampling scheme for Non-replacement case for different values of C_d

Table 6.2 shows the effect of C_d on joint inventory and quality policy for non-replacement case. It is evident that increase in C_d makes the sampling plan more tight, from no inspection to 100% inspection. It is seen that for $C_d = 15$ and $C_d = 40$ the inspection policy jumps to $n = 87$ and $c = 2$, the inspection plan with these values does not satisfy the risk constraints therefore the sampling plan given by the risk equations is adopted. $*$ in table corresponds to OC curve shown in figure (6.1) for with and without risk constrains.

Table 6.3 shows the effect of A on joint inventory and quality policy. As intuitive,

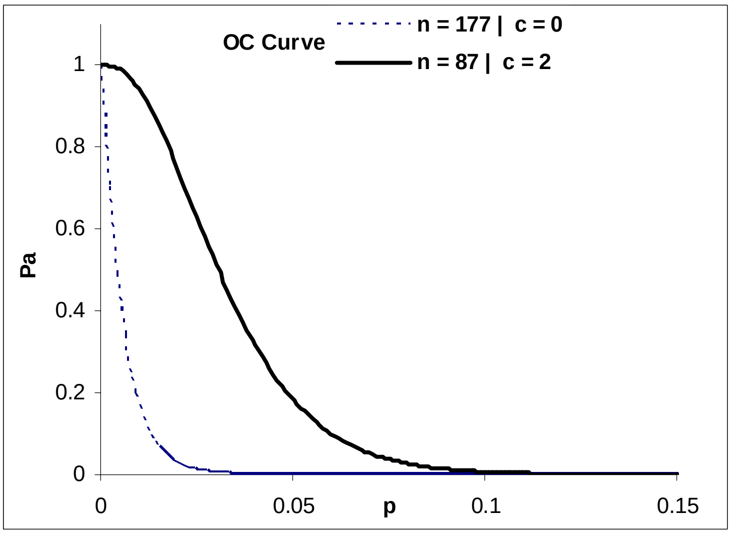

Figure 6.1: OC curve for with and without risk constrains

A	W risk					W/O risk				
	Q^*	r^*	K	n	c	Q^*	r^*	K	n	c
25	1414.34	49.82	153562.1	87	2	1414.39	49.82	153560.7	141	2
50	2000.26	49.75	155025.0	213	6	2000.26	49.75	155025.0	213	6
75	2449.81	49.69	156148.7	269	9	2449.81	49.69	156148.7	269	9
100	2828.80	49.64	157096.0	318	12	2828.80	49.64	157096.0	318	12
125	3162.70	49.60	157930.7	381	15	3162.70	49.60	157930.7	381	15
150	3464.56	49.56	158685.2	409	17	3464.56	49.56	158685.2	409	17

Table 6.3: Expected cost, Order quantity, Reorder point and Sampling scheme for Non-replacement case for different values of A

it is evident that increase in A increases the order quantity and thereby also effects the sampling plan.

Table 6.4 shows the effect of h on joint inventory and quality policy. For these values of h, no sampling plan invokes risk constrains. It is just an incident and combination of parameters of incoming quality (θ and ϕ) that the ordering quantity in replacement case is half to that of in non-replacement case. Reordering point is

h	W/O Replacement					With Replacement				
	Q^*	r^*	K	n	c	Q^*	r^*	K	n	c
1	5477.40	49.86	152763.1	268	9	2738.69	49.86	202763.3	181	5
3	3162.53	49.76	154768.1	260	9	1581.26	49.76	204817.9	185	5
5	2449.81	49.69	156148.7	269	9	1224.90	49.69	206247.8	185	5
7	2070.57	49.63	157271.2	269	9	1035.28	49.63	207419.2	186	5
9	1826.16	49.58	158241.8	267	9	913.08	49.58	208438.8	177	5
11	1651.91	49.54	159109.6	263	9	825.95	49.54	209355.3	175	5
13	1519.61	49.50	159901.5	265	9	759.80	49.50	210195.8	186	5
15	1414.75	49.46	160634.6	271	9	707.37	49.46	210977.5	182	5

Table 6.4: Expected cost, Order quantity, Reorder point and Sampling scheme for Non-replacement and Replacement cases for different values of h

hardly affected

π	W/O Replacement					With Replacement				
	Q^*	r^*	K	n	c	Q^*	r^*	K	n	c
0.5	2461.84	37.69	156166.8	268	9	1230.92	37.69	206217.8	179	5
1	2455.65	43.86	156157.5	268	9	1227.82	43.86	206233.2	185	5
3	2451.55	47.95	156151.3	270	9	1225.77	47.95	206243.4	182	5
5	2450.73	48.77	156150.1	266	9	1225.36	48.77	206245.5	185	5
10	2450.12	49.38	156149.2	265	9	1225.05	49.38	206247.0	184	5
20	2449.81	49.69	156148.7	269	9	1224.90	49.69	206247.9	185	5
30	2449.71	49.79	156148.6	269	9	1224.85	49.79	206248.0	182	5
40	2449.66	49.84	156148.5	265	9	1224.82	49.84	206248.1	177	5

Table 6.5: Expected cost, Order quantity, Reorder point and Sampling scheme for Non-replacement and Replacement cases for different values of π

Table 6.5 shows the effect of π on joint inventory and quality policy. For these values of π, no sampling plan invokes risk constraints. It can be seen that as π is increased r^* also increases. As penalty on stockouts increases the system tends to lower the number of stockouts thereby increasing the reorder level.

Table 6.6 shows the effect of D on joint inventory and quality policy for replacement and non-replacement cases. As intuitive, it is evident that increase in D increases the order quantity and thereby also effects the sampling plan

D	W/O Replacement					With Replacement				
	Q^*	r^*	K	n	c	Q^*	r^*	K	n	c
10000	1096.14	49.32	32764.6	268	9	547.99	49.45	42862.2	180	5
20000	1549.69	49.52	63898.5	259	9	774.79	49.61	83997.0	188	5
40000	2191.25	49.66	125502.4	265	9	1095.58	49.72	165601.4	188	5
60000	2683.58	49.72	186733.1	269	9	1341.75	49.77	246832.4	183	5
80000	3098.65	49.76	247770.6	267	9	1549.29	49.80	327870.2	184	5
100000	3464.34	49.78	308684.7	272	9	1732.14	49.82	408785.0	182	5

Table 6.6: Expected cost, Order quantity, Reorder point and Sampling scheme for Non-replacement and Replacement cases for different values of D

μ	W/O Replacement					With Replacement				
	Q^*	r^*	K	n	c	Q^*	r^*	K	n	c
5	2449.57	9.94	156128.4	264	9	1224.78	9.93	206148.4	183	5
10	2449.63	19.88	156133.5	266	9	1224.81	19.87	206173.2	181	5
20	2449.75	39.76	156143.6	269	9	1224.87	39.75	206222.9	174	5
30	2449.88	59.63	156153.8	270	9	1224.93	59.63	206272.6	183	5
40	2450.00	79.51	156164.0	264	9	1224.99	79.51	206322.3	179	5
50	2450.12	99.39	156174.2	262	9	1225.05	99.38	206372.0	186	5

Table 6.7: Expected cost, Order quantity, Reorder point and Sampling scheme for Non-replacement and Replacement cases for different values of μ

Table 6.7 shows the effect of μ on joint inventory and quality policy for replacement and non-replacement cases. μ has direct effect on r^* and very slight effect on sampling plan. It is seen that the reorder point is sensitive only to the parameters directly relating to it.

C_i	W risk					W/O risk				
	Q^*	r^*	K	n	c	Q^*	r^*	K	n	c
1.0	1244.89	49.69	181248.0	100 % Insp.		1244.89	49.69	181248.0	100 % Insp.	
1.2	1224.89	49.69	191248.0	100 % Insp.		1224.89	49.69	191248.0	221	0
1.6	1224.90	49.69	211247.7	171	5	1224.90	49.69	211247.7	171	5
2.0	1224.90	49.69	231247.4	87	2	1224.93	49.69	231246.6	141	6
9.0	1225.10	49.69	581234.4	87	2	1355.87	49.66	567582.9	59	19
13	1224.89	49.69	756247.9	No Insp.		2119.90	49.47	699720.1	87	45
15	1224.89	49.69	756247.9	No Insp.		1224.89	49.69	756247.9	No Insp.	

Table 6.8: Expected cost, Order quantity, Reorder point and Sampling scheme for Replacement case for different values of C_i.

Table 6.8 shows the effect of C_i on joint inventory and quality policy for replacement case . As intuitive, it is evident that increase in C_i reduces the sample size. Whenever any constraint is violated we go back to inspection plan given by risk equations. Reorder point is hardly affected.

C_d	W risk					W/O risk				
	Q^*	r^*	K	n	c	Q^*	r^*	K	n	c
5	1224.86	49.75	131248.1	No Insp.		1224.86	49.75	131248.1	No Insp.	
15	1224.92	49.69	206246.4	87	2	1226.04	49.69	206206.6	149	18
25	1224.90	49.69	206247.7	87	2	1224.92	49.69	206247.2	165	7
30	1224.87	49.75	206247.9	174	5	1224.87	49.75	206247.9	174	5
32	1224.87	49.75	206248.0	205	5	1224.87	49.75	206248.0	205	5
34	1224.86	49.75	206248.1	100 % Insp.		1224.86	49.75	206248.1	100 % Insp.	

Table 6.9: Expected cost, Order quantity, Reorder point and Sampling scheme for Replacement case for different values of C_d

Tables 6.9 shows the effect of C_d on joint inventory and quality policy for replacement case. As intuitive, it is evident that increase in C_d makes the sampling plan more tight, from no inspection to 100% inspection.

A	W risk					W/O risk				
	Q^*	r^*	K	n	c	Q^*	r^*	K	n	c
25	707.19	49.82	203660.1	100 % Insp.		707.19	49.82	203660.0	126	1
50	1000.12	49.75	205124.4	100 % Insp.		1000.12	49.75	205124.3	150	3
75	1224.87	49.75	206247.9	174	5	1224.87	49.75	206247.9	174	5
100	1414.36	49.71	207195.1	209	7	1414.36	49.71	207195.1	209	7
125	1581.30	49.68	208029.6	251	9	1581.30	49.68	208029.6	251	9
150	1732.23	49.65	208784.0	283	11	1732.23	49.65	208784.0	283	11

Table 6.10: Expected cost, Order quantity, Reorder point and Sampling scheme for Replacement case for different values of A

Tables 6.10 shows the effect of A on joint inventory and quality policy. As expected, it is evident that increase in A has direct effect on the ordering quantity and thereby effecting the sampling plan and slightly reorder point.

C_r	W risk					W/O risk				
	Q^*	r^*	K	n	c	Q^*	r^*	K	n	c
1	1224.90	49.69	106247.9	195	4	1224.90	49.69	106247.9	195	4
3	1224.89	49.69	156248.0	100 % Insp.		1224.89	49.69	156248.0	100 % Insp.	
5	1224.90	49.69	206247.9	185	5	1224.90	49.69	206247.9	185	5
7	1224.91	49.69	256247.6	179	6	1224.91	49.69	256247.6	179	6
10	1224.90	49.69	331247.7	87	2	1224.92	49.69	331247.2	168	7
15*	1224.91	49.69	456247.1	87	2	1225.02	49.69	456243.6	149	10
20	1224.92	49.69	581246.4	87	2	1226.04	49.69	581206.6	149	18
25	1224.93	49.69	706245.8	87	2	1251.97	49.69	705113.6	132	37
28	1224.90	49.69	756247.9	No Insp.		1224.90	49.69	756247.9	NoInsp.	

Table 6.11: Expected cost, Order quantity, Reorder point and Sampling scheme for replacement case for different values of C_r.

Table 6.11 shows the effect of C_r on joint inventory and quality policy for replacement case. It does not affect much on order quantity but its effect on sampling plan is evident. As it is stated earlier whenever any risk constraint is violated we go back to inspection plan given by risk constraints, this provide protection to both customer and producer. $*$ in table corresponds to OC curve shown in figure (6.2) for with and without risk constrains. It can be seen from the figure that the inspection plan without risk constrains is too loose. Even lot with 7.5% non conforming units have 80% chance of acceptance. Therefore inspection plan with risk constraints is used to give a good level of protection to customer.

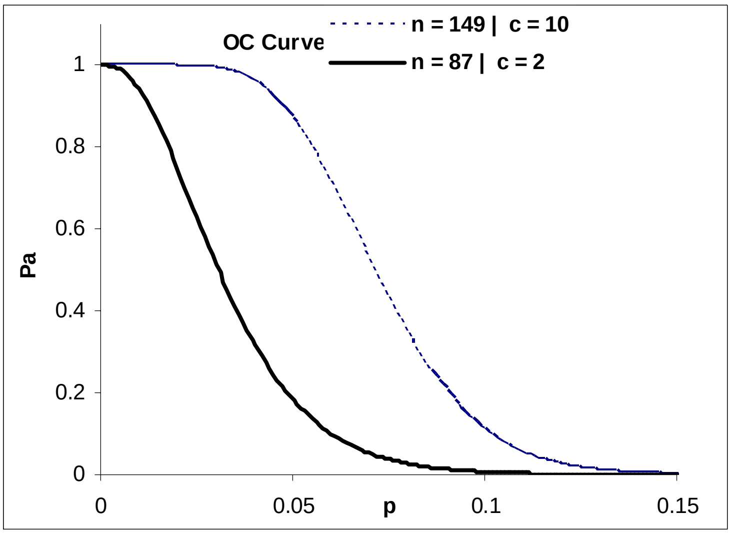

Figure 6.2: OC curve for with and without risk constrains

6.5 Conclusion

Joint inventory and inspection model is developed in this chapter by incorporating producer and consumer risk; Q-r model with inspection policy. Assumption of deterministic demand is relaxed. Optimal ordering quantity, reorder point and parameters of inspection plan are calculated. Two cases are examined i.e with and without replacement of defective items found in inspection and in screening.

Chapter 7

Conclusion and Future Research

7.1 Summary

In this thesis, relevant literature in the area of inventory and quality control is reviewed. Four models has been developed. Two from producers point of view relating to production process and two from retailers point of view relating to inventory control and acceptance policy.

Chapter 1 provides a brief introduction to inventory and quality control and states the problem under consideration in this thesis. Literature in the area of inventory and quality control is reviewed along with Yeh et.al. [1] and Jobe and Hanna's [2] models in chapter 2. In chapter 3 a model is developed for a system following a general probability distribution. Optimal production run length and optimal inspection policy has been derived for such a system, were its products are sold with warranty.

This model extends the work done by Yeh et.al. [1] to the case where the process follow a general probability distribution and inspection is carried out throughout the production period at definite intervals. The developed model is tailored for different distributions. Sensitivity analysis to the developed model is conducted to study the effect of model parameters on its solution. The model developed in chapter 3 is extended to a case with preventive maintenance. Effect of preventive maintenance on the production run and inspection scheduled are studied. Sensitivity analysis with respect to investment in PM and warranty cost are presented in chapter 4. Joint optimal inventory and quality policy for retailer is derived. Models are developed for deterministic and stochastic incoming quality and for replacement and non-replacement of defective items. Sensitivity analysis of the developed model is conducted with respect to quantity, inspection policy and cost parameters in chapter 5. The model developed in chapter 5 is extended to a case with stochastic demand i.e. (Q,r) policy. here again models are developed for deterministic and stochastic incoming quality and for replacement and non-replacement of defective items. Sensitivity analysis of the developed model is conducted with respect to quantity, inspection policy and cost parameters in chapter 6.

Thus, the work done in this thesis can be summarized as follows:

- Four models of inventory and quality control are developed. Two for producer and two for retailer.

- In the first extension, the assumption of exponential distribution is relaxed and process inspection is introduced in the model by Yeh et.al. [1].
- Preventive maintenance is introduced instead of only inspection in the model developed in chapter 3.
- Joint inventory and quality policy model is developed for EOQ policy.
- In the fourth extension, the model developed in chapter 5 is extended to a case with stochastic demand i.e. (Q,r) policy.

7.2 Recommendations for Future Research

The research work in this thesis can be extended to the following areas.

- The model developed in chapter 3 can be extended to find optimal warranty period, knowing the loss of goodwill if items are not repaired.
- The model in chapter 4 can be extended to find the optimal investment in preventive maintenance.
- The model in chapter 4 can also be extended to find optimal warranty period.
- Models in chapter 5 and 6 can be extended to find optimal producer and consumer risks, that are taken as fixed in this thesis.

- The developed models can be extended to the case where the products deteriorates with time.

- Models in chapter 5 and 6 can be extended to other inventory policies.

- Models in chapter 5 and 6 can be extended for optimal repair policy, which are taken as two different policies in this work.

- The Assumption of perfect inspection in models of chapter 5 and 6 can be relaxed and models with inspection errors can be developed.

Appendix A

Computer Code for Objective I of the Thesis

```
    program golden_section
weibull distribution
    real :: a,b,mu1,lemda,alpha_const,epsilon,stop_at,profit_mu1,
    &profit_lemda,plo,pgo,t1lo,t1go,tpgo,tplo
    integer :: n,l
    open(unit=200, file='results.txt',status='UNKNOWN')
      l=0
    n=1
5   a=0;b=3;alpha_const=0.618;stop_at=0.001
10  mu1=a+alpha_const*(b-a)
    lemda=a+(1-alpha_const)*(b-a)
    epsilon= b-a
    if(epsilon > s top_at) then
        call objf(l,n,mu1,profit_mu1,tp_mu1)
        call objf(l,n,lemda,profit_lemda,tp_lemda)
        if (profit_lemda >= profit_mu1)then
            b=mu1
        else
            a=lemda
        endif
        goto 10
    else
```

```
        plo=profit_mu1
        t1lo=mu1
        tplo=tp_mu1
        if(n==1) then
            pgo=plo
            t1go=t1lo
            tpgo=tplo
            n=n+1
            goto 5
        else
            if(plo>=pgo) then
                pgo=plo
                t1go=t1lo
                tpgo=tplo
                n=n+1
                goto 5
            else
                l=1
            call objf(l,n-1,t1go,profit,tp)
                print*,'optimal n=',n-1
                print*,'optimal t1=',t1go
                print*,'optimal tp=',tpgo
                print*,'optimal profit=',pgo
            endif
        endif
    endif
    end
c   objective function
    subroutine objf(l,n,t1,profit,tp)
    use msimsl
    real::nu
    v=20.0
    s=15.0
    pi_2=0.65
    pi_1=0.15
    wp=7
    R_2=(wp/2)**2
    R_1=(wp/6)**2
    C_m=1
    C_r=20
    cr1=1
    theta=1
    beeta=2
    C_p=10.0
    A=1000
    nu=20.0
    P=600
    d=400
    h=1
```

```
      total=0.0
      sum=0.0
      total2=0.0
      sum2=0.0
      tp=n**(1/beeta)*t1
      do 1 i=1,n
      total=i**(1/beeta)*t1*(exp((t1/theta)**beeta)-1)-theta*
      &(gami(1+1/beeta,i*(t1/theta)**beeta)-
       &gami(1+1/beeta,(i-1)*(t1/theta)**beeta))/exp(-i*(t1/theta)**beeta)
        sum=total+sum
1     continue
      do 2 i=1,n-1
      total2=i**(1/beeta)*t1*(exp((t1/theta)**beeta)-1)-theta*
      &(gami(1+1/beeta,i*(t1/theta)**beeta)-
       &gami(1+1/beeta,(i-1)*(t1/theta)**beeta))/exp(-i*(t1/theta)**beeta)
        sum2=total2+sum2
2     continue
      qc=(v-s)*sum/tp
      sic=(A+(n-1)*nu)/(P*tp)
      hc=(P-d)*tp*h/(2*d)
      rc=(C_r*(n-1)*(1-1/Exp((t1/theta)**beeta))
      &+cr1*sum2)/(P*tp)
      wc=(((pi_2-pi_1)*sum/tp+pi_1)*(R_2-R_1)+R_1)*C_m
      profit1=v-qc-sic-hc-rc-wc-C_p
      if (l==1) then
      write(200,*)wp,t1,n,tp,sic,hc,qc,rc,wc,profit1
      PRINT*,theta,t1,n,tp,sic,hc,qc,rc,wc,profit1
      endif
      profit=v-((v-s)+(pi_2-pi_1)*(R_2-R_1)*C_m)*sum/
      &(n**(1/beeta)*t1)-C_p-(P-d)*n**(1/beeta)*t1*h/(2*d)
       &-(A+(n-1)*nu+C_r*(n-1)*(1-1/Exp((t1/theta)**beeta))+cr1*sum2)
       &/(P*n**(1/beeta)*t1)-C_m*(pi_1*(R_2-R_1)+R_1)
        end
```

Appendix B

Computer Code for Objective II of the Thesis

```
      program obj2
c ojective 2     with pm new
    real :: a,b,mu1,lemda,alpha_const,epsilon,stop_at,profit_mu1,
    &profit_lemda,plo,pgo,t1lo,t1go,tpgo,tplo
    integer :: n,l
    open(unit=200, file='results.txt',status='UNKNOWN')
    l=0
    n=1
5   a=0;b=1;alpha_const=0.618;stop_at=0.00001
10  mu1=a+alpha_const*(b-a)
    lemda=a+(1-alpha_const)*(b-a)
    epsilon= b-a
    if(epsilon > stop_at) then
        call objf(l,n,mu1,profit_mu1,tp_mu1)
        call objf(l,n,lemda,profit_lemda,tp_lemda)
        if (profit_lemda > profit_mu1)then
            b=mu1
        else
            a=lemda
        endif
        goto 10
    else
        plo=profit_mu1
        t1lo=mu1
        tplo=tp_mu1
        if(n==1) then
            pgo=plo
            t1go=t1lo
            tpgo=tplo
            n=n+1
            goto 5
        else
```

```
            if(plo>=pgo) then
                pgo=plo
                t1go=t1lo
                tpgo=tplo
                n=n+1
                goto 5
            else
                l=1
            call objf(l,n-1,t1go,profit,tp)
                print*,'optimal n=',n-1
                print*,'optimal t1=',t1go
                print*,'optimal tp=',tpgo
                print*,'optimal profit=',pgo
c               write(200,*)n-1,t1go,tpgo,pgo
            endif
        endif
    endif
    end
c   objective function
    subroutine objf(l,n,t1,profit,sumh)
    use msimsl
    real, dimension(n+1) :: b,w,hh,y
    real::nu,ni,lam1,lam2,wp
    v=20.0
    s=15.0
    pi_2=0.65
    pi_1=0.15
    C_m=1
    C_r=20
    cr1=1
    theta=1
    beeta=2
    C_p=10.0
    A=1000
    nu=400.0
    cppm=800
    ni=0.9
    P=600
    d=400
    h=1
    wp=7
    lam1=6
    lam2=2
    if((n-1)>=1) then
    beeta1=1.5+(1-nu/cppm)*0.5
    else
    beeta1=2
    endif
    R_2=(wp/lam2)**beeta1
```

```
      R_1=(wp/lam1)**beeta1
        sumh=0.0
      total1=0.0
      sum1=0.0
      total2=0.0
      sum2=0.0
      total3=0.0
      sum3=0.0
      y(0)=0.0
      w(0)=0.0
      y(1)=t1
        hh(1)=y(1)
      b(1)=1-nu/cppm
      w(1)=b(1)*y(1)
      sumh=0.0
      do  i=2,n
      b(i)=ni**(i-1)*(1-nu/cppm)
      hh(i)=(w(i-1)**beeta+hh(1)**beeta)**(1/beeta)-w(i-1)
      y(i)=w(i-1)+hh(i)
        w(i)=b(i)*y(i)
      end do
      do 1 i=1,n
      sumh=sumh+hh(i)
      total1=(y(i)*(exp(-(w(i-1)/theta)**beeta)
      &-exp(-(y(i)/theta)**beeta))-
       &theta*(gami(1+1/beeta,(y(i)/theta)**beeta)
       &-gami(1+1/beeta,(w(i-1)/theta)**beeta)))/
       &exp(-(w(i-1)/theta)**beeta)
        sum1=total1+sum1
1     continue
      do 2 i=1,n-1
      total2=exp(-(y(i)/theta)**beeta)/
      &exp(-(w(i-1)/theta)**beeta)
        sum2=total2+sum2
        total3=(y(i)*(exp(-(w(i-1)/theta)**beeta)
      &-exp(-(y(i)/theta)**beeta))-
       &theta*(gami(1+1/beeta,(y(i)/theta)**beeta)
       &-gami(1+1/beeta,(w(i-1)/theta)**beeta)))/
       &exp(-(w(i-1)/theta)**beeta)
        sum3=total3+sum3
2     continue
      qc=(v-s)*sum1/sumh
      spmc=(A+(n-1)*nu)/(P*sumh)
      hc=(P-d)*sumh*h/(2*d)
      rc=(C_r*(n-1-sum2)+cr1*sum3)/(P*sumh)
      wc=(((pi_2-pi_1)*sum1/sumh+pi_1)*(R_2-R_1)+R_1)*C_m
      profit=v-((v-s)+(pi_2-pi_1)*(R_2-R_1)*C_m)*sum1/
      &sumh-C_p-(P-d)*sumh*h/(2*d)
       &-(A+(n-1)*nu+C_r*(n-1-sum2)+cr1*sum3)
```

```
 &/(P*sumh)-C_m*(pi_1*(R_2-R_1)+R_1)
profit1=v-qc-spmc-hc-rc-wc-C_p
if (l==1) then
write(200,*)wp,hh(1),n,sumh,spmc,hc,qc,rc,wc,profit1
print*,theta,hh(1),n,sumh,spmc,hc,qc,rc,wc,profit1
endif
  end
```

Appendix C

Computer Code for Objective III Non-replacement case

```
      program ob3withoutreplacement
c    basic non replacement model   final dec 2
    use msimsl
    common n,c
    integer n,c,cgo,c1,n1
    real p1,p2,cr,pr
*****given prducer and coustomer risk************
    alpha=0.05
    beeta=0.15
    p1=0.01
    p2=0.06
      gpr=1-alpha
    gcr=beeta
*****calculation for 100% inspection and no inspection****
    n=0
    c=0
    call obj(n,c,qnoinsp,cnoinsp)
    n=0
    c=1
      call obj(n,c,qfinsp,cfinsp)
    if(cnoinsp<cfinsp)then
      tcgo=cnoinsp
        qgo=qnoinsp
        ngo=0
        cgo=0
    else
    tcgo=cfinsp
        qgo=qfinsp
        ngo=qfinsp
         cgo=0
    endif
***********search over n and c ******
```

```
    c=0
5   n=c+1
    call obj(n,c,qu,tc)
    tcp=tc
    tclo=tcp
    qp=qu
    np=n
    cp=c
10      n =n+1 c   print*,c,n,qu,qlo,qgo
        call obj(n,c,qu,tc)
    if(qu>=n)then
      if (tc<=tcp) then
        tcp=tc
        qp=qu
        np=n
        cp=c
        goto 10
    else
        tclo=tcp
        qlo=qp
        nlo=np
        clo=cp
     if(tclo<tcgo)  then
        tcgo=tclo
        qgo=qlo
        ngo=nlo
        cgo=clo
        c=c+1
    goto 5
    else
    goto 100
    endif
    endif
    else
        tclo=tcp
        qlo=qp
        nlo=np
        clo=cp
         if(tclo<tcgo)  then
        tcgo=tclo
        qgo=qlo
        ngo=nlo
        cgo=clo
        c=c+1
        goto 5
        else
        goto 100
        endif
        endif
```

```
********* cheaking for agreed producer and customer risk*** 100
if(abs(ngo-qgo)>1) then
        if(ngo>0) then
        pr=BINDF(cgo,ngo,p1)
        cr=BINDF(cgo,ngo,p2)
        else
        pr=0
        cr=0
        endif
        else
        goto 200
        endif
    if(1-pr<=1-gpr)then
    if(cr<=gcr)then
    n1=0
    c1=0
    goto 200
    else
    goto 150
    endif
    else
    goto 150
    endif
****if agreed risk are not satisfied by optimal plan, search for
new n and c following given risk equations******** 150     do 15
n1=1,400
        do 20 c1=0,n1
        rhs1=BINDF(c1,n1,p1)
        rhs2=BINDF(c1,n1,p2)
    if(abs(rhs1-gpr)<=0.04*gpr)then
    if(abs(rhs2-gcr)<=0.04*gcr)then
    goto 180
    endif
    endif
20  continue 15  continue **** solution obtained by solving risk
equations**** 180     call obj(n1,c1,qu,tc)
        print*,'optimal sampling plan doesnot satisfy agreed risk'
        print*,'optimal n=',ngo
        print*,'optimal c=',cgo
        print*,'alpha=',1-pr
        print*,'beeta=',cr
        print*,'quantity=',qgo
        print*,'operating cost=',tcgo
        print*,' as per agreed risk '
        print*,'n=',n1
        print*,'c=',c1
        print*,'order quantity=',qu
        print*,'operating cost=',tc
****to cheak wheather sampling plan with agreed risk is costing
```

```
more than
no inspection and 100% inspection*****
        if(tc>min(cnoinsp,cfinsp)) then
        if(cnoinsp<cfinsp) then
        print*,'its better to go for no inspection'
        print*,'order quantity=',qnoinsp
        print*,'operating cost=',cnoinsp
        else
        print*,'its better to go for 100% inspection'
        print*,'order quantity=',qfinsp
        print*,'operating cost=',cfinsp
        endif
        endif
        goto 300
*** solution  noispection***** 200     if(tcgo==cnoinsp) then
        print*,'no need of ispection'
        print*,'order quantity=',qgo
        print*,'operating cost=',tcgo
        goto 300
        endif
***solution  100% inspection******
        if(tcgo==cfinsp) then
        print*,'100% ispection'
        print*,'order quantity=',qgo
        print*,'operating cost=',tcgo
        goto 300
        endif
*** solution  sampling inspection   ************
        print*,'optimal n=',ngo
        print*,'optimal c=',cgo
        print*,'alpha=',1-pr
        print*,'beeta=',cr
        print*,'quantity=',qgo
        print*,'operating cost=',tcgo
300     print*,'solution for EOQ without replacment'
        end
c   objective function
        subroutine obj(n,c,aeq,ak)
    use msimsl
        integer n,c,x
            real lamsum,sisum,fx,pnx,q,aeq,ak,k,te,fi,x1,y,z
        a=75
        d=50000
        h=5.0
        ca=7
        cd=24
        te=0.8
        fi=2
        pbar=te/(te+fi)
```

```
        lamsum=0.0
        sisum=0.0
    if (n>0) then
        do 5 x=0,c
            pnx=(te+x)/(te+fi+n)
            fx=binom(n,x)*beta(te+x,n+fi-x)/beta(te,fi)
            fsi = (cd*pnx-ca)*fx
            lam=pnx*fx
            sisum=sisum+fsi
            lamsum=lamsum+lam
5           continue
        x1=2*a*d-2*d*n*sisum+h*n**2*lamsum**2
        y=2*d*ca+2*d*sisum-2*h*(1-pbar+lamsum)*n*lamsum
        z=h*(1-pbar+lamsum)**2
        r=1-pbar+lamsum
    if(z*(r**2*x1+n*lamsum*r*y+n**2*lamsum**2*z)>=0.0) then
    q=(n*lamsum*z+sqrt(z*(r**2*x1+n*lamsum*r*y
    &+n**2*lamsum**2*z)))/(r*z)
    k=(x1+y*q+z*q**2)/(2*(q*r-n*lamsum))
        else
        q=0
        endif
        aeq=q
        ak=k
    else
        if (n==c) then
            aeq=sqrt(2*a*d/h)
            ak=sqrt(2*a*d*h)+cd*pbar*d
        else
            aeq=sqrt(2*a*d/h)/(1-pbar)
            ak=sqrt(2*a*d*h)+ca*d/(1-pbar)
        endif
    endif
        end
```

Appendix D

Computer Code for Objective III Replacement case

```
    use msimsl
    common n,c
    integer n,c,cgo,c1,n1
    real p1,p2,cr,pr
*****given prducer and coustomer risk************
    alpha=0.05
    beeta=0.15
    p1=0.01
    p2=0.06
      gpr=1-alpha
    gcr=beeta
*****calculation for 100% inspection and no inspection****
    n=0
    c=0
    call obj(n,c,qnoinsp,cnoinsp)
    n=0
    c=1
      call obj(n,c,qfinsp,cfinsp)
    if(cnoinsp<cfinsp)then
      tcgo=cnoinsp
        qgo=qnoinsp
        ngo=0
        cgo=0
    else
    tcgo=cfinsp
        qgo=qfinsp
        ngo=qfinsp
         cgo=0
    endif
***********search over n and c ******
    c=0
5   n=c+1
```

```
    call obj(n,c,qu,tc)
    tcp=tc
    tclo=tcp
    qp=qu
    np=n
    cp=c
10      n =n+1 c   print*,c,n,qu,qlo,qgo
        call obj(n,c,qu,tc)
    if(qu>=n)then
      if (tc<=tcp) then
        tcp=tc
        qp=qu
        np=n
        cp=c
        goto 10
    else
        tclo=tcp
        qlo=qp
        nlo=np
        clo=cp
     if(tclo<tcgo)  then
        tcgo=tclo
        qgo=qlo
        ngo=nlo
        cgo=clo
        c=c+1
    goto 5
    else
    goto 100
    endif
    endif
    else
        tclo=tcp
        qlo=qp
        nlo=np
        clo=cp
         if(tclo<tcgo)  then
        tcgo=tclo
        qgo=qlo
        ngo=nlo
        cgo=clo
        c=c+1
        goto 5
        else
        goto 100
        endif
        endif
********* cheaking for agreed producer and customer risk*** 100
if(abs(ngo-qgo)>1) then
```

```
        if(ngo>0) then
        pr=BINDF(cgo,ngo,p1)
        cr=BINDF(cgo,ngo,p2)
        else
        pr=0
        cr=0
        endif
        else
        goto 200
        endif
    if(1-pr<=1-gpr)then
    if(cr<=gcr)then
    n1=0
    c1=0
    goto 200
    else
    goto 150
    endif
    else
    goto 150
    endif
****if agreed risk are not satisfied by optimal plan, search for
new n and c following given risk equations******** 150     do 15
n1=1,400
        do 20 c1=0,n1
        rhs1=BINDF(c1,n1,p1)
        rhs2=BINDF(c1,n1,p2)
    if(abs(rhs1-gpr)<=0.04*gpr)then
    if(abs(rhs2-gcr)<=0.04*gcr)then
    goto 180
    endif
    endif
20  continue 15  continue **** solution obtained by solving risk
equations**** 180     call obj(n1,c1,qu,tc)
        print*,'optimal sampling plan doesnot satisfy agreed risk'
        print*,'optimal n=',ngo
        print*,'optimal c=',cgo
        print*,'alpha=',1-pr
        print*,'beeta=',cr
        print*,'quantity=',qgo
        print*,'operating cost=',tcgo
        print*,' as per agreed risk '
        print*,'n=',n1
        print*,'c=',c1
        print*,'order quantity=',qu
        print*,'operating cost=',tc
****to cheak wheather sampling plan with agreed risk is costing
more than
no inspection and 100% inspection*****
```

```
        if(tc>min(cnoinsp,cfinsp)) then
        if(cnoinsp<cfinsp) then
        print*,'its better to go for no inspection'
        print*,'order quantity=',qnoinsp
        print*,'operating cost=',cnoinsp
        else
        print*,'its better to go for 100% inspection'
        print*,'order quantity=',qfinsp
        print*,'operating cost=',cfinsp
        endif
        endif
        goto 300
*** solution  noispection***** 200     if(tcgo==cnoinsp) then
        print*,'no need of ispection'
        print*,'order quantity=',qgo
        print*,'operating cost=',tcgo
        goto 300
        endif
***solution  100% inspection******
        if(tcgo==cfinsp) then
        print*,'100% ispection'
        print*,'order quantity=',qgo
        print*,'operating cost=',tcgo
        goto 300
        endif
*** solution  sampling inspection   ************
        print*,'optimal n=',ngo
        print*,'optimal c=',cgo
        print*,'alpha=',1-pr
        print*,'beeta=',cr
        print*,'quantity=',qgo
        print*,'operating cost=',tcgo
300     print*,'solution for EOQ with replacment'
        end
c   objective function
        subroutine obj(n,c,aeq,ak)
    use msimsl
        integer n,c,x
            real sisum,fx,pnx,q,aeq,ak,k,te,fi,cr
        a=75
        d=50000
        h=5.0
        ca=1.5
        cr=2
        cd=24
        te=0.8
        fi=2.0
        pbar=te/(te+fi)
        sisum=0.0
```

```
    if (n>0) then
        do 5 x=0,c
            pnx=(te+x)/(te+fi+n)
            fx=binom(n,x)*beta(te+x,n+fi-x)/beta(te,fi)
            fsi = ((cd-cr)*pnx-ca)*fx
            sisum=sisum+fsi
5           continue c   print*,sisum
        if((a-n*sisum)>0)then
            q=sqrt(2*d*(a-n*sisum)/h)
c   k=sqrt(2*(a-n*sisum)*d*h)+2*(ca+cr*pbar+sisum)*d
            k=d*(a-n*sisum)/q+(ca+cr*pbar+sisum)*d+h*q/2
            aeq=q
            ak=k
        else
             aeq=0
             ak=10000000
        endif
      else
        if (n==c) then
            aeq=sqrt(2*a*d/h)
            ak=sqrt(2*a*d*h)+cd*pbar*d
        else
            aeq=sqrt(2*a*d/h)
            ak=sqrt(2*a*d*h)+(ca+cr*pbar)*d
        endif
    endif
        end
```

Appendix E

Computer Code for Objective IV Non-replacement case

```
      program ob4withoutreplacement
c      Q-r, non replacement model   new
      use msimsl
      common n,c
      integer n,c,cgo,c1,n1
      real p1,p2,cr,pr
*****given prducer and coustomer risk************
      alpha=0.05
      beeta=0.1
      p1=0.01
      p2=0.06
        gpr=1-alpha
      gcr=beeta
*****calculation for 100% inspection and no inspection****
      n=0
      c=0
      call obj(n,c,qnoinsp,rnoinsp,cnoinsp)
c     print*,qnoinsp
      n=0
      c=1
        call obj(n,c,qfinsp,rfinsp,cfinsp)
c     print*,qfinsp
      if(cnoinsp<cfinsp)then
        tcgo=cnoinsp
          qgo=qnoinsp
          rgo=rnoinsp
          ngo=0
          cgo=0
      else
          tcgo=cfinsp
          qgo=qfinsp
          rgo=rfinsp
```

```
        ngo=qfinsp
        cgo=0
    endif
***********search over n and c ******
    c=0
5   n=c+1
    call obj(n,c,qu,r,tc)
    tcp=tc
    tclo=tcp
    qp=qu
    rp=r
    np=n
    cp=c
10      n =n+1 c   print*,c,n,qu,qlo,qgo
        call obj(n,c,qu,r,tc)
    if(qu>=n)then
      if (tc<=tcp) then
        tcp=tc
        qp=qu
        rp=r
        np=n
        cp=c
        goto 10
    else
        tclo=tcp
        qlo=qp
        rlo=rp
        nlo=np
        clo=cp
     if(tclo<tcgo)  then
        tcgo=tclo
        qgo=qlo
        rgo=rlo
        ngo=nlo
        cgo=clo
        c=c+1
    goto 5
    else
    goto 100

    endif
    endif
    else
        tclo=tcp
        qlo=qp
        rlo=rp
        nlo=np
        clo=cp
         if(tclo<tcgo)  then
```

```
        tcgo=tclo
        qgo=qlo
        rgo=rlo
        ngo=nlo
        cgo=clo
        c=c+1
        goto 5
        else
        goto 100
        endif
        endif
********* cheaking for agreed producer and customer risk*** 100
if(abs(ngo-qgo)>1) then
        if(ngo>0) then
        pr=BINDF(cgo,ngo,p1)
        cr=BINDF(cgo,ngo,p2)
        else
        pr=0
        cr=0
        endif
        else
        goto 200
        endif
    if(1-pr<=1-gpr)then
    if(cr<=gcr)then
    n1=0
    c1=0
    goto 200
    else
    goto 150
    endif
    else
    goto 150
    endif
****if agreed risk are not satisfied by optimal plan, search for
new n and c following given risk equations******** 150
if(ngo==0)  goto   200
        do 15 n1=1,1000
        do 20 c1=0,n1
        rhs1=BINDF(c1,n1,p1)
        rhs2=BINDF(c1,n1,p2)
    if(abs(rhs1-gpr)<=0.04*gpr)then
    if(abs(rhs2-gcr)<=0.04*gcr)then
    goto 180
    endif
    endif
20  continue%
15  continue%
**** solution obtained by solving risk equations****%
```

```
180     call obj(n1,c1,qu,r,tc)
        print*,'optimal sampling plan doesnot satisfy agreed risk'
        print*,'optimal n=',ngo
        print*,'optimal c=',cgo
        print*,'alpha=',1-pr
        print*,'beeta=',cr
        print*,'quantity=',qgo
        print*,'reorder point=',rgo
        print*,'operating cost=',tcgo
        print*,' as per agreed risk '
        print*,'n=',n1
        print*,'c=',c1
        print*,'order quantity=',qu
        print*,'reorder point=',r
        print*,'operating cost=',tc
****to cheak wheather sampling plan with agreed risk is costing
more than no inspection and 100\% inspection*****
        if(tc>min(cnoinsp,cfinsp)) then
        if(cnoinsp<cfinsp) then
        print*,'its better to go for no inspection'
        print*,'order quantity=',qnoinsp
        print*,'reorder point=',rnoinsp
        print*,'operating cost=',cnoinsp
        else
        print*,'its better to go for 100% inspection'
        print*,'order quantity=',qfinsp
        print*,'reorder point=',rfinsp
        print*,'operating cost=',cfinsp
        endif
        endif
        goto 300
*** solution  noispection*****%
 200     if(tcgo==cnoinsp) then
        print*,'no need of ispection'
        print*,'order quantity=',qgo
        print*,'reorder point=',rgo
        print*,'operating cost=',tcgo
        goto 300
        endif
***solution  100\% inspection******
        if(tcgo==cfinsp) then
        print*,'100\% ispection'
        print*,'order quantity=',qgo
        print*,'reorder point=',rgo
        print*,'operating cost=',tcgo
        goto 300
        endif
*** solution  sampling inspection   ************
        print*,'optimal n=',ngo
```

```
        print*,'optimal c=',cgo
        print*,'alpha=',1-pr
        print*,'beeta=',cr
        print*,'quantity=',qgo
        print*,'reorder point=',rgo
        print*,'operating cost=',tcgo
300     print*,'solution for (Q-r) without replacment'
        print*,cfinsp,cnoinsp
        end
c   objective function
        subroutine obj(n,c,eq,er,k)
        integer n,c,x
             real lamsum,sisum,fx,pnx,q,q0,eq,er,k,te,fi,x1,y,z,lam,fsi
         real  v,sbar,mu,pi,h,cd
c       print*,n,c,pbar
        a=75.0
        d=50000
        ca=1.5
        cd=30
        h=5
        pi=20
c       b=50
        mu=5
        b=2*mu
        te=5.0
        fi=5.0
        pbar=te/(te+fi)
        lamsum=0.0
        sisum=0.0
    if (n>0) then
        do 1 x=0,c
            pnx=(te+x)/(te+fi+n)
            fx=binom(n,x)*beta(te+x,n+fi-x)/beta(te,fi)
            fsi = (cd*pnx-ca)*fx
            lam=pnx*fx
            sisum=sisum+fsi
            lamsum=lamsum+lam
1       continue c       print*,lamsum,sisum,lam
        r=0
        sbar=0
        x1=2*d*(a+pi*sbar)-2*d*n*sisum+h*n**2*lamsum**2-2*h*(r-mu)*n*
    &   lamsum
        y=2*d*ca+2*d*sisum-2*h*(1-pbar+lamsum)*n*lamsum+2*h*(r-mu)*
    &   (1-pbar+lamsum)
        z=h*(1-pbar+lamsum)**2
        v=1-pbar+lamsum
        if(z*(v**2*x1+n*lamsum*v*y+n**2*lamsum**2*z)>=0.0) then
            q=(n*lamsum*z+sqrt(z*(v**2*x1+n*lamsum*v*y
    &       +n**2*lamsum**2*z)))/(v*z)
```

```
        else
            q=0
        endif
c   print*,q 2
       q0=q
        r=b-(b*h*(q*(1-pbar+lamsum)-n*lamsum)/(pi*d))
        sbar=r**2/(2*b)-r+b/2
        x1=2*d*(a+pi*sbar)-2*d*n*sisum+h*n**2*lamsum**2-2*h*(r-mu)*n*
    &   lamsum
        y=2*d*ca+2*d*sisum-2*h*(1-pbar+lamsum)*n*lamsum+2*h*(r-mu)*
    &   (1-pbar+lamsum)
        z=h*(1-pbar+lamsum)**2
        v=1-pbar+lamsum
        if(z*(v**2*x1+n*lamsum*v*y+n**2*lamsum**2*z)>=0.0) then
            q=(n*lamsum*z+sqrt(z*(v**2*x1+n*lamsum*v*y
    &       +n**2*lamsum**2*z)))/(v*z)
        else
            q=0
        endif
        if(abs(q0-q)<1) then
            eq=q
            er=r
        else
            goto  2
        endif
c       k=(x1+q*y+q**2*z)/(2*(q*v-n*lamsum))
        k=(2*d*(a+pi*sbar+q*ca+(eq-n)*sisum)+h*(eq*(1-pbar+lamsum)-n*
    &lamsum)**2+2*(er-mu)*(eq*(1-pbar+lamsum)-n*lamsum))/
     &(2*(eq*(1-pbar+lamsum)-n*lamsum))
    else
        if (c==0) then
            r=0
            sbar=0
            if (2*d*(a+pi*sbar)/h<0) then
                q=0
                else
                q=sqrt(2*d*(a+pi*sbar)/h)
            endif
3           q0=q
            r=b-(b*h*(q*(1-pbar+lamsum)-n*lamsum)/(pi*d))
            sbar=r**2/(2*b)-r+b/2
            if (2*d*(a+pi*sbar)/h<0) then
                q=0
                else
                q=sqrt(2*d*(a+pi*sbar)/h)
            endif
            if(abs(q-q0)<=1) then
                eq=q
                er=r
```

```
            else
                goto 3
            endif
            k=(a+pi*sbar)*d/eq+h*(eq/2+er-mu)+cd*d*pbar
        else
            if (2*d*(a+pi*sbar)/h<0) then
                q=0
            else
                q=sqrt(2*d*(a+pi*sbar)/h)/(1-pbar)
            endif
4           q0=q
            r=b-(b*h*(q*(1-pbar+lamsum)-n*lamsum)/(pi*d))
            sbar=r**2/(2*b)-r+b/2
            if (2*d*(a+pi*sbar)/h<0) then
                q=0
            else
                q=sqrt(2*d*(a+pi*sbar)/h)/(1-pbar)
            endif
            if(abs(q-q0)<=1) then
                eq=q
                er=r
            else
                goto 4
            endif
            k=(a+pi*sbar)*d/eq+h*(q/2+er-mu)+ca*d/(1-pbar)
        endif
    endif
    end
```

Appendix F

Computer Code for Objective IV Replacement case

```
      program ob4with replacement
c      Q-r,  replacement model   new
      use msimsl
      common n,c
      integer n,c,cgo,c1,n1
      real p1,p2,cr,pr
*****given prducer and coustomer risk************
      alpha=0.05
      beeta=0.1
      p1=0.01
      p2=0.06
        gpr=1-alpha
      gcr=beeta
*****calculation for 100% inspection and no inspection****
      n=0
      c=0
      call obj(n,c,qnoinsp,rnoinsp,cnoinsp)
c     print*,qnoinsp
      n=0
      c=1
        call obj(n,c,qfinsp,rfinsp,cfinsp)
c     print*,qfinsp
      if(cnoinsp<cfinsp)then
        tcgo=cnoinsp
          qgo=qnoinsp
          rgo=rnoinsp
          ngo=0
          cgo=0
      else
          tcgo=cfinsp
          qgo=qfinsp
          rgo=rfinsp
```

```
        ngo=qfinsp
        cgo=0
    endif
***********search over n and c ******
    c=0
5   n=c+1
    call obj(n,c,qu,r,tc)
    tcp=tc
    tclo=tcp
    qp=qu
    rp=r
    np=n
    cp=c
10      n =n+1 c   print*,c,n,qu,qlo,qgo
        call obj(n,c,qu,r,tc)
    if(qu>=n)then
      if (tc<=tcp) then
        tcp=tc
        qp=qu
        rp=r
        np=n
        cp=c
        goto 10
    else
        tclo=tcp
        qlo=qp
        rlo=rp
        nlo=np
        clo=cp
     if(tclo<tcgo)  then
        tcgo=tclo
        qgo=qlo
        rgo=rlo
        ngo=nlo
        cgo=clo
        c=c+1
    goto 5
    else
    goto 100
    endif
    endif
    else
        tclo=tcp
        qlo=qp
        rlo=rp
        nlo=np
        clo=cp
         if(tclo<tcgo)  then
        tcgo=tclo
```

```
        qgo=qlo
        rgo=rlo
        ngo=nlo
        cgo=clo
        c=c+1
        goto 5
        else
        goto 100
        endif
        endif
********* cheaking for agreed producer and customer risk*** 100
if(abs(ngo-qgo)>1) then
        if(ngo>0) then
        pr=BINDF(cgo,ngo,p1)
        cr=BINDF(cgo,ngo,p2)
        else
        pr=0
        cr=0
        endif
        else
        goto 200
        endif
    if(1-pr<=1-gpr)then
    if(cr<=gcr)then
    n1=0
    c1=0
    goto 200
    else
    goto 150
    endif
    else
    goto 150
    endif
****if agreed risk are not satisfied by optimal plan, search for
new n and c following given risk equations******** 150
if(ngo==0)  goto   200
        do 15 n1=1,1000
        do 20 c1=0,n1
        rhs1=BINDF(c1,n1,p1)
        rhs2=BINDF(c1,n1,p2)
    if(abs(rhs1-gpr)<=0.04*gpr)then
    if(abs(rhs2-gcr)<=0.04*gcr)then
    goto 180
    endif
    endif
20  continue%
15  continue%
**** solution obtained by solving risk equations****%
180     call obj(n1,c1,qu,r,tc)
```

```
        print*,'optimal sampling plan doesnot satisfy agreed risk'
        print*,'optimal n=',ngo
        print*,'optimal c=',cgo
        print*,'alpha=',1-pr
        print*,'beeta=',cr
        print*,'quantity=',qgo
        print*,'reorder point=',rgo
        print*,'operating cost=',tcgo
        print*,' as per agreed risk '
        print*,'n=',n1
        print*,'c=',c1
        print*,'order quantity=',qu
        print*,'reorder point=',r
        print*,'operating cost=',tc
****to cheak wheather sampling plan with agreed risk is costing
more than no inspection and 100\% inspection*****
        if(tc>min(cnoinsp,cfinsp)) then
        if(cnoinsp<cfinsp) then
        print*,'its better to go for no inspection'
        print*,'order quantity=',qnoinsp
        print*,'reorder point=',rnoinsp
        print*,'operating cost=',cnoinsp
        else
        print*,'its better to go for 100% inspection'
        print*,'order quantity=',qfinsp
        print*,'reorder point=',rfinsp
        print*,'operating cost=',cfinsp
        endif
        endif
        goto 300
*** solution  noispection***** %
200     if(tcgo==cnoinsp) then
        print*,'no need of ispection'
        print*,'order quantity=',qgo
        print*,'reorder point=',rgo
        print*,'operating cost=',tcgo
        goto 300
        endif
***solution  100\% inspection******
        if(tcgo==cfinsp) then
        print*,'100% ispection'
        print*,'order quantity=',qgo
        print*,'reorder point=',rgo
        print*,'operating cost=',tcgo
        goto 300
        endif
*** solution  sampling inspection   ************
        print*,'optimal n=',ngo
        print*,'optimal c=',cgo
```

```
        print*,'alpha=',1-pr
        print*,'beeta=',cr
        print*,'quantity=',qgo
        print*,'reorder point=',rgo
        print*,'operating cost=',tcgo
300     print*,'solution for (Q-r) without replacment'
        print*,cfinsp,cnoinsp
        end
c   objective function
        subroutine obj(n,c,eq,er,ak)
        use msimsl
        integer n,c,x
            real sisum,fx,pnx,q,eq,ak,k,te,fi,cr,mu
        a=75.0
        d=50000
        h=5.0
        ca=13
        cr=5
        cd=30
        pi=20
        mu=25
        b=2*mu
        te=5.0
        fi=5.0
        pbar=te/(te+fi)
        sisum=0.0
    if (n>0) then
        do 5 x=0,c
            pnx=(te+x)/(te+fi+n)
            fx=binom(n,x)*beta(te+x,n+fi-x)/beta(te,fi)
            fsi = ((cd-cr)*pnx-ca)*fx
            sisum=sisum+fsi
5           continue c   print*,sisum
        r=0
        sbar=0
        if((a+pi*sbar-n*sisum)>0)then
            q=sqrt(2*d*(a+pi*sbar-n*sisum)/h)
            k=d*(a+pi*sbar-n*sisum)/q+(ca+cr*pbar+h*(r-mu)/d+sisum)*d
    &       +h*q/2
        else
             q=0
             k=10000000
        endif
2       q0=q
        r=b-(b*h*q/(pi*d))
        sbar=r**2/(2*b)-r+b/2
        if((a+pi*sbar-n*sisum)>0)then
            q=sqrt(2*d*(a+pi*sbar-n*sisum)/h)
            k=d*(a+pi*sbar-n*sisum)/q+(ca+cr*pbar+h*(r-mu)/d+sisum)*d
```

```
     &       +h*q/2
        else
             q=0
             k=10000000
        endif
        if(abs(q0-q)<1) then
            eq=q
            er=r
            ak=k
        else
            goto  2
        endif
      else
        r=0
        sbar=0
        if (2*d*(a+pi*sbar)/h<0) then
            q=0
        else
            q=sqrt(2*d*(a+pi*sbar)/h)
        endif
3       q0=q
        r=b-(b*h*q/(pi*d))
        sbar=r**2/(2*b)-r+b/2
        if (2*d*(a+pi*sbar)/h<0) then
            q=0
        else
            q=sqrt(2*d*(a+pi*sbar)/h)
        endif
        if(abs(q-q0)<=1) then
            eq=q
            er=r
        else
            goto 3
        endif
        if (c==0) then
            ak=(a+pi*sbar)*d/eq+h*(eq/2+er-mu)+cd*d*pbar
        else
            ak=(a+pi*sbar)*d/eq+h*(eq/2+er-mu)+ca*d+cr*d*pbar
        endif
    endif
    end
```

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Vitaé

Syed Mubasher Noman was born in Hyderabad, India on July 28, 1975. He received Bachelor's of Engineering in Mechanical with specialization in Production Engineering from Osmania University in 1996. He joined Al-Zamil Glass Industry, which is one of the biggest glass industries in Saudi Arabia as a Production and quality in-charge in 1997. He joined the Department of Systems Engineering, King Fahd University of Petroleum and Minerals (KFUPM), as a Research Assistant in Fall 2000. He received the Master of Science degree in Systems Engineering from KFUPM in 2003.