Extensions in Multicharacteristics Repeat inspection plans

by

Iftikhar Ali Nadeem

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

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In Partial Fulfillment of the
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This thesis, written by

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Date: Feb. 16th, 1993

Dedicated to

My Mother

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All praise be to Allah, the Lord of the worlds. May peace and blessings be upon Mohammad the last of the messengers and his family. I thank Allah for His limitless help and guidance.

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التخصـــــــص : **هندسة النظم والأساليب**

تاريخ منح الدرجة : يناير ١٩٩٣م

خلال عملية الفحص عادة مايرتكب الفاحص نوعين شائعين من الاخطاء خلال حكمه على جودة المنتج . ويعرف هذان النوعان من الخطأ باسم النوع الأول ، و النوع الثاني . وتبحث هذه الأطروحة في تأثير هذين النوعين من الخطأ على خطط الفحص المتكرر للأجزاء الحرجه . وتحسن هذه الرسالة النماذج الموجودة للفحص بتطوير خطط الفحص المتوفرة وذلك بالتركيز على الحالات التي تكون فيها العلاقة بين معدلات الخلل في خواص الأجزاء معتمدة احصائياً على بعضها البعض .

ونظراً للحقيقة المعروفة والتى تنص على ان احتمالات الخطأ تعتمد على جودة المنتج ، فإن هذه الدراسة تقترح خطوات جديدة مطورة لتقدير قيمة النوع الأول والثاني من اخطاء الفحص وذلك من خلال معرفة مسبقة للجودة الأساسية . ومن ثم استخدام هذه الخطوات المطورة للربط بين السلوك الديناميكي لأخطاء الفحص وعملية الفحص المتكرر . وتوضح نتائج الدراسة ان هذه الاخطاء والاعتمادية الإحصائية لها اثر كبير ولذلك لابد من مراعاتها عند تصميم خطط الفحص .

درجة ماجستير في العلوم
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THESIS ABSTRACT

FULL NAME OF STUDENT IFTIKHAR ALI NADEEM

TITLE OF STUDY EXTENSIONS IN MULTICHARACTERISTICS REPEAT INSPECTION PLANS

MAJOR FIELD SYSTEMS ENGINEERING

DATE OF DEGREE JANUARY, 1993

In the process of inspection, an inspector is likely to make Type–I and Type–II errors in his judgement about the product quality. This thesis investigates the effect of Type–I and Type–II errors on multicharacteristic repeat inspection plans for critical components. The practicality of inspection models already developed was enhanced by modifying them for the cases where characteristics' defective rates are statistically dependent. Based on the fact that error probabilities are a function of incoming quality, procedures are suggested for estimating the Type–I and Type–II errors for a given incoming quality. These procedures are then utilized to incorporate the dynamic behavior of inspection errors into the repeat inspection models. Results indicate that the effect of errors and statistical dependency are significant and should be incorporated in the design of the inspection plans.

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CHAPTER 1
INTRODUCTION

1.1 INTRODUCTION

Inspection is defined as the function of comparing or determining the conformance of products to established specifications. Inspection is often used to evaluate the quality of purchased and/or manufactured items. Traditionally, inspection is represented as a simple flow chart or decision tree (Figure 1.1) in which the decision process is represented by a box, incoming units and inspection criteria. Instructions are being represented by incoming arrows, and the action taken is represented by a set of outgoing arrows.

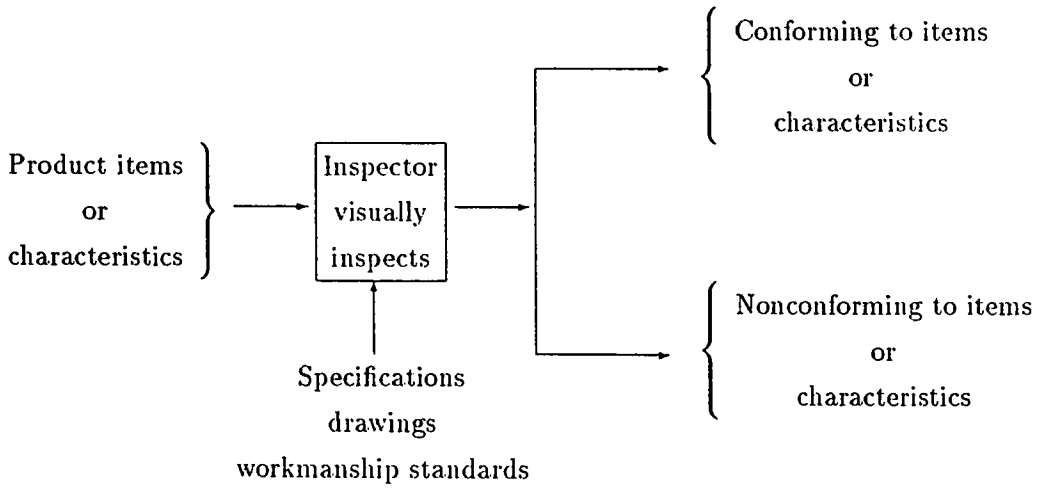


Figure 1.1. Basic Inspection task model (Wallack and Adams, 1969)

Inspection tasks can be classified into three basic categories: tasks involving visual scanning, tasks involving measurements, and tasks involving monitoring of a process.

The accuracy of the inspection process is a necessity to better production quality; however, the inspection accuracy is influenced by a number of factors. These factors can be categorized into three major groups:

1. Inspector-related factors: e.g. age, experience, sex, etc.
2. Task-related factors: e.g. task pacing, task perception, etc.
3. Environmental and organizational factors: e.g. illumination, temperature, motivation, etc.

Industry quality control is accomplished by Statistical Process Control (SPC) or Acceptance Sampling. Statistical Process Control involves the use of control charts, process capability studies, etc.; however, acceptance sampling is done to accept or reject a lot. Since sampling involves only a part of the product characteristics or lot, needs fewer inspectors, and incurs less handling damages. It is more economical; however, sampling has a risk of accepting a bad item or rejecting a good item. Sampling needs added planning and documentation, but provides less information about the product quality.

The emphasis of this thesis work will be on a branch of inspection plans, known as complete repeat inspection plans. Complete inspection plans are becoming increasingly important in the area of quality control due to the growth in automatic manufacturing systems that make complete inspection inexpensive and reliable. Complete inspection plans are also instituted to guard against human inspection errors. These human errors could be of two types: Type–I error (rejecting a good

component instead of accepting it) and Type–II error (accepting a bad component instead of rejecting it). The implications of these inspection errors could be catastrophic in the event of a critical component failure. A critical component has more than one characteristic, and its failure could lead to a system failure causing a catastrophy or a very high cost. In order to protect complex systems from these kinds of failures, one should institute repeat inspections, which reduce inspection errors.

The inspection plan and the models representing this plan in this thesis are developed for critical components which have several characteristics with known incoming quality. A component is classified as non-defective only if all the characteristics meet the quality specifications. The probabilities of Type–I error and Type–II error are assumed to be known. Three different types of costs are considered: (i) cost due to false rejection of a non-defective component, (ii) cost due to false acceptance of a defective component, and, finally, (iii) cost of inspection. The estimates for these costs are assumed to be available in industry.

The inspection plan studied in this thesis is shown in *Figure 1.2*, and it is applied as follows: an inspector inspects one particular characteristic for each component entering the inspection process; then all the accepted components go to the second inspector, who inspects the second characteristic. This chain of inspection continues until all the characteristics are inspected once. This completes one cycle of inspection. All accepted components, if necessary, go to the next cycle of inspection, and the process is repeated a total of n times before the com-

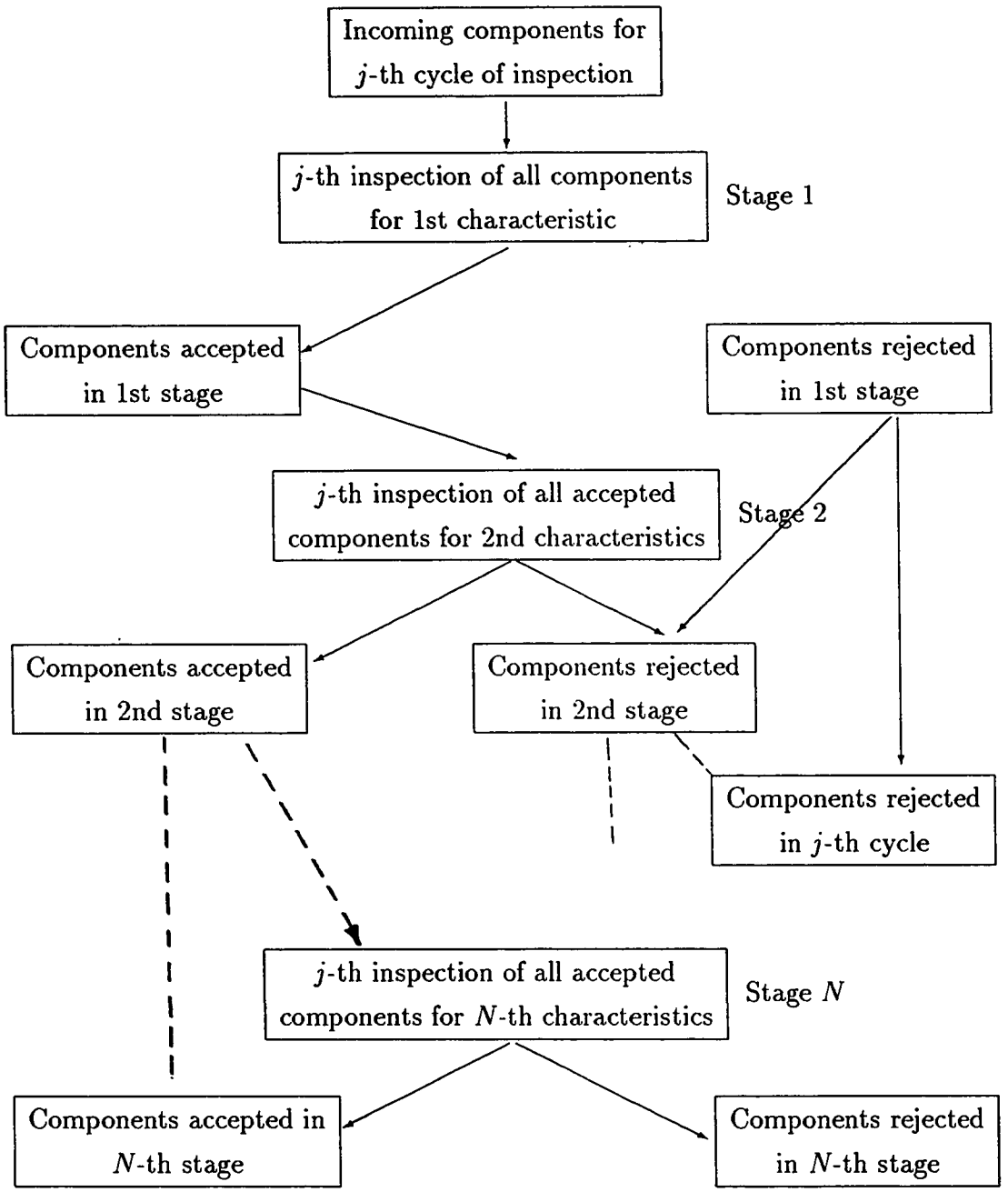


Fig 1.2 Inspection Plan for j-th cycle
$j = 1, 2, \ldots, n$

ponent is finally accepted. Here, n is the optimal number of inspections necessary to minimize the cost per accepted component or to minimize the probability of accepting a defective component. Finally, the accepted components will be those accepted in n-th cycle, and the totality of rejected components will be the sum of those rejected in the 1st, 2nd, ..., n-th cycles. Based on the above plan, several models were developed by Raouf(1983) and Duffuaa and Raouf(1989), to obtain the optimal n.

1.2 OBJECTIVES OF THE THESIS

1. To investigate the effect of inspection errors on performance measures of complete multicharacteristic inspection plans.

2. To extend the models for the multicharacteristic inspection plans for a situation where characteristic's defective rates are statistically dependent.

3. To examine the effect of inspection errors for the case of dependency.

4. To suggest procedures, for estimating inspection errors under varying incoming quality.

5. To extend models for multicharacteristic inspection for those cases where the inspection errors are varying due to the process of inspection.

6. To compare the results of the 1st plan with that of the 5th on real industrial situations.

1.3 PLAN OF THE PROPOSED WORK

The research work proposed for the thesis is based on the model developed by Duffuaa and Raouf, (1989). It will begin with investigating the effect of inspection errors on performance measures of complete inspection plans, such as Average Total Inspection (ATI), Expected Total Cost (ETC) and Average Outgoing Quality (AOQ). This will be accomplished by varying Type–I and Type–II errors and observing the effect on the performance measures.

Then the models will be extended to situations where the characteristic's defective rates are statistically dependent. This necessitates the knowledge of joint probability mass function ($j.p.m.f.$) of the random variables. Using the $j.p.m.f.$ we can obtain the individual marginal probability mass function. Since the joint and marginal mass function varies from cycle to cycle, the values for the individual random variable marginal mass function will be updated using Bayes's theorem. After the inspection of the first characteristic, the marginal of other characteristics must be updated prior to inspecting them. This will be accomplished by updating the joint probability mass function at each stage of the inspection. This would be followed by procedures for estimating inspection errors under varying incoming quality. It will be done based on the fact that inspection errors are a function of incoming quality (Raouf and El–Feituri, 1093). The procedures suggested above for the estimation of inspection errors will then be utilized to model the variation of errors during the process of inspection; then, it will be modified to reflect this

variation. Finally, a comparison of the results obtained with and without varying the inspection errors will be presented.

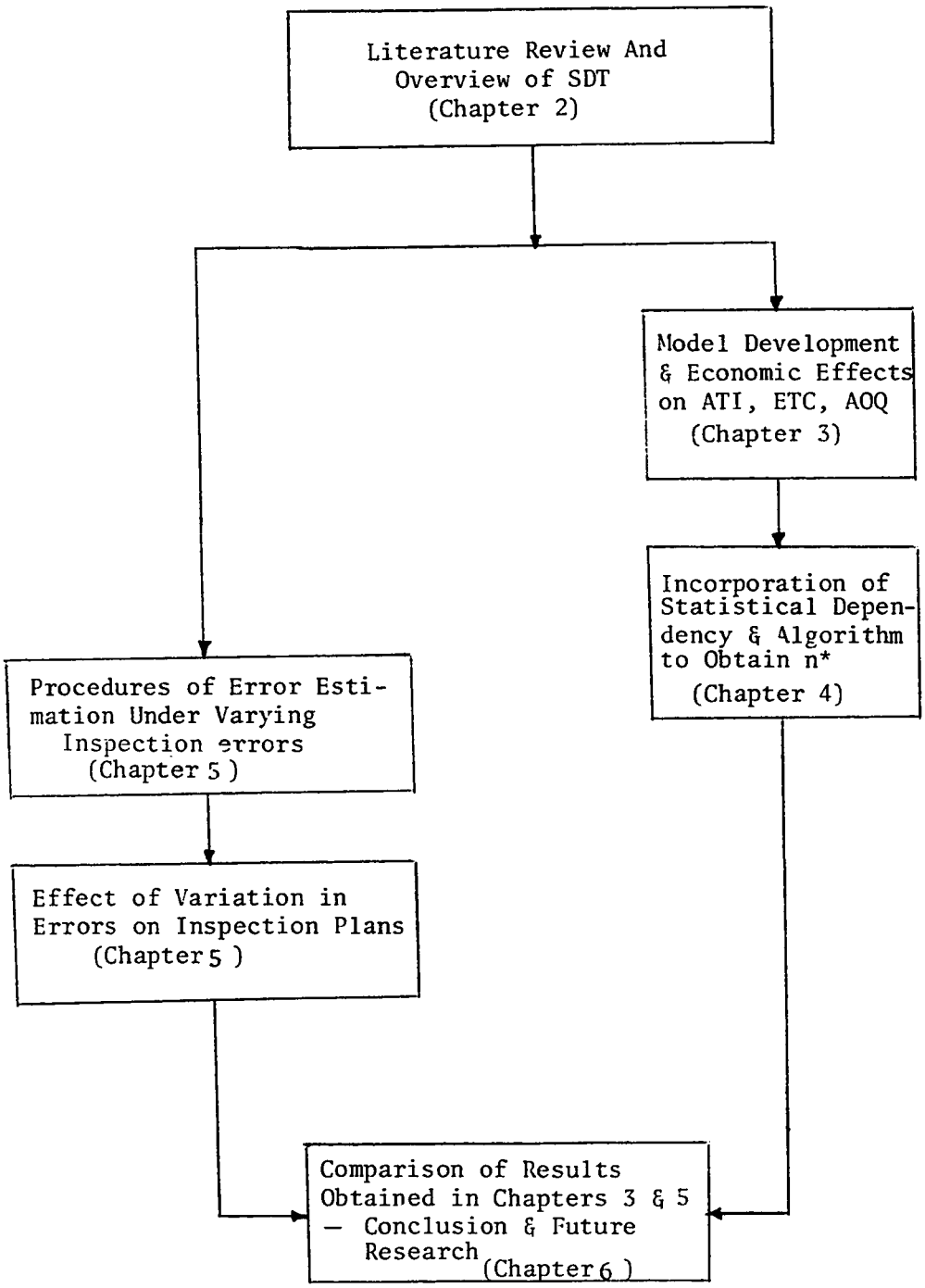
1.4 THESIS ORGANIZATION

This thesis is presented in seven chapters. In chapter 2, some background on inspection error in quality control, extensive review of the literature relevent to the sampling and inspection plans under inspection error, and a brief description of Signal Detection Theory (SDT) are presented. Chapter 3 investigates the effect of inspection errors on the performance measures of complete inspection plans: such as ATI, ETC, AOQ by varying Type-I error and Type–II error.

In chapter 4, the model presented in chapter 3 is modified to situations where the characteristics' defective rate is statistically dependent. An algorithm, to obtain the optimal number of inspections, with an example is presented to demonstrate the results of the model on the performance measures.

Procedures for estimating inspector error under varying incoming quality are proposed in chapter 5. In this chapter, the results of the procedures proposed are compared with those existing in the literature. This chapter also evaluates the effect of variation in inspection errors on which deal with inspection plans. It does this by utilizing the procedures proposed to incorporate the change in the inspection errors during the process of inspection.

FLOW CHART OF THESIS ORGANIZATION



Chapter 5 also compares the models in chapters 3 and 5 to show the difference in performance measures for an inspection plan with and without the error variation. Chapter 6 gives conclusions, and directions for future research.

CHAPTER 2

BACKGROUND AND LITERATURE REVIEW

2.1 INTRODUCTION

In the process of inspection, an inspector makes "accept" or "reject" decisions about the product conformance, based on the predetermined standards of quality characteristics. The human inspector is subject to make Type–I error and Type–II error in his judgement about the product quality. The error of judgement made by human observers have been studied for a long time by human factor specialists. Not until around 1970, did quality control researchers start considering inspection error, and begin attempting to understand and to model their implications on plans of quality control.

Signal Detection Theory (SDT) has been used to model the senstivity of the inspector which could be an industrial inspector. The detection performance of an observer can be evaluated by a graph called a Reciever Operating Characteristic (ROC) curve. This chapter gives background material that would be useful in following chapters, and reviews the relevant literature on acceptance sampling and complete inspection plans which are subject to inspection error.

The chapter has been organized in the following manner. Section 2 presents an overview of inspection error in quality control and literature which is relevent to the

inspection error modeling in sampling, and complete inspection plans are reviewed in section 3. An overview of SDT is presented in section 4, while conclusions are contained in section 5.

2.2 INSPECTION ERROR IN QUALITY CONTROL

The basic assumption in the design of inspection plans in traditional quality control is that the inspection process is carried free of error. This is not always true, and can be misleading. In statistical process control, control charts are used for drawing conclusions about the probability of a process going out of statistical control. The prediction power and usefulness of control charts are directly affected by measurement errors.

Inspection errors have been shown to have adverse effects on the protection provided by sampling plans. The probability of acceptance of a lot is affected by a combination of Type–I and Type–II errors. In industry repeat inspection plans are instituted for the inspection of multicharacteristic critical components to guard against these inspection errors, but not much work has been done to study the effect of inspection errors on complete inspection plans.

2.3 LITERATURE REVIEW

Harris (1968) conducted an experiment on a set of 32 defective items employing

80 naive inspectors to observe the inspection error effect. He concluded that inspection accuracy decreased with reduction in defect rate. Green and Swets (1974) studied the effects of inspection error by using Signal Detection Theory (SDT), and by using a graph called the Receiver Operating Characteristic (ROC) curve. Jaraidi (1983) studied the economic design of sampling plans subject to inspection error. He used SDT to model an inspection task and to evaluate capabilities of an industrial inspector in order to design more cost effective single sampling plans that incorporate inspection error. Raouf and Elfeituri (1983) listed the factors that may affect the inspection accuracy, and they studied the effect on inspection accuracy by varying the incoming quality, the task complexity and the inspection rate. Shor and Raz (1988) indicated the human factors that cause inspection errors. Chandra and Susan (1980) present a method of optimizing total inspection cost assuming that measurement error is a random variable $\sim N(0, \sigma^2)$ while the true value of the product $\sim (\mu, \sigma^2)$. They suggested that to obtain the best estimator of the measurement of interest, an optimum number of replicates should be used. Dharale (1981) claims the distribution of defectives, due to errors in 100% inspected lots, follows negative binomial under the assumption that failure to detect a defective unit by inspectors is described as Poisson distribution. Ayoub et al (1970) presented formulas for Average Outgoing Quality (AOQ) and Average Total Inspection (ATI) for a single sampling plan under inspection error. Collins et al (1972) relaxed the assumption of perfect inspection of replacement for (AOQ), and allowed defective replacement in the formulation of ATI. Bennet et al (1974) investigated the effect of errors on a single sampling plan with known

incoming quality distributions. Case et al (1975) showed the effect of errors on quality control systems using AOQ and ATI as quality cost measures. They also present economic comparison, of with and without errors, under simulated (OC) curve behaviors. Mei et al (1975) assumed that the measurement error distribution (often normal) has a known mean (bias) and a standard deviation (imprecision). Also, the lot distribution and measurement error distribution are independent. A comparison plan could be used to enhance the protection against variable acceptance sampling risk. The author, however, does not propose any cost criteria to quantify the measurement error effect. Dorris and Foote (1978) surveyed the effect of inspection errors on standard quality control methods. The (OC) curve shape changes, as does the protection that an (OC) curve may afford for a sampling plan when inspector errors are included in models. Rosenshine and Soars (1978) presented a comparison of two optimal inspection procedures in the presence of errors. They concluded that the Bayes procedure is better, but these procedures hold under the basic assumption of exponential populations. Tang (1987) investigates the economic and the statistical impacts of inspector errors on complete inspection plans. Tang borrows Taguchi's concept of quadratic loss function to determine the expected quality cost per item after inspection. Maqsoodloo (1987) provided rectified equations for performance measures (P_a, AOQ, ATI) in the presence of inspection error in a multistage sampling plan. He uses the work of Collins et al (1973) as a basis of his study. Shin and Sunil (1992) studied the effect of varying inspection errors on single sampling plans. They assumed that inspection errors are functions of the sequence of the item number in the sample or the lot, and

used AOQ and $ALIC$ (Average Lot Inspection Cost) as performance measures to evaluate the effects of the errors. The conclusion of this study was that the design of sampling plans must be reconsidered to include the effect of inspection errors. Roauf et al (1983) developed the initial model for determining the optimal number of repeat inspections from multicharacteristic components to minimizing the total cost per accepted component due to Type–I error, Type-II error, and cost of inspection. A rule is proposed for minimizing the cost of inspection within each inspection cycle. A proof of this rule was later provided by Duffuaa and Raouf (1989). Duffuaa and Raouf (1987) extended the model for statistically dependent multicharacteristic components. They developed a procedure, under the assumption that probability of a characteristic being defective depends upon the probability of other characteristics being defective, for finding the optimal sequence for inspection in each inspection cycle.

Duffuaa and Raouf (1989) developed three mathematical optimization models for multicharacteristics repeat inspection. The first model (cost minimization model) minimizes the total cost due to Type-I error, Type–II error and repeat inspections to determine the optimal number of repeat inspections. The second model (probability minimizing model) reduces the probability of accepting a defective component. The third model (the satisfying model) determines a satisfying solution by specifying an upper limit for total inspection cost and for the probability of accepting a defective component.

Duffuaa (1992) examined the effect of inspector errors on the expected total cost of a complete repeat inspection plan for critical components. Also, Duffuaa and

Nadeem (1992) developed an extension of the model proposed in Raouf et al, 1983 for components whose characteristics' defective rates are statistically dependent.

In the literature, no significant work is done on the effect of inspector errors on the performance measures of repeat multicharacteristics' inspection plans. Also, the case of characteristics' defective rate has not been treated extensively. In addition, all models, which are presented in the area of multicharacteristic inspection, incorporate the effect of inspector error, but they assume the errors are fixed. It has been documented in the literature that inspector errors are a function of incoming quality (Raouf and Elfeituri, 1983). Therefore, this is a limitation in these models, and they need to be extended to rectify this limitation. In this thesis, we propose models under varying inspector errors.

2.4 SIGNAL DETECTION THEORY

The development of signal detection theory started around 1945. The fundamentals of the theory were formally developed and introduced by Peterson, Birdsall, and Fox (1954). Since then, the theory has found its usefulness in such diverse areas as radar communications, medical diagnoses, the psychophysics of human perception, and industrial quality control.

According to the signal detection model, an observer bases his decision on the information available prior to the presentation of the stimulus, the information content of the stimulus, the sensory analyzer mechanism, and the consequences of each decision. A stimulus may contain noise alone or it may contain a signal super-

imposed on the noise. The observer's response is "yes" or "no", corresponding to his belief that a signal has been present or not. This situation results in a two-by-two layout as shown in Table 2.1

Table 2.1: Two-by-two Layout

		Response	
		Yes	No
Stimulus	Signal-plus-noise	Correct decision (HIT)	Miss
	Noise alone	False alarm	Correct decision

The stimulus presented is mapped into a decision axis, on which the probability distributions for the noise alone, and signal plus noise are defined. An optimal observer chooses a criterion point on this axis as the boundary between "yes" or "no" responses, in order to optimize an objective function.

The detection performance of an observer can be evaluated by a graph called the Receiver Operating Characteristic (ROC) curve. The vertical axis on the ROC curve is "Probability of Hit", $P(HIT)$ and the horizontal axis is the, "Probability of False Alarm", $P(FA)$. The (ROC) curve for an observer can be generated either by a two-choice experiment in which the payoff matrix and prior probabilities are manipulated, or by a rating method in which the observer expresses his degree of confidence in the stimulus which is a result of the noise alone or a signal-plus-noise. Fig. 2.1 shows the chance diagonal on which $P(HIT) = P(FA)$ representing random performance where the decision is independent of the stimulus presented.

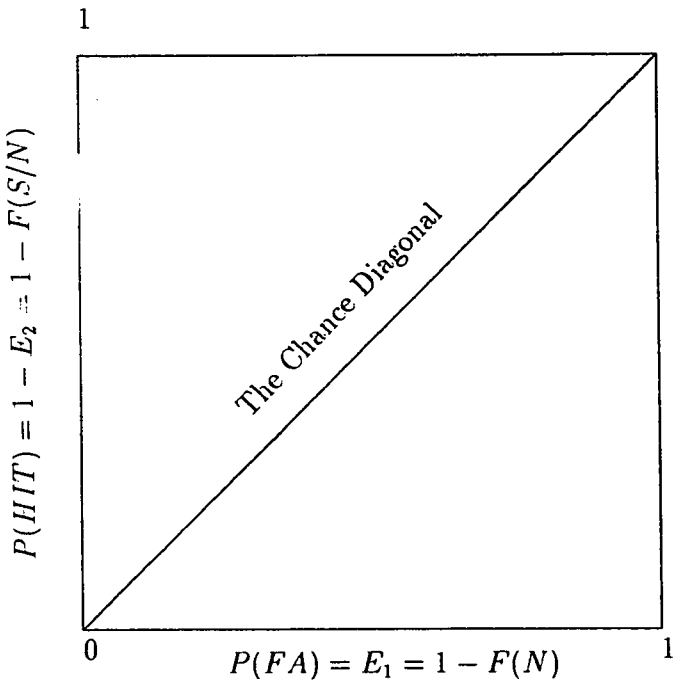


Figure 2.1: The Chance Diagonal

2.5 CONCLUSION

The implications of inspection errors must be considered while designing a plan of quality control. SDT can be used to model the sensitivity of an industrial inspector. In literature, inspection errors have been considered for sampling plans but no significant work has been done on the effect of these errors which deal with complete inspection plans. This is the subject of the next chapter.

CHAPTER 3

THE EFFECT OF INSPECTOR'S ERRORS ON REPEAT INSPECTION PLANS

3.1 INTRODUCTION

The error of judgement made by inspectors has been studied for a long time by human factor specialists. Not until about 1970, did quality control researchers start considering inspection errors and begin attempting to understand and to model their implications (Heikes and Montogomery, 1981). This problem has been approached from several angles in the last several years, and an attempt has been made to incorporate inspection errors into quality control techniques. Dorris and Foote (1978) surveyed the state of the knowledge in this area and found that much work remains to be done in order to adequately understand and deal with this phenomenon.

Existing models represent a fairly wide variety of situations and process behaviors (e.g., Collins and Case (1976); Tenenbien (1970); Heikes (1974). The effect of inspection error on measures like Average Outgoing Quality (AOQ) and Average Total Inspection (ATI) has been studied for sampling plans by several researchers (e.g. Case et al (1975); Beigel (1974)).

Inspection errors have been shown to have adverse effects on the protection provided by sampling plans. In most earlier studies only Type–II error (rejecting a good component) was considered, but the practical significance of Type–I errors was also questioned. The impact of inspection errors (both Type–I and Type–I) is more adverse while dealing with critical components with more than one characteristic. This calls for enforcement of repeat inspection plans in order to guard against any kind of inspection error.

Measuring inspection errors and modeling for them, represents two areas of challenging research, each depending upon one another. The content of this chapter is specified to illustrate the significant effects of the inspector's errors on repeat inspection plans, especially the model described in (Duffuaa and Raouf, 1989). This will be done by studying the effect on the inspection plans performance measures such as AOQ, ATI and ETC. This is accomplished by varying Type–I and Type–II errors for a given situation and observing the behavior of the performance measure.

This chapter, section 2, describes the model, and section 3 contains the basic relationships in the model. A detailed development of the model is provided in section 4. Section 5 presents economic effects of inspection errors, and section 6 proposes a methodology for incorporating the effect of error change in inspection plans. Conclusions are contained in section 7.

3.2 MODEL DESCRIPTION

The model is developed for components containing several characteristics for inspection with known incoming quality p_i. A component is classified as nondefective only if all the characteristics meet the quality specifications. The probabilities of Type–I error, P_{1i} and Type–II error P_{2i} are assumed to be known. Three different types of costs are considered: (1) cost due to false rejection of a nondefective component, $C_r(ii)$ cost due to false acceptance of defective component, C_a, and, finally (iii) cost of inspection, C_i. The estimates for C_r, C_a, and C_i are assumed to be available in the industry.

The inspection plan applied is as follows: an inspector inspects one particular characteristic for each component entering the inspection process. All the accepted components go to the second inspector, who inspects the second characteristic. This chain of inspection continues until all the characteristics are inspected once. This completes one cycle of inspection. All accepted components, if necessary, go to the next cycle of inspection, and the process is repeated a total of n times before the component is finally accepted. Here n is the optimal number of inspections necessary to minimize the total cost per accepted component. Finally, the accepted components will be those which are accepted in the n-th cycle, and the totality of rejected components will be the sum of those rejected in the 1st, 2nd, ..., n-th cycles. In order to model this inspection plan, the following notation is employed.

Notation

M_j	Number of components entering the j-th cycle of inspection
N	Number of characteristics in each component to be inspected
P_i	Probability of i-th characteristic in the sequence of inspection being defective entering the inspection
PG	Probability of a component being nondefective entering the inspection
PGC	Probability of a component being defective entering the inspection, the complement of PG
P_{1i}	Probability of classifying the i-th nondefective characteristic in the sequence of inspection as defective (Type–I error)
P_{2i}	Probability of classifying the i-th defective characteristic in the sequence of inspection as nondefective (Type–II error)
$P_i(j)$	Probability of i-th characteristic in the sequence of inspection being defective entering the j-th cycle
$PG(j)$	Probability of a component being nondefective entering the j-th cycle
$PGC(j)$	Probability of a component being defective entering the j-th cycle, complement of $PG(j)$
$M_{i,j}$	Number of components entering the i-th stage of inspection in the j-th cycle
$PG_{i,j}$	Probability of a component being nondefective in the i-th stage of the j-th cycle

$FR_{i,j}$	Number of falsely rejected components in the i-th stage of the j-th cycle
$FA_{i,j}$	Number of falsely accepted components in the i-th stage of the j-th cycle
$CA_{i,j}$	Number of correctly accepted components in the i-th stage of the j-th cycle
$R_{i,j}$	Rate of rejection of components due to i-th characteristics in the sequence of inspection in the j-th cycle
$A(j)$	Number of accepted components in the j-th cycle
$CFR(j)$	Cost of false rejection in the j-th cycle
$CFA(j)$	Cost of false acceptance in the j-th cycle
$CI(j)$	Cost of inspection in the j-th cycle
$TCFR$	Total cost of false rejection
$TCFA$	Total cost of false acceptance
TCI	Total cost of inspection
TA	Total number of accepted components
$E(tc)\|_j$	Expected total cost per accepted component after j cycles of inspection
$E(\ )$	Expected value of the argument inside the parentheses

3.3 BASIC RELATIONSHIPS IN THE MODEL

The probability of the i-th characteristics being defective will vary from cycle to cycle. The relationship between $P_i(j)$ and P_i is given below.

$$P_i(1) = P_i \tag{3.1}$$

Using Baye's theorem

$$P_i(2) = P_i P_{2i}/[P_i P_{2i} + (1 - P_i)(1 - P_{1i})] \tag{3.2}$$

Similarly

$$P_i(3) = P_i(2)P_{2i}/[P_i(2)P_{2i} + (1 - P_i(2))(1 - P_{1i})] \tag{3.3}$$

and from the symmetry of expressions (2) and (3) we get

$$P_i(j) = P_i(j-1)P_{2i}/[P_i(j-1)P_{2i} + (1 - P_i(j-1)(1 - P_{1i})] \tag{3.4}$$

The probability of a characteristic being defective changes in each cycle; hence the probability of a component being nondefective also changes. It is given below:

$$PG = \prod_{i=1}^{N}(1 - P_i) \tag{3.5}$$

The probability of a component being defective is

$$PGC = 1 - PG \tag{3.6}$$

Clearly,

$$PG(1) = PG = \prod_{i=1}^{N}(1 - P_i(1)) \tag{3.7}$$

The probability of a component being nondefective entering the j-th cycle is

$$PG(j) = \prod_{i=1}^{N}(1 - P_i(j)) \tag{3.8}$$

The probability of a component being defective entering the j-th cycle is

$$PGC(j) = 1 - PG(j) \tag{3.9}$$

When there is no inspection, the expected total cost per accepted component will simply be the cost of false acceptance of all the defective components

$$E(tc)|_{j=0} = C_a(1 - PG) \tag{3.10}$$

The expected total cost per accepted component, after n cycles of inspection, can be written as

$$E(tc)|_{j=n} = [TCFR + TCFA + TCI]/TA \tag{3.11}$$

where $TCFR, TCFA, TCI$, and TA are as defined earlier.

3.4 COST MINIMIZATION MODEL

The objective of this model is to determine the optimal inspection plan for multicharacteristic components. The model minimizes the total cost per accepted component resulting from Type-I errors, Type-II errors, and cost of inspection. Given the basic relationships in the previous section, a mathematical expression for expected total cost per accepted component will be obtained. Our objective is to minimize this cost subject to the relationships governing this situation.

In order to derive the cost of inspection after n cycles of inspections, analysis of cycle 1 of inspection is necessary. All the components entering the cycle 1 go to the first inspector, who inspects the first characteristic in each component in order to classify it as defective or nondefective. This is the first stage of inspection.

Stage 1 in cycle 1: Number of components entering this stage is

$$M_{1,1} = M_1 \tag{3.12}$$

The probability of a component being nondefective is

$$PG_{1,1} = PG \tag{3.13}$$

E (number of falsely rejected components) is

$$\begin{aligned} FR_{1,1} &= M_{1,1}PG_{1,1}P_{11} \\ &= M_1PGP_{11} \end{aligned} \tag{3.14}$$

E (number of falsely accepted components) is

$$\begin{aligned} FA_{1,1} &= M_{1,1}[P_1P_{21} + (1 - PG_{1,1} - P_1)(1 - P_{11})] \\ &= M[P_1P_{21} + (1 - PG - P_1)(1 - P_{11})] \end{aligned} \tag{3.15}$$

E (number of correctly accepted components) is

$$\begin{aligned} CA_{1,1} &= M_{1,1}PG_{1,1}(1 - P_{11}) \\ &= MPG(1\,P_{11}) \end{aligned} \tag{3.16}$$

All accepted components in this stage go to the second inspector who inspects the second characteristic of each component in order to classify it as defective or nondefective.

Stage 2 of the first cycle

$$\begin{aligned} M_{2,1} &= FA_{1,1} + CA_{1,1} \\ &= M_1[P_1P_{21} + (1 - P_1)(1 - P_{11})] \end{aligned} \tag{3.17}$$

$$\begin{aligned} PG_{2,1} &= CA_{1,1}/M_{2,1} \\ &= PG(1 - P_{11})/[P_1P_{21} + (1 - P_1)(1 - P_{11})] \end{aligned} \tag{3.18}$$

$$FR_{2,1} = M_{2,1}PG_{2,1}P_{12}$$

$$= M_1 PG(1 - P_{11})P_{12} \tag{3.19}$$

$$\begin{aligned} FA_{2,1} &= M_{2,1}[P_2 P_{22} + (1 - PG_{2,1} - P_2)(1 - P_{12})] \\ &= M_1[P_2 P_{22} + (1 - PG_{2,1} - P_2)(1 - P_{12})] \end{aligned} \tag{3.20}$$

$$\begin{aligned} CA_{2,1} &= M_{2,1} PG_{2,1}(1 - P_{12}) \\ &= M_1 PG \prod_{i=1}^{2}(1 - P_{1i}) \end{aligned} \tag{3.21}$$

By symmetry, we can obtain stage N of the first cycle

$$M_{N,1} = M_1 \prod_{i=1}^{N-1} [P_i P_{2i} + (1 - P_i)(1 - P_{1i})] \tag{3.22}$$

$$PG_{N,1} = PG \prod_{i=1}^{N-1} [(1 - P_{1i})/(P_i P_{2i} + (1 - P_i)(1 - P_{1i})] \tag{3.23}$$

$$FR_{N,1} = M_1 PG \prod_{i=1}^{N-1} (1 - P_{1i}) P_{1N} \tag{3.24}$$

$$\begin{aligned} FA_{N,1} &= M_1 \prod_{i=1}^{N-1} [P_1 P_{2i} + (1 - P_i)(1 - P_{1i})] \\ &\times [P_N P_{2N} + (1 - PG_{N,1} - P_N)(1 - P_{1N})] \end{aligned} \tag{3.25}$$

$$CA_{N,1} = M_1 PG \prod_{i=1}^{N}(1 - P_{1i}) \tag{3.26}$$

This completes one cycle of inspection, and the result of this cycle is described by the following equations. Number of accepted components after completing the first cycle is,

$$A(1) = FA_{N,1} + CA_{N,1} \tag{3.27}$$

Cost of false rejection is

$$CFR(1) = C_r \sum_{i=1}^{N}(FR_{i,1}) \tag{3.28}$$

Cost of false acceptance is

$$CFA(1) = C_a(FA_{N,1}) \tag{3.29}$$

Cost of inspection is

$$CI(1) = \sum_{i=1}^{N} C_i M_{i,1} \tag{3.30}$$

E (total cost per accepted components after one cycle of inspection is)

$$E(tc)|_{j=1} = [CFR(1) + CFA(1) + CI(1)]/A(1) \tag{3.31}$$

where $CFR(1)$, $CFA(1)$, $CI(1)$, and $A(1)$ are given by equations (3.28), (3.29), (3.30) and (3.27), respectively.

Before proceeding to the second cycle, it was shown in Raouf et al, 1983 that the manner in which characteristics are ordered for inspection affects the cost of inspection; in Duffuaa and Raouf, 1990 a rule is given and proved to be optimal for minimizing the cost of inspection within each inspection cycle. In minimizing the cost, this rule should be applied in each cycle. The rule says, "at inspection cycle j, compute the ratio $C_i/R_{i,j}$ for all i; then, for each component, first inspect the characteristic with the least ratio, and lastly, inspect the one with the highest ratio." This value ensures that $CI(j)$ is minimized within each cycle.

From the analysis of cycle 1, it can easily be seen that after this cycle we can compute the new values of $P_i(2)$, $PG(2)$, M_2 and proceed in the same manner as in the first cycle to compute the cost of false rejection, cost of false acceptance and

cost of inspection in this cycle. Hence, by symmetry, we can obtain the results of the n-th cycle.

$$A(n) = FA_{N,n} + CA_{i,n} \tag{3.32}$$

$$CFR(n) = C_r \sum_{i=1}^{N} (FR_{i,N}) \tag{3.33}$$

$$CFA(n) = C_a(FA_{N,n}) \tag{3.34}$$

$$CI(n) = \sum_{i=1}^{N} C_i M_{i,n} \tag{3.35}$$

The ratio used to determine the optimal ordering of characteristics in the n-th is $C_i/R_{i,n} i = 1, \ldots, N$ where

$$R_{i,n} = P_i(n)(1 - P_{2i}) + (1 - P_i(n))P_{1i} \tag{3.36}$$

After n cycles of inspection we must determine the total cost of inspection per accepted component, which consists of: total cost of false rejection $TCFR$, total cost of false acceptance $TCFA$, and total cost of inspection.

$$TCFR = \sum_{j=1}^{n} [CFR(j)] \tag{3.37}$$

$$TCFA = CFA(n) \tag{3.38}$$

$$TCI = \sum_{j=1}^{n} [CI(j)] \tag{3.39}$$

Total accepted components

$$TA = A(n) \tag{3.40}$$

The above equations (3.1) through (3.38) provide the basic relationship for the model; the purpose is to find the value of n which minimizes the expected total

cost per accepted component. The above model can be stated as

$$\text{Min } E(tc)|_{j=n} \tag{3.41}$$

The following is an algorithm for finding a local optimal n.

Algorithm

Step 1 Determine the PG and $E(tc)|_{j=0}$ from equations (3.5) and (3.10); respectively, set $j = 1$.

Step 2 Compute $P_i(j), PG(j), PG_{N,j}, M_j$ and $C_i/R_{i,j}$ for $i = 1, 2, \ldots, N$ using equations (3.4), (3.8), (3.23), (3.27), (3.36), respectively. Arrange the ratios $C_i/R_{i,j}$ $(i = 1, 2, \ldots, N)$ in order of decreasing magnitude. This is the optimal sequence for the j-th cycle and has been shown in (Duffuaa and Raouf, 1990).

Step 3 Rearrange the probabilities P_i, P_{1i}, P_{2i}, and the inspection cost C_i according to the optimal sequence obtained in step 2.

Step 4 Compute $A(j), CFR(j), CFA(j)$, and $CI(j)$ using equations (3.32), (3.33), (3.34), and (3.35) respectively.

Step 5 Compute $TCFR, TCFA, TCI$, and TA from equations (3.37), (3.38), (3.39), and (3.40) respectively.

Step 6 Compute $E(tc)|_j$, using equation (3.11).

Step 7 If $E(tc)|_j$ is less than $E(tc)|_{j-1}$, set $j = j + 1$ and go to step 2; otherwise STOP $(n = j - 1)$.

3.5 ECONOMIC EFFECTS OF INSPECTION ERRORS FOR THE INDEPENDENT CASE

To examine the effects of Type–I errors and Type–II errors, a batch of 100 components, each with 3 characteristics was inspected with incoming quality of

0.1, 0.2 and 0.3 for each characteristic, respectively. The type–I error and Type–II error were varied from 0.01 to 0.15 to observe the effect on expected total cost per accepted component (ETC) at an optimal number of inspection; however, cost of inspection, cost of false rejection and cost of false acceptance were used as fixed values of \$100, \$500, and \$100,000 per component respectively. The results are shown in the Table 3.1.

3.5.1 Effects of Inspection Errors on Average Total Inspection (ATI)

The Average Total Inspection (ATI) represents the total number of characteristics inspected per inspection. This includes the number of correctly as well as falsely accepted components during the course of inspection. For an inspection plan ATI can be expressed as:

$$ATI = \sum_{j=1}^{n} \left(M_j \sum_{k=1}^{n} \prod_{i=1}^{k-1} \left[{}^{j}P_i P_{2i} + \left(1 - {}^{j}P_i\right)\left(1 - P_{1i}\right)\right)\right)$$

ATI is also the function of the sequence of inspection at each inspection cycle.

The relationship between Type–I error and Average Total Inspection (ATI) is linearly decreasing, while ATI increases as E_2 increases exhibiting an almost linear behavior as shown in figure 3.1 and figure 3.2.

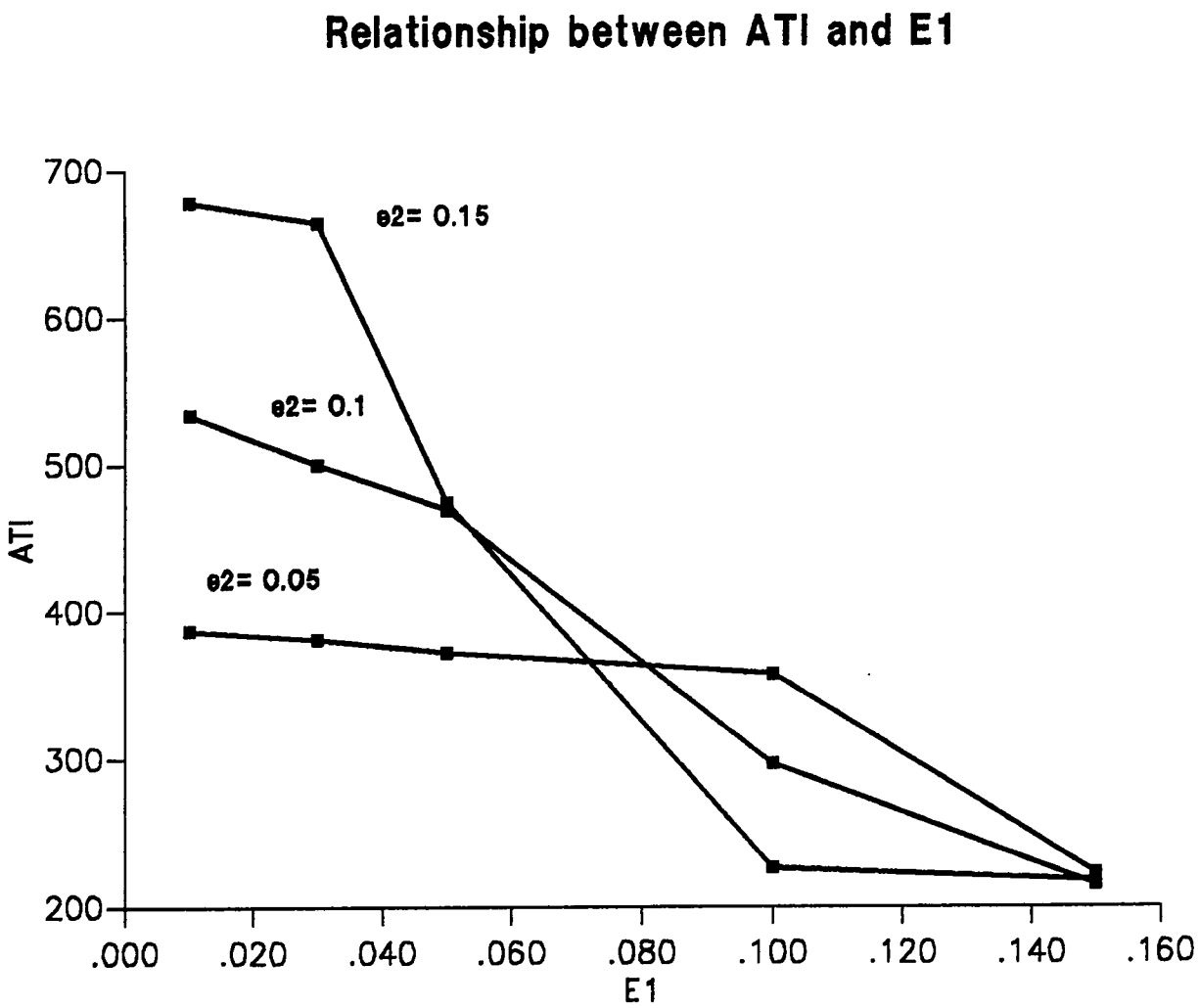

Relationship between ATI and E1
700
600
500
400
300
200
ATI
e2= 0.15
e2= 0.1
e2= 0.05
.000
.020
.040
.060
.080
.100
.120
.140
.160
E1

FIGURE 3.1

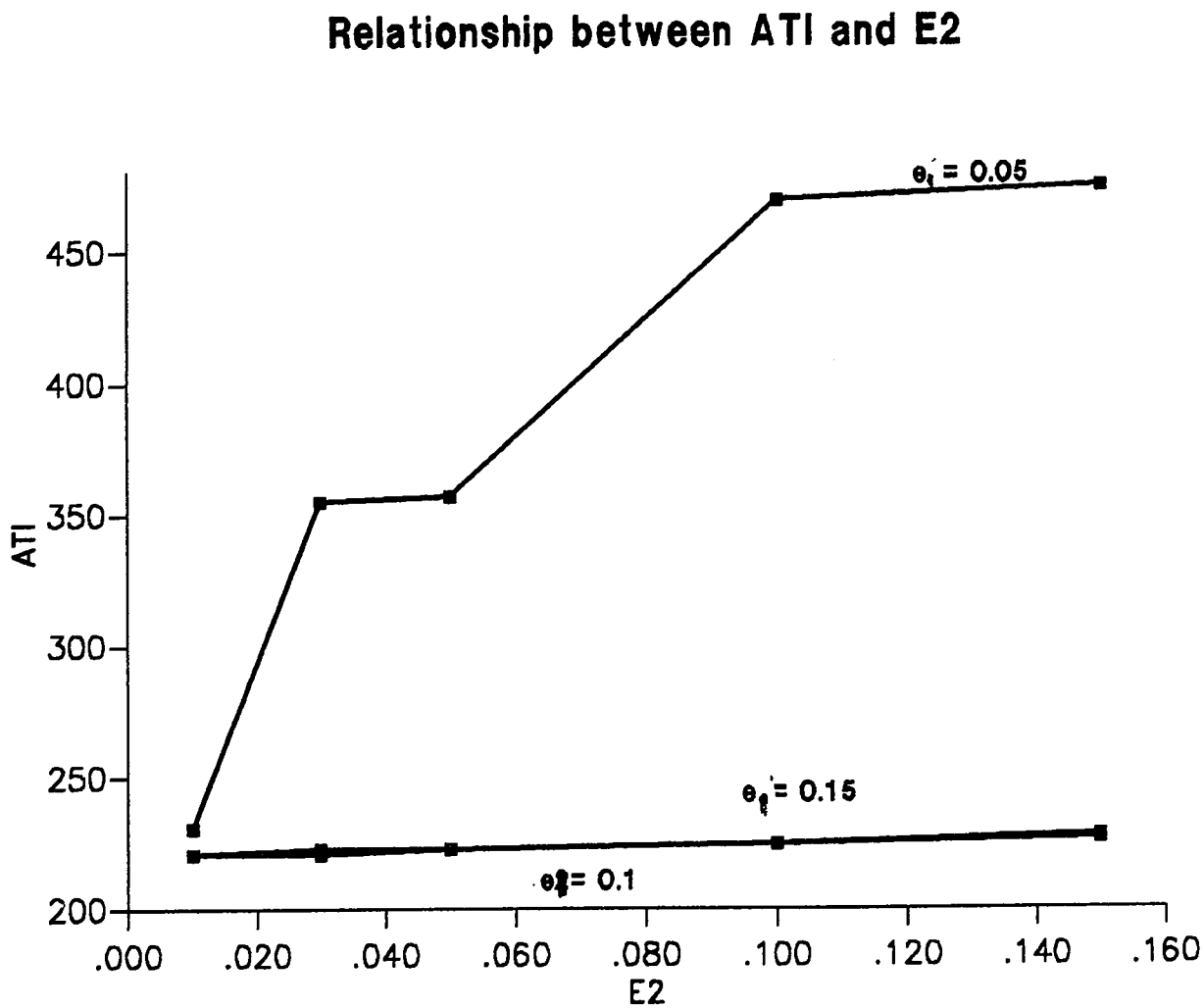

Relationship between ATI and E2
ATI
450
400
350
300
250
200
.000
.020
.040
.060
.080
.100
.120
.140
.160
E2
= 0.05
= 0.15
= 0.1

FIGURE 3.2

3.5.2 Effects of Inspection Errors on Expected Total Cost (ETC)

There is a significant effect of inspection errors on Expected Total Cost (ETC) per inspection. ETC is the sum of inspection cost, cost of false rejection, cost of false acceptance, per accepted component. Figures 3.3 and 3.4 illustrate the ETC as a function of Type–I error and Type–II error respectively. As intuitively expected, Type–I error has more drastic effects on ETC. This can be observed by comparing the slopes of each figure. The slope in case of Type–I error is considerably higher than that of Type–II error.

3.5.3 Effects of Inspection Errors on Average Outgoing Quality (AOQ)

The performance measure 'Average Outgoing Quality' (AOQ) is defined by the ratio:

$$AOQ = \frac{\text{expected number of defective components remaining after inspection}}{\text{total number of components in the lot}}.$$

Figures 3.5 and 3.6 show the effect of Type–I and Type–II errors on AOQ. Evidently, AOQ sharply increases with an increase in Type–II error. However, the effect of Type–I error on AOQ is relatively less significant.

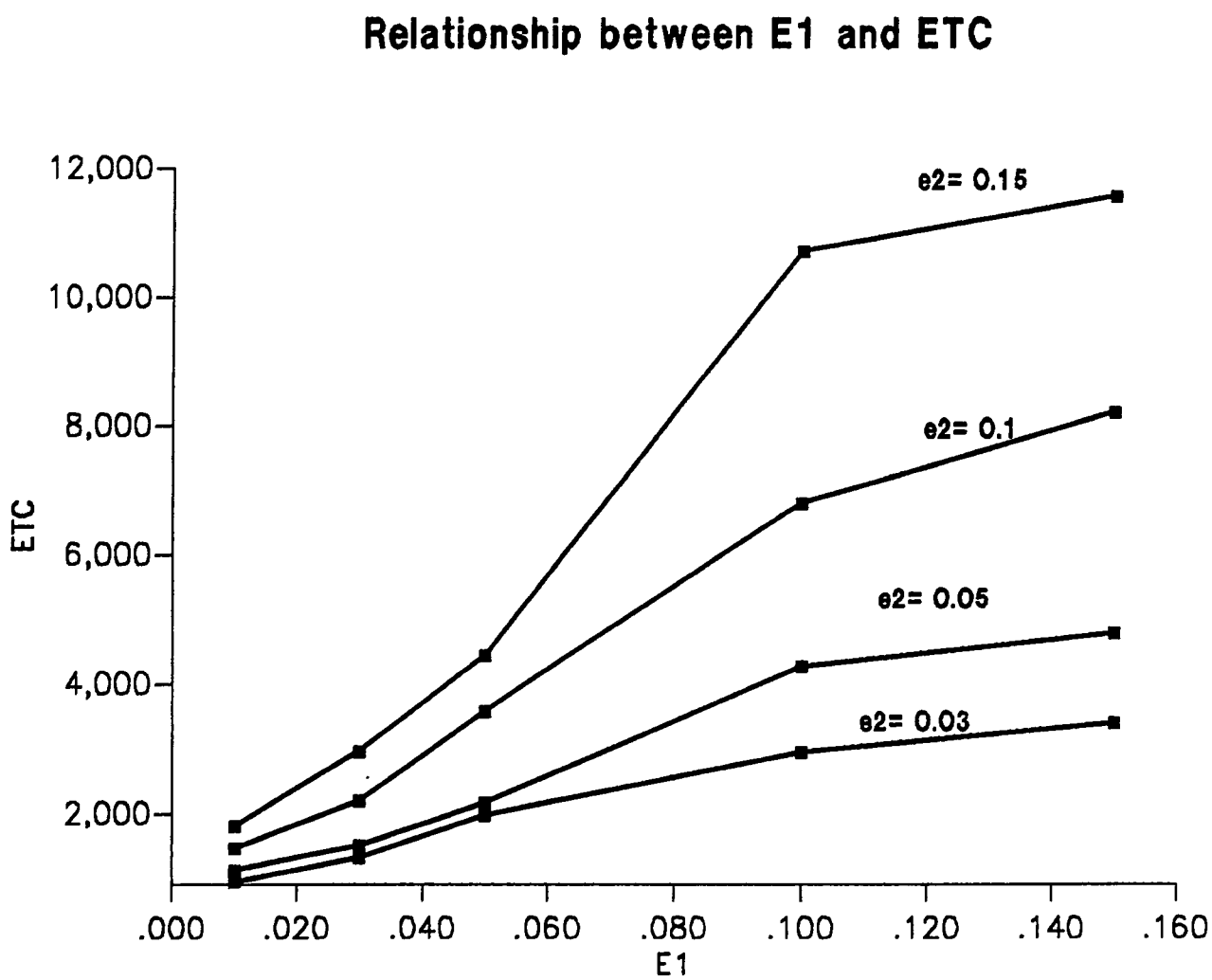

Relationship between E1 and ETC
12,000
10,000
8,000
6,000
4,000
2,000
ETC
.000
.020
.040
.060
.080
.100
.120
.140
.160
E1
e2= 0.15
e2= 0.1
e2= 0.05
e2= 0.03

FIGURE 3.3

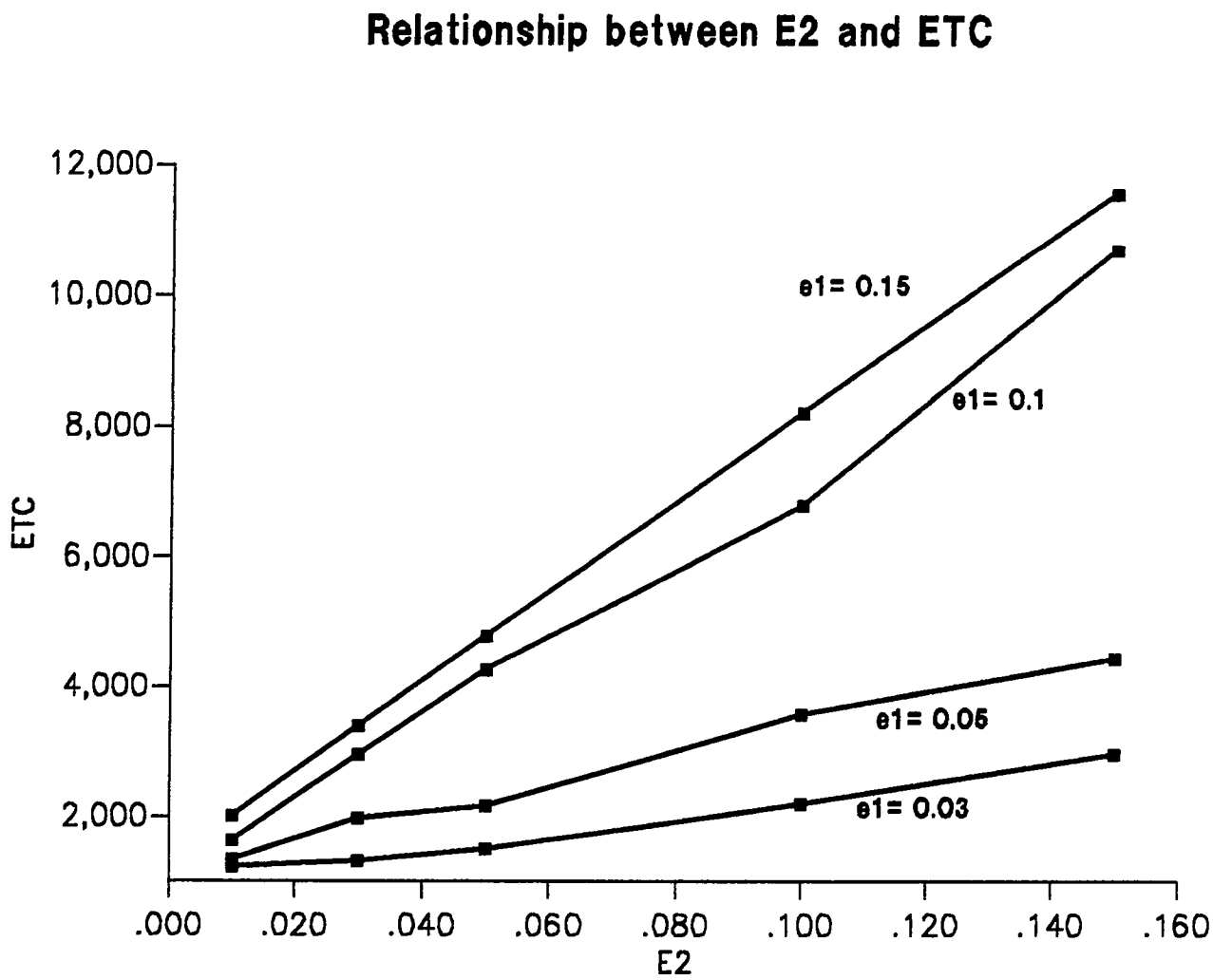

Relationship between E2 and ETC
12,000
10,000
8,000
6,000
4,000
2,000
ETC
e1= 0.15
e1= 0.1
e1= 0.05
e1= 0.03
.000
.020
.040
.060
.080
.100
.120
.140
.160
E2

FIGURE 3.4

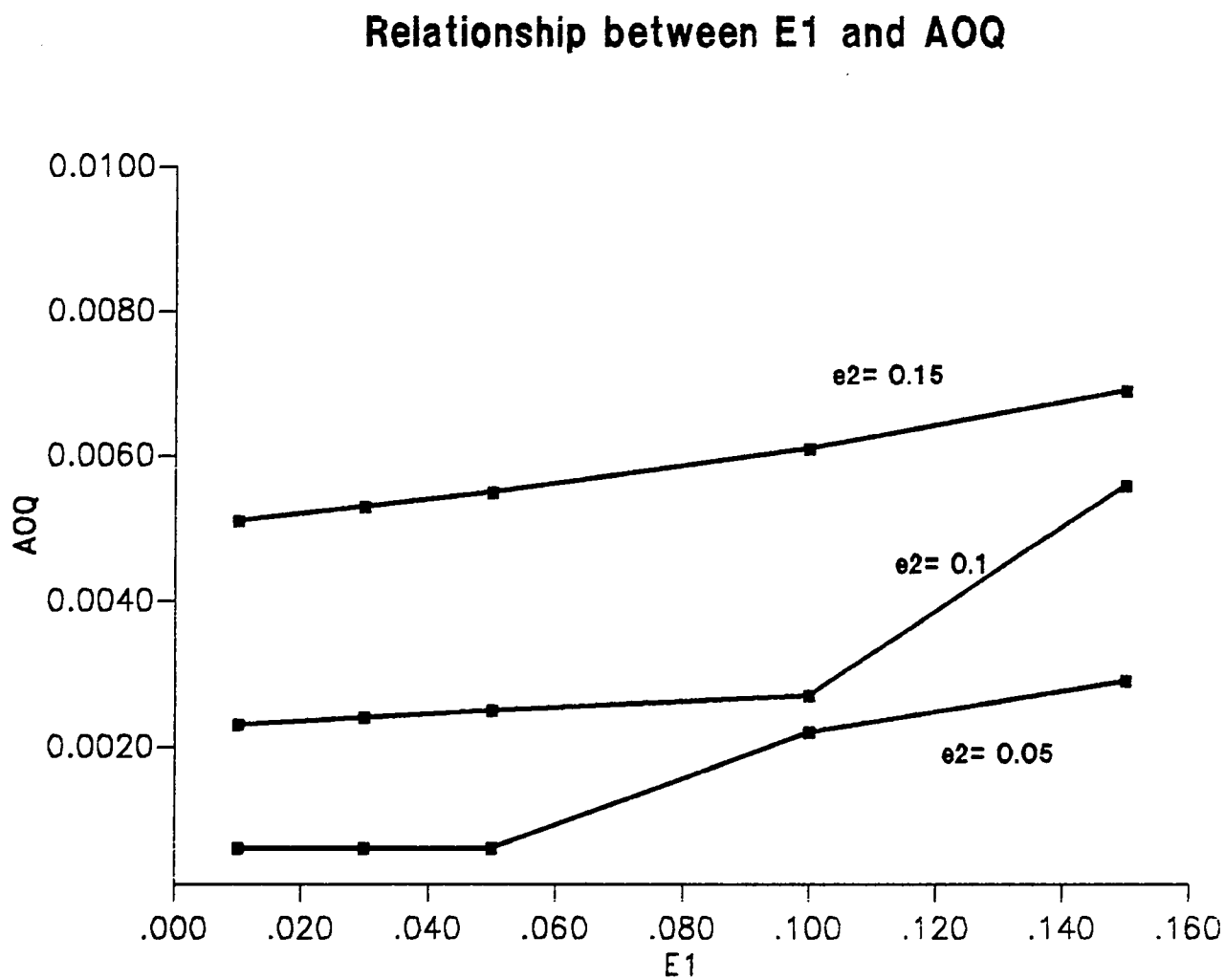

Relationship between E1 and AOQ
0.0100
0.0080
0.0060
0.0040
0.0020
AOQ
e2= 0.15
e2= 0.1
e2= 0.05
.000
.020
.040
.060
.080
.100
.120
.140
.160
E1

FIGURE 3.5

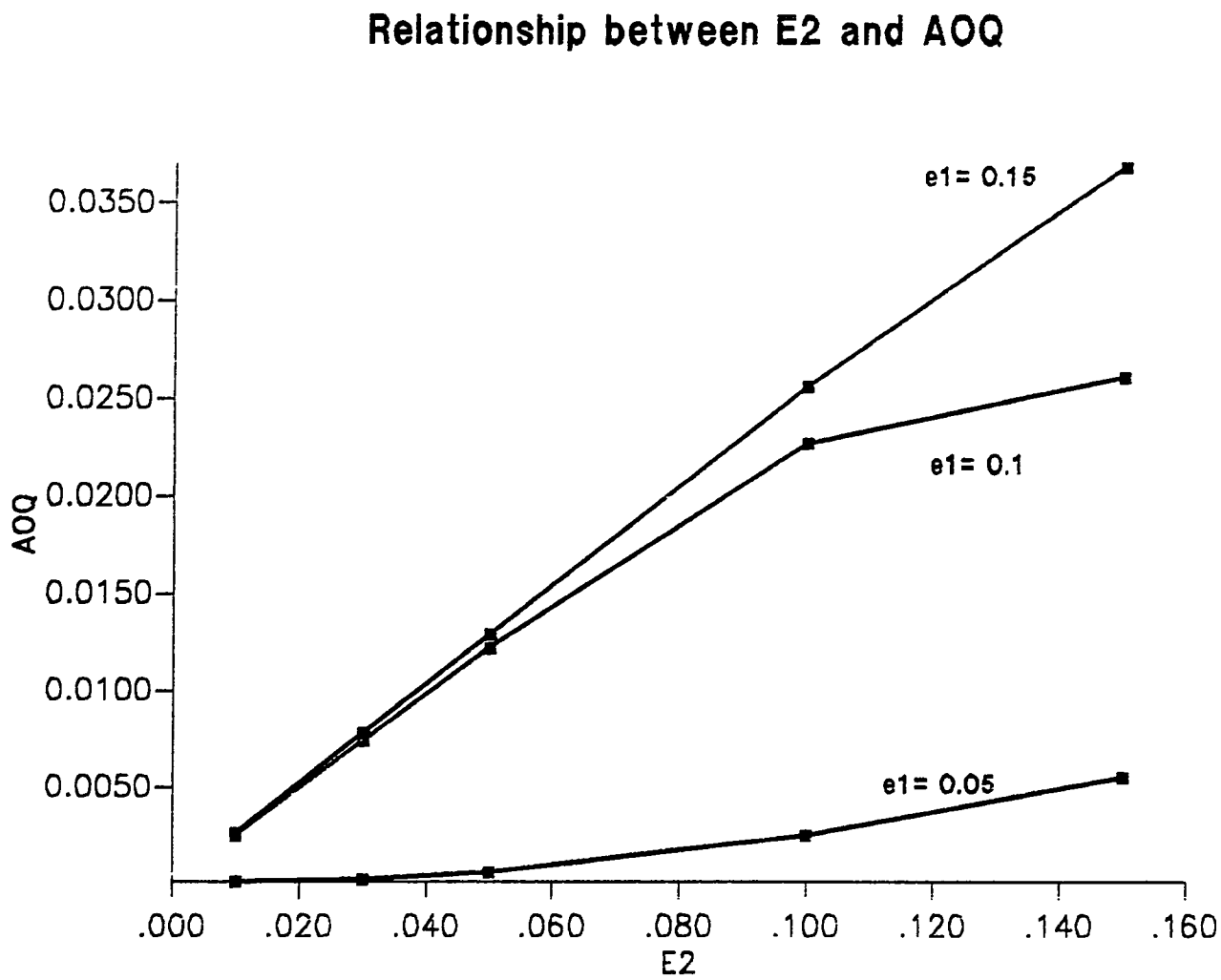

Relationship between E2 and AOQ
AOQ
0.0350
0.0300
0.0250
0.0200
0.0150
0.0100
0.0050
.000
.020
.040
.060
.080
.100
.120
.140
.160
E2
e1= 0.15
e1= 0.1
e1= 0.05

FIGURE 3.6

Table 3.1: Effect Of The Error On The Inspection Plan

	e_1	e_2	1-PG	ETC	CI	CFR	CFA	A(n)	n^*
1	0.00	0.01	0.0000	400.41	400.34	0.02	0.05	81	2
2	0.00	0.03	0.0002	402.18	401.00	0.59	0.59	81	2
3	0.00	0.05	0.0006	407.33	401.77	2.78	2.78	81	2
4	0.00	0.10	0.0022	425.92	403.71	0.00	22.21	81	2
5	0.00	0.15	0.0050	480.71	405.76	0.00	74.95	81	2
6	0.01	0.00	0.0000	441.08	410.00	31.08	0.00	76	2
7	0.01	0.01	0.0000	441.08	410.00	31.07	0.01	77	2
8	0.01	0.03	0.0002	442.68	411.00	31.07	0.61	76	2
9	0.01	0.05	0.0006	445.92	412.00	31.08	2.84	76	2
10	0.01	0.10	0.0023	468.05	414.09	31.07	22.89	76	2
11	0.01	0.15	0.0051	524.52	416.24	31.05	77.23	76	2
12	0.03	0.00	0.0000	532.15	431.89	100.26	0.00	67	2
13	0.03	0.01	0.0000	532.57	432.29	100.26	0.02	67	2
14	0.03	0.03	0.0002	534.00	433.09	100.26	0.64	67	2
15	0.03	0.05	0.0006	537.20	433.93	100.25	3.02	71	2
16	0.03	0.10	0.0024	560.74	436.17	100.24	24.33	67	2
17	0.03	0.15	0.0053	620.80	438.51	100.17	82.13	67	2
18	0.05	0.00	0.0000	635.66	455.48	180.18	0.00	60	2
19	0.05	0.01	0.0000	636.09	455.91	180.18	0.00	60	2
20	0.05	0.03	0.0002	637.67	465.80	180.18	0.69	60	2
21	0.05	0.05	0.0060	641.12	457.72	180.18	3.22	59	3
22	0.05	0.10	0.0025	666.03	460.00	180.14	25.89	59	3
23	0.05	0.15	0.0055	729.06	462.00	180.03	87.03	60	3
24	0.10	0.00	0.0000	837.23	575.15	262.07	0.00	53	2
25	0.10	0.01	0.0025	839.80	575.00	262.07	2.73	53	2
26	0.10	0.03	0.0074	863.60	576.91	262.01	24.68	53	2
27	0.10	0.05	0.0122	908.38	577.95	261.89	68.53	53	2
28	0.10	0.10	0.0027	1001.35	530.18	440.70	30.47	43	3
29	0.10	0.15	0.0061	1076.63	533.45	440.38	102.80	43	3
30	0.15	0.00	0.0000	1131.32	673.48	457.84	0.00	42	2
31	0.15	0.01	0.0026	1134.88	674.00	457.82	3.06	42	2
32	0.15	0.05	0.0078	1160.95	675.57	457.72	27.66	42	2
33	0.15	0.05	0.0129	1211.09	676.77	457.49	76.83	42	2
34	0.15	0.10	0.0256	1142.39	679.12	456.43	306.84	42	2
35	0.15	0.15	0.0069	1569.21	622.48	824.73	122.00	31	3

3.6 METHODOLOGY FOR INCORPORATING ERROR CHANGE IN THE COMPLETE INSPECTION PLAN

The optimal complete inspection plan is developed using the optimization model presented in Section 2. Type–I and Type–II errors are assumed known and fixed. In reality these errors are functions of incoming quality. In Jaraidi, 1983, a model is developed to express Type–I error (P_{1i}) and Type–II error (P_{2i}) as a function of incoming quality.

$$P_{1i} = f_{1i}(P_i) \tag{3.42}$$

$$P_{2i} = f_{2i}(P_i) \tag{3.43}$$

The above function can be incorporated in the model given in Section 2. After the inspection of each characteristic, the error is updated for each characteristic using equations (3.42) and (3.43) before the next inspection. In this fashion the model incorporates the change in Type–I and Type–II errors, and hence an inspection plan results. This will be addressed in more detail in chapter 5.

3.7 CONCLUSION

The chief purpose of this chapter was to illustrate the effects of Type–I error (E_1) and Type–II errors (E_2) on repeat inspection plans. This effect was quantified by observing the change in (ETC), (AOQ) and ATI under the varied values of Type–I error (E_1) and Type–II error (E_2). It was observed that as Type–I error

increases, the (ETC) also increases at a faster rate than in the case of Type-II error where the (ETC) does increase, but at a very slow rate. This indicates that (E_1) leaves more serious effects on ETC than that of (E_2). Similarly, Average Outgoing Quality (AOQ) also increases as (E_2) goes up, but it does not change significantly as (E_1) varies. Nevertheless, the combined effect of increasing E_1 and (E_2) elevates the AOQ values more than (E_2) does it alone.

The average total inspection is illustrated as a function of Type–I error and Type–II error. ATI decreases as Type–I error increases. This is obvious since an increment in probability of rejecting a good component will reduce the number of components to be inspected in the subsequent inspection cycles. However, we observed the opposite effect on ATI as Type–II error was increased. The relationship between ATI and E_2 is more or less linear and directly proportional.

CHAPTER 4

A STATISTICALLY DEPENDENT MULTICHARACTERISTIC INSPECTION PLAN

4.1 Introduction

In many situations components have characteristics which depend on each other. This implies a dependency in defective rates of characteristics. For example, a welded joint has four key quality characteristics: tensile strength, fatigue strength, toughness and fracture toughness. The quality of these four characteristics is dependent on one another.

All the models presented in the area of multicharacteristic inspection plans assume that the characteristic's rate is independent. There is a need to modify these models for the case where the characteristics' defective rate is statistically dependent. This requires knowledge about the joint probability mass function ($u.p.m.f.$) of the random variables representing characteristics' defective rate. Using the $j.p.m.f.$ we can obtain the individual marginal probability mass function. Since the marginal probability mass function varies from cycle to cycle, the values for the individual random variable marginal mass function must be updated using Bayes's theorem. After the inspection of the first characteristic, the marginal of others must be updated prior to inspecting them. Using the above concepts, the models will be modified to handle dependency case.

In this chapter, the models in Duffuaa and Raouf, 1989 are extended, and the rules for updating the quality of dependent characteristics are proposed. These rules are consistent with basic probability rules. An alternative scheme for sequencing characteristic for inspection is suggested. In section 2, the model is described. Section 3 contains the detailed development of the model. Section 4 presents a methodology for obtaining the optimal number of inspections, and an example which demonstrates the results of the new model. Section 5 evaluates the effects of the dependency model on the inspection performance measures: AOQ, ETC and ATI. Conclusion are contained in section 6.

4.2 MODEL DESCRIPTION

The model is developed for components with several characteristics. A component is rejected if one of its characteristics is found to be defective. A component is accepted if all of its characteristics meet the quality specifications. We denote a random variable X_i which takes the value 0 if characteristic i is defective, and 1 if characteristic i is nondefective. The joint probability density function of the multivariate random variable $X = (x_1, x_2, ..., x_N)$ is assumed to be known or to be estimated emperically from the data. The probability of a type I error P_{1i}, and type II error P_{2i} is assumed to be known. Three types of costs are considered: (i) cost due to false rejection of a nondefective component, C_r, (ii) cost due to false acceptance of a defective component, C_a, and (iii) cost of inspection, C_i. The estimates of C_r, C_a and C_i are assumed to be known.

The inspection plan which is applied is as follows: an inspector inspects one particular characteristic for each component that enters the inspection process. Then all the accepted components are passed on to the second inspector who inspects the second characteristic. Again, the accpeted components are passed on to the third inspector who inspects the third characteristic. This pattern of inspection continues untill all characteristics are inspected once. This completes one cycle of inspection. Each cycle consists of N stages of inspections. Components which are accepted in the first cycle, pass to the second cycle of inspection if needed. These cycles of inspection are repeated n times before a component is accepted (n is unknown and we would like to determine n in order to minimize total expected cost per accepted component). Those components which complete n inspection cycles without being rejected are the accepted ones. The rejected components will be the sum of all those rejected in all inspection cycles. Determining the optimal n will provide the optimal inspection plan.

Notations

M_j	Number of components entering the jth cycle of inspection
N	Number of characteristics in each component to be inspected
X_i	A discrete random variable which takes value 0 if characteristic i is defective and 1 if it is nondefective;
$P(x_1, x_2, ...x_N)$	The joint probability mass function of the random variables X_i , i=1,....N at the start of the inspection

$P_i(x_i)$	The marginal probability mass function of the random variable X_i
$P_i(0)$	The probability of the ith characteristic is defective entering the first inspection cycle
${}^{j}P(x_1, x_2, ..., x_N)$	The joint probability mass function of the random variables X_i for a component entering the jth cycle of inspection
${}^{kj}p_i(x_i)$	The marginal probability density function of the random variable X_i for a component in j^{th} cycle entering the kth stage of inspection
$M_{i,j}$	Number of components entering the ith stage of inspection in the jth inspection cycle
$PG_{i,j}$	Probability of a component being nondefective entering the ith stage of the jth cycle
PGC_{ij}	Probability of a component being defective entering the ith stage of the jth cycle
$FR_{i,j}$	Number of falsely rejected components in the ith stage of the jth inspection cycle
$FA_{i,j}$	Number of falsely accepted components in the ith stage of the jth cycle
$CA_{i,j}$	Number of correctly accepted components in the ith stage of the jth cycle
$R_{i,j,k}$	Rate of rejection of components due to the ith characteristic at the k_{th} stage of the j_{th} cycle
A(j)	Number of accepted components in the jth cycle

CFR(j)	Cost of false rejection in the jth cycle
CFA(j)	Cost of false acceptance in the jth cycle
CI(j)	Cost of inspection in the jth cycle
TCFR	Total cost of false rejection
TCFA	Total cost of false acceptance
${}^{kj}P(x_1, x_2, ..., x_N)$	The joint probability mass function for the random variable X_i i=1,...,N for a component in the jth cycle entering the kth stage of inspection
TCI	Total cost of inspection
TA	Total number of accepted components
$E(tc)\|_j$	Expected total cost per accepted component after j cycles of inspection
E( )	Expected value of the argument inside the parenthesis

4.3 MODEL DEVELOPMENT

At the start of inspection, we know the joint probability mass function (j.p.m.f) for the random variable $(X_1, X_2, ..., X_N)$. Using the j.p.m.f, we can obtain the individual marginal probability mass functions:

$$P_i(x_i) = \sum_{x_1}\sum_{x_2}\sum_{x_3}...\sum_{x_{i-1}}\sum_{x_i}...\sum_{x_N} P(x_1, x_2, ..., x_N) \tag{4.1}$$

The joint and marginal mass functions will vary from cycle to cycle

$${}^{1}P_i(0) = P_i(0) \tag{4.2}$$

Using Bayes's theorem

$$
\begin{aligned}
{}^{2}P_i(0) &= P_i(0)P2i/[P_i(0)P_{2i} + (1 - P_i(0))(1 - P_{1i})] && (4.3)\\
{}^{2}P_i(1) &= 1 -^{2} P_i(0) && (4.4)
\end{aligned}
$$

Equations (4.2) and (4.3) provide the updated values for the individual random variable marginal mass function. In general, the marginal probability mass function for the i–th characteristic at the j–th cycle of inspection is :

$$
\begin{aligned}
{}^{j}P_i(0) &= {}^{j-1}P_i(0)P_{2i}/[{}^{j-1}P_i(0)P_{2i} + (1 -^{j-1} P_i)(1 - P1i)] && (4.5)\\
{}^{j}P_i(1) &= 1 -^{j} P_i(0)
\end{aligned}
$$

After the first inspection of characteristic i, $P_i(x_i)$ can be obtained. Due to the statistical dependency between i and the other characteristics, we can not proceed and inspect other characteristics. The marginals of other characteristics must be updated prior to inspecting them. This is accomplished by updating the joint probability mass function in the following manner. After inspecting characteristic i at the first stage of cycle 1 the rule for updating the joint probability mass function is

$$
\begin{aligned}
{}^{1,1}P(x_1, x_2, ...x_n) &= P(x_1, x_2, ..., x_n)\\
{}^{2,1}P(x_1, x_2, ..., x_n) &= {}^{1,1}P(x_1, x_2, ..., x_n)\frac{{}^{2}P_i(x_i)}{{}^{1}P_i(x_i)} && (4.6)
\end{aligned}
$$

I.e. we multiply the old joint probability mass function by the updated marginal mass function for characteristic i (which is just inspected at the first stage in the first cycle) and then we divide by the old marginal density function of the

inspected characteristic. It is important to show that the resulting mass function is a probability mass function which is going to be shown at the end of this section. After obtaining the updated joint mass function, we can obtain the marginal for each characteristic. Then we can proceed to inspect the second characteristic and so on until a cycle of inspection is completed. At the end of the cycle, we can compute the probability that the component is non-defective, which is given by:

$$PG_{N,1} =^{N,1} P(1,1,1,...,1) \tag{4.7}$$

The expected total cost per accepted component after n cycles of inspection is given as

$$E(tc)|_{j=n} = [TCFR + TCFA + TC1]/TA \tag{4.8}$$

If no inspection is performed we incur only the cost of false acceptance

$$E(tc)|_{j=0} = C_a(1 - P(1,1,1,...,1)) \tag{4.9}$$

The objective is to determine the optimal n. Prior to presenting the procedure of how to find the optimal n, we prove the following lemma which shows that the updated function given in equation (4.6) is a probability mass function.

Lemma. *Let us assume we are at the kth stage of the inspection in the jth cycle and at stage k-1 characteristic i was inspected then the following updated function is a probability mass function provided ${}^{k-1,j}P(x_1, x_2, ..., x_N)$ is a probability mass function.*

$${}^{k,j}P(x_1, x_2, ..., x_N) =^{k-1,j} P(x_1, x_2, ..., x_N)\frac{{}^{k,j}P_i(x_i)}{{}^{k-1,j}P(x_i)} \tag{4.10}$$

Proof:

$$^{k-1,j}P(x_1, x_2, ..., x_N), ^{k,j}P(x_i), ^{k-1,j}P(x_i) \geq 0$$

Therefore,

$$^{k,j}P(x_1, x_2, ..., x_N) \geq 0.$$

Also,

$$\begin{aligned}\sum_{x_1}\sum_{x_2}, ..., \sum_{x_N} {}^{k-1,j}P(x_1, ..., x_N)\frac{^{k,j}P(x_i)}{^{k-1,j}P(x_i)} &= \sum_{x_i}\frac{^{k,j}P(x_i)}{^{k-1,j}P(x_i)}\sum_{x_1}\sum_{x_2}\cdots\sum_{x_{i-1}}\sum_{x_{i+1}} {}^{k-1,j}P(x_1, x_2, ..., x_N) \\ &= \sum_{x_i}\frac{^{k-1,j}P(x_i)^{k-1,j}P(x_i)}{^{k-1,j}P(x_i)} \qquad (4.11) \\ &= \sum_{x_i} {}^{k,j}P(x_i) = 1\end{aligned}$$

Therefore the updated function is a probability mass function.

4.3.1 Analysis of the jth cycle

Stage 1

Let M_j be the number of components entering the jth cycle of inspection

$$M_{1,j} = M_j$$

The probability of a component being nondefective is

$$PG_{i,j} = PG_{N,j-1} = ^{N,j-1}P(1, 1, 1, ..., 1) \qquad (4.12)$$

E [number of falsely rejected components)

$$FR_{1,j} = M_{1,j}PG_{1,j}[1 - ^{N,j-1}P_1(0)]P_{11} \qquad (4.13)$$

E [number of falsely accepted components]

$$FA_{1,j} = M_{1,j}[1 - PG_{1,j}]^{N,j-1} P_1(0) P_{21} \tag{4.14}$$

E [number of correctly accepted components]

$$CA_{1,j} = M_{1,j} PG_{1,j}[1 -^{N,j-1} P_1(0)](1 - P_{11}) \tag{4.15}$$

All accepted components in this stage proceed to the second stage to inspect the second characteristic.

$$M_{2,j} = FA_{1,j} + CA_{1,j} \tag{4.16}$$

$$PG_{2,j} = {}^{2,j}P(1,1,1,...,1) \tag{4.17}$$

$$FR_{2,j} = M_{2,j} PG_{2,j}[1 -^{2,j} P_2(0)] P_{12} \tag{4.18}$$

$$FA_{2,j} = M_{2,j}(1 - PG_{2,j})^{2,j} P_2(0) P_{22} \tag{4.19}$$

$$CA_{2,j} = M_{2,j} PG_{2,j}[1 -^{2,j} P_2(0)](1 - P_{12}) \tag{4.20}$$

Similarly for the ith stage of the jth cycle,

$$M_{i,j} = FA_{i-1,j} + CA_{i-1,j} \tag{4.21}$$

$$PG_{i,j} = {}^{i,j}P(1,1,1,...,1) \tag{4.22}$$

$$FR_{i,j} = M_{i,j} PG_{i,j}[1 -^{i,j} P_i(0)] P_{12} \tag{4.23}$$

$$FA_{i,j} = M_{i,j}(1 - PG_{i,j})^{i,j} P_i(0) P_{2i} \tag{4.24}$$

$$CA_{i,j} = M_{i,j} PG_{i,j}[1 -^{i,j} P_i(0)](1 - P_{1i}) \tag{4.25}$$

From the symmetry of the expressions, we can write

$$M_{N,j} = FA_{N-1,j} + CA_{N-1,j} \tag{4.26}$$

$$FR_{N,j} = M_{N,j} PG_{N,j}[1 -^{N,j} P_N(0)]P_{1N} \tag{4.27}$$

$$FA_{N,j} = M_{N,j}(1 - PG_{N,j})^{N,j} P_N(0) P_{2N} \tag{4.28}$$

$$CA_{N,j} = M_{N,j} PG_{N,j}[1 -^{N,j} P_N(0)](1 - P_{1N}) \tag{4.29}$$

The number of components entering the first stage of the $(j+1)$ cycle is

$$M_{j+1} = M_{1,j+1} = CA_{N,j} + FA_{N,j} \tag{4.30}$$

By replacing M_j by M_{j+1} and the updated version of the joint and marginal probability mass functions the equations governing the number of falsely rejected, falsely accepted and correctly accepted for each cycle can be obtained. The following costs are associated with the jth cycle of inspection.

Cost of false rejection for the jth cycle is

$$CFR(j) = C_r \sum_{i=1}^{N} FR_{i,j} \tag{4.31}$$

Cost of false acceptance for the jth cycle

$$CFA(j) = C_a FA_{N,j} \tag{4.32}$$

Cost of inspection for the jth cycle

$$CI(j) = \sum_{i=1}^{N} C_i M_{i,j} \tag{4.33}$$

4.3.2 Minimization of Inspection Cost in Cycle $j, CI(j)$

The sequence in which characteristics are inspected influences the expected cost of inspection. The characteristic with lower cost and higher probability of being defective should be inspected first. Duffuaa and Raouf (1990) proposed an optimal rule for sequencing characteristics for inspection for the case in which the characteristics' defective rates are statistically independent. In general, the rule may not be true for the dependent case. In this paper, we propose the following sequencing rules: At the beginning of cycle j, compute the following ratio:

$$r_{i,j} = C_i / R_{i,j} \tag{4.34}$$

where

$$R_{i,j} =^{N,j-1} P_i(0)(1 - P_{2i}) + (1 -^{N,j-1} P_i(0))P_{1i} \tag{4.35}$$

order the ratios in an increasing order. Employ this order for inspection in this cycle.

Rule 2: at the kth stage of the jth inspection cycle k-1 characteristics have been inspected. For the remaining compute the following ratio:

$$r_{i,j,k} = C_i / R_{i,j,k} \tag{4.36}$$

where

$$R_{i,j} =^{N,j-1} P_i(0)(1 - P2_i) + (1 -^{k-1,j} P_i(0))P_{1i} \tag{4.37}$$

Select the characteristic with the lowest ratio for inspection at the kth stage. At stage 1 use the ratio given in equations (4.31) and (4.32). The general expression

for the total expected cost per expected component after n cycles of inspection is given by equation (4.7). To determine the expected cost, it is required to determine the total cost of false rejection TCFR, the total cost of false acceptance TCFA, and the total cost of inspection and the number of expected components, TA.

$$TCFR = \sum_{j=1}^{n}[CFR(j)] \tag{4.38}$$

$$TCFA = CFA(n) = CFA_{N,n} \tag{4.39}$$

$$TCI = \sum_{j=1}^{n}[CI(j)] = \sum_{j=1}^{n}\sum_{i=1}^{N} C_i M_{i,j} \tag{4.40}$$

$$TA = CA_{N,n} + FA_{N,n} \tag{4.41}$$

Substituting TCFR, TCFA, TCI and TA in equation (4.7) we obtain the total expected cost at the end of the nth cycle. Next an algorithm is presented to find n.

4.4. AN ALGORITHM FOR DETERMINING THE OPTIMAL INSPECTION PLAN

Alogrithm

Step 1 : Compute ${}^{j}P_i(0), M_j$ for $i = 1, 2, ..., N$ from equations (4.5) and (4.30). Select the ith characteristic for inspection based on the ratios in equation (4.36). Repeat this until all characteristics are inspected.

Step 2 : Compute $FR_{i,j}$ and $M_{i,j}$ for each i and $FA_{N,j}$ from equations (4.23), (4.21) and (4.28) respectively.

Step 4 : Compute CFR(j), CFA(j) and CI(j) from equations (4.31), (4.32) and (4.33) respectively.

Step 5 : Compute TCFR, TCFA, TCI and TA using equations (4.38), (4.39), (4.40) and (4.41) respectively.

Step 6 : Compute $E(tc)|_j$ from equation (4.8).

Step 7 : If $E(tc)|_j < E(tc)|_{j-1}$, set $j = j + 1$ and go to 2 otherwise stop, n=j-1

Following is an example to demonstrate the developed model and the proposed algorithm. The same example is solved using the Raouf et al. (1983) model which assumes independence. The results are shown below. The model presented in section 3 depicts the process of repeat inspections for critical components with multicharacteristics having dependent defective rates. The output of the model is an optimal inspection plan. A program was developed (see Appendix–A) implementing the above algorithm. The results of the following example are given below. In the example, we have 100 components, each with three characteristics. The joint probability mass function for the defective rates is P(0,0,0) = 0.05, P(0,0,1) = 0.05, P(0,1,0) = 0.05, P(0,1,1) = 0.1, P(1,0,0) = 0.05, P(1,0,1) = 0.05, P(1,1,0) = 0.15, P(1,1,1) =0.5. Other input values are, C_i=100, C_a=100,000, C_r=1000, P_{1i}=0.01 and P_{2i}=0.015. The results of the model are as follows:

Expected Total Cost (ETC) without inspection = 50,000.00

——Starting the inspection(First Cycle)———

PG(1) = 0.9909216

ETC(1)= 1429.04

A(1) = 48.95

——end of cycle 1(Beginning Cycle 2)————

PG(2) = 0.9998594

ETC(2)= 906.86

A(2) = 47.08

——end of cycle 2(beginning Cycle 3)————

PG(3) = 0.9999937

ETC(3)= 1285.18

A(3) = 45.68

————end of cycle 3 ————————

The optimum number of inspection (n^*) = 2

The results of Raouf et al. model (1983) for the same example assuming independence are as follows: Expected Total Cost before inspection = 58,000.00

——Starting the inspection (First cycle)———

PG(1) = 0.9848253

ETC(1)= 2083.61

A(1) = 41.38

——end of cycle 1 (beginning of cycle 2)————

PG(2) = 0.997678

ETC(2)= 955.07

A(2) = 39.55

——end of cycle 2 (beginning of cycle 3)————

PG(3) = 0.999765

ETC(3)= 719.61

A(3) = 38.37

——end of cycle 3 (beginning of cycle 4)————

PG(4) = 0.999997678

ETC(4)= 749.61

A(4) = 38.15

——end of cycle 4————————————

The optimum number of cycle (n^*) = 3

The model in this chapter suggests a complete inspection with three stages and two cycles. On other hand, the Raouf et al. model (1983) suggests a different plan with three stages and three cycles. The accepted component from the inspection plan suggested by the model has less probability of being defective; therefore, it is more reliable. The results of the model in this paper indicate that the effect of statistical dependency is significant, and must be incorporated in the design of the inspection plan.

4.5 ECONOMIC EFFECTS OF INSPECTION ERRORS FOR THE DEPENDENT CASE

The effects of inspection errors (Type–I error and Type–II error) are examined on a batch of 100 components, each with three characteristics. The joint probability mass function for the defective rates is $P(0,0,0) = 0.5$, $P(0,0,1) = 0.05$, $P(0,1,0) = 0.05$, $P(1,1,0) = 0.13$, $P(1,0,0) = 0.05$, $P(1,0,1) = 0.05$, $P(0,1,1) = 0.15$, $P(1,1,1) = 0.5$. The values of Type–I error and Type–II error were varied from 0.00 to 0.15, while $C_a = 100,000$ component, $C_r = 500$ component and $C_i = 100$ component for each characteristic were fixed. The effects of the inspection errors on Expected Total Cost (ETC) per component and Average Total Inspection (ATI) are observed as shown in Table 4.1. Moreover, the behavior of Average Outgoing Quality (AOQ) under a varied average incoming quality using different pairs of Type–I and Type–II errors is also presented.

4.5.1 Effects of Inspection Errors on Expected Total Cost (ETC)

Expected Total Cost (ETC) is calculated per accepted component, and it includes the cost of false acceptance, cost of false rejection and cost of inspection per accepted component. ETC increases as Type–I error (E_1) increases, but it stabilizes at a lower value of E_2 as shown in Figure 4.1. However, it increases drastically with higher values of E_2 (e.g., $E_2 = 0.15$). Similarly, ETC goes up as Type–II error (E_2) increases. Figure 4.2 shows that the combined effect of E_1 and

Table 4.1 EFFECT OF ERROR ON DEPENDENCY MODEL

	e_1	e_2	PG	ETC	CI	CFR	CFA	$A(n)$	n^*
1	0.00	0.01	0.9999365	784.17	782.26	0.00	1.91	50.002	2
2	0.00	0.03	0.999456	836.215	786.68	–	49.40	50.027	2
3	0.00	0.05	0.998496	1021.73	790.59	0.00	231.13	50.075	2
4	0.00	0.10	0.999395	1295.52	1104.86	0.00	190.67	50.029	3
5	0.00	0.15	0.999689	1560.69	1420.30	0.00	150.38	50.015	4
6	0.01	0.00	0.9999921	521.86	490.43	30.61	0.76	48.52	1
7	0.01	0.01	0.9999355	881.45	813.89	64.52	1.89	47.08	2
8	0.01	0.03	0.9994476	933.51	819.36	65.61	49.46	47.09	2
9	0.01	0.05	0.998464	1119.65	822.54	65.57	231.4	47.15	2
10	0.01	0.10	0.9993772	1459.63	1169.29	97.93	192.52	45.70	3
11	0.01	0.15	0.9996775	1808.225	1528.27	126.71	153.35	44.33	4
12	0.03	0.00	0.99999737	608.455	512.62	95.68	0.193	45.63	1
13	0.03	0.01	0.993819	1223.604	510.57	95.09	618.19	45.91	1
14	0.03	0.03	0.999422	1316.53	888.07	378.88	49.62	41.67	2
15	0.03	0.05	0.9984024	1502.31	893.51	378.88	231.51	41.71	2
16	0.03	0.10	0.9993367	2200.68	1314.43	689.09	196.92	38.04	3
17	0.03	0.15	0.9996482	2970.50	1780.15	1029.5	160.81	34.70	4
18	0.05	0.00	0.99999809	702.83	536.42	166.37	0.165	42.86	1
19	0.05	0.01	0.9936894	1330.42	534.07	165.29	631.06	43.14	1
20	0.05	0.03	0.999398	1979.83	966.25	963.76	49.59	36.78	2
21	0.05	0.05	0.999624	2165.35	972.21	963.76	231.45	36.82	2
22	0.05	0.10	0.9992943	3581.23	1391.08	1894.19	201.23	31.53	3
23	0.05	0.15	0.99763208	4436,61	1504.93	1891.19	1041.41	31.58	3
24	0.10	0.00	0.99999809	975.89	603.95	371.71	0.17	36.45	1
25	0.10	0.00	0.99333918	1636.59	601.31	369.28	666.15	36.89	1
26	0.10	0.03	0.980066	2953.42	595.80	364.32	1993.33	37.19	1
27	0.10	0.05	0.9668577	4363.93	590.49	359.48	3314.99	37.69	1
28	0.10	0.10	0.9925989	6792.16	1229.14	3661.03	1901.98	26.77	2
29	0.10	0.15	0.90187805	10710.35	562.88	335.24	9812.32	40.415	1
30	0.15	0.00	0.999998808	1314.64	686.13	628.13	0.092	30.71	1
31	0.15	0.01	0.9929468	2012.05	682.97	623.96	705.35	30.92	1
32	0.15	0.03	0.9788985	3401.35	676.34	–	2110.61	31.36	1
33	0.15	0.05	0.9649207	4783.68	669.52	606.31	3508.17	31.82	1
34	0.15	0.10	0.9393149	8205.75	652.69	584.53	6968.52	33.006	1
35	0.15	0.15	0.89623642	11575.39	635.93	563.13	10376.70	34.26	1

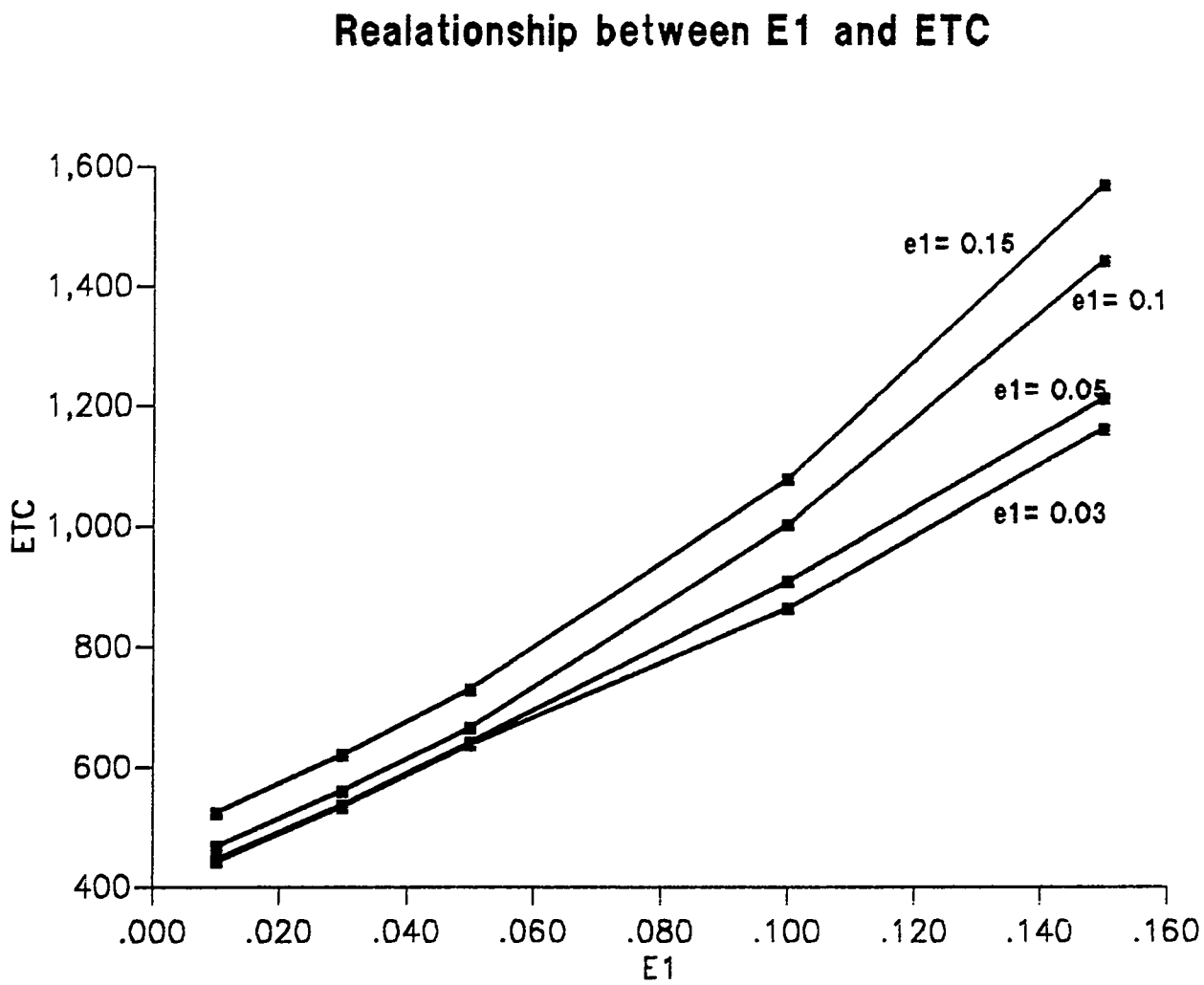

Realationship between E1 and ETC
1,600
1,400
1,200
1,000
800
600
400
ETC
.000
.020
.040
.060
.080
.100
.120
.140
.160
E1
e1= 0.15
e1= 0.1
e1= 0.05
e1= 0.03

FIGURE 4.1

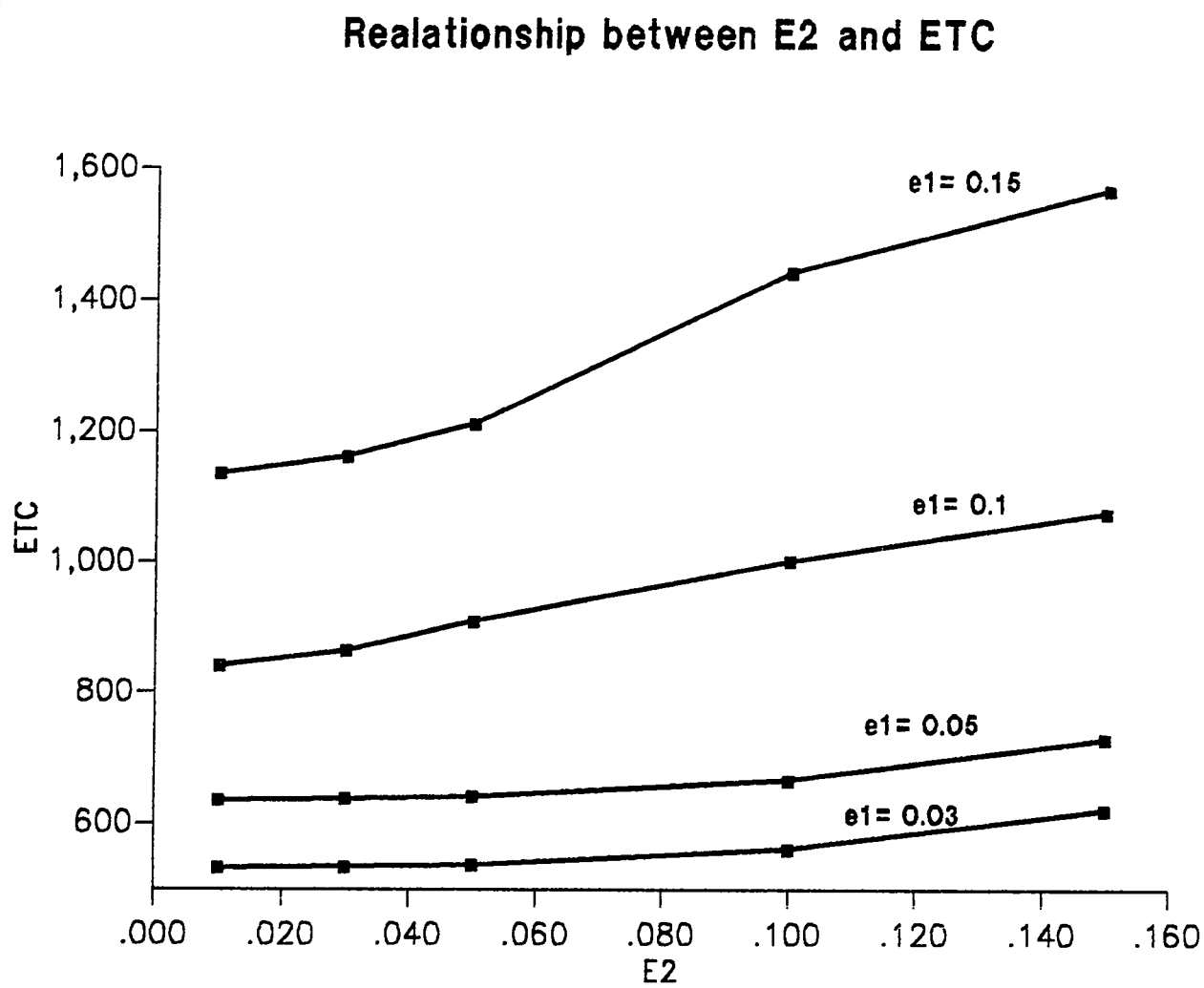

Realationship between E2 and ETC
1,600
1,400
1,200
1,000
800
600
ETC
e1= 0.15
e1= 0.1
e1= 0.05
e1= 0.03
.000
.020
.040
.060
.080
.100
.120
.140
.160
E2

FIGURE 4.2

E_2 is more serious, and it shows that ETC elevates in an almost linear trend. The slope of the expected cost function in case of E_2 is higher than that of E_1.

4.5.2 Effects of Inspection Errors on Average Total Inspection (ATI).

Average Total Inspection (ATI) is the expected total number of inspections performed to reach an optimal inspection plan. The relationship is expressed in Section 3.5.1. ATI is also the function of the sequence of inspection at each inspection cycle. ATI decreases sharply as Type–I error (E_1) increases. This inversely proportional relationship is shown in Figure 4.3. On the other hand, ATI increases as Type–II error increases as exhibited in Figure 4.4. However, ATI is observed to be very stable at a higher value of E_1 (e.., $E_1 = 0.15$, $E_2 = 0.1$) because the probability of rejecting a good component is also high.

4.5.3 The Behavior of Average Outgoing Quality (AOQ) under the Varied Values of Incoming Quality

There is no significant effect on AOQ under the different values of incoming quality. The AOQ was observed almost unchanged when incoming quality was increased from 0.5 to 0.515. Nonetheless, AOQ is very much affected by the inspection errors. The relationship of AOQ and Average Incoming Quality (AIQ) is shown in Figure 4.5 under different pairs of E_1 and E_2. Evidently, E_1 has more adverse effects than E_2 because when E_2 was increased from 0.05 to 0.1 with a fixed value $E_2 = 0.15$, AOQ increased from 0.012 to 0.098. The change in AOQ

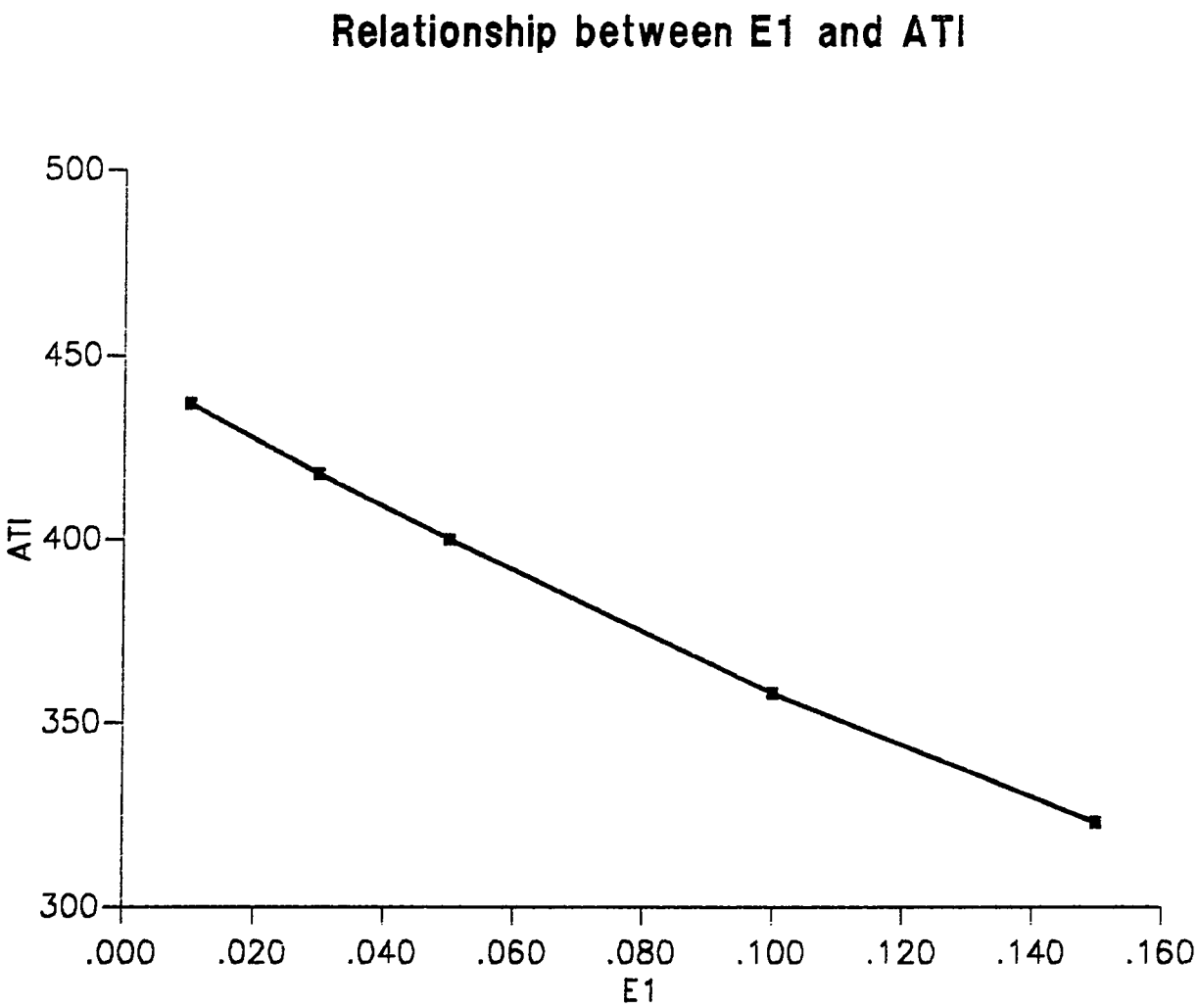

Relationship between E1 and ATI
500
450
400
350
300
ATI
.000
.020
.040
.060
.080
.100
.120
.140
.160
E1

FIGURE 4.3

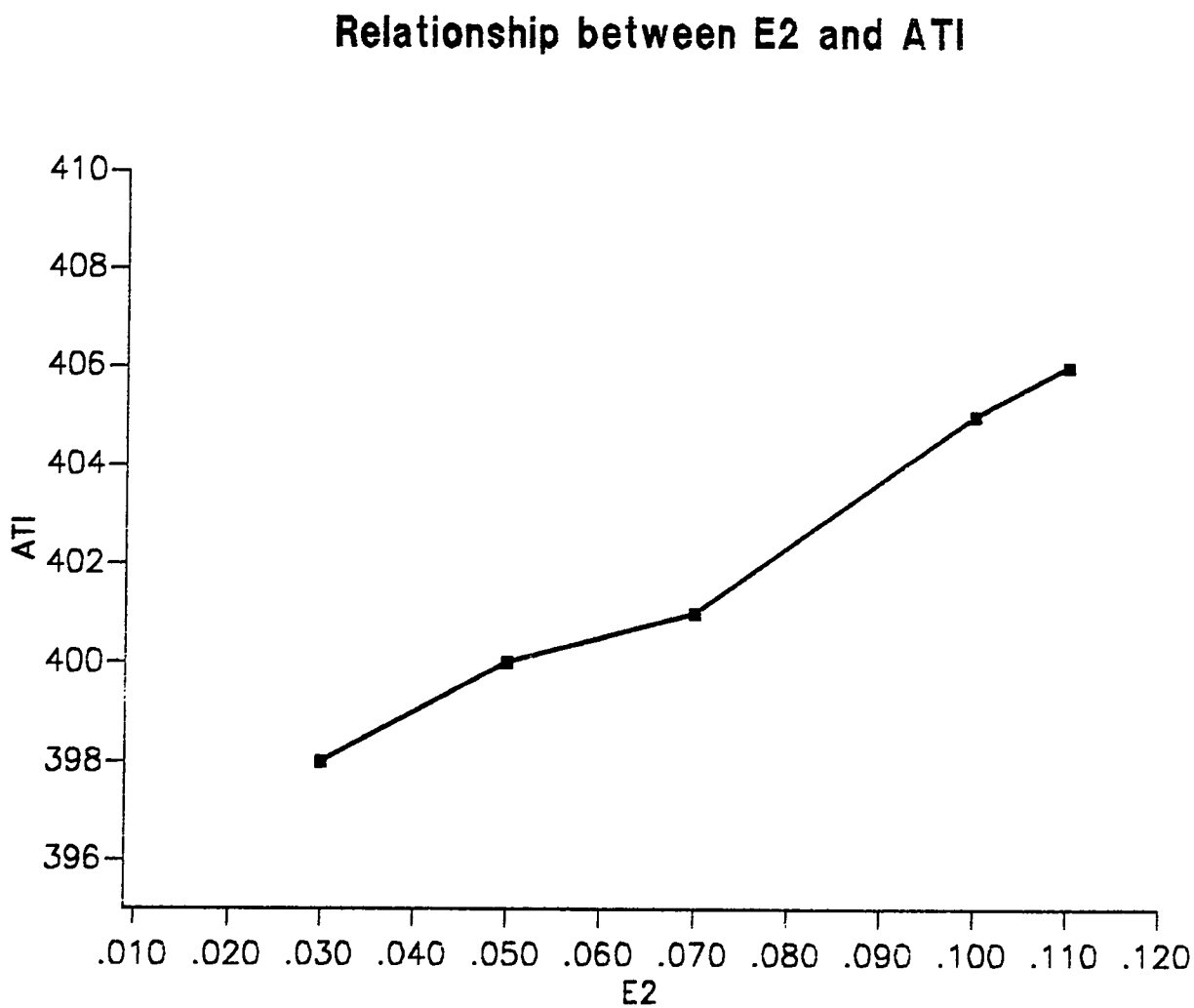

Relationship between E2 and ATI
410
408
406
404
402
400
398
396
ATI
.010 .020 .030 .040 .050 .060 .070 .080 .090 .100 .110 .120
E2

FIGURE 4.4

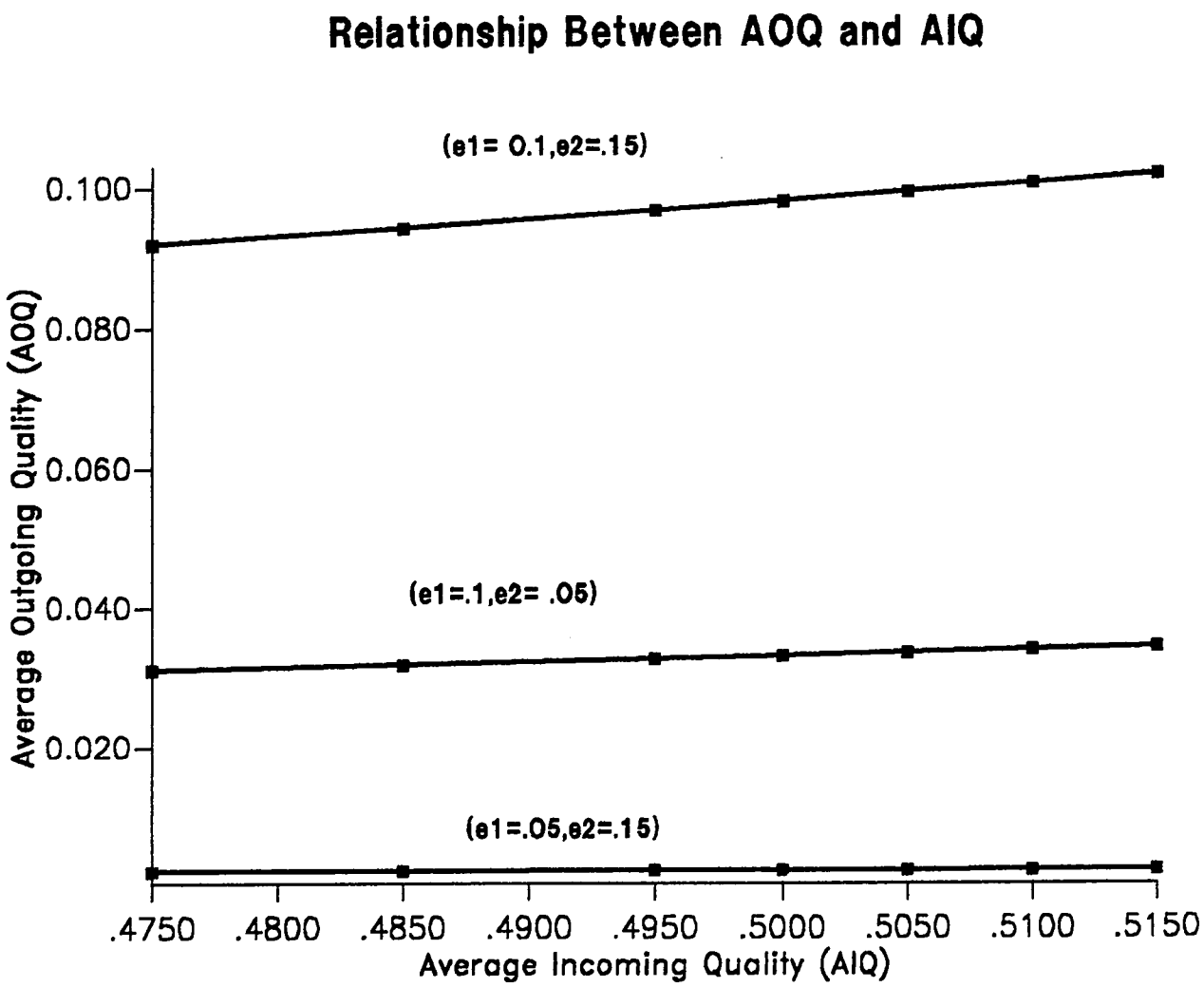

Relationship Between AOQ and AIQ
(e1= 0.1,e2=.15)
(e1=.1,e2= .05)
(e1=.05,e2=.15)
0.100
0.080
0.060
0.040
0.020
Average Outgoing Quality (AOQ)
.4750
.4800
.4850
.4900
.4950
.5000
.5050
.5100
.5150
Average Incoming Quality (AIQ)

FIGURE 4.5

in case of E_2 for a similar value was lesser than that of E_1.

4.6 CONCLUSION

In this chapter, the models developed by Duffuaa and Raouf (1989) have been extended to the case where characteristic defective rate is statistically dependent. The necessary software to run the models is also developed. The effect of dependency is major and must be incorporated in the inspection plan as shown in this chapter.

The application of the model is demonstrated by an example. The result of the model is an optimal inspection plan which can be used for controlling the quality of critical components.

The effect of inspection errors on ETC, ETI and AOQ is investigated. ETC increases as the Type–I error and Type–II error increase, but the slope of the cost function in the case of Type–II error is higher than that of Type–I error. Type–II error leaves very adverse effects on ATI, while it is just opposite in the case of Type–I error. We also found that AOQ is not very much sensitive to incoming quality as it was observed to be stable under different values of incoming quality. The results obtained in chapter 3 and 4 are consistent with the ones in the literature obtained for sampling plans.

CHAPTER 5

REPEAT INSPECTION PLANS UNDER VARYING INSPECTION ERRORS

5.1 INTRODUCTION

In the process of inspection, a characteristic's defective rate decreases each time it is inspected. This is discussed in detail in chapter 3. The inspection errors (Type–I error and Type–II error) are largely dependent on incoming quality (Raouf and Elfeturi, 1983). In general, it has been found that an increase in incoming quality results in a decreased Type–II error probability and increased Type–I error rate (Harris, 1968). In complete inspection plans, the defective rate of a characteristic is updated each time it is inspected. Since the inspection error probabilities are also a function of characteristics' defective rate, therefore, error probabilities are also needed to be updated at each stage of the inspection plan. It has been shown in the literature that Type–I error increases as a characteristic's defective rate is increased. On the other hand, Type–II error decreases with an increase in fraction defectives in a given lot. In order to update the error change at each stage of an inspection plan, there is a need to estimate the error probability for modeling of varying inspection errors.

Signal Detection Theory (SDT) has been found very useful in modeling the performance of an industrial inspector (Wallack and Adams, 1969). (In chapter 2, an overview of SDT is given). In the context of complete inspection plans, the

incoming quality is equivalent to the probability of occurrence of a signal. Jaraiedi (1983) has offered two procedures of inspection error estimation using SDT and ROC curves. ROC curve is a means to evaluate and compare the performance of inspectors for any given value of costs and value associated with the accept–reject type of decisions made by inspectors. The errors of judgement made by inspectors have a direct impact on the overall cost of quality control systems. The ROC curve can be used to quantify the inspection error probabilities and the cost of committing these errors. With a given fraction defective (P), an estimate of Type–I (E_1) and Type–II (E_2) errors can be obtained from the ROC curve. This leads to the development of a functional relationship between fraction defectives, Type–I error (E_1), and Type–II error (E_2).

The functional relationships developed between P and E_1, and P and E_2 can be used to incorporate the effect of varying inspection errors into the repeat inspection models. The significance of this effect can be highlighted by comparing the results of the models with and without varying the inspection errors.

In this chapter, we are proposing procedures to estimate the Type–I error and Type–II error as a function of incoming quality of a characteristic entering an inspection cycle of a multicharacteristic inspection plan. Section 2 of the chapter describes the work done by Jaraiedi (1983) on error probabilities estimation. Section 3 gives the proposed procedure for ROC and errors estimation while section 4 highlights the importance of error variation in inspection plans. Section 5

evaluates the effect of inspection error on repeat inspection plans under varying inspection errors. Section 6 provides a comparison between the results of the models proposed in chapter 3, and the results of those presented in the models under varying inspection errors. Section 7 concludes this chapter.

5.2 RELATIONSHIP BETWEEN 'P' AND ERROR PROBABILITIES AS GIVEN BY JARAIEDI (1983)

Jaraiedi used Receiver's Operating Characteristic (ROC) curve to develop a functional relationship between defective rate (P) and probability of Type–I error and Type–II error.

The ROC analysis of the detection task has, where the observation interval is constant, revealed that a change in P results in a significant change in the level of detection of defects (Fox and Haslegrave, 1969), and causes the observation to shift along the ROC curve due to criterion adjustments made by inspectors. In such a case, one can assume that the length of the ROC curve travelled due to change in P which is proportional to the amount of the change. An increase in P causes a shift towards the upper corner of the ROC space, while a decrease in P has the opposite effect. Based on this assumption, a method for estimation of two types of error probabilities is developed.

Given two unknown defect rates and their corresponding points on the ROC curve, we see that the problem is how to find the point on the curve which cor-

responds to a value of P between the two known points. To estimate the Type–I and Type–II error probabilities at a certain incoming fraction defective P which is between the two known points p_1 and p_2, the length of the curve between x_1, corresponding to p_1, and x_2, corresponding to p_2 can be used. Using the assumption that distance travelled on the curve is proportional to the change in signal rate, it can be seen from Figure 5–1 that

$$\frac{S_{x_1,x_2}}{S_{x_1,x_p}} = \frac{p_2 - p_1}{p - p_1}$$

$$\frac{h(x_2) - h(x_1)}{h(x_p - h(x_1)} = \frac{p_2 - p_1}{p - p_1}$$

Solving for $h(x_p)$ the result is

$$h(x_p) = \frac{p - p_1}{p_2 - p_1}\ [h(x_2) - h(x_1)] + h(x_1)$$

Hence, the Type–I error probability, E_1, becomes:

$$E_1 = h^{-1}(x_p)$$

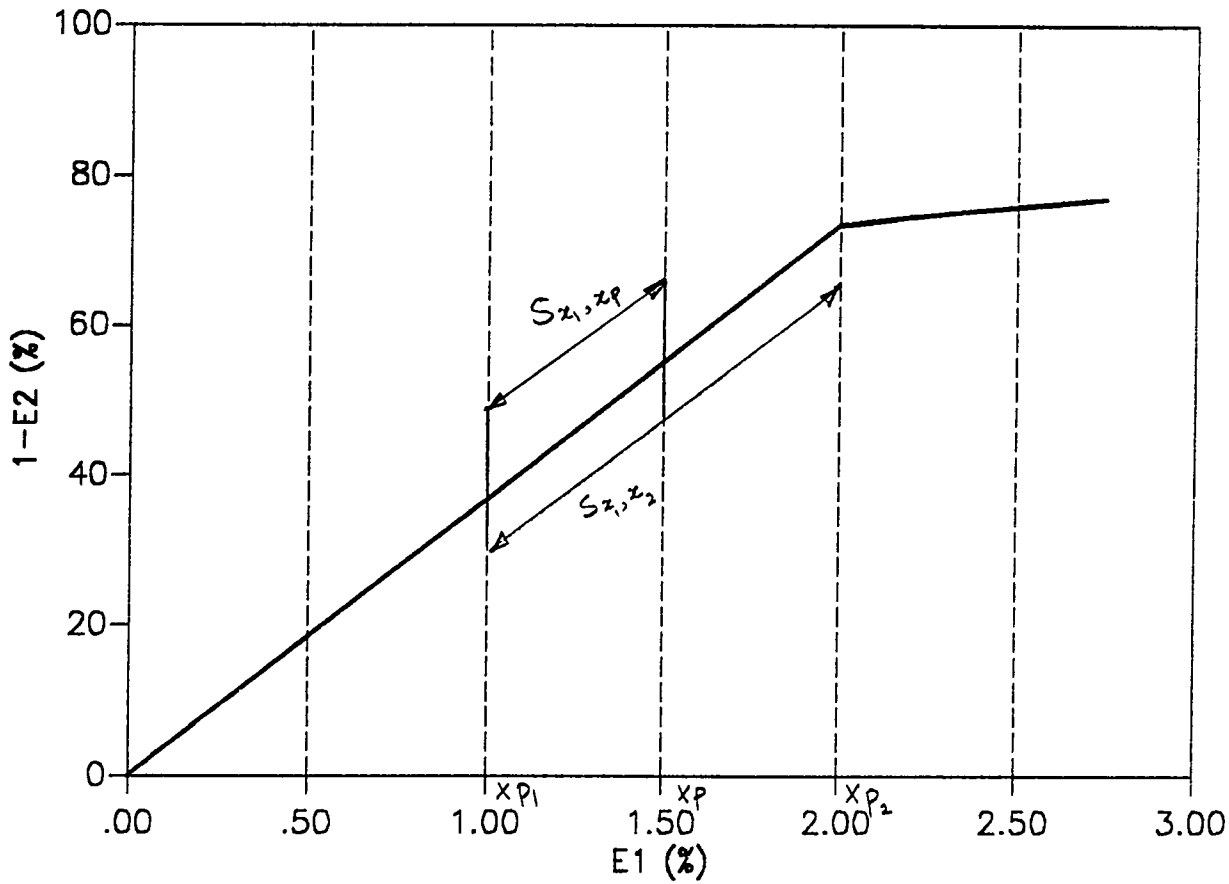
and, given the equation for ROC curve, the Type–II error probability is given as:

$$E_2 = 100 - f(E_1)$$

Using a stepwise regression and fractional power function approach at $\alpha = 0.05$ the functional form of $E_1 = f(p')$, Jaraiedi found that estimates for Type–I Type–II errors as a function of P are as follows

$$E_1 = 16.54 + 1.6p - 37.8p^{1/2} + 105.38p^{1/3} - 84.98p^{1/5}$$

Fig.5.1 Computation of Unknown Error Probabilities



for Type–II error, the relationship found is as

$$\ln(E_2) = -0.56p + 16.52p^{1/2} - 41.94p^{1/3} + 29.52p^{1/5}$$

5.3 PROCEDURE FOR ESTIMATION OF ERROR PROBABILITIES

The procedures presented are based on the *ROC* curve which is fitted between probability of false alarm (Type–I error) and the probability of hit (Type–II error). The steps of the procedure are as follows:

1. Use a constrained regression model for fitting the *ROC* curve. The functions selected for the fit should reflect the properties of the *ROC* curve which are

$$Y' = \frac{dY}{dX} > 0 \qquad \text{for } 0 < X < 100$$

and

$$Y'' = \frac{d^2Y}{d^2X} < 0 \qquad \text{for } 0 < X < 100$$

where Y' (first derivative) guarantees that the curve does not peak in the $(0, 100)$ range and Y'' (second derivative) ensures that it is a concave down in the same region.

2. Use the method in section 5.2 to generate data for E_1, E_2 and P from *ROC* curve.

3. Develope a constrained regression model to estimate E_1 and E_2 as a function of P. The model should reflect the expected behavior of E_1 and E_2 versus

P.

The problem was formulated as a constrained regression model. It is a non-linear programming problem:

$$\text{Min} \sum_{i=1}^{n} [(1 - E_2) - f(E_1)]^2$$

s.t.

$$f'(E_1) \geq 0$$

$$f''(E_1) \leq 0$$

$$f(0) = 0$$

$$f(100) = 100$$

$$n = 1, 2, \ldots, 4.$$

The data set (Appendix-D) used to find the best fit for ROC curve was adopted from the results of an experiment reported by Harris (1968). The following parameter estimates were obtained giving the best fit using GINO (General INteractive Optimizer):

$$A_1 = -54.6177$$

$$A_2 = 127.3531$$

$$N_1 = -10.6549$$

$$N_2 = 74.7161$$

The function that describes the best fit was of the following form:

$$P(\text{Hit}) = A_1 P(FA)^{1/N_1} + A_2 P(FA)^{1/N_2}$$

or

$$[1 - E_2] = A_1 (E_1)^{1/N_1} + A_2 (E_1)^{1/N_2}$$

Hence, the *ROC* equation becomes

$$Y(X) = -54.6177\, X^{-0.093862} + 1273531\, X^{0.013384}$$

The Jaraiedi's *ROC* equation was also modeled as GINO problem and was found to have more error than that of ours. Therefore, ours was a better fit.

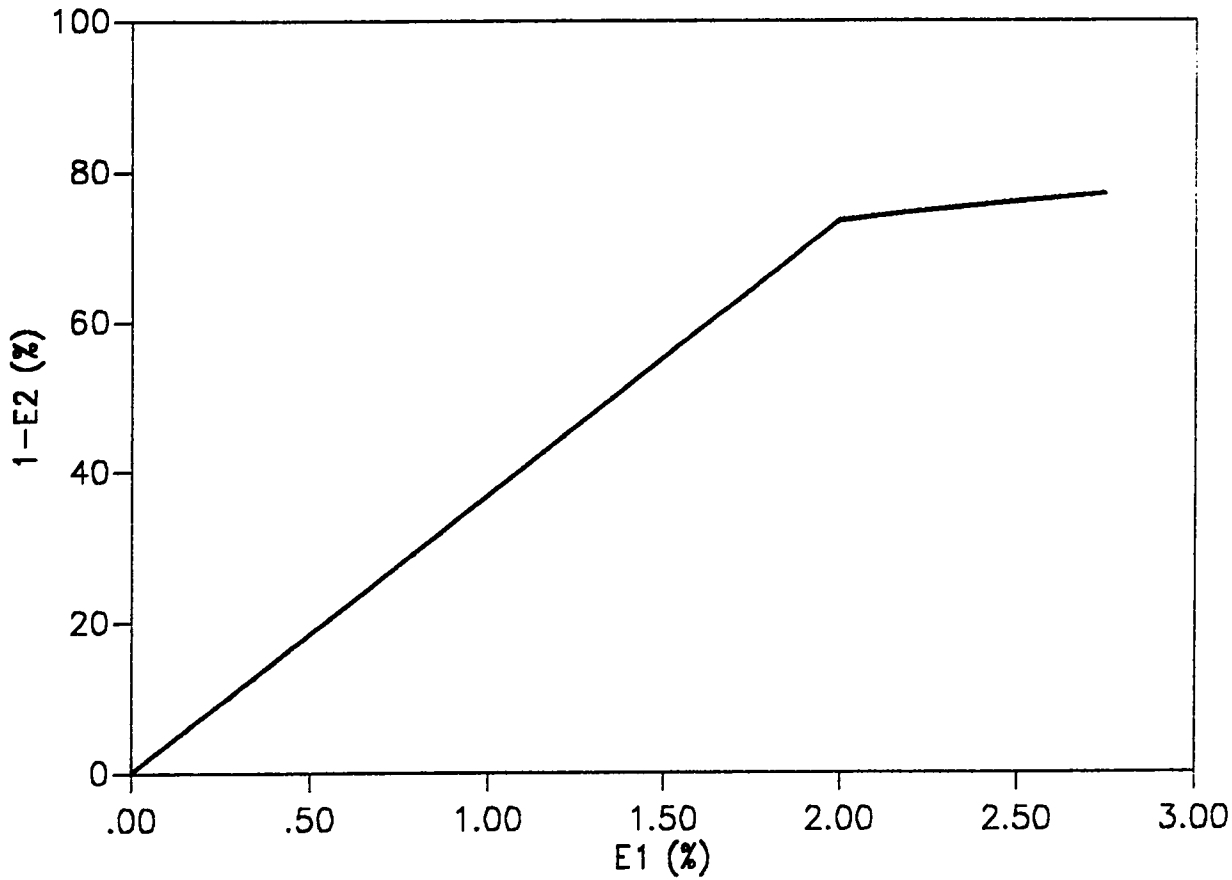
Once the *ROC* curve has been fitted, many points corresponding to Type–I and Type–II error can be generated following this method as described in the previous section. A Fortran program was developed (Appendix–B) that uses the numerical integration method to generate the desired points for a given defective rate as given in Table 5–1 below:

Table 5.1: Corresponding E_1 and E_2 for a given P

P	$E_1(\%)$	$E_2(\%)$
0.01	2.001	26.668
0.02	2.047	26.418
0.025	2.188	25.683
0.03	2.375	24.765
0.04	2.750	23.102

Figure 5.2 shows the *ROC* curve obtained. The following non–linear model was used to find the estimates of the functional relationships between P and E_1 and P and E_2.

Fig.5.2 Plot of ROC Curve



Model for E_1

$$\min \sum_{i=1}^{n} [E_{1i} - f_1(P_i)]^2$$

s.t.

$$f_1'(P_i) \geq 0 \qquad \text{[increasing function]}$$

$$f_1(P_i) \leq 0.5$$

$$f_1(P_i) \geq 0$$

$$n = 1, 2, \ldots 5$$

Model for E_2

$$\min \sum_{i=1}^{n} [E_{2i} - f_2(P_i)]^2$$

s.t.

$$f_2'(P_i) \leq 0 \qquad \text{[decreasing function]}$$

$$f_2(P_i) \leq 0.5$$

$$f_2(P_i) \geq 0$$

$$n = 1, 2, \ldots 5$$

f_1 and f_2 are restricted from the class of monotone polynomial functions. Using the data in Table 5.1 and GINO, the following two polynomials have been found the best to describe the data:

$$E_1 = 1.977 - 0.049p + 0.0449p^2 + 0.00419p^3 - 0.000041p^4$$

and

$$E_2 = 26.786 + 0.2789p - 0.2675p^2 - 0.0122p^3 + 0.00083p^4$$

Figures 5.3 and 5.4 illustrate the behavior of the two functions. These functions better describe the relationships between the error probabilities and incoming quality. The functions given by Jaraiedi, however, show a reasonable behavior for a very small range of data, representing incoming quality.

5.4 INSPECTION PLANS UNDER VARYING INSPECTION ERROR

The models developed previously assume fixed Type–I and Type–II errors through the inspection process. It has been documented in the literature that the errors are a function of incoming quality (Raouf and El-Fatouri, 1983) and in general, Type–I error increases as incoming quality (P) increases and Type–II error decreases with an increase in P (Harris. 1968). Each time a characteristic is inspected, the incoming quality for the next inspection changes. We add to the models, that after each characteristic is inspected the errors are estimated as a function of the new P. Then they are used for the next inspection. The rest of the equations and the algorithms are as before in the models. Next, we examine the effect of this on the inspection models.

5.5 EFFECT OF INSPECTION ERRORS ON REPEAT INSPECTION PLANS UNDER VARYING INSPECTION ERRORS

The effect of inspection errors (E_1 & E_2) has been evaluated on repeat inspection plans under the varying incoming quality by employing the functional

Fig.5.3 Relationship between P and E1

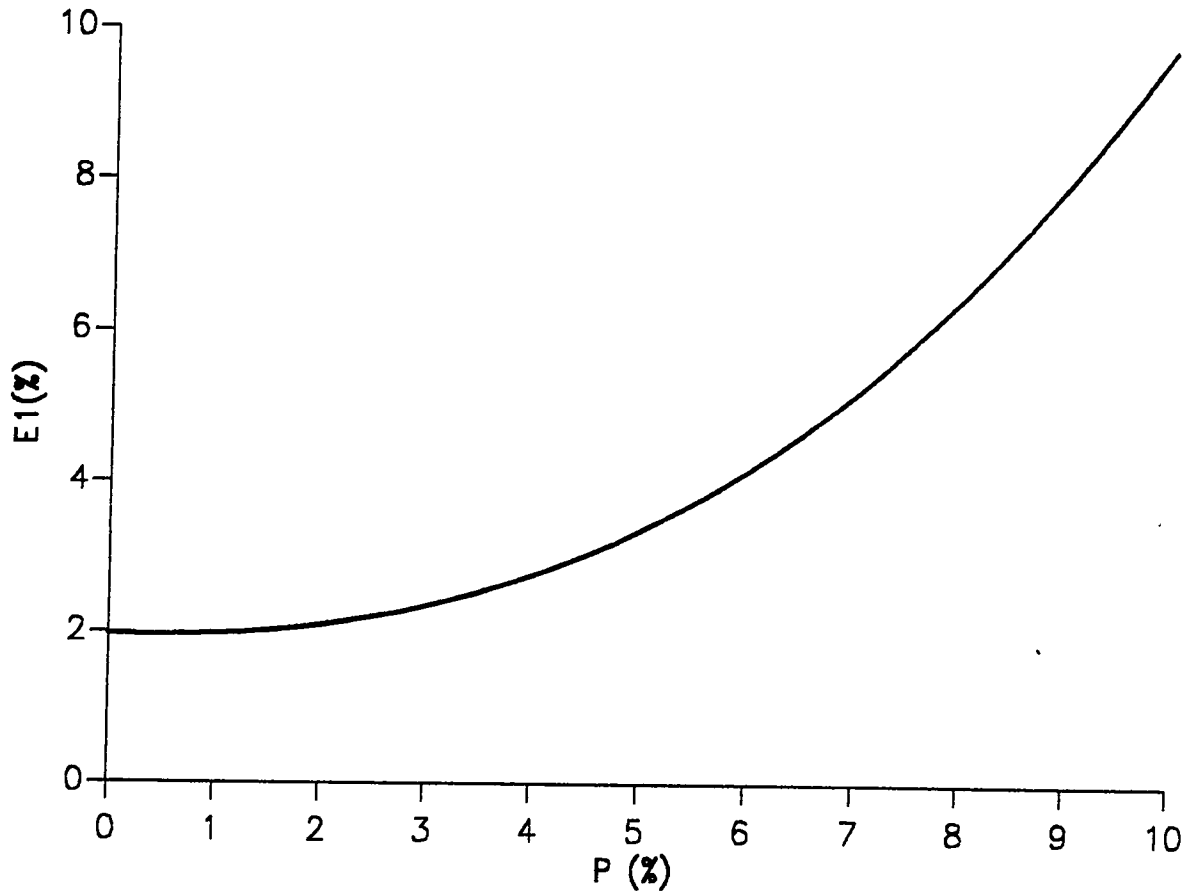
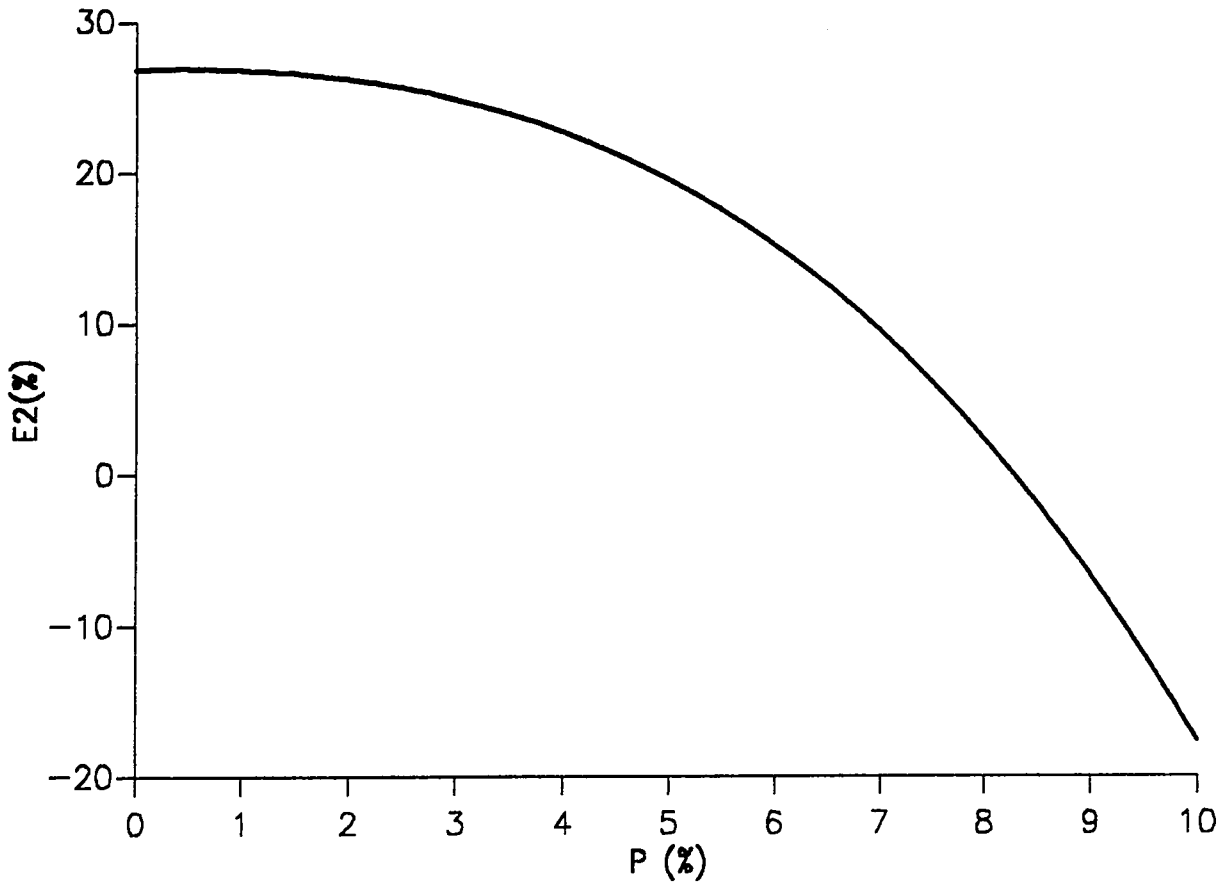


Fig.5.4 Relationship between P and E2



relationships between P and E_1, and P and E_2 as proposed in last section. The effect is quantified using the performance measures: expected Total Cost (ETC), Average Outgoing Quality Control (AOQ) and Average Total Inspection (ATI). The input data used is the same as in section 3.5. A Fortran program used to evaluate the effect of an error change is attached as Appendix–C. Figures 5.3 and 5.4 show the relationship between P and E_1 and P and E_2 respectively.

5.5.1 Effect of Inspection Errors on ETC Under Varying E_1 and E_2

Inspection errors leave a significant effect on Expected Total Cost (ETC) which is the sum of inspection cost, cost of false rejection due to Type–I error and cost of false acceptance due to Type–II error per accepted component. The cost of inspection is a single most dominant component in ETC as shown in Table 5.2. Figures 5.5 and 5.6 illustrate the behavior of ETC as E_1 and E_2 increases respectively. The effect of Type–I error is more drastic than that of Type–II error. It is observed that the combined effect of the two errors is much more serious than any one of them can make alone.

5.5.2 Effect of Inspection Errors on AOQ Under Varying E_1 and E_2

The Average Outgoing Quality (AOQ) is another important performance measure of inspection plans which had been defined in section 3.5.3. It is the percent of defectives left in the lot at the end of an inspection. Figures 5.7 and 5.8 show

Fig.5.5 Relationship between ETC and E1

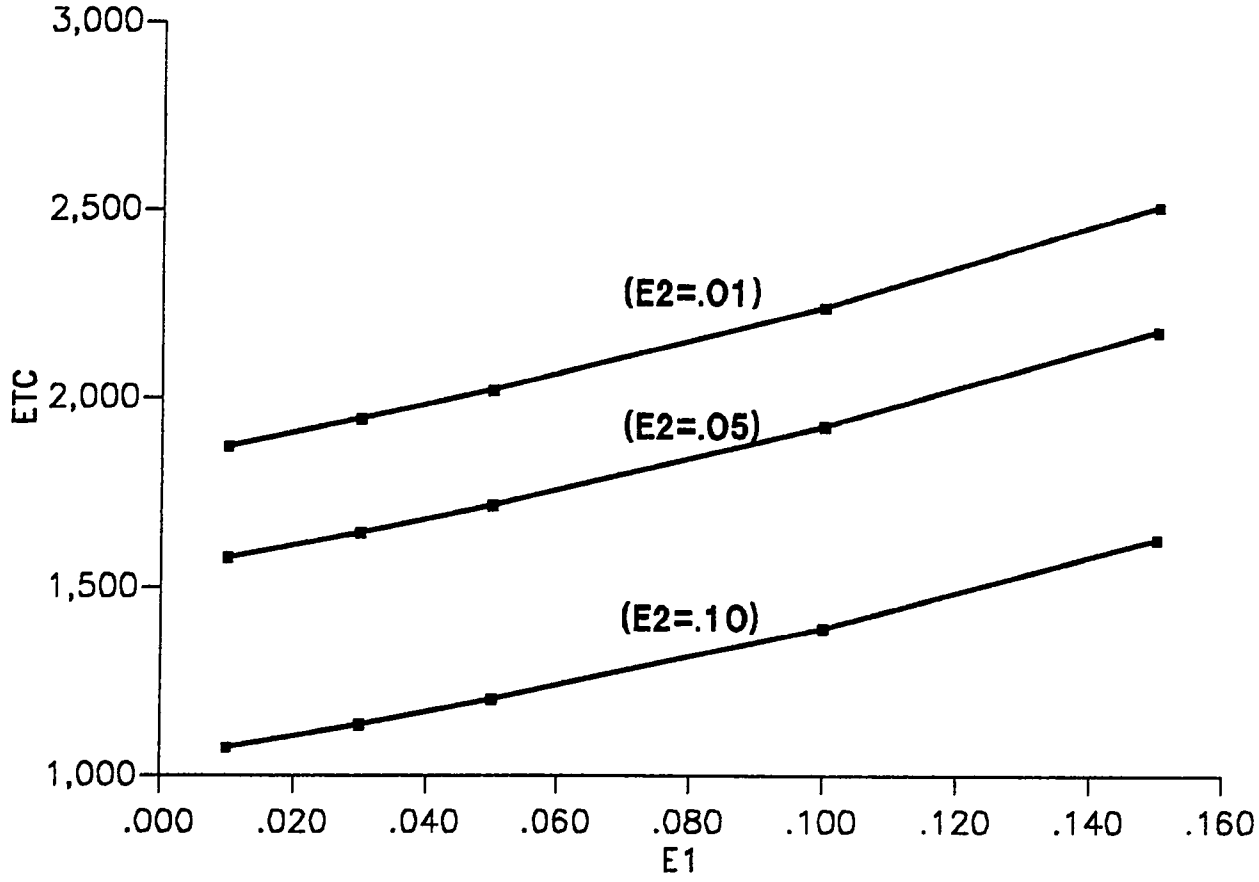


Fig.5.6 Relationship between ETC and E2

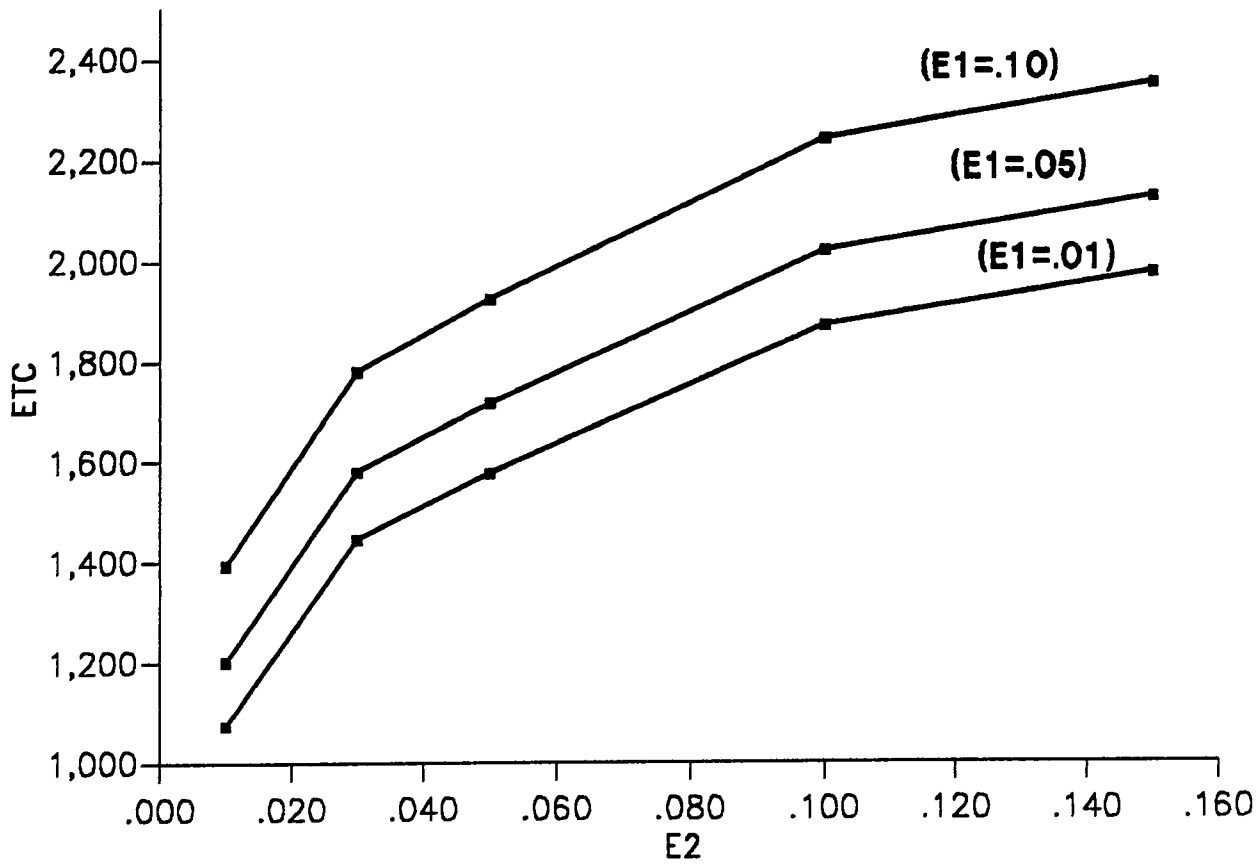


Fig.5.7 Relationship between AOQ and E1

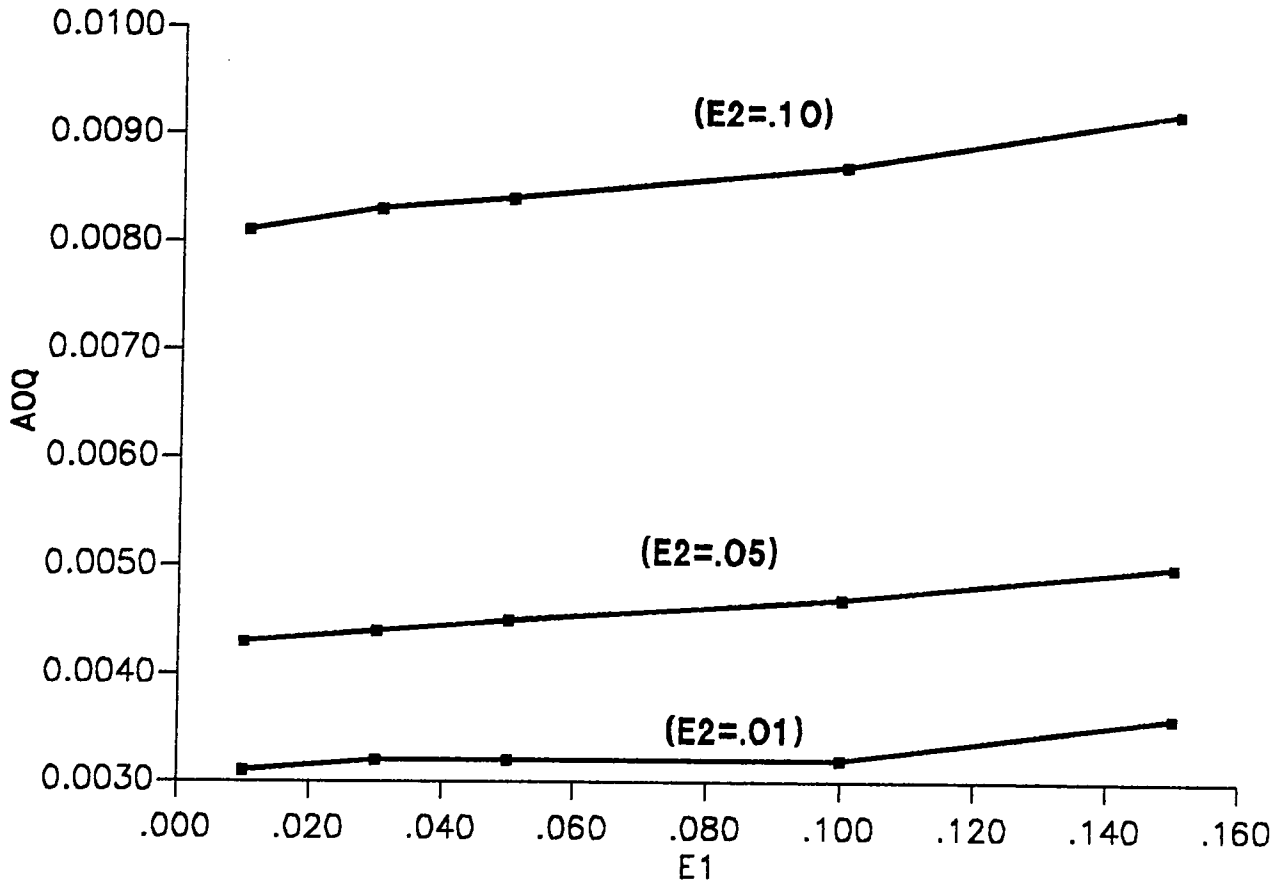
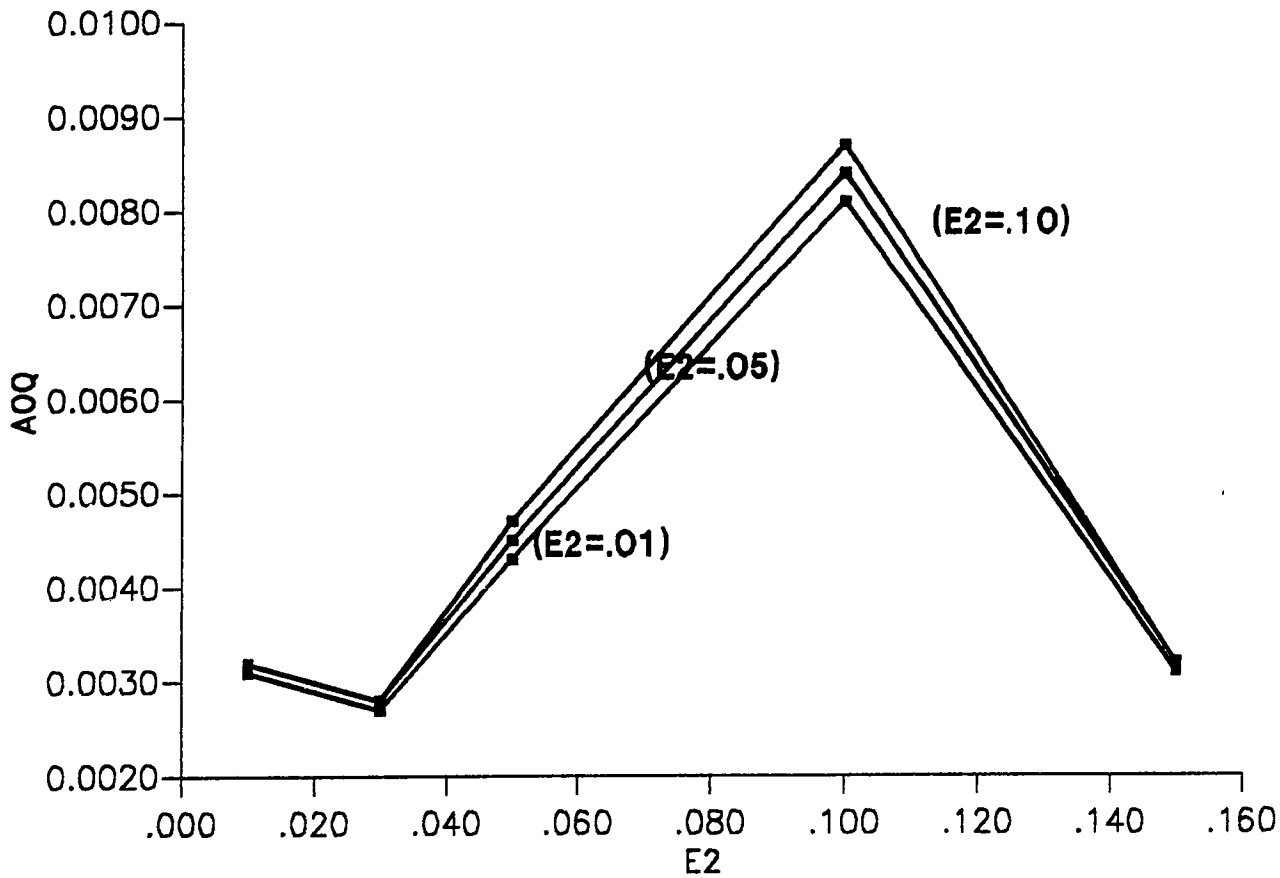


Fig.5.8 Relationship between AOQ and E2



the effects of Type–I error and Type–II error on AOQ respectively. Clearly, the effect of E_1 is almost linearly increasing, but in case of E_2, it exhibits a non-linear relationship with AOQ.

5.5.3 Effect of Inspection Errors on ATI Under Varying Error E_1 and E_2

Average Total Inspection (ATI) is the average number of times a component undergoes the inspection process by different inspectors. Each inspector inspects one characteristic. The mathematical expression for ATI is given in section 3.5.1. The value of ATI decreases as E_1 goes up as shown in Figure 5.9. The relationship is almost linear, but it attains higher value under higher probability of E_1. In case of E_2, the behavior is just opposite as that of E_1. The Figure 5.10 shows that with an increased probability of E_2, the value of ATI increases, but the growth pattern is very slow.

5.6 COMPARISON OF RESULTS OF INSPECTION PLANS WITH AND WITHOUT VARYING INSPECTION ERRORS

Inspection errors are functions of inspection errors (P). Functional relationships between P and Type–I error (E_1), and P and Type–II error (E_2) have been proposed in section 3 of this chapter. These relationships, when incorporated into the inspection models of repeat inspection plans, help evaluate the effect of varying inspection errors on the inspection plans. Table 5.2 shows the effect of inspection errors on inspection plans under varying inspection errors. The comparison of the

Fig.5.9 Relationship between ATI and E1

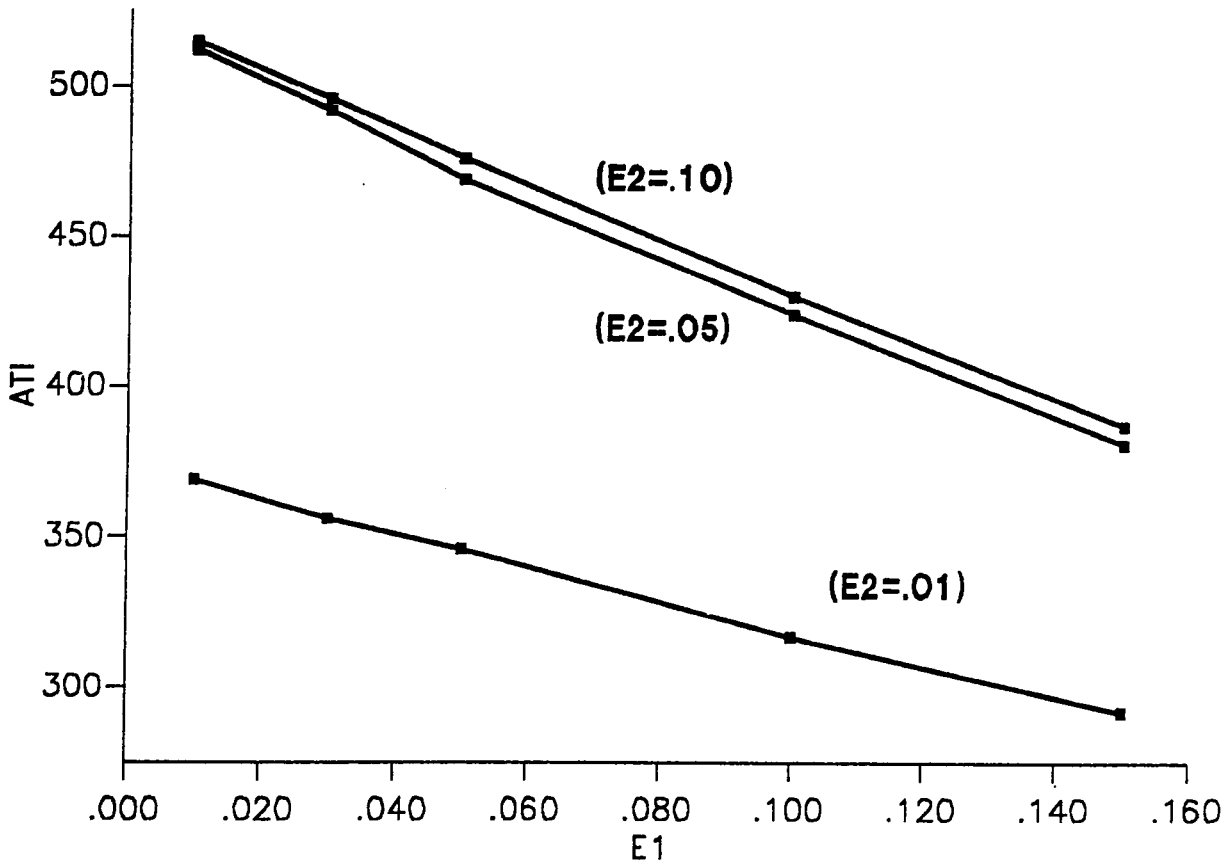


Fig.5.10 Relationship between ATI and E2

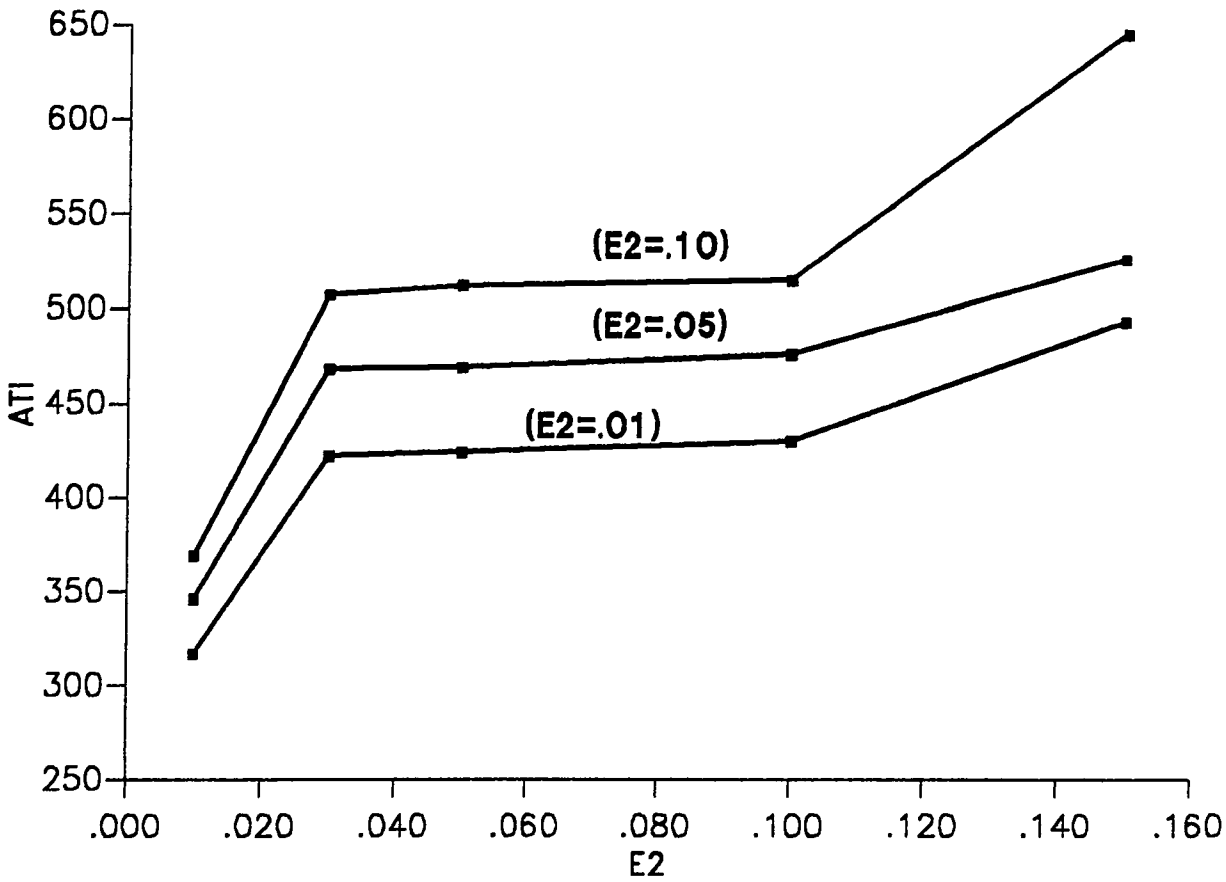


Table 5.2: Effect Of The Error On The Inspection Plan Under Varying Inspection Errors

	e_1	e_2	$1-PG$	ETC	CI	CFR	CFA	$A(n)$	n^*
1	0.00	0.01	0.0031	1044.90	789.93	30.99	223.98	48	2
2	0.00	0.03	0.0026	1413.62	1159.18	52.81	257.99	45	3
3	0.00	0.05	0.0041	1544.01	1166.09	64.75	313.16	45	3
4	0.00	0.10	0.0081	1836.39	1187.68	69.93	578.78	44	3
5	0.00	0.15	0.0031	1941.54	1612.05	117.14	212.33	41	4
6	0.01	0.00	0.1001	473.73	458.43	15.31	0.00	49	1
7	0.01	0.01	0.0032	1074.03	683.99	36.35	306.35	47	2
8	0.01	0.03	0.0027	1444.20	1170.62	81.32	192.25	43	3
9	0.01	0.05	0.0043	157587	1177.66	82.03	316.17	43	3
10	0.01	0.10	0.0081	1870.48	1199.77	87.45	583.27	43	3
11	0.01	0.15	0.0031	1975.54	1491.96	123.68	308.95	41	4
12	0.03	0.00	0.1001	527.39	318.65	29.27	0.00	53	1
13	0.03	0.01	0.0032	1135.62	823.04	81.60	230.90	43	2
14	0.03	0.03	0.0027	1508.91	1194.77	117.98	196.16	41	3
15	0.03	0.05	0.0044	164321	1202.09	118.73	322.38	41	3
16	0.03	0.10	0.0083	1942.43	1225.29	124.72	592.41	41	3
17	0.03	0.15	0.0032	2047.52	1655.05	177.62	214.84	37	4
18	0.05	0.00	0.1001	585.45	502.27	83.17	0.00	43	1
19	0.05	0.01	0.0032	1202.06	847.18	119.11	235.76	41	2
20	0.05	0.03	0.0028	1578.75	1220.72	157.78	200.24	38	3
21	0.05	0.05	0.0045	1715.78	1228.35	138.60	328.35	38	3
22	0.05	0.10	0.0084	2019.72	1252.75	165.2	601.74	38	3
23	0.05	0.15	0.0032	1215.32	1686.48	222.44	216.40	35	4
24	0.10	0.00	0.1001	752.97	567.09	185.87	0.00	37	1
25	0.10	0.01	0.0032	1392.82	916.07	227.89	248.85	35	2
26	0.10	0.03	0.0029	1779.42	1294.77	273.47	273.46	33	3
27	0.10	0.05	0.0047	1923.86	1303.29	274.50	346.06	33	3
28	0.10	0.10	0.0087	2240.32	1331.22	283.21	625.89	32	3
29	0.10	0.15	0.0032	2349.66	1776.57	353.44	219.66	30	4
30	0.15	0.00	0.1001	960.20	646.03	314.16	0.00	31	1
31	0.15	0.01	0.0036	1627.17	999.94	363.76	263.46	29	2
32	0.15	0.03	0.0036	2026.30	1384.92	417.99	223.40	27	3
33	0.15	0.05	0.0050	2179.01	1394.55	419.35	365.11	27	3
34	0.15	0.10	0.0092	2509.16	1427.01	431.07	651.09	27	3
35	0.15	0.15	0.0034	2627.31	1887.08	518.55	221.67	25	4

results in Table 5.2 with those contained in Table 3.1 underlines the significance of the varying E_1 and E_2. It is clear from Table 5.2 that the optimal number (n^*) of cycle(s) required for an inspection plan is higher than that in Table 3.1. This leads to higher Expected Total Cost (ETC). The substantial increase in ETC is also due to higher cost of false acceptance, since the probability of Type–II error (E_2) goes higher as the probability of inspection errors goes down. Another interesting change was observed in the value of Average Outgoing Quality (AOQ). This was expected because an increase in Type–II error would decelerate the decreasing rate of P, and the net effect would be a slower growth of PG (Probability of a good component entering the inspection cycle). We witnessed higher value of AOQ in inspection under varying inspection errors. In a nutshell, the varying inspection errors posted adverse effects on the inspection plans; nonetheless, it presented a close–to–real picture of the business of repeat inspection plans.

5.7 CONCLUSION

The ROC curve can be used to estimate the error probabilities. This chapter has provided the procedure to estimate the probability of Type–I and Type–II errors from the ROC curve. These estimates then led to the development of functional relationships between P and E_1, and P and E_2. These functional relationships were utilized to evaluate the dynamic effect of error probabilities on repeat inspection plans. A comparison of the results of inspection plans with and without varying errors showed a significant change in performance measures of

inspection plans. Even though this change is not positive, it reflects a real fact of inspection plans. It is recommended to incorporate the effect of varying errors in the design of inspection plans.

CHAPTER 6

CONCLUSIONS

6.1 SUMMARY AND CONCLUSIONS

The objective of this research was to model and evaluate the effect of inspection plans. This work was motivated by the realization that the inspection function is error-prone, and the assumption of perfect inspection in most practical applications is not valid. The emphasis of the research work was on a branch of inspection plans known as complete inspection plans which are instituted to guard against the human inspection errors. The implications of these errors could be catastrophic in the event of a critical component failure.

This thesis is an extension of the inspection models developed by Duffuaa and Raouf (1989). However, the literature review suggests that not much work has been done to study the effect of inspection errors on complete inspection plans. The effect of the two inspection errors, Type–I error and Type–II error was investigated. The change in the performance measures, Expected Total Cost (ETC), Average Outgoing Quality (AOQ) and Average Total Inspection (ATI) were quantified under the varied values of Type–I error (E_1) and Type–II error (E_2). It was observed that as E_1 increased the RTC increased at a faster rate than it did in case of E_2. Similarly, AOQ also increased as E_2 went up, but the

change in case of E_1 was not very significant. However, ATI decreased as E_1 was increased and it showed opposite effect in case of E_2.

The practicality of inspection models which are already developed was enhanced by modifying them for the cases where characteristics' defective rate is statistically dependent. This was accompalished by using the knowledge about the joint probability mass function ($j.p.m.f.$) of the random variables representing characteristics' defective rate. The rules for updating the quality of dependent characteristics were proposed which were consistent with basic probability rules. An alternative scheme for sequencing characteristics was suggested along with a methodology proposed for obtaining the optimal number of inspections. The effect of the inspection errors on dependency model was evaluated by observing the behaviors of the ETC, AOQ and ATI. The results were found consistent with the ones in the literature obtained for sampling plans.

It is documented in the literature that error probabilities are a function of incoming quality (P). Signal Detection Theory (SDT) can be applied to evaluate the performance of an inspector. The ROC gives the relationship between the two error probabilities. The GINO computer package was used to find the parameters of the ROC curve which was later used to find Type–I error and Type–II error which corresponds to a given probability of incoming quality. It was assumed that the distance travelled on the ROC curve due to a change in P is proportional to the amount of the change that has taken place. A computer program was written

to perform the necessary estimation procedures via numerical analysis methods.

A functional relationship between incoming fraction defective and the two types of errors was developed and found more accurate than those in the literature. A non–linear regression model under constraints was solved using GINO to estimate the parameters of these relationships. Then the two functional relationships were incorporated into the repeat inspection models to evaluate the effect of inspection errors under varied inspection errors on the three performance measures of inspection plans.

The results of this research can be applied to many practical situations where the cost of committing inspection errors is substantial. The performance of inspectors can be compared based on their estimated *ROC* curve. This can be used in selection and training of inspectors when inspection tasks are being designed or evaluated.

6.2 FUTURE RESEARCH

Most of the results presented in this research were extended attempts to address some of the problems associated with inspection error modeling in repeat inspection plans. Some of the following suggestions regard possible research directions in the future.

1. A different scheme describing the inspection process of repeat inspection can

be evaluated.

2. The process of data generation for parameter estimation of *ROC* curve can be refined. This would improve the accuracy for assessment of error probabilities; furthermore, the effect of job–related factors (such as task pace, product type, training and experience of inspectors, environmental factors, etc.) need to be investigated.

3. The effect of functional relationships between incoming fraction defectives and error probabilities can be incorporated into the inspection models under statistical dependency.

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C                 **************                                        SUP00010
C                 * APPENDIX-A *                                        SUP00020
C                 **************                                        SUP00030
C                                                                       SUP00040
C     THIS PROGRAM FINDS OPTIMAL 'N' UNDER STATISTICAL                  SUP00050
C     DEPENDENCY FOR REPEAT INSPECTION MODELS                           SUP00060
C*****************************************************************     SUP00070
      REAL CP(1024),PG(0:10,12),OP(10,10),P1(10),P2(10),SUM(10)         SUP00080
      REAL CI(0:9),TC(40),CA(40,40),CFA(40),FR(40,40),CR1(40,40)        SUP00090
      REAL CR(40),TCFR(0:40),TCFA(0:40),TCI(0:40),CFR(40),FA(40,40)     SUP00100
      REAL ETC(40),A1(40),CI1(40,40),M1(10,10),P(40)                    SUP00110
      INTEGER T,ILI(10),ILX(10,10)                                      SUP00120
      READ(5,*) M1(1,1)                                                 SUP00130
      READ(5,*) N                                                       SUP00140
      JL=N                                                              SUP00150
      IL=2**JL                                                          SUP00160
      DO 5 J=1,JL                                                       SUP00170
      SUM(J)=0.0                                                        SUP00180
    5 CONTINUE                                                          SUP00190
      DO 10 I=1,IL                                                      SUP00200
      READ(5,*) CP(I)                                                   SUP00210
   10 CONTINUE                                                          SUP00220
      READ(5,*)CFA(1)                                                   SUP00230
      READ(5,*)CFR(1)                                                   SUP00240
      READ(5,*)PG0                                                      SUP00250
      PG(0,1)=PG0                                                       SUP00260
      WRITE(6,301)'=====================SUMMARY OF THE RESULTS======   SUP00270
     *============'                                                     SUP00280
  301 FORMAT(15X,A)                                                     SUP00290
      WRITE(6,*)'                                                 '     SUP00300
      WRITE(6,*)'                                                 '     SUP00310
      WRITE(6,*)'                                                 '     SUP00320
                                                                        SUP00330
      ETCBI=(1-PG(0,1))*CFA(1)                                          SUP00340
      WRITE(6,302)'THE EXPECTED COST PER COMPONENT BEFPRE INSPECTION    SUP00350
     * =',ETCBI                                                        SUP00360
  302 FORMAT(15X,A,F15.3)                                               SUP00370
      WRITE(6,*)'                                                 '     SUP00380
      WRITE(6,*)'                                                 '     SUP00390
      WRITE(6,*)'                                                 '     SUP00400
      WRITE(6,303)'----------------STARTING THE INSPECTION-----------'  SUP00410
  303 FORMAT(20X,A)                                                     SUP00420
      WRITE(6,*)'                                                 '     SUP00430
      WRITE(6,*)'                                                 '     SUP00440
                                                                        SUP00450
      DO 20 I=1,N                                                       SUP00460
   20 READ(5,*)P1(I)                                                    SUP00470
      DO 25 I=1,N                                                       SUP00480
   25 READ(5,*)P2(I)                                                    SUP00490
      DO 30 I=1,N                                                       SUP00500
   30 READ(5,*)TC(I)                                                    SUP00510
      READ(5,*)PGG                                                      SUP00520
      ILI(1)=IL/2                                                       SUP00530
      DO 35 J=2,JL                                                      SUP00540
       ILI(J)=ILI(J-1)/2                                                SUP00550
```

```
35    CONTINUE                                                          SUP00560
      DO 40 J=1,JL                                                      SUP00570
       K=0                                                              SUP00580
       KX=ILI(J)                                                        SUP00590
       DO 45 I=1,IL/2                                                   SUP00600
        ILX(I,J)=I+K                                                    SUP00610
        ILX(I+IL/2,J)=I+ILI(J)                                          SUP00620
        IF(I.EQ.ILI(J)) THEN                                            SUP00630
         K=ILI(J)                                                       SUP00640
         ILI(J)=ILI(J)+KX                                               SUP00650
        ENDIF                                                           SUP00660
45     CONTINUE                                                         SUP00670
       DO 50 I=1,IL/2                                                   SUP00680
        SUM(J)=SUM(J)+CP(ILX(I,J))                                      SUP00690
50     CONTINUE                                                         SUP00700
40    CONTINUE                                                          SUP00710
      DO 200 T=1,5                                                      SUP00720
       DO 100 L=1,JL                                                    SUP00730
        DO 55 J=1,JL                                                    SUP00740
         OP(L,J)=SUM(J)                                                 SUP00750
         WRITE(6,*) 'SUM ',SUM(J)                                       SUP00760
55      CONTINUE                                                        SUP00770
        P(L)=OP(L,L)*P2(L)                                              SUP00780
        P(L)=P(L)/(OP(L,L)*P2(L)+(1-OP(L,L))*(1-P1(L)))                 SUP00790
        WRITE(6,*) 'P(L)',P(L)                                          SUP00800
        SUM(L)=P(L)                                                     SUP00810
        DO 60 I=1,IL                                                    SUP00820
         IF(I.LE.IL/2) THEN                                             SUP00830
          CP(ILX(I,L))=CP(ILX(I,L))*(P(L)/OP(L,L))                      SUP00840
          WRITE(6,*) CP(ILX(I,L))                                       SUP00850
         ELSE                                                           SUP00860
          CP(ILX(I,L))=CP(ILX(I,L))*((1-P(L))/(1-OP(L,L)))              SUP00870
          WRITE(6,*) CP(ILX(I,L))                                       SUP00880
         ENDIF                                                          SUP00890
60      CONTINUE                                                        SUP00900
        PG(L,T)=CP(IL)                                                  SUP00910
        WRITE(6,*) 'PG = ',PG(L,T)                                      SUP00920
        DO 65 J=1,JL                                                    SUP00930
         IF(J.NE.L) SUM(J)=0.0                                          SUP00940
65      CONTINUE                                                        SUP00950
        DO 70 J=1,JL                                                    SUP00960
         DO 75 I=1,IL/2                                                 SUP00970
          IF(J.NE.L) SUM(J)=SUM(J)+CP(ILX(I,J))                         SUP00980
75       CONTINUE                                                       SUP00990
70      CONTINUE                                                        SUP01000
100   CONTINUE                                                          SUP01010
      L=1                                                               SUP01020
      FR(L,T)=M1(L,T)*PG(L,T)*P1(L)                                     SUP01030
      WRITE(6,*)'FR(1,1)=',FR(L,T)                                      SUP01040
      FA(L,T)=M1(L,T)*((OP(L,L)*P2(L))                                  SUP01050
     + +(1-PG(L,T)-OP(L,L))*(1-P1(L)))                                  SUP01060
      WRITE(6,*)OP(L,L)                                                 SUP01070
      WRITE(6,*)'FA(1,1)=',FA(L,T)                                      SUP01080
      CA(L,T)=M1(L,T)*PG(L,T)*(1-P1(L))                                 SUP01090
      WRITE(6,*)'CA(1,1)=',CA(L,T)                                      SUP01100
```

```
      DO 101 L=2,N                                                      SUP01110
       M1(L,T)=FA(L-1,T)+CA(L-1,T)                                      SUP01120
       WRITE(6,*)'THE VALUE OF M=',M1(L,T)                              SUP01130
       FR(L,T)=M1(L,T)*PG(L-1,T)*P1(L)                                  SUP01140
       CA(L,T)=M1(L,T)*PG(L-1,T)*(1-P1(L))                              SUP01150
       FA(L,T)=M1(L,T)*((OP(L,L)*P2(L))                                 SUP01160
     +  +(1-PG(L-1,T)-OP(L,L))*(1-P1(L)))                               SUP01170
       WRITE(6,*)'OP(L,L)=',OP(L,L),L                                   SUP01180
       WRITE(6,*)'FA(L,T)=',FA(L,T)                                     SUP01190
       WRITE(6,*)'CA(L,T)=',CA(L,T)                                     SUP01200
101   CONTINUE                                                          SUP01210
                                                                        SUP01220
102      FORMAT(20X,A,I3,I3,A,F14.4)                                    SUP01230
C    CALCULATING TOTAL ACCEPTED COMPONENTS IN TTH CYCLE                 SUP01240
                A1(T)=FA(N,T)+CA(N,T)                                   SUP01250
103      FORMAT(20X,A,I2,A,F14.3)                                       SUP01260
C    CALCULATING THE NUMBER OF COMPONENT ENTERING NEXT CYCLE            SUP01270
              M1(1,T+1)=A1(T)                                           SUP01280
C    CALCULATING THE COST OF FALSE ACCEPTANCE                           SUP01290
               CFA(T)=CFA(1)*FA(N,T)                                    SUP01300
           L=1                                                          SUP01310
               CR1(L,T)=FR(L,T)                                         SUP01320
           DO 9 L=2,N                                                   SUP01330
9               CR1(L,T)=CR1(L-1,T)+FR(L,T)                             SUP01340
C     CALCULATING THE COST OF FALSE REJECTION                           SUP01350
           CFR(T)=CFR(1)*CR1(N,T)                                       SUP01360
C       CALCULATING THE COST OF INSPECTION                              SUP01370
               L=1                                                      SUP01380
             CI1(L,T)=TC(L)*M1(L,T)                                     SUP01390
              DO 23 L=2,N                                               SUP01400
23           CI1(L,T)=CI1(L-1,T)+TC(L)*M1(L,T)                          SUP01410
             CI(T)=CI1(N,T)                                             SUP01420
C     CALCULATING THE TOTAL COST FOR TTH CYCLE                          SUP01430
             TCFR(0)=0.0                                                SUP01440
             TCI(0)=0.0                                                 SUP01450
             TCFR(T)=TCFR(T-1)+CFR(T)                                   SUP01460
             TCFA(T)=CFA(T)                                             SUP01470
             TCI(T)=TCI(T-1)+CI(T)                                      SUP01480
      WRITE(6,*)'TCFR',TCFR(T)                                          SUP01490
      WRITE(6,*)'TCFA',TCFA(T)                                          SUP01500
      WRITE(6,*)'TCI',TCI(T)                                            SUP01510
      WRITE(6,*)'A=',A1(T)                                              SUP01520
C     CALCULATING EXPECTED TOTAL COST PER ACCEPTED COMPONENT            SUP01530
             ETC(T)=(TCI(T)+TCFA(T)+TCFR(T))/A1(T)                      SUP01540
      WRITE(6,306) 'ETC(',T,')=',ETC(T)                                 SUP01550
306      FORMAT(20X,A,I2,A,F20.3)                                       SUP01560
     WRITE(6,104)'------------END OF CYCLE',T,'--------------------'    SUP01570
104  FORMAT (20X,A,I2,A)                                                SUP01580
C    TESTING FOR MINIMIZING THE EXPECTED TOTAL COST                     SUP01590
     IF(T.EQ.1)GOTO 200                                                 SUP01600
C     IF(ETC(T).GT.ETC(T-1)) GOTO 500                                   SUP01610
C    IF(T.GE.2)THEN                                                     SUP01620
      PG111 = PG(L-1,T)-PG(L-1,T-1)                                     SUP01630
      WRITE(7,*)'PG111=',PG111                                          SUP01640
      IF((PG(L-1,T)-PG(L-1,T-1)).LE.PGG) GOTO 500                       SUP01650
```

```
C     ENDIF                                                          SUP01660
200   CONTINUE                                                       SUP01670
500   WRITE(6,*)'THE OPTIMUM NUMBER OF INSPECTION ',T-1              SUP01680
      STOP                                                           SUP01690
      END                                                            SUP01700
```

```
C                 **************                                    CUR00010
C                 * APPENDIX-B *                                    CUR00020
C                 **************                                    CUR00030
C     THIS PROGRAM GENERATES POINTS ON THE ROC CURVE                CUR00040
C*********************************************************          CUR00050
      EXTERNAL LENGTH                                               CUR00060
      REAL A1,A2,N1,N2,X1,X2,RP,LP,LENP,LENM,LENGTH,PON,E2          CUR00070
      COMMON /TRANS/ A1,A2,N1,N2                                    CUR00080
      OPEN(1,FILE='CURVE.DATA',STATUS='OLD')                        CUR00090
      READ(1,*) A1,A2,N1,N2                                         CUR00100
      READ(1,*) X1,X2                                               CUR00110
      READ(1,*) P1,P2,P                                             CUR00120
      EPS=2                                                         CUR00130
      LENP=LENGTH(X1,X2)*((P-P1)/(P2-P1))                           CUR00140
      WRITE(6,*) 'TOTAL LENGTH = ',LENGTH(X1,X2)                    CUR00150
      WRITE(6,*) 'LENGTH TO P = ',LENP                              CUR00160
      LP=X1                                                         CUR00170
      PON=(X1+X2)/2                                                 CUR00180
      RP=X2                                                         CUR00190
    1 LENM=LENGTH(X1,PON)                                           CUR00200
      WRITE(6,*) 'POINT IS AT ->',PON,'THE LENGTH IS ->',LENM       CUR00210
      IF(ABS(LENM-LENP).GT.EPS) THEN                                CUR00220
       IF(LENM.GT.LENP) THEN                                        CUR00230
        PON=(LP+PON)/2                                              CUR00240
        GOTO 1                                                      CUR00250
       ELSE                                                         CUR00260
        PON=(PON+RP)/2                                              CUR00270
        GOTO 1                                                      CUR00280
       ENDIF                                                        CUR00290
      ENDIF                                                         CUR00300
      N1=1.0/N1                                                     CUR00310
      N2=1.0/N2                                                     CUR00320
      E1=PON                                                        CUR00330
      E2=100.0-A1*E1**(N1)-A2*E1**(N2)                              CUR00340
      WRITE(6,*) '     E1        E2       '                         CUR00350
      WRITE(6,*) '----------------------'                           CUR00360
      WRITE(6,2) E1,E2                                              CUR00370
    2 FORMAT(2X,F7.3,2X,F7.3)                                       CUR00380
      CLOSE(1)                                                      CUR00390
      STOP                                                          CUR00400
      END                                                           CUR00410
                                                                    CUR00420
      REAL FUNCTION LENGTH(X1,X2)                                   CUR00430
      EXTERNAL F                                                    CUR00440
      REAL A1,A2,N1,N2,X1,X2,XI,H,SUM                               CUR00450
      COMMON /TRANS/ A1,A2,N1,N2                                    CUR00460
      N=10                                                          CUR00470
      H=(X2-X1)/(N-1)                                               CUR00480
      SUM=0.5*(F(X1)+F(X2))                                         CUR00490
      DO 10 I=2,N-1                                                 CUR00500
       XI=(I-1)*H                                                   CUR00510
       SUM=SUM+F(XI)                                                CUR00520
   10 CONTINUE                                                      CUR00530
      LENGTH=H*SUM                                                  CUR00540
      RETURN                                                        CUR00550
```

```
      END                                                       CUR00560
                                                                CUR00570
      FUNCTION F(X)                                             CUR00580
      EXTERNAL G                                                CUR00590
      REAL A1,A2,N1,N2,X                                        CUR00600
      COMMON /TRANS/ A1,A2,N1,N2                                CUR00610
      F=1/(N1*N2)*((N1**2)*(N2**2)+G(X))**(0.5)                 CUR00620
      RETURN                                                    CUR00630
      END                                                       CUR00640
                                                                CUR00650
      FUNCTION G(X)                                             CUR00660
      REAL A1,A2,N1,N2,X                                        CUR00670
      COMMON /TRANS/ A1,A2,N1,N2                                CUR00680
      G=(A1*N2*X**((1-N1)/N1)+A2*N1*X**((1-N2)/N2))**2          CUR00690
      RETURN                                                    CUR00700
      END                                                       CUR00710
```

```
C                 **************                                        MOD00010
C                 * APPENDIX-C *                                        MOD00020
C                 **************                                        MOD00030
C     THIS PROGRAM FINDS OPTIMAL'N' AND INCORPORATES THE                MOD00040
C     EFFECT OF TYPE-I ERROR AND TYPE-II ERROR INTO THE                 MOD00050
C     REPEAT MULTICHARACTERISTICS INSPECTION PLANS                      MOD00060
C*************************************************************          MOD00070
      REAL A(40),TC(40),B(40),H,R(40),C(40),P(40),P1(40)                MOD00080
      REAL P2(40),M(40),ETC(40),TCFA(40),TCFR(40),CI(40),CA(40)         MOD00090
      REAL PG(40),Y,EPS,FA(40),FR(40),CFA,CFR,MI,SORT,DIFF,PGG          MOD00100
      INTEGER I,J,JJ,K,L,N,MRAT(40),ISORT                               MOD00110
      OPEN(1,FILE='XXM.DATA',STATUS='OLD')                              MOD00120
      READ(1,*) MI,N                                                    MOD00130
      WRITE(6,*) 'M(1) =',MI                                            MOD00140
      WRITE(6,*) 'N =',N                                                MOD00150
      READ(1,*) CFA,CFR                                                 MOD00160
      WRITE(6,*) 'CFA =',CFA                                            MOD00170
      WRITE(6,*) 'CFR =',CFR                                            MOD00180
      READ(1,*) (P1(I),I=1,N)                                           MOD00190
      WRITE(6,*) (P1(I),I=1,N)                                          MOD00200
      READ(1,*) (P2(I),I=1,N)                                           MOD00210
      WRITE(6,*) (P2(I),I=1,N)                                          MOD00220
      READ(1,*) (P(I),I=1,N)                                            MOD00230
      WRITE(6,*) (P(I),I=1,N)                                           MOD00240
      READ(1,*) (C(I),I=1,N)                                            MOD00250
      WRITE(6,*) (C(I),I=1,N)                                           MOD00260
C     READ(1,*) PGG                                                     MOD00270
      WRITE(6,*) 'PGG =',PGG                                            MOD00280
      WRITE(6,500) '----------STARTING THE INSPECTION----------------' MOD00290
      WRITE(6,500) ' '                                                  MOD00300
      J=0                                                               MOD00310
      JJ=1                                                              MOD00320
      TCFRS=0.0                                                         MOD00330
      CISUM=0.0                                                         MOD00340
      PG(1)=0.0                                                         MOD00350
      M(J+1)=MI                                                         MOD00360
      ETCL=1.0E20                                                       MOD00370
C ORDERING THE CHARACTERISTICS ACCORDING TO THE LOWEST RATIO OF         MOD00380
C C(I)/R(I)                                                             MOD00390
100   WRITE(6,501) 'CYCLE NUMBER ===> ',JJ                              MOD00400
      DO 1 I=1,N                                                        MOD00410
1     MRAT(I)=I                                                         MOD00420
      J=J+1                                                             MOD00430
      DO 3 I=1,N                                                        MOD00440
      R(I)=P(I)*(1-P2(I))+(1-P(I))*P1(I)                                MOD00450
3     B(I)=C(I)/R(I)                                                    MOD00460
      DO 5 I=1,N                                                        MOD00470
       DO 4 K=1,N                                                       MOD00480
        IF(B(I).LT.B(K)) THEN                                           MOD00490
         SORT=B(I)                                                      MOD00500
         B(I)=B(K)                                                      MOD00510
         B(K)=SORT                                                      MOD00520
         ISORT=MRAT(I)                                                  MOD00530
         MRAT(I)=MRAT(K)                                                MOD00540
         MRAT(K)=ISORT                                                  MOD00550
```

```
        ENDIF                                                           MOD00560
 4     CONTINUE                                                         MOD00570
 5    CONTINUE                                                          MOD00580
      WRITE(6,501) 'THE SEQUENCE AT CYCLE ',JJ                          MOD00590
      DO 6 I=1,N                                                        MOD00600
 6     WRITE(6,505) MRAT(I)                                             MOD00610
      DO 7 I=1,N                                                        MOD00620
       PG(I+1)=1-P(1)                                                   MOD00630
       DO 2 K=2,N                                                       MOD00640
 2     PG(I+1)=(1-P(K))*PG(I+1)                                         MOD00650
       WRITE(6,504) 'PG =',PG(I+1)                                      MOD00660
       DIFF=PG(I+1)-PG(I)                                               MOD00670
 C     IF(DIFF.LE.PGG) GOTO 102                                         MOD00680
       II=MRAT(I)                                                       MOD00690
       WRITE(6,502) 'MIN RATIO IS AT',II,'CHARACTERISTICS'              MOD00700
 C FINISHING THE SEQUENCE OF CHARACTERISTICS                            MOD00710
       FR(I)=M(I)*PG(I+1)*P1(II)                                        MOD00720
       FA(I)=M(I)*(P(II)*P2(II)+(1-PG(I+1)-P(II))*(1-P1(II)))           MOD00730
       CA(I)=M(I)*PG(I+1)*(1-P1(II))                                    MOD00740
       A(I)=FA(I)+CA(I)                                                 MOD00750
       TCFR(I)=CFR*FR(I)+TCFRS                                          MOD00760
       TCFA(I)=CFA*FA(I)                                                MOD00770
       CI(I)=M(I)*C(II)+CISUM                                           MOD00780
       TCFRS=TCFR(I)                                                    MOD00790
       CISUM=CI(I)                                                      MOD00800
       WRITE(6,503) 'A(',I,') =',A(I)                                   MOD00810
       WRITE(6,503) 'TCFR(',I,') =',TCFR(I)/A(I)                        MOD00820
       WRITE(6,503) 'TCFA(',I,') =',TCFA(I)/A(I)                        MOD00830
       WRITE(6,503) 'CI(',I,') =',CI(I)/A(I)                            MOD00840
       P(II)=P(II)*P2(II)/(P(II)*P2(II)+(1-P(II))*(1-P1(II)))           MOD00850
       WRITE(6,503) 'P(',II,') =',P(II)                                 MOD00860
       P(II)=P(II)*100                                                  MOD00870
 C     UPDATING TYPE-I ERROR (P1) AND TYPE-II ERROR (P2)                MOD00880
 C     AFTER UPDATING INCOMING QUALITY (P).                             MOD00890
 C                                                                      MOD00900
       P1(II)=(1.99774-.04907*P(II)+.044922*P(II)**2                    MOD00910
     * +.004194*P(II)**3-.000041*P(II)**4)*0.01                         MOD00920
       WRITE(6,503) 'P1(',II,') =',P1(II)                               MOD00930
       P2(II)=(27.78646+.278897*P(II)-.267551*(P(II)**2)                MOD00940
     * -.012174*(P(II)**3)+.000829*(P(II))**4)*0.01                     MOD00950
       WRITE(6,503) 'P2(',II,') =',P2(II)                               MOD00960
       P(II)=P(II)*.01                                                  MOD00970
       M(I+1)=A(I)                                                      MOD00980
      WRITE(6,503) 'M(',I,') =',M(I)                                    MOD00990
 7    CONTINUE                                                          MOD01000
      M(1)=A(N)                                                         MOD01010
      ETC(JJ)=(TCFR(N)+TCFA(N)+CI(N))/A(N)                              MOD01020
      WRITE(6,503) 'ETC(',J,') =',ETC(J)                                MOD01030
      IF(ETC(JJ).GT.ETCL) GOTO 102                                      MOD01040
      ETCL=ETC(JJ)                                                      MOD01050
      WRITE(6,501) 'END OF CYCLE ',JJ                                   MOD01060
      WRITE(6,500) ' '                                                  MOD01070
      JJ=JJ+1                                                           MOD01080
      GOTO 100                                                          MOD01090
 102  CLOSE(1)                                                          MOD01100
```

```
      WRITE(6,501) 'END OF CYCLE ',JJ                                   MOD01110
      WRITE(6,501) 'OPTIMUM FOUND AT CYCLE ===> ',JJ-1                  MOD01120
500   FORMAT(20X,A)                                                     MOD01130
501   FORMAT(20X,A,1X,I2)                                               MOD01140
502   FORMAT(20X,A,1X,I3,1X,A)                                          MOD01150
503   FORMAT(20X,A,I2,A,1X,F18.9)                                       MOD01160
504   FORMAT(20X,A,F12.9)                                               MOD01170
505   FORMAT(23X,I2)                                                    MOD01180
      STOP                                                              MOD01190
      END                                                               MOD01200
```

Appendix–D

The data set used for fitting the *ROC* curve was adapted from a visual inspection experiment reported by Harris (1968).

P	Defects Detected	False Reports
0.0025	0.575	0.850
0.0100	0.715	0.395
0.0400	0.750	0.179
0.1600	0.840	0.075

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