Extensions in Multicharacteristics
Repeat inspection plans

by

Iftikhar Ali Nadeem

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

SYSTEM ENGINEERING

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Extensions in multicharacteristics repeat inspection plans

Nadeem, Iftikhar Ali, M.S.

King Fahd University of Petroleum and Minerals (Saudi Arabia), 1993
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REPEAT INSPECTION PLANS

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KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
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This thesis, written by

Iftikhar Ali Nadeem

under the direction of his thesis committee, and approved by all the members, has been presented to and accepted by the Dean, College of Graduate Studies, in partial fulfillment of the requirement for the degree of

MASTER OF SCIENCE IN SYSTEMS ENGINEERING

Thesis Committee

Chairman (Dr. S.O. Duffuaa)

Member (Dr. Abdul Raouf)

Member (Dr. M. Ben Daya)

Member (Dr. J. E. Al-Alwani)

Department Chairman

Dean, College of Graduate Studies

Date: Feb. 16th, 1998
Dedicated to

My Mother
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All praise be to Allah, the Lord of the worlds. May peace and blessings be upon Mohammad the last of the messengers and his family. I thank Allah for His limitless help and guidance.

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ملخص الرسالة

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خلال عملية الفحص عادة ما يرتبط الفاحص نوعين شائعين من الخطأ خلال حكمه على جودة المنتج، ويعرف هذان النوعان من الخطأ باسم النوع الأول، النوع الثاني. وتبحث هذه الأطروحة في تأثير هذين النوعين من الخطأ على خطط الفحص المتكرر للإجراءات المرجحة، وتحسين هذه الرسالة النماذج الموجودة للفحص بتطوير خطط الفحص المتوقعة وذلك بالتركيز على الحالات التي تكون فيها العلاقة بين معدلات الخلل في خواص الأجزاء معتمدة إحصائياً على بعضها البعض.

ولنظرًا للحقيقة المعروفة والتي تنص على أن احتمالات الخطأ تعتمد على جودة المنتج، فإن هذه الدراسة تقترح خطوات جديدة مطورة لتقدير قيمة النوع الأول والثاني من اخطاء الفحص وذلك من خلال معرفة مسبقة للجودة الأساسية. ومن ثم استخدام هذه الخطوات المطورة للربط بين السلوك الديناميكي لأخطاء الفحص وعملية الفحص المتكرر. وتوضح نتائج الدراسة أن هذه الخطط والاعتمادية الإحصائية لها اثر كبير ولذلك لابد من مراجعتها عند تصميم خطط الفحص.

درجة ماجستير في العلوم
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المملكة العربية السعودية - الظهران
يناير 1993م
THESIS ABSTRACT

FULL NAME OF STUDENT    IFTIKHAR ALI NADEEM

TITLE OF STUDY          EXTENSIONS IN MULTICHARACTERISTICS REPEAT INSPECTION PLANS

MAJOR FIELD             SYSTEMS ENGINEERING

DATE OF DEGREE          JANUARY, 1993

In the process of inspection, an inspector is likely to make Type–I and Type–II errors in his judgement about the product quality. This thesis investigates the effect of Type–I and Type–II errors on multicharacteristic repeat inspection plans for critical components. The practicality of inspection models already developed was enhanced by modifying them for the cases where characteristics' defective rates are statistically dependent. Based on the fact that error probabilities are a function of incoming quality, procedures are suggested for estimating the Type–I and Type–II errors for a given incoming quality. These procedures are then utilized to incorporate the dynamic behavior of inspection errors into the repeat inspection models. Results indicate that the effect of errors and statistical dependency are significant and should be incorporated in the design of the inspection plans.

MASTER OF SCIENCE DEGREE

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

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CHAPTER 1
INTRODUCTION

1.1 INTRODUCTION

Inspection is defined as the function of comparing or determining the conformance of products to established specifications. Inspection is often used to evaluate the quality of purchased and/or manufactured items. Traditionally, inspection is represented as a simple flow chart or decision tree (Figure 1.1) in which the decision process is represented by a box, incoming units and inspection criteria. Instructions are being represented by incoming arrows, and the action taken is represented by a set of outgoing arrows.

![Diagram](https://via.placeholder.com/150)

**Figure 1.1. Basic Inspection task model (Wallack and Adams, 1969)**

Inspection tasks can be classified into three basic categories: tasks involving visual scanning, tasks involving measurements, and tasks involving monitoring of a process.
The accuracy of the inspection process is a necessity to better production quality; however, the inspection accuracy is influenced by a number of factors. These factors can be categorized into three major groups:

1. Inspector-related factors: e.g. age, experience, sex, etc.

2. Task-related factors: e.g. task pacing, task perception, etc.

3. Environmental and organizational factors: e.g. illumination, temperature, motivation, etc.

Industry quality control is accomplished by Statistical Process Control (SPC) or Acceptance Sampling. Statistical Process Control involves the use of control charts, process capability studies, etc.; however, acceptance sampling is done to accept or reject a lot. Since sampling involves only a part of the product characteristics or lot, needs fewer inspectors, and incurs less handling damages. It is more economical; however, sampling has a risk of accepting a bad item or rejecting a good item. Sampling needs added planning and documentation, but provides less information about the product quality.

The emphasis of this thesis work will be on a branch of inspection plans, known as complete repeat inspection plans. Complete inspection plans are becoming increasingly important in the area of quality control due to the growth in automatic manufacturing systems that make complete inspection inexpensive and reliable. Complete inspection plans are also instituted to guard against human inspection errors. These human errors could be of two types: Type–I error (rejecting a good
component instead of accepting it) and Type-II error (accepting a bad component instead of rejecting it). The implications of these inspection errors could be catastrophic in the event of a critical component failure. A critical component has more than one characteristic, and its failure could lead to a system failure causing a catastrophe or a very high cost. In order to protect complex systems from these kinds of failures, one should institute repeat inspections, which reduce inspection errors.

The inspection plan and the models representing this plan in this thesis are developed for critical components which have several characteristics with known incoming quality. A component is classified as non-defective only if all the characteristics meet the quality specifications. The probabilities of Type-I error and Type-II error are assumed to be known. Three different types of costs are considered: (i) cost due to false rejection of a non-defective component, (ii) cost due to false acceptance of a defective component, and, finally, (iii) cost of inspection. The estimates for these costs are assumed to be available in industry.

The inspection plan studied in this thesis is shown in Figure 1.2, and it is applied as follows: an inspector inspects one particular characteristic for each component entering the inspection process; then all the accepted components go to the second inspector, who inspects the second characteristic. This chain of inspection continues until all the characteristics are inspected once. This completes one cycle of inspection. All accepted components, if necessary, go to the next cycle of inspection, and the process is repeated a total of n times before the com-
Fig 1.2  Inspection Plan for $j$-th cycle

$j = 1, 2, \ldots, n$
ponent is finally accepted. Here, $n$ is the optimal number of inspections necessary to minimize the cost per accepted component or to minimize the probability of accepting a defective component. Finally, the accepted components will be those accepted in $n$-th cycle, and the totality of rejected components will be the sum of those rejected in the 1st, 2nd, . . . , $n$-th cycles. Based on the above plan, several models were developed by Raouf (1983) and Duffuaa and Raouf (1989), to obtain the optimal $n$.

1.2 OBJECTIVES OF THE THESIS

1. To investigate the effect of inspection errors on performance measures of complete multicharacteristic inspection plans.

2. To extend the models for the multicharacteristic inspection plans for a situation where characteristic's defective rates are statistically dependent.

3. To examine the effect of inspection errors for the case of dependency.

4. To suggest procedures, for estimating inspection errors under varying incoming quality.

5. To extend models for multicharacteristic inspection for those cases where the inspection errors are varying due to the process of inspection.

6. To compare the results of the 1st plan with that of the 5th on real industrial situations.
1.3 PLAN OF THE PROPOSED WORK

The research work proposed for the thesis is based on the model developed by Duffuaa and Raouf, (1989). It will begin with investigating the effect of inspection errors on performance measures of complete inspection plans, such as Average Total Inspection (ATI), Expected Total Cost (ETC) and Average Outgoing Quality (AOQ). This will be accomplished by varying Type–I and Type–II errors and observing the effect on the performance measures.

Then the models will be extended to situations where the characteristic’s defective rates are statistically dependent. This necessitates the knowledge of joint probability mass function (j.p.m.f.) of the random variables. Using the j.p.m.f. we can obtain the individual marginal probability mass function. Since the joint and marginal mass function varies from cycle to cycle, the values for the individual random variable marginal mass function will be updated using Bayes’s theorem. After the inspection of the first characteristic, the marginal of other characteristics must be updated prior to inspecting them. This will be accomplished by updating the joint probability mass function at each stage of the inspection. This would be followed by procedures for estimating inspection errors under varying incoming quality. It will be done based on the fact that inspection errors are a function of incoming quality (Raouf and El–Feituri, 1093). The procedures suggested above for the estimation of inspection errors will then be utilized to model the variation of errors during the process of inspection; then, it will be modified to reflect this
variation. Finally, a comparison of the results obtained with and without varying the inspection errors will be presented.

1.4 THESIS ORGANIZATION

This thesis is presented in seven chapters. In chapter 2, some background on inspection error in quality control, extensive review of the literature relevant to the sampling and inspection plans under inspection error, and a brief description of Signal Detection Theory (SDT) are presented. Chapter 3 investigates the effect of inspection errors on the performance measures of complete inspection plans: such as ATI, ETC, AOQ by varying Type-I error and Type-II error.

In chapter 4, the model presented in chapter 3 is modified to situations where the characteristics' defective rate is statistically dependent. An algorithm, to obtain the optimal number of inspections, with an example is presented to demonstrate the results of the model on the performance measures.

Procedures for estimating inspector error under varying incoming quality are proposed in chapter 5. In this chapter, the results of the procedures proposed are compared with those existing in the literature. This chapter also evaluates the effect of variation in inspection errors on which deal with inspection plans. It does this by utilizing the procedures proposed to incorporate the change in the inspection errors during the process of inspection.
FLOW CHART OF THESIS ORGANIZATION

Literature Review And Overview of SDT (Chapter 2)

- Model Development & Economic Effects on ATI, ETC, AOQ (Chapter 3)
- Incorporation of Statistical Dependency & Algorithm to Obtain n* (Chapter 4)
- Procedures of Error Estimation Under Varying Inspection Errors (Chapter 5)
- Effect of Variation in Errors on Inspection Plans (Chapter 5)

Comparison of Results Obtained in Chapters 3 & 5 - Conclusion & Future Research (Chapter 6)
Chapter 5 also compares the models in chapters 3 and 5 to show the difference in performance measures for an inspection plan with and without the error variation. Chapter 6 gives conclusions, and directions for future research.
CHAPTER 2

BACKGROUND AND LITERATURE REVIEW

2.1 INTRODUCTION

In the process of inspection, an inspector makes "accept" or "reject" decisions about the product conformance, based on the predetermined standards of quality characteristics. The human inspector is subject to make Type-I error and Type-II error in his judgement about the product quality. The error of judgement made by human observers have been studied for a long time by human factor specialists. Not until around 1970, did quality control researchers start considering inspection error, and begin attempting to understand and to model their implications on plans of quality control.

Signal Detection Theory (SDT) has been used to model the sensitivity of the inspector which could be an industrial inspector. The detection performance of an observer can be evaluated by a graph called a Reciever Operating Characteristic (ROC) curve. This chapter gives background material that would be useful in following chapters, and reviews the relevant literature on acceptance sampling and complete inspection plans which are subject to inspection error.

The chapter has been organized in the following manner. Section 2 presents an overview of inspection error in quality control and literature which is relevant to the
inspection error modeling in sampling, and complete inspection plans are reviewed in section 3. An overview of SDT is presented in section 4, while conclusions are contained in section 5.

2.2 INSPECTION ERROR IN QUALITY CONTROL

The basic assumption in the design of inspection plans in traditional quality control is that the inspection process is carried free of error. This is not always true, and can be misleading. In statistical process control, control charts are used for drawing conclusions about the probability of a process going out of statistical control. The prediction power and usefulness of control charts are directly affected by measurement errors.

Inspection errors have been shown to have adverse effects on the protection provided by sampling plans. The probability of acceptance of a lot is affected by a combination of Type-I and Type-II errors. In industry repeat inspection plans are instituted for the inspection of multicharacteristic critical components to guard against these inspection errors, but not much work has been done to study the effect of inspection errors on complete inspection plans.

2.3 LITERATURE REVIEW

Harris (1968) conducted an experiment on a set of 32 defective items employing
80 naive inspectors to observe the inspection error effect. He concluded that inspec-
tion accuracy decreased with reduction in defect rate. Green and Swets (1974) studied the effects of inspection error by using Signal Detection Theory \((SDT)\), and by using a graph called the Receiver Operating Characteristic \((ROC)\) curve. Jaraidi (1983) studied the economic design of sampling plans subject to inspection error. He used \(SDT\) to model an inspection task and to evaluate capabilities of an industrial inspector in order to design more cost effective single sampling plans that incorporate inspection error. Raouf and El Feituri (1983) listed the factors that may affect the inspection accuracy, and they studied the effect on inspection accuracy by varying the incoming quality, the task complexity and the inspection rate. Shor and Raz (1988) indicated the human factors that cause inspection errors. Chandra and Susan (1980) present a method of optimizing total inspection cost assuming that measurement error is a random variable \(\sim N(0, \sigma^2)\) while the true value of the product \(\sim (\mu, \sigma^2)\). They suggested that to obtain the best estimator of the measurement of interest, an optimum number of replicates should be used. Dharale (1981) claims the distribution of defectives, due to errors in 100% inspected lots, follows negative binomial under the assumption that failure to detect a defective unit by inspectors is described as Poisson distribution. Ayoub et al (1970) presented formulas for Average Outgoing Quality \((AOQ)\) and Average Total Inspection \((ATI)\) for a single sampling plan under inspection error. Collins et al (1972) relaxed the assumption of perfect inspection of replacement for \((AOQ)\), and allowed defective replacement in the formulation of \(ATI\). Bennet et al (1974) investigated the effect of errors on a single sampling plan with known
incoming quality distributions. Case et al (1975) showed the effect of errors on quality control systems using $AOQ$ and $ATI$ as quality cost measures. They also present economic comparison, of with and without errors, under simulated ($OC$) curve behaviors. Mei et al (1975) assumed that the measurement error distribution (often normal) has a known mean (bias) and a standard deviation (imprecision). Also, the lot distribution and measurement error distribution are independent. A comparison plan could be used to enhance the protection against variable acceptance sampling risk. The author, however, does not propose any cost criteria to quantify the measurement error effect. Dorris and Foote (1978) surveyed the effect of inspection errors on standard quality control methods. The ($OC$) curve shape changes, as does the protection that an ($OC$) curve may afford for a sampling plan when inspector errors are included in models. Rosenshine and Soars (1978) presented a comparison of two optimal inspection procedures in the presence of errors. They concluded that the Bayes procedure is better, but these procedures hold under the basic assumption of exponential populations. Tang (1987) investigates the economic and the statistical impacts of inspector errors on complete inspection plans. Tang borrows Taguchi's concept of quadratic loss function to determine the expected quality cost per item after inspection. Maqsoodloo (1987) provided rectified equations for performance measures ($P_e, AOQ, ATI$) in the presence of inspection error in a multistage sampling plan. He uses the work of Collins et al (1973) as a basis of his study. Shin and Sunil (1992) studied the effect of varying inspection errors on single sampling plans. They assumed that inspection errors are functions of the sequence of the item number in the sample or the lot, and
used $AOQ$ and $ALIC$ (Average Lot Inspection Cost) as performance measures to evaluate the effects of the errors. The conclusion of this study was that the design of sampling plans must be reconsidered to include the effect of inspection errors. Roauf et al (1983) developed the initial model for determining the optimal number of repeat inspections from multicharacteristic components to minimizing the total cost per accepted component due to Type–I error, Type–II error, and cost of inspection. A rule is proposed for minimizing the cost of inspection within each inspection cycle. A proof of this rule was later provided by Duffuaa and Raouf (1989). Duffuaa and Raouf (1987) extended the model for statistically dependent multicharacteristic components. They developed a procedure, under the assumption that probability of a characteristic being defective depends upon the probability of other characteristics being defective, for finding the optimal sequence for inspection in each inspection cycle.

Duffuaa and Raouf (1989) developed three mathematical optimization models for multicharacteristics repeat inspection. The first model (cost minimization model) minimizes the total cost due to Type-I error, Type-II error and repeat inspections to determine the optimal number of repeat inspections. The second model (probability minimizing model) reduces the probability of accepting a defective component. The third model (the satisfying model) determines a satisfying solution by specifying an upper limit for total inspection cost and for the probability of accepting a defective component.

Duffuaa (1992) examined the effect of inspector errors on the expected total cost of a complete repeat inspection plan for critical components. Also, Duffuaa and
Nadeem (1992) developed an extension of the model proposed in Raouf et al, 1983 for components whose characteristics' defective rates are statistically dependent.

In the literature, no significant work is done on the effect of inspector errors on the performance measures of repeat multicharacteristics' inspection plans. Also, the case of characteristics' defective rate has not been treated extensively. In addition, all models, which are presented in the area of multicharacteristic inspection, incorporate the effect of inspector error, but they assume the errors are fixed. It has been documented in the literature that inspector errors are a function of incoming quality (Raouf and Elseituri, 1983). Therefore, this is a limitation in these models, and they need to be extended to rectify this limitation. In this thesis, we propose models under varying inspector errors.

2.4 SIGNAL DETECTION THEORY

The development of signal detection theory started around 1945. The fundamentals of the theory were formally developed and introduced by Peterson, Birdsall, and Fox (1954). Since then, the theory has found its usefulness in such diverse areas as radar communications, medical diagnoses, the psychophysics of human perception, and industrial quality control.

According to the signal detection model, an observer bases his decision on the information available prior to the presentation of the stimulus, the information content of the stimulus, the sensory analyzer mechanism, and the consequences of each decision. A stimulus may contain noise alone or it may contain a signal super-
imposed on the noise. The observer's response is "yes" or "no", corresponding to his belief that a signal has been present or not. This situation results in a two-by-two layout as shown in Table 2.1

Table 2.1: Two-by-two Layout

<table>
<thead>
<tr>
<th>Stimulus</th>
<th>Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal-plus-noise</td>
<td>Correct decision (HIT)</td>
</tr>
<tr>
<td>Noise alone</td>
<td>False alarm</td>
</tr>
</tbody>
</table>

The stimulus presented is mapped into a decision axis, on which the probability distributions for the noise alone, and signal plus noise are defined. An optimal observer chooses a criterion point on this axis as the boundary between "yes" or "no" responses, in order to optimize an objective function.

The detection performance of an observer can be evaluated by a graph called the Receiver Operating Characteristic (ROC) curve. The vertical axis on the ROC curve is "Probability of Hit", $P(HIT)$ and the horizontal axis is the, "Probability of False Alarm", $P(FA)$. The (ROC) curve for an observer can be generated either by a two-choice experiment in which the payoff matrix and prior probabilities are manipulated, or by a rating method in which the observer expresses his degree of confidence in the stimulus which is a result of the noise alone or a signal-plus-noise. Fig. 2.1 shows the chance diagonal on which $P(HIT) = P(FA)$ representing random performance where the decision is independent of the stimulus presented.
2.5 CONCLUSION

The implications of inspection errors must be considered while designing a plan of quality control. SDT can be used to model the sensitivity of an industrial inspector. In literature, inspection errors have been considered for sampling plans but no significant work has been done on the effect of these errors which deal with complete inspection plans. This is the subject of the next chapter.
CHAPTER 3

THE EFFECT OF INSPECTOR'S ERRORS ON REPEAT INSPECTION PLANS

3.1 INTRODUCTION

The error of judgement made by inspectors has been studied for a long time by human factor specialists. Not until about 1970, did quality control researchers start considering inspection errors and begin attempting to understand and to model their implications (Heikes and Montogomery, 1981). This problem has been approached from several angles in the last several years, and an attempt has been made to incorporate inspection errors into quality control techniques. Dorris and Foote (1978) surveyed the state of the knowledge in this area and found that much work remains to be done in order to adequately understand and deal with this phenomenon.

Existing models represent a fairly wide variety of situations and process behaviors (e.g., Collins and Case (1976); Tenenbien (1970); Heikes (1974). The effect of inspection error on measures like Average Outgoing Quality (AOQ) and Average Total Inspection (ATI) has been studied for sampling plans by several researchers (e.g. Case et al (1975); Beigel (1974)).
Inspection errors have been shown to have adverse effects on the protection provided by sampling plans. In most earlier studies only Type-II error (rejecting a good component) was considered, but the practical significance of Type-I errors was also questioned. The impact of inspection errors (both Type-I and Type-I) is more adverse while dealing with critical components with more than one characteristic. This calls for enforcement of repeat inspection plans in order to guard against any kind of inspection error.

Measuring inspection errors and modeling for them, represents two areas of challenging research, each depending upon one another. The content of this chapter is specified to illustrate the significant effects of the inspector's errors on repeat inspection plans, especially the model described in (Duffuaa and Raouf, 1989). This will be done by studying the effect on the inspection plans performance measures such as A0Q, ATI and ETC. This is accomplished by varying Type-I and Type-II errors for a given situation and observing the behavior of the performance measure.

This chapter, section 2, describes the model, and section 3 contains the basic relationships in the model. A detailed development of the model is provided in section 4. Section 5 presents economic effects of inspection errors, and section 6 proposes a methodology for incorporating the effect of error change in inspection plans. Conclusions are contained in section 7.
3.2 MODEL DESCRIPTION

The model is developed for components containing several characteristics for inspection with known incoming quality $p_i$. A component is classified as nondefective only if all the characteristics meet the quality specifications. The probabilities of Type-I error, $P_{1i}$, and Type-II error $P_{2i}$ are assumed to be known. Three different types of costs are considered: (1) cost due to false rejection of a nondefective component, $C_r$, (ii) cost due to false acceptance of defective component, $C_a$, and, finally (iii) cost of inspection, $C_i$. The estimates for $C_r$, $C_a$, and $C_i$ are assumed to be available in the industry.

The inspection plan applied is as follows: an inspector inspects one particular characteristic for each component entering the inspection process. All the accepted components go to the second inspector, who inspects the second characteristic. This chain of inspection continues until all the characteristics are inspected once. This completes one cycle of inspection. All accepted components, if necessary, go to the next cycle of inspection, and the process is repeated a total of $n$ times before the component is finally accepted. Here $n$ is the optimal number of inspections necessary to minimize the total cost per accepted component. Finally, the accepted components will be those which are accepted in the $n$-th cycle, and the totality of rejected components will be the sum of those rejected in the 1st, 2nd, ..., $n$-th cycles. In order to model this inspection plan, the following notation is employed.
Notation

\( M_j \) \hspace{1cm} \text{Number of components entering the } j\text{-th cycle of inspection}

\( N \) \hspace{1cm} \text{Number of characteristics in each component to be inspected}

\( P_i \) \hspace{1cm} \text{Probability of } i\text{-th characteristic in the sequence of inspection being defective entering the inspection}

\( PG \) \hspace{1cm} \text{Probability of a component being nondefective entering the inspection}

\( PGC \) \hspace{1cm} \text{Probability of a component being defective entering the inspection, the complement of } PG

\( P_{1i} \) \hspace{1cm} \text{Probability of classifying the } i\text{-th nondefective characteristic in the sequence of inspection as defective (Type-I error)}

\( P_{2i} \) \hspace{1cm} \text{Probability of classifying the } i\text{-th defective characteristic in the sequence of inspection as nondefective (Type-II error)}

\( P_{i(j)} \) \hspace{1cm} \text{Probability of } i\text{-th characteristic in the sequence of inspection being defective entering the } j\text{-th cycle}

\( PG(j) \) \hspace{1cm} \text{Probability of a component being nondefective entering the } j\text{-th cycle}

\( PGC(j) \) \hspace{1cm} \text{Probability of a component being defective entering the } j\text{-th cycle, complement of } PG(j)

\( M_{i,j} \) \hspace{1cm} \text{Number of components entering the } i\text{-th stage of inspection in the } j\text{-th cycle}

\( PG_{i,j} \) \hspace{1cm} \text{Probability of a component being nondefective in the } i\text{-th stage of the } j\text{-th cycle}
\( FR_{i,j} \) Number of falsely rejected components in the \( i \)-th stage of the \( j \)-th cycle

\( FA_{i,j} \) Number of falsely accepted components in the \( i \)-th stage of the \( j \)-th cycle

\( CA_{i,j} \) Number of correctly accepted components in the \( i \)-th stage of the \( j \)-th cycle

\( R_{i,j} \) Rate of rejection of components due to \( i \)-th characteristics in the sequence of inspection in the \( j \)-th cycle

\( A(j) \) Number of accepted components in the \( j \)-th cycle

\( CFR(j) \) Cost of false rejection in the \( j \)-th cycle

\( CFA(j) \) Cost of false acceptance in the \( j \)-th cycle

\( CI(j) \) Cost of inspection in the \( j \)-th cycle

\( TCFR \) Total cost of false rejection

\( TCFA \) Total cost of false acceptance

\( TCI \) Total cost of inspection

\( TA \) Total number of accepted components

\( E(tc)_{ij} \) Expected total cost per accepted component after \( j \) cycles of inspection

\( E(\ ) \) Expected value of the argument inside the parentheses

### 3.3 BASIC RELATIONSHIPS IN THE MODEL

The probability of the \( i \)-th characteristics being defective will vary from cycle to cycle. The relationship between \( P_i(j) \) and \( P_i \) is given below.

\[
P_i(1) = P_i
\]

(3.1)

Using Baye's theorem

\[
P_i(2) = \frac{P_i P_{2i}}{P_i P_{2i} + (1 - P_i)(1 - P_{1i})}
\]

(3.2)
Similarly

\[ P_i(3) = P_i(2)P_{2i}/[P_i(2)P_{2i} + (1 - P_i(2))(1 - P_{1i})] \]  \hspace{1cm} (3.3)

and from the symmetry of expressions (2) and (3) we get

\[ P_i(j) = P_i(j - 1)P_{2i}/[P_i(j - 1)P_{2i} + (1 - P_i(j - 1))(1 - P_{1i})] \]  \hspace{1cm} (3.4)

The probability of a characteristic being defective changes in each cycle; hence the probability of a component being nondefective also changes. It is given below:

\[ PG = \prod_{i=1}^{N}(1 - P_i) \]  \hspace{1cm} (3.5)

The probability of a component being defective is

\[ PG_C = 1 - PG \]  \hspace{1cm} (3.6)

Clearly,

\[ PG(1) = PG = \prod_{i=1}^{N}(1 - P_i(1)) \]  \hspace{1cm} (3.7)

The probability of a component being nondefective entering the \( j \)-th cycle is

\[ PG(j) = \prod_{i=1}^{N}(1 - P_i(j)) \]  \hspace{1cm} (3.8)

The probability of a component being defective entering the \( j \)-th cycle is

\[ PG_C(j) = 1 - PG(j) \]  \hspace{1cm} (3.9)

When there is no inspection, the expected total cost per accepted component will simply be the cost of false acceptance of all the defective components

\[ E(tc)_{j=0} = C_a(1 - PG) \]  \hspace{1cm} (3.10)
The expected total cost per accepted component, after \( n \) cycles of inspection, can be written as

\[
E(tc)_{j=n} = \frac{TCFR + TCFA + TCI}{TA}
\]  \hspace{1cm} (3.11)

where \( TCFR, TCFA, TCI \), and \( TA \) are as defined earlier.

### 3.4 COST MINIMIZATION MODEL

The objective of this model is to determine the optimal inspection plan for multicharacteristic components. The model minimizes the total cost per accepted component resulting from Type-I errors, Type-II errors, and cost of inspection. Given the basic relationships in the previous section, a mathematical expression for expected total cost per accepted component will be obtained. Our objective is to minimize this cost subject to the relationships governing this situation.

In order to derive the cost of inspection after \( n \) cycles of inspections, analysis of cycle 1 of inspection is necessary. All the components entering the cycle 1 go to the first inspector, who inspects the first characteristic in each component in order to classify it as defective or nondefective. This is the first stage of inspection.

Stage 1 in cycle 1: Number of components entering this stage is

\[
M_{1,1} = M_1
\]  \hspace{1cm} (3.12)

The probability of a component being nondefective is

\[
PG_{1,1} = PG
\]  \hspace{1cm} (3.13)
$E$ (number of falsely rejected components) is

$$FR_{1,1} = M_{1,1}PG_{1,1}P_{11}$$
$$= M_1PGP_{11}$$ \hspace{1cm} (3.14)

$E$ (number of falsely accepted components) is

$$FA_{1,1} = M_{1,1}[P_1P_{21} + (1 - PG_{1,1} - P_1)(1 - P_{11})]$$
$$= M[P_1P_{21} + (1 - PG - P_1)(1 - P_{11})]$$ \hspace{1cm} (3.15)

$E$ (number of correctly accepted components) is

$$CA_{1,1} = M_{1,1}PG_{1,1}(1 - P_{11})$$
$$= MPG(1 - P_{11})$$ \hspace{1cm} (3.16)

All accepted components in this stage go to the second inspector who inspects the second characteristic of each component in order to classify it as defective or nondefective.

Stage 2 of the first cycle

$$M_{2,1} = FA_{1,1} + CA_{1,1}$$
$$= M_{1,1}[P_1P_{21} + (1 - P_1)(1 - P_{11})]$$ \hspace{1cm} (3.17)

$$PG_{2,1} = CA_{1,1}/M_{2,1}$$
$$= PG(1 - P_{11})/[P_1P_{21} + (1 - P_1)(1 - P_{11})]$$ \hspace{1cm} (3.18)

$$FR_{2,1} = M_{2,1}PG_{2,1}P_{12}$$
\[ M_N = M_1 \prod_{i=1}^{N-1} \left( P_i P_{2i} + (1 - P_i)(1 - P_{1i}) \right) \]  (3.22)

\[ PG_N = PG \prod_{i=1}^{N-1} \left( (1 - P_{1i})/(P_i P_{2i} + (1 - P_i)(1 - P_{1i})) \right) \]  (3.23)

\[ FR_{N,1} = M_1 PG \prod_{i=1}^{N-1} (1 - P_{1i}) P_{1N} \]  (3.24)

\[ FA_{N,1} = M_1 \prod_{i=1}^{N-1} \left( P_i P_{2i} + (1 - P_i)(1 - P_{1i}) \right) \]
\[ \times \left[ P_N P_{2N} + (1 - PG_{N,1} - P_N)(1 - P_{1N}) \right] \]  (3.25)

\[ CA_{N,1} = M_1 PG \prod_{i=1}^{N} (1 - P_{1i}) \]  (3.26)

This completes one cycle of inspection, and the result of this cycle is described by the following equations. Number of accepted components after completing the first cycle is,

\[ A(1) = FA_{N,1} + CA_{N,1} \]  (3.27)

Cost of false rejection is

\[ CFR(1) = C_r \sum_{i=1}^{N} (FR_{i,1}) \]  (3.28)
Cost of false acceptance is

\[ CFA(1) = C_a(FA_{N,1}) \] (3.29)

Cost of inspection is

\[ CI(1) = \sum_{i=1}^{N} C_i M_{i,1} \] (3.30)

\( E \) (total cost per accepted components after one cycle of inspection is)

\[ E(tc)|_{j=1} = [CFR(1) + CFA(1) + CI(1)]/A(1) \] (3.31)

where \( CFR(1), CFA(1), CI(1) \) and \( A(1) \) are given by equations (3.28), (3.29), (3.30) and (3.27), respectively.

Before proceeding to the second cycle, it was shown in Raouf et al, 1983 that the manner in which characteristics are ordered for inspection affects the cost of inspection; in Duffuaa and Raouf, 1990 a rule is given and proved to be optimal for minimizing the cost of inspection within each inspection cycle. In minimizing the cost, this rule should be applied in each cycle. The rule says, “at inspection cycle \( j \), compute the ratio \( C_i/R_{i,j} \) for all \( i \); then, for each component, first inspect the characteristic with the least ratio, and lastly, inspect the one with the highest ratio.” This value ensures that \( CI(j) \) is minimized within each cycle.

From the analysis of cycle 1, it can easily be seen that after this cycle we can compute the new values of \( P_i(2), PG(2), M_2 \) and proceed in the same manner as in the first cycle to compute the cost of false rejection, cost of false acceptance and
cost of inspection in this cycle. Hence, by symmetry, we can obtain the results of
the n-th cycle.

\[ A(n) = FA_{N,n} + CA_{i,n} \quad (3.32) \]

\[ CFR(n) = C_r \sum_{i=1}^{N} (FR_{i,N}) \quad (3.33) \]

\[ CFA(n) = C_a (FA_{N,n}) \quad (3.34) \]

\[ CI(n) = \sum_{i=1}^{N} C_i M_{i,n} \quad (3.35) \]

The ratio used to determine the optimal ordering of characteristics in the n-th
is \( C_i/R_{i,n} \), \( i = 1, \ldots, N \) where

\[ R_{i,n} = P_i(n)(1 - P_{2i}) + (1 - P_i(n))P_{1i} \quad (3.36) \]

After \( n \) cycles of inspection we must determine the total cost of inspection per
accepted component, which consists of: total cost of false rejection \( TCFR \), total
cost of false acceptance \( TCFA \), and total cost of inspection.

\[ TCFR = \sum_{j=1}^{n} [CFR(j)] \quad (3.37) \]

\[ TCFA = CFA(n) \quad (3.38) \]

\[ TCI = \sum_{j=1}^{n} [CI(j)] \quad (3.39) \]

Total accepted components

\[ TA = A(n) \quad (3.40) \]

The above equations (3.1) through (3.38) provide the basic relationship for the
model; the purpose is to find the value of \( n \) which minimizes the expected total
cost per accepted component. The above model can be stated as

\[ \text{Min } E(tc)|_{j=n} \quad (3.41) \]

The following is an algorithm for finding a local optimal \( n \).

**Algorithm**

**Step 1** Determine the \( PG \) and \( E(tc)|_{j=0} \) from equations (3.5) and (3.10), respectively, set \( j = 1 \).

**Step 2** Compute \( P_i(j), PG(j), PG_{N,i}, M_j \) and \( C_i/R_{ij} \) for \( i = 1, 2, \ldots, N \) using equations (3.4), (3.8), (3.23), (3.27), (3.36), respectively. Arrange the ratios \( C_i/R_{ij} (i = 1, 2, \ldots, N) \) in order of decreasing magnitude. This is the optimal sequence for the \( j \)-th cycle and has been shown in (Duffuaa and Raouf, 1990).

**Step 3** Rearrange the probabilities \( P_i, P_{ii}, P_{2i}, \) and the inspection cost \( C_i \) according to the optimal sequence obtained in step 2.

**Step 4** Compute \( A(j), CFR(j), CFA(j), \) and \( CI(j) \) using equations (3.32), (3.33), (3.34), and (3.35) respectively.

**Step 5** Compute \( TCFR, TCFA, TCI \), and \( TA \) from equations (3.37), (3.38), (3.39), and (3.40) respectively.

**Step 6** Compute \( E(tc)|_{j} \), using equation (3.11).

**Step 7** If \( E(tc)|_{j} \) is less than \( E(tc)|_{j-1} \), set \( j = j + 1 \) and go to step 2; otherwise STOP \( (n = j - 1) \).

**3.5 ECONOMIC EFFECTS OF INSPECTION ERRORS FOR THE INDEPENDENT CASE**

To examine the effects of Type-I errors and Type-II errors, a batch of 100 components, each with 3 characteristics was inspected with incoming quality of
0.1, 0.2 and 0.3 for each characteristic, respectively. The type-I error and Type-II error were varied from 0.01 to 0.15 to observe the effect on expected total cost per accepted component (ETC) at an optimal number of inspection; however, cost of inspection, cost of false rejection and cost of false acceptance were used as fixed values of $100, $500, and $100,000 per component respectively. The results are shown in the Table 3.1.

3.5.1 Effects of Inspection Errors on Average Total Inspection (ATI)

The Average Total Inspection (ATI) represents the total number of characteristics inspected per inspection. This includes the number of correctly as well as falsely accepted components during the course of inspection. For an inspection plan ATI can be expressed as:

\[
ATI = \sum_{j=1}^{n} \left( M_j \sum_{k=1}^{n} \prod_{i=1}^{k-1} \left[ \frac{j}{P_i P_{2i}} + \left( 1 - \frac{j}{P_i} \right) \left( 1 - P_{1i} \right) \right] \right)
\]

ATI is also the function of the sequence of inspection at each inspection cycle.

The relationship between Type-I error and Average Total Inspection (ATI) is linearly decreasing, while ATI increases as \( E_2 \) increases exhibiting an almost linear behavior as shown in figure 3.1 and figure 3.2.
Relationship between ATI and E1
FIGURE 3.2

Relationship between ATI and E2

\( \theta_1 = 0.05 \)
\( \theta_2 = 0.15 \)
\( \theta_3 = 0.1 \)
3.5.2 Effects of Inspection Errors on Expected Total Cost \((ETC)\)

There is a significant effect of inspection errors on Expected Total Cost \((ETC)\) per inspection. \(ETC\) is the sum of inspection cost, cost of false rejection, cost of false acceptance, per accepted component. Figures 3.3 and 3.4 illustrate the \(ETC\) as a function of Type-I error and Type-II error respectively. As intuitively expected, Type-I error has more drastic effects on \(ETC\). This can be observed by comparing the slopes of each figure. The slope in case of Type-I error is considerably higher than that of Type-II error.

3.5.3 Effects of Inspection Errors on Average Outgoing Quality \((AOQ)\)

The performance measure 'Average Outgoing Quality' \((AOQ)\) is defined by the ratio:

\[
    AOQ = \frac{\text{expected number of defective components remaining after inspection}}{\text{total number of components in the lot}}.
\]

Figures 3.5 and 3.6 show the effect of Type-I and Type-II errors on \(AOQ\). Evidently, \(AOQ\) sharply increases with an increase in Type-II error. However, the effect of Type-I error on \(AOQ\) is relatively less significant.
Relationship between E2 and ETC
Figure 3.5

Relationship between $E_I$ and $A_OQ$
Relationship between EZ and A00
Table 3.1: Effect Of The Error On The Inspection Plan

<table>
<thead>
<tr>
<th>$e_1$</th>
<th>$e_2$</th>
<th>1-PG</th>
<th>ETC</th>
<th>CI</th>
<th>CFR</th>
<th>CFA</th>
<th>$A(n)$</th>
<th>$n^*$</th>
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<tbody>
<tr>
<td>1</td>
<td>0.00</td>
<td>0.01</td>
<td>0.0000</td>
<td>400.41</td>
<td>400.34</td>
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<td>401.00</td>
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<td>0.59</td>
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<td>0.0006</td>
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<td>2.78</td>
<td>81</td>
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<td>0.00</td>
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<td>0.0000</td>
<td>532.57</td>
<td>432.29</td>
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<td>67</td>
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<tr>
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<td>0.03</td>
<td>0.03</td>
<td>0.0002</td>
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3.6 METHODOLOGY FOR INCORPORATING ERROR CHANGE IN THE COMPLETE INSPECTION PLAN

The optimal complete inspection plan is developed using the optimization model presented in Section 2. Type-I and Type-II errors are assumed known and fixed. In reality these errors are functions of incoming quality. In Jaraidi, 1983, a model is developed to express Type-I error \( P_{1i} \) and Type-II error \( P_{2i} \) as a function of incoming quality.

\[
P_{1i} = f_{1i}(P_i) \tag{3.42}
\]

\[
P_{2i} = f_{2i}(P_i) \tag{3.43}
\]

The above function can be incorporated in the model given in Section 2. After the inspection of each characteristic, the error is updated for each characteristic using equations (3.42) and (3.43) before the next inspection. In this fashion the model incorporates the change in Type-I and Type-II errors, and hence an inspection plan results. This will be addressed in more detail in chapter 5.

3.7 CONCLUSION

The chief purpose of this chapter was to illustrate the effects of Type-I error \( E_1 \) and Type-II errors \( E_2 \) on repeat inspection plans. This effect was quantified by observing the change in \( ETC \), \( AOQ \) and \( ATI \) under the varied values of Type-I error \( E_1 \) and Type-II error \( E_2 \). It was observed that as Type-I error
increases, the \((ETC)\) also increases at a faster rate than in the case of Type-II error where the \((ETC)\) does increase, but at a very slow rate. This indicates that \((E_1)\) leaves more serious effects on \(ETC\) than that of \((E_2)\). Similarly, Average Outgoing Quality \((AOQ)\) also increases as \((E_2)\) goes up, but it does not change significantly as \((E_1)\) varies. Nevertheless, the combined effect of increasing \(E_1\) and \((E_2)\) elevates the \(AOQ\) values more than \((E_2)\) does it alone.

The average total inspection is illustrated as a function of Type-I error and Type-II error. \(ATI\) decreases as Type-I error increases. This is obvious since an increment in probability of rejecting a good component will reduce the number of components to be inspected in the subsequent inspection cycles. However, we observed the opposite effect on \(ATI\) as Type-II error was increased. The relationship between \(ATI\) and \(E_2\) is more or less linear and directly proportional.
CHAPTER 4
A STATISTICALLY DEPENDENT
MULTICHA RACTERISTIC INSPECTION PLAN

4.1 Introduction

In many situations components have characteristics which depend on each other. This implies a dependency in defective rates of characteristics. For example, a welded joint has four key quality characteristics: tensile strength, fatigue strength, toughness and fracture toughness. The quality of these four characteristics is dependent on one another.

All the models presented in the area of multicharacteristic inspection plans assume that the characteristic’s rate is independent. There is a need to modify these models for the case where the characteristics’ defective rate is statistically dependent. This requires knowledge about the joint probability mass function (u.p.m.f.) of the random variables representing characteristics’ defective rate. Using the j.p.m.f. we can obtain the individual marginal probability mass function. Since the marginal probability mass function varies from cycle to cycle, the values for the individual random variable marginal mass function must be updated using Bayes’s theorem. After the inspection of the first characteristic, the marginal of others must be updated prior to inspecting them. Using the above concepts, the models will be modified to handle dependency case.
In this chapter, the models in Duffuaa and Raouf, 1989 are extended, and the rules for updating the quality of dependent characteristics are proposed. These rules are consistent with basic probability rules. An alternative scheme for sequencing characteristic for inspection is suggested. In section 2, the model is described. Section 3 contains the detailed development of the model. Section 4 presents a methodology for obtaining the optimal number of inspections, and an example which demonstrates the results of the new model. Section 5 evaluates the effects of the dependency model on the inspection performance measures: \( AOQ \), \( ETC \) and \( ATI \). Conclusion are contained in section 6.

4.2 MODEL DESCRIPTION

The model is developed for components with several characteristics. A component is rejected if one of its characteristics is found to be defective. A component is accepted if all of its characteristics meet the quality specifications. We denote a random variable \( X_i \) which takes the value 0 if characteristic \( i \) is defective, and 1 if characteristic \( i \) is nondefective. The joint probability density function of the multivariate random variable \( X = (x_1, x_2, ..., x_N) \) is assumed to be known or to be estimated emperically from the data. The probability of a type I error \( P_{1i} \), and type II error \( P_{2i} \) is assumed to be known. Three types of costs are considered: (i) cost due to false rejection of a nondefective component, \( C_r \), (ii) cost due to false acceptance of a defective component, \( C_a \), and (iii) cost of inspection, \( C_i \). The estimates of \( C_r, C_a \) and \( C_i \) are assumed to be known.
The inspection plan which is applied is as follows: an inspector inspects one particular characteristic for each component that enters the inspection process. Then all the accepted components are passed on to the second inspector who inspects the second characteristic. Again, the accepted components are passed on to the third inspector who inspects the third characteristic. This pattern of inspection continues until all characteristics are inspected once. This completes one cycle of inspection. Each cycle consists of $N$ stages of inspections. Components which are accepted in the first cycle, pass to the second cycle of inspection if needed. These cycles of inspection are repeated $n$ times before a component is accepted ($n$ is unknown and we would like to determine $n$ in order to minimize total expected cost per accepted component). Those components which complete $n$ inspection cycles without being rejected are the accepted ones. The rejected components will be the sum of all those rejected in all inspection cycles. Determining the optimal $n$ will provide the optimal inspection plan.

Notations

$M_j$ Number of components entering the $j^{th}$ cycle of inspection

$N$ Number of characteristics in each component to be inspected

$X_i$ A discrete random variable which takes value 0 if characteristic $i$ is defective and 1 if it is nondefective;

$P(x_1, x_2, \ldots, x_N)$ The joint probability mass function of the random variables $X_i$, $i=1, \ldots, N$ at the start of the inspection
$P_i(x_i)$  The marginal probability mass function of the random variable $X_i$

$P_i(0)$  The probability of the $i^{th}$ characteristic is defective entering the first inspection cycle

$jP(x_1, x_2, ..., x_N)$  The joint probability mass function of the random variables $X_i$ for a component entering the $j^{th}$ cycle of inspection

$k_j p_i(x_i)$  The marginal probability density function of the random variable $X_i$ for a component in $j^{th}$ cycle entering the $k^{th}$ stage of inspection

$M_{i,j}$  Number of components entering the $i^{th}$ stage of inspection in the $j^{th}$ inspection cycle

$PG_{i,j}$  Probability of a component being nondefective entering the $i^{th}$ stage of the $j^{th}$ cycle

$PGC_{i,j}$  Probability of a component being defective entering the $i^{th}$ stage of the $j^{th}$ cycle

$FR_{i,j}$  Number of falsely rejected components in the $i^{th}$ stage of the $j^{th}$ inspection cycle

$FA_{i,j}$  Number of falsely accepted components in the $i^{th}$ stage of the $j^{th}$ cycle

$CA_{i,j}$  Number of correctly accepted components in the $i^{th}$ stage of the $j^{th}$ cycle

$R_{i,j,k}$  Rate of rejection of components due to the $i^{th}$ characteristic at the $k^{th}$ stage of the $j^{th}$ cycle

$A(j)$  Number of accepted components in the $j^{th}$ cycle
CFR(j) Cost of false rejection in the j\textsuperscript{th} cycle
CFA(j) Cost of false acceptance in the j\textsuperscript{th} cycle
CI(j) Cost of inspection in the j\textsuperscript{th} cycle
TCFR Total cost of false rejection
TCFA Total cost of false acceptance

\( kj P(x_1, x_2, \ldots, x_N) \) The joint probability mass function for the random variable

\( X_i \) \text{ i=1,\ldots,N } for a component in the j\textsuperscript{th} cycle entering

the k\textsuperscript{th} stage of inspection

TCI Total cost of inspection

TA Total number of accepted components

\( E(tc)_{ij} \) Expected total cost per accepted component after j cycles

of inspection

E( ) Expected value of the argument inside the parenthesis

\section*{4.3 MODEL DEVELOPMENT}

At the start of inspection, we know the joint probability mass function (j.p.m.f) for the random variable \((X_1, X_2, \ldots, X_N)\). Using the j.p.m.f, we can obtain the individual marginal probability mass functions:

\[
P_i(x_i) = \sum_{x_1} \sum_{x_2} \sum_{x_3} \ldots \sum_{x_{i-1}} \sum_{x_i} \ldots \sum_{x_N} P(x_1, x_2, \ldots, x_N)
\]  \hspace{1cm} (4.1)

The joint and marginal mass functions will vary from cycle to cycle

\[
1P_i(0) = P_i(0)
\]  \hspace{1cm} (4.2)
Using Bayes’s theorem

\[
^2P_i(0) = P_i(0)P_{2i}/[P_i(0)P_{2i} + (1 - P_i(0))(1 - P_{2i})] \tag{4.3}
\]

\[
^2P_i(1) = 1 - ^2P_i(0) \tag{4.4}
\]

Equations (4.2) and (4.3) provide the updated values for the individual random variable marginal mass function. In general, the marginal probability mass function for the \(i\)-th characteristic at the \(j\)-th cycle of inspection is:

\[
^jP_i(0) = ^{j-1}P_i(0)P_{2i}/[^{j-1}P_i(0)P_{2i} + (1 - ^{j-1}P_i)(1 - P_{2i})] \tag{4.5}
\]

\[
^jP_i(1) = 1 - ^jP_i(0)
\]

After the first inspection of characteristic \(i\), \(P_i(x_i)\) can be obtained. Due to the statistical dependency between \(i\) and the other characteristics, we cannot proceed and inspect other characteristics. The marginals of other characteristics must be updated prior to inspecting them. This is accomplished by updating the joint probability mass function in the following manner. After inspecting characteristic \(i\) at the first stage of cycle 1 the rule for updating the joint probability mass function is

\[
^{1,1}P(x_1, x_2, ..., x_n) = P(x_1, x_2, ..., x_n)
\]

\[
^{2,1}P(x_1, x_2, ..., x_n) = ^{1,1}P(x_1, x_2, ..., x_n)^{2P_i(x_i)}P_i(x_i) \tag{4.6}
\]

I.e. we multiply the old joint probability mass function by the updated marginal mass function for characteristic \(i\) (which is just inspected at the first stage in the first cycle) and then we divide by the old marginal density function of the
inspected characteristic. It is important to show that the resulting mass function is a probability mass function which is going to be shown at the end of this section. After obtaining the updated joint mass function, we can obtain the marginal for each characteristic. Then we can proceed to inspect the second characteristic and so on until a cycle of inspection is completed. At the end of the cycle, we can compute the probability that the component is non-defective, which is given by:

$$PG_{N,1} = \frac{N!}{1!} P(1,1,1,...,1)$$  \hspace{1cm} (4.7)

The expected total cost per accepted component after n cycles of inspection is given as

$$E(tc)|_{j=n} = \frac{TCFR + TCFR + TC1}{TA}$$  \hspace{1cm} (4.8)

If no inspection is performed we incur only the cost of false acceptance

$$E(tc)|_{j=0} = C_a(1 - P(1,1,1,...,1))$$  \hspace{1cm} (4.9)

The objective is to determine the optimal n. Prior to presenting the procedure of how to find the optimal n, we prove the following lemma which shows that the updated function given in equation (4.6) is a probability mass function.

**Lemma.** Let us assume we are at the kth stage of the inspection in the jth cycle and at stage k-1 characteristic i was inspected then the following updated function is a probability mass function provided $$k-1,j P(x_1,x_2,...,x_N)$$ is a probability mass function.

$$k,j P(x_1,x_2,...,x_N) = \frac{k,j P_i(x_i)}{k-1,j P(x_i)}$$  \hspace{1cm} (4.10)
Proof:

\[ k^{-1,j} P(x_1, x_2, ..., x_N), k^j P(x_i), k^{-1,j} P(x_i) \geq 0 \]

Therefore,

\[ k^j P(x_1, x_2, ..., x_N) \geq 0. \]

Also,

\[
\sum_{x_1} \cdots \sum_{x_N} k^{-1,j} P(x_1, ..., x_N) \frac{k^j P(x_i)}{k^{-1,j} P(x_i)} = \sum_{x_i} k^{-1,j} P(x_i) \sum_{x_2} \cdots \sum_{x_{i-1}} k^{-1,j} P(x_1, x_2, ..., x_{i-1}) = \sum_{x_i} k^{-1,j} P(x_i) \]

\[
= \sum_{x_i} k^j P(x_i) = 1 \tag{4.11}
\]

Therefore the updated function is a probability mass function.

4.3.1 Analysis of the \( j^{th} \) cycle

Stage 1

Let \( M_j \) be the number of components entering the \( j^{th} \) cycle of inspection

\[ M_{1,j} = M_j \]

The probability of a component being nondefective is

\[ PG_{i,j} = PG_{N,j-1} = N^{j-1} P(1,1,1,...,1) \tag{4.12} \]

E [number of falsely rejected components]

\[ FR_{1,j} = M_{1,j} PG_{1,j} [1 - N^{j-1} P_1(0)] P_{11} \tag{4.13} \]
\[ FA_{1,j} = M_{1,j}[1 - PG_{1,j}]^{N_j - 1}P_1(0)P_{21} \]  

(4.14)

\[ CA_{1,j} = M_{1,j}PG_{1,j}[1 - N_j^{-1} P_1(0)](1 - P_{11}) \]  

(4.15)

All accepted components in this stage proceed to the second stage to inspect the second characteristic.

\[ M_{2,j} = FA_{1,j} + CA_{1,j} \]  

(4.16)

\[ PG_{2,j} = 2^j P(1,1,1,\ldots,1) \]  

(4.17)

\[ FR_{2,j} = M_{2,j}PG_{2,j}[1 - 2^j P_2(0)]P_{12} \]  

(4.18)

\[ FA_{2,j} = M_{2,j}(1 - PG_{2,j})2^j P_2(0)P_{22} \]  

(4.19)

\[ CA_{2,j} = M_{2,j}PG_{2,j}[1 - 2^j P_2(0)](1 - P_{12}) \]  

(4.20)

Similarly for the \( i^{th} \) stage of the \( j^{th} \) cycle,

\[ M_{i,j} = FA_{i-1,j} + CA_{i-1,j} \]  

(4.21)

\[ PG_{i,j} = i^j P(1,1,1,\ldots,1) \]  

(4.22)

\[ FR_{i,j} = M_{i,j}PG_{i,j}[1 - i^j P_i(0)]P_{12} \]  

(4.23)

\[ FA_{i,j} = M_{i,j}(1 - PG_{i,j})i^j P_i(0)P_{2i} \]  

(4.24)

\[ CA_{i,j} = M_{i,j}PG_{i,j}[1 - i^j P_i(0)](1 - P_{1i}) \]  

(4.25)

From the symmetry of the expressions, we can write

\[ M_{N,j} = FA_{N-1,j} + CA_{N-1,j} \]  

(4.26)
\[ FR_{N,j} = M_{N,j} PG_{N,j}[1 - N^j P_N(0)] P_{1N} \]  
(4.27)

\[ FA_{N,j} = M_{N,j} (1 - PG_{N,j})^N P_N(0) P_{2N} \]  
(4.28)

\[ CA_{N,j} = M_{N,j} PG_{N,j}[1 - N^j P_N(0)](1 - P_{1N}) \]  
(4.29)

The number of components entering the first stage of the \((j + 1)\) cycle is

\[ M_{j+1} = M_{1,j+1} = CA_{N,j} + FA_{N,j} \]  
(4.30)

By replacing \(M_j\) by \(M_{j+1}\) and the updated version of the joint and marginal probability mass functions the equations governing the number of falsely rejected, falsely accepted and correctly accepted for each cycle can be obtained. The following costs are associated with the \(j^{th}\) cycle of inspection.

Cost of false rejection for the \(j^{th}\) cycle is

\[ CFR(j) = C_{r} \sum_{i=1}^{N} FR_{i,j} \]  
(4.31)

Cost of false acceptance for the \(j^{th}\) cycle

\[ CFA(j) = C_{a} FA_{N,j} \]  
(4.32)

Cost of inspection for the \(j^{th}\) cycle

\[ CI(j) = \sum_{i=1}^{N} C_i M_{i,j} \]  
(4.33)
4.3.2 Minimization of Inspection Cost in Cycle $j, CI(j)$

The sequence in which characteristics are inspected influences the expected cost of inspection. The characteristic with lower cost and higher probability of being defective should be inspected first. Duffuaa and Raouf (1990) proposed an optimal rule for sequencing characteristics for inspection for the case in which the characteristics’ defective rates are statistically independent. In general, the rule may not be true for the dependent case. In this paper, we propose the following sequencing rules: At the beginning of cycle $j$, compute the following ratio:

$$r_{i,j} = C_i/R_{i,j}$$  \hspace{1cm} (4.34)

where

$$R_{i,j} = N_{j-1} P_i(0)(1 - P_{2i}) + (1 - N_{j-1} P_i(0)) P_{1i}$$  \hspace{1cm} (4.35)

order the ratios in an increasing order. Employ this order for inspection in this cycle.

Rule 2: at the $k^{th}$ stage of the $j^{th}$ inspection cycle $k-1$ characteristics have been inspected. For the remaining compute the following ratio:

$$r_{i,j,k} = C_i/R_{i,j,k}$$  \hspace{1cm} (4.36)

where

$$R_{i,j} = N_{j-1} P_i(0)(1 - P_{2i}) + (1 - k^{-1} P_{i}(0)) P_{1i}$$  \hspace{1cm} (4.37)

Select the characteristic with the lowest ratio for inspection at the $k^{th}$ stage. At stage 1 use the ratio given in equations (4.31) and (4.32). The general expression
for the total expected cost per expected component after $n$ cycles of inspection is given by equation (4.7). To determine the expected cost, it is required to determine the total cost of false rejection TCFR, the total cost of false acceptance TCFA, and the total cost of inspection and the number of expected components, TA.

$$TCFR = \sum_{j=1}^{n}[CFR(j)]$$  \hspace{1cm} (4.38)

$$TCFA = CFA(n) = CFA_{N,n}$$  \hspace{1cm} (4.39)

$$TCI = \sum_{j=1}^{n}[CI(j)] = \sum_{j=1}^{n} \sum_{i=1}^{N} C_i M_{i,j}$$  \hspace{1cm} (4.40)

$$TA = CA_{N,n} + FA_{N,n}$$  \hspace{1cm} (4.41)

Substituting TCFR, TCFA, TCI and TA in equation (4.7) we obtain the total expected cost at the end of the $n^{th}$ cycle. Next an algorithm is presented to find $n$.

### 4.4. AN ALGORITHM FOR DETERMINING THE OPTIMAL INSPECTION PLAN

Algorithm

Step 1 : Compute $^jP_i(0), \ M_j$ for $i = 1, 2, ..., N$ from equations (4.5) and (4.30). Select the $i^{th}$ characteristic for inspection based on the ratios in equation (4.36). Repeat this until all characteristics are inspected.

Step 2 : Compute $FR_{i,j}$ and $M_{i,j}$ for each $i$ and $FA_{N,j}$ from equations (4.23), (4.21) and (4.28) respectively.
Step 4: Compute CFR(j), CFA(j) and CI(j) from equations (4.31), (4.32) and (4.33) respectively.

Step 5: Compute TCFR, TCFA, TCI and TA using equations (4.38), (4.39), (4.40) and (4.41) respectively.

Step 6: Compute $E(tc)|_j$ from equation (4.8).

Step 7: If $E(tc)|_j < E(tc)|_{j-1}$, set $j = j + 1$ and go to 2 otherwise stop, $n=j-1$

Following is an example to demonstrate the developed model and the proposed algorithm. The same example is solved using the Raouf et al. (1983) model which assumes independence. The results are shown below. The model presented in section 3 depicts the process of repeat inspections for critical components with multicharacteristics having dependent defective rates. The output of the model is an optimal inspection plan. A program was developed (see Appendix–A) implementing the above algorithm. The results of the following example are given below. In the example, we have 100 components, each with three characteristics. The joint probability mass function for the defective rates is $P(0,0,0) = 0.05$, $P(0,0,1) = 0.05$, $P(0,1,0) = 0.05$, $P(0,1,1) = 0.1$, $P(1,0,0) = 0.05$, $P(1,0,1) = 0.05$, $P(1,1,0) = 0.15$, $P(1,1,1) = 0.5$. Other input values are, $C_l=100$, $C_a=100,000$, $C_r=1000$, $P_{1i}=0.01$ and $P_{2i}=0.015$. The results of the model are as follows:
Expected Total Cost (ETC) without inspection = 50,000.00

---Starting the inspection (First Cycle)---

\[ PG(1) = 0.9909216 \]

ETC(1) = 1429.04

\[ A(1) = 48.95 \]

---end of cycle 1 (Beginning Cycle 2)---

\[ PG(2) = 0.9998594 \]

ETC(2) = 906.86

\[ A(2) = 47.08 \]

---end of cycle 2 (beginning Cycle 3)---

\[ PG(3) = 0.9999937 \]

ETC(3) = 1285.18

\[ A(3) = 45.68 \]

---end of cycle 3---

The optimum number of inspection \((n^*) = 2\)

The results of Raouf et al. model (1983) for the same example assuming independence are as follows: Expected Total Cost before inspection = 58,000.00

---Starting the inspection (First cycle)---

\[ PG(1) = 0.9848253 \]

ETC(1) = 2083.61

\[ A(1) = 41.38 \]

---end of cycle 1 (beginning of cycle 2)---
PG(2) = 0.997678
ETC(2) = 955.07
A(2) = 39.55
———end of cycle 2 (beginning of cycle 3)———

PG(3) = 0.999765
ETC(3) = 719.61
A(3) = 38.37
———end of cycle 3 (beginning of cycle 4)———

PG(4) = 0.999997678
ETC(4) = 749.61
A(4) = 38.15
———end of cycle 4———

The optimum number of cycle ($n^*$) = 3

The model in this chapter suggests a complete inspection with three stages and two cycles. On other hand, the Raouf et al. model (1983) suggests a different plan with three stages and three cycles. The accepted component from the inspection plan suggested by the model has less probability of being defective; therefore, it is more reliable. The results of the model in this paper indicate that the effect of statistical dependency is significant, and must be incorporated in the design of the inspection plan.
4.5 ECONOMIC EFFECTS OF INSPECTION ERRORS FOR THE DEPENDENT CASE

The effects of inspection errors (Type–I error and Type–II error) are examined on a batch of 100 components, each with three characteristics. The joint probability mass function for the defective rates is \( P(0,0,0) = 0.5, \quad P(0,0,1) = 0.05, \quad P(0,1,0) = 0.05, \quad P(1,1,0) = 0.13, \quad P(1,0,0) = 0.05, \quad P(1,0,1) = 0.05, \quad P(0,1,1) = 0.15, \quad P(1,1,1) = 0.5. \) The values of Type–I error and Type–II error were varied from 0.00 to 0.15, while \( C_a = 100,000 \) component, \( C_r = 500 \) component and \( C_i = 100 \) component for each characteristic were fixed. The effects of the inspection errors on Expected Total Cost (ETC) per component and Average Total Inspection (ATI) are observed as shown in Table 4.1. Moreover, the behavior of Average Outgoing Quality (AOQ) under a varied average incoming quality using different pairs of Type–I and Type–II errors is also presented.

4.5.1 Effects of Inspection Errors on Expected Total Cost (ETC)

Expected Total Cost (ETC) is calculated per accepted component, and it includes the cost of false acceptance, cost of false rejection and cost of inspection per accepted component. ETC increases as Type–I error \( (E_1) \) increases, but it stabilizes at a lower value of \( E_2 \) as shown in Figure 4.1. However, it increases drastically with higher values of \( E_2 \) (e.g., \( E_2 = 0.15 \)). Similarly, ETC goes up as Type–II error \( (E_2) \) increases. Figure 4.2 shows that the combined effect of \( E_1 \) and
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Figure 4.1

Relationship between ET and ETC
$E_2$ is more serious, and it shows that $ETC$ elevates in an almost linear trend. The slope of the expected cost function in case of $E_2$ is higher than that of $E_1$.

### 4.5.2 Effects of Inspection Errors on Average Total Inspection ($ATI$)

Average Total Inspection ($ATI$) is the expected total number of inspections performed to reach an optimal inspection plan. The relationship is expressed in Section 3.5.1. $ATI$ is also the function of the sequence of inspection at each inspection cycle. $ATI$ decreases sharply as Type-I error ($E_1$) increases. This inversely proportional relationship is shown in Figure 4.3. On the other hand, $ATI$ increases as Type-II error increases as exhibited in Figure 4.4. However, $ATI$ is observed to be very stable at a higher value of $E_1$ (e.g., $E_1 = 0.15$, $E_2 = 0.1$) because the probability of rejecting a good component is also high.

### 4.5.3 The Behavior of Average Outgoing Quality ($AOQ$) under the Varied Values of Incoming Quality

There is no significant effect on $AOQ$ under the different values of incoming quality. The $AOQ$ was observed almost unchanged when incoming quality was increased from 0.5 to 0.515. Nonetheless, $AOQ$ is very much affected by the inspection errors. The relationship of $AOQ$ and Average Incoming Quality ($AIQ$) is shown in Figure 4.5 under different pairs of $E_1$ and $E_2$. Evidently, $E_1$ has more adverse effects than $E_2$ because when $E_2$ was increased from 0.05 to 0.1 with a fixed value $E_2 = 0.15$, $AOQ$ increased from 0.012 to 0.098. The change in $AOQ$
Relationship Between AOQ and AIQ
in case of \( E_2 \) for a similar value was lesser than that of \( E_1 \).

4.6 CONCLUSION

In this chapter, the models developed by Duffuua and Raouf (1989) have been extended to the case where characteristic defective rate is statistically dependent. The necessary software to run the models is also developed. The effect of dependency is major and must be incorporated in the inspection plan as shown in this chapter.

The application of the model is demonstrated by an example. The result of the model is an optimal inspection plan which can be used for controlling the quality of critical components.

The effect of inspection errors on \( ETC \), \( ETI \) and \( AOQ \) is investigated. \( ETC \) increases as the Type-I error and Type-II error increase, but the slope of the cost function in the case of Type-II error is higher than that of Type-I error. Type-II error leaves very adverse effects on \( ATI \), while it is just opposite in the case of Type-I error. We also found that \( AOQ \) is not very much sensitive to incoming quality as it was observed to be stable under different values of incoming quality. The results obtained in chapter 3 and 4 are consistent with the ones in the literature obtained for sampling plans.
CHAPTER 5

REPEAT INSPECTION PLANS UNDER VARYING INSPECTION ERRORS

5.1 INTRODUCTION

In the process of inspection, a characteristic's defective rate decreases each time it is inspected. This is discussed in detail in chapter 3. The inspection errors (Type-I error and Type-II error) are largely dependent on incoming quality (Raouf and Elfeturi, 1983). In general, it has been found that an increase in incoming quality results in a decreased Type-II error probability and increased Type-I error rate (Harris, 1968). In complete inspection plans, the defective rate of a characteristic is updated each time it is inspected. Since the inspection error probabilities are also a function of characteristics' defective rate, therefore, error probabilities are also needed to be updated at each stage of the inspection plan. It has been shown in the literature that Type-I error increases as a characteristic's defective rate is increased. On the other hand, Type-II error decreases with an increase in fraction defectives in a given lot. In order to update the error change at each stage of an inspection plan, there is a need to estimate the error probability for modeling of varying inspection errors.

Signal Detection Theory (SDT) has been found very useful in modeling the performance of an industrial inspector (Wallack and Adams, 1969). (In chapter 2, an overview of SDT is given). In the context of complete inspection plans, the
incoming quality is equivalent to the probability of occurrence of a signal. Jaraiedi (1983) has offered two procedures of inspection error estimation using SDT and ROC curves. ROC curve is a means to evaluate and compare the performance of inspectors for any given value of costs and value associated with the accept-reject type of decisions made by inspectors. The errors of judgement made by inspectors have a direct impact on the overall cost of quality control systems. The ROC curve can be used to quantify the inspection error probabilities and the cost of committing these errors. With a given fraction defective ($P$), an estimate of Type-I ($E_1$) and Type-II ($E_2$) errors can be obtained from the ROC curve. This leads to the development of a functional relationship between fraction defectives, Type-I error ($E_1$), and Type-II error ($E_2$).

The functional relationships developed between $P$ and $E_1$, and $P$ and $E_2$ can be used to incorporate the effect of varying inspection errors into the repeat inspection models. The significance of this effect can be highlighted by comparing the results of the models with and without varying the inspection errors.

In this chapter, we are proposing procedures to estimate the Type-I error and Type-II error as a function of incoming quality of a characteristic entering an inspection cycle of a multicharacteristic inspection plan. Section 2 of the chapter describes the work done by Jaraiedi (1983) on error probabilities estimation. Section 3 gives the proposed procedure for ROC and errors estimation while section 4 highlights the importance of error variation in inspection plans. Section 5
evaluates the effect of inspection error on repeat inspection plans under varying inspection errors. Section 6 provides a comparison between the results of the models proposed in chapter 3, and the results of those presented in the models under varying inspection errors. Section 7 concludes this chapter.

5.2 RELATIONSHIP BETWEEN 'P' AND ERROR PROBABILITIES AS GIVEN BY JARAIEDI (1983)

Jaraiedi used Receiver's Operating Characteristic (ROC) curve to develop a functional relationship between defective rate (P) and probability of Type-I error and Type-II error.

The ROC analysis of the detection task has, where the observation interval is constant, revealed that a change in P results in a significant change in the level of detection of defects (Fox and Haslegrave, 1969), and causes the observation to shift along the ROC curve due to criterion adjustments made by inspectors. In such a case, one can assume that the length of the ROC curve travelled due to change in P which is proportional to the amount of the change. An increase in P causes a shift towards the upper corner of the ROC space, while a decrease in P has the opposite effect. Based on this assumption, a method for estimation of two types of error probabilities is developed.

Given two unknown defect rates and their corresponding points on the ROC curve, we see that the problem is how to find the point on the curve which cor-
responds to a value of $P$ between the two known points. To estimate the Type-I and Type-II error probabilities at a certain incoming fraction defective $P$ which is between the two known points $p_1$ and $p_2$, the length of the curve between $x_1$, corresponding to $p_1$, and $x_2$, corresponding to $p_2$ can be used. Using the assumption that distance travelled on the curve is proportional to the change in signal rate, it can be seen from Figure 5-1 that

$$\frac{S_{x_1x_2}}{S_{x_1x_p}} = \frac{p_2 - p_1}{p - p_1}$$

$$\frac{h(x_2) - h(x_1)}{h(x_p) - h(x_1)} = \frac{p_2 - p_1}{p - p_1}$$

Solving for $h(x_p)$ the result is

$$h(x_p) = \frac{p - p_1}{p_2 - p_1} [h(x_2) - h(x_1)] + h(x_1)$$

Hence, the Type-I error probability, $E_1$, becomes:

$$E_1 = h^{-1}(x_p)$$

and, given the equation for ROC curve, the Type-II error probability is given as:

$$E_2 = 100 - f(E_1)$$

Using a stepwise regression and fractional power function approach at $\alpha = 0.05$ the functional form of $E_1 = f(p')$, Jaraiedi found that estimates for Type-I Type-II errors as a function of $P$ are as follows

$$E_1 = 16.54 + 1.6p - 37.8p^{1/2} + 105.38p^{1/3} - 84.98p^{1/5}$$
Fig. 5.1
Computation of Unknown Error Probabilities
for Type–II error, the relationship found is as

\[
\ln(E_2) = -0.56p + 16.52p^{1/2} - 41.94p^{1/3} + 29.52p^{1/5}
\]

5.3 PROCEDURE FOR ESTIMATION OF ERROR PROBABILITIES

The procedures presented are based on the ROC curve which is fitted between probability of false alarm (Type–I error) and the probability of hit (Type–II error). The steps of the procedure are as follows:

1. Use a constrained regression model for fitting the ROC curve. The functions selected for the fit should reflect the properties of the ROC curve which are

\[
Y'' = \frac{dY}{dX} > 0 \quad \text{for } 0 < X < 100
\]

and

\[
Y''' = \frac{d^2Y}{d^2X} < 0 \quad \text{for } 0 < X < 100
\]

where \( Y' \) (first derivative) guarantees that the curve does not peak in the \((0, 100)\) range and \( Y'' \) (second derivative) ensures that it is a concave down in the same region.

2. Use the method in section 5.2 to generate data for \( E_1, E_2 \) and \( P \) from ROC curve.

3. Develop a constrained regression model to estimate \( E_1 \) and \( E_2 \) as a function of \( P \). The model should reflect the expected behavior of \( E_1 \) and \( E_2 \) versus
The problem was formulated as a constrained regression model. It is a non-linear programming problem:

\[
\text{Min} \sum_{i=1}^{n} [(1 - E_2) - f(E_1)]^2
\]

s.t.

\[
\begin{align*}
    & f'(E_1) \geq 0 \\
    & f''(E_1) \leq 0 \\
    & f(0) = 0 \\
    & f(100) = 100 \\
    & n = 1, 2, \ldots, 4.
\end{align*}
\]

The data set (Appendix-D) used to find the best fit for ROC curve was adopted from the results of an experiment reported by Harris (1968). The following parameter estimates were obtained giving the best fit using GINO (General INteractive Optimizer):

\[
\begin{align*}
    A_1 &= -54.6177 \\
    A_2 &= 127.3531 \\
    N_1 &= -10.6549 \\
    N_2 &= 74.7161
\end{align*}
\]
The function that describes the best fit was of the following form:

\[ P(\text{Hit}) = A_1 P(FA)^{1/N_1} + A_2 P(FA)^{1/N_2} \]

or

\[ [1 - E_2] = A_1(E_1)^{1/N_1} + A_2(E_1)^{1/N_2} \]

Hence, the \( ROC \) equation becomes

\[ Y(X) = -54.6177 X^{-0.093862} + 1273531 X^{0.013384} \]

The Jaraiedi's \( ROC \) equation was also modeled as GINO problem and was found to have more error than that of ours. Therefore, ours was a better fit.

Once the \( ROC \) curve has been fitted, many points corresponding to Type-I and Type-II error can be generated following this method as described in the previous section. A Fortran program was developed (Appendix-B) that uses the numerical integration method to generate the desired points for a given defective rate as given in Table 5-1 below:

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Table 5.1: Corresponding \( E_1 \) and \( E_2 \) for a given \( P \)

Figure 5.2 shows the \( ROC \) curve obtained. The following non-linear model was used to find the estimates of the functional relationships between \( P \) and \( E_1 \) and \( P \) and \( E_2 \).
Model for $E_1$

$$\min \sum_{i=1}^{n} [E_{1i} - f_1(P_i)]^2$$

s.t.

$$f'_1(P_i) \geq 0 \quad \text{[increasing function]}$$

$$f_1(P_i) \leq 0.5$$

$$f_1(P_i) \geq 0$$

$$n = 1, 2, \ldots 5$$

Model for $E_2$

$$\min \sum_{i=1}^{n} [E_{2i} - f_2(P_i)]^2$$

s.t.

$$f'_2(P_i) \leq 0 \quad \text{[decreasing function]}$$

$$f_2(P_i) \leq 0.5$$

$$f_2(P_i) \geq 0$$

$$n = 1, 2, \ldots 5$$

$f_1$ and $f_2$ are restricted from the class of monotone polynomial functions. Using the data in Table 5.1 and GINO, the following two polynomials have been found the best to describe the data:

$$E_1 = 1.977 - 0.049p + 0.0449p^2 + 0.00419p^3 - 0.000041p^4$$
and

\[ E_2 = 26.786 + 0.2789p - 0.2675p^2 - 0.0122p^3 + 0.00083p^4 \]

Figures 5.3 and 5.4 illustrate the behavior of the two functions. These functions better describe the relationships between the error probabilities and incoming quality. The functions given by Jaraiedi, however, show a reasonable behavior for a very small range of data, representing incoming quality.

5.4 INSPECTION PLANS UNDER VARYING INSPECTION ERROR

The models developed previously assume fixed Type-I and Type-II errors through the inspection process. It has been documented in the literature that the errors are a function of incoming quality (Raouf and El-Fatouri, 1983) and in general, Type-I error increases as incoming quality \( (P) \) increases and Type-II error decreases with an increase in \( P \) (Harris, 1968). Each time a characteristic is inspected, the incoming quality for the next inspection changes. We add to the models, that after each characteristic is inspected the errors are estimated as a function of the new \( P \). Then they are used for the next inspection. The rest of the equations and the algorithms are as before in the models. Next, we examine the effect of this on the inspection models.

5.5 EFFECT OF INSPECTION ERRORS ON REPEAT INSPECTION PLANS UNDER VARYING INSPECTION ERRORS

The effect of inspection errors \((E_1 \ & \ E_2)\) has been evaluated on repeat inspection plans under the varying incoming quality by employing the functional
FIG. 6.3 Relationship between p and E₁
Fig. 5.4  Relationship between P and E2
relationships between $P$ and $E_1$, and $P$ and $E_2$ as proposed in last section. The effect is quantified using the performance measures: expected Total Cost ($ETC$), Average Outgoing Quality Control ($AOQ$) and Average Total Inspection ($ATI$). The input data used is the same as in section 3.5. A Fortran program used to evaluate the effect of an error change is attached as Appendix-C. Figures 5.3 and 5.4 show the relationship between $P$ and $E_1$ and $P$ and $E_2$ respectively.

5.5.1 Effect of Inspection Errors on $ETC$

Under Varying $E_1$ and $E_2$

Inspection errors leave a significant effect on Expected Total Cost ($ETC$) which is the sum of inspection cost, cost of false rejection due to Type-I error and cost of false acceptance due to Type-II error per accepted component. The cost of inspection is a single most dominant component in $ETC$ as shown in Table 5.2. Figures 5.5 and 5.6 illustrate the behavior of $ETC$ as $E_1$ and $E_2$ increases respectively. The effect of Type-I error is more drastic than that of Type-II error. It is observed that the combined effect of the two errors is much more serious than any one of them can make alone.

5.5.2 Effect of Inspection Errors on $AOQ$

Under Varying $E_1$ and $E_2$

The Average Outgoing Quality ($AOQ$) is another important performance measure of inspection plans which had been defined in section 3.5.3. It is the percent of defectives left in the lot at the end of an inspection. Figures 5.7 and 5.8 show
Figure 6.6 Relationship between ETC and Et
Figure 6.7 Relationship between A0Q and E1

(E2=0.1) (E2=0.5) (E2=1.0)
Fig 5.8
Relationship between AOQ and E2
the effects of Type-I error and Type-II error on $AOQ$ respectively. Clearly, the
effect of $E_1$ is almost linearly increasing, but in case of $E_2$, it exhibits a non-linear
relationship with $AOQ$.

5.5.3 Effect of Inspection Errors on $ATI$
Under Varying Error $E_1$ and $E_2$

Average Total Inspection ($ATI$) is the average number of times a component
undergoes the inspection process by different inspectors. Each inspector inspects
one characteristic. The mathematical expression for $ATI$ is given in section 3.5.1.
The value of $ATI$ decreases as $E_1$ goes up as shown in Figure 5.9. The relationship
is almost linear, but it attains higher value under higher probability of $E_1$. In case
of $E_2$, the behavior is just opposite as that of $E_1$. The Figure 5.10 shows that with
an increased probability of $E_2$, the value of $ATI$ increases, but the growth pattern
is very slow.

5.6 COMPARISON OF RESULTS OF INSPECTION
PLANS WITH AND WITHOUT VARYING
INSPECTION ERRORS

Inspection errors are functions of inspection errors ($P$). Functional relation-
ships between $P$ and Type-I error ($E_1$), and $P$ and Type-II error ($E_2$) have been
proposed in section 3 of this chapter. These relationships, when incorporated into
the inspection models of repeat inspection plans, help evaluate the effect of varying
inspection errors on the inspection plans. Table 5.2 shows the effect of inspection
errors on inspection plans under varying inspection errors. The comparison of the
Fig. 5.9
Relationship between ATI and E1

(E2 = 0.01)

(E2 = 0.05)

(E2 = 0.10)
Fig. 5.10
Relationship between ATI and E2

(E2 = .01)
(E2 = .05)
(E2 = .10)
<table>
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<th>$e_1$</th>
<th>$e_2$</th>
<th>$1 - PG$</th>
<th>$ETC$</th>
<th>$CI$</th>
<th>$CFR$</th>
<th>$CFA$</th>
<th>$A(n)$</th>
<th>$n^*$</th>
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results in Table 5.2 with those contained in Table 3.1 underlines the significance of the varying $E_1$ and $E_2$. It is clear from Table 5.2 that the optimal number ($n^*$) of cycle(s) required for an inspection plan is higher than that in Table 3.1. This leads to higher Expected Total Cost ($ETC$). The substantial increase in $ETC$ is also due to higher cost of false acceptance, since the probability of Type–II error ($E_2$) goes higher as the probability of inspection errors goes down. Another interesting change was observed in the value of Average Outgoing Quality ($AOQ$). This was expected because an increase in Type–II error would decelerate the decreasing rate of $P$, and the net effect would be a slower growth of $PG$ (Probability of a good component entering the inspection cycle). We witnessed higher value of $AOQ$ in inspection under varying inspection errors. In a nutshell, the varying inspection errors posted adverse effects on the inspection plans; nonetheless, it presented a close-to-real picture of the business of repeat inspection plans.

5.7 CONCLUSION

The $ROC$ curve can be used to estimate the error probabilities. This chapter has provided the procedure to estimate the probability of Type–I and Type–II errors from the $ROC$ curve. These estimates then led to the development of functional relationships between $P$ and $E_1$, and $P$ and $E_2$. These functional relationships were utilized to evaluate the dynamic effect of error probabilities on repeat inspection plans. A comparison of the results of inspection plans with and without varying errors showed a significant change in performance measures of
inspection plans. Even though this change is not positive, it reflects a real fact of inspection plans. It is recommended to incorporate the effect of varying errors in the design of inspection plans.
CHAPTER 6

CONCLUSIONS

6.1 SUMMARY AND CONCLUSIONS

The objective of this research was to model and evaluate the effect of inspection plans. This work was motivated by the realization that the inspection function is error-prone, and the assumption of perfect inspection in most practical applications is not valid. The emphasis of the research work was on a branch of inspection plans known as complete inspection plans which are instituted to guard against the human inspection errors. The implications of these errors could be catastrophic in the event of a critical component failure.

This thesis is an extension of the inspection models developed by Duffuaa and Raouf (1989). However, the literature review suggests that not much work has been done to study the effect of inspection errors on complete inspection plans. The effect of the two inspection errors, Type–I error and Type–II error was investigated. The change in the performance measures, Expected Total Cost (ETC), Average Outgoing Quality (AOQ) and Average Total Inspection (ATI) were quantified under the varied values of Type–I error ($E_1$) and Type–II error ($E_2$). It was observed that as $E_1$ increased the $RTC$ increased at a faster rate than it did in case of $E_2$. Similarly, $AOQ$ also increased as $E_2$ went up, but the
change in case of $E_1$ was not very significant. However, $ATI$ decreased as $E_1$ was increased and it showed opposite effect in case of $E_2$.

The practicality of inspection models which are already developed was enhanced by modifying them for the cases where characteristics' defective rate is statistically dependent. This was accomplished by using the knowledge about the joint probability mass function ($j.p.m.f.$) of the random variables representing characteristics' defective rate. The rules for updating the quality of dependent characteristics were proposed which were consistent with basic probability rules. An alternative scheme for sequencing characteristics was suggested along with a methodology proposed for obtaining the optimal number of inspections. The effect of the inspection errors on dependency model was evaluated by observing the behaviors of the $ETC$, $AOQ$ and $ATI$. The results were found consistent with the ones in the literature obtained for sampling plans.

It is documented in the literature that error probabilities are a function of incoming quality ($P$). Signal Detection Theory ($SDT$) can be applied to evaluate the performance of an inspector. The $ROC$ gives the relationship between the two error probabilities. The GINO computer package was used to find the parameters of the $ROC$ curve which was later used to find Type–I error and Type–II error which corresponds to a given probability of incoming quality. It was assumed that the distance travelled on the $ROC$ curve due to a change in $P$ is proportional to the amount of the change that has taken place. A computer program was written
to perform the necessary estimation procedures via numerical analysis methods.

A functional relationship between incoming fraction defective and the two types of errors was developed and found more accurate than those in the literature. A non-linear regression model under constraints was solved using GINO to estimate the parameters of these relationships. Then the two functional relationships were incorporated into the repeat inspection models to evaluate the effect of inspection errors under varied inspection errors on the three performance measures of inspection plans.

The results of this research can be applied to many practical situations where the cost of committing inspection errors is substantial. The performance of inspectors can be compared based on their estimated $ROC$ curve. This can be used in selection and training of inspectors when inspection tasks are being designed or evaluated.

6.2 FUTURE RESEARCH

Most of the results presented in this research were extended attempts to address some of the problems associated with inspection error modeling in repeat inspection plans. Some of the following suggestions regard possible research directions in the future.

1. A different scheme describing the inspection process of repeat inspection can
be evaluated.

2. The process of data generation for parameter estimation of \( ROC \) curve can be refined. This would improve the accuracy for assessment of error probabilities; furthermore, the effect of job-related factors (such as task pace, product type, training and experience of inspectors, environmental factors, etc.) need to be investigated.

3. The effect of functional relationships between incoming fraction defectives and error probabilities can be incorporated into the inspection models under statistical dependency.
***
* APPENDIX-A *
***

THIS PROGRAM FINDS OPTIMAL 'N' UNDER STATISTICAL DEPENDENCY FOR REPEAT INSPECTION MODELS

REAL CP(1024), PG(0:10,12), OP(10,10), P1(10), P2(10), SUM(10)
REAL C(0:9), TC(40), CA(40,40), CFA(40), FR(40,40), CR1(40,40)
REAL CR(40), TCFR(0:40), TCFA(0:40), TCI(0:40), CFR(40), FA(40,40)
REAL ETC(40), A1(40), C11(40,40), M1(10,10), P(40)
INTEGER T, IL1(10), ILX(10,10)
READ(5,*), M1(1,1)
READ(5,*), N
JL=N
IL=2**JL
DO 5 J=1, JL
SUM(J)=0.0
5 CONTINUE
DO 10 I=1, IL
READ(5,*), CP(I)
10 CONTINUE
READ(5,*), CFA(1)
READ(5,*), CFR(1)
READ(5,*), PFO
PG(0,1)=PFO
WRITE(6,301) '==================================SUMMARY OF THE RESULTS======'

301 FORMAT(15X,A)
WRITE(6,*), ' ' ' ' ' ' ' ' ' ' '
WRITE(6,*), ' ' ' ' ' ' ' ' ' ' '
WRITE(6,*), ' ' ' ' ' ' ' ' ' ' '
ETCB=1-PG(0,1)*CFA(1)
WRITE(6,302) 'THE EXPECTED COST PER COMPONENT DIFFERENT INSPECTION *
* = ', ETCB

302 FORMAT(15X,A,F15.3)
WRITE(6,*), ' ' ' ' ' ' ' ' ' ' '
WRITE(6,*), ' ' ' ' ' ' ' ' ' ' '
WRITE(6,*), ' ' ' ' ' ' ' ' ' ' '
WRITE(6,*), ' ' ' ' ' ' ' ' ' ' '
WRITE(6,303) '----------------------------------------STARTING THE INSPECTION-------------------'

303 FORMAT(20X,A)
WRITE(6,*), ' ' ' ' ' ' ' ' ' ' '
WRITE(6,*), ' ' ' ' ' ' ' ' ' ' '
DO 20 I=1, N
READ(5,*), P1(I)
DO 25 I=1, N
READ(5,*), P2(I)
DO 30 I=1, N
READ(5,*), TC(I)
READ(5,*), PFO
IL1(I)=IL1/I2
DO 35 J=2, JL
IL1(J)=IL1(J-1)/2
20 CONTINUE
25 CONTINUE
30 CONTINUE
35 CONTINUE
CONTINUE
DO 40 J=1,JL
  K=0
  KX=ILI(J)
DO 45 I=1,IL/2
  ILX(I,J)=I+K
  ILX(I+IL/2,J)=I+ILI(J)
IF(1.EQ.ILI(J)) THEN
  K=ILI(J)
  ILI(J)=ILI(J)+KX
ENDIF
45 CONTINUE
DO 50 I=1,IL/2
  SUM(J)=SUM(J)+CP(ILX(I,J))
50 CONTINUE
DO 200 T=1,5
  DO 100 L=1,JL
    DO 55 J=1,JL
      OP(L,J)=SUM(J)
      WRITE(6,*) 'SUM ',SUM(J)
    55 CONTINUE
    P(L)=OP(L,L)*P2(L)
    P(L)=P(L)/(OP(L,L)*P2(L)+(1-OP(L,L))*(1-P1(L)))
    WRITE(6,*) 'P(L)',P(L)
  100 SUM(L)=P(L)
    DO 60 I=1,IL
      IF((1.EQ.IL/2) THEN
      CP(ILX(I,L))=CP(ILX(I,L))*(P(L)/OP(L,L))
      WRITE(6,*) CP(ILX(I,L))
      ELSE
      CP(ILX(I,L))=CP(ILX(I,L))*((1-P(L))/(1-OP(L,L)))
      WRITE(6,*) CP(ILX(I,L))
    ENDIF
60 CONTINUE
PG(L,T)=CP(IL)
WRITE(6,*) 'PG = ',PG(L,T)
DO 65 J=1,JL
  IF(J.NE.L) SUM(J)=0.0
65 CONTINUE
DO 70 J=1,JL
  DO 75 I=1,IL/2
      IF(J.NE.L) SUM(J)=SUM(J)+CP(ILX(I,J))
  75 CONTINUE
70 CONTINUE
100 CONTINUE
L=1
FR(L,T)=M1(L,T)*PG(L,T)*P1(L)
WRITE(6,*) 'FR',FR(L,T)
FA(L,T)=M1(L,T)*((OP(L,L)*P2(L))
  +(1-PG(L,T)-OP(L,L))*(1-P1(L)))
WRITE(6,*) OP(L,L)
WRITE(6,*) FA(L,T)
CA(L,T)=M1(L,T)*PG(L,T)*(1-P1(L))
WRITE(6,*) CA(L,T)
DO 101 L=2,N
   M1(L,T)=FA(L-1,T)+CA(L-1,T)
   WRITE(6,*)'THE VALUE OF M=,'M1(L,T)
   FR(L,T)=M1(L,T)*PG(L-1,T)*P1(L)
   CA(L,T)=M1(L,T)*PG(L-1,T)*(1-P1(L))
   FA(L,T)=M1(L,T)*((OP(L,L)*P2(L))
      + (1-PG(L-1,T)-OP(L,L))*(1-P1(L))
   WRITE(6,*)'OP(L,L)=,'OP(L,L),L
   WRITE(6,*)'FA(L,T)=,'FA(L,T)
   WRITE(6,*)'CA(L,T)=,'CA(L,T)
101 CONTINUE

102 FORMAT(20X,A,13,13,A,F14.4)
C CALCULATING TOTAL ACCEPTED COMPONENTS IN TTH CYCLE
   A1(T)=FA(N,T)+CA(N,T)
103 FORMAT(20X,A,12,A,F14.3)
C CALCULATING THE NUMBER OF COMPONENT ENTERING NEXT CYCLE
   M1(1,T+1)=A1(T)
C CALCULATING THE COST OF FALSE ACCEPTANCE
   CFA(T)=CFA(1)*FA(N,T)
   L=1
   CR1(L,T)=FR(L,T)
   DO 9 L=2,N
      CR1(L,T)=CR1(L-1,T)+FR(L,T)
9 CALCULATING THE COST OF FALSE REJECTION
   CFR(T)=CFR(1)*CR1(N,T)
C CALCULATING THE COST OF INSPECTION
   L=1
   CI1(L,T)=TC(L)*M1(L,T)
   DO 23 L=2,N
      CI1(L,T)=CI1(L-1,T)+TC(L)*M1(L,T)
   C(T)=CI1(N,T)
23 CALCULATING THE TOTAL COST FOR TTH CYCLE
   TCFR(0)=0.0
   TCI(0)=0.0
   TCFR(T)=TCFR(T-1)+CFR(T)
   TCFA(T)=CFA(T)
   TCI(T)=TCI(T-1)+C(T)
   WRITE(6,*)'TCFR,TCFR(T)
   WRITE(6,*)'TCFA,TCFA(T)
   WRITE(6,*)'TCI,TCI(T)
   WRITE(6,*)'A=',A1(T)
C CALCULATING EXPECTED TOTAL COST PER ACCEPTED COMPONENT
   EG(T)=TCI(T)+TCFA(T)+TCFR(T)/A1(T)
   WRITE(6,306) 'ETC(\',T,')=',EG(T)
306 FORMAT(20X,A,12,A,F20.3)
C TESTING FOR MINIMIZING THE EXPECTED TOTAL COST
   IF(T.EQ.1)GOTO 200
6 FORMAT(20X,A,12,A)
   IF(ETC(T).GT.ETC(T-1)) GOTO 500
   IF(T.GE.2)THEN
      PG11 = PG(L-1,T)-PG(L-1,T-1)
      WRITE(7,*)'PG11=','PG11
     IF((PG(L-1,T)-PG(L-1,T-1)).LE.PG) GOTO 500

SUP01110
SUP01120
SUP01130
SUP01140
SUP01150
SUP01160
SUP01170
SUP01180
SUP01190
SUP01200
SUP01210
SUP01220
SUP01230
SUP01240
SUP01250
SUP01260
SUP01270
SUP01280
SUP01290
SUP01300
SUP01310
SUP01320
SUP01330
SUP01340
SUP01350
SUP01360
SUP01370
SUP01380
SUP01390
SUP01400
SUP01410
SUP01420
SUP01430
SUP01440
SUP01450
SUP01460
SUP01470
SUP01480
SUP01490
SUP01500
SUP01510
SUP01520
SUP01530
SUP01540
SUP01550
SUP01560
SUP01570
SUP01580
SUP01590
SUP01600
SUP01610
SUP01620
SUP01630
SUP01640
SUP01650
C     ENDIF
200   CONTINUE
500   WRITE(6,*)'THE OPTIMUM NUMBER OF INSPECTION ',T-1
       STOP
       END
C ***************
C * APPENDIX-B *
C ***************
C THIS PROGRAM GENERATES POINTS ON THE ROC CURVE
C***************
EXTERNAL LENGTH
REAL A1,A2,N1,N2,X1,X2,RP,LP,LENP,LENM,LENGTH,PON,E2
COMMON /TRANS/ A1,A2,N1,N2
OPEN(1,FILE='CURVE.DATA',STATUS='OLD')
READ(1,*) A1,A2,N1,N2
READ(1,*) X1,X2
READ(1,*) P1,P2,P
EPS=2
LENP=LENGTH(X1,X2)*((P-P1)/(P2-P1))
WRITE(6,*) 'TOTAL LENGTH = ',LENGTH(X1,X2)
WRITE(6,*) 'LENGTH TO P = ',LENP
LP=X1
PON=(X1+X2)/2
RP=X2
LENM=LENGTH(X1,PON)
WRITE(6,*) 'POINT IS AT ->',PON,'THE LENGTH IS ->',LENM
IF(ABS(LENM-LENP).GT.EPS) THEN
  IF(LENM.GT.LENP) THEN
    PON=(LP+PON)/2
    GOTO 1
  ELSE
    PON=(PON+RP)/2
    GOTO 1
  ENDIF
ENDIF
N1=1.0/N1
N2=1.0/N2
E1=PON
E2=100.0-A1*E1**2*(N1)-A2*E1**2*(N2)
WRITE(6,*)  ' E1  E2  '
WRITE(6,*) '--------------------------'
WRITE(6,2) E1,E2
2 FORMAT(2X,F7.3,2X,F7.3)
CLOSE(1)
STOP
END

EXTERNAL LENGTH(X1,X2)
EXTERNAL F
REAL A1,A2,N1,N2,X1,X2,X1,H,SUM
COMMON /TRANS/ A1,A2,N1,N2
N=10
H=(X2-X1)/(N-1)
SUM=0.5*(F(X1)+F(X2))
DO 10 I=2,N-1
  XI=(I-1)*H
  SUM=SUM+F(XI)
10 CONTINUE
LENGTH=H*SUM
RETURN
END

FUNCTION F(X)
EXTERNAL G
REAL A1,A2,N1,N2,X
COMMON /TRANS/ A1,A2,N1,N2
F=1/(N1*N2)*((N1**2)*(N2**2)+G(X))**(0.5)
RETURN
END

FUNCTION G(X)
REAL A1,A2,N1,N2,X
COMMON /TRANS/ A1,A2,N1,N2
G=(A1*N2*X**((1-N1)/N1)+A2*N1*X**((1-N2)/N2))**2
RETURN
END
C ***************
C * APPENDIX-C *
C ***************
C THIS PROGRAM FINDS OPTIMAL 'N' AND INCORPORATES THE
C EFFECT OF TYPE-I ERROR AND TYPE-II ERROR INTO THE
C REPEAT MULTICLASS CHARACTERISTICS INSPECTION PLANS
C**********************************************************************************************
REAL A(40), TC(40), B(40), R(40), C(40), P(40), P1(40)
REAL M(40), ETC(40), TCFA(40), TCFR(40), C1(40), CA(40)
REAL PG(40), Y, EPS, FA(40), FR(40), CFA, CFR, MI, SORT, DIFF, PGG
INTEGER I, J, JJ, K, L, N, MRAT(40), ISORT
OPEN(1, FILE='XMEM.DAT', STATUS='OLD')
READ(1, *) MI, N
WRITE(6, *) 'N(1) = ', MI
WRITE(6, *) 'N = ', N
READ(1, *) CFA, CFR
WRITE(6, *) 'CFA = ', CFA
WRITE(6, *) 'CFR = ', CFR
READ(1, *) (P(I), I=1, N)
WRITE(6, *) (P(I), I=1, N)
READ(1, *) (P2(I), I=1, N)
WRITE(6, *) (P2(I), I=1, N)
READ(1, *) (P1(I), I=1, N)
WRITE(6, *) (P1(I), I=1, N)
READ(1, *) PGG
WRITE(6, *) 'PGG = ', PGG
WRITE(6,500) '------------STARTING THE INSPECTION--------------'
WRITE(6,500) ' '
J=0
JJ=1
TCFRS=0.0
CISUM=0.0
PG(1)=0.0
M(J+1)=MI
ETCL=1.0E20
C ORDERING THE CHARACTERISTICS ACCORDING TO THE LOWEST RATIO OF
C C(I)/R(I)
100 WRITE(6,501) 'CYCLE NUMBER ==> ', JJ
    DO 1 I=1, N
        MRAT(I)=1
    J=J+1
    DO 3 I=1, N
        R(I)=P(I)*(1-P2(I))+(1-P(I))*P1(I)
3    B(I)=C(I)/R(I)
    DO 5 I=1, N
        DO 4 K=1, N
            IF(B(I).LT.B(K)) THEN
                SORT=B(I)
                B(I)=B(K)
                B(K)=SORT
                ISORT=MRAT(I)
                MRAT(I)=MRAT(K)
                MRAT(K)=ISORT
            
4        END IF
5      END DO
1    END DO
    END
ENDIF

4 CONTINUE

5 CONTINUE

WRITE(6,501) 'THE SEQUENCE AT CYCLE ',JJ
DO 6 I=1,N

6 WRITE(6,505) MRAT(1)
DO 7 I=1,N
PG(I+1)=1-P(1)
DO 2 K=2,N
PG(I+1)=(1-P(K))*PG(I+1)
WRITE(6,504) 'PG = ',PG(I+1)
DIFF=PG(I+1)-PG(I)
IF(DIFF.LE.PGG) GOTO 102
I=MRAT(1)
WRITE(6,502) 'MIN RATIO IS AT ',I,' CHARACTERISTICS'

C FINISHING THE SEQUENCE OF CHARACTERISTICS

FR(1)=M(1)*PG(I+1)*PI(11)
FA(1)=M(1)*(P(11)*P2(11)+((1-PG(I+1)-P(11))*((1-P(11))))
CA(1)=M(1)*PG(I+1)*((1-P(11)))
A(1)=FA(1)+CA(1)
TCFR(1)=CFR*FR(1)+TCFRS
TCFA(1)=CFA*FA(1)
CI(1)=M(1)*CI(1)+CISUM
TCFRS=TCFR(1)
CISUM=CI(1)
WRITE(6,503) 'A( , , ) = ',A(1)
WRITE(6,503) 'TCFR( , , ) = ',TCFR(1)/A(1)
WRITE(6,503) 'TCFA( , , ) = ',TCFA(1)/A(1)
WRITE(6,503) 'CI( , , ) = ',CI(1)/A(1)
PI(11)=P(11)*P2(11)/(P(11)*P2(11)+(1-P(11))*((1-P(11))))
WRITE(6,503) 'P( , , ) = ',P(11)
P(11)=P(11)**100

C UPDATING TYPE-I ERROR (P1) AND TYPE-II ERROR (P2)

C AFTER UPDATING INCOMING QUALITY (P).

P1(11)=(1.99774-.04907*P(11)+.049222*P(11)**2)
* +.000194*P(11)**3+.000041*P(11)**4)*.01
WRITE(6,503) 'P1( , , , ) = ',P1(11)
P2(11)=(27.78646+.278897*P(11)-.267551*(P(11)**2)
* -.012174*(P(11)**3)+.000829*(P(11)**4))*.01
WRITE(6,503) 'P2( , , , ) = ',P2(11)
P(11)=P(11)**.01
M(I+1)=A(I)
WRITE(6,503) 'M( , , ) = ',M(1)

7 CONTINUE

M(1)=A(N)
ETC(JJ)=(TCFR(N)+TCFA(N)+CI(N))/A(N)
WRITE(6,503) 'ETC( , , ) = ',ETC(JJ)
IF(ETC(JJ).LT.ETC(1)) GOTO 102
ETC=ETC(JJ)
WRITE(6,501) 'END OF CYCLE ',JJ
WRITE(6,500) ''
JJ=JJ+1
GOTO 100

102 CLOSE(1)
WRITE(6,501) 'END OF CYCLE ' , JJ
WRITE(6,501) 'OPTIMUM FOUND AT CYCLE ==> ' , JJ-1
500 FORMAT(20X,A) MOD01110
501 FORMAT(20X,A,1X,I2) MOD01120
502 FORMAT(20X,A,1X,13,1X,A) MOD01130
503 FORMAT(20X,A,12,A,1X,F18.9) MOD01140
504 FORMAT(20X,A,F12.9) MOD01150
505 FORMAT(23X,I2) MOD01160
STOP MOD01170
END MOD01180
Appendix D

The data set used for fitting the ROC curve was adapted from a visual inspection experiment reported by Harris (1968).

<table>
<thead>
<tr>
<th>$P$</th>
<th>Defects Detected</th>
<th>False Reports</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0025</td>
<td>0.575</td>
<td>0.850</td>
</tr>
<tr>
<td>0.0100</td>
<td>0.715</td>
<td>0.395</td>
</tr>
<tr>
<td>0.0400</td>
<td>0.750</td>
<td>0.179</td>
</tr>
<tr>
<td>0.1600</td>
<td>0.840</td>
<td>0.075</td>
</tr>
</tbody>
</table>
REFERENCES


Vita

Iftikhar Ali Nadeem

Born at Sahiwal, Pakistan

Received Bachelor’s degree in Industrial Engineering from Dawood College of Engineering and Technology, Karachi, Pakistan in 1986.

Completed Master’s degree at KFUPM, Dhahran, Saudi Arabia in January, 1993.