Real and Reactive Power Wheeling Cost
Based on the Marginal Cost Theory

by

Qasim Mohammed Allawati

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

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DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
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إهداء

هذه الرساله مهداه إلى ...

روح والدتي العزيزة ... ووالدي العزيز ...

ولزوجتي وبناتي آلاء .. و طوعه

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Nomenclature

$C_i(P_{Gi})$	operating cost of producing P_{Gi} units of real power at the generating plant at bus i
$C_i(Q_{Gi})$	operating cost of producing Q_{Gi} units of reactive power at the generating plant at bus i
N	set of bus indices
P_{Gi}	active power generation at bus i
P_{Si}	active power selling at bus i
P_{di}	active power demand at bus i
P_{Bi}	active power buying at bus i
Q_{Gi}	reactive power generation at bus i
Q_{Si}	reactive power selling at bus i
Q_{di}	reactive power demand at bus i
Q_{Bi}	reactive power buying at bus i
V_i	voltage magnitude at bus i
Y_{ij}	element of i row and j column in admittance matrix of transmission network
θ_{ij}	phase angle of Y_{ij}
δ_i	voltage angle at bus i
NG	set of generation bus indices
NC	set of capacitor bank indices
NT	set of transformer indices
NL	set of transmission line indices
V_i^{min}	minimum voltage level at bus i
V_i^{max}	maximum voltage level at bus i

P_{Gi}^{min}	minimum active power output at generation bus i
P_{Gi}^{max}	maximum active power output at generation bus i
Q_{Gi}^{min}	minimum reactive power output at generation bus i
Q_{Gi}^{max}	maximum reactive power output at generation bus i
T_i^{min}	minimum tapping ratio at transformer i
T_i	tapping ratio at transformer i
T_i^{max}	maximum tapping ratio at transformer i
I_i	current magnitude at transmission line i
I_i^{max}	maximum current magnitude at transmission line i
ω_P	active power wheeling rate
ω_Q	reactive power wheeling rate
MCP_B	marginal cost of active power at buyer bus
MCP_S	marginal cost of active power at seller bus
MCQ_B	marginal cost of reactive power at buyer bus
MCQ_S	marginal cost of reactive power at seller bus
W	total wheeling rate
P_B	active power demand at buyer bus
P_S	active power demand at seller bus

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ABSTRACT

Wheeling is the transmission of active power and reactive power from one utility to another through the transmission network of a third party. Wheeling rates of active and reactive power are important issues that need to be addressed. This thesis presents a non linear optimization model based on the marginal cost theory to set wheeling rates.

The model is a modified Optimal Power flow. The objective function is to minimize the active and reactive power generation costs of the wheeling utility and the capital costs of the wheeling facilities. The model consists of equality and inequality constraints that are non linear in nature. Reactive power generation costs, transmission Losses and transmission line stability limits are included in the model.

Two Case studies have been used to test the model. The first test system consists of an eight bus network. The second case is a practical system of three of the Gulf Cooperation Council states (GCC) of Oman, United Arab Emirates and Qatar. Wheeling rates for the transfer of the power from Oman to Qatar, via the transmission network of UAE, and vice versa has been determined.

ARABIC ABSTRACT

الموجـــز

يعتبر الربط بين شبكات الكهرباء إحـدى الوسـائل المتبعـة حاليـا لتخفيـف الأحمـال والضغوط على خطوط هذه الشبكات . كما تعتبر الدوليه أحد الوسائل المتبعة للربط . تعرف الدوليه بنقل الطاقه الكهربائية من شبكه إلى أخرى بواسطة طرف ثالث . يتم نقل الطاقه من البائع إلى المشتري بواسطة خطوط نقل شبكة الناقل (الطرف الثالث). التعرفه الاستهلاكية هي إحدى المواضيع التي يجب دراستها والاتفاق عليها لتحديد الرسوم التي ستفرضها الشركة الناقلة للطاقه على البائع والمشتري والتي يجب الاتفاق عليها بين الأطراف الثلاثة.

في هذه الرساله تم اعداد نموذج أمثل لتحديد رسوم نقل الطاقه بين الاطراف الثلاثة بحيث يكون الهدف تقليل مصاريف التشغيل في شبكة الناقل وفي نفس الوقت الالتزام بالقيود المفروضة على الشبكه. فعلى سبيل المثال لا الحصر يجب التقيد بالحد الاعلى لطاقة الخطوط من حيث قدرتها على تحمل الطاقه المحمولة عليها . وقد تضمن النموذج معادلات خسائر خطوط النقل واستقرارية الشبكه نتيجة لنقل الطاقه .تم اختبار النموذج المعد على شبكتين مختلفتين . الشبكه الأولى مكونه من ثمانية قضبان. والشبكه الثانية تضمنت دوائر الإمارات العربية المتحدة وعمان وقطر . حيث تم تحديد أسعار نقل الطاقه من قطر إلى عمان وبالعكس باستخدام شبكة الإمارات .

CHAPTER 1

INTRODUCTION

1.1 Energy Trading

Due to the rapid growth in urban and rural areas, high demand of power has become one of the challenges that electric utilities industry face today. To meet this high demand some utilities select to interconnect with neighboring systems. There are several reasons which make electric system utilities interconnect with neighboring systems. Interconnected power systems are more reliable and the loss of load probability is reduced. If a unit is forced out of the system, the load can be taken by other units that supply the spinning reserve of the system. The interconnected system is a better system to operate. The most important reason, however, is the better economics of operation that can be achieved by operating an interconnected system [1].

1.2 Available Types of Interchange

Below are some of the available types of the interchange between interconnected power systems [1]:

1.2.1 Capacity Interchange

A capacity agreement is signed between two utilities when one utility does not have enough surplus to cover its own load due to the loss of some of its units. This sort of interchange takes place , only, in an emergency case.

1.2.2. Diversity Interchange

This type of interchange takes place when two large systems serve two different areas that span different time zones. The two systems may peak at different times. In this case the power is exchanged from one system, where the other pays back during its off-peak periods. Also, one system may peak in the summer due to air conditioning load and the other may peak in the winter due to the heating load. The summer peaking system will help the winter peaking system and vice versa.

1.2.3. Energy Banking

This form of interchange takes place when a hydro system is interconnected to a thermal system. The hydro system will sell energy to the thermal system during high water runoff periods. The hydro system will import energy during low runoff periods. The agreement is made between the two systems on banking principle basis. One system deposits energy when it has a surplus and will withdraw only the deposited amount when it is needed.

1.2.4 Emergency Power Interchange

It is usual for power systems to have generation failures. In such circumstances, agreement with neighboring systems to supply the power is required. Since this sort of agreements is on emergency cases, the interchange rates are high.

1.2.5. Inadvertent power exchange

Automatic Generation Control (AGC) systems of interconnected utilities may have errors in controlling the amount of energy interchanged. This can cause significant, accumulated amount of energy. This is resolved by paying back the same amount of energy in future times.

1.2.6 Power Pools

Power Pools are formed when several utilities agree to interchange power. Usually, a central dispatch center is formed to control the power flow between the interconnected utilities. Below are some of the advantages of power pools,

1. Minimize operating costs.
2. Perform a system-wide unit commitment.
3. Minimize the reserves being carried throughout the system.
4. Coordinate maintenance scheduling to minimize costs and maximize reliability by sharing reserves during maintenance periods.
5. Maximize the benefits of emergency procedures.

1.3 Wheeling

Wheeling is the transmission of electrical active power and reactive power from a seller to a buyer through a transmission network owned by a third party [2] .Wheeling is defined as the use of transmission or distribution facilities of a system to transmit the power to another entity or entities [3]. The seller and the buyer use the transmission network of the wheeling utility to transfer the power between them. Figure 1.1 represents the concept of wheeling where S represents the seller, B the buyer, and K_i the wheeler. The buyer and the seller can select to wheel through different wheelers. Wheeling can take several forms and it does not, necessarily, have to be between two utilities. The forms of wheeling are summarized as follows [3]:

1.3.1 Utility to Utility : From one utility to another for bulk power wheeling via the transmission network of an intervening utility.

1.3.2. Utility to private user : a private user (such as an industrial customer) purchases energy from a utility that does not service his geographical location.

1.3.3. Private generator to utility : a private generator sells to a large utility whose service does not cover the geographical location of the generator.

1.3.4. Private generator to a private user: both are selling and buying through an intervening utility.

When wheeling takes place load flow laws apply. The three parties networks become one system where no utility has the control over the power flow. In other words, it is not a specific set of electrons will be sent from the seller to the buyer. The Seller increases his generation through Automatic Generation Control by the same amount of power required by the buyer and the buyer decreases his generation by the same amount [8]. A seller and a buyer, sometimes, have the choice to select among many different wheeling utilities as indicated by Figure 1.1. There are several factors which may force a seller and a buyer to select an appropriate wheeler. Two of these are stated below :

1. A certain utility might not be able to offer a wheeling service to the seller and the buyer at a specific time due to its own reasons.

2. Cost or wheeling rate is an important factor in deciding which wheeling utility to be selected.

The second factor is very important because wheeling may increase or decrease losses in the wheeling utility's network. This is simply due to the extra flow of power in its transmission network.

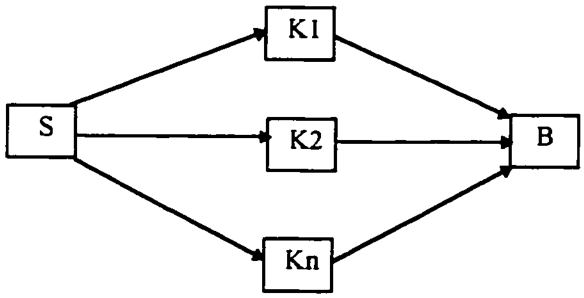

Fig. 1.1 Multi - Area Wheeling Topology

1.3.5. Wheeling Rates

Wheeling rates are defined as the charge of active and reactive power that both the seller and the buyer pay to the wheeler. The need to set wheeling rates, that are adequate for all parties, is a necessity. A wheeling utility may build a transmission system to be used for power exchange. In addition the network losses may increase. The rates should reflect a revenue for all parties involved. Below are some of the transmission cost pricing methods [4],

1. Short-Run Incremental cost pricing : This pricing methodology entails evaluating and assigning the operating costs associated with a new transmission transaction. It should be noted that these costs can be negative.
2. Long-Run Incremental cost pricing : This pricing methodology entails evaluating all long-run costs (operating and reinforcement costs) necessary to accommodate a transmission transaction and assigning such costs to that transaction.
3. Short-Run Marginal cost pricing: in this pricing methodology, the marginal operating cost of the power system due to a transmission transaction is calculated first.
4. Long-Run Marginal cost pricing: In this pricing methodology the marginal operating costs of the power system are used to determine the prices for a transmission transaction.

1.4 Literature Survey

The subject of wheeling rates has been addressed by several researchers. This section presents a review of the literature on setting up wheeling rates:

1.4.1 Rates Based on Marginal Theory without assuming existence of spot pricing

This approach is presented in reference [3]. The wheeling rates are evaluated without assuming the existence of a spot price based energy market place. The spot price is a dynamic price that is influenced by several factors such as fuel , operation and maintenance costs. The spot price can be set for different period of times, such as days, weeks, months or sometimes a year provided the costs can be obtained a head of time. In this method, the direct evaluation of the maintenance and the quality of supply costs are avoided. It is assumed that the power losses will be considered as a part of the inflow from the seller who will increase his generation slightly to recover the losses incurred through wheeling. It is important to note here that wheeling power on heavy loaded lines increases losses and vice versa.

The wheeling rate is expressed as :

$$\omega = \frac{\text{Change in (operating costs of wheeling utility)}}{\text{Change in (amount of energy being wheeled)}} \qquad (1.1)$$

The derivative of equation 1.1 will be evaluated subject to constraints, such as energy balance, Kirchhof's law, and line flow limits.

1.4.2 A load flow based method for calculating embedded, incremental, and marginal costs of the transmission capacity.

Reference [4] addresses the allocation of transmission capacity based on load flow. It estimates the usage of each transmission facility with cost defined on a per facility basis.

a) Embedded cost : it is defined as the revenue requirements needed to pay for all existing facilities plus any new facilities added to the power system during the life of the contract for transmission service.

The annual embedded cost can be calculated as follows :

$$ECC = \sum_{f \varepsilon F} \frac{\Delta MWf, I \ * \ ECf}{\sum_{s \varepsilon S} \Delta MWf, S} \tag{1.2}$$

where

ECC is the annual Embedded Capacity Cost

$\Delta MW_{f,I}$ is the change in megawatt flow due to the contracted transmission # I on facility f

EC_f is the annual embedded cost of facility f which is the sum of depreciation, embedded cost of capital, taxes and expenses.

S,F the sets of all sales S and facilities F in a given year.

b. Incremental Cost : this is the revenue requirements needed to pay for any new facilities that are specifically attributed to the transmission service. It is represented by :

$$ICC = \sum_{y \varepsilon Y} \sum_{f \varepsilon FI} \frac{\Delta MWf, I, y \ * \ ICf, y}{\sum_{s \varepsilon S} \Delta MWf, S, y} * PWFy \tag{1.3}$$

where

ICC is the total incremental capacity costs across the life of the contract.

$\Delta MW_{f,I,y}$ is the change in megawatt flow due to the contracted transmission service on incremental facility f for year y.

$\Delta MW_{f,S,y}$ is the change in megawatt flow due to all transmission service on facility f for all incremental customers s in year y that requires he this incremental facility.

$IC_{f,y}$ is the incremental cost of facility f in year y in which is the sum of depreciation on facility f, incremental cost of capital, incremental taxes and incremental expenses.

$F_{I,S,Y}$ sets of incremental facilities, incremental customer sales, and service life years of each incremental facility, respectively.

PWFy is the appropriate present worth factor

c. Marginal cost : It is defined as the revenue requirements needed to pay for any new capacity on the transmission system.

The annual marginal cost can be calculated as follows :

$$MCC = \sum_{f \varepsilon FN} \frac{\Delta MWf, I \ * \ MCf}{\sum_{s \varepsilon SM} \Delta MWf, S} \qquad (1.4)$$

where

MCC is the annual Marginal Capacity Cost

$\Delta MW_{f,I}$ is the change in megawatt flow due to the contracted transmission service on new facility f.

$\Delta MW_{f,S}$ is the change in megawatt flow due to transmission service on new facility f for all marginal sales S.

MC_f is the cost of new facility f which is the sum of depreciation on facility f ,marginal cost of capital, marginal taxes and marginal expenses for any year of the transaction.

F_N, S_M the sets of new facilities all marginal sales S in a given year.

d. Boundary Flow Method or Power Allocation Method (PAM)

This method equates the impact of a sale on the transmission system to the gross change in real power outflow from the system caused by sale. Mathematically [4].

$$\text{PAM COST} = \frac{\sum_{t \varepsilon T}(\text{Flow final}, t - \text{Flow initial}, t)}{(\text{Magnitude of transaction})} * \text{ARR} \qquad (1.5)$$

where

ARR is the total transmission annual revenue requirements for the system.

T is sets of all ties

e. Generalized Flowmile methods

This group of methods measures the amount of transmission capacity used by summing the products of each facility length and the change in flow quantity on the transmission facility caused by a sale. Mathematically,

$$\text{ARs},t = \sum_{f \varepsilon F} \frac{\Delta F \text{MWf},S,t * \text{Mf}}{\sum_{f \varepsilon F} \Delta R \text{MWf},S,t * \text{Mf}} * \text{ARRt} \qquad (1.8)$$

where

$AR_{s,t}$ is the annual revenue for transmission service from sales s on transmission system t.

$FMW_{f,s,t}$ is the power flow on line f due to transmission sales s on transmission system t.

M_f is the length in miles of line f.

$RMW_{f,s,t}$ is the power rating or flow on line f from all sales s on transmission system t.

ARR_t is the total transmission annual revenue requirements for transmission owner t, usually at an embedded cost basis.

and

$$\text{The Marginal Cost} = \frac{\sum \text{Flowmiles} * \text{ARR}}{\text{total flow miles}} \tag{1.9}$$

1.4.3 Real-time pricing of reactive power

Reference [5] addresses the issue of real-time pricing of reactive power using a modified optimal power flow model . The model consists of two parts an objective function and constraints. The objective function is to minimize the utility's operating costs while satisfying a set of constraints. The complete model is described below:

Minimize $C = \Sigma\ C_i(P_{Gi})$

subject to the following equality and inequality constraints :

$$P_{Gi} - P_{di} = \Sigma V_i V_j Y_{ij} \operatorname{Cos}(\theta_{ij} + \delta_j - \delta_i)$$

$$Q_{Gi} - Q_{di} = -\Sigma V_i V_j Y_{ij} \operatorname{Sin}(\theta_{ij} + \delta_j - \delta_i)$$

$$V_i^{min} \leq V_i \leq V_i^{max} \qquad i \in N$$

$$P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max} \qquad i \in NG$$

$$Q_{Gi}^{min} \leq Q_{Gi} \leq Q_{Gi}^{max} \qquad i \in NG$$

$$T_i^{min} \leq T_i \leq T_i^{max} \qquad i \in NT$$

$$I_i \leq I_i^{max} \qquad i \in NL$$

The real time prices of real power are obtained using the marginal cost at bus i. The active and reactive power wheeling rates will be obtained by the Lagrange multipliers of the equality constraints. The marginal costs are defined as

$$_{i} = \frac{\partial}{\partial pd_i} \text{ [total cost of providing electricity to all customers subject to the operational constraints]} \quad (1.10)$$

$$MCQ_i = \frac{\partial}{\partial Qd_i} \text{ [total cost of providing electricity to all customers subject to the operational constraints]} \quad (1.11)$$

The derivative of equations 1.10 and 1.11 will be evaluated subject to constraints, such as energy balance, Kirchhof's law, and line flow limits.

1.4.4 Pricing reactive power conveyance

Reference [6] describes a method based on marginal cost pricing and is implemented using a modified optimal power flow. This method is similar to the method described by [5], but is applied to a wheeling case. It states that although reactive power production cost is small compared to the active power generation, it can not simply be ignored. The model is similar to the model described in section 1.3.3. The wheeling rates for active and reactive power are given as :

$$\omega_P = MCP_B - MCP_S \quad (1.13)$$

$$\omega_Q = MCQ_B - MCQ_S \quad (1.14)$$

$$W_P = P_B\, \omega_P \quad (1.15)$$

$$W_Q = Q_B\, \omega_Q \quad (1.16)$$

$$W = W_P + W_Q \quad (1.17)$$

1.4.5 Application of OPF for reactive power pricing as applied to the NGC system

Reference [7] discusses the pricing of reactive power supply. It is stated that reactive power cost is a capital expenditure issue. The minimization of net Var cost is considered to be an important issue that needs to be achieved. The paper discusses the case of the National Grid Company (NGC) of England. The company is using an Optimal Power Flow Package to decide reactive power pricing. The package, however, does not accommodate the NGC requirements. Therefore, the paper describes a reactive power pricing problem, the modeling requirements, and the resulting extensions made to the OPF formulation and package. The paper formulated the pricing problem as a form of economic reactive power dispatch. In order to account for the transmission network restrictions, the model was formulated as a security constrained optimal power flow which incorporates all NGC's required modeling features.

1.4.6 Rates based on Marginal Cost Theory

The short-run costs of wheeling are defined as the marginal (incremental) costs of the last Mwh or Mvarh of wheeled energy [8]. The short-run marginal wheeling costs can be computed from the marginal costs of electricity at the buses where it enters and leaves the wheeling utility, and

$$\text{Ideal Wheeling Rate} = \text{Marginal Cost of Wheeling}$$

The ideal wheeling rate varies as the spot prices of electricity change. It recognizes the transmission constraints. For example, if wheeling causes an overload, the ideal wheeling rate increases to discourage it . If wheeling causes a reduction in losses, the ideal rate can even be negative. It is a characteristic of marginal cost pricing that the wheeling utility will at least recover its incremental operating costs, and will make a profit. However, there is no assurance that this utility will recover its imbedded capital costs(cost of building transmission system,.. ,etc.). In addition there is no guarantee that ideal wheeling rate will provide the funds which may be needed for wheeling. In fact, it may over-cover or under-cover capital. Therefore another solution should be introduced which should adjust the ideal wheeling rate to give the appropriate capital recovery:

$$\text{Revenue Reconciled Wheeling Rate} = \text{Ideal Wheeling} + \text{Revenue Reconciliation Adjustment}$$

The concept of revenue reconciliation is very complex. To date, there is no sound and good theory which tells how much capital should be recovered. Arguments also can be made for including capital recovery for generating plant, if wheeling causes a utility to lose electricity sales for which it had constructed power plants. That is, if the utility has an "obligation to serve" the buyer or seller of the wheeled power. Once the capital to be recovered has been decided, a revenue reconciliation adjustment can be made to recover it.

Reference [8] discusses the importance of wheeling especially in countries planning to privatize the electric sector. It uses the same method of [3] for setting wheeling rates.

1.5 Thesis Objective and Research

More research in the area of reactive power generation is required. Reactive power generation cost which is similar to active power generation cost was not addressed in previous works. Power losses, an important factor during wheeling, were not addressed in the previous work covered in this thesis. A wheeling utility might reschedule its dispatching pattern to account for the presence of the system losses caused by wheeling. Finally, the stability limits of interconnecting transmission lines were not addressed.

This thesis is directed toward the achievement of the following,

1. To build a model that will add the cost function of reactive power generation to the objective function.

2. To include Power system losses and line stability limits as extra constraints of the model .

3. To test the model on an IEEE 8 bus system and study the effects of tightening the constraints on the wheeling rates.

4. To add reactive power sources to some buses and study its effect on improving the rates.

5. To implement the model on a real system (GCC States) to show the adequacy of the model for practical systems.

1.6 Thesis Description

This thesis is divided into six chapters. Chapter 1 introduces the energy trading concept and wheeling. It also presents a literature survey on the subject of wheeling rates. Chapter 2 presents the general optimization model. Since the problem to be solved is non linear, a non-linear programming model is explained in details in this chapter. Chapter 3 contains the complete non linear model of setting wheeling rates. It is based on the marginal cost theory. Chapter 4 presents the results of the IEEE-eight bus test system. Chapter 5 applies the model obtained in chapter 3 to the networks of three of the Gulf Cooperation Council States. It also proposes modifications to the existing system. Chapter 6 presents general conclusions and recommendations for future extension to this thesis work.

CHAPTER 2

THE OPTIMIZATION MODEL

2.1 Introduction

One of the useful tools used in power systems for economic dispatch and least cost operation is optimization. Power system engineers try to obtain the most economical solution and at the same time abide with system rules and constraints. Two of the common methods used to solve any model optimally is linear and non-linear programming. Focus of this thesis will be on the later because the power system is non-linear by nature. Any optimization model consists, basically, of two main components [9] :

a) An objective function : $f(x)$, and

b) A set of system constraints : h , g

2.2 Non-Linear Programming Model

A non-linear programming method will be employed. The model will be formulated as follows [9] :

Minimize f (x) (2.1)

subject to $h_i (x) = 0$ (2.2)

for i = 1, ..., m

$g_i (x) < 0$ (2.3)

for i = 1,....., n

The above problem will be solved for values of $x_1,...x_k$ that satisfy the constraints while minimizing the objective function f (x) . In this model f is called the objective function or the criterion function. The first set of constraints $h_i (x) = 0$ for i =1,...,m is called the equality constraints and the second set $g_i (x) < 0$ for i = 1,...,n is called the inequality constraints. The vector $x \in X$ that satisfy all of the equality and inequality constraints is called a feasible solution to the problem. Therefore, a nonlinear programming model is solved to obtain a feasible point x' so that f (x) > f (x') for every feasible point x. The point x' is called an optimal solution or a solution to the problem. When more than one optimum solution exists , they are called alternative optimal solutions [9].

2.2.1 Lagrange Function

Sometimes it is required to get an extreme value for the objective function which is either maximum or minimum. In this case there are some necessary conditions that must be satisfied. This is simply done by adding the constraint function after multiplying it by an undetermined multiplier. This function is called the Lagrange function and represented Mathematically [1],

$$L(x,\lambda,\ \mu) = f(x) + \sum_{i=1}^{n} \lambda_i h_i(x) + \sum_{i=1}^{m} \mu_i g_i(x) \tag{2.4}$$

where

n : the dimension of the equality constraints vector.

m : the dimension of the inequality constraints vector.

λ_i : the equality constraints multiplier.

μ_i : the inequality constraints multiplier.

The necessary conditions for an extreme value of the objective function are met by taking the first derivative of the function L with respect to each of the independent variables and set all of these derivatives to zero.

The Lagrange function can be rewritten for inequality constraints of the form $a_i^{min} \leq g_i(x^0) \leq a_i^{max}$ as [5] ,

$$L(x, \lambda, m) = f(x) + \sum_{i=1}^{n} \lambda_i h_i(x) - \sum_{i=1}^{m} m_i^{min} + (g_i(x) - a_i^{min}) + \sum_{i=1}^{m} m_i^{max} (g_i(x) - a_i^{max}) \quad (2.5)$$

2.2.2 Karush - Khun - Tucker Conditions

The optimum solution is obtained when certain conditions of the Lagrange function are satisfied. They are called the Khun-Tucker conditions. For a vector x of dimension N , the point (x^0, λ^0, μ^0) is an optimum point if [1] ,

1. $\frac{\partial L}{\partial x_i}(x^0, \lambda^0, \mu^0) = 0$ (2.6)

2. $h_i(x^0) = 0$ for i = 1,...., Nh (2.7)

3. $g_i(x^0) \leq 0$ for i = 1,....., Ng (2.8)

4. $\mu_i^0 \, g_i^0 (x0) = 0$ for $I = 1, 2, \ldots, Ng$

$\mu_i^0 \geq 0$ (2.9)

The above model will be used to solve the economic dispatch problem presented in chapter 3. The Lagrange multipliers λ_I of the quality constraints represent the cost of producing an extra MWh of energy [1].

Wheeling rates can be obtained using an optimization model. The model consists of three utilities : the seller , the buyer and the wheeler. The system will be solved as one network. The objective will be to minimize the operating costs of the generating units for active power, reactive power. A quadratic function typical for operating costs will be used for every generator in the system. The capital cost of the wheeling equipment such as newly constructed transmission lines, power transformer, converters, inverts, etc. will be a constant value. The quality and inequality constraints will be added to the model which is explained in details in chapter 3. The quality constraints will consists of the Kirchoff's laws that regulate the power flow in the whole network. The inequality constraints such as generator loading limits, transmission lines thermal capacity, minimum and maximum voltage at every bus and transformer tap limits will be imposed on the model used to solve the problem.

CHAPTER 3

Modified OPF Model for setting Wheeling Rates

3.1 Introduction

In this chapter, a modified Optimal Power Flow (OPF) model will be presented. This model will be used to derive the wheeling rates for both active and reactive power. The cost of providing an extra Mwh will be the Lagrange multipliers corresponding to the equality constraints of the Kirchoff's laws.

3.2 Model Formulation

It is assumed that three utilities agree to exchange power or trade energy. The three utilities are a seller, a wheeler and a buyer. It is understood that all agree on minimizing the operational cost involved due to wheeling. This is done to maximize their benefits and at the same time comply with the system operational constraints. The following model is developed and expressed mathematically as,

$$\text{Minimize } C = \Sigma [C_i (P_{Gi}) + C_i (Q_{Gi}) + C_w] \quad (3.1)$$

Subject to,

$$P_{Gi} + P_{Si} - P_{di} - P_{Bi} = \sum_{i=1}^{N} V_i V_j Y_{ij} \cos (\theta_{ij} + \delta_j - \delta_i) \quad (3.2)$$

$$Q_{Gi} + Q_{Si} - Q_{di} - Q_{Bi} = - \sum_{i=1}^{N} V_i V_j Y_{ij} \sin (\theta_{ij} + \delta_j - \delta_i) \quad (3.3)$$

$$V_i^{min} \leq V_i \leq V_i^{max} \qquad i \in N \quad (3.4)$$

$$P_{Gi}^{min} \leq P_{Gi} \leq P_{gi}^{max} \qquad i \in NG \quad (3.5)$$

$$Q_{Gi}^{min} \leq Q_{Gi} \leq Q_{gi}^{max} \qquad i \in NG \quad (3.6)$$

$$T = \text{closest integer to} \left[\frac{V_{spec} - V_i}{V_i * \text{Tap Step}} \right] \quad (3.7)$$

$$P_{ij} \leq V_i V_j Y_{ij} \cos (\theta_{ij} + \delta_j - \delta_i) - V_i^2 \; Y_{ij} \cos (\theta_{ij})$$

(3.8)

$$P_{Rmax} = \frac{1}{Z} [V_r V_s - A V_r^2 \cos(\theta_z - \theta_A)]$$

(3.9)

$$P_{loss} = \sum_{i=1}^{N} \sum_{j=1}^{N} V_i V_j Y_{ij} \cos (\delta_i - \delta_j - \theta_{ij}) \quad (3.10)$$

$$\sum_{i=1}^{N} P_R - \sum_{i=1}^{N} Pi + Ploss = 0 \qquad (3.11)$$

The model developed is detailed below,

3.2.1 Cost Function

It is understood that the three utilities involved in this energy trade are interested in maximizing their profits. Also, the buyer is interested in minimizing the cost involved in buying power from a seller in order to minimize his customers' expenses. This is an economic dispatch problem. The objective is to minimize the cost involved due to wheeling and at the same time to comply with system operational constraints. The objective function can be modeled as :

$$\text{Minimize } C = \Sigma\, [\, C_i(P_{Gi}) + C_i(Q_{Gi}) + C_w\,] \qquad (3.12)$$

In the above function $C_i(P_{Gi})$ represents the cost function of producing a P_{Gi} unit of power at bus i. This function can be obtained from the system's generators manufacturer(s) or from the generating plants staff who maintain the input-output curve of every unit. Figure 3.1 represents an input-output curve of a generator for active power. It shows the relationship between the output of a generator in MW and the incremental cost in \$/Mwh for a typical generator. $C_i(Q_{Gi})$ represents the cost function of generating Q_{Gi} unit of reactive power at bus i. This function can be obtained from the generating plants and it is shown in figure 3.2. Another form of $C_i(Q_{Gi})$ cost function could be the cost involved in operating a Static Var Compensator (SVC) or Static Shunt Capacitor (SSC). Switching of a reactive power source will involve an added cost of operation. This function can be linear or even quadratic depending on the reactive source involved. The last part of the objective function is C_w. It represents the capital cost involved. For example, in a case of three isolated neighboring systems it could represent the capital investment required for installing wheeling facilities. The facilities include interconnecting transmission lines and power transformers at the wheeler's border buses.

3.2.2 Power Flow Equations

The power flow is regulated by kirchoff's laws, which determine the flow of power throughout the network. This set of equations represents the equality constraints that must be complied with. For the wheeling system, the power flow equations are given as:

$$P_{Gi} + P_{Si} - P_{di} - P_{Bi} = \sum_{i=1}^{N} V_i V_j Y_{ij} \cos(\theta_{ij} + \delta_j - \delta_i) \qquad (3.13)$$

$$Q_{Gi} + Q_{Si} - Q_{di} - Q_{Bi} = -\sum_{i=1}^{N} V_i V_j Y_{ij} \sin(\theta_{ij} + \delta_j - \delta_i) \qquad (3.14)$$

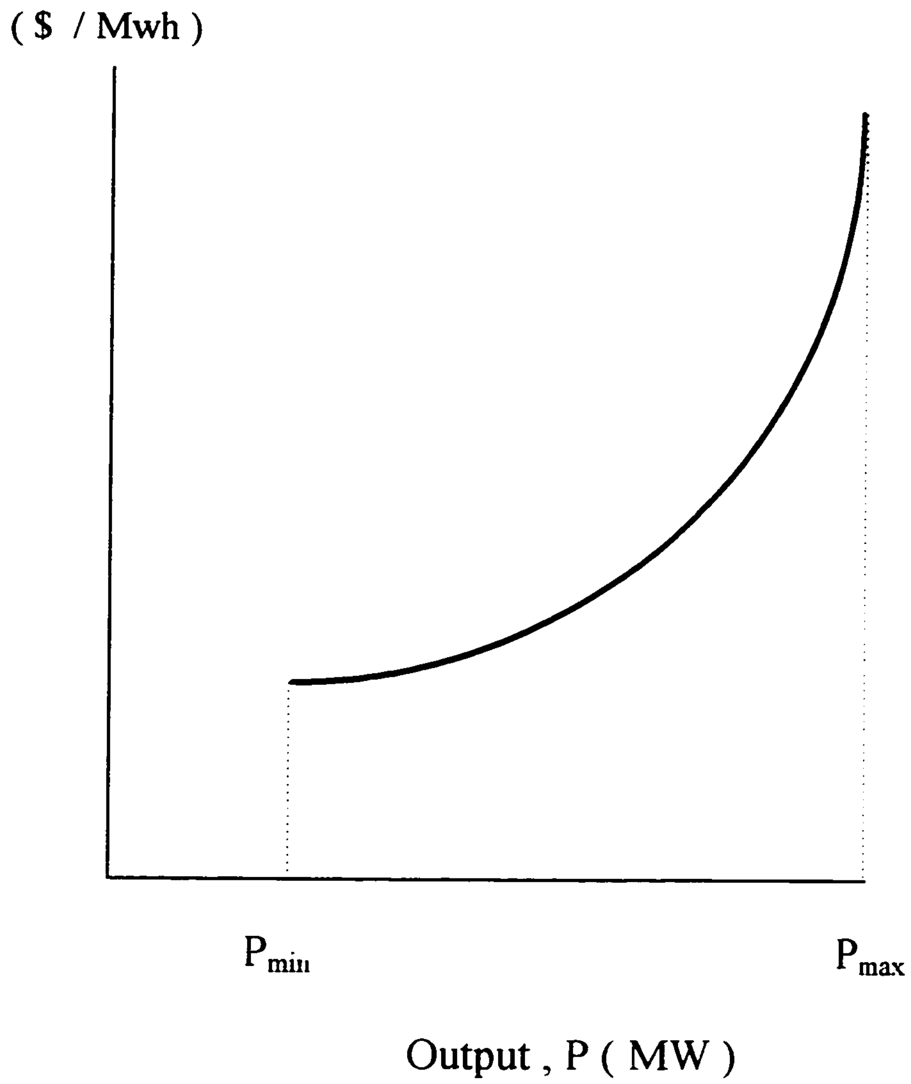

Fig. 3.1 Active Power Input-output curve of a typical generator

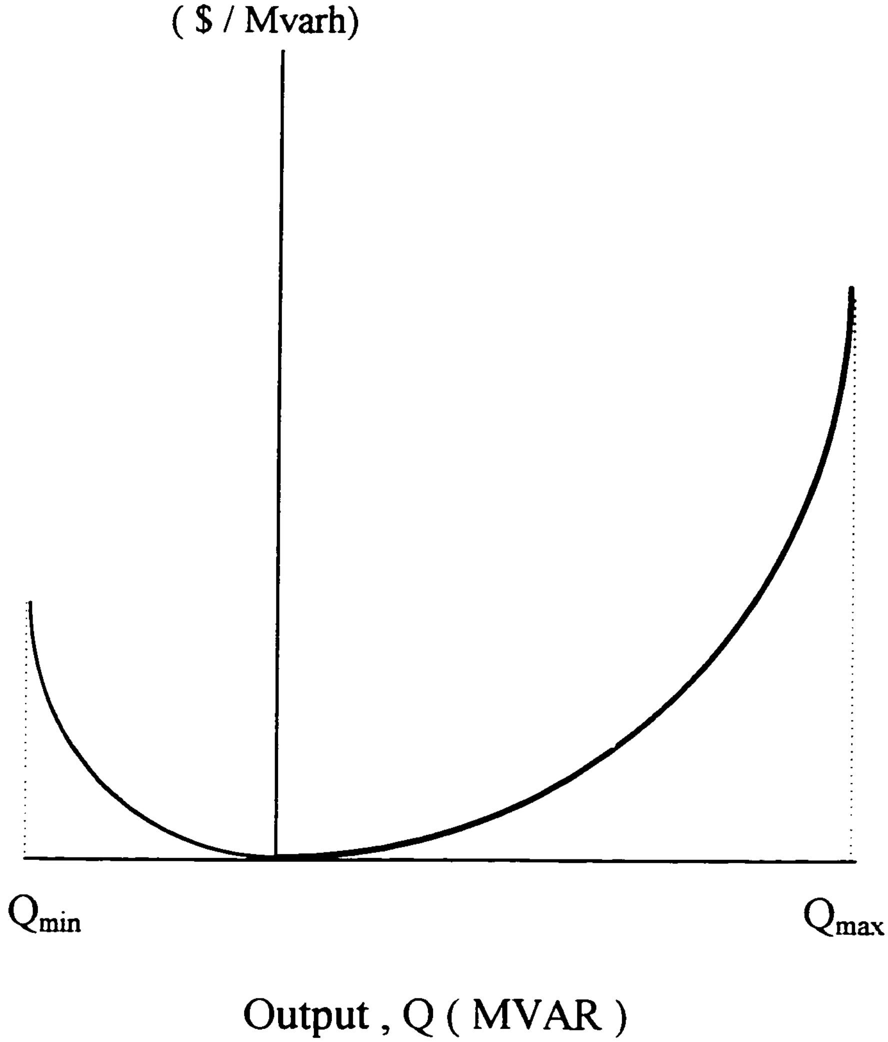

Fig. 3.2 Reactive Power Input-output curve of a typical generator

When solving the optimization model, the Lagrange multipliers corresponding to equations 3.13 and 3.14 represent the marginal cost of providing an extra MWh or MVArh of energy at bus i. The marginal costs at the seller and buyer buses are of importance since they are part of the wheeling rate for both active and reactive power. The rates are shown below as :

$$\omega_P = MCP_B - MCP_S \quad (3.15)$$

$$\omega_Q = MCP_Q - MCP_Q \quad (3.16)$$

$$W_P = P_B \, \omega_P \quad (3.17)$$

$$W_Q = Q_B \, \omega_Q \quad (3.18)$$

$$W = W_P + W_Q \quad (3.19)$$

3.2.3 Voltage Limits

The minimum and maximum voltage limits will be set. These constraints limit the voltage in a certain range. It is not necessary to have the same voltage range for all buses. For example, the interconnecting bus voltages can be set at higher limits (i.e. 0.95 and 1.1 P.U.) and other system buses at lower limits (i.e. 0.9 and 1.05). The slack bus is usually set at 1.0 P.U., but it may also have maximum and minimum ranges. The voltage constraints can be included in the model as,

$$V_i^{min} \leq V_i \leq V_i^{max} \qquad i \in N \tag{3.20}$$

3.2.4 Generation Constraints

Maximum and minimum generator limits are also set. The generator limits are directly related to the marginal costs. Similar to voltage limits, generation limits are usually expressed as maximum and minimum for every generator in the system. Mathematically they are represented as ,

$$P_{Gi}^{min} \leq P_{Gi} \leq P_{gi}^{max} \qquad i \in NG \tag{3.21}$$

For the reactive generation there are two cases. The first case is similar to the active power case where limits are expressed in the same fashion as active power limits:

$$Q_{Gi}^{min} \leq Q_{Gi} \leq Q_{gi}^{max} \quad \boldsymbol{i \in NG} \tag{3.22}$$

In the second case, a source of reactive power is switched at a specific bus. The minimum reactive generation will be zero. The limits are expressed as:

$$0 \leq Q_{Ci} \leq Q_{ci}^{max} \quad i \in NC \tag{3.23}$$

3.2.5 Transformer Tap Changer Limits

The transformer's tap changer limits are,

$$T_i^{min} \leq T_i \leq T_i^{max} \quad i \in NT \tag{3.24}$$

The tap changer position is obtained by following two steps:

i) The complete model is solved and the bus voltage corresponding to the tap changer side is obtained.

ii) A mathematical formula is used to obtain the tap changer position to the closest integer.

Mathematically the tap position is expressed as:

$$T = \text{closest integer to} \left[\frac{V_{spec} - V_i}{V_i * \text{Tap Step}} \right] \quad (3.25)$$

V_{spec} represents the voltage required at that bus and V_i the voltage obtained from the nonlinear model at bus i.

3.2.6 Transmission Limits

Transmission line limits can be defined as the maximum power or current that a line can transmit under some specified conditions. The amount of power that can be transmitted on a line connecting bus i to bus j can not exceed some limits due to the design and load restrictions. Two types of limits govern the power flow on a transmission line. Thermal limit dominates for shorter lines usually less than eighty kilometers. The stability limit will be considered for transmission lines that are longer than eighty kilometers. The thermal limit is expressed as ,

$$I_i \leq I_i^{max} \qquad i \in NL \quad (3.26)$$

and in terms of power transfer thermal limit between bus i and j is,

$$P_{ij} = V_i\ V_j\ Y_{ij} \, Cos\,(\,\theta_{ij} + \delta_j - \delta_i\,) - V_i^2\ \ Y_{ij}\, Cos\,(\,\theta_{ij}\,) \qquad (3.27)$$

The stability limits can be represented as,

$$P_{Rmax} = \frac{1}{Z'}[\, V_r\, V_s - A\, V_r^2\, Cos(\theta_z - \theta_A)\,] \qquad (3.28)$$

$$A = 1 + YZ'/2 = \ A\ \angle\theta_A \qquad (3.29)$$

V_r and V_s represent the receiving end and sending end voltages, respectively and Y and Z' represent the shunt admittance and line impedance, respectively.

3.2.7 Transmission Losses

The presence of the losses will increase the cost function since it must be compensated for by the generating units. Wheeling may increase or decrease losses depending on whether the wheeled power is in the same direction of the wheeler's load or not. Mathematically the total system losses are expressed as,

$$P_{loss} = \sum_{i=1}^{N} \sum_{j=1}^{N} V_i\ V_j\ Y_{ij}\, Cos\,(\delta_i - \delta_j - \theta_{ij}\,) \qquad (3.30)$$

3.2.8 Power Balance Constraint

Since losses are included in the model it can be simply stated that the generation equals the load. The total generated power $\sum_{i=1}^{N} Pi$ by all generating units will equal the total load at all buses plus total system losses. Mathematically,

$$\sum_{i=1}^{N} P_R - \sum_{i=1}^{N} Pi + Ploss = 0 \qquad (3.31)$$

3.4 Method Of Solution

The model developed in sec. 3.2 will be solved using GINO (General Interactive Optimizer). GINO uses the Generalized Reduced Gradient Method (GRG) for generating improving feasible directions. This method was developed by Abadie and Carpentier which handles nonlinear constraints [14]. It depends on reducing the dimension of the problem by representing all the variables in terms of an independent subset of the variables. The general algorithm of the gradient-based iterative method is described in [14] as follows :

1. Compute the gradient of f at the current point (x_c , y_c),

   $\nabla f(x_c, y_c)$.

2. If (xc , yc) is close enough to being optimal, stop.

3. Compute a search direction d using $\nabla f(x_c, y_c)$ and perhaps other information.

4. Determine how far to move along the direction d, starting from (x_c, y_c) , and move this distance along d to a new point (x_{new} , y_{new}). Replace (x_c, y_c) by (x_{new} , y_{new}) and return to step 1.

CHAPTER 4

Application of the Modified OPF to a simple Power System

4.1 Introduction

In this chapter the mathematical model developed in chapter 3 will be implemented on the test system presented in Fig. 4.1. The technical data of the IEEE 8 bus system is given in [6]. The network consists of three utilities. The seller and the buyer are located on buses 5 and 6, respectively. It is assumed that a generator exists on bus 5 (entry bus) and power is extracted on bus 6 (exit bus).

4.2 Model Formulation

The modified OPF model developed in chapter 3 is used. The objective is to minimize the operational cost of the wheeling utility subject to the operational constraints. The constraints include voltage limits, line capacity limits and generation limits. Transmission lines stability limits and active and reactive power balance are also included. A base power of 100 MVA and voltage of 400 Kv are used in the study. The objective function consists of two major parts. The cost function of the of active and reactive power generation. It is assumed that reactive power will have a cost function similar to the active power cost function. The cost of producing an extra MW is higher than the cost of generating a MVAR. The reactive power cost function may involve the operational cost of a variable reactive power source such as a static Var compensator (SVC) or a shunt capacitor bank. It is assumed that the wheeler has its own load of 300 MW, 100 MW and 50 MW at buses 4 ,7 and 8. Different Values of the wheeled power are studied ranging from 20 MW to 140 MW in a 20 MW step.

4.2.1 Modified OPF Model

The model is described below as follows,

Objective Function :

Minimize $C = 50\,P_1 + 50\,P_1^2 + 75\,P_2 + 75\,P_2^2 + 50\,P_3 + 50\,P_3^2 + 0.50\,Q_1 + 0.50\,Q_1^2 + 0.75\,Q_2 + 0.75\,Q_2^2 + 0.50\,Q_3 + 0.5\,Q_3^2$

Load Flow Equations :

$$P_{Gi} + P_{Si} - P_{di} - P_{Bi} = \sum_{j=1}^{N} V_i\, V_j\, Y_{ij} \cos(\theta_{ij} + \delta_j - \delta_i)$$

$$Q_{Gi} + Q_{Si} - Q_{di} - Q_{Bi} = -\sum_{j=1}^{N} V_i V_j Y_{ij} \sin(\theta_{ij} + \delta_j - \delta_i)$$

$i = 1,2,\ldots,8$

Generator's Limits :

$0 \leq P_{G1} \leq 5.0$

$0 \leq P_{G2} \leq 2.0$

$0 \leq P_{G3} \leq 2.0$

$0 \leq Q_{G1} \leq 3.75$

$0 \leq Q_{G2} \leq 2.0$

$0 \leq Q_{G3} \leq 2.0$

Voltage Limits :

$0.95 \leq V_i \leq 1.05$

where $i = 1,2,\ldots, 8$

Transmission Line Thermal Limits :

$I_i \leq 500$ A $\quad i \in NL$

or

$I_i \leq 250$ A $\quad i \in NL$

Transmission Line Stability Limits :

It was assumed that only line 3 -6 is long and stability limit of this line need to be considered in the model . Therefore, the formula used is :

$P_{36} \leq 9.5$ P.U.

Power Balance equation :

$$\sum_{i=1}^{N} P_R - \sum_{i=1}^{N} P_i + P_{loss} = 0 \quad , N = 8$$

Transmission Losses :

$$P_{loss} = \sum_{i=1}^{N} \sum_{j=1}^{N} V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \quad , N = 8$$

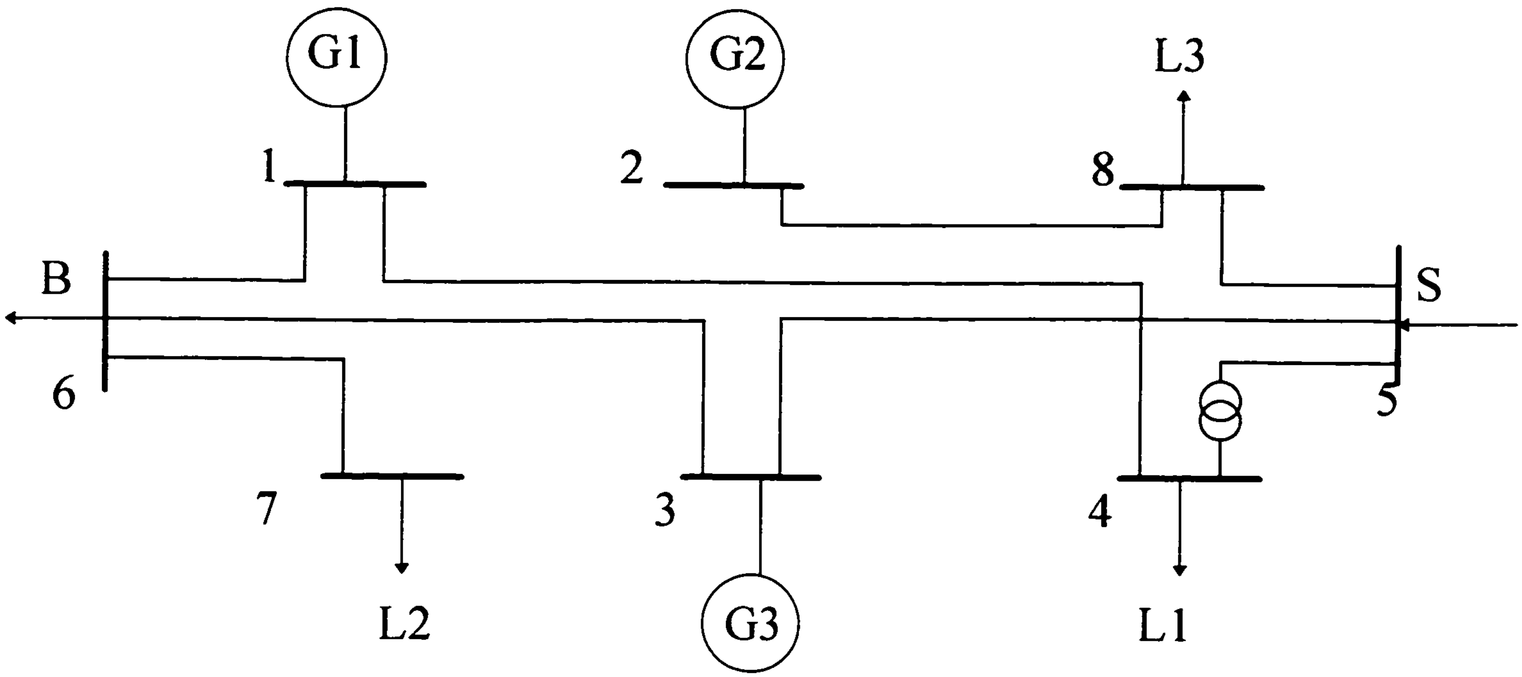

Fig. 4.1 Test System of Case Study 1

4.3 Case Studies

The system presented above will be studied for different operating conditions. Different cases will be presented in order to evaluate the effects of changing the operating constraints on the wheeling rates.

4.3.1 Case 1

In this case the wheeler's network was studied prior to wheeling. The wheeler's load is kept constant at 300 MW, 100 MW and 50 MW at buses 4 ,7 and 8 with a power factor of 0.8. The voltage limits are 0.9 and 1.1 P.U. The cost function for this network was found to be $ 692.98 with 23.3 MW transmission losses.

4.3.1 Case 2

To examine the accuracy of the model developed , the model was used on a previous work done in [6]. The same loads of case 1 are used. Table 4.1 illustrates the results of the previous work. Table 4.2 illustrates the output of the model developed in chapter 3. Figures 4.2 and 4.3 present a comparison of the wheeling rates of active power and reactive power respectively. There were some similarities and differences in the inequality binding constraints. The differences could be due to the lack of complete data of the model. The details of the transformer were not provided in reference 6.

Power Wheeled (MW)	25	50	75	100	125	150	175	200
BINDING INEQUALITY CONSTRAINTS (Previous Work)	V2(max) V3(max)	V2(max) V3(max)	V2(max) V3(max) Q2(min)	V1(max) V2(max) V3(max) Q2(min)	V1(max) V3(min) Q2(min)	V1(max) V3(max) Q2(min) T(min)	V5(max) Q2(min) T(min)	V5(max) Q2(min) T(min) P3(max)
MCPB($/MWH)	2.38	2.41	2.44	2.47	2.51	2.55	2.62	2.72
MCPS($/MWH)	2.21	2.18	2.15	2.13	2.12	2.11	2.03	2.03
MCQB($/MWH)	0.027	0.033	0.039	0.046	0.054	0.062	0.075	0.092
MCQS($/MWH)	0.005	-0.002	-0.021	-0.059	-0.092	-0.126	-0.249	-0.3
WP ($/MWH)	0.17	0.23	0.29	0.34	0.39	0.44	0.59	0.69
WQ ($/MVARH)	0.022	0.035	0.06	0.105	0.146	0.188	0.324	0.392
TOTAL RATE($)	4.6625	12.8125	25.125	41.875	62.4375	87.15	145.775	196.8

Table 4.1 Wheeling Rates of Reference [6]

Power Wheeled (MW)	25	50	75	100	125	150	175	200
BINDING INEQUALITY CONSTRAINTS (Thesis model)	V2(max) V3(max) T(min)	V2(max) V3(max) T(min)	V5(max) Q2(min) T(min)	V5(max) Q2(min) T(min)	Q2(min) Q1(max)	V5(max) Q2(min) Q1(max)	V5(max) Q2(min) Q1(max)	V5(max) Q2(min) Q1(max) V7(min)
MCPB($/MWH)	2.48	2.5	2.53	2.57	2.61	2.66	2.71	2.85
MCPS($/MWH)	2.26	2.25	2.24	2.24	2.23	2.24	2.24	2.2
MCQB($/MWH)	0.033	0.04	0.049	0.063	0.089	0.119	0.152	0.348
MCQS($/MWH)	0.012	0.01	-0.019	-0.027	-0.035	-0.043	-0.053	-0.136
WP ($/MWH)	0.22	0.25	0.29	0.33	0.38	0.42	0.47	0.65
WQ ($/MVARH)	0.021	0.03	0.068	0.09	0.124	0.162	0.205	0.484
TOTAL RATE($)	5.89375	13.625	25.575	39.75	59.125	81.225	109.156	202.6

Table 4.2 Wheeling Rates of Thesis Model

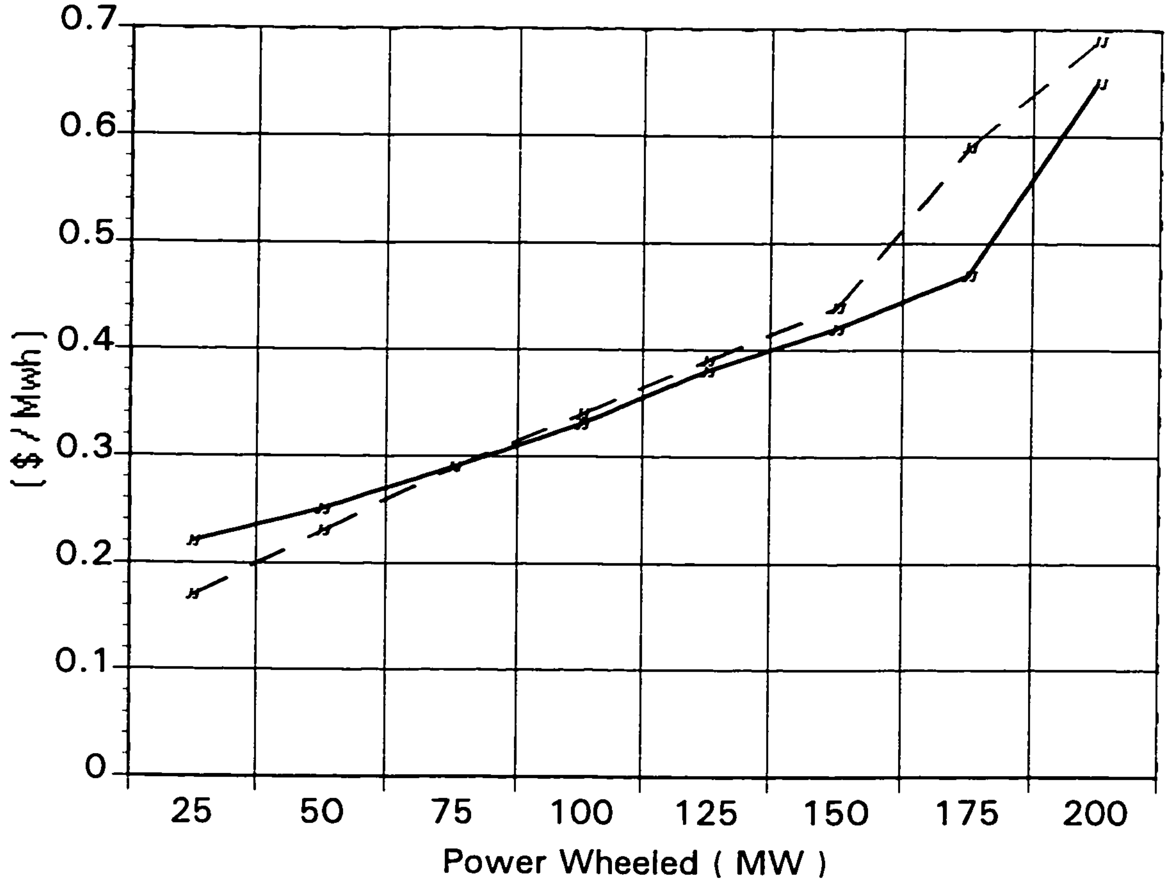

Fig. 4.2 Active Power Wheeling Rates Comparison of Reference 6 and thesis model

----- Reference 6

___ Thesis Model

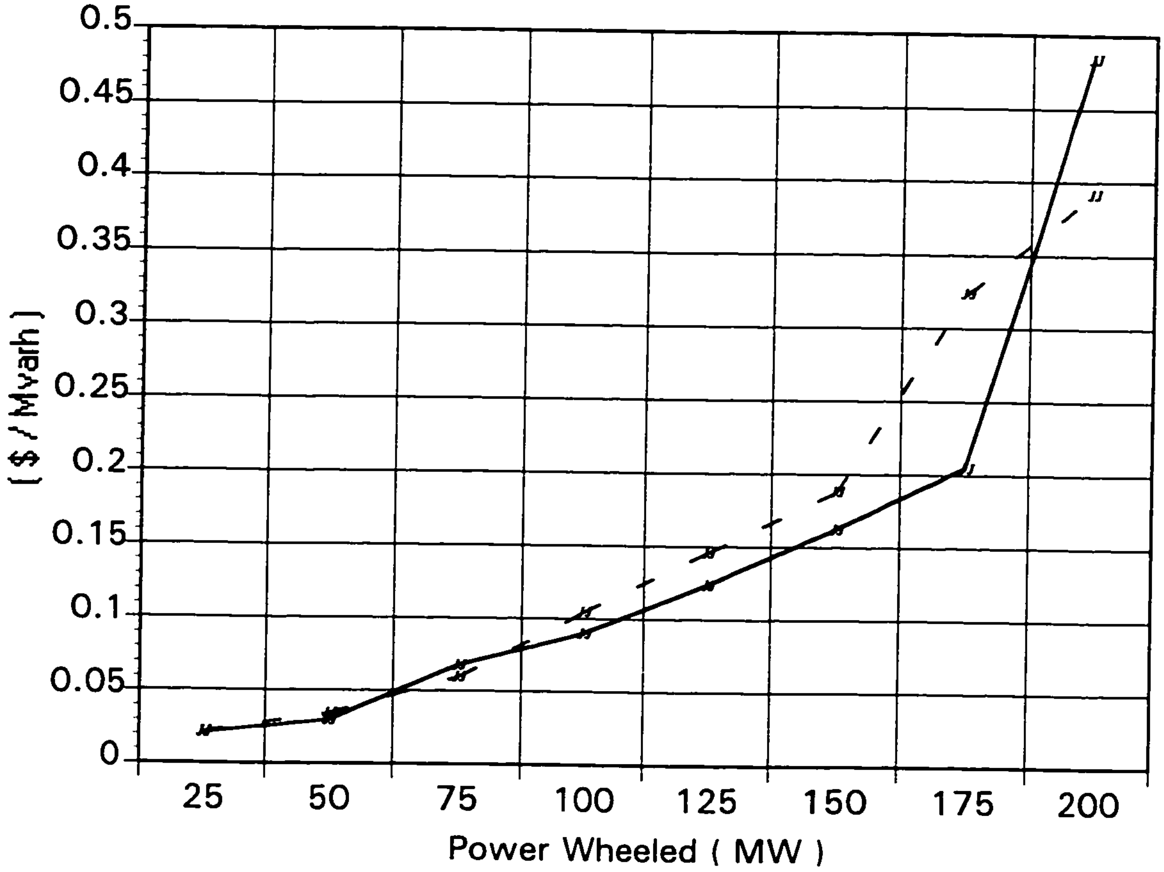

Fig. 4.3 Reactive Power Wheeling Rates Comparison of Reference 6 and thesis model

----- Reference 6

___ Thesis Model

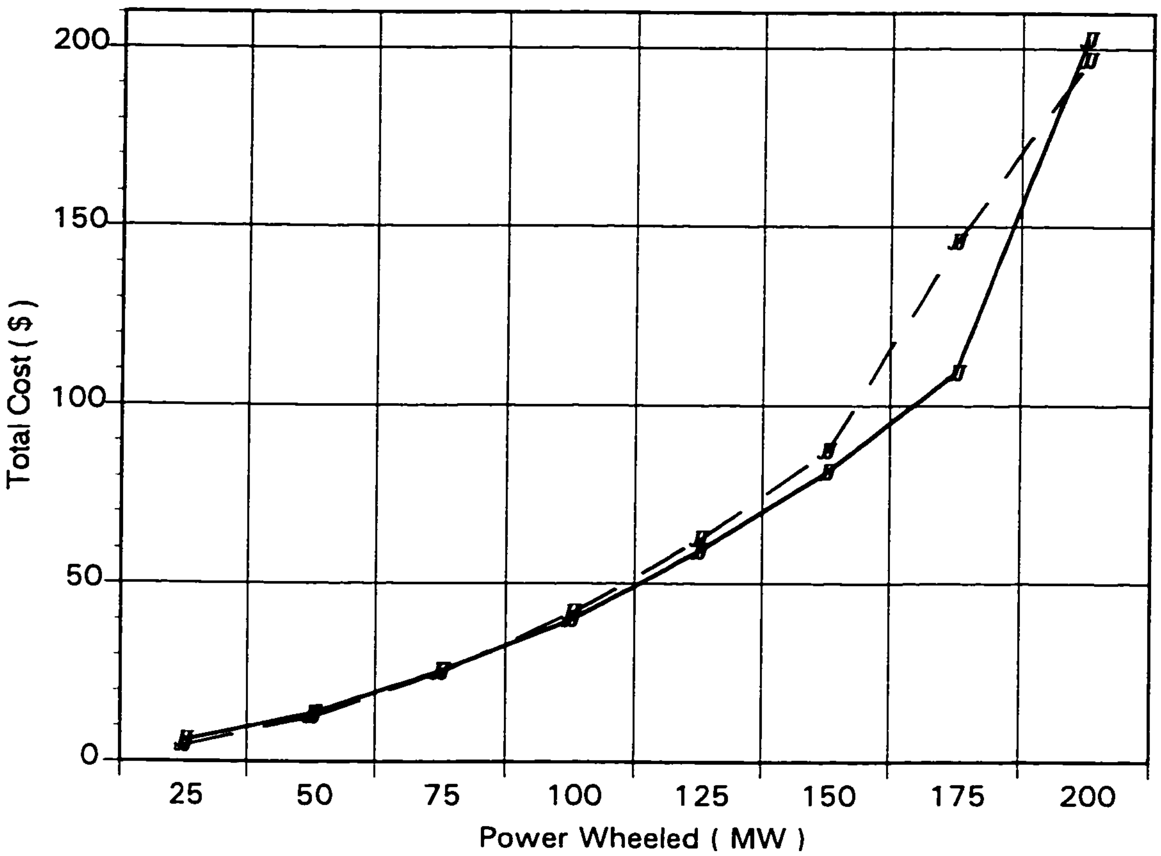

Fig. 4.4 Total Wheeling Rates Comparison of Reference 6 and thesis model

----- Reference 6

___ Thesis Model

4.3.3 Case 3

In this case, the cost function of reactive power and the equation of the stability limit were not included in the model. The voltage limits of 0.9 and 1.1 P.U. are used at all the buses. Table 4.3 presents the wheeling rates for active and reactive power, the binding constraints, system losses and the behavior of the wheeler's cost function. Figures 4.5 - 4.7 present the change of wheeling rates of active and reactive power and the change of the objective function for different values of wheeled power. It is observed that wheeler's cost function and power losses increased. This is reflected as an increase in the wheeling rates. Table 4.3 illustrates the wheeler's net benefits from wheeling.

POWER WHEELED(MW)	20	40	60	80	100	120	140
BINDING INEQUALITY CONSTRAINTS	V2(max) V3(max)	V2(max) V3(max)	V3(max) V3(max) Q2(min) T(min)	V5(max) Q2(min) T(min)	V5(max) Q1(max) Q2(min) T(min)	V5(max) Q1(max) Q2(min)	V5(max) Q1(max) Q2(min)
MCPB($/MWH)	2.47	2.49	2.52	2.54	2.57	2.6	2.64
MCPS($/MWH)	2.26	2.25	2.24	2.24	2.23	2.24	2.24
WP ($/MWH)	0.21	0.24	0.28	0.3	0.34	0.36	0.4
MCQB($/MWH)	0.032	0.038	0.044	0.051	0.062	0.083	0.106
MCQS($/MWH)	0.013	0.011	-0.013	-0.021	-0.027	-0.033	-0.039
WQ ($/MWH)	0.019	0.027	0.057	0.072	0.089	0.116	0.145
TOTAL RATE ($)	4.485	10.41	19.365	28.32	40.675	53.64	71.225
LOSSES (MW)	23.2	25.2	27.4	30.02	32.97	36.3	40
WHEELER COST($)	686.7	691.5	698.04	703.8	711.2	719.8	729.4
WHEELER Net Benefit ($)	0.175	1.3	3.715	6.91	11.865	16.23	24.215

Table 4.3 Wheeling Rates Comparison & Wheeler's Cost Change (Case 3)

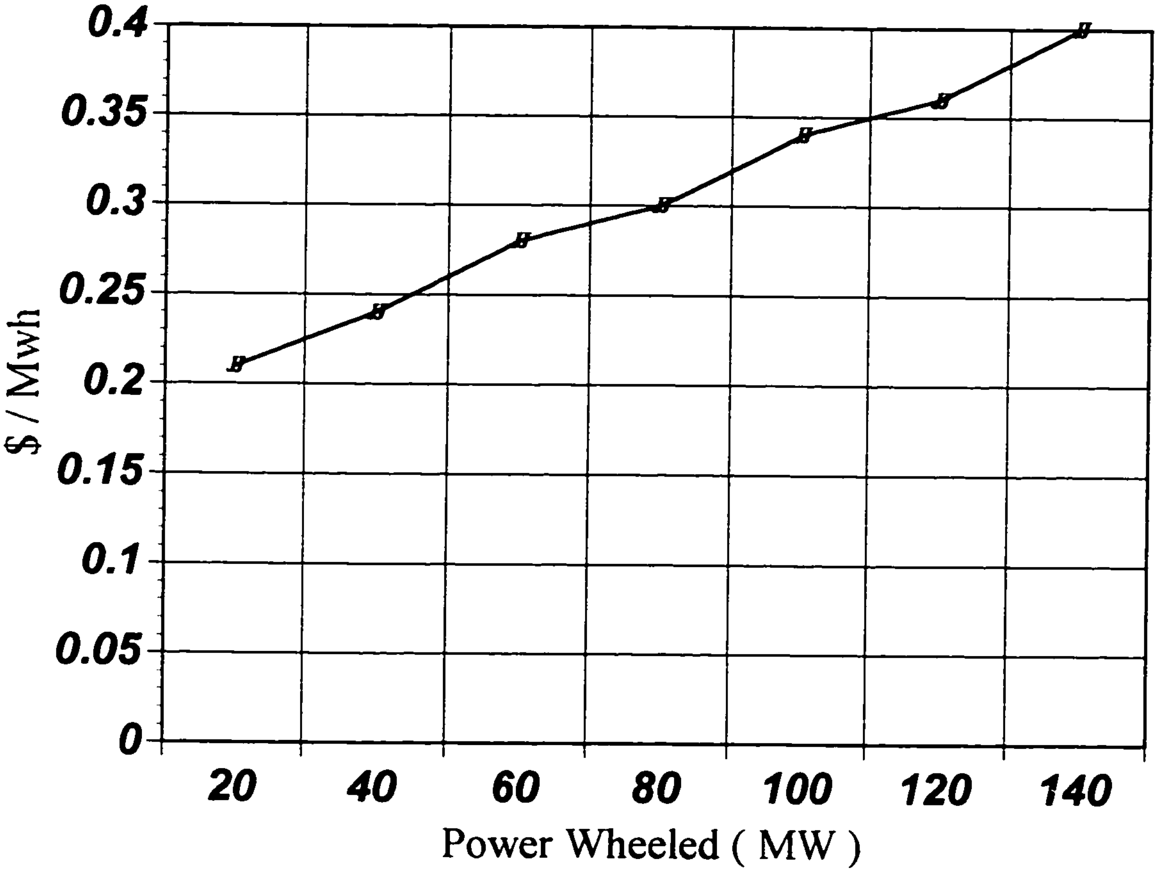

Fig. 4.5Variation of Active Power Wheeling rates (Case 3)

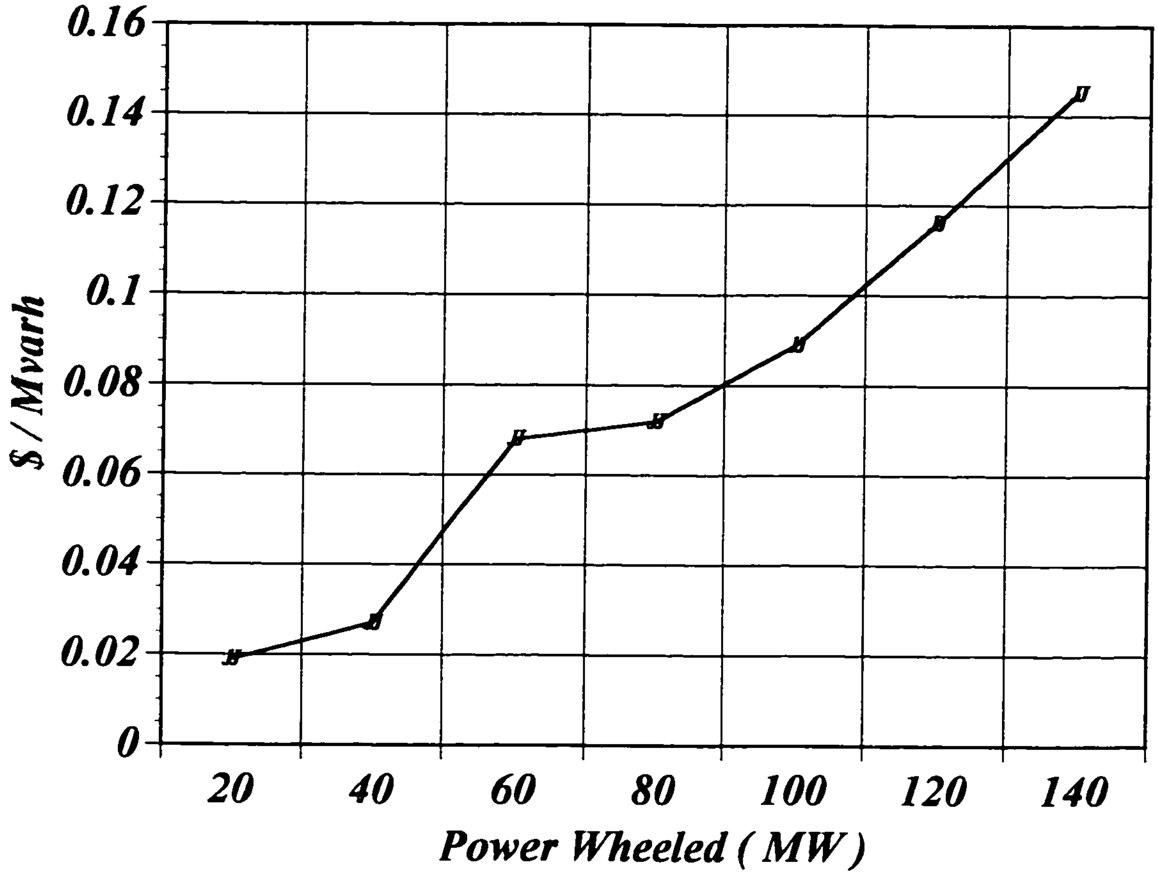

Fig. 4.6 Variation of Reactive Power Wheeling rates (Case 3)

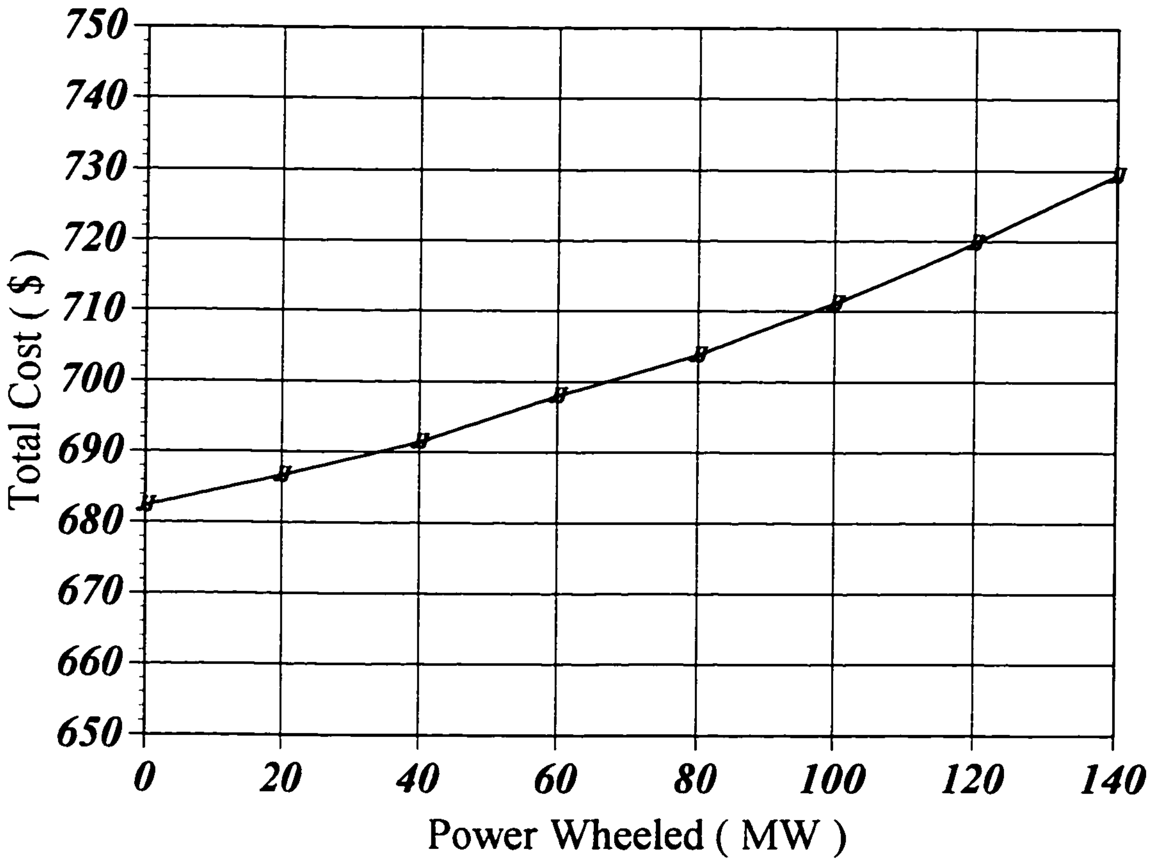

Fig. 4.7 Variation of Wheeler's Total Cost (Case 3)

4.3.4 Case 4

This case is similar to case 3 except that the cost function of the reactive power and the stability limit of line 3-6 are included. Table 4.4 and figures 4.8 - 4.10 present the output for this case. The effect of the reactive power generation cost is observed in this case. Table 4.4 illustrates the wheeling rates variation. It can be observed that the addition of the reactive power cost function will change the reactive power wheeling rates. It has a minor effect on the active power wheeling rates.

Power Wheeled (MW)	20	40	60	80	100	120	140	200
BINDING INEQUALITY CONSTRAINTS			V2(max)	V2(max) V4(min)	V2(max) V4(min)	V2(max) V4(min)	V2(max) V4(min) Q1(max)	Q1(max) Q2(min) V4(min)
MCPB($/MWH)	2.48	2.51	2.53	2.56	2.58	2.61	2.64	3.16
MCPS($/MWH)	2.25	2.26	2.25	2.25	2.25	2.25	2.24	1.92
WP ($/MWH)	0.23	0.25	0.28	0.31	0.33	0.36	0.4	1.24
MCQB($/MWH)	0.059	0.066	0.073	0.08	0.09	0.1	0.12	1.049
MCQS($/MWH)	0.017	0.015	0.012	0.01	0.009	0.007	0.007	-0.444
WQ ($/MWH)	0.042	0.051	0.061	0.07	0.081	0.093	0.113	1.493
TOTAL RATE($)	5.23	11.53	19.545	29	39.075	51.57	67.865	471.95
LOSSES (MW)	25.1	27.1	29.3	31.8	34.59	37.58	41.03	47.8
WHEELER COST($)	697.81	703.3	709.42	716.24	723.7	731.95	740.97	784.35
WHEELER Net Benefit ($)	0.4	1.21	3.105	5.74	8.355	12.6	19.875	380.58

Table 4.4 Whceling Rates Comparison & Wheeler's Cost Change (Case 4)

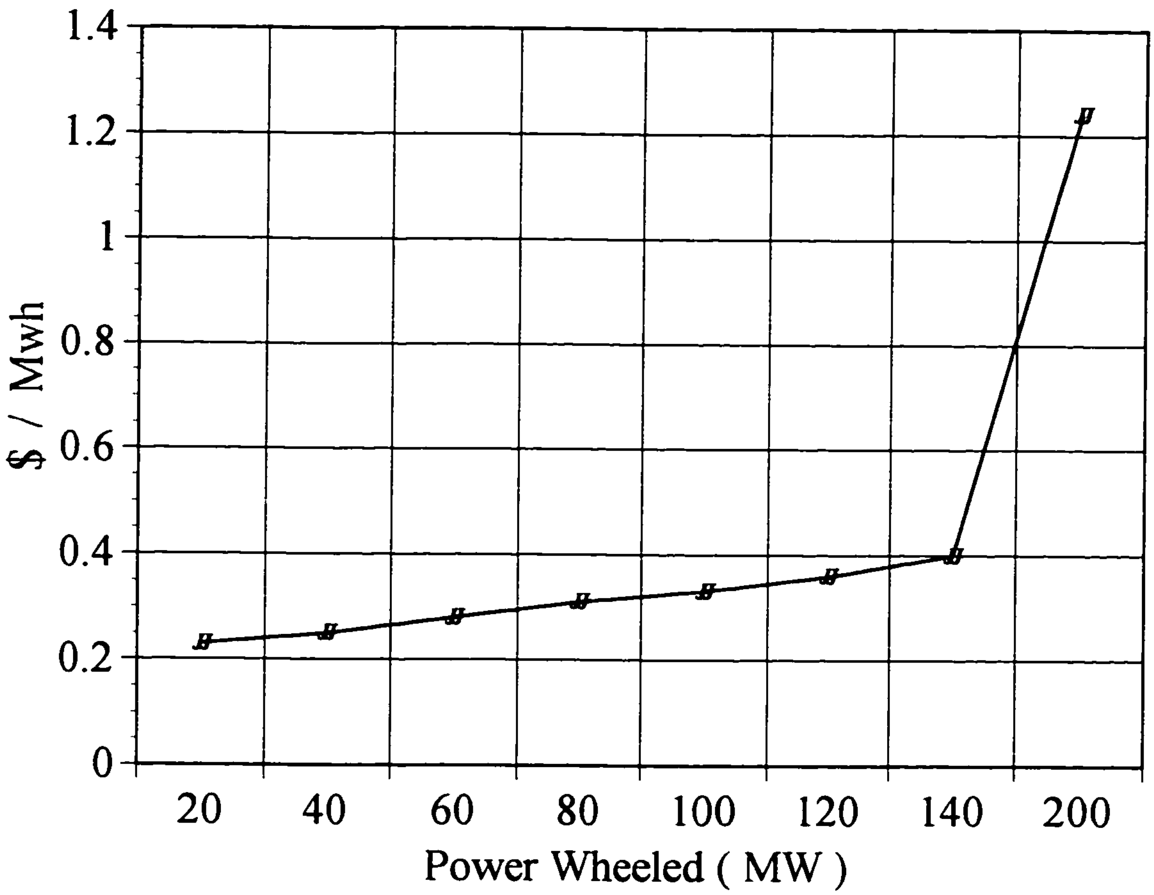

Fig. 4.8 Comparison of Active Power Wheeling rates (Case 4)

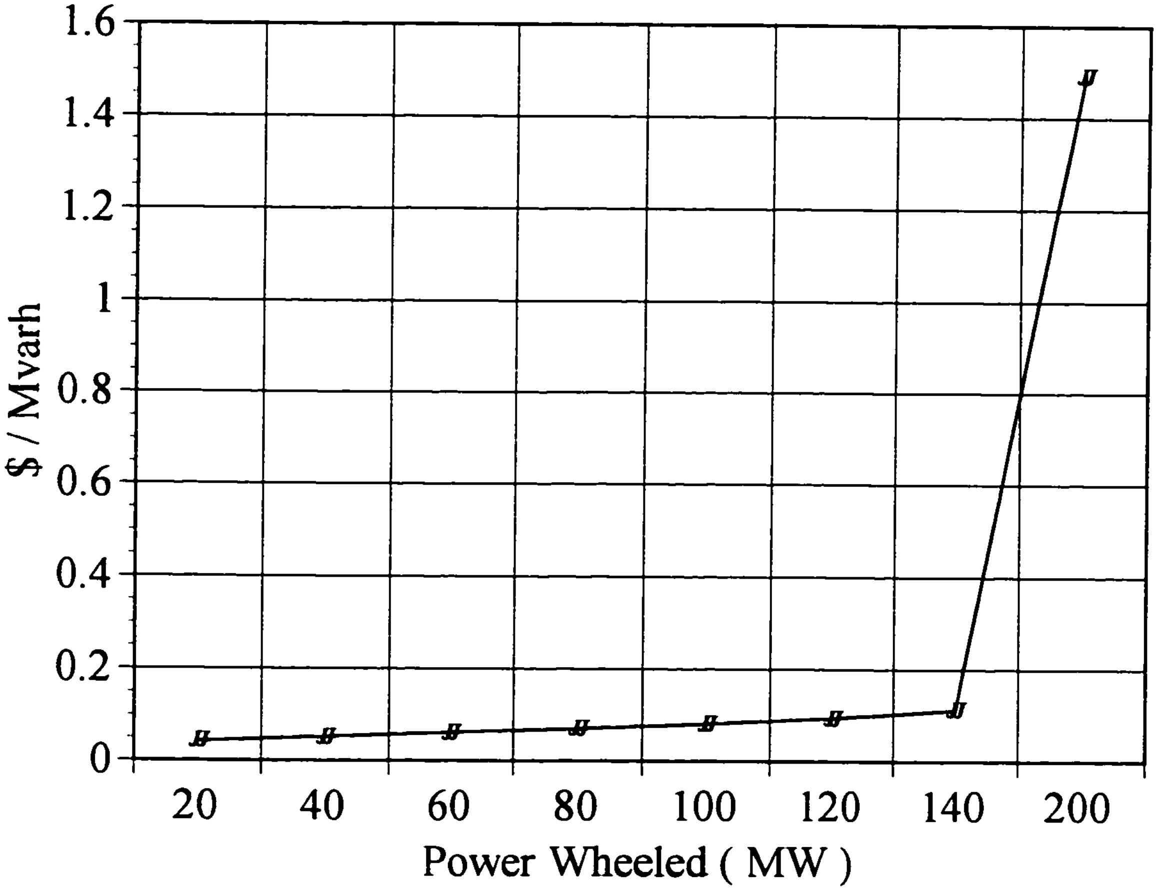

Fig. 4.9 Comparison of Reactive Power Wheeling rates (Case 4)

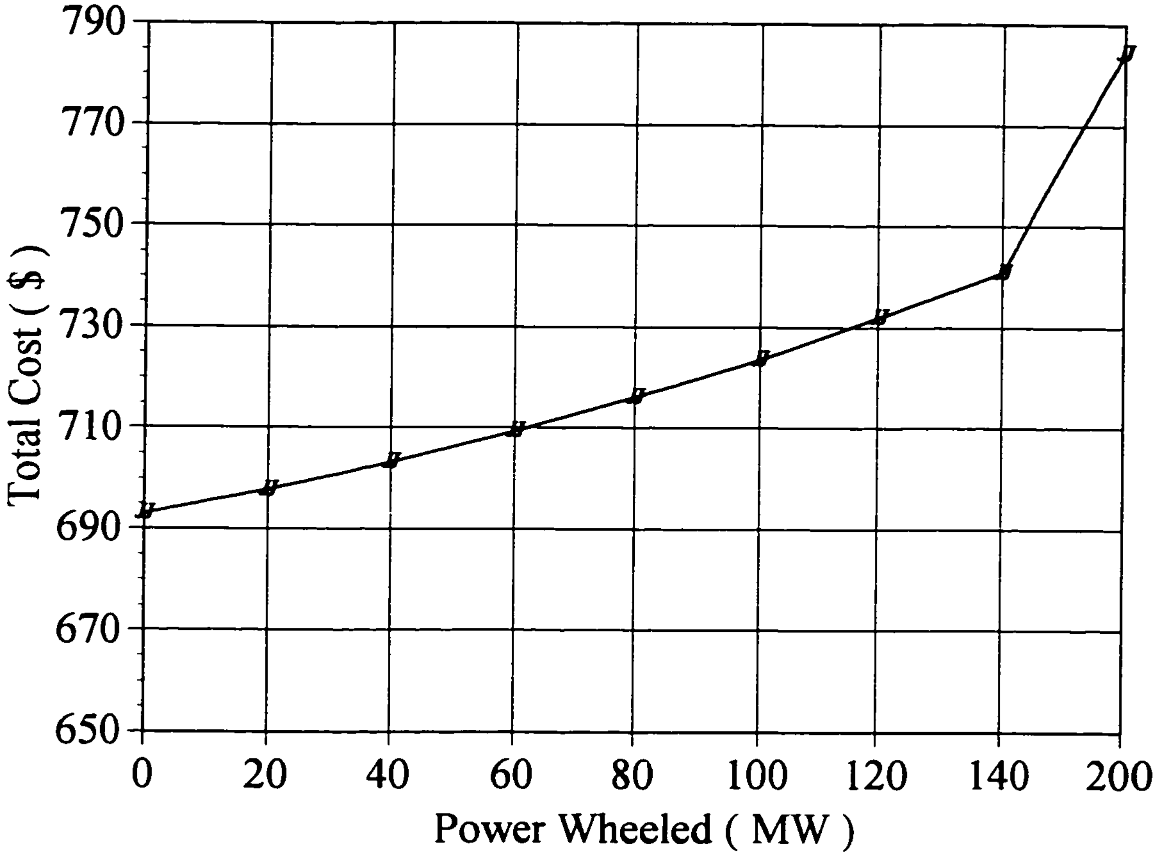

Fig. 4.10 Variation of Wheeler's Total Cost (Case 4)

4.3.5 Case 5

The voltage constraints are made narrower in this case. The voltage limits are changed to 0.95 and 1.05 P.U. It was observed that as wheeled power increased the wheeling rates increased significantly. The wheeler's objective function also increased. As matter of fact, the rates for reactive power became higher than for the active power. This could be explained by the strong relationship between bus voltages and reactive power. As the voltage lower limit is increased, a high reactive power support is required. This leads to higher reactive power wheeling charges. It is observed that bus 7 is a critical bus . When the minimum limit of the voltage at bus 7 was reduced to 0.9 P.U., the wheeling rates and the objective function dropped significantly. Table 4.5 and figures 4.11 - 4.13 present the output of this case.

POWER WHEELED(MW)	20	40	60	80	100	120	140
BINDING INEQUALITY CONSTRAINTS	T(min) V2(max) V3(max)	V2(max) Q1(max) V3(max) V7(min)	V5(max) V7(min) Q1(max) Q2(min)	V5(max) V7(min) Q1(max) Q2(min)	V5(max) Q2(min) V7(min) T (min)	V5(max) Q2(min) V5(max) Q1(max)	V5(max) Q2(min) V5(max) Q1(max) T (min)
MCPB($/MWH)	2,52	2,69	3,27	3,997	2,61	3,05	3,8
MCPS($/MWH)	2,25	2,17	1,63	1,11	2,23	1,9	1,35
WP ($/MWH)	0,27	0,52	1,64	2,887	0,38	1,15	2,45
MCQB($/MWH)	0,098	0,5	1,67	3,15	0,104	0,995	2,54
MCQS($/MWH)	0,033	0,102	-0,81	-1,53	-0,021	-0,45	-1,18
WQ ($/MWH)	0,065	0,398	2,48	4,68	0,125	1,445	3,72
TOTAL RATE($)	6,375	32,74	210	511,76	47,375	268,05	733,6
LOSSES (MW)	25,5	26,2	22,99	20,33	36,6	34,8	30,6
WHEELER COST($)	701,3	709,6	749,1	848,2	729,7	749,7	824,4
WHEELER Net Benefit ($)	2,055	20,12	157,88	360,54	14,655	215,33	606,18

Table 4.5 Wheeling Rates Comparison & Wheeler's Cost Change (Case 5)

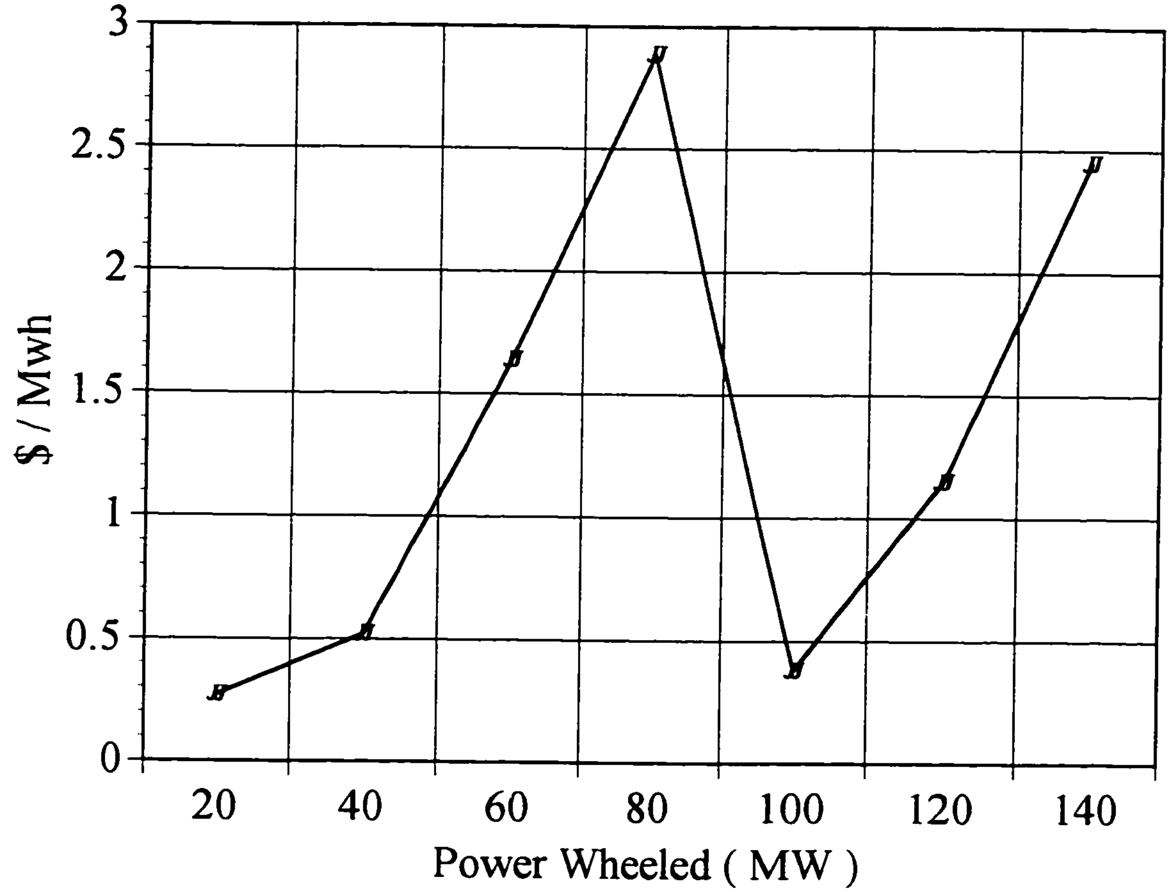

Fig. 4.11 Comparison of Active Power Wheeling rates (Case 5)

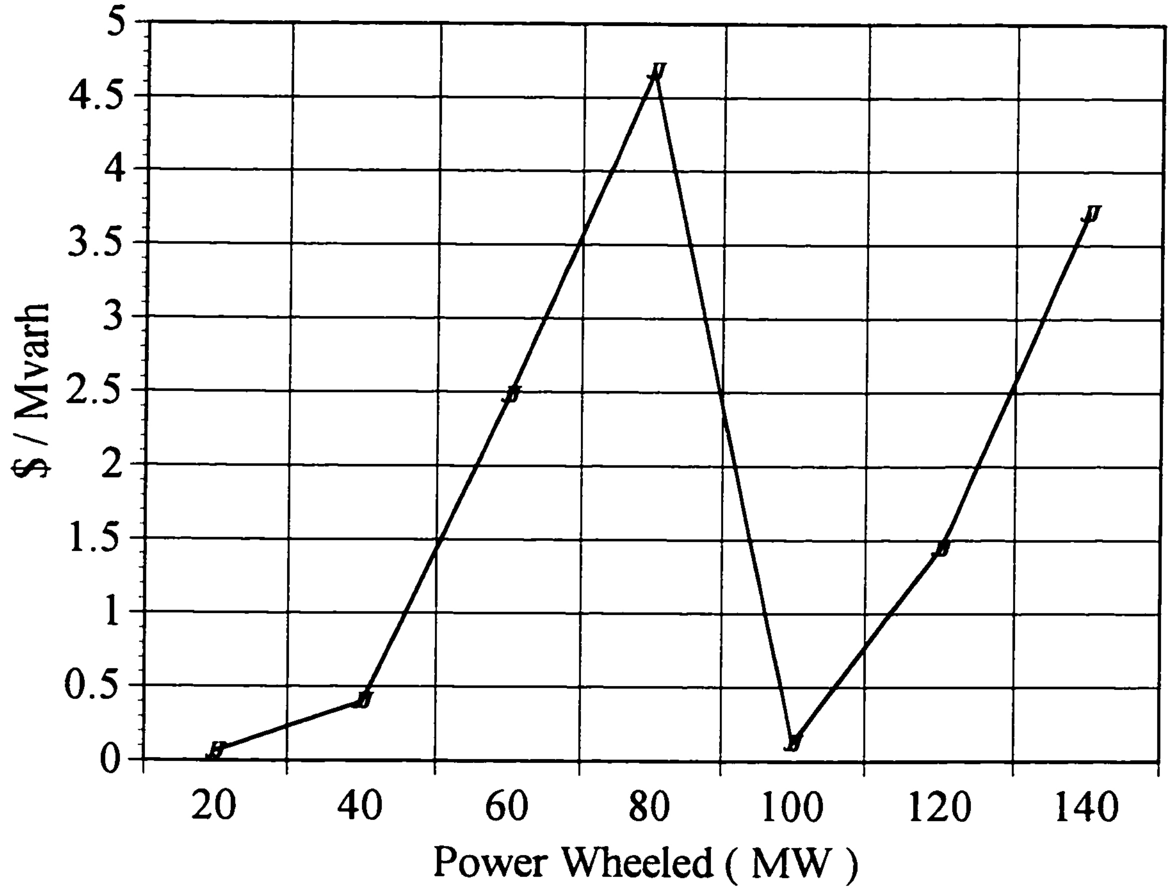

Fig. 4.12 Comparison of Reactive Power Wheeling rates (Case 5)

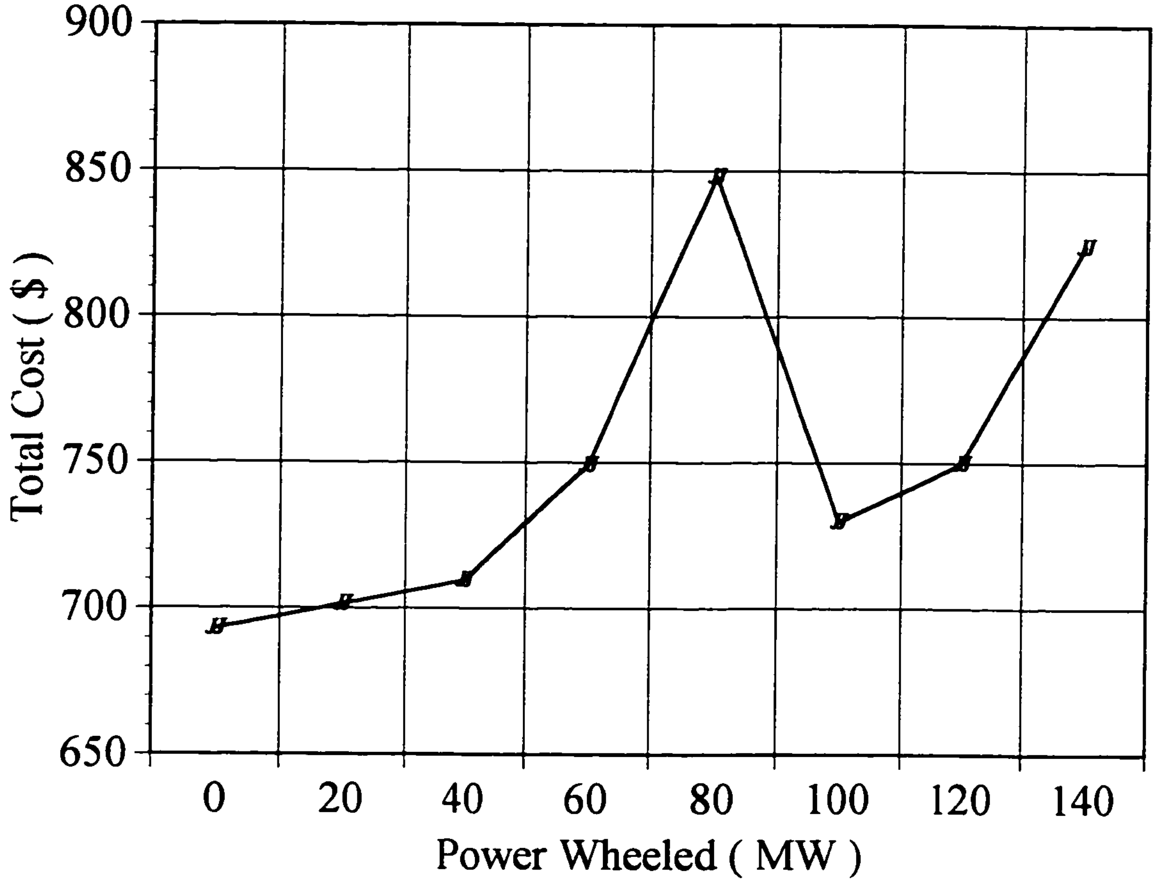

Fig. 4.13 Variation of Wheeler's Total Cost (Case 5)

4.3.6 Case 6

In this case the transmission lines thermal limits were reduced to 500 A instead of full line capacity of 1000 A . For some values of wheeled power some of the lines reached their full capacity. This is reflected as a higher wheeling rates. Therefore, it can be concluded that if some transmission line thermal limits are reached, the wheeling rates can be higher. The results for this case are shown in Table 4.6 and figures 4.14 - 4.16.

WHEELED(MW)	20	40	60	80	100	120	140
BINDING INEQUALITY CONSTRAINTS	V2(max) V7(min)	V2(max) V3(max) V7(min) Q1(max)	V3(max) V5(max)	V5(max) V7(min) Q1(max) Q2(min)	V5(max) Q1(max) Q2(min)	V5(max) Q1(max) Q2(min)	V5(max) V7(min) Q1(max) Q2(min)
MCPB($/MWH)	2,5	2,69	2,545	2,58	2,61	3,05	3,8
MCPS($/MWH)	2,25	2,17	2,25	2,24	2,23	1,9	1,36
WP ($/MWH)	0,25	0,52	0,295	0,34	0,38	1,15	2,44
MCQB($/MWH)	0,098	0,497	0,084	0,094	0,104	0,995	2,54
MCQS($/MWH)	0,033	0,102	0,021	-0,014	-0,021	-0,453	-1,19
WQ ($/MWH)	0,065	0,395	0,063	0,108	0,125	1,448	3,73
TOTAL RATE ($)	5,975	32,65	20,535	33,68	47,375	268,32	733,25
LOSSES (MW)	25,5	26,2	30,4	33,38	36,6	34,8	30,6
WHEELER COST($)	701,3	709,6	713,05	720,8	729,7	749,7	824,4
WHEELER Net Benefit ($)	0,975	19,35	3,785	9,18	13,975	214,92	605,15

Table 4.6 Wheeling Rates Comparison & Wheeler's Cost Change (case 6)

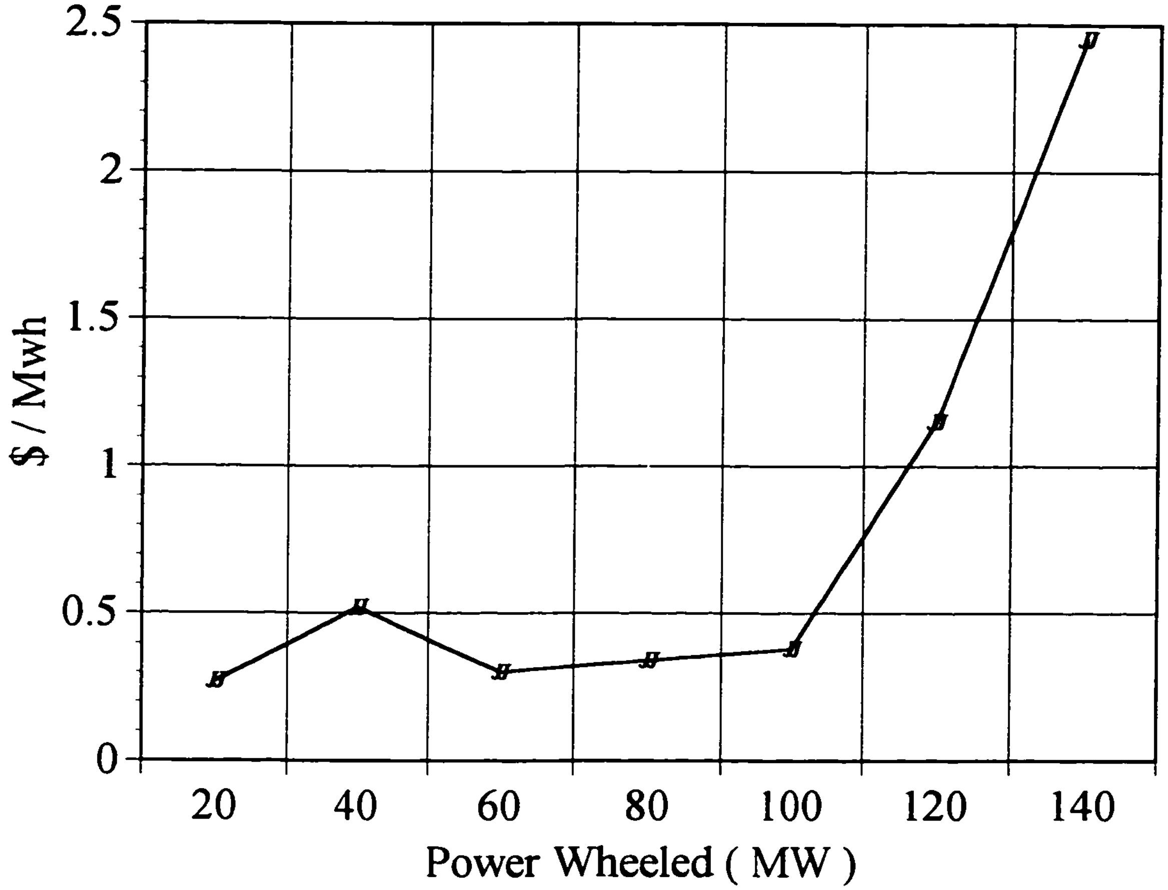

Fig. 4.14 Comparison of Active Power Wheeling rates (Case 6)

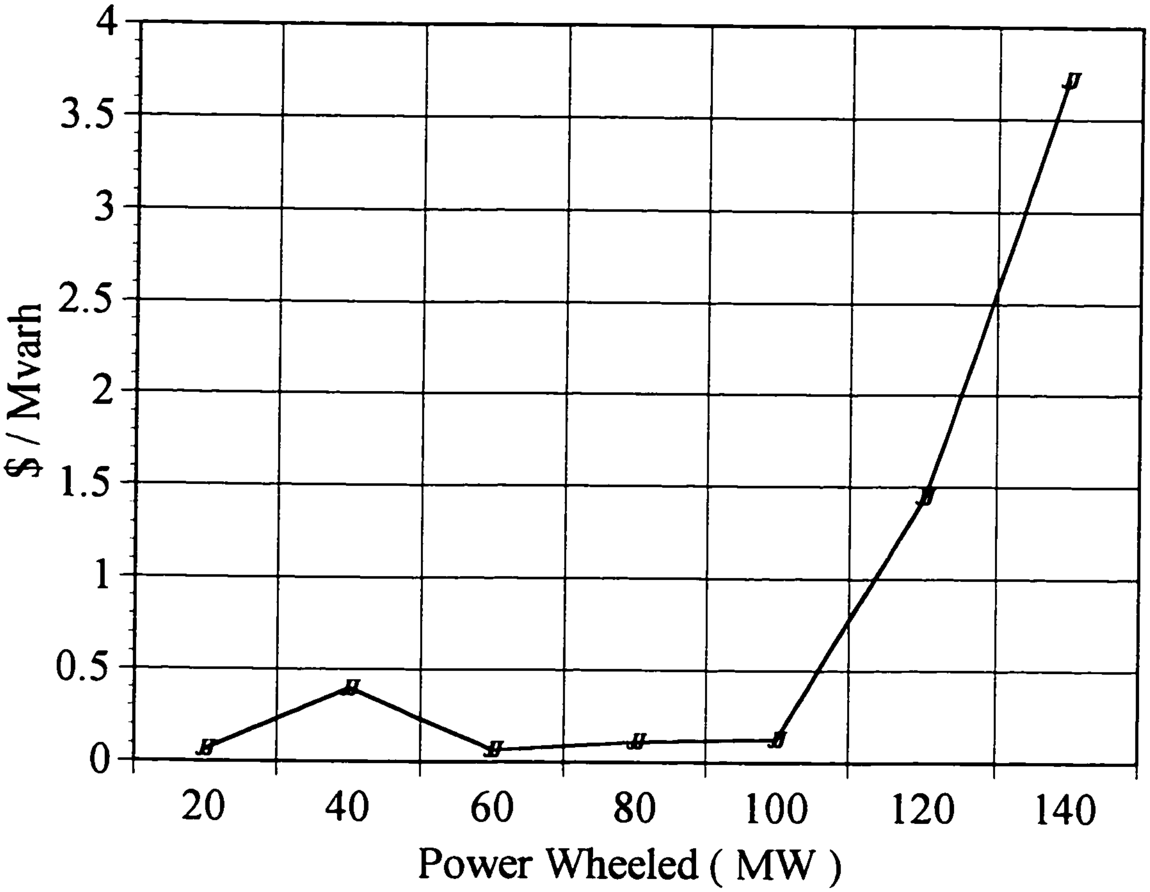

Fig. 4.15 Comparison of Reactive Power Wheeling rates (Case 6)

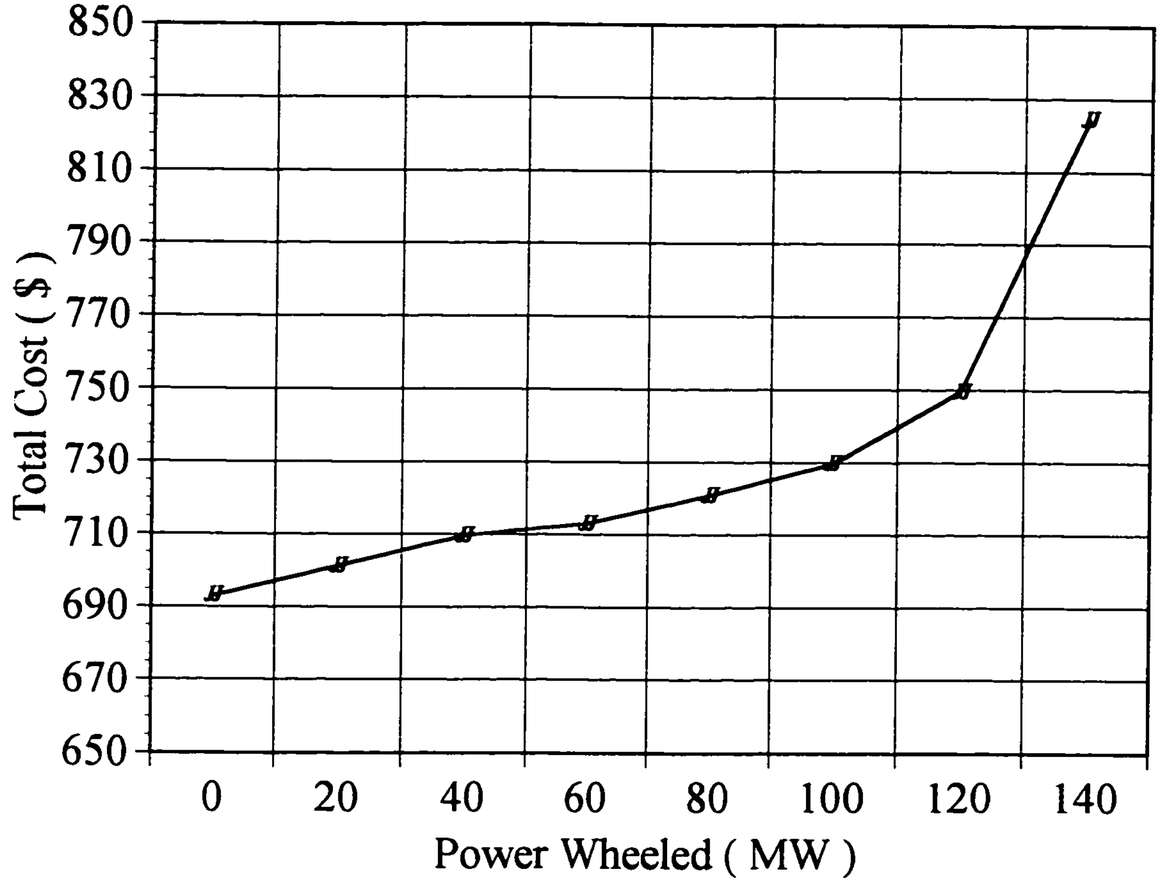

Fig. 4.16 Variation of Wheeler's Total Cost (Case 6)

4.3.7 Case7

This case is investigated with the addition of a variable reactive power source of 0 to 20 MVAR on bus 8. Although the rates were lower than the previous cases, the effect of lowering the minimum voltage limit at bus 7 was more significant on the objective function. As a matter of fact, the optimum rate was reached without the use of the 20 MVAR reactive power source. It can be concluded that location of capacitor banks has to be chosen in an optimal manner. This is illustrated in Table 4.7 and figures 4.17- 4.19.

POWER WHEELED(MW)	20	40	60	80	100	120	140
BINDING INEQUALITY CONSTRAINTS	V2(max) V3(max)	V3(max) V7(min) Q1(max)	V2(max) V3(max) Q2(min) T(max) Q7(min)	V2(max) V5(max) Q1(max) Q2(min) Q7(min)	V5(max) Q7(min) Q2(min) T(max)	V5(max) Q1(max) Q2(min) T(max) Q7(min)	V5(max) V7(min) Q1(max) Q2(min) Q7(min)
MCPB($/MWH)	2.5	2.68	2.55	2.56	2.61	3.045	3.8
MCPS($/MWH)	2.25	2.15	2.25	2.22	2.23	1.89	1.35
WP ($/MWH)	0.25	0.53	0.3	0.34	0.38	1.155	2.45
MCQB($/MWH)	0.096	0.442	0.084	0.093	0.104	0.995	2.54
MCQS($/MWH)	0.03	-0.012	0.013	-0.018	-0.021	-0.45	-1.19
WQ ($/MWH)	0.066	0.454	0.071	0.111	0.125	1.445	3.73
TOTAL RATE ($)	5.99	34.82	21.195	33.86	47.375	268.65	734.65
LOSSES (MW)	25.6	26.2	22.99	20.34	36.54	34.8	30.6
WHEELER COST($)	700.9	709.2	712.8	720.8	729.7	749.7	824.4
WHEELER Net Benefit ($)	-1.93	18.6	1.375	6.04	10.655	211.93	603.23

Table 4.7 Wheeling Rates Comparison & Wheeler's Cost Change (Case 7)

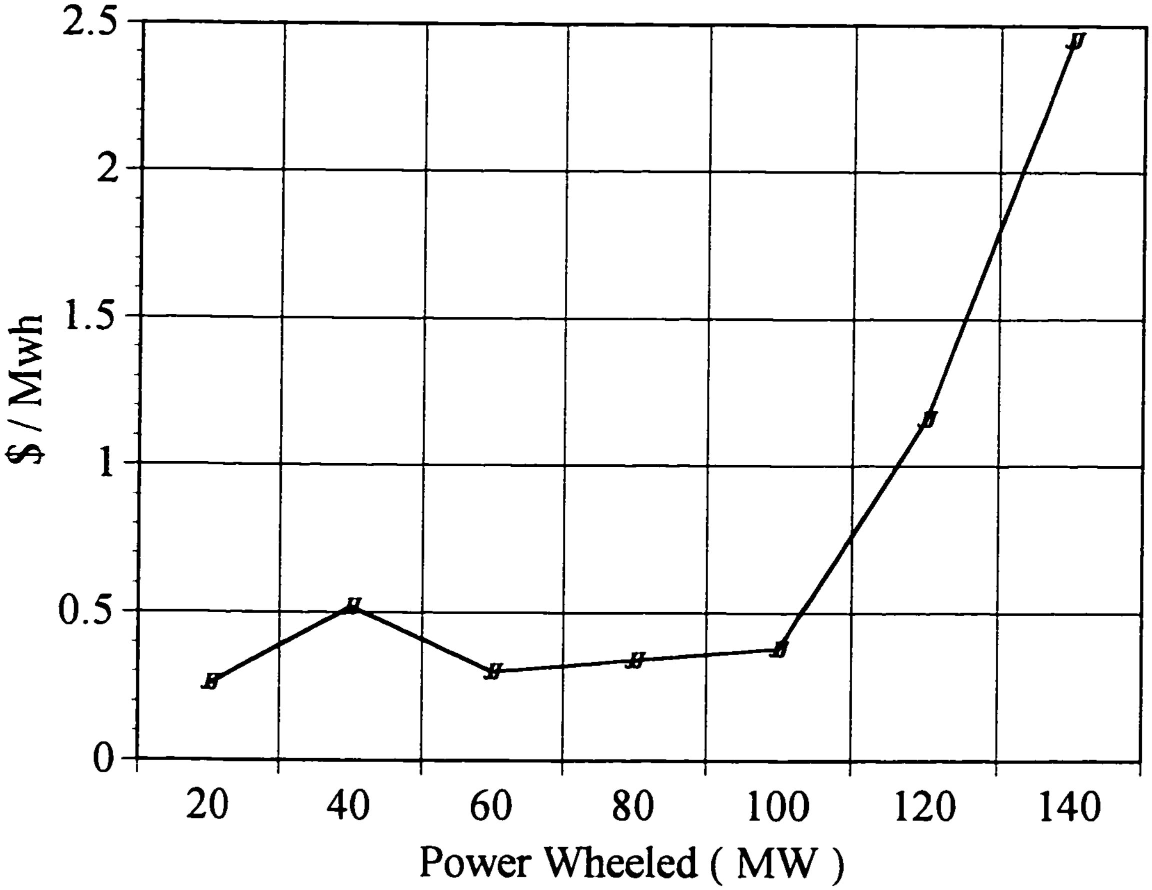

Fig. 4.17 Comparison of Active Power Wheeling rates (Case 7)

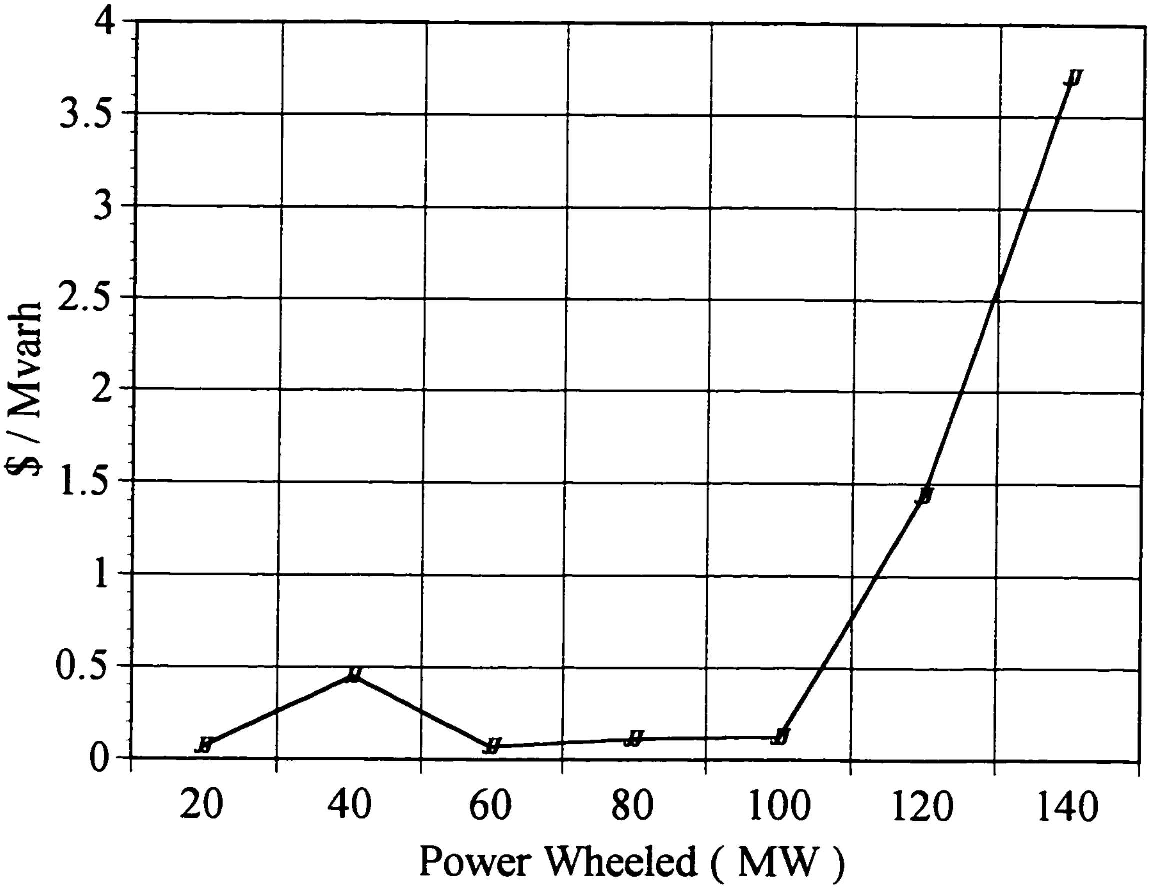

Fig. 4.18 Comparison of Reactive Power Wheeling rates (Case 7)

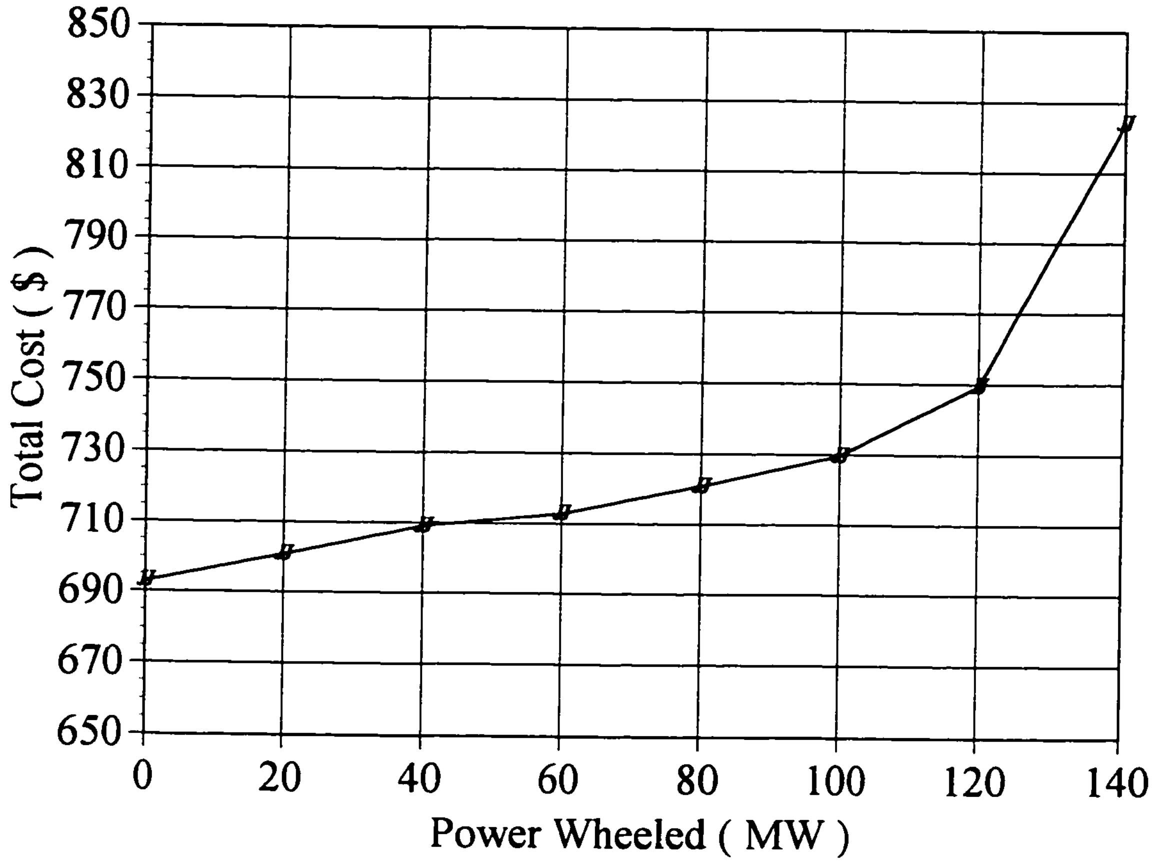

Fig. 4.19 Variation of Wheeler's Total Cost (Case 7)

4.3.8 Case 8

This case is identical to case 7 except that the reactive power source is put on bus 7. The effect of installing this source on bus 7 had a better result than the previous case despite the high rates at the 140 MW case. The optimum was always reached with full utilization of the 20 MVAR source. Again, in this case it was required to reduce the bus 7 minimum voltage limits to 0.9 P.U. It was noticed that bus 7 is very critical and its voltage magnitude controls the wheeling rates. Therefore, a higher value of shunt compensation can be added to control this bus voltage in order to have lower wheeling rates. The output of this case is shown in Table 4.8 and figures 4.20 - 4.22.

POWER WHEELED(MW)	20	40	60	80	100	120	140
BINDING INEQUALITY CONSTRAINTS	V2(max) V3(max) T(min) Q7(max)	V2(max) V3(max) T(min) V7(min)	V5(max) Q2(min) V7(min) T(min)	V5(max) T(min) V7(min) Q1(max) Q2(min)	V5(max) Q2(max) T(min)	V5(max) Q2(min)	V5(max) Q1(max) Q2(min)
MCPB($/MWH)	2.49	2.53	2.6	3.28	2.6	2.64	3.06
MCPS($/MWH)	2.25	2.24	2.18	1.633	2.23	2.23	1.91
WP ($/MWH)	0.24	0.29	0.42	1.647	0.37	0.41	1.15
MCQB($/MWH)	0.059	0.084	0.165	1.61	0.092	0.102	0.943
MCQS($/MWH)	0.024	0.027	-0.085	-0.8	-0.02	-0.03	-0.444
WQ ($/MWH)	0.035	0.057	0.25	2.41	0.112	0.132	1.387
Total Wheeling Rate ($)	5.325	13.31	36.45	276.36	45.4	61.08	306.635
LOSSES (MW)	24.6	26.6	28.6	24.12	35.1	38.6	36.9
WHEELER COST($)	697.03	702.97	711.3	751.5	725.2	734.8	754.7
WHEELER Net Benefit ($)	1.275	3.32	18.13	217.84	13.18	19.26	244.915

Table 4.8 Wheeling Rates Comparison & Wheeler's Cost Change (Case 8)

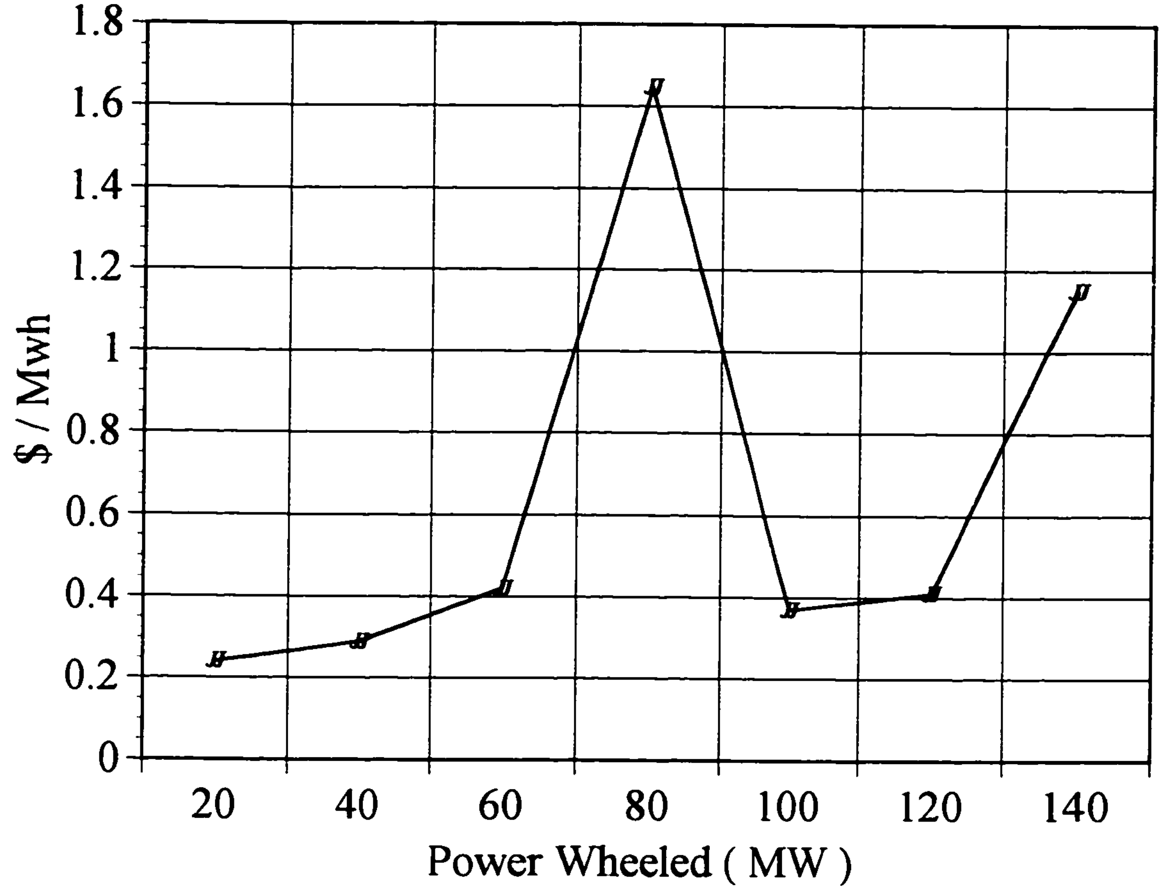

Fig. 4.20 Comparison of Active Power Wheeling rates (Case 8)

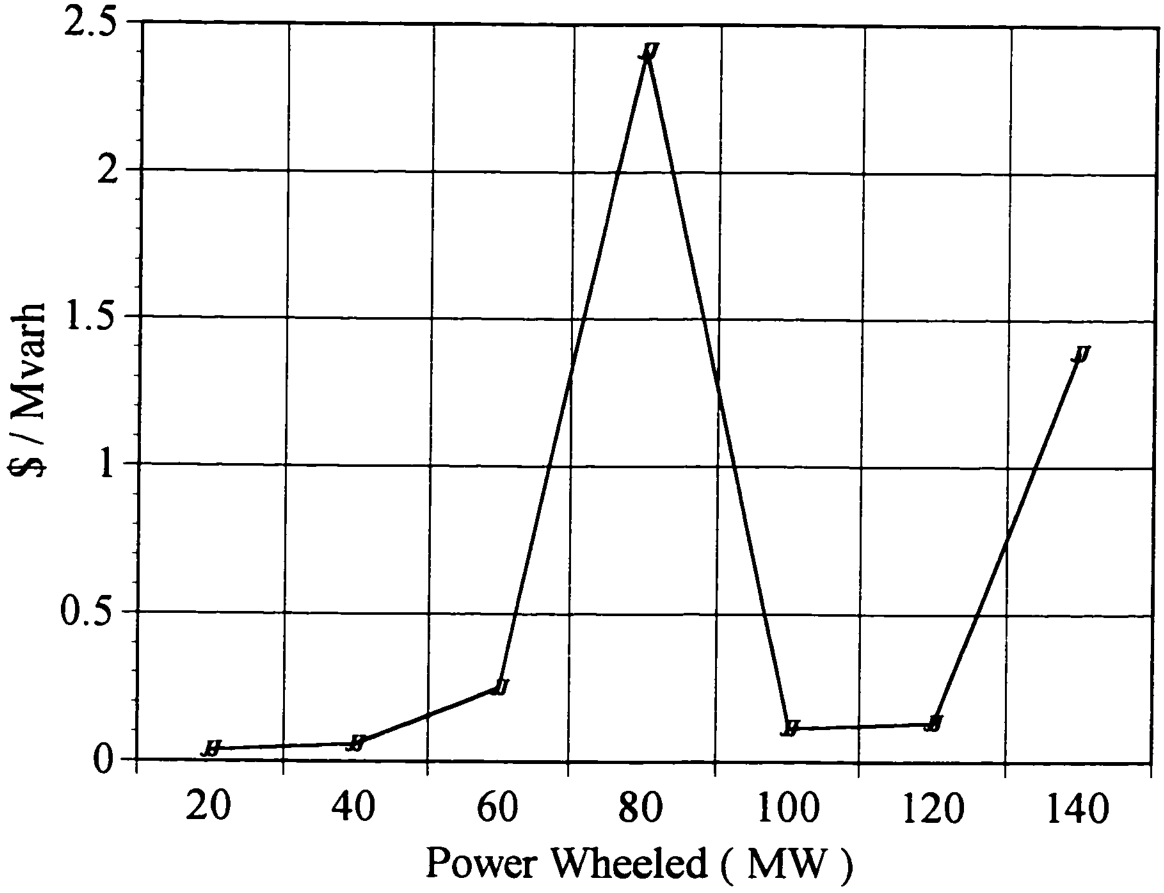

Fig. 4.21 Comparison of Reactive Power Wheeling rates (Case 8)

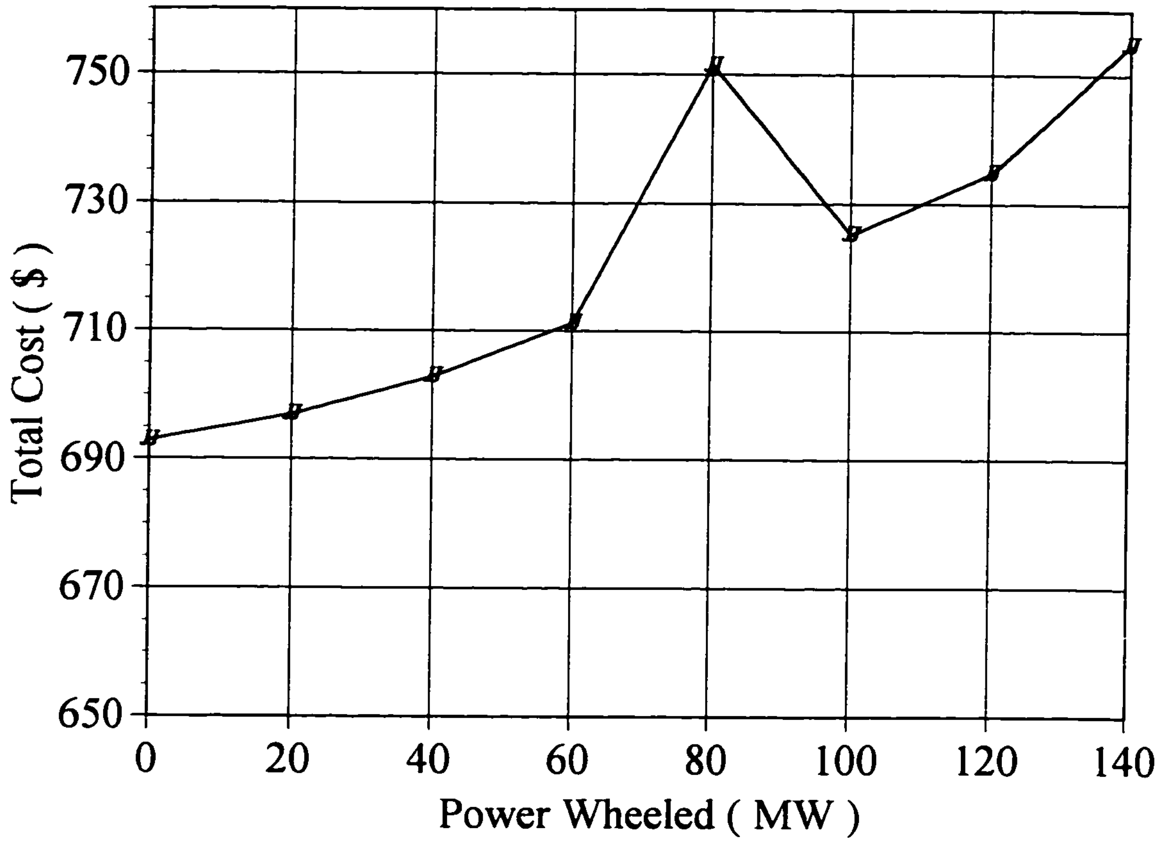

Fig. 4.22 Variation of Wheeler's Total Cost (Case 8)

4.3.9 Case 9

This case is similar to the above case except that the reactive power source is put on bus 5. The effect of installing this source on this bus was not significant for higher values of wheeled power . The results of this case are consistent with the findings of the previous case. The location of shunt compensation should be selected optimally.The effect of increasing some bus voltages is very critical for having lower wheeling rates. This is shown in table 4.9. In some cases the effect of reducing bus 7 minimum voltage constraint to 0.9 P.U. was more significant than the presence of this reactive power source. Figures 4.23- 4.25 present the wheeling rates and the cost function changes.

POWER WHEELED(MW)	20	40	60	80	100	120	140
BINDING INEQUALITY CONSTRAINTS	V2(max) V3(max) V7(min)	V2(max) V3(max) V7(min)	Q1(max) Q2(min) V5(max) V7(min)	Q1(max) Q2(min) V5(max) V7(min)	Q2(min) V5(max)	T(min) Q1(max) Q2(min) V5(max) V7(min)	Q1(max) Q2(min) V5(max)
MCPB($/MWH)	2.52	2.69	3.27	3.99	2.61	3.04	3.8
MCPS($/MWH)	2.24	2.17	1.63	1.11	2.23	1.9	1.35
WP ($/MWH)	0.28	0.52	1.64	2.88	0.38	1.14	2.45
MCQB($/MWH)	0.1	0.5	1.67	3.15	0.164	0.995	2.54
MCQS($/MWH)	0.033	0.1	-0.81	-1.15	-0.021	-0.45	-1.18
WQ ($/MWH)	0.067	0.4	2.48	4.3	0.185	1.445	3.72
Total Wheeling Rate ($)	6.605	32.8	210	488.4	51.875	266.85	733.6
LOSSES (MW)	25.5	26.2	23	20.3	36.64	34.8	30.6
WHEELER COST($)	701.3	709.6	749.08	848.2	729.7	749.7	824.4
WHEELER Net Benefit ($)	-1.715	16.18	153.9	333.18	15.155	210.13	602.18

Table 4.9 Wheeling Rates Comparison & Wheeler's Cost Change (Case 9)

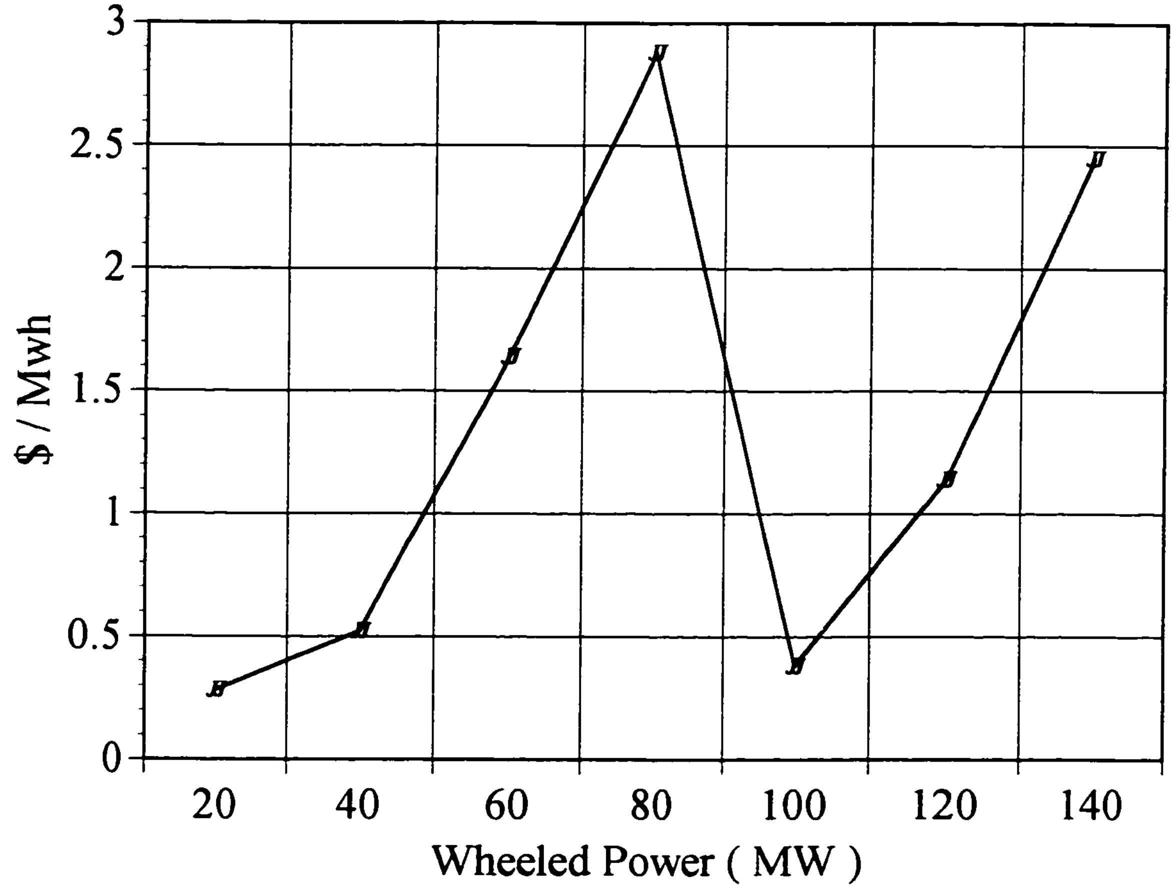

Fig. 4.23 Comparison of Active Power Wheeling rates (Case 9)

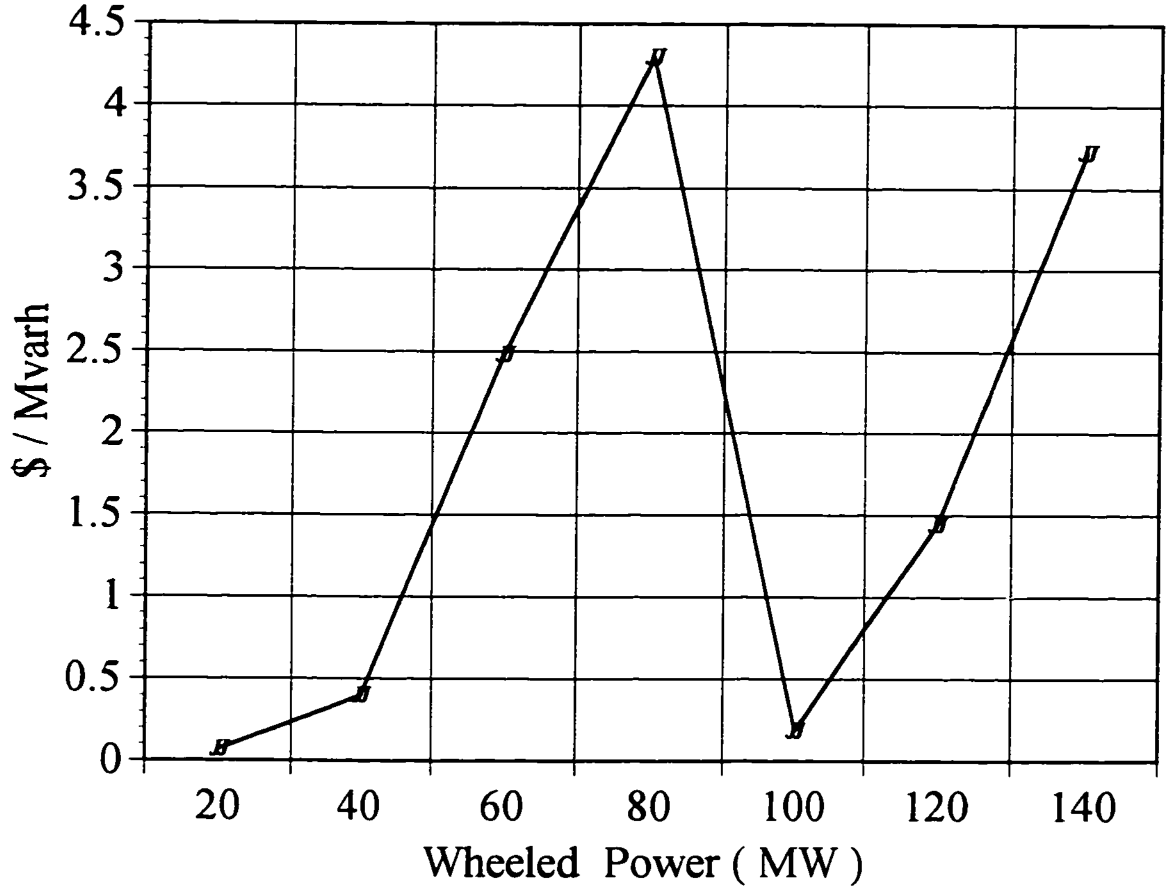

Fig. 4.24 Comparison of Reactive Power Wheeling rates (Case 9)

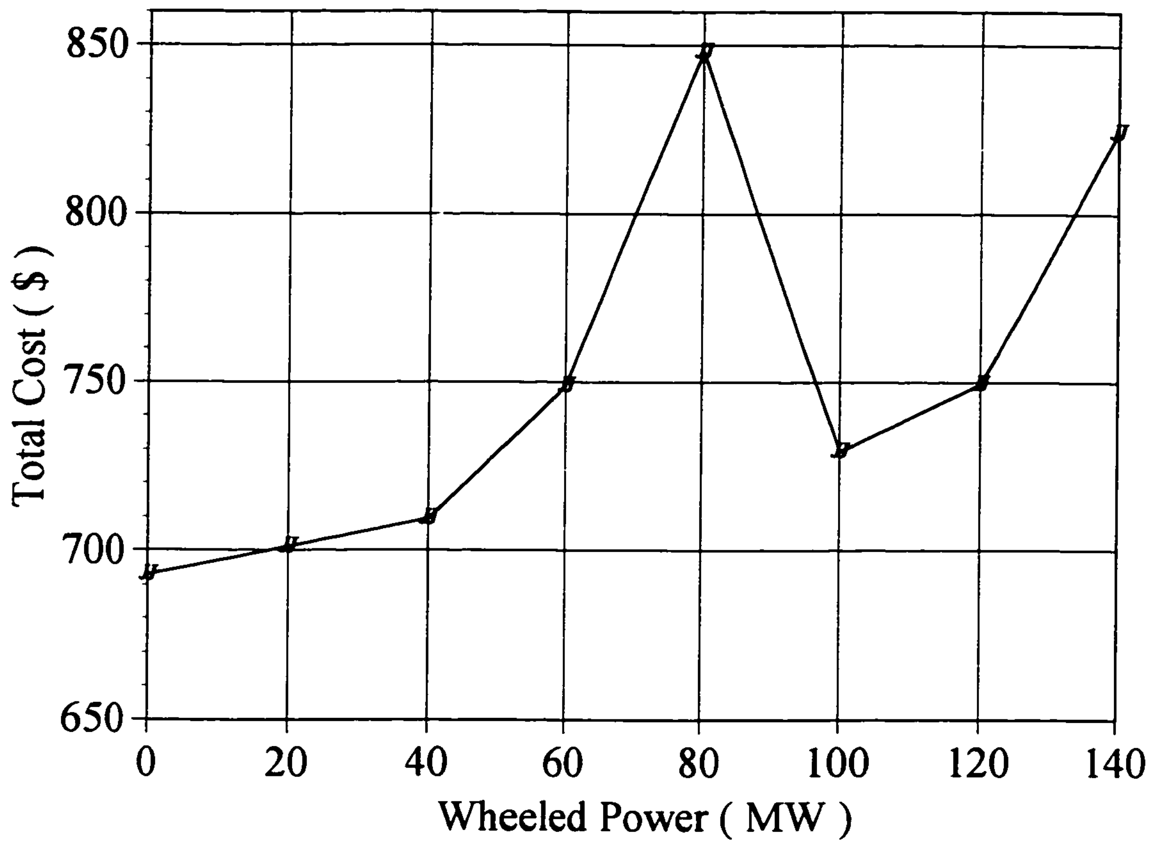

Fig. 4.25 Variation of Wheeler's Total Cost (Case 9)

4.3.10 Case 10

The voltage and the transmission line limits constraints were kept at 0.9 and 1.1 P.U. and 500 A respectively. A reactive power source of 20 MVAR was installed on bus 7. The effect of shunt compensation is observed clearly in this case. It was observed that the presence of this source did have some effect on the wheeling rates as compared to case 2. Its effect is observed at the case of 200 MW power exchange. The wheeling charges dropped from $1.893 to $ 0.22 . This is due to the fact that the presence of this reactive power source kept bus 7 voltage above 0.9 P.U. This resulted in lower wheeling rate. These results are illustrated in Table 4.10 and figures 4.26- 4.28.

POWER WHEELED(MW)	20	40	60	80	100	120	140
BINDING INEQUALITY CONSTRAINTS	V2(max) V3(max) V7(min)	V2(max) V3(max) V7(min)	V5(max) V7(min) Q1(max) Q2(min)	V5(max) V7(min) Q2(min) Q3(min)	P2(min) V1(max) V5(max) V7(min) Q2(min)	V5(max) V7(min) Q1(max) Q2(min) T(max)	V5(max) V7(min) Q1(max) Q2(min)
MCPB($/MWH)	2.49	2.52	2.93	3.59	6.37	2.69	3.42
MCPS($/MWH)	2.25	2.24	1.9	1.31	0.4	2.19	1.63
WP ($/MWH)	0.24	0.28	1.03	2.28	5.97	0.5	1.79
MCQB($/MWH)	0.03	0.048	0.883	1.17	8.43	0.19	1.71
MCQS($/MWH)	0.016	0.018	-0.448	-2.6	-4.02	-0.093	-0.815
WQ ($/MWH)	0.014	0.03	1.331	3.77	12.45	0.283	2.525
Total Wheeling Rate ($)	5.01	12.1	121.695	408.6	1530.75	85.47	515.725
LOSSES (MW)	25.5	27.5	25.9	22.2	19.9	38.8	33.6
WHEELER COST($)	692.37	697.9	714.63	785.66	926.7	728	772.8
WHEELER Net Benefit ($)	5.62	7.18	100.045	315.92	1297.03	50.45	435.905

Table 4.10 Wheeling Rates Comparison & Wheeler's Cost Change (Case 10)

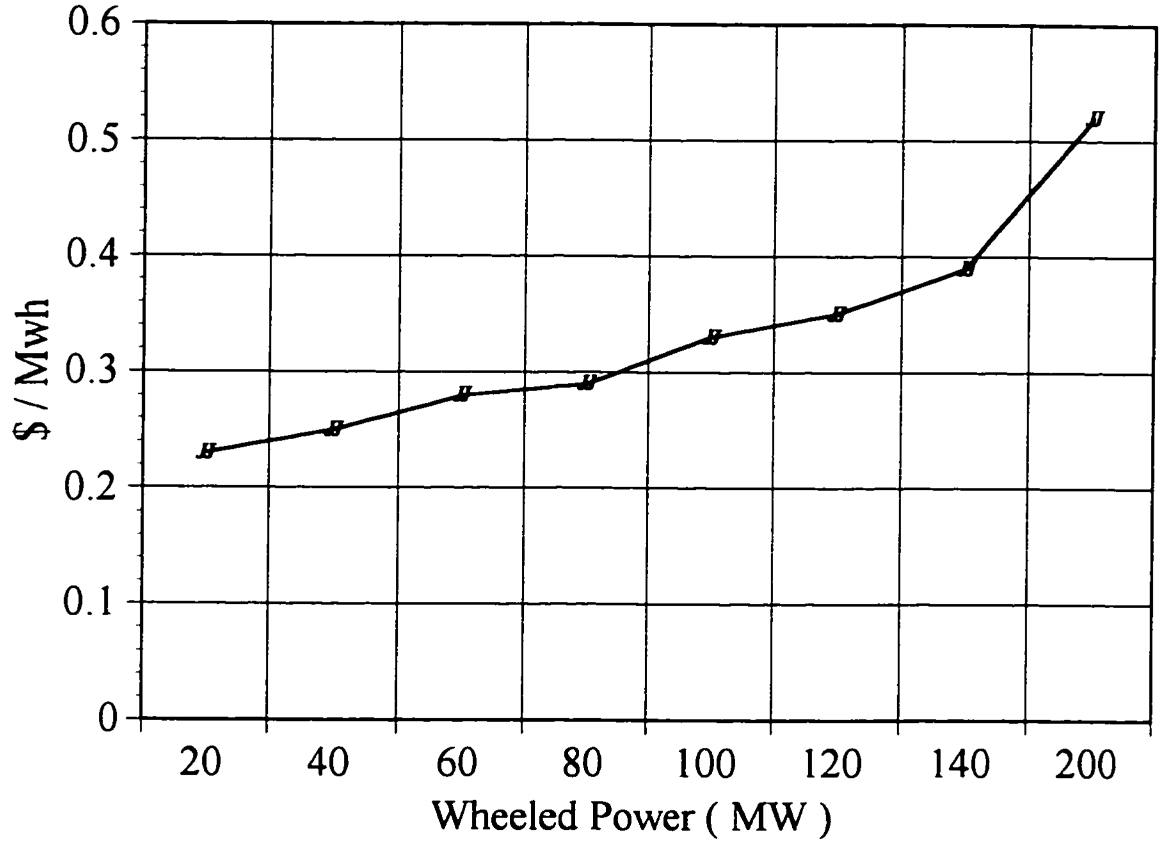

Fig. 4.26 Comparison of Active Power Wheeling rates (Case 10)

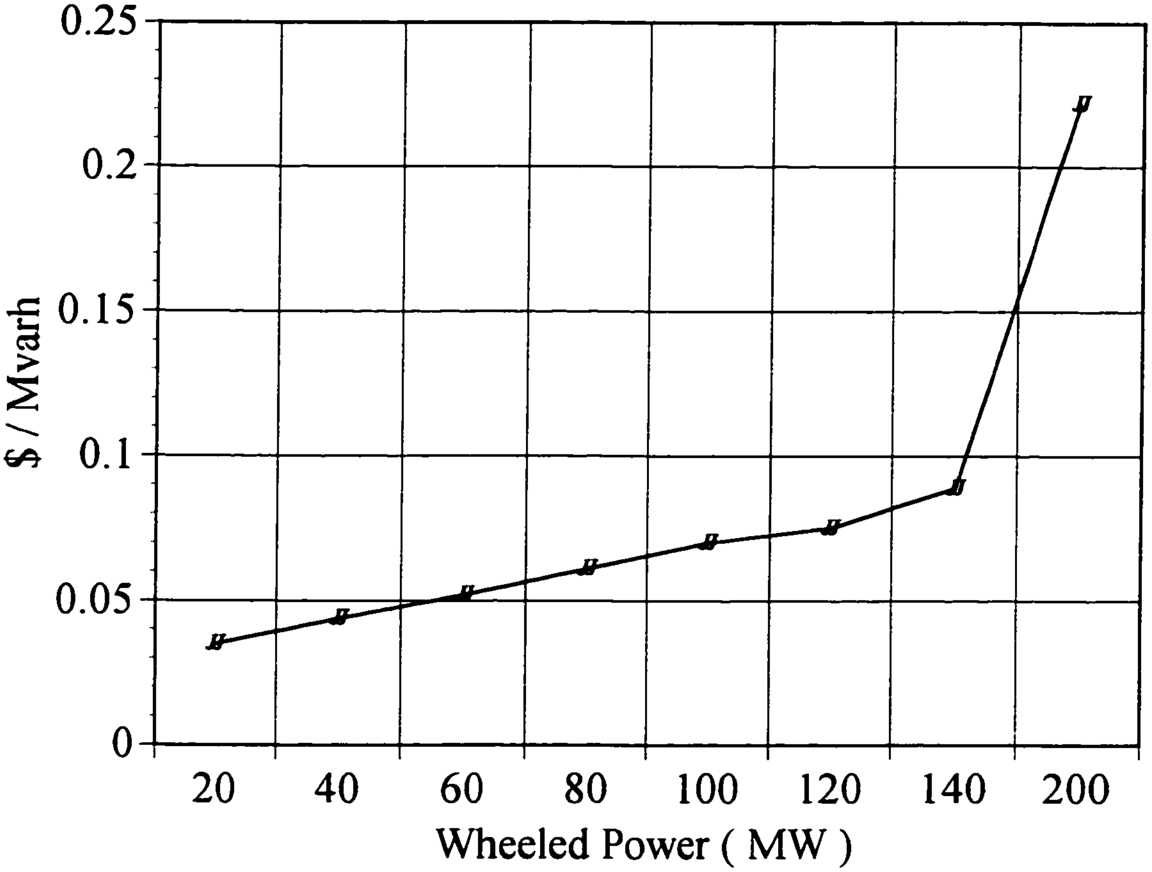

Fig. 4.27 Comparison of Reactive Power Wheeling rates (Case 10)

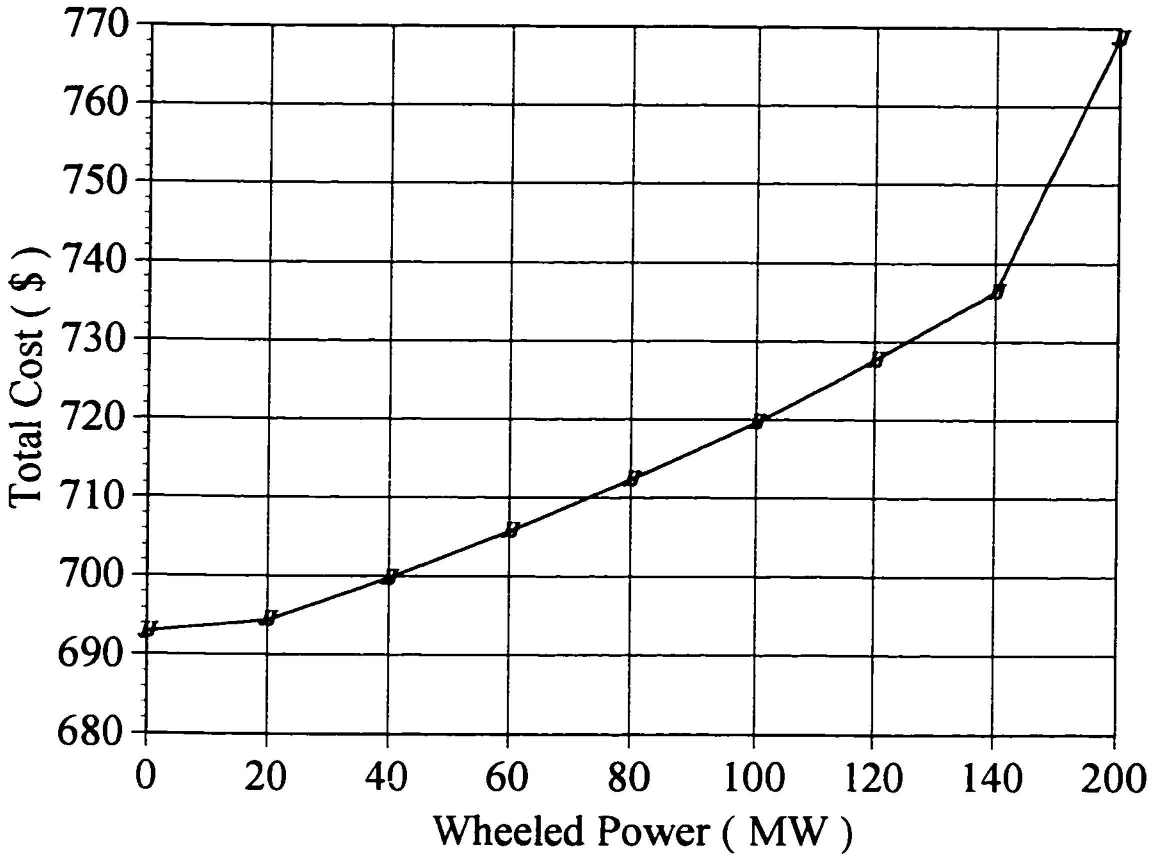

Fig. 4.28 Variation of Wheeler's Total Cost (Case 10)

4.3.11 Case 11

The wheeler's load can have some effects on the total wheeling charges. This is observed in this case where the basic case was repeated with lower voltage limits and line capacity constraints. The wheeler's load on the three buses were decreased by 15%. The wheeling prices and wheeler's cost function have decreased reflecting this change. Both the active power and the reactive wheeling rates varied significantly. It can be concluded that off-peak wheeling transactions can lead to lower wheeling charges. In effect, this reduces the costs for both the seller and the buyer. The results are shown in Table 4.11 and figures 4.29 - 4.31.

POWER WHEELED(MW)	20	40	60	80	100	120	140	200
BINDING INEQUALITY CONSTRAINTS				V2(max)	V2(max)	V2(max)	V2(max)	Q1(max) Q2(min)
MCPB($/MWH)	2.48	2.5	2.52	2.54	2.57	2.59	2.63	2.75
MCPS($/MWH)	2.25	2.25	2.24	2.25	2.24	2.24	2.24	2.23
WP ($/MWH)	0.23	0.25	0.28	0.29	0.33	0.35	0.39	0.52
MCQB($/MWH)	0.051	0.057	0.063	0.07	0.077	0.08	0.093	0.17
MCQS($/MWH)	0.016	0.013	0.011	0.009	0.007	0.005	0.004	-0.052
WQ ($/MWH)	0.035	0.044	0.052	0.061	0.07	0.075	0.089	0.22
Total Wheeling Rate ($)	5.125	11.32	19.14	26.86	38.25	48.75	63.945	137
LOSSES (MW)	23.96	25.9	28.1	30.5	33.2	36.17	39.3	51.4
WHEELER COST($)	694.5	699.8	705.8	712.43	719.7	727.7	736.4	768.8
WHEELER Net Benefit ($)	3.605	4.5	6.32	7.41	11.53	14.03	20.525	61.18

Table 4.11 Wheeling Rates Comparison & Wheeler's Cost Change (Case 11)

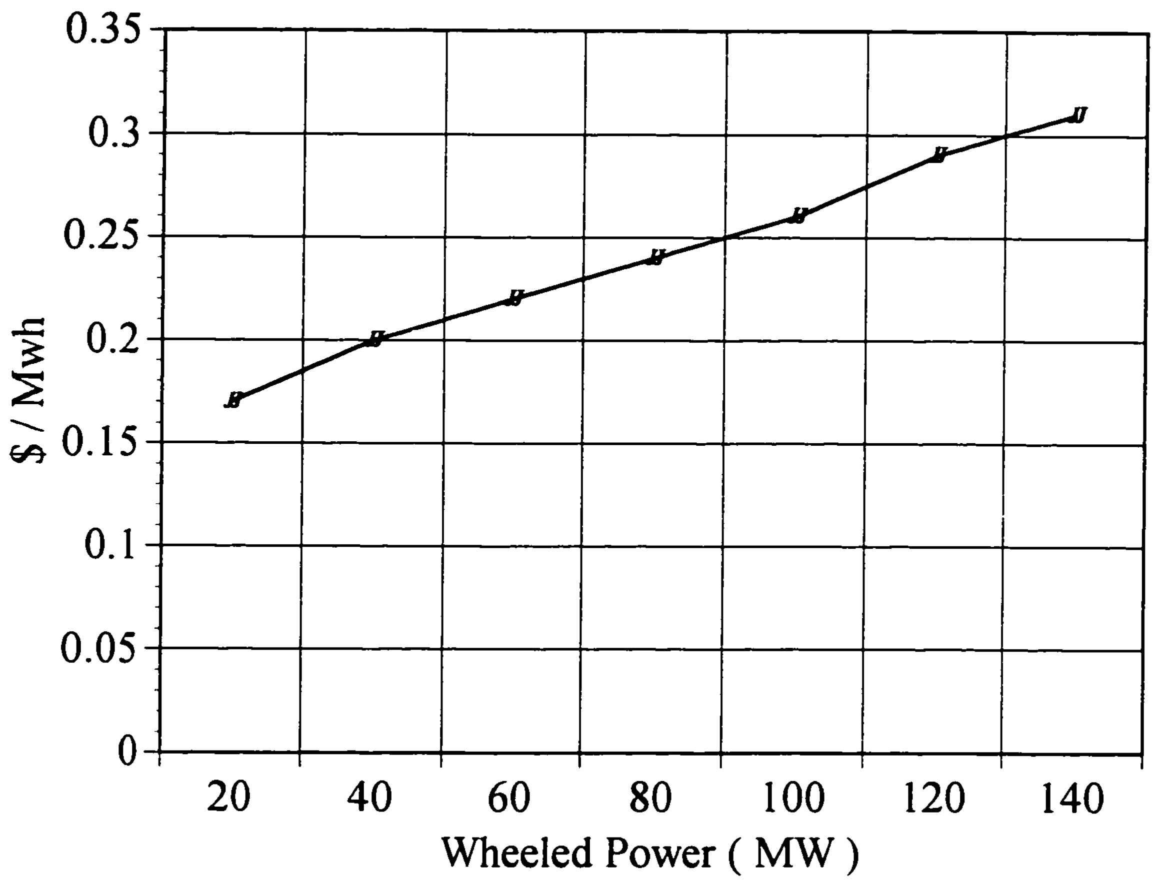

Fig. 4.29 Comparison of Active Power Wheeling rates (Case 11)

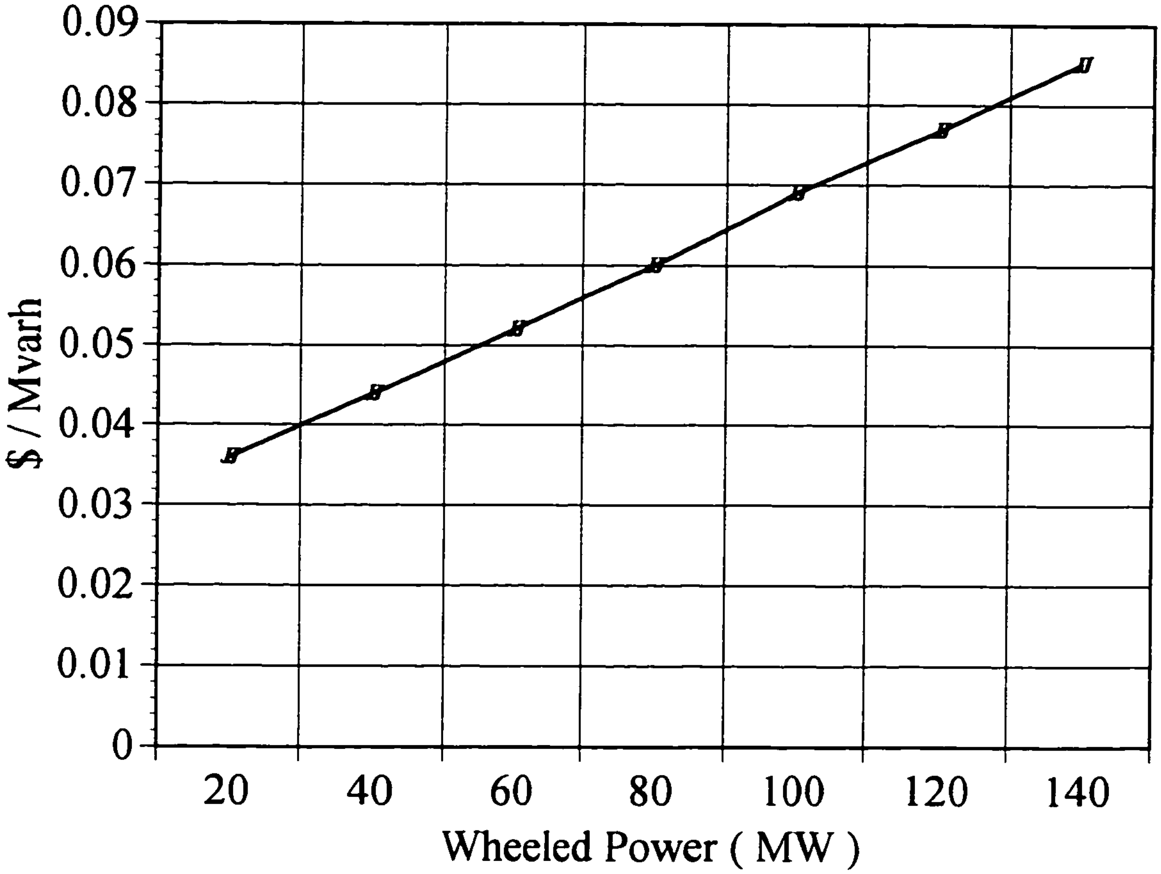

Fig. 4.30 Comparison of Reactive Power Wheeling rates (Case 11)

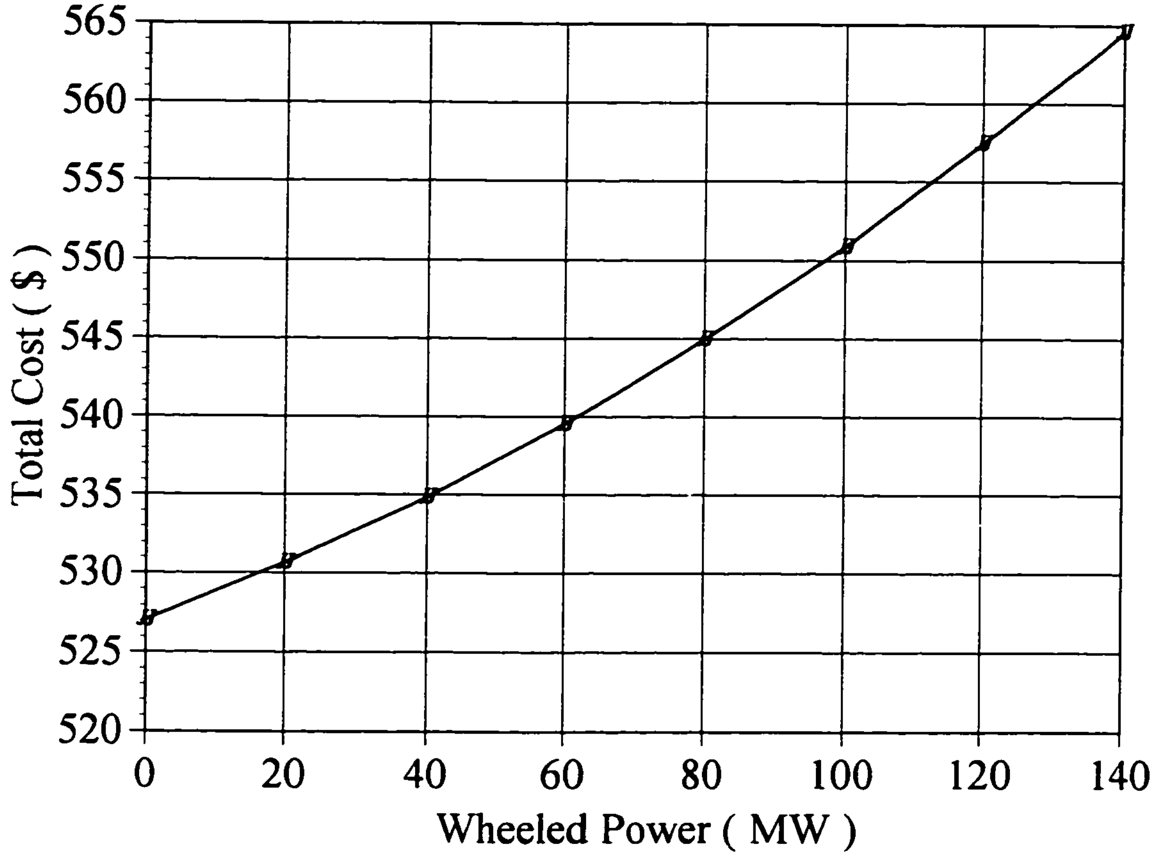

Fig. 4.31 Variation of Wheeler's Total Cost (Case 11)

4.3.13 Case 12

The effect of high wheeler's load is studied in this case . The wheeler's load on the three buses was increased by 15%. Wheeling Prices and wheeler's cost function have increased. Both the active and reactive power wheeling rates were high. Therefore, it could be concluded that in some cases it might not be economical to wheel power at peak times. The results are shown in table 4.12 and figures 4.32 - 4.34.

POWER WHEELED(MW)	20	40	60	80	100	120	140
BINDING INEQUALITY CONSTRAINTS	V2(max)	V2(max)	V2(max)	V2(max)	V2(max)	V2(max) T(min)	V2(max) T(min)
MCPB($/MWH)	2,82	2,85	2,88	2,92	2,95	3	3,38
MCPS($/MWH)	2,53	2,53	2,52	2,52	2,53	2,53	2,33
WP ($/MWH)	0,29	0,32	0,36	0,4	0,42	0,47	1,05
MCQB($/MWH)	0,074	0,087	0,109	0,134	0,16	0,187	1,21
MCQS($/MWH)	0,025	0,023	0,023	0,022	0,02	0,022	0,19
WQ ($/MWH)	0,049	0,064	0,086	0,112	0,14	0,165	1,02
TOTAL RATE ($)	6,535	14,72	25,47	33,32	52,5	71,25	254,1
LOSSES (MW)	33,2	35,5	38,2	41,3	26,5	29,3	32,3
WHEELER COST($)	890,1	897,1	905,1	914,1	924,2	935,3	955,7
WHEELER Net Benefit ($)	0,335	1,52	4,27	3,12	12,2	19,85	182,3

Table 4.12 Wheeling Rates Comparison & Wheeler's Cost Change (Case 12)

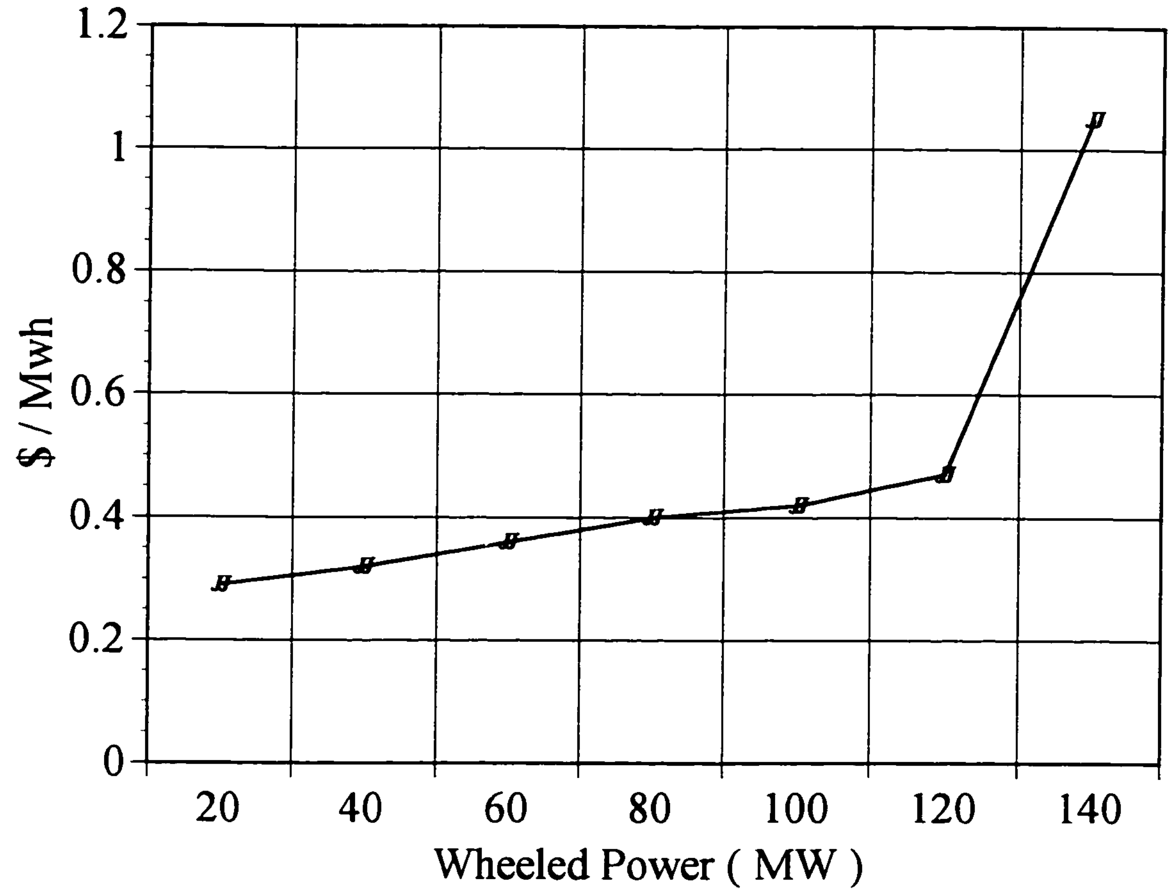

Fig. 4.32 Comparison of Active Power Wheeling rates (Case 12)

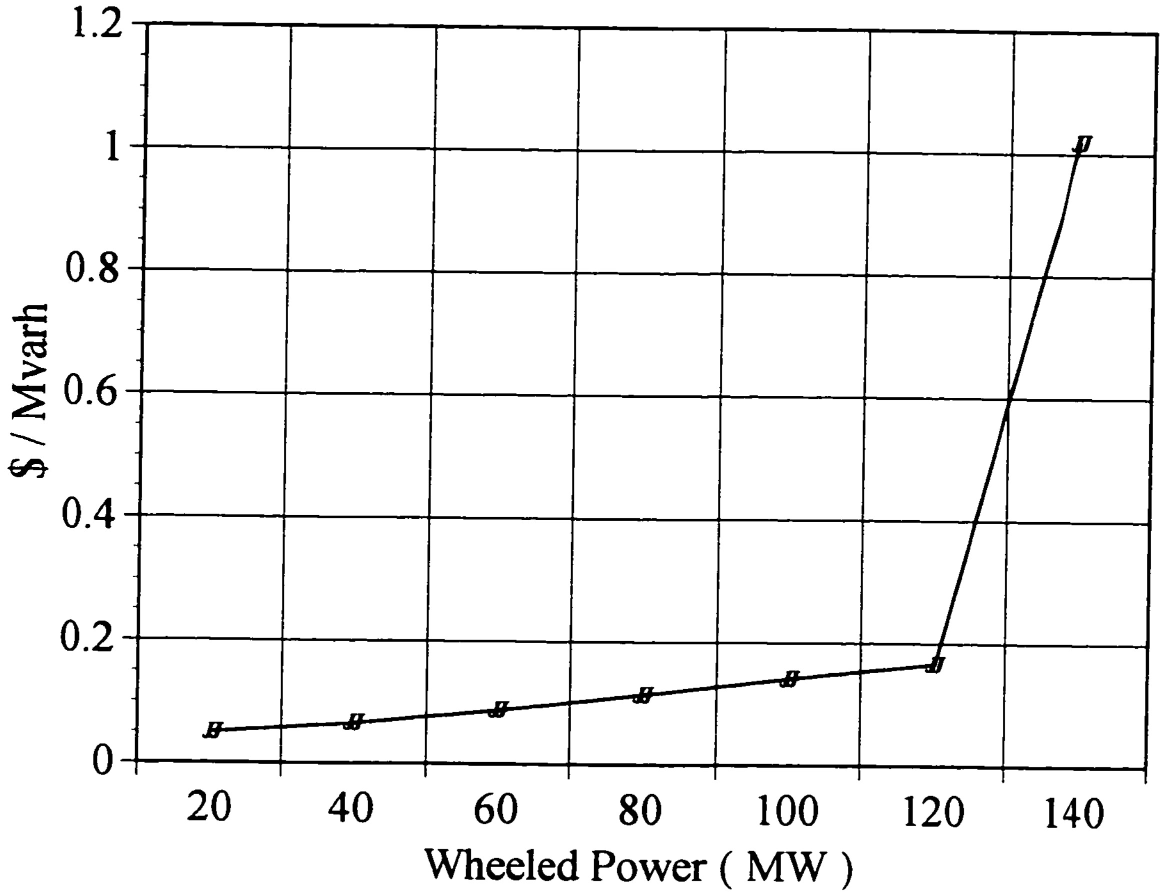

Fig. 4.33 Comparison of Reactive Power Wheeling rates (Case 12)

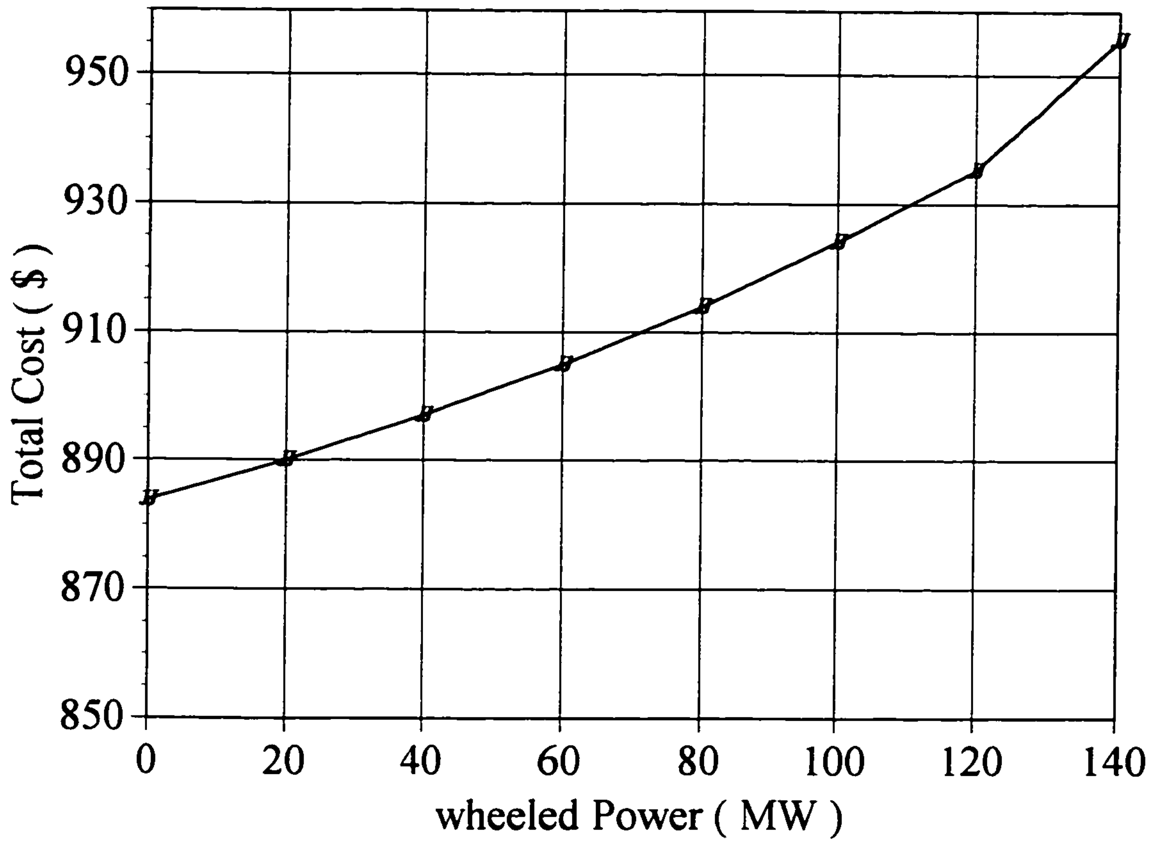

Fig. 4.34 Variation of Wheeler's Total Cost (Case 12)

4.3.14 Comparison of Wheeling Rates

Wheeling rates for one value of wheeled power is presented here. The case of 120 MW is presented. The effect of tightening the operational constraints causes an increase in the total charges. Fig. 4.38 shows the results of eight different operational cases that reflect the changes. They are summarized as follows:

1. The total wheeler's load is reduced to 15%. Voltages are at 0.90 and 1.1 P.U., stability and thermal limits are included (Case 11).

2. The wheeler load is 100%. Voltages are at 0.9 and 1.1 P.U. (Case 3).

3. The wheeler load is 100%. All the constraints are included. Capacitor of 20 Mvar is on bus 7 with voltage limits in 0.9 and 1.1 P.U. (Case 8).

4. Similar to 3 , but voltage limits at 0.95 and 1.05 P.U. (Case 9).

5. The wheeler load is 100%. Reactive power generation cost and stability limits are included. Voltage at bus 7 is at 0.9 P.U. (case 4).

6. The wheeler load is increased 15%. Operational constraints similar to case 1 (Case 12).

7. Capacitors of 20 Mvar is installed at bus 6 with same constraints as case 4. Voltage at bus 7 is 0.9 P.U. (Case 10).

8. The wheeler load is 100%. Voltage at all buses are below 0.95 and 1.05 P.U. All other constraints are included.

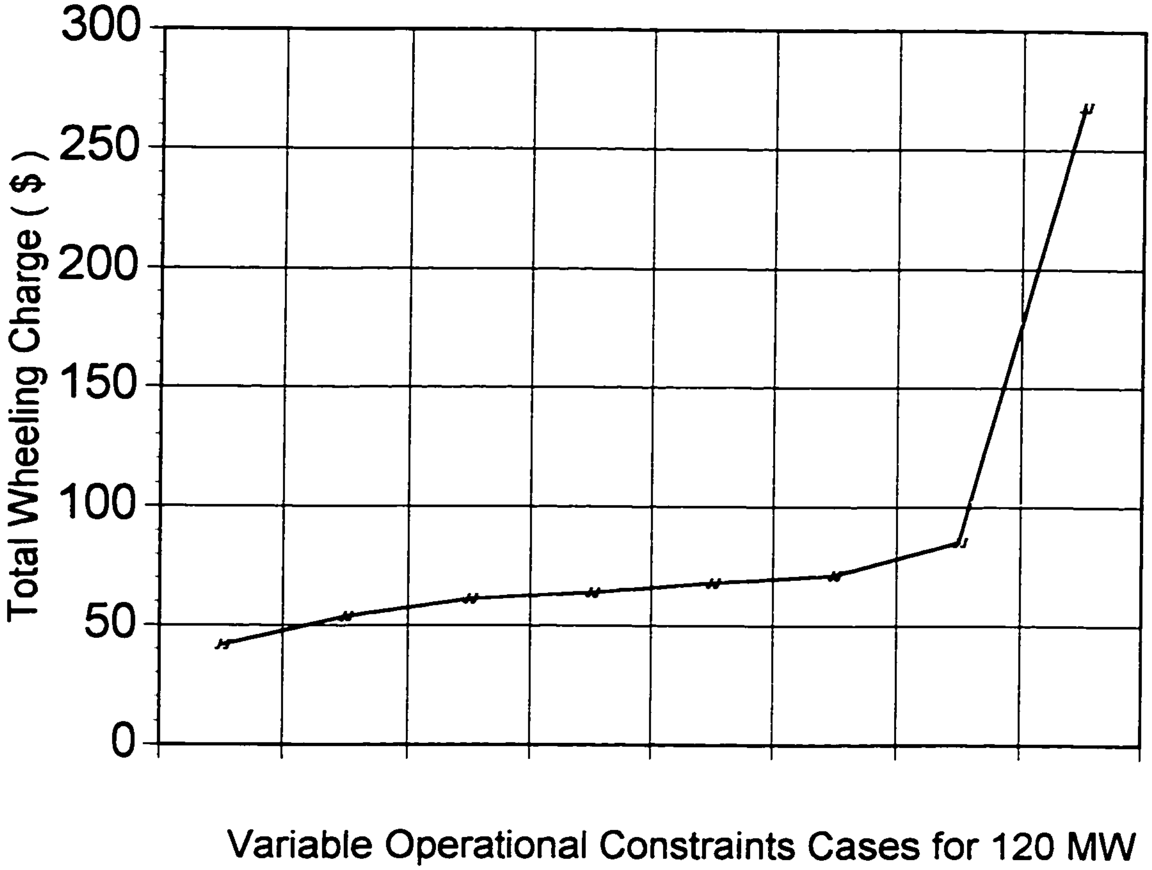

Fig. 4.35 Variable Operational Cases for 120 MW wheeled power under different operational constraints

CHAPTER 5

Application of the modified OPF Model to a practical System

5.1 Introduction

In this Chapter the model developed in chapter 3 will be used on a practical system. The High voltage network of three of the Gulf Cooperation Council States namely Oman, Qatar and UAE is studied. At present no interconnecting lines exist between the three utilities. Two 220 KV transmission lines between Alain Power Station of UAE and Sohar Substation of Oman are suggested. Two 400 KV lines between Salwa S/S of Qatar and Hadwany S/S of UAE are used. The reason for choosing a 400 KV voltage level is the long distance between Qatar and UAE. Figure 5.1 illustrates this system where Oman and Qatar on buses 8 and 9 respectively are exchanging the role of the seller and the buyer.

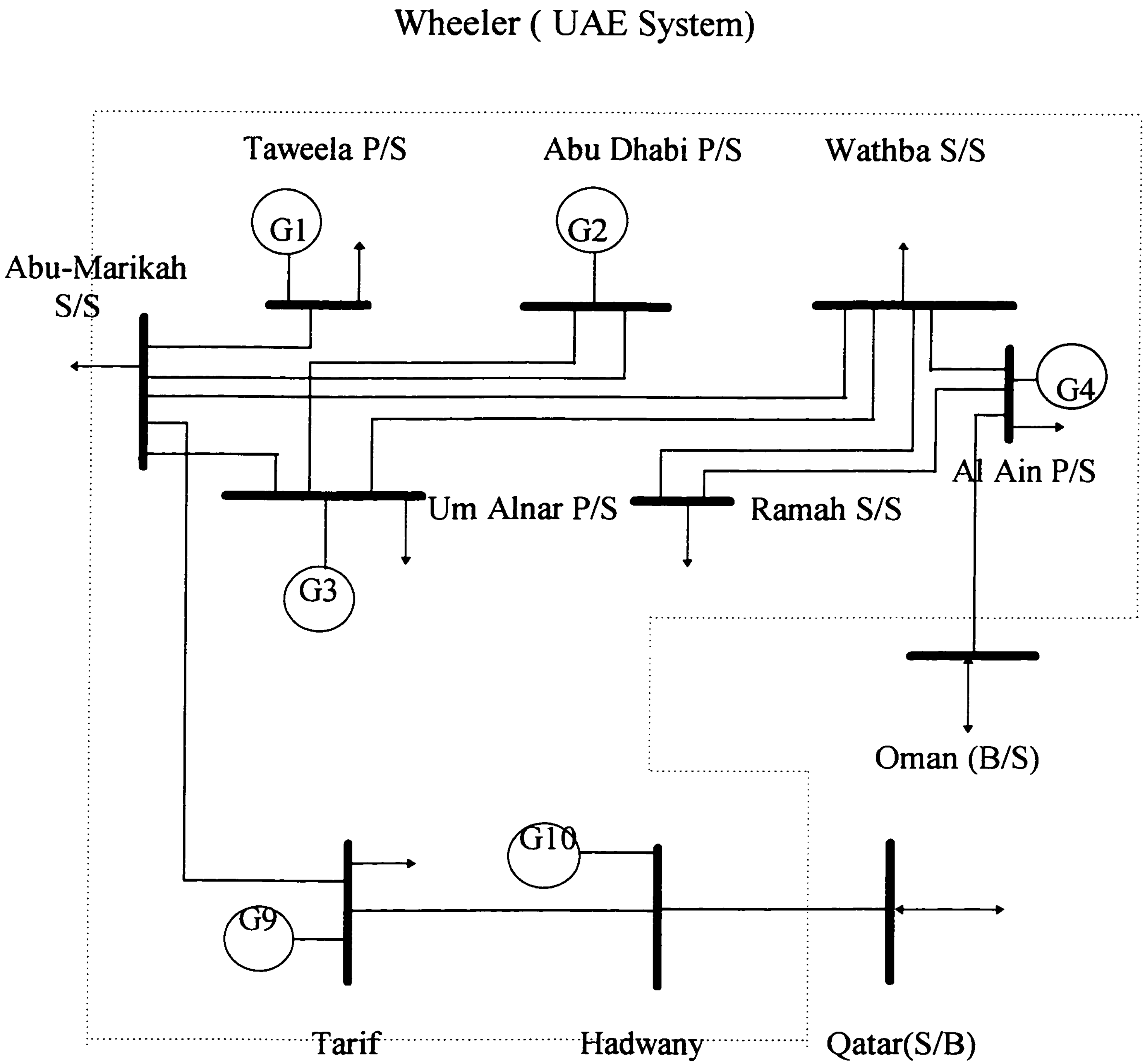

Fig. 5.1 Network Single Line Diagram of Case Study (2)

5.2 Case Studies

The original system of UAE is shown in [27]. Due to the limitations of GINO which is limited in size, the system of Dubai was not included in the model. Figure 5.1 represents the reduced model. Three different Cases are studied for this system to decide the wheeling rates for both active and reactive power. Details of this system parameters are included in [27]. It is assumed that the wheeler's system (UAE) has a total load of 1600 MW with a power factor of 0.8.

5.2.1 Case 1

In this case the UAE System is modeled separately prior to wheeling. The voltage at all the buses are restricted to be in the range of 0.95 and 1.05 P.U. . The optimal generation cost for this system was found to be $ 2608.92 prior to wheeling with 19.71 MW losses.

5.2.2 Case 2

The first wheeling case is run with Oman importing power from Qatar. Two constraint for the line stability limits are added to the model to insure the stability of the double circuit 400 KV line connecting Qatar to UAE and the 220

KV connecting Alain to Oman. This study examined the flow of power from 25 to 150 MW in increments of 25 MW each. Table 5.1 illustrates the results of this wheeling case. It was not possible to transfer more than 125 MW. At 150 MW the solution was infeasible, due to the violation of the lower limit of the voltage constraint at Oman's bus. In the last case of 125 MW the cost function increased from $ 2685.635 to $ 2981.537. This was reflected as a high increase in the total wheeling cost. The power flow is in the same direction of the wheeler's load. Another reason is the 220 KV line is used to interconnect the two systems compared to the 400 KV line between Qatar and UAE.

POWER WHEELED(MW)	25	50	75	100	125
BINDING INEQUALITY CONSTRAINTS	V11(max) Q2(max) Q10(min) Q1(max) Q4(max)	V11(max) Q2(max) Q10(min) Q1(max)	V11(max) Q2(max) Q10(min) Q4(max)	V11(max) Q2(max) Q10(min) Q4(max)	V8(min) V11(max) Q1(max) Q3(max) Q4(max) Q10(min)
MCPOman($/MWH)	2.800	2.861	2.936	3.406	7.464
MCPQatar($/MWH)	2.406	2.384	2.363	2.222	1.141
WP ($/MWH)	0.394	0.477	0.573	1.184	6.323
MCQOman($/MWH)	0.2668	0.32024	0.392	1.817	16.703
MCQQatar($/MWH)	0.0963	0.07017	0.0379	-0.57846	-6.297
WQ ($/MWH)	0.1705	0.25007	0.3541	2.39546	23
TOTAL RATE ($)	13.046875	33.202625	62.915625	298.0595	2946.6625
LOSSES (MW)	22.1	25.2	29.06	29.76	19.98
WHEELER COST($)	2620.7	2635.47	2654.17	2685.635	2981.537
WHEELER Net Benefit ($)	1.266875	6.652625	17.665625	221.3445	2574.0455

Table 5.1 Wheeling Rates for Power transfer from Qatar to Oman

5.2.3 Case Study 3

Case study 2 is repeated with the assumption that Oman is exporting power to Qatar. The results of this case are illustrated in table 5.2. In this case it was possible to transfer up to 350 MW. In the low values of transmitted power (25 - 50 MW) , wheeling rates were negative. The total benefits of the wheeler were positive. At these values cost function and system losses were less than the ones prior to wheeling. The low wheeling rates are consistent with the fact that wheeling could reduce operational costs and losses. The seller and buyer are rewarded. The wheeled power flow is in the opposite direction of the wheeler's original load.

POWER WHEELED(MW)	25	50	75	100	125	150
BINDING INEQUALITY CONSTRAINTS	Q2(max) Q10(min) Q1(max) Q4(max)	Q2(max) Q10(min) Q1(max)	Q1(max) Q2(max) Q10(min)	Q2(max) Q10(min)	Q2(max) Q10(min)	Q2(max) Q10(min)
MCPOman($/MWH)	2.713	2.677	2.644	2.618	2.588	2.556
MCPQatar($/MWH)	2.445	2.464	2.481	2.503	2.520	2.544
WP ($/MWH)	0.268	0.213	0.163	0.115	0.068	0.012
MCQOman($/MWH)	0.2288	0.197	0.2101	0.2246	0.2398	0.2641
MCQQatar($/MWH)	0.1602	0.07017	0.2009	0.1901	0.1802	0.1488
WQ ($/MWH)	0.0686	0.12683	0.0092	0.0345	0.0596	0.1153
TOTAL RATE ($)	7.98625	15.40613	12.7425	14.0875	14.0875	14.77125
LOSSES (MW)	17.96	16.64	15.74	15.23	15.15	15.65
WHEELER COST($)	2599.91	2593.2	2588.42	2585.35	2583.96	2584.57
WHEELER Net Benefit ($)	16.99625	31.12613	33.2425	37.6575	39.0475	39.12125

Table 5.2 Wheeling Rates for Power transfer from Oman to Qatar

5.2.3 Case 4

Wheeling causes changes in the power flow. The power flow before and after wheeling is different. Figures 5.2 and 5.3 represent the variation of the power flow change of case 2 for the minimum wheeled power of 25 MW and the maximum of 150 MW, respectively. Table 5.3 presents the variation of the wheeler's cost function, power losses and the variation of the power flow. It could be concluded that system losses are increasing as wheeled power increases and so is the cost function. The generation dispatch is also changing where sometimes more units are brought into service in order to satisfy all operational constraints. Figure 5.2 represents a comparison of the variation of the power flow for the no wheeling and the 25 MW wheeeling cases. The cost function and the rates are increasing as wheeled power increases. This is due to the generation rescheduling. Highwer generation cost units are brought into service.

Power Wheeled (MW)	0	25	50	75	100	125	150
Wheeler's Cost ($)	2608.92	2599.9	2593.2	2588.41	2585.3	2583.936	2584.57
PG1	217.7	217.7	217.4	217.165	216.9	216.88	216.93
PG2	485.2	484.01	482.9	482.12	481.3	480.9	480.6
PG3	300	300	300	300	300	300	300
PG4	196.2	194	191.9	19.38	188.9	187.23	185.59
PG9	224.2	224.9	225.47	226.04	226.67	227.35	228.1
PG10	194.69	195.88	197.4	198.65	199.9	201.39	203
P(LOSS)	19.72	17.95	16.64	15.75	15.23	15.15	15.64
P15	-82.8	-76.17	-77.13	-76.4	-83.1	85.65	-82.75
P23	279.5	247	249.12	235.6	584.38	226.47	225.6
P25	28.1	34.46	34.96	44.02	42.44	49.03	59.95
P35	0.52	7.56	7.78	14.76	-17.73	18.9	26.4
P36	255.7	24.42	231.94	222.3	141.69	200.35	189.85
P46	-69.98	-59.05	-47.92	-49.05	-36.55	-14.11	-2.78
P47	66.3	77.6	89.93	66.13	101.03	153.57	133.9
P48	0	-24.72	-48.86	-74.39	-100.1	121.1	-147.79
P56	157.2	140	132.07	116.02	117.49	96.6	79.46
P59	410.8	-389.9	-366	-330.21	-344.28	301.57	-276.6
P67	136.8	125.26	113.8	104.27	101.44	91.11	65.21
P910	-192.6	-169.05	-144.07	-149.89	-122.16	-69.28	-50.94
P1011	0	24.8	50.16	75.17	95.79	125.59	150.88

Table 5.3 Comparison of Objective function and Power Flow Change

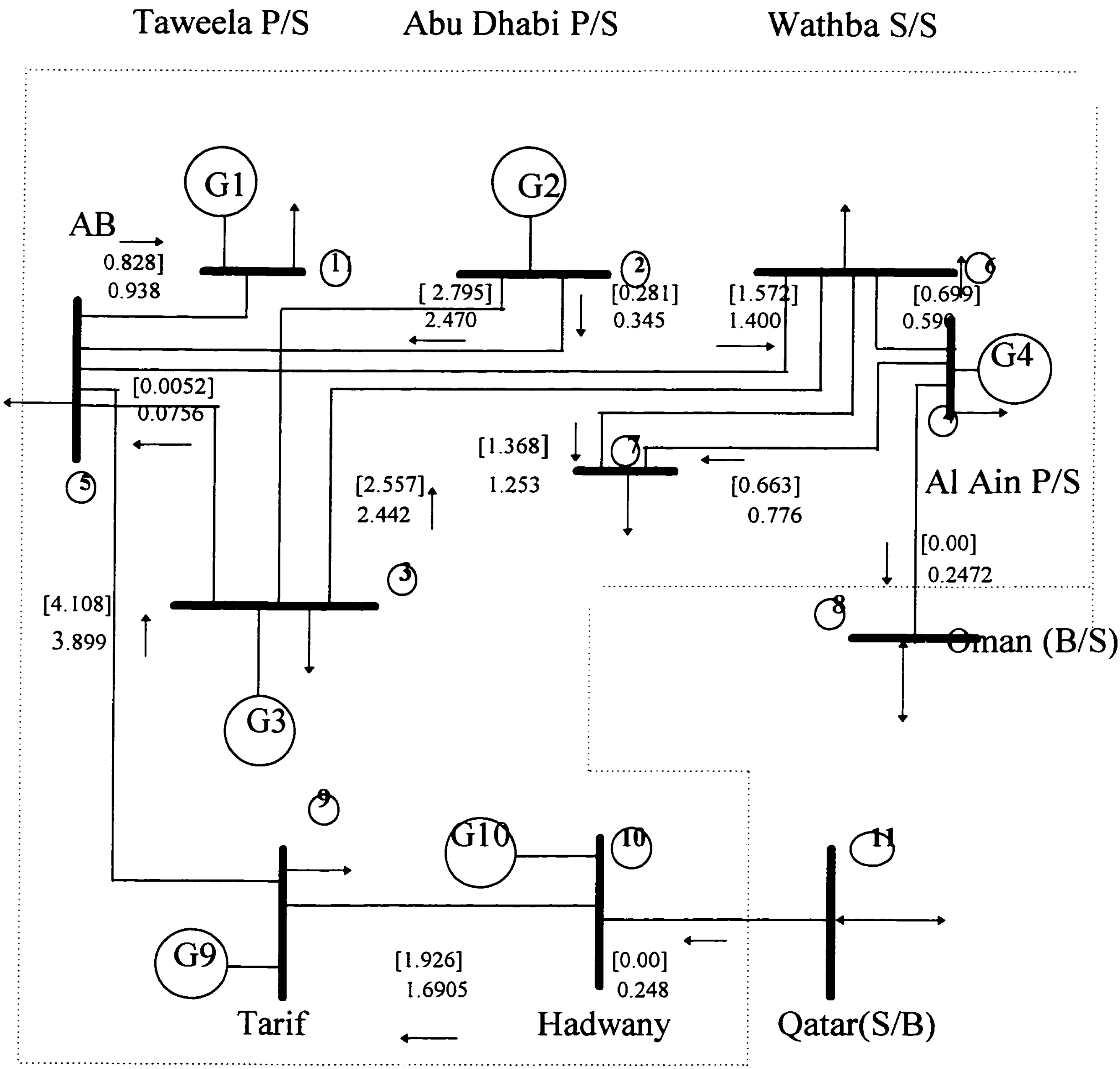

Fig. 5.2 Comparison of power flow change before and after wheeling for 25 MW Power Transfer from Qatar to Oman

[ ] : Power Flow before wheeling

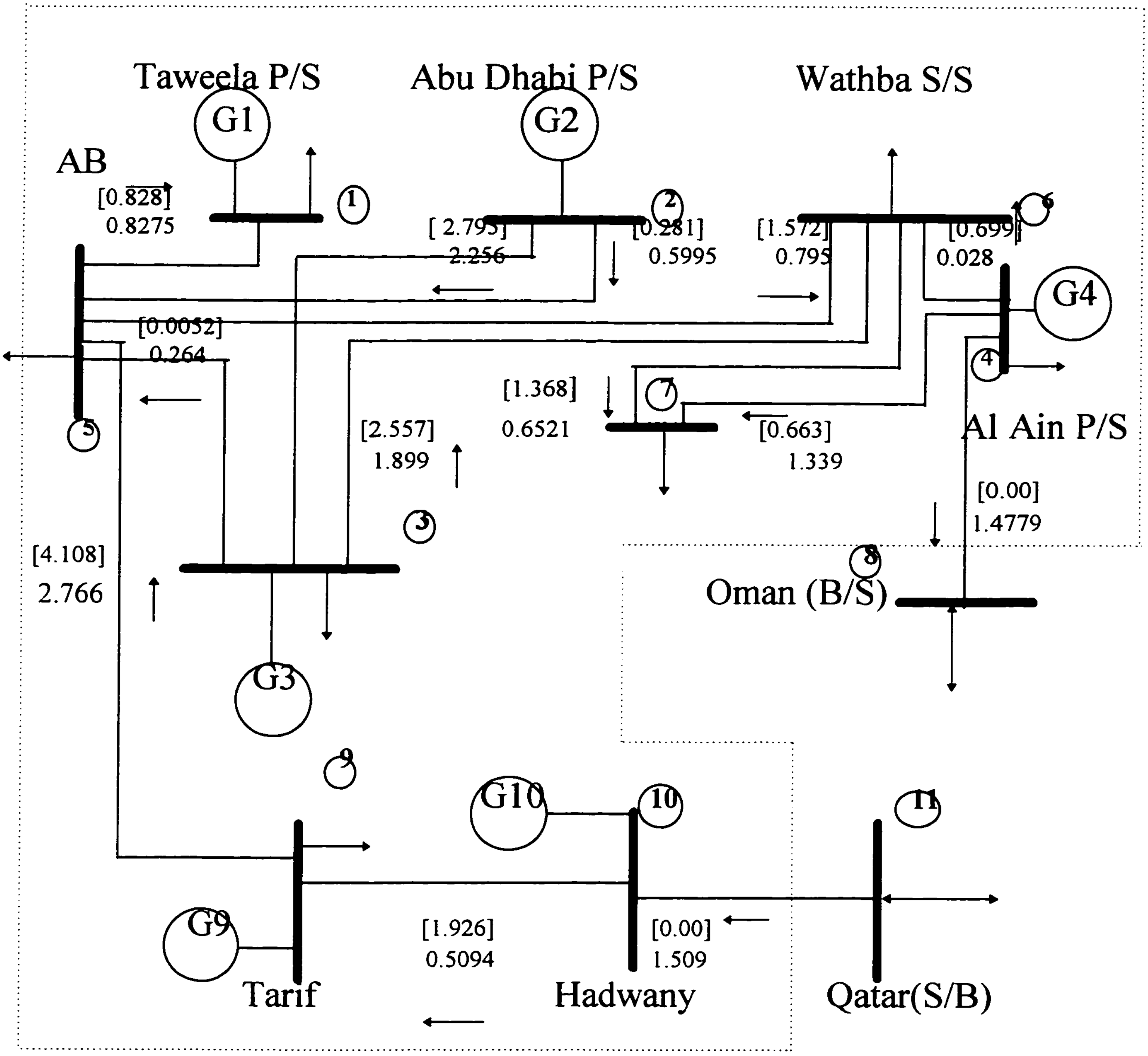

Fig. 5.3 Comparison of power flow change before and after wheeling for 150 MW Power Transfer from Qatar to Oman

[ ] : Power Flow before wheeling

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

In this thesis a modified Optimal Power Flow model for setting wheeling rates was proposed. The marginal cost theory was used to set the wheeling charges. The model included the following constraints :

1. Power Flow equations.
2. Bus voltage limits.
3. Transmission lines thermal and stability limits.
4. Active power and reactive power generation Costs.
5. Wheeler's Transmission Losses.
6. Power Balance equations.

Two case studies were used to test the proposed model. In the first case the effect of tightening the constraints were examined. The followings were concluded :

1. The addition of the reactive power generation costs can affect the wheeling rates and can not be ignored.

2. The transmission losses must be included in the model. If they are ignored misleading conclusions might result. This is due to the fact that power loss formula consists of all voltage buses and line admittances.

3. Tightening the constraints may lead to higher wheeling rates.

4. Minimum losses of wheeler's do not always mean lower production costs.

5. The reactive power wheeling costs can be higher than the active power rates.

6. The installation of shunt capacitors or static Var compensators might lower the wheeling rates.

7. The location of the seller and the buyer affect the wheeling rates. For example, the power charges for a power wheeled from a seller to a buyer do not necessarily equal the reveres charges.

8. The effect of reducing or relaxing some of the constraints (i.e. bus voltages) may have a better effect than installing a reactive power source.

In the second case the practical system of Oman, Qatar and UAE was examined for wheeling. Four transmission lines were proposed to interconnect the three utilities. Exchange of the buyer and seller location was examined.

6.2 Recommendations

The following are some suggestions to extend this thesis work,

1. A modification to this model to check the optimum location of reactive power sources, instead of trial and error procedure, is needed.
2. An automatic check of the effect of relaxing some of the constraints may lead to significant reduction of wheeling charges.

3. This model can be modified to include the wheeler's load changes.

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APPENDIX I

MODEL:

! The Objective Function is Introduced Here where it contains the active and reactive power generation costs.

1) MIN= 50 * P1 + 50 * P1 ^ 2 + 75 * P2 + 75 * P2 ^ 2 + 50 * P3 + 50 * P3 ^ 2 + .5 * Q1 + .5 * Q1 ^ 2 + .05 * Q2 + .05 * Q2 ^ 2 + .5 * Q3 + 0.5 * Q3 ^ 2 ;

! The Power Flow (Kirchoff's equations) of active and reactive power are
! introduced as the first set of constraints.

2) 0 = 100 * (P1 - V1 ^ 2 * Y11 * COS(D11) - V1 * V6 * Y16 * COS(D16 + D6 - D1) - V4 * V1 * Y14 * COS(D14 + D4 - D1)) ;

3) 0 = 100 * (P2 - V2 * V8 * Y28 * COS(D28 + D8 - D2) - V2 ^ 2 * Y22 *COS(D22)) ;

4) 0 = 100 * (P3 - V3 ^ 2 * Y33 * COS(D33) - V3 * V5 * Y35 * COS(D35 + D5 - D3) - V3 * V6 * Y36 * COS(D36 + D6 - D3)) ;

5) 0 = 100 * (- 3*.85 - V4 * V1 * Y14 * COS(D14 + D1 - D4) - V4 * V4 * Y44 * COS(D44) - V4 * V5 * Y45 * COS(D45 + D5 - D4)) ;

6) 0 = 100 * (- V5 * V4 * Y45 * COS(D45 + D4 - D5) - V5 * V8 * Y58 * COS(D58 + D8 - D5) - V3 * V5 * Y35 * COS(D35 + D3 - D5) - V5 ^ 2 * Y55 * COS(D55)) ;

7) 0 = 100 * (- V6 * V6 * Y66 * COS(D66) - V6 * V1 * Y16 * COS(D16 + D1 - D6) - V6 * V7 * Y67 * COS(D67 + D7 - D6) - V6 * Y36 * V3 * COS(D36 + D3 - D6)) ;

8) 0 = 100 * (- 1*.85 - V6 * V7 * Y67 * COS(D67 + D6 - D7) - V7 ^ 2 * Y77 * COS(D77)) ;

9) 0 = 100 * (- .50*.85 - V2 * V8 * Y28 * COS(D28 + D2 - D8) - V8 * V5 * Y58 * COS(D58 + D5 - D8) - V8 ^ 2 * Y88 * COS(D88)) ;

```
10) 100 * (.75 - Q1 - V1 ^ 2 * Y11 * SIN( D11 ) - V1 * V6 * Y16 * SIN(
    D16 + D6 - D1 ) - V4 * V1 * Y14 * SIN( D14 + D4 - D1 ) ) = 0 ;

11) 100 * ( - Q2 - V2 * V8 * Y28 * SIN( D28 + D8 - D2 ) - V2 ^ 2 * Y22
   * SIN( D22 ) ) = 0 ;

12) 100 * ( - Q3 - V3 ^ 2 * Y33 * SIN( D33 ) - V3 * V5 * Y35 * SIN(
    D35 + D5 - D3 ) - V3 * V6 * Y36 * SIN( D36 + D6 - D3 ) ) = 0 ;

13) 100 * ( 2.25*.85  - V4 * V1 * Y14 * SIN( D14 + D1 - D4 ) - V4 * V4
    * Y44 * SIN( D44 ) - V4 * V5 * Y45 * SIN( D45 + D5 - D4 ) ) = 0 ;

14) 100 * ( - V5 * V4 * Y45 * SIN( D45 + D4 - D5 ) - V5 * V8 * Y58 *
    SIN( D53 + D8 - D5 ) - V3 * V5 * Y35 * SIN( D35 + D3 - D5 ) - V5
    ^ 2  * Y55 * SIN( D55 ) ) = 0 ;

15) 100 * (  - V6 * V6 * Y66 * SIN( D66 ) - V6 * V1 * Y16 * SIN( D16
    + D1 - D6 ) - V6 * V7 * Y67 * SIN( D67 + D7 - D6 ) - V6 * Y36 *
    V3 * SIN( D36 + D3 - D6 ) ) = 0 ;

16) 100 * ( .75*.85  - V6 * V7 * Y67 * SIN( D67 + D6 - D7 ) - V7 ^ 2 *
    Y77 * SIN( D77 ) ) = 0 ;

17) 100 * ( .375*.85  - V2 * V8 * Y28 * SIN( D28 + D2 - D8 ) - V8 * V5
    Y58 * SIN( D58 + D5 - D8 ) - V8 ^ 2 * Y88 * SIN( D88 ) ) = 0 ;

! The first set of inequality constraints is introduced. It contains
generations limit of active and reactive power.

18) P1 > 0 ;

19) P1 < 5.0 ;

20) P2 > 0 ;

21) P2 < 2.00 ;
```

22) P3 > 0 ;

23) P3 < 2.00 ;

24) Q1 > 0.0 ;

25) Q1 > 3.75 ;

26) Q2 > 0.0 ;

27) Q2 < 2.0 ;

28) Q3 > 0.0 ;

29) Q3 < 2.0;

! The line admittance is introduced here.

30) Y11 = 92.3 ;

31) Y22 = 26.42

32) Y33 = 45.42 ;

33) Y44 = 71.46 ;

34) Y55 = 67.6 ;

35) Y66 = 69.3 ;

36) Y77 = 24.39 ;

37) Y88 = 52.83 ;

38) D11 = - 1.196 ;

39) D22 = - 1.358 ;

40) D33 = - 1.2689 ;

41) D44 = - 1.0556 ;

42) D55 = - 1.2217 ;

43) D66 = - 1.1123 ;

44) D77 = - 0.8203 ;

45) D88 = - 1.358 ;

46) Y45 = 12.194 ;

47) Y67 = 24.37 ;

48) Y35 = 31.235 ;

49) Y28 = 26.417 ;

50) Y16 = 32.208 ;

51) Y36 = 14.315 ;

52) Y14 = 60.41 ;

53) Y58 = 26.417 ;

54) D45 = 2.4859 ;

55) D67 = 2.3213 ;

56) D35 = 1.8233 ;

57) D28 = 1.7837 ;

58) D16 = 1.8314 ;

59) D36 = 1.9832 ;

60) D14 = 2.0074 ;

61) D58 = 1.7837 ;

! Voltage limits at all system buses is introduced here.

62) V1 > 0.9 ;

63) V1 < 1.1 ;

64) V2 > 0.9 ;

65) V2 < 1.1 ;

66) V3 > 0.9 ;

67) V3 < 1.1 ;

68)V4 > 0.9 ;

69) V4 < 1.1 ;

70) V5 > 0.9 ;

71) V5 < 1.1;

72) V6 > 0.9 ;

73) V6 < 1.1;

74) V7 > 0.9;

75) V7 < 1.1;

76) V8 > 0.9;

```
77) V8 < 1.1;

! Transmission Line thermal Limits

78) V4 * V5 * Y45 * COS( D45 + D5 - D4 ) - V4 ^ 2 * Y45 * COS( D45 )
    <  6;
79) V6 * V7 * Y67 * COS( D67 + D7 - D6 ) - V6 ^ 2 * Y67 * COS( D67 )
    < 6;
80) V3 * V5 * Y35 * COS( D35 + D5 - D3 ) - V3 ^ 2 * Y35 * COS( D35 )
    < 6;
81) V2 * V8 * Y28 * COS( D28 + D8 - D2 ) - V2 ^ 2 * Y28 * COS( D28 )
    < 6;
82) V1 * V6 * Y16 * COS( D16 + D6 - D1 ) - V1 ^ 2 * Y16 * COS( D16 )
    < 6;
83) V3 * V6 * Y36 * COS( D36 + D6 - D3 ) - V3 ^ 2 * Y36 * COS( D36 )
    <  6;
84) V1 * V4 * Y14 * COS( D14 + D4 - D1 ) - V1 ^ 2 * Y14 * COS( D14 )
    < 4.2 ;
85) V5 * V8 * Y58 * COS( D58 + D8 - D5 ) - V5 ^ 2 * Y58 * COS( D58 )
    <   6;

86) PLOSS = V1 ^ 2 * Y11 * COS( - D11 ) + V1 * V6 * Y16 * COS( - D16
- D6 + D1 ) + V4 * V1 * Y14 * COS( - D14 - D4 + D1 ) + V2 * V8 * Y28 *
COS( - D28 - D8 + D2 ) + V2 ^ 2 * Y22 * COS( - D22 ) + V3 ^ 2 * Y33 *
   COS( - D33 ) + V3 * V5 * Y35 * COS( - D35 - D5 + D3 ) + V3 * V6 *
    Y36 * COS( - D36 - D6 + D3 ) + V4 * V1 * Y14 * COS( - D14 - D1 +
  D4 ) + V4 * V4 * Y44 * COS( - D44 ) + V4 * V5 * Y45 * COS( - D45 -
  D5 + D4 ) + V5 * V4 * Y45 * COS( - D45 - D4 + D5 ) + V5 * V8 * Y58
  * COS(- D58 - D8 + D5 ) + V3 * V5 * Y35 * COS( - D35 - D3 + D5 ) +
  V5 ^ 2 * Y55 * COS( - D55 ) + V6 * V1 * Y16 * COS( - D16 - D1 + D6
  ) + V6 * V7 * Y67 * COS( - D67 - D7 + D6 ) + V6 * Y36 * V3 * COS( -
  D36 - D3 + D6 ) + V6 * V6 * Y66 * COS( - D66 ) + V6 * V7 * Y67 *
  COS( - D67 - D6 + D7 ) + V7 ^ 2 * Y77 * COS( - D77 ) + V2 * V8 *
  Y28 * COS( - D28 - D2 + D8 ) + V8 * V5 * Y58 * COS( - D58 - D5 +
  D8 ) + V8 ^ 2 * Y88   COS( - D88 ) ;

! Transmission Line Connecting Bus 3 and 6 Stability limits.
```

```
87) ( V3 * V6 / 0.07 ) - ( 0.998 * V6 ^ 2 / 0.0698 ) * COS( 1.157 ) < 9.5;

! Tap Changer position of transformer at bus 4

88) T = ( V - V4 ) / ( V4 * 0.025 ) ;

89) T > - 4 ;

90) T < 4 ;

91) V > 0.9 ;

92) V < 1.1 ;

! Power Balance Equation

93) P1 + P2 + P3 - PLOSS = 4.5;

END

   LEAVE
```

Appendix II

Model of UAE system (Case 2)

```
MODEL:

1) MIN= 45 * P1 + 50 * P1 ^ 2 + 55 * P2 + 20 * P2 ^ 2 + 35 * P3
     + 30 * P3 ^ 2 + 40 * P4 + 60 * P4 ^ 2 + 14.5 * Q1 + 1.3 * Q1 ^ 2
     + 12.90 * P2 + 0.9 * Q2 ^ 2 + 17 * Q3 + 3.7 * Q3 ^ 2 + 13.5 * Q4 +
     1.9 * Q4 ^ 2 + 38 * P9 + 47 * P9 ^ 2 + 56 * P10 + 48 * P10 ^ 2
     + 12.49 * Q9 + 1.2 * Q9 ^ 2 + 23.56 * Q10 + 1.37 * Q10 ^ 2 ;

2) 100 * ( 3 - P1 + V1 ^ 2 * 143.704 * COS( - 1.39382 ) + V1 * V5 *
     143.704 * COS( 1.74776 + D5 - D1 ) )=0 ;

3) 100 * ( 2 - P2 + V2 * V3 * 794.298 * COS( 1.6902 + D3 - D2 ) +
     V2 ^ 2 * 866.04 * COS( - 1.44659 ) + V2 * V5 * 71.852 * COS( 1.74778
     + D5 - D2 ) )=0 ;

4) 100 * ( 3 - P3 + V2 * V3 * 794.298 * COS( 1.6902 + D2 - D3 ) +
     V3 ^ 2 * 959.44 * COS( - 1.4419 ) + V3 * V5 * 49.49 * COS( 1.7449 +
     D5 - D3 ) + V3 * V6 * 115.86 * COS( 1.7454 + D6 - D3 ) )=0 ;

5) 100 * ( 2 - P4 + V4 * V6 * 12.05 * COS( 1.7882 + D6 - D4 ) + V4
     ^ 2 * 75.7657 * COS( - 1.3658 ) + V4 * V7 * 40.167 * COS( 1.7895 +
     D7 - D4 ) + V4 * V8 * 23.557 * COS( 1.746 + D8 - D4 ) )=0 ;

6) 100 * ( 2 + V1 * V5 * 143.704 * COS( 1.747757 + D1 - D5 ) + V5 ^
     2 * 363.38 * COS( - 1.39895 ) + V6 * V5 * 71.3 * COS( 1.75 + D6 - D5
     ) + V2 * V5 * 71.852 * COS( 1.74776 + D2 - D5 ) + V3 * V5 * 49.49 *
     COS( 1.7449 + D3 - D5 ) + V9 * V5 * 27.094 * COS( 1.6785 + D9 - D5 )
     )=0 ;

7) 100 * ( 2 + V6 ^ 2 * 217.83 * COS( - 1.3886 ) + V6 * V4 * 12.05
     * COS( 1.7882 + D4 - D6 ) + V6 * V3 * 115.86 * COS( 1.7454 + D3 - D6
     ) + V6 * 71.3 * V5 * COS( 1.75 + D5 - D6 ) + V6 * V7 * 18.63 * COS(
     1.789 + D7 - D6 ) )=0 ;

8) 100 * ( 2 + V4 * V7 * 40.18 * COS( 1.7893 + D4 - D7 ) + V7 ^ 2 *
     58.806 * COS( - 1.3524 ) + V6 * V7 * 18.63 * COS( 1.789 + D6 - D7 )
     )=0 ;

9) 100 * ( .25 + V4 * V8 * 23.557 * COS( 1.746 + D4 - D8 ) + V8 ^ 2
     * 23.556 * COS( - 1.39557 ) )=0 ;

10) 100 * ( - P9 + V5 * V9 * 27.094 * COS( 1.6785 + D5 - D9 ) + V9 ^
     2 * 60.444 * COS( - 1.42635 ) + V9 * V10 * 33.38 * COS( 1.7453 + D10
     - D9 ) )=0 ;

11) 100 * ( - P10 + V10 * V9 * 33.38 * COS( 1.7453 + D9 - D10 ) +
     V10 ^ 2 * 71.5605 * COS( - 1.4319 ) + V11 * V10 * 38.2164 * COS(
     1.6785 + D11 - D10 ) )=0 ;

12) 100 * (-.25 + V10 * V11 * 38.2164 * COS( 1.6785 + D10 - D11 ) +
```

13) 0 = 100 * (- 2.1 + Q1 + V1 ^ 2 * 143.704 * SIN(- 1.39382) + V1 * V5 * 143.704 * SIN(1.74776 + D5 - D1)) ;

14) 0 = 100 * (- 1.5 + Q2 + V2 * V3 * 794.298 * SIN(1.6902 + D3 - D2) + V2 ^ 2 * 866.04 * SIN(- 1.44659) + V2 * V5 * 71.852 * SIN(1.74778 + D5 - D2)) ;

15) 0 = 100 * (- 2.1 + Q3 + V2 * V3 * 794.298 * SIN(1.6902 + D2 - D3) + V3 ^ 2 * 959.44 * SIN(- 1.4419) + V3 * V5 * 49.49 * SIN(1.7449 + D5 - D3) + V3 * V6 * 115.86 * SIN(1.7454 + D6 - D3)) ;

16) 0 = 100 * (- 1.5 + Q4 + V4 * V6 * 12.05 * SIN(1.7882 + D6 - D4) + V4 ^ 2 * 75.7657 * SIN(- 1.3658) + V4 * V7 * 40.167 * SIN(1.7895 + D7 - D4) + V4 * V8 * 23.557 * SIN(1.746 + D8 - D4)) ;

17) 0 = 100 * (- 1.5 + V1 * V5 * 143.704 * SIN(1.747757 + D1 - D5) + V5 ^ 2 * 363.38 * SIN(- 1.39895) + V6 * V5 * 71.3 * SIN(1.75 + D6 - D5) + V2 * V5 * 71.852 * SIN(1.74776 + D2 - D5) + V3 * V5 * 49.49 * SIN(1.7449 + D3 - D5) + V9 * V5 * 27.094 * SIN(1.6785 + D9 - D5)) ;

18) 0 = 100 * (- 1.5 + V6 ^ 2 * 217.83 * SIN(- 1.3886) + V6 * V4 * 12.05 * SIN(1.7882 + D4 - D6) + V6 * V3 * 115.86 * SIN(1.7454 + D3 - D6) + V6 * 71.3 * V5 * SIN(1.75 + D5 - D6) + V6 * V7 * 18.63 * SIN(1.789 + D7 - D6)) ;

19) 0 = 100 * (-1.5 + V4 * V7 * 40.18 * SIN(1.7893 + D4 - D7) + V7 ^ 2 * 58.806 * SIN(- 1.3524) + V6 * V7 * 18.63 * SIN(1.789 + D6 - D7)) ;

20) 0 = 100 * (-.1875 + V4 * V8 * 23.557 * SIN(1.746 + D4 - D8) + V8 ^ 2 * 23.556 * SIN(- 1.39557)) ;

21) 0 = 100 * (Q9 + V5 * V9 * 27.094 * SIN(1.6785 + D5 - D9) + V9 ^ 2 * 60.444 * SIN(- 1.42635) + V9 * V10 * 33.38 * SIN(1.7453 + D10 - D9)) ;

22) 0 = 100 * (Q10 + V10 * V9 * 33.38 * SIN(1.7453 + D9 - D10) + V10 ^ 2 * 71.5605 * SIN(- 1.4319) + V11 * V10 * 38.2164 * SIN(1.6785 + D11 - D10)) ;

23) 0 = 100 * (.1875 + V10 * V11 * 38.2164 * SIN(1.6785 + D10 - D11) + V11 ^ 2 * 38.216 * SIN(- 1.4631)) ;

24) P1 > 0 ;

25) P1 < 4 ;

26) P2 > 0 ;

27) P2 < 5 ;

28) P3 > 0 ;

29) P3 < 3 ;

30) P4 > 0 ;

31) $P4 < 4$;

32) $P9 > 0$;

33) $P9 < 3$;

34) $P10 > 0$;

35) $P10 < 4$;

36) $Q9 > 0$;

37) $Q9 < 3$;

38) $Q10 > 0$;

39) $Q10 < 3.5$;

40) $Q1 > 0$;

41) $Q1 < 3.1$;

42) $Q2 > 0$;

43) $Q2 < 3.8$;

44) $Q3 > 0$;

45) $Q3 < 2.5$;

46) $Q4 > 0$;

47) $Q4 < 3.1$;

48) $V1 > 0.9$;

49) $V1 < 1.1$;

50) $V2 > 0.9$;

51) $V2 < 1.1$;

52) $V3 > 0.9$;

53) $V3 < 1.1$;

54) $V4 > 0.9$;

55) $V4 < 1.1$;

56) $V5 > 0.9$;

57) $V5 < 1.1$;

58) $V6 > 0.9$;

60) V7 > 0.9 ;

61) V7 < 1.1 ;

62) V8 > 0.9 ;

63) V8 < 1.1 ;

64) V9 > 0.9 ;

65) V9 < 1.1 ;

66) V10 > 0.9 ;

67) V10 < 1.1 ;

68) V11 > 0.9 ;

69) V11 < 1.1 ;

70) PL = V1 ^ 2 * 143.704 * COS(1.39382) + V1 * V5 * 143.704 * COS(- 1.74776 - D5 + D1) + V2 * V5 * 71.852 * COS(- 1.74778 - D5 + D2) + V2 * V3 * 794.298 * COS(- 1.6902 - D3 + D2) + V2 ^ 2 * 866.04 * COS(1.44659) + V3 ^ 2 * 959.44 * COS(1.4419) + V3 * V2 * 794.297 * COS(- 1.6902 - D2 + D3) + V3 * V6 * 115.86 * COS(- 1.7454 - D6 + D3) + V3 * V5 * 49.49 * COS(- 1.7449 - D5 + D3) + V4 * V7 * 40.167 * COS(- 1.7895 - D7 + D4) + V4 * V4 * 75.7657 * COS(1.3658) + V4 * V8 * 23.557 * COS(- 1.746 - D8 + D4) + V4 * V6 * 12.054 * COS(- 1.7884 - D6 + D4) + V5 * V1 * 143.704 * COS(- 1.74776 - D1 + D5) + V5 * V2 * 71.852 * COS(- 1.74777 - D2 + D5) + V5 * V9 * 27.094 * COS(- 1.6784 - D9 + D5) + V3 * V5 * 49.49 * COS(- 1.7449 - D3 + D5) + V5 ^ 2 * 363.38 * COS(1.398958) + V6 * V5 * 71.302 * COS(- 1.75 - D6 + D5) + V6 * V3 * 115.85 * COS(- 1.7454 - D3 + D6) + V6 * V4 * 12.054 * COS(- 1.7884 - D4 + D6) + V6 * 71.302 * V5 * COS(- 1.75 - D5 + D6) + V6 * V6 * 217.826 * COS(1.3886) + V6 * V7 * 18.63 * COS(- 1.7887 - D7 + D6) + V6 * V7 * 18.63 * COS(- 1.7887 - D6 + D7) + V7 * V4 * 40.167 * COS(- 1.7895 - D4 + D7) + V7 ^ 2 * 58.805 * COS(1.3524) + V4 * V8 * 23.557 * COS(- 1.746 - D4 + D8) + V8 ^ 2 * 23.5567 * COS(1.39557) + V9 ^ 2 * 60.444 * COS(1.4263) + V5 * V9 * 27.094 * COS(- 1.6784 - D5 + D9) + V9 * V10 * 33.38 * COS(- 1.7452 - D10 + D9) + V10 ^ 2 * 71.5605 * COS(1.4319) + V9 * V10 * 33.38 * COS(- 1.7452 - D9 + D10) + V10 * V11 * 38.2164 * COS(- 1.67849 - D11 + D10) + V11 ^ 2 * 38.216 * COS(1.4631) + V10 * V11 * 38.2164 * COS(- 1.6785 - D10 + D11) ;

71) P1 + P2 + P3 + P4 + P9 + P10 - PL = 16 ;

72) Q1 + Q2 + Q3 + Q4 + Q9 + Q10 > 12 ;

73) V5 * V9 * 27.094 * COS(1.6784) - V5 ^ 2 * 27.094 * COS(1.6784) < 13.2 ;

74) V4 * V8 / 0.085 - 0.9787 * V4 ^ 2 / .085 * COS(1.39558 - 0.003883) < 13 ;

75) V10 * V11 / .0865 - 0.95125 * V10 ^ 2 / .0865 * COS(1.4631 - 0.1035

END

LEAVE

IMAGE EVALUATION TEST TARGET (QA-3)

150mm

6″

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