Modelling and Antiswing Control of Overhead Cranes

by

Zafar Haidar Khan

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

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In

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Modelling and antiswing control of overhead cranes

Khan, Zafar Haider, M.S.

King Fahd University of Petroleum and Minerals (Saudi Arabia), 1993

U·M·I
300 N. Zeeb Rd.
Ann Arbor, MI 48106

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ZAFAR HAIDER KHAN

under the direction of his Thesis Advisor, and approved by his Thesis Committee, has been presented to and accepted by the Dean, College of Graduate Studies, in partial fulfillment of the requirements for the degree of

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Thesis Committee

Chairman (Dr. K. A. F. MOUSTAFA)

Member (Dr. A. Z. AL-GARNI)

Member (Dr. M. E. EMARA-SHABAIK)

Member (Dr. Y. A. R. KHULIEF)

Department Chairman

Dean, College of Graduate Studies

Date : 28-2-93

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"In the name of Allah (God), Most Gracious, Most Merciful. Read, In the name of thy Lord and Cherisher, Who created.Created man from a [*leech-like*] clot. Read, and thy Lord Is Most Bountiful, He Who taught [*the use of*] the pen. Taught man that Which he knew not. Nay, but man doth Transgress all bounds. In that he looketh Upon himself as self-sufficient. Verily, to thy Lord Is the return [*of all*]. " (The Holy QURAN, Surah no. 96)

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NOMENCLATURE

g	:	acceleration due to gravity (m/sec^2)
H	:	weighting matrix
I_1	:	mass moment of inertia of the bridge motor ($Kg.m^2$)
I_2	:	mass moment of inertia of the trolley motor ($Kg.m^2$)
I_3	:	mass moment of inertia of the hoisting motor ($Kg.m^2$)
J	:	Performance Index
L	:	Lagrangian function
l	:	length of the cable (m)
M	:	mass of the load (Kg)
m_1	:	mass of the bridge (Kg)
m_2	:	mass of the trolley (Kg)
r_1	:	radius of the bridge motor pinion (m)
r_2	:	radius of the trolley motor pinion (m)
r_3	:	radius of the hoisting drum (m)
Q	:	vector of generalized forces
q	:	generalized coordinate
R	:	weighting matrix
T	:	kinetic energy of the system
T_x	:	driving torque generated by the the bridge drive motor (N-m)
T_y	:	driving torque generated by the the trolley drive motor (N-m)

T_l	:	driving torque generated by the the hoisting motor (N-m)
V	:	potential energy of the system
x_g, y_g, z_g	:	x, y, and z coordinates of the load
x, y	:	x, and y coordinates of the trolley

Greek Symbols

ψ	:	oscillation of the plane determined by the cable and the vertical axis through the suspension point
θ	:	swing in the plane of motion

THESIS ABSTRACT

NAME OF STUDENT	*: ZAFAR HAIDER KHAN*
TITLE OF STUDY	*: Modelling and Anti-Swing Control of Overhead Cranes*
MAJOR FIELD	*: Mechanical Engineering*
DATE OF DEGREE	*: January, 1993*

A nonlinear dynamical model for an overhead crane is first developed. The most general motion, i.e. simultaneous travel, traverse, and hoisting, is considered. Two different mathematical models corresponding to torque control and acceleration control are derived from the general model. Linearized models, by expanding the nonlinear equations about nominal operating conditions using Taylor's series, are also presented. Optimization techniques are used to obtain a suboptimal feedback control scheme, so that an object can be transferred to a desired position as quickly as possible while minimizing the sways of the load during transfer and at the end as well. Mathematically, a boundary value problem with constraints both in the control and state variables is analyzed. A new computational technique is employed for computing the suboptimal control and several numerical results are presented. Computer simulations show that the control strategy works well.

MASTER OF SCIENCE DEGREE

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

Dhahran , Saudi Arabia

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خلاصة الرسالة

اسم الطالب ظفر حيدر خان
عنوان الرسالة أستنباط نموذج رياضي لرافعة علوية وطريقة للتحكم في نمايل أحمالها
التخصص هندسة ميكانيكية
تاريخ منح الدرجة يناير ١٩٩٣م

الغرض من هذا البحث هو استنباط نموذج حركي غير خطي ليمثل حركة رافعة علوية وايجاد افضل طريقة للتحكم الجزئي في حركتها . ويأخذ النموذج في الاعتبار كل الحركات المتوقعة مثل الانتقال الخطي والعرضي والعلوي . وبعد ذلك تم اشتقاق نموذجين رياضيين من النوع الخطي احدهما للتحكم في الرافعة عن طريق تغيير العزم والآخر عن طريق تغيير التسارع . وهذان النموذجان تم اشتقاقهما بأستعمال صيغ المتواليات لفك النموذج الغير خطي . ولقد تم استعمال طرق الحلول المثلى للحصول على افضل تحكم جزئي والذي يهيئ حمل الرافعة لكي يُنقل للموقع المطلوب باسرع وقت وبأقل تأرجح ممكن ، ولقد تم ايضاً تقديم طريقه عدديه جديده لحساب التحكم الجزئي الافضل حيث أثبتت المحاكاه الرقميه باستخدام الحاسب الالي فعالية خطة التحكم المقترحه .

درجة الماجستير في العلوم
جامعة الملك فهد للبترول والمعادن
الظهران - المملكة العربية السعودية
يناير ١٩٩٣م

Chapter 1

INTRODUCTION

Cranes are indispensable for transferring heavy loads from one place to another. They are commonly used in ports, factories, and construction sites. There are different types of cranes such as overhead cranes, gantry cranes, rotary cranes, jib and pillar cranes, manually-operated cranes, nuclear plant cranes, stacker cranes and various special purpose cranes [1-4] . Each type of crane has its own application area.

The overhead cranes are among those types which are most widely used in several work places. These usually consist of a bridge which can travel longitudinally on rails laid over gantry girders, properly supported over columns, and a trolley which traverses along the bridge crosswise to the direction of the bridge motion. The trolley houses a hoisting mechanism for load hoisting/lowering [5]. In general, the task performed by an overhead crane, in which load is picked, raised, moved to target and placed down, may be divided into three phases:

1. Hoisting the object.
2. Transporting the object which includes longitudinal and transverse motion.
3. Lowering the object.

Thus the fundamental motions of an overhead crane can be described as: object hoisting or lowering, travel (bridge motion), and traverse (trolley motion).

In order to increase the productivity of the system, it is necessary that all these motions of crane should take place at higher speeds. If the travelling and transversing motions take place at higher speeds without a proper control then there are chances of swinging of the suspended object. This swing is undesirable because it could cause serious damages to the system and the surroundings as well. Also, if this swing of the suspended object continues up to the end of the transfer then it becomes quite difficult to place the object at the desired location. Therefore, for a crane system designer, it is a fundamental requirement to seek a satisfactory control method to minimize the swing during the complete process of load transfer.

There are different control methods which can be used to suppress the swing of the object. In some installations, it is the only skill of operator that is used to bring the object to the desired position and have it stationary during the journey and at the end as well. Open-loop methods such as continuous swing control and discontinuous swing control are also used in some applications [6-7], however, due to many disadvantages, the open-loop control schemes are found to be unsuitable in real working environment, especially in applications where accuracy is of prime importance. One of the main disadvantages of an open-loop control system is that it cannot make any compensation for external disturbances such as wind etc.

To avoid the problems caused by open-loop control systems, in most of the work places, closed-loop control systems are being employed. Different types of closed-loop control methods which can be used are optimal control, variable structure control and linear feedback control.

In the present study, the modelling problem of overhead cranes is addressed. The most general case of crane motions i.e. simultaneous travelling of bridge, traversing of trolley, and load hoisting is considered. This type of motion has not been reported, to the best of our knowledge, in the literature of the relevant field. Most of the reported work, addresses only the traversing motion of the trolley. In references [8-10] the simultaneous motions of hoisting of the load and traversing of the trolley are considered. Recently, Moustafa and Ebeid [11] developed a nonlinear model that takes into account the simultaneous travelling of the bridge and the traversing of the trolley. However, their model did not consider the hoisting of the load.

A general nonlinear model is derived in the present study by using the Lagrange's equation of motion to describe the dynamics of crane motion. For the purpose of industrial applications, two different models namely: acceleration-control model and torque-control model are then derived from the general nonlinear model. A linearization technique is then carried out by expanding the nonlinear model about a nominal operating point by using Taylor's Series. To obtain the linearized models the nominal operating conditions are described. One of the derived models is used to formulate an optimized feedback scheme to control the motion of the crane so that the suspended load arrives at its final destination in a state of rest. Also the transfer is required to occur in the shortest possible time with minimum load swing. It should be noted that the control schemes, normally described in literature, are either open-loop time-optimal or linearized feedback controls.

In this study, a detailed description of an optimized feedback control including the performance index selection criteria, control constraints, and state-variable constraints is provided. A numerical optimization technique, used for solving the given control problem, is also discussed. The organization of this study is in the following manner:

Chapter 1 describes the general aspects of an overhead crane. A literature review relative to the modelling and the control of the considered cranes is discussed in detail. A description of the objectives of the present work is also mentioned. In Chapter 2, a detailed derivations of the dynamic model of the considered overhead crane is provided. Linearized models are developed in Chapter 3. In Chapter 4, an anti-swing optimized feedback control scheme is presented. Results and discussion are presented in Chapter 5. Firstly, the simulations of the nonlinear-torque control and the acceleration-control models are presented without applying any antiswing control scheme. Secondly, the simulations of the nonlinear acceleration-control model, after applying the optimization control scheme, are discussed.

1.1 LITERATURE SURVEY

A lot of research work related to the modelling and control of cranes to suppress the swinging of the suspended objects have been reported by many researchers. Alzinger [6] and Carbon [7] discussed two open-loop methods of swing control . One of these methods, which is the simplest one, is known as continuous swing control. In this method, as shown in Figure 1.1, the trolley velocity ramps at a constant rate for absolutely one period of the pendulum

formed by the cable and the load. It can be proved mathematically that if there is no swing at the beginning of the load transfer operation then there will be no swing at the end. The second method which is known as discontinuous swing control uses the maximum acceleration and deceleration rates attainable by the trolley motor, as shown in Figure 1.2 . Another open-loop control scheme was presented by Starr [12] for transporting suspended objects with a path controlled robot manipulator such that there is no swing at the target position. The method presented by him required a manipulator capable of constant-velocity straight line motion and an acceleration time that is small compared to the natural period of the suspended object.

A great economical saving can be achieved by reducing the load transfer time. Therefore, a significant amount of work, related to the time-optimal control of cranes under various operating conditions, have been carried out by many researchers [13-19]. In most of the cases, Pontryagin's maximum principal [20] was used to find the optimal control solutions. Martenson [8] derived two different mathematical models corresponding to torque control and acceleration control and presented time-optimal control strategies. Auernig and Troger [10] presented a time-optimal control scheme for overhead cranes by considering a planar motion due to hoisting of the load and traversing of the trolley for a fixed position of the crane. In their mathematical description, they included both types of motors used in operating systems i.e. the speed controlled and the torque directed motors. They used an extension of the Pontryagin's maximum principle in the mathematical formulation of the optimal control problem. The control scheme presented by them was to suppress the swing of the load at the end of the transfer only.

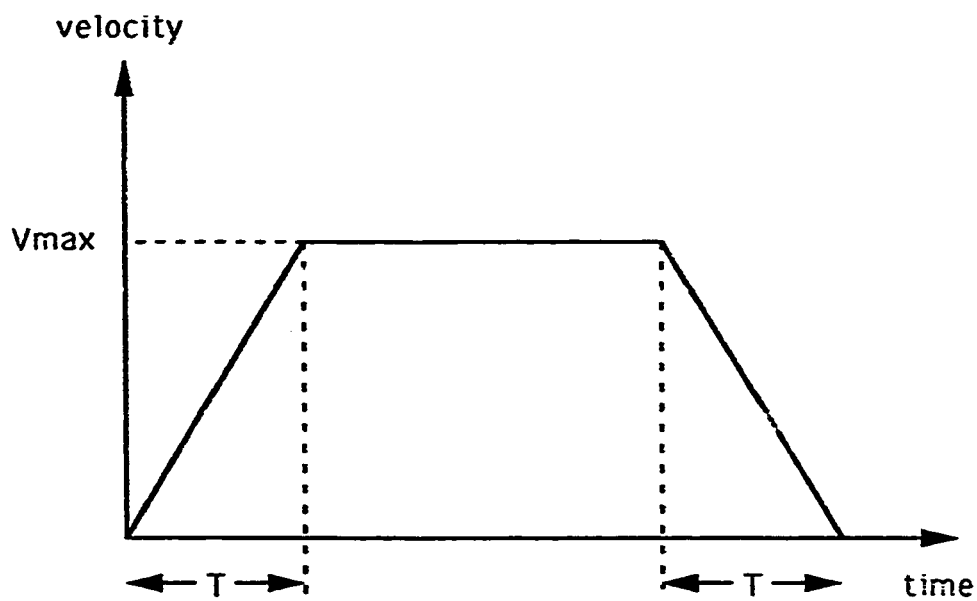


Figure 1.1: Continuous swing control.

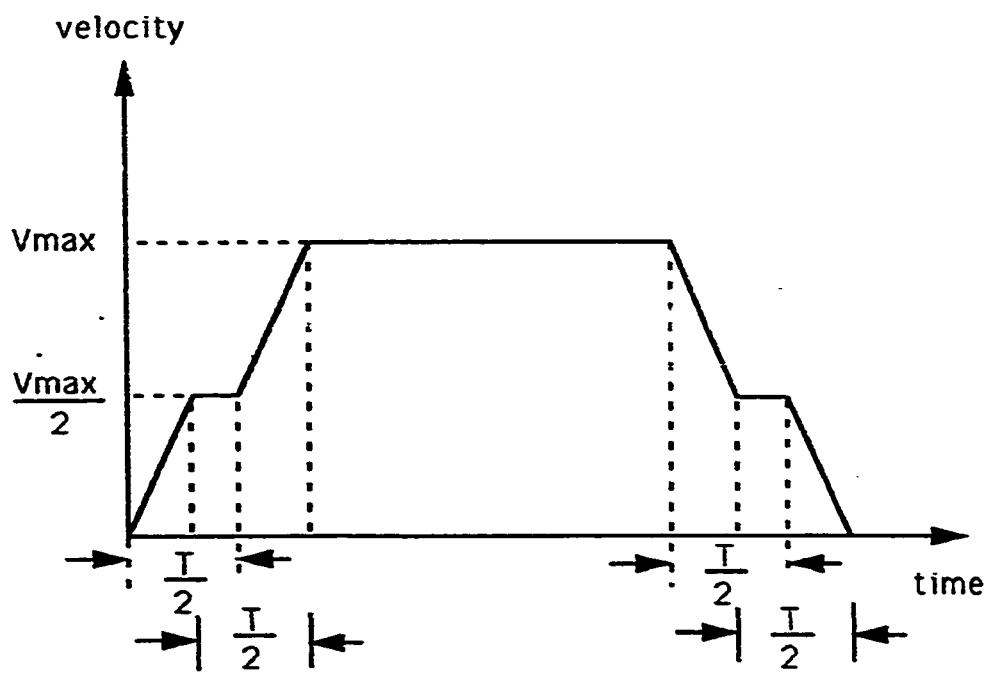


Figure 1.2: Discontinuous swing control.

Sakawa et al. [4] developed a nonlinear model for a rotary crane and applied an open-loop plus feedback optimal control scheme to minimize the swing of the load during the transfer, as well as the swing at the end of the transfer. They considered only two kinds of motion namely rotation and load hoisting. The boom angle was kept constant. A modification to this rotary crane's model was made by Sakawa and Nakazumi [21] by including the third motion i.e. boom hoisting. The major drawback of their scheme was that it suppressed the swing only at the target position and there was no control to minimize the swing during the transfer of the load. Yoshimoto and Sakawa [22]. and Sato and Sakawa [23] further modified the rotary crane model presented in ref. [21] by adding the flexibility effects. Hara et al. [24] derived a non-linear model for a jib crane and applied a semi-optimal control scheme to suppress the swing of the load during and at the end of the transport. They also treated the control of the extending or shrinkage motion of the crane boom.

Zinober [25] described a closed-loop method. known as variable structure control, for overhead cranes. This method uses a switching surface and avoids the limit cycles inherent in the time-optimal techniques by rotating the switching surface. This results in a self-adaptive control system which is obviously sub-optimal. A single cycle automatic operation system for an overhead crane was developed by Ohnishi et al. [26]. They achieved an anti-swing control for transporting the steel coils in cold strip mills. Sakawa and Shindo [9] derived a dynamical model for a container crane for cargo handling work at ports. Their aim was to minimize the swing of the container during as well as at the end of the transfer without paying any consideration to the transfer time. The type of

control. of the motion of the trolley, used was position control i.e. the usage of a speed-controlled motor. They divided the whole motion of the crane into five sections consisting of vertical motions, horizontal motions, and diagonal motions. The optimal control of the diagonal motion was computed by a different algorithm [27], which was developed by themselves.

Manson [28] presented an idealized model for an overhead crane and applied time-optimal control scheme. The method presented was useful for comparing the other sub-optimal solutions based on more complex and less idealized model. However this model was not feasible for practical applications. Van de Ven [29] presented a feasibility study of the time-optimal control of a hoisting crane. He described two methods for an open-loop control system, one based on pontryagin's maximum principal and the other based on the pattern of the phase trajectories. He also derived two methods to determine the time optimal switching intervals in a closed-loop control systems. In the first method an analytical approach was used to calculate the switching surfaces, whereas, in the second method a fast model was used to predict the time-optimal switching intervals. Karihaloo and Parberg [30] discussed the optimal control problem of a dynamical system representing a gantry crane. Their aim was to find the optimal control for getting the suspended mass to arrive at its final destination in the state of rest. They assumed that the distance over which and the time in which the suspended mass to be transported were known to them for application of the control scheme.

Moustafa and Ebeid [11] presented a dynamic model of an overhead crane which allows simultaneous travel and transverse motions. They developed an

anti-swing control system to transport an object along a defined path in such a way that the undesired effects due to simultaneous travel and transverse motions (swing in the plane of motion and oscillation of the plane formed by the cables and the vertical axis through the suspension point about an equilibrium position) could be minimized. A linearized state space model about an equilibrium state was derived and used as a basis for designing a feedback control system which specified the input torques to the motor at every instant. Ebeid et al. [31] presented a non-linear electro-mechanical model describing the dynamical behavior of overhead cranes. They represented each of the two driving motors (bridge motor and the trolley motor) by the classical fifth order model of induction machines where electrical transients as well as mechanical transients were considered. In their model the stator voltage was used to drive the motor. Moustafa and Emara-Shabaik [32] developed a feedback control strategy to minimise the swing of the suspended object for the overhead crane model presented in [11] by using pole assignment. They used the singular perturbation technique to obtain a reduced order model. A Proportional-Derivative (PD) controller was designed by them to achieve the goal.

Herteau and Desantis [33] described the design and implementation of a microprocessor-based adaptive controller for a crane system. Their proposal controller was made of a state regulator block and a gain tuning module. The adjustment of the regulator gains as a function of the parameters value was made by means of a modified pole placement criterion. They considered only transverse motion of trolley. Ridout [34] presented a linear feedback anti-swing control system for an overhead crane. The control loop presented by him used

negative feedback of the trolley position and velocity signals, and positive feedback of the cable angle. The linear feed-back strategy was modified by him by implementing variable damping [35]. The modified control strategy using an error measure derived from both the trolley position error and the rope angle error signals. He devised a contour map approach (based on computer simulations) to shape the damping function and to develop a general tuning strategy for the system.

Strip [36] presented a general strategy for swing-free transport of suspended objects. The restriction imposed by Starr's method [12] i.e. the period of acceleration must be small as compared to the period of the object, was removed by his strategy. The author concluded that to get the object stationary at target, the crane must begin to decelerate at the same rate as it was accelerated, starting at a distance from its target equal to the distance travelled while accelerating. Jones and Petterson [37] presented the mathematics describing the oscillation damped trajectories for simply suspended payloads using controlled acceleration. A specific implementation using a CIMCORP XR6100 gantry robot was also described. One of the drawbacks in their method was the requirement of a system that is able to produce a constant acceleration profile because many commercially available programmable cranes and manipulators do not have the ability to generate a constant acceleration profile.

A great deal of research related to the crane automation is being done at Helsinky University of Technology by a number of researchers in collaboration with other research institutes, industries, and government organizations [38-44]. Their basic aim is to achieve a whole set of new control and measuring methods,

whereby the system designer or the operator can choose an appropriate method for his specific problem. For this purpose they are using a pilot scale gantry crane based on the model described by the Ackermann [45]. Among these researchers, Marttinen and Virkkunen [38] analysed the dynamical behavior of the model on the frequency domain. Virkkunen and Marttinen [39] described the details of construction and the instrumentation system of the pilot crane. An open-loop minimum time strategy was used until the load came closer to the target and then a linear quadratic controller was used to bring the load to the target in order to avoid the large control signal changes in the vicinity of the target. Rintanen et al. [40] concentrated on data processing and the requirements for the control application software. They presented the hardware and software implementation based on Intel's Bitbus Boards and discussed some related technical problems from the crane control point of view. Vaha and Marttinen [41] introduced the basic conventional control methods and their possibilities to damp the load swing. To show the characteristics of the Proportional-Integral-Derivative (PID)-type of control strategies they used rootlocus method, which indicated that the performance was not satisfactory. They have briefly presented some suboptimal control strategies. Marttinen [42] applied a pole-placement algorithm for anti-swing control of the pilot gantry. An adaptive control scheme was proposed, which was necessary to take into account the varying cable length. Marttinen et al. [43] described various features related to the velocity-controlled model of the pilot crane for educational experimentation. They considered modelling issues and compared different identification methods. They also studied three control schemes namely: minimum-time control strategy, PID-

controller design with the root locus method [41], and polynomial pole placement and its adaptive applications to only two kinds of motions (hoisting and traversing). Virkkunen et al. [44] surveyed the main results obtained by many researchers from the working on the said pilot crane. In almost all of the above mentioned work at the Helsinky University of Technology, only traverse motion of trolley was considered. Also, they reduced the nonlinear model to the linear one for applying the control schemes. Caron et al. [46] presented a reference model control approach for overhead cranes, taking into account the lengths of the suspension cable. In order to permit fast travel of suspended objects and to minimize the resulting oscillation magnitude, they first used a reference model for a constant cable length and then introduced a linearizing control in the case of a variable length to achieve similar behavior. Two outputs (the swing of the load and the position of the trolley) were controlled by means of one input (the reference speed of the trolley motor).

Yoshida and Kawabe [47] derived a saturating control law, which satisfied a constrained input condition and gave an upper bound of a given quadratic performance index using a guaranteed cost control method. The practical aspects of the scheme were also discussed by an experimental examination in a real crane system. The control law developed by them showed the dependence on initial conditions, therefore, the rule of trial and error using a computer was necessary to optimize the control law. Butler et al. [48] presented a method of reference model decomposition as an extension of model reference adaptive control and applied it to a gantry crane scale model. They considered the crane as a single-input, single-output system with the input voltage at the trolley motor as the

control signal and the position of the load as the feedback information for the controller. Yamada et al. [49] developed a fuzzy controller for the roof crane. They also made a comparison between the fuzzy controller and the suboptimal controller using switching velocity method. They considered only transverse motion of the trolley and used trolley acceleration as the control input.

1.2 OBJECTIVES OF THE PROPOSED RESEARCH

The first objective of the present research is to derive nonlinear models those represent the dynamics of overhead cranes which carries out simultaneous bridge travel, trolley traverse, and load hoisting or lowering motions. In practical situations, these motions induce two types of undesirable effects, namely, swing in plane of motion and oscillations of the plane formed by the rope and the vertical axis through the suspension point about an equilibrium position.

The second objective is to obtain linearized models, by expanding the nonlinear models about nominal operating point using the Taylor series expansion, which can be used in future work to apply the well-developed linear control theories.

The third objective is to develop an anti-swing control scheme for the nonlinear acceleration-control model to minimize the above mentioned swing and oscillations during the transfer process. The load is required to be transferred in minimum time and to arrive at its final destination without any swing and oscillation. The control scheme uses the optimized feedback control methods by selecting a suitable performance measure and considering all the physical constraints on the control and state variables.

Chapter 2

DYNAMICAL MODEL OF THE OVERHEAD CRANE

Consider the model of the overhead crane as shown in Figure 2.1.

For simplicity, assume the following

- The elastic deformability of the crane is negligible and it is assumed that all elements are of infinite stiffness.
- Dissipative effects like rolling resistance and losses in the drive mechanism are negligible.
- Effects of wind forces are negligible.
- The load, assumed to be concentrated at a point, is hanging on a massless cable
- The change of the cable length due to swing of the load is negligible.

The system will have five degrees of freedom namely; travelling motion of the bridge (x), traversing motion of trolley (y), swing in the plane of motion (θ), oscillation of the plane (ψ) determined by the cable and the vertical axis through the suspension point about an equilibrium position, and the length of the cable (l).

The position vector of the point of suspension, O with respect to fixed axes coordinate system is

$$\mathbf{r}_o = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}. \tag{2.1}$$

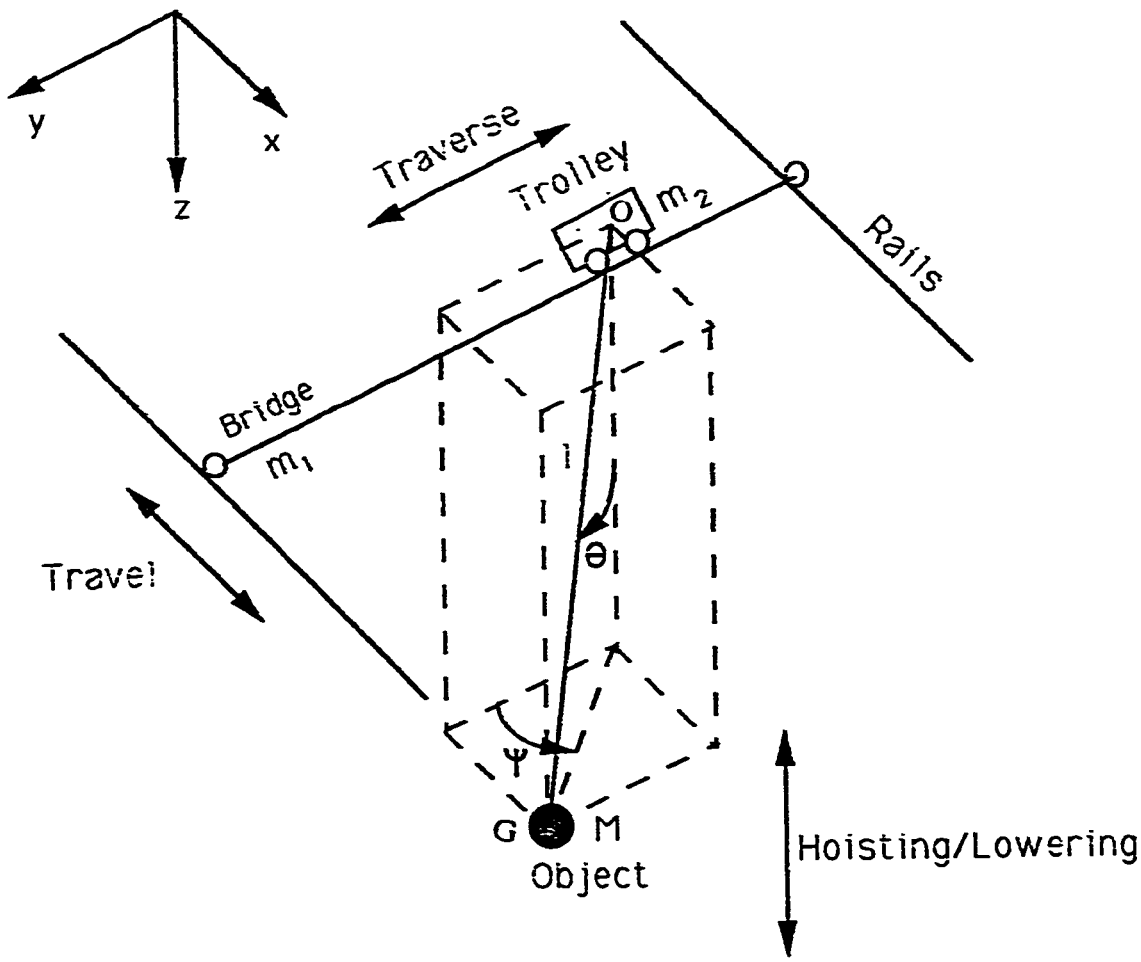


Figure 2.1: Simplified model of the overhead crane.

We can write the position vector of the material point that represents the load, G, by using the moving spherical coordinates θ and ψ to indicate the absolute position of G, as

$$\mathbf{r}_G = x_G\mathbf{i} + y_G\mathbf{j} + z_G\mathbf{k}. \tag{2.2}$$

where,

$$\begin{aligned} x_G &= x + l\sin\theta\sin\psi \\ y_G &= y + l\sin\theta\cos\psi \\ z_G &= z + l\cos\theta \end{aligned} \tag{2.3}$$

The kinetic energy of the system is

$$\begin{aligned} T = {} & \frac{1}{2}M(\dot{x}_G^2 + \dot{y}_G^2 + \dot{z}_G^2) + \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2(\dot{x}^2 + \dot{y}^2) \\ & + \frac{1}{2}I_1(\dot{x}/r_1)^2 + \frac{1}{2}I_2(\dot{y}/r_2)^2 + \frac{1}{2}I_3(\dot{l}/r_3)^2 \end{aligned} \tag{2.4}$$

and the potential energy of the system is

$$V = -Mgl\cos\theta, \tag{2.5}$$

where I_1 is the mass moment of inertia of the bridge motor, I_2 is the mass moment of inertia of the trolley motor, I_3 is the mass moment of inertia of the hoisting motor, l is the length of the cable, M is the mass of the load, m_1 is the mass of the bridge, m_2 is the mass of the trolley, r_1 is the radius of the bridge motor pinion, r_2 is the radius of the trolley motor pinion, and r_3 is the radius of the hoisting drum.

The Lagrangian function is given as

$$L = T - V.$$

Substituting the values of x_G, y_G, and z_G from equations (2.3) and simplifying, we get

$$
\begin{aligned}
L = \frac{1}{2} M\{ & (\dot{x}^2 + \dot{y}^2 + \dot{l}^2) + l^2 \dot{\theta}^2 \\
& + l^2 \dot{\psi}^2 \sin^2\theta + 2\dot{x}\dot{l}\sin\theta \sin\psi \\
& + 2\dot{x} l \dot{\psi} \sin\theta \cos\psi + 2\dot{x} l \dot{\theta} \cos\theta \sin\psi \\
& + 2\dot{y}\dot{l}\sin\theta \cos\psi + 2\dot{y} l \dot{\psi} \sin\theta \sin\psi \\
& + 2\dot{y} l \dot{\theta} \cos\theta \cos\psi \} + \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 (\dot{x}^2 + \dot{y}^2) \\
& + \frac{1}{2} I_1 (\dot{x}/r_1)^2 + \frac{1}{2} I_2 (\dot{y}/r_2)^2 + \frac{1}{2} I_3 (\dot{l}/r_3)^2 \\
& + M g l \cos\theta.
\end{aligned}
$$

The Lagrange's equations of motion are given in Reference [50] as

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = Q_k$$

Q_k and q_k are the vectors of the generalized forces and the generalized coordinate, respectively.

Taking x, y, θ, ψ and l as the generalized coordinates, the equations of motion can be derived as follows:

$$
\begin{aligned}
& \ddot{x} + b_1 \ddot{l} \sin\theta \sin\psi + b_1 l \ddot{\psi} \sin\theta \cos\psi \\
& + b_1 l \ddot{\theta} \cos\theta \sin\psi - b_1 l (\dot{\theta}^2 + \dot{\psi}^2) \sin\theta \sin\psi \\
& + 2 b_1 l \dot{\theta} \dot{\psi} \cos\theta \cos\psi + 2 b_1 \dot{l} \dot{\theta} \cos\theta \sin\psi \\
& + 2 b_1 \dot{l} \dot{\psi} \sin\theta \cos\psi = F_1
\end{aligned}
\tag{2.6}
$$

$$\ddot{y} + b_2 \ddot{l} \sin\theta \cos\psi - b_2 l \ddot{\psi} \sin\theta \sin\psi$$
$$+ b_2 l \ddot{\theta} \cos\theta \cos\psi - b_2 l (\dot{\theta}^2 + \dot{\psi}^2) \sin\theta \cos\psi$$
$$- 2 b_2 l \dot{\theta} \dot{\psi} \cos\theta \sin\psi + 2 b_2 \dot{l} \dot{\theta} \cos\theta \cos\psi$$
$$- 2 b_2 \dot{l} \dot{\psi} \sin\theta \sin\psi = F_2 \tag{2.7}$$

$$l \ddot{\theta} + 2 \dot{l} \dot{\theta} - \frac{1}{2} l \dot{\psi}^2 \sin(2\theta) + g \sin\theta$$
$$+ \ddot{x} \sin\psi \cos\theta + \ddot{y} \cos\psi \cos\theta = 0 \tag{2.8}$$

$$l \sin\theta \ddot{\psi} + 2 \dot{l} \sin\theta \dot{\psi} + 2 l \dot{\psi} \dot{\theta} \cos\theta$$
$$+ \ddot{x} \cos\psi - \ddot{y} \sin\psi = 0 \tag{2.9}$$

$$\ddot{l} - b_3 l \dot{\theta}^2 - b_3 l \dot{\psi}^2 \sin^2\theta - b_3 g \cos\theta$$
$$+ b_3 \ddot{x} \sin\theta \sin\psi + b_3 \ddot{y} \sin\theta \cos\psi = F_3 \tag{2.10}$$

where we have defined

$$F_1 = F_x / M_1$$
$$F_2 = F_y / M_2$$
$$F_3 = F_l / M_3$$
$$F_x = T_x / r_1$$
$$F_y = T_y / r_2$$
$$F_l = T_l / r_3$$
$$M_1 = M + m_1 + m_2 + I_1 / r_1^2$$

$$M_2 = M + m_2 + I_2/r_2^2$$

$$M_3 = M + I_3/r_3^2$$

$$b_1 = M/M_1$$

$$b_2 = M/M_2$$

and

$$b_3 = M/M_3 .$$

T_x, T_y, and T_l are the driving torques generated by the bridge drive motor, the trolley drive motor, and the hoisting motor, respectively: F_1, F_2, F_3, are the respective normalized driving forces.

Equations (2.6) - (2.10) represent the nonlinear dynamical model of the overhead crane in its general form. However, two types of crane models are usually considered for control applications. The first one, which is most common in industry, is known as acceleration control model [43] , and the second is known as torque control model [38]. In the first model, the available control variables are supposed to be the acceleration of the bridge, the trolley, and the winch. Whereas, in the second model the control variables are related to the torques of the electric driving motors of the bridge, the trolley, and the winch. In the second model, the dynamic properties of the crane depend on the mass and the oscillation of the load influences the motion of the trolley.

As far as the industrial applications are concerned, the acceleration control models are preferred in places where accuracy is of prime importance. Also, from the operating point of view, they are easier to control. The

drawback of using the acceleration-control model is that the required instrumentation is very expensive. On the other hand, torque-control models are very complex. Therefore, it is quite difficult to develop a practical control scheme for these models. As compared to acceleration-control model, the instrumentation required for the torque-control model is less expensive [3].

The mathematical formulation of the two models in the state space form is given below.

2.1 ACCELERATION-CONTROL MODEL

Assuming the available control variables to be the accelerations of the bridge, the trolley, and the winch, i.e.,

$$\begin{aligned} u_1 &= \ddot{x} \\ u_2 &= \ddot{y} \\ u_3 &= \ddot{l} \end{aligned} \tag{2.11}$$

Introducing the state variables $x_1, \ldots, x_{10}$ as

$$\begin{aligned} x_1 &= x \\ x_2 &= \dot{x} \\ x_3 &= y \\ x_4 &= \dot{y} \\ x_5 &= \theta \\ x_6 &= \dot{\theta} \\ x_7 &= \psi \end{aligned} \tag{2.12}$$

$$x_8 = \dot{\psi}$$

$$x_9 = l$$

$$x_{10} = \dot{l}$$

Then by definition and from equations (2.8) and (2.9) we get the following state space representation

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = u_1$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = u_2$$

$$\dot{x}_5 = x_6$$

$$\dot{x}_6 = -2\frac{x_{10}}{x_9}x_6 + \frac{1}{2}x_8^2\sin(2x_5) - \frac{g}{x_9}\sin x_5 - \frac{u_1}{x_9}\cos x_5 \sin x_7 - \frac{u_2}{x_9}\cos x_5 \cos x_7$$

$$\dot{x}_7 = x_8 \tag{2.13}$$

$$\dot{x}_8 = -2\frac{x_{10}}{x_9}x_8 - 2x_8 x_6 \cot x_5 - \frac{u_1}{x_9}\frac{\cos x_7}{\sin x_5} + \frac{u_2}{x_9}\frac{\sin x_7}{\sin x_5}$$

$$\dot{x}_9 = x_{10}$$

$$\dot{x}_{10} = u_3$$

The initial conditions are chosen as

$$
\begin{aligned}
x_1(0) &= \ldots = x_4(0) = 0 \\
x_5(0) &= \theta_i \\
x_6(0) &= \dot{\theta}_i \\
x_7(0) &= \psi_i \\
x_8(0) &= \dot{\psi}_i \\
x_9(0) &= l_s \\
x_{10}(0) &= 0
\end{aligned}
\tag{2.14}
$$

Where l_s is the length of the cable at the starting of the operation. θ_i, ψ_i are their initial swing angle, and $\dot{\theta}_i$, $\dot{\psi}_i$ are the initial rates of change. The required final states are

$$
\begin{aligned}
x_1(t_f) &= X \\
x_2(t_f) &= 0 \\
x_3(t_f) &= Y \\
x_4(t_f) &= 0 \\
x_5(t_f) &= 0 \\
x_6(t_f) &= 0 \\
x_7(t_f) &= 0 \\
x_8(t_f) &= 0 \\
x_9(t_f) &= l_f \\
x_{10}(t_f) &= 0
\end{aligned}
\tag{2.15}
$$

Where X, Y are the desired final positions of the bridge and the trolley respectively, and l_f is the length of the cable at the target position.

It can be seen from the above model that the position of the trolley and the load cable length are simple to control, since these parts of the model contain only pure integrations. Nevertheless, the swing of the load and the oscillations of the plane are governed by highly nonlinear equations and a simple analytical solution to the problem is not possible [8].

2.2 TORQUE-CONTROL MODEL

For obtaining the nonlinear torque control model, the nonlinear general model is written as

$$[A]\ddot{X} = [B]$$

or

$$\ddot{X} = [A]^{-1}[B] \tag{2.16}$$

where $\ddot{X} = [\ddot{x}\ \ddot{y}\ \ddot{\theta}\ \ddot{\psi}\ \ddot{l}]^T$ and $[A]$ and $[B]$ matrices are given in Appendix A. The matrix inversion and multiplication is done by using Mathematica [51]. Now assuming the available control to be the torques of the driving motors of the bridge, the trolley, and the winch, i.e.,

$$\begin{aligned} u_1 &= F_1 \\ u_2 &= F_2 \\ u_3 &= F_3 \end{aligned} \tag{2.17}$$

Then by definition and from equation (2.16) we get the following state space representation for the torque model

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f_1(x_5, x_6, x_7, x_8, x_9, x_{10}, u_1, u_2, u_3) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= f_2(x_5, x_6, x_7, x_8, x_9, x_{10}, u_1, u_2, u_3) \\
\dot{x}_5 &= x_6 \\
\dot{x}_6 &= f_3(x_5, x_6, x_7, x_8, x_9, x_{10}, u_1, u_2, u_3) \\
\dot{x}_7 &= x_8 \\
\dot{x}_8 &= f_4(x_5, x_6, x_7, x_8, x_9, x_{10}, u_1, u_2, u_3) \\
\dot{x}_9 &= x_{10} \\
\dot{x}_{10} &= f_5(x_5, x_6, x_7, x_8, x_9, x_{10}, u_1, u_2, u_3)
\end{aligned}
\tag{2.18}
$$

Complete description of $f_1(\cdot), \ldots, f_5(\cdot)$ is given in appendix A. The initial conditions are the same as for the acceleration-control model. The mutual coupling between the load and the trolley can be noticed in this model by observing the influence of the oscillations of the load on the motion of the trolley. The magnitudes of the control variables u_1 u_2 and u_3 and the mass ratios b_1, b_2, and b_3 determine the strength in the coupling.

Chapter 3

LINEARIZED MODELS

Linearization is commonly used to analyze a nonlinear system for small departures from an operating point. Its stability and transient nature can be quantified for comparison with the system requirements. When the perturbations are small enough, their products are almost negligible, so discarding them reduces the system dynamics to linear equations in the perturbed variables [52].

The linearization of the nonlinear dynamical equation is done by expanding them into a Taylor series about nominal operating conditions and neglecting the terms of second and higher orders.

3.1 NOMINAL OPERATING CONDITIONS

Let the nominal operating conditions be denoted by x_o, y_o, l_o, θ_o, and ψ_o. The corresponding nominal operating forces are denoted by F_1^o, F_2^o, and F_3^o. Now the relations for θ_o, ψ_o, F_1^o, F_2^o, and F_3^o in terms of $\dot{x}_o$, $\dot{y}_o$, and $\dot{l}_o$ (which in actual conditions are selected according to the desired transportation route) can be obtained by substituting

$$\ddot{\theta}(t) = \dot{\theta}(t) = 0$$

$$\ddot{\psi}(t) = \dot{\psi}(t) = 0 \tag{3.1}$$

in equations (2.6) - (2.10)

The resulting equations are:

$$\ddot{x}_o + b_1 \ddot{l}_o \theta_o \sin\psi_o = F_1^o \tag{3.2}$$

$$\ddot{y}_o + b_2 \ddot{l}_o \theta_o \cos\psi_o = F_2^o \tag{3.3}$$

$$g\,\theta_o + \ddot{x}_o \sin\psi_o + \ddot{y}_o \cos\psi_o = 0 \tag{3.4}$$

$$\ddot{x}_o \cos\psi_o - \ddot{y}_o \sin\psi_o = 0 \tag{3.5}$$

$$\ddot{l}_o - b_3 g + b_3 \ddot{x}_o \theta_o \sin\psi_o + b_3 \ddot{y}_o \theta_o \cos\psi_o = F_3^o \tag{3.6}$$

Solving these equations simultaneously, we get

$$\theta_o = -\frac{a_o}{g} \tag{3.7}$$

$$\psi_o = \tan^{-1}\left(\frac{\ddot{x}_o}{\ddot{y}_o}\right) \tag{3.8}$$

$$F_1^o = \ddot{x}_o \left(1 - \frac{b_1 \ddot{l}_o}{g}\right) \tag{3.9}$$

$$F_2^o = \ddot{y}_o \left(1 - \frac{b_2 \ddot{l}_o}{g}\right) \tag{3.10}$$

$$F_3^o = \ddot{l}_o - b_3 \left(g + \frac{a_1^{\,2}}{g}\right) \tag{3.11}$$

where

$$a_o = \sqrt{\ddot{x}_o^2 + \ddot{y}_o^2} \tag{3.12}$$

In practice the acceleration of the bridge, the trolley, and the winch are normally known according to the desired transport route. This will be discussed in more detail in this study. The nominal operating values θ_o, ψ_o, F_1^o, F_2^o, and F_3^o, can then be calculated from equations (3.7) - (3.11), respectively.

3.2 LINEARIZED GENERAL MODEL

Let the small perturbation in a function x(t) be denoted by

$$\delta x(t) = x(t) - x_o. \tag{3.13}$$

Then by expanding the nonlinear equations (2.6) - (2.10) in Taylor series about the nominal operating conditions, neglecting the higher order terms (order of two and more) and taking $\sin\theta_o = \theta_o$ and $\cos\theta_o = 1$ (as θ_o is small), the linearized equations of motion can be written as

$$\begin{aligned}
&\delta\ddot{x} + b_1\theta_o\sin\psi_o\delta\ddot{l} + b_1 l_o\theta_o\cos\psi_o\delta\ddot{\psi} \\
&\quad + b_1 l_o\sin\psi_o\delta\ddot{\theta} + 2b_1\dot{l}_o\sin\psi_o\delta\dot{\theta} \\
&\quad + 2b_1\dot{l}_o\theta_o\cos\psi_o\delta\dot{\psi} = \delta F_1
\end{aligned} \tag{3.14}$$

$$\begin{aligned}
&\delta\ddot{y} + b_2\theta_o\cos\psi_o\delta\ddot{l} - b_2 l_o\theta_o\sin\psi_o\delta\ddot{\psi} \\
&\quad + b_2 l_o\cos\psi_o\delta\ddot{\theta} + 2b_2\dot{l}_o\cos\psi_o\delta\dot{\theta} \\
&\quad + 2b_2\dot{l}_o\theta_o\sin\psi_o\delta\dot{\psi} = \delta F_2
\end{aligned} \tag{3.15}$$

$$\begin{aligned}
&l_o\delta\ddot{\theta} + 2\dot{l}_o\,\delta\dot{\theta} + g\,\delta\theta \\
&\quad + \sin\psi_o\,\delta\ddot{x} + \cos\psi_o\,\delta\ddot{y} = 0
\end{aligned} \tag{3.16}$$

$$\begin{aligned}
&l_o\theta_o\delta\ddot{\psi} + 2\dot{l}_o\theta_o\,\delta\dot{\psi} + g\theta_o\,\delta\theta \\
&\quad + \cos\psi_o\,\delta\ddot{x} - \sin\psi_o\,\delta\ddot{y} = 0
\end{aligned} \tag{3.17}$$

$$\delta\ddot{l} + b_3\theta_o\sin\psi_o\,\delta\ddot{x} + b_3\theta_o\cos\psi_o\,\delta\ddot{y} = \delta F_3 \tag{3.18}$$

In the following subsections, the linearized acceleration- and torque-control models are derived from the linearized general model described by equations (3.14) - (3.18).

3.3 LINEARIZED ACCELERATION-CONTROL MODEL

By replacing the control and state variables of equations (2.11) and (2.12) by their perturbed values about the nominal operating conditions, we can obtain, by using equations (3.16) and (3.17) , the linearized acceleration control model as

$$
\begin{aligned}
\delta\dot{x}_1 &= \delta x_2 \\
\delta\dot{x}_2 &= \delta u_1 \\
\delta\dot{x}_3 &= \delta x_4 \\
\delta\dot{x}_4 &= \delta u_2 \\
\delta\dot{x}_5 &= \delta x_6 \\
\delta\dot{x}_6 &= -2\frac{\dot{l}_o}{l_o}\delta x_6 - \frac{g}{l_o}\delta x_5 - \frac{\sin\psi_o}{l_o}\delta u_1 - \frac{\cos\psi_o}{l_o}\delta u_2 \\
\delta\dot{x}_7 &= \delta x_8 \\
\delta\dot{x}_8 &= -\frac{g}{l_o}\delta x_5 - 2\frac{\dot{l}_o}{l_o}\delta x_8 - \frac{\cos\psi_o}{l_o\theta_o}\delta u_1 + \frac{\sin\psi_o}{l_o\theta_o}\delta u_2 \\
\delta\dot{x}_9 &= \delta x_{10} \\
\delta\dot{x}_{10} &= \delta u_3
\end{aligned}
\tag{3.19}
$$

the initial conditions are

$$\delta x_1(0) = \ldots = \delta x_4(0) = 0$$

$$\delta x_5(0) = \delta\theta_i$$

$$\delta x_6(0) = \delta\dot{\theta}_i$$

$$\delta x_7(0) = \delta\psi_i$$

$$\delta x_8(0) = \delta\dot{\psi}_i$$

$$\delta x_9(0) = 0$$

$$\delta x_{10}(0) = 0$$

Similar to the nonlinear acceleration control model, the position of the trolley and the load cable are simple to control. Moreover, the swing angles of the load are not as complex as in nonlinear acceleration-control model. It should be noted that the perturbation notation, δ, appearing in equation (3.19) will be dropped in later development if there is no cause for confusion.

3.4 LINEARIZED TORQUE-CONTROL MODEL

For obtaining the linearized torque-control model, the linearized general model is written as

$$[C]\,\delta\ddot{X} = [E]\,\delta\dot{X} + [F]\,\delta X + [B]\,\delta F$$

or

$$\delta\ddot{X} = [C]^{-1}[E]\,\delta\dot{X} + [C]^{-1}[F]\,\delta X + [C]^{-1}[K]\,\delta F \tag{3.20}$$

where

$$\delta\ddot{X} = [\,\delta\ddot{x}\ \ \delta\ddot{y}\ \ \delta\ddot{\theta}\ \ \delta\ddot{\psi}\ \ \delta\ddot{l}\,]^T$$

$$\delta\dot{X} = [\,\delta\dot{x}\ \ \delta\dot{y}\ \ \delta\dot{\theta}\ \ \delta\dot{\psi}\ \ \delta\dot{l}\,]^T$$

$$\delta X = [\,\delta x\ \ \delta y\ \ \delta\theta\ \ \delta\psi\ \ \delta l\,]^T$$

$$\delta F = [\,\delta F_1\ \ \delta F_2\ \ 0\ \ 0\ \ \delta F_3\,]^T$$

and the matrices $[C]$, $[E]$, $[F]$, and $[K]$, are given in Appendix B. The matrix inversion and multiplication is done by using Mathematica [51]. Now assuming the available controls as

$$\delta u_1 = \delta F_1$$

$$\delta u_2 = \delta F_2$$

$$\delta u_3 = \delta F_3$$

and by using the equation (3.20) and the same definitions of states variables as used in previous section, the state space representation of the linearized torque-control model is given as

$$\begin{aligned}
\delta\dot{x}_1 &= \delta x_2 \\
\delta\dot{x}_2 &= d_1\delta x_5 + d_2\delta x_7 + d_3\delta x_6 + d_4\delta x_8 + e_1\delta u_1 + e_2\delta u_2 + e_3\delta u_3 \\
\delta\dot{x}_3 &= \delta x_4 \\
\delta\dot{x}_4 &= d_5\delta x_5 + d_6\delta x_7 + d_7\delta x_6 + d_8\delta x_8 + e_4\delta u_1 + e_5\delta u_2 + e_6\delta u_3 \\
\delta\dot{x}_5 &= \delta x_6 \\
\delta\dot{x}_6 &= d_9\delta x_5 + d_{10}\delta x_7 + d_{11}\delta x_6 + d_{12}\delta x_8 + e_7\delta u_1 + e_8\delta u_2 + e_9\delta u_3 \\
\delta\dot{x}_7 &= \delta x_8 \\
\delta\dot{x}_8 &= d_{13}\delta x_5 + d_{14}\delta x_7 + d_{15}\delta x_6 + d_{16}\delta x_8 + e_{10}\delta u_1 + e_{11}\delta u_2 + e_{12}\delta u_3 \\
\delta\dot{x}_9 &= \delta x_{10} \\
\delta\dot{x}_{10} &= d_{17}\delta x_5 + d_{18}\delta x_7 + d_{19}\delta x_6 + d_{20}\delta x_8 + e_{13}\delta u_1 + e_{14}\delta u_2 + e_{15}\delta u_3
\end{aligned} \tag{3.21}$$

where the coefficients $d_1, \ldots, d_{20}$ and $e_1, \ldots, e_{15}$ are composed of parameters such as $b_1, b_2, b_3, l_o, \dot{l}_o, \ddot{l}_o, \theta_o$, and ψ_o. The complete description of these coefficients is given in Appendix B. In later developments, the perturbation notation, δ, will be dropped from the linearized model if there is no cause for confusion.

Although, the linearized torque-control model presented above is not as complex as the nonlinear torque-control model presented in chapter 2, it is still quite difficult to develop a suitable optimized control scheme for this model.

Chapter 4

CONTROL SCHEME

It has been stated earlier that the second objective of this study is to develop a control scheme for transporting an object from some initial position to a desired position with minimum oscillations, in minimum time. In other words, the control problem under consideration consists of finding the controls u_1, u_2, and u_3, which transfer the dynamical system from the initial state (2.14) to the final state (2.15), while a given objective function is minimized and a number of constraints both on control and state variables are satisfied. In order to solve this problem, a numerical technique, based on an optimization method, is used to provide the optimized feedback laws for the swing control.

Before going on to determine the optimized feedback control scheme a discussion on why the knowledge of the optimized feedback control vector $\mathbf{u}^*(\mathbf{x}, t)$, where $\mathbf{x}$ is the state variable vector, is desired instead of the open-loop control vector $\mathbf{u}^*(t)$ [53].

The open-loop control vector $\mathbf{u}^*(t)$ is determined as the optimized control to take the system between specified initial and terminal states. Now, there are problems of engineering interest, for example, regulator problems, for which the possible range of initial conditions is very large. To provide optimized system response over the set of initial conditions that might be encountered using the open-loop control solution would require determining $\mathbf{u}^*(t)$ for each possible ini-

tial state. This obviously can be impractical. By contrast, if the feedback control $\mathbf{u}^*(\mathbf{x}, t)$ is known, the optimized control is determined at any time by knowledge of the current state $\mathbf{x}$.

At the other extreme, consider a problem in which only one initial state is to be expected. Here, the open-loop solution $\mathbf{u}^*(t)$ would appear to have more meaning. However, even in this case, feedback information is required to overcome the effects of errors and disturbances which change the system response from the nominal trajectory $\mathbf{x}(t)$. If $\mathbf{u}^*(t)$ is followed without correction in the presence of these disturbances, the resulting trajectory cannot be expected to be either optimized or to satisfy the terminal conditions. Again, if the feedback control $\mathbf{u}^*(\mathbf{x}, t)$ is known, this difficulty would not occur. Knowledge of the current state $\mathbf{x}$ and the time t would suffice to determine the optimized control without reference to whether the state $\mathbf{x}(t)$ resulted from a disturbance from the open-loop optimized trajectory

A control scheme is acceptable only in the case when it provides the optimized feedback laws, while minimizing a given performance index and satisfying certain constraints. In the following sections a detailed description about the performance Index and the control- and state-variable-constraints is given.

4.1 PERFORMANCE INDEX

Classical design techniques have been successfully applied to linear, time-invariant, single-input single-output systems with zero initial conditions. Typical performance criteria are system response to a step or ramp input - characterized by rise time, settling time, peak overshoot, and steady-state accuracy,as well as

the frequency response of the system which is characterized by gain and phase margins, peak amplitude, and bandwidth. Classical techniques have proved to be successful in many applications; however for complex systems performance objectives can not be described in classical terms.

4.1.1 PERFORMANCE MEASURES FOR CONTROL OPTIMIZATION PROBLEMS

The "control optimization problem" is to find a control $\mathbf{u}^* \in U$ which causes the system

$$\dot{\mathbf{x}}(t) = \mathbf{a}(\mathbf{x}(t), \mathbf{u}(t), t) \tag{4.1}$$

to follow a trajectory $\mathbf{x}^* \in X$ that minimizes the performance measure

$$J = h(\mathbf{x}(t_f), t_f) + \int_{t_o}^{t_f} g(\mathbf{x}(t), \mathbf{u}(t), t)\, dt \tag{4.2}$$

Some typical control problems are discussed below to provide some physical motivation for the selection of a performance measure [54-56].

MINIMUM-TIME PROBLEMS

The problem is to transfer a system from an arbitrary initial state $\mathbf{x}(t_o) = x_o$ to specified target set S in minimum time.

The performance measure to be minimized is

$$\begin{aligned} J &= t_f - t_o \\ &= \int_{t_o}^{t_f} dt \end{aligned} \tag{4.3}$$

with t_f the first instant of time when $\mathbf{x}(t)$ and S intersect.

TERMINAL CONTROL PROBLEMS

The problem is to minimize the deviation of the final state of a system from its desired value $\mathbf{r}(t_f)$.

A possible performance measure is

$$J = \sum_{i=1}^{n} [x_i(t_f) - r_i(t_f)]^2 . \tag{4.4}$$

Since positive and negative deviations are equally undesirable, the error is squared. Absolute values could also be used, but the quadratic form in the above equation is easier to handle mathematically. Using matrix notation, we have

$$J = [\mathbf{x}(t_f) - \mathbf{r}(t_f)]^T [\mathbf{x}(t_f) - \mathbf{r}(t_f)] , \tag{4.5}$$

or this can be written as

$$J = \|\mathbf{x}(t_f) - \mathbf{r}(t_f)\|^2$$

where $\|\mathbf{x}(t_f) - \mathbf{r}(t_f)\|$ is the norm of the vector $[\mathbf{x}(t_f) - \mathbf{r}(t_f)]$. A real symmetric positive semi-definite (n x n) weighting matrix **H** can be inserted, to allow greater generality, as

$$J = [\mathbf{x}(t_f) - \mathbf{r}(t_f)]^T \mathbf{H} [\mathbf{x}(t_f) - \mathbf{r}(t_f)] . \tag{4.6}$$

This quadratic form can also be written as

$$J = \|\mathbf{x}(t_f) - \mathbf{r}(t_f)\|^2_{\mathbf{H}} .$$

Suppose that **H** is a diagonal matrix. The assumption that **H** is positive semi-definite implies that all of the diagonal elements are nonnegative. By

adjusting the element values, the relative importance of the deviation of each of the states from their desired values can be weighted. Thus, by increasing h_{ii}, the ith element of the matrix **H**, we attach more significance to deviation of $x_i(t_f)$ from its desired value; by making h_{ii} zero, it is indicated that the final value of x_i is of no concern whatsoever. The elements of **H** should also be adjusted to normalize the numerical values encountered.

TRACKING PROBLEMS

The problem is to maintain the system state $\mathbf{x}(t)$ as close as possible to the desired state $\mathbf{r}(t)$ in the interval $[t_o, t_f]$

As a performance measure we select

$$J = \int_{t_o}^{t_f} \|\mathbf{x}(t) - \mathbf{r}(t)\|^2_{\mathbf{R}(t)} dt, \tag{4.7}$$

where $\mathbf{R}(t)$ is a real symmetric (n x n) matrix that is positive semi-definite for all $t \in [t_o, t_f]$. The elements of the matrix **R** are selected to weigh the relative importance of the different components of the state vector and to normalize the numerical values of the deviations. For example, if **R** is a constant diagonal matrix and q_{ii} is zero, this indicates that deviations of x_i are of no concern.

4.1.2 SELECTING A PERFORMANCE MEASURE

In selecting a performance measure the designer attempts to define a mathematical expression which when minimized indicates that the system is performing in the most desirable manner. Thus, choosing a performance measure implies a representation of the system's physical requirements in the form of mathematical terms. If the performance measure truly reflects desired system performance, the trajectory selected by the designer should yield a smaller value

of **J** . In the sequel, u^* will be referred to as an optimized control and x^* as an optimized trajectory.

In the present study, the purpose of the control scheme is to transport an object from an initial position to a target one under the following conditions :

a) minimizing the load swings and the oscillations at the final time:

b) minimizing the load swings and the oscillations in the transfer process

c) making the transfer time as short as possible.

Therefore, a performance index which takes into account all the above stated three problems, i.e. the terminal control problem, tracking problem, and the minimum-time problem is used. This is given as

$$\mathbf{J} = \frac{1}{2}\left[\left\{\mathbf{x}(t_f) - \mathbf{x}^f\right\}^T \mathbf{H}\left\{\mathbf{x}(t_f) - \mathbf{x}^f\right\} + \int_{t_o}^{t_f} \left\{ 1 + \mathbf{x}(t)^T \mathbf{R}\mathbf{x}(t)\right\} dt \right] \tag{4.8}$$

where $(\,.\,)^T$ means the transpose of (.), $\mathbf{x}^f$ is the objective state vector. **H** and **R** are the weighting matrices.

As the swing angles and their rate of change are of prime and equal importance, in the present study **R** and **H** are chosen as diagonal matrices, with elements $h_{ii} = r_{ii} = 1$, for i = 5, 6, 7, 8, and zero otherwise. The resulting Performance Index is thus given as

$$\mathbf{J} = \frac{1}{2}\left[x_5^2(t_f) + x_6^2(t_f) + x_7^2(t_f) + x_8^2(t_f) + \int_{t_o}^{t_f} \left\{ 1 + x_5^2(t) + x_6^2(t) + x_7^2(t) + x_8^2(t)\right\} dt \right]$$

4.2 CONTROL- AND STATE VARIABLES-CONSTRAINTS

Due to the limited effects of the driving motors, the control variables as well as the bridge, trolley, and hoisting velocities are bounded. These constraints, in general, depend on the characteristics of the electric motors. In this study we have, for simplicity, assumed that the constraints are simple magnitude limits.

4.2.1 CONTROL-CONSTRAINTS

Since the torque of the electrical driving motors is limited, it is natural to assume that the magnitude of the accelerations are bounded. In this case it is assumed that

$$u_{1min} \le u_1 \le u_{1max}$$

$$u_{2min} \le u_2 \le u_{2max}$$

$$u_{3min} \le u_3 \le u_{3max}.$$

4.2.2 STATE VARIABLES-CONSTRAINTS

The velocities of the bridge, the trolley and the winch are bounded due to practical limitations. It is assumed that these variables, respectively, satisfy the following constraints

$$\dot{x}_{min} \le x_2 \le \dot{x}_{max}$$

$$\dot{y}_{min} \le x_4 \le \dot{y}_{max}$$

$$\dot{l}_{min} \le x_{10} \le \dot{l}_{max}$$

for all t such that $0 \le t \le t_f$.

The terminal constraints on state variables are

$$x_1(t_f) = X$$

$$x_2(t_f) = 0$$

$$x_3(t_f) = Y$$

$$x_4(t_f) = 0$$

$$x_5(t_f) = 0$$

$$x_6(t_f) = 0$$

$$x_7(t_f) = 0$$

$$x_8(t_f) = 0$$

$$x_9(t_f) = l_f$$

$$x_{10}(t_f) = 0$$

where X, Y, and l_f are the final positions of the bridge, the trolley, and the load, respectively.

4.3 SIMULATOR AND OPTIMIZATION PROCEDURE

In order to find the control law, which minimizes the given performance index J and satisfies all the constraints, extensive numerical computation was done by using a simulation and optimization package. The package uses Gear method in simulator [57] and Feasible Sequential Quadratic Programming (FSQP) in the optimization which is based on the routines developed by Zhou and Tits [58]. The package was developed by Al-Garni and Nizami at KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS, DHAHRAN [59].

The package is self-contained and has both the simulator and the optimization routines, clubbed together through an interface code which can provide the control law and the other parameters It is capable of solving stiff differential

equation systems without committing a single step error larger than the specified value. It has variable step size control, so as to minimize computer time. The optimization part is a set of FORTRAN subroutines for the minimization of the maximum of a set of smooth objective functions subject to nonlinear equality and inequality constraints, linear equality and inequality constraints, and simple bound on the variables.

The algorithm used in this package is based on Sequential Quadratic Programming (SQP) iteration, modified so as to generate feasible iterates. The merit function is the objective function. An Armijo- type line search is used to generate an initial feasible point when required. After obtaining feasibility, either (i) an Armijo-type line search may be used, yielding a monotone decrease of the objective function at each iteration: or (ii) a nonmonotone line search may be selected, forcing a decrease of the objective function within at most four iterations. In the monotone line search scheme, the SQP direction is first "tilted" if nonlinear constraints are present to yield a feasible direction, then possibly "bent" to ensure that close to a solution the step of one is accepted, a requirement for superliner convergence. The nonmonotone line search scheme achieves a superlinear convergence with no bending of the search direction thus avoiding functions evaluations at auxiliary points and subsequent solution of an additional quadratic program.

The controls u_1, u_2, and u_3 are formulated as a combination of the state variables x_5 and x_7. These state variables are selected for feedback because they can be easily measured and also their values are required at every moment by the control scheme so that the corresponding control actions can be taken.

Mathematically, the controls are represented as

$$u_1 = c_1 a_1(t) + c_2 x_5(t)$$

$$u_2 = c_3 a_2(t) + c_4 x_7(t)$$

$$u_3 = c_5 a_3(t)$$

where a_1, a_2, and a_3 are some suitably selected control inputs for travelling, traversing and hoisting motions and the coefficients $c_1, c_2, \ldots, c_5$ are to be calculated by the optimization program with the satisfaction of the constraints.

This form of control law, i.e. linear with time-invariant optimization coefficients ($c_1, c_2, \ldots, c_5$), is chosen to achieve the desired objectives without causing any further complications. However, if the required results are not achieved, more involved forms of nonlinear control law can be used.

Chapter 5

RESULTS AND DISCUSSION

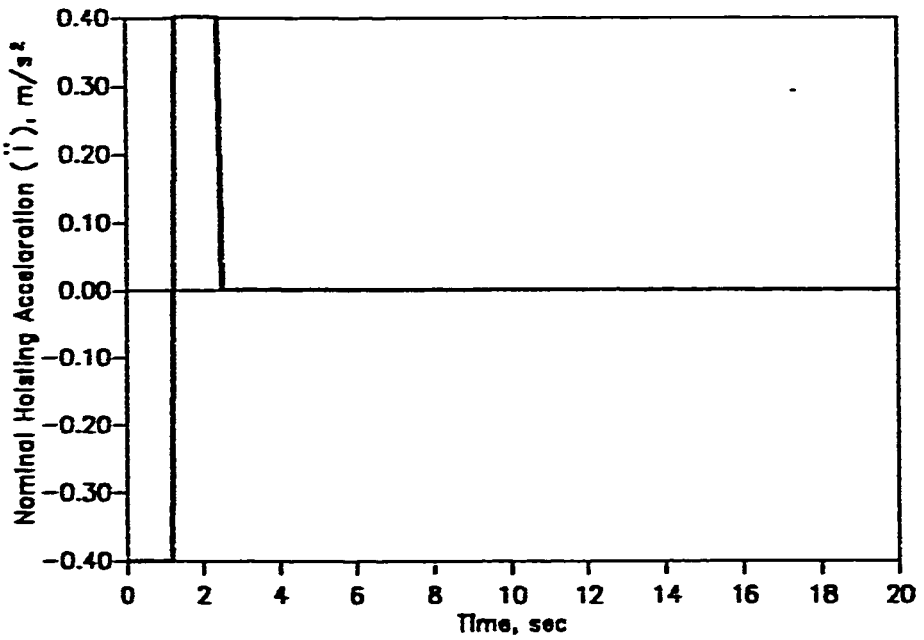
The performance of the overhead crane is numerically simulated by using the nonlinear models developed in chapter 2. The simulations are carried out for a typical overhead crane for the following four different cases.

1. Nonlinear acceleration-control model (a simple case)
2. Nonlinear acceleration-control model (complete transport route)
3. Nonlinear torque-control model (complete transport route)
4. Nonlinear acceleration-control model (optimized control scheme)

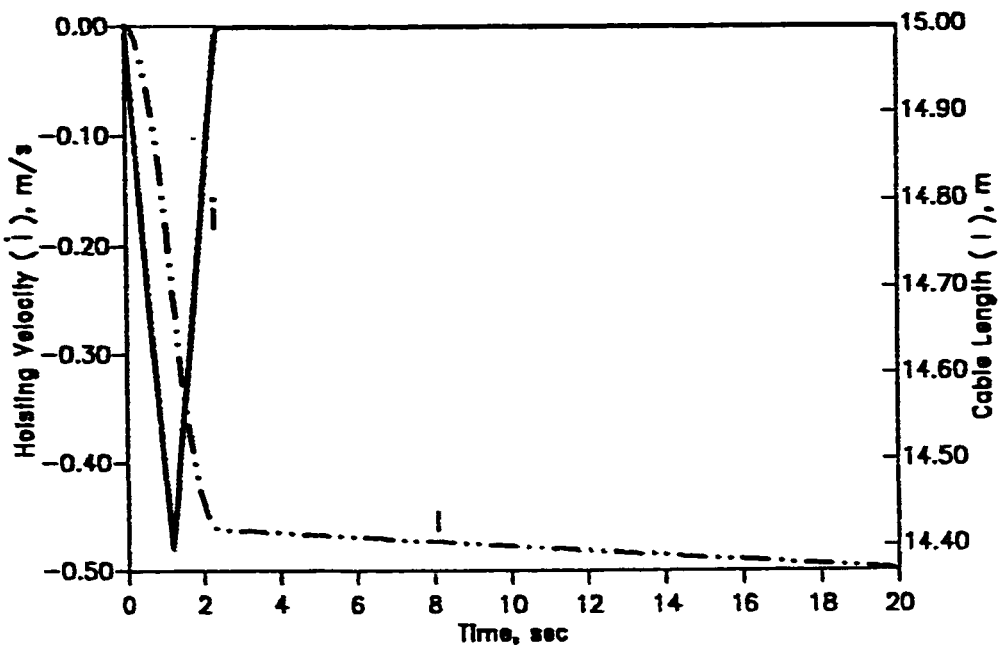
The first three cases are simulated by applying nominal control inputs. Whereas in the fourth case, optimization results are simulated.

5.1 CASE 1

In this case, simulations are carried out to show the effects of the simultaneous hoisting and trolley traversing motions on the load trajectory for an acceleration control model. The travelling of the bridge is not taken into account and therefore there is no oscillation i.e., $\psi = \dot{\psi} = 0$ throughout the crane motion. The initial length of the rope is taken as 15m and all other initial conditions are assumed to be zero. Figures 5.1 and 5.2 show the motions of the crane for this case, and are described in the following steps: (these steps are commonly used in practical applications)

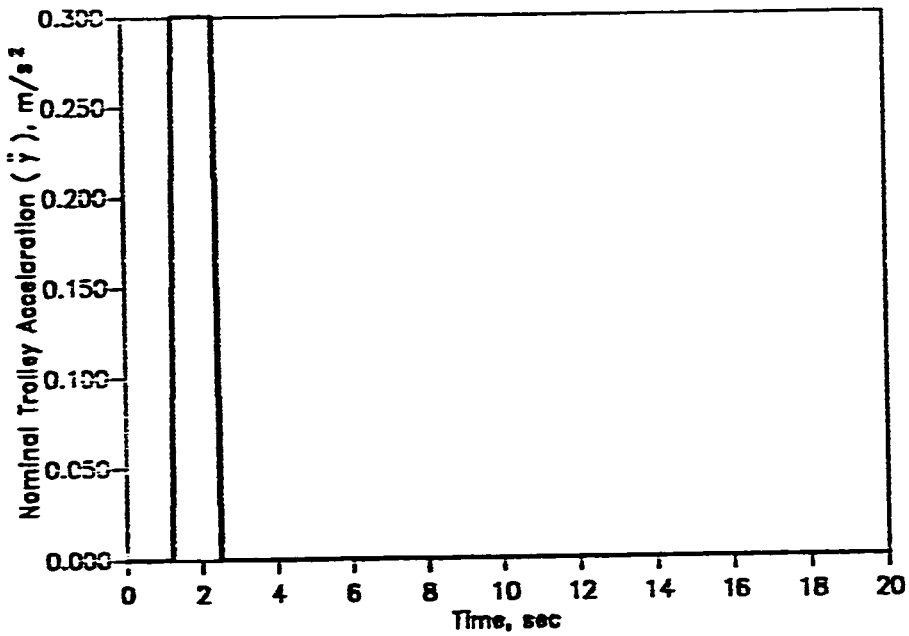


(a)

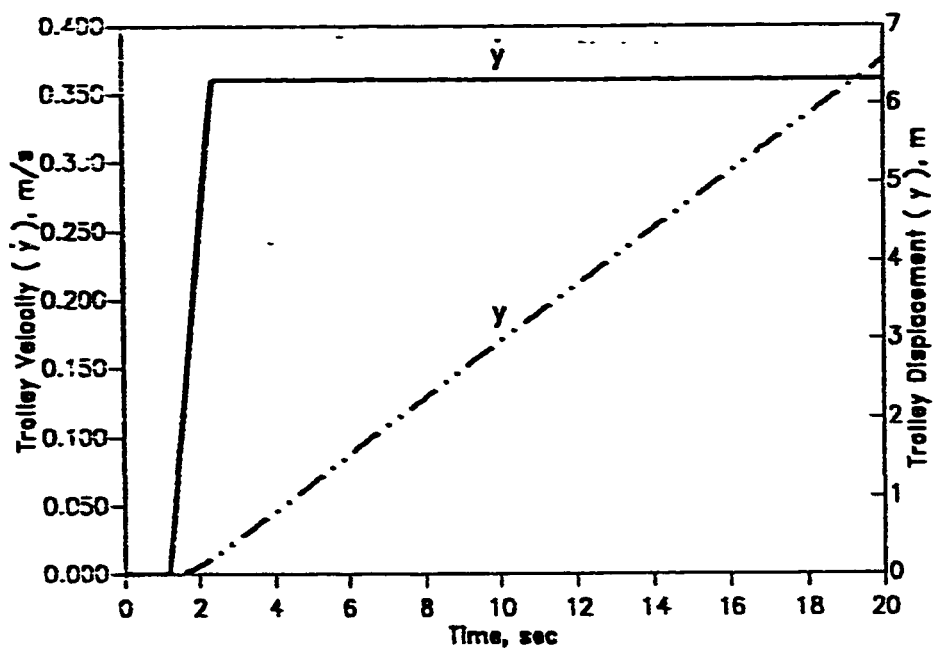


(b)

Figure 5.1: Hoisting Motion - Case 1.
(a) Nominal Hoisting Acceleration
(b) Hoisting Velocity and Cable Length



(a)



(b)

Figure 5.2: Trolley Motion - Case 1.
(a) Nominal Trolley Acceleration
(b) Trolley Velocity and Displacement

1- hoisting of the load with constant acceleration of magnitude 0.4 m/s^2 for a time interval of 0 - 1.2 s in order to achieve a velocity of 0.48 m/s.

2- hoisting with a constant deceleration of magnitude 0.4 m/s^2 and trolley traversing with a constant acceleration of magnitude 0.3 m/s^2, simultaneously, up to a point where the hoisting speed becomes zero and the trolley acquires a traversing speed of 0.36 m/s within a time interval of 1.2 - 2.4 s.

3- trolley traversing with a constant speed of 0.36 m/s without load hoisting for a time interval of 2.4 - 20.0 s

It can be observed from Fig. 5.3 that during step 1 swing of the load does not take place, which agrees with the intuition that the hoisting motion alone does not produce any swing θ. During step 2, swing starts due to the acceleration of the trolley and continues with the same magnitude throughout the step 3.

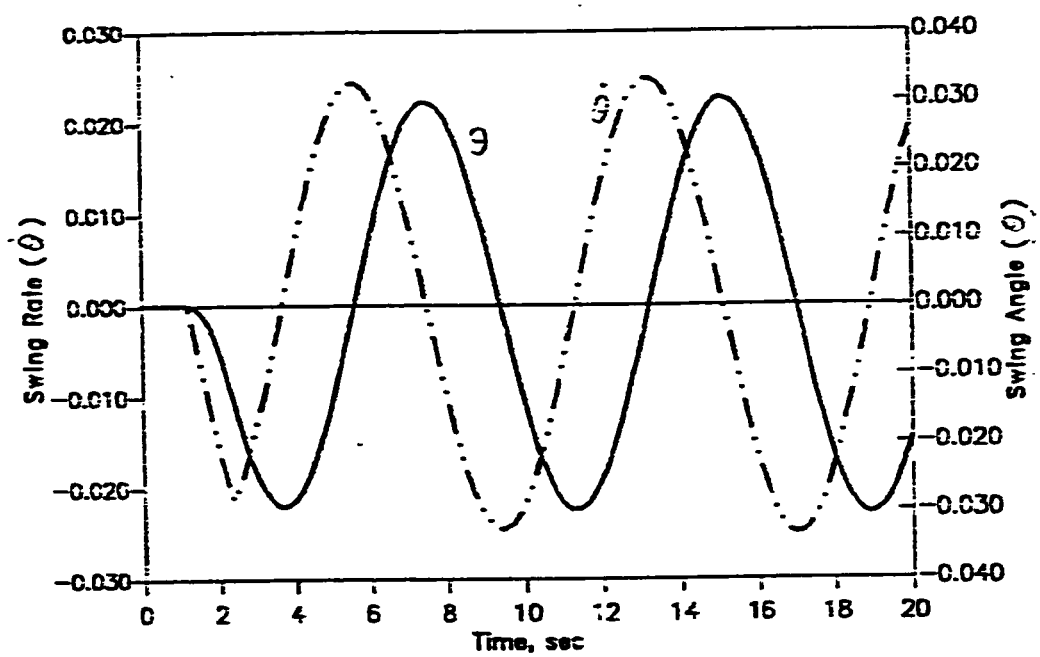


Figure 5.3: Swing Angle and Swing Rate -Case I

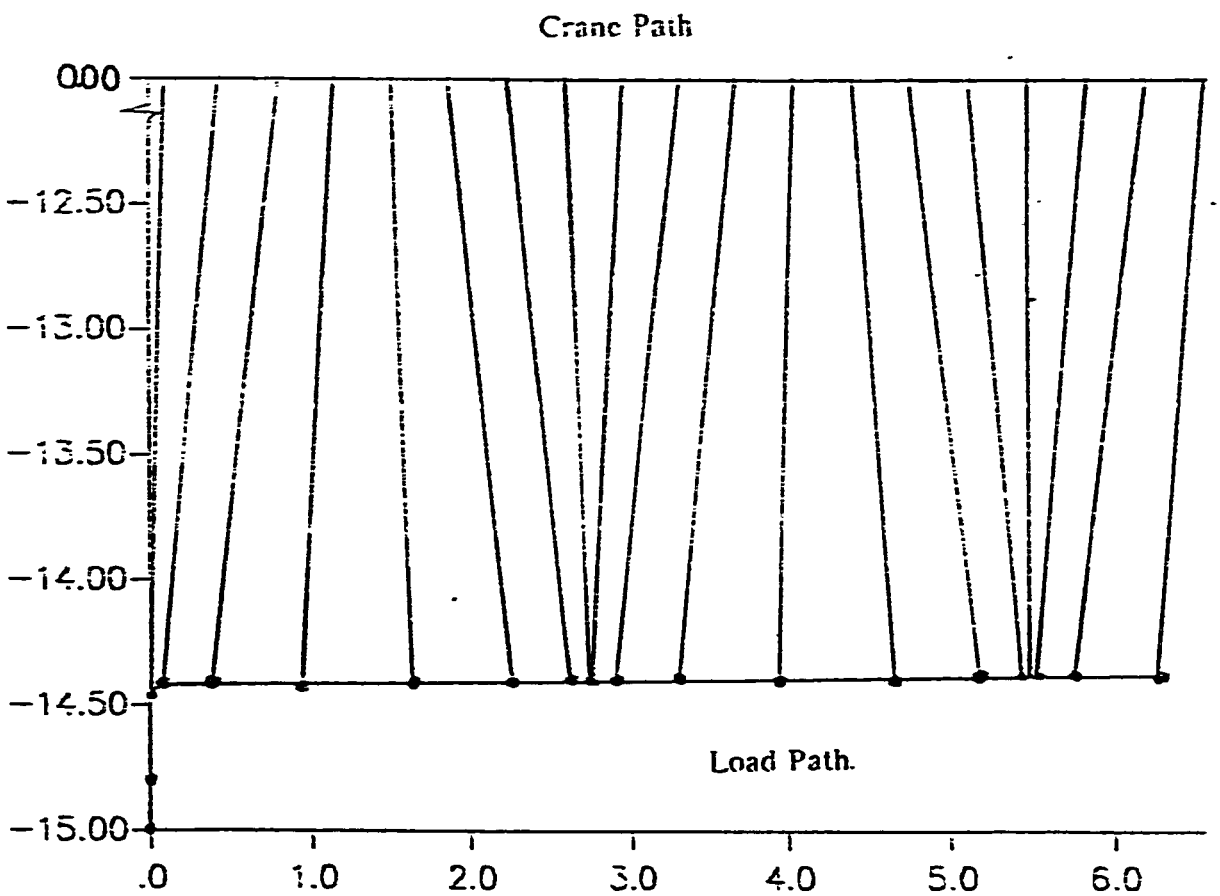


Figure 5.4: Crane and Load Paths - Case 1

5.2 CASE 2

In this case a complete transport route of an acceleration control model is simulated by applying the nominal control inputs where the three crane motion, namely travelling, traversing, and hoisting occur simultaneously. It should be noted that case 1 considers only crane traversing and hoisting of the load to show the effect of acceleration on load swing. The complete transport route can be described in the following steps, as shown in Figs. 5.5 to 5.7.

1- hoisting of the load with constant acceleration of magnitude 0.4 m/s^2 for a time interval of 0 - 1.2 s in order to achieve a velocity of 0.48 m/s.

2- hoisting with a constant deceleration of magnitude 0.4 m/s^2 and trolley traversing with a constant acceleration of magnitude 0.3 m/s^2, simultaneously, up to a point where the hoisting speed becomes zero and the trolley acquires a traversing speed of 0.36 m/s within a time interval of 1.2 - 2.4 s.

3- simultaneous travelling of bridge with a constant acceleration and the traversing of the trolley with uniform velocity for a time interval of 2.4 - 6.0 s which make the velocity of the bridge reaches to 1.08 m/s, while the velocity of the trolley remains the same i.e. 0.36 m/s.

4- simultaneous travelling with uniform speed of 1.08 m/s and traversing with a constant speed of 0.36 m/s for a time interval of 6 - 7 s.

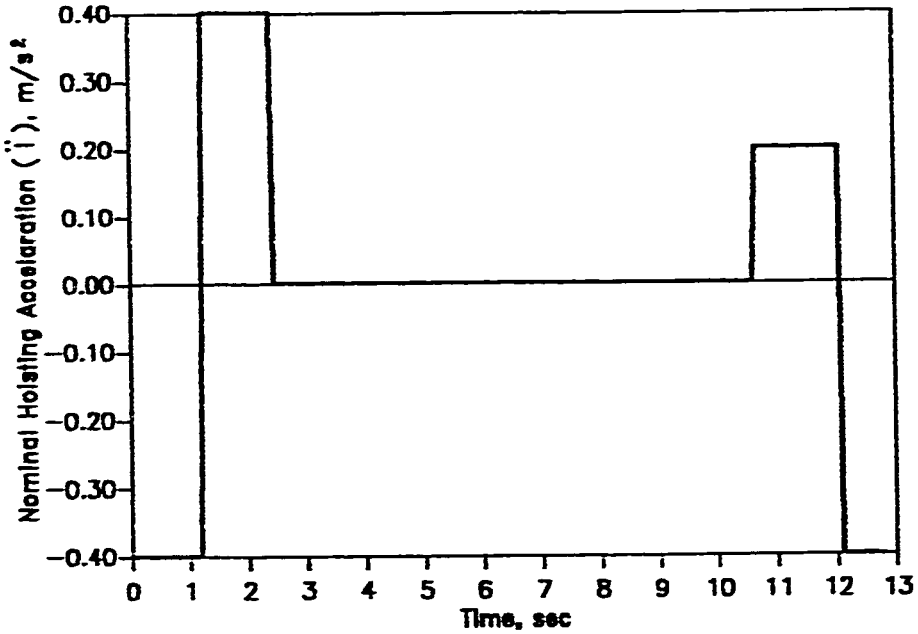
5- simultaneous travelling with constant deceleration of magnitude 0.3 m/s^2 and traversing with a uniform speed of 0.36 m/s, which makes the travelling speed to become zero within a time interval of 7 - 10.6 s.

6- simultaneous lowering with constant acceleration of magnitude 0.2 m/s^2 and traversing with constant deceleration of magnitude 0.25 m/s^2, which

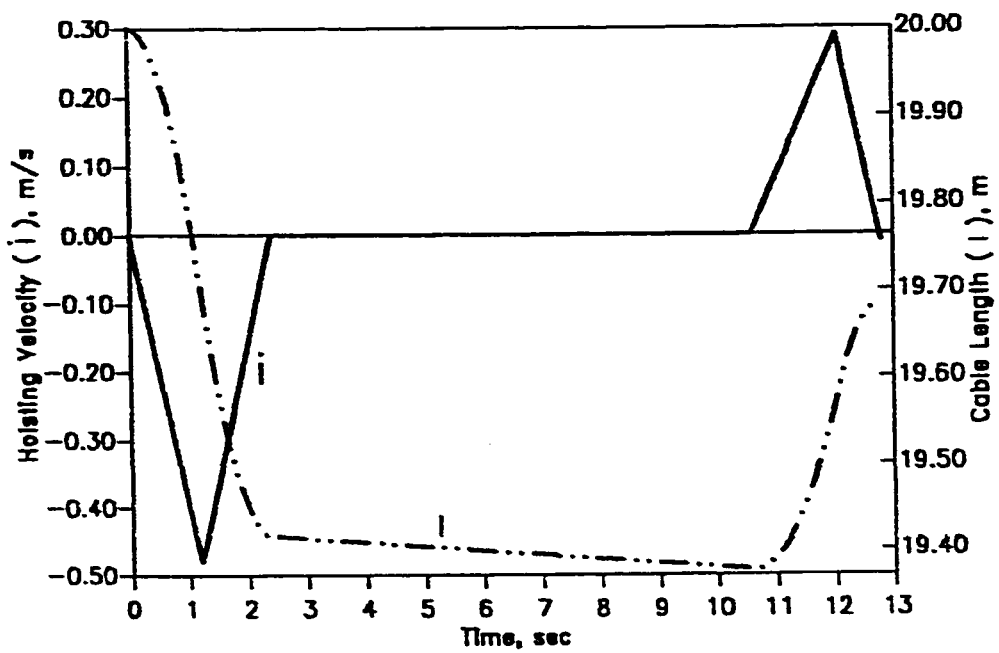
make the hoisting speed equal to 0.29 m/s and traversing speed equal to zero within a time interval of 10.6 - 12.05 s.

7- lowering of the load at a constant deceleration of magnitude 0.4 m/s^2 for a time interval of 12.05 - 12.8s in order to acquire vertical load velocity equals to zero.

It can be noticed from the simulation results, Figs. 5.5 - 5.8 that the simultaneous bridge and trolley motion produces the swings θ and the oscillations ψ. The oscillations increase at a high rate and particularly, in part 5 of the motion, where the values are very large. This shows the necessity of controlling the crane motion to kill the load swing during transport as well as at the final destination. This results in safe transportation of the load, more effective operation and greater handling capacity.

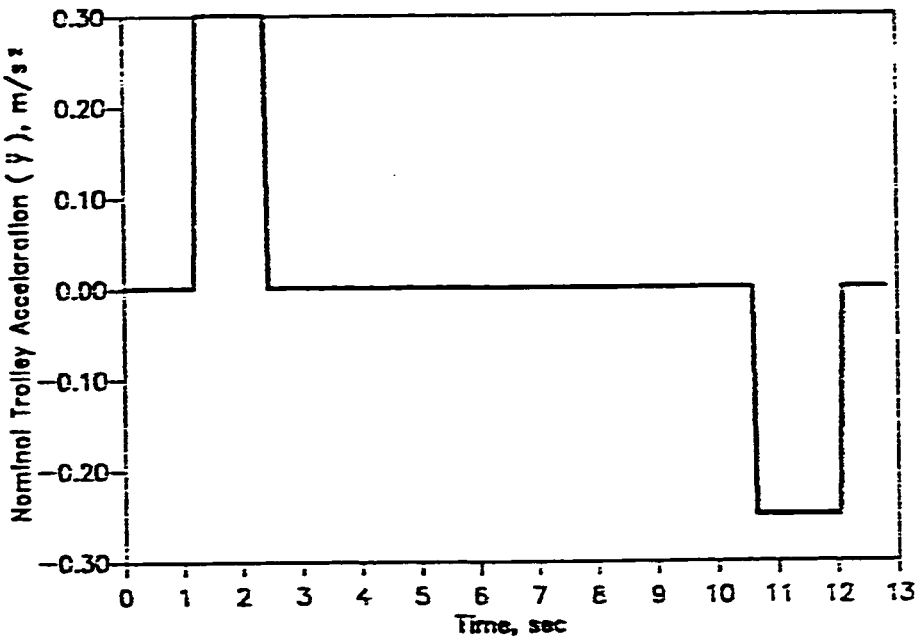


(a)

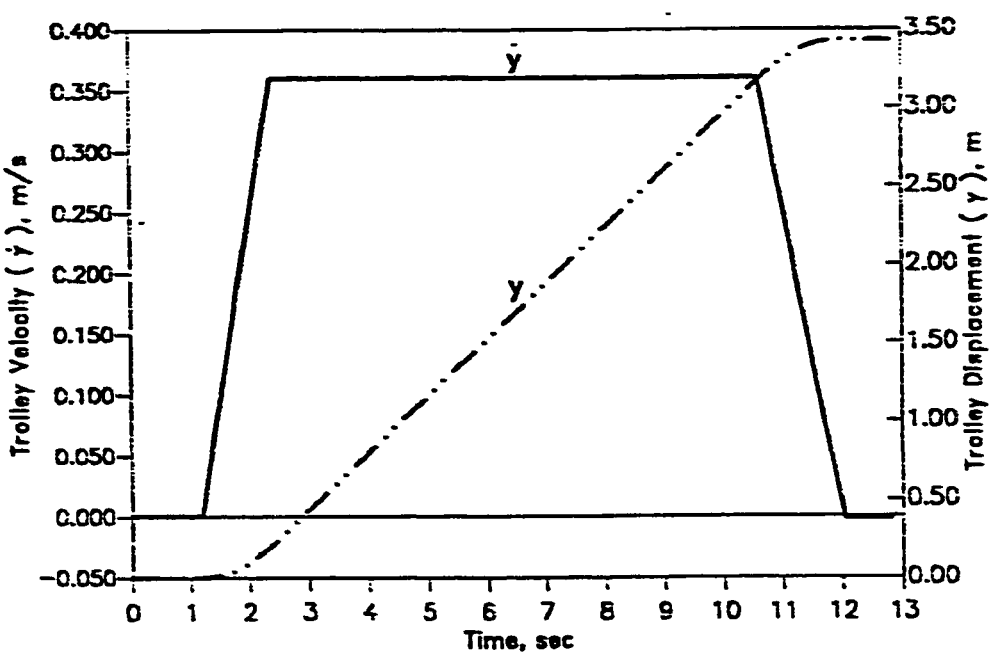


(b)

Figure 5.5: Hoisting Motion-Case 2.
(a) Nominal Hoisting Acceleration
(b) Hoisting Velocity and Cable Length

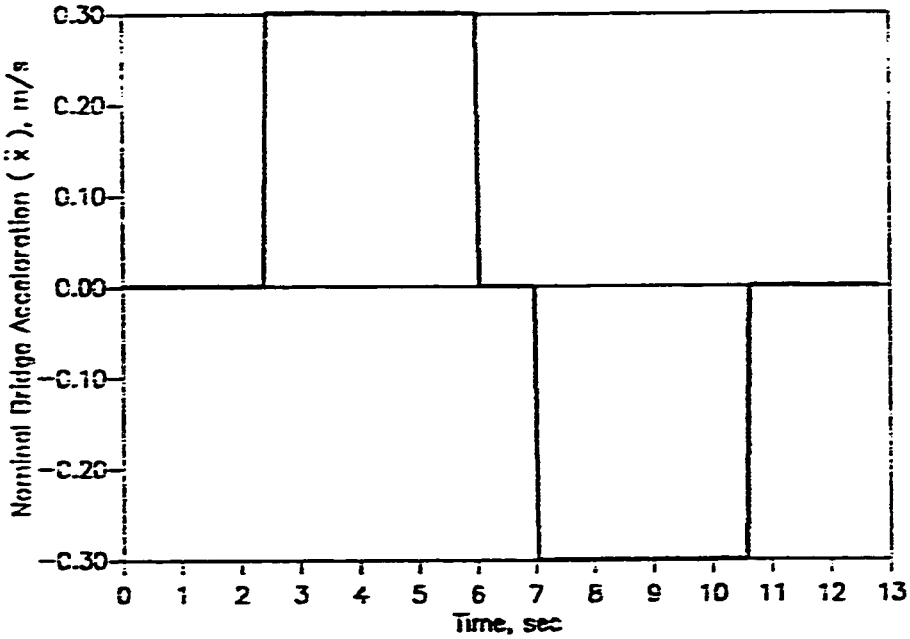


(a)



(b)

Figure 5.6: Trolley Motion-Case 2.
(a) Nominal Trolley Acceleration
(b) Trolley Velocity and Displacement



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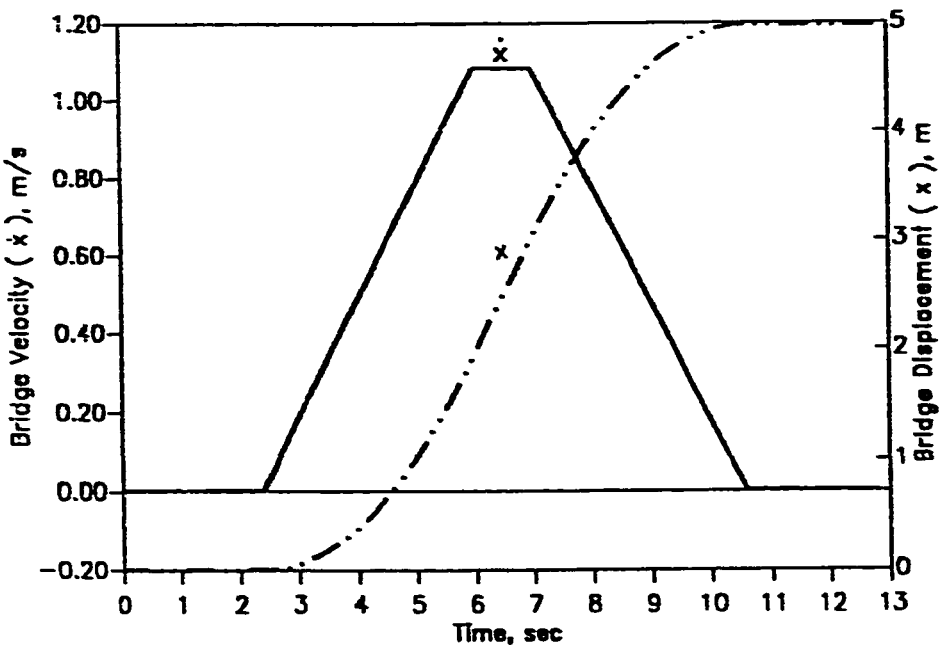
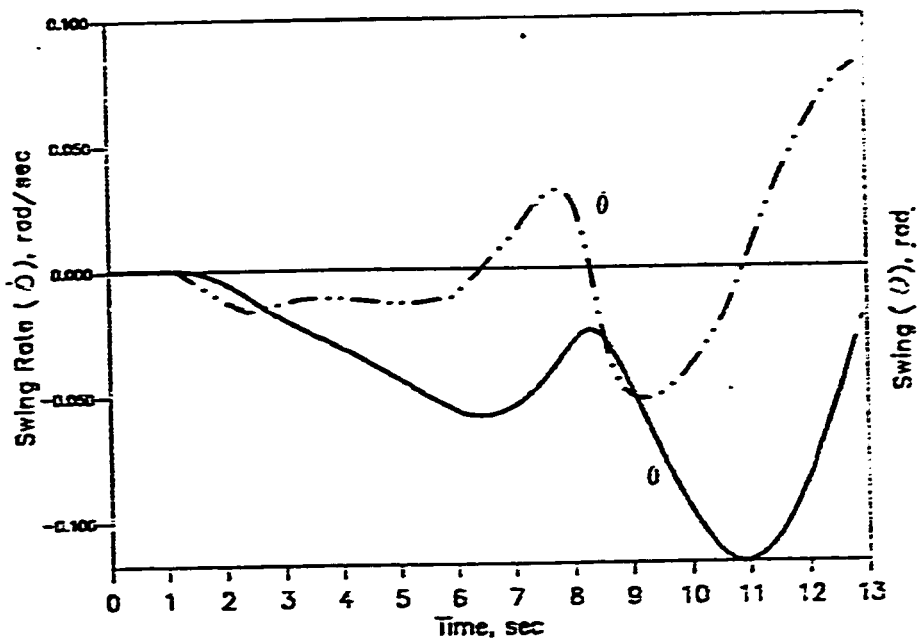
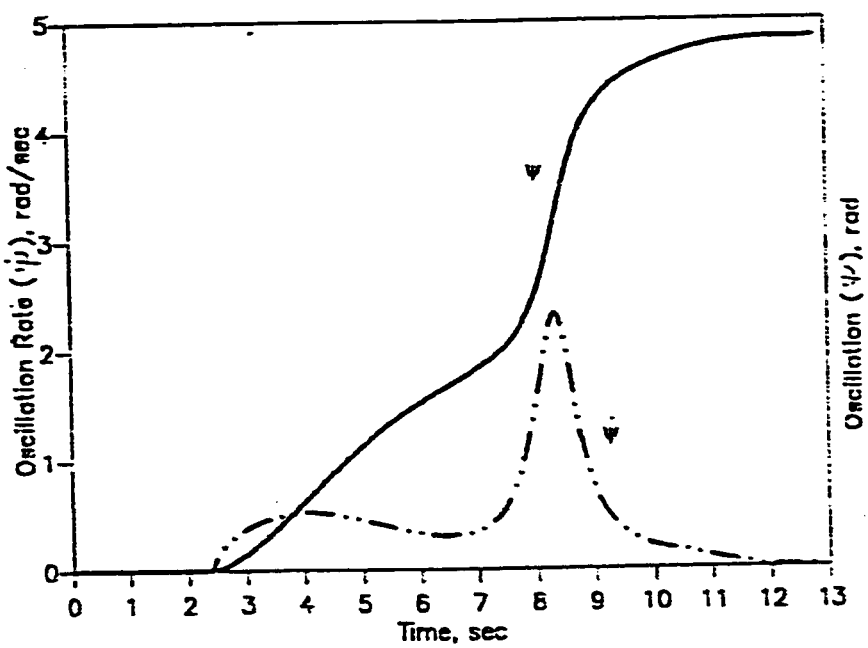


(b)

Figure 5.7: Bridge Motion-Case 2.
(a) Nominal Bridge Acceleration
(b) Bridge Velocity and Displacement



(a)



(b)

Figure 5.8: Load Sways - Case 2
(a) Swing Angle and Swing Rate
(b) Oscillation and Rate of Oscillation

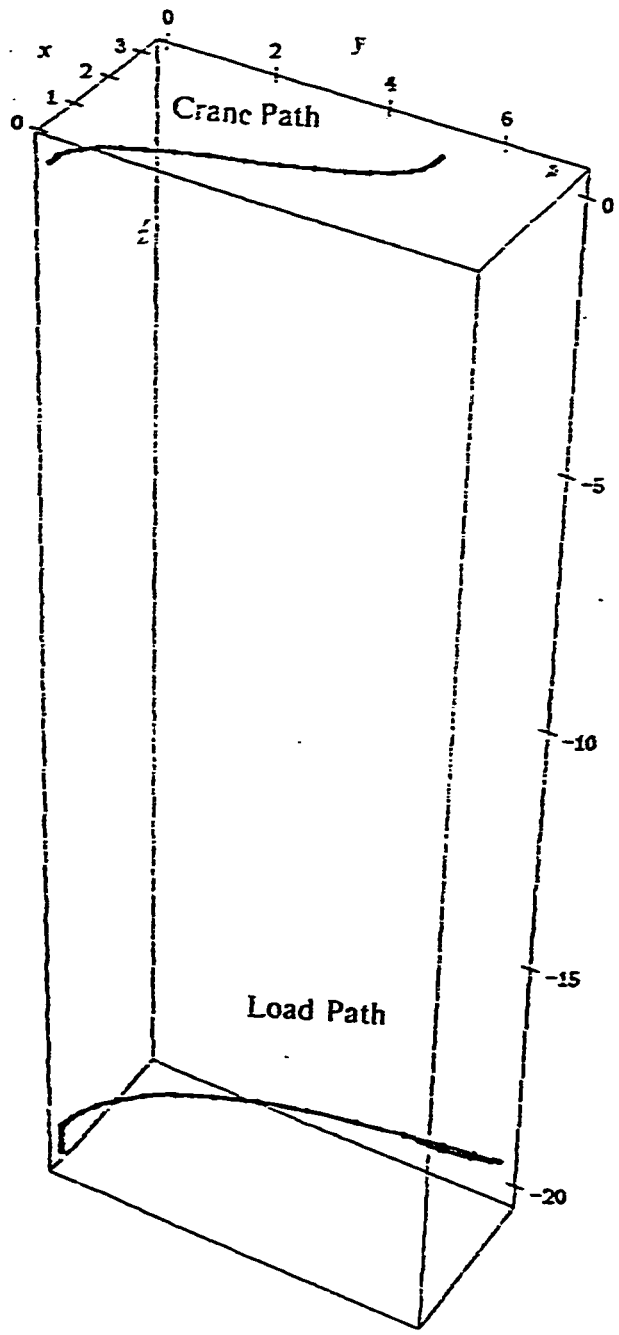


Figure 5.9: Crane and Load Paths - Case 2

5.3 CASE 3

In this case, a complete transport route of a torque-control model is simulated by applying the nominal control inputs which are the nominal driving torques of the trolley, the bridge, and the hoisting motors. In this model, the crane dynamics depend upon the masses of the load, the trolley, and the bridge. Therefore, typical crane data and relevant computations are presented below [2]:

$m_1 = 13{,}000\ Kg$

$m_2 = 4{,}000\ Kg$

$M = 10{,}000\ Kg$

$I_1 = 2.0\ Kg.m^2$

$I_2 = 1.0\ Kg.m^2$

$I_3 = 20.0\ Kg.m^2$

$r_1 = 2.0 \cdot 10^{-2}\ m$

$r_2 = 1.5 \cdot 10^{-2}\ m$

$r_3 = 1.8 \cdot 10^{-2}\ m$

The nominal torque inputs are calculated from the desired nominal values of accelerations of the bridge, the trolley, and the winch. These nominal values are selected according to the desired transport route as given in Table 5.1. The corresponding torques of the bridge motor, the trolley motor, and the winch motor are calculated by using equations (3.9) and (3.11), and are shown in Table 5.2. The complete transport route in this case consists of the following steps and is shown in Figs. 5.10 - 5.12

Table 5.1 Nominal crane accelerations.

Step	Time Interval (Sec.)	Acceleration of Bridge $\ddot{x}_b$ m/s^2	Acceleration of Trolley $\ddot{x}_t$ m/s^2	Acceleration of Winch $\ddot{l}_c$ m/s^2
1	0 - 1.2	0.00	0.00	-0.40
2	1.2 - 2.4	-0.23	-0.23	0.40
3	2.4 - 9.0	0.00	0.00	0.00
4	9.0 - 10.2	0.23	0.23	0.40
5	10.2 - 11.4	0.00	0.00	-0.40

Table 5.2 Nominal torques of the crane driving motors.

Step	Time Interval (Sec.)	Torque of Bridge F_b m/s^2	Torque of Trolley F_t m/s^2	Torque of Winch F_c m/s^2
1	0 - 1.2	0.000	0.000	-1.768
2	1.2 - 2.4	-0.227	-0.225	-0.969
3	2.4 - 9.0	0.000	0.000	-1.368
4	9.0 - 10.2	0.227	0.225	-0.969
5	10.2 - 11.4	0.000	0.000	-1.768

1- Hoisting of the load with a constant accelerating torque of 1.768 m/s^2 for a time interval of 0 - 1.2 s in order to achieve a hoisting speed of 0.48 m/s.

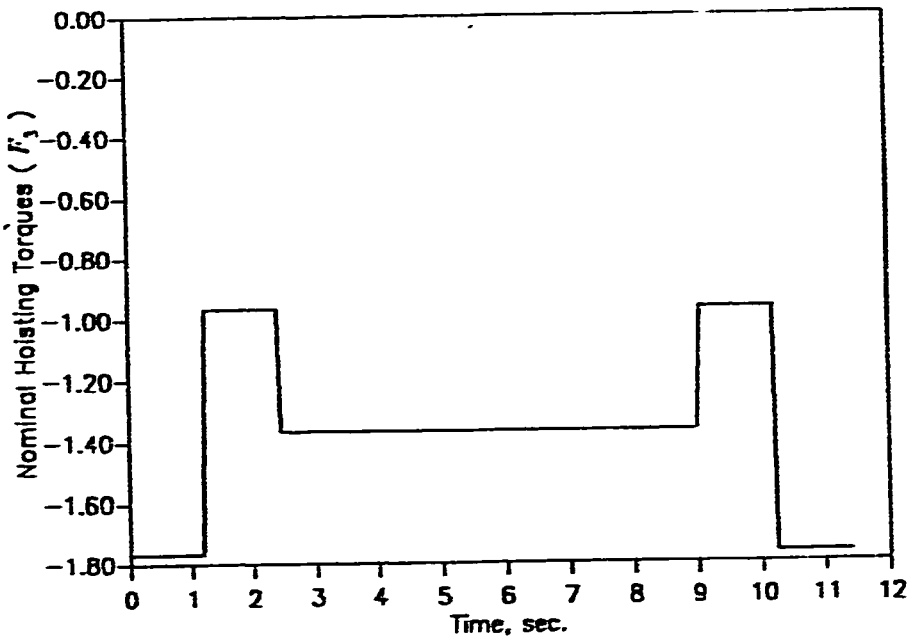
2- Simultaneous hoisting with a constant decelerating torque of 0.969 m/s^2 travelling with a constant accelerating torque of 0.227 m/s^2, and traversing with a constant accelerating torque of 0.225 m/s^2, for a time interval of 1.2 - 2.4s, resulting the final speeds of the hoisting equal to zero, the travelling attains 0.376 m/s, and the traversing acquires 0.506 m/s.

3- Simultaneous travelling and traversing with zero torque; a torque of magnitude 1.368 m/s^2 is constantly applied by a hoisting motor in order to maintain the height of the load against the effects of gravity for a time interval of 2.4-9.0 s.

4- Simultaneous travelling with a decelerating torque of magnitude 0.227 m/s^2, traversing with a decelerating torque of magnitude 0.225 m/s^2, and hoisting with an accelerating torque of 0.969 m/s^2 for a time interval of 9.0 to 10.2 s.

5- Lowering of the load with a decelerating torque of 1.768 m/s^2 in order to acquire the hoisting speed equal to zero within a time interval of 10.2 to 11.4 s.

The simulation results for this case are shown in Figs. 5.10 to 5.13. It can be observed from these results that during hoisting, there is no swing θ nor oscillation ψ of the load, indicating that hoisting itself does not produce any swings or oscillations. The value of the swing θ at the target position is also large, indicating the need of a proper control scheme to reduce it. It can also be noticed that the simultaneous motion of the bridge and the trolley produces the oscilla-



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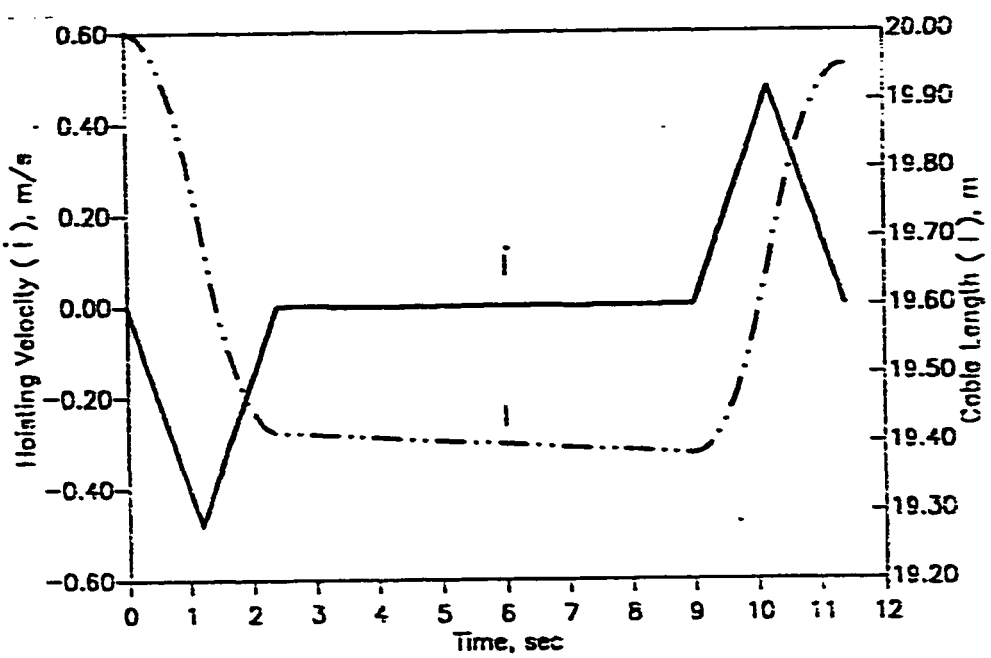
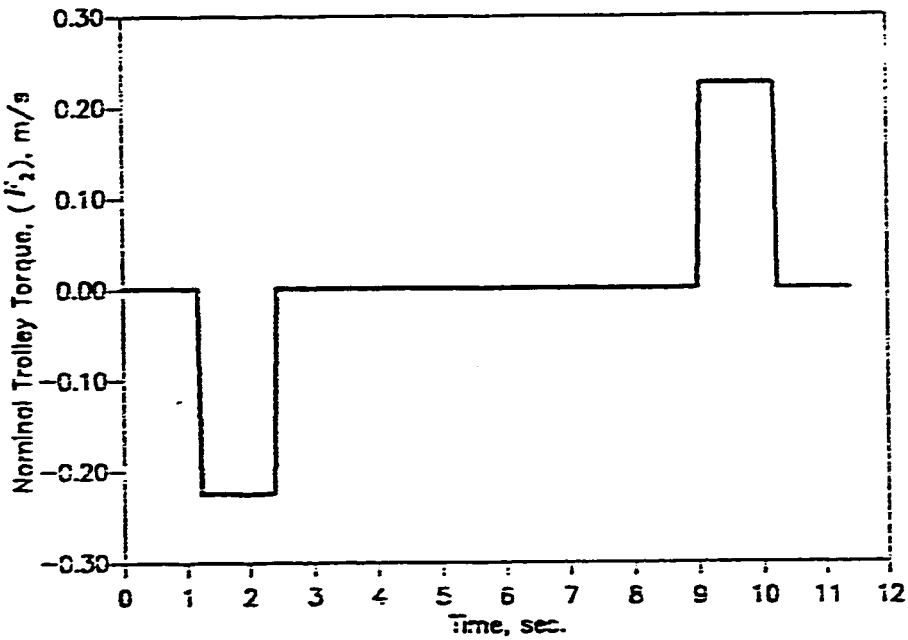


(b)

Figure 5.10: Hoisting Motion-Case 3.
(a) Nominal Hoisting Torque
(b) Hoisting Velocity and Cable Length



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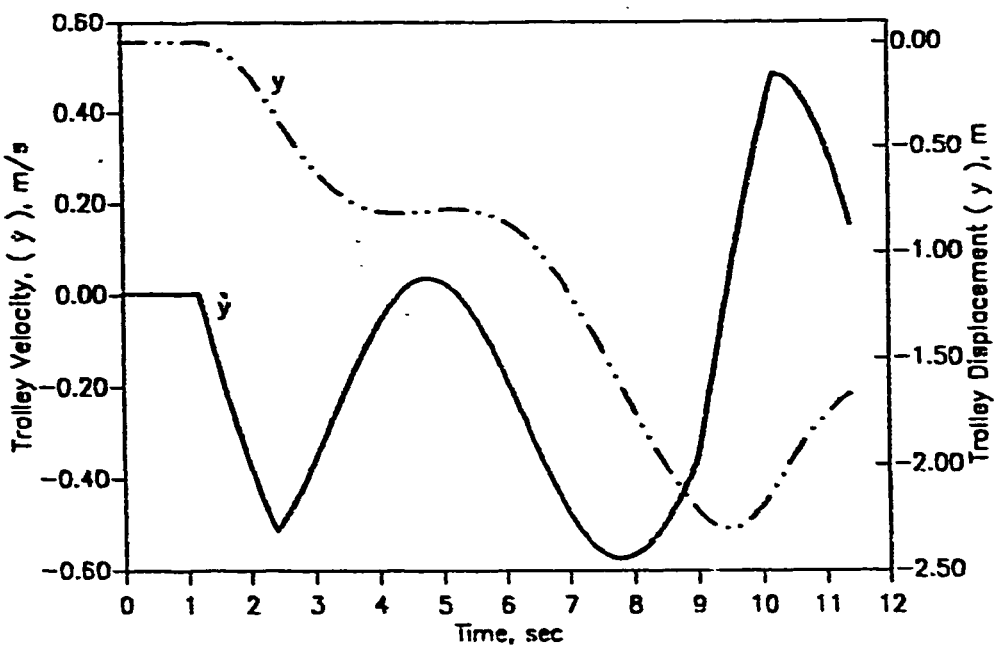
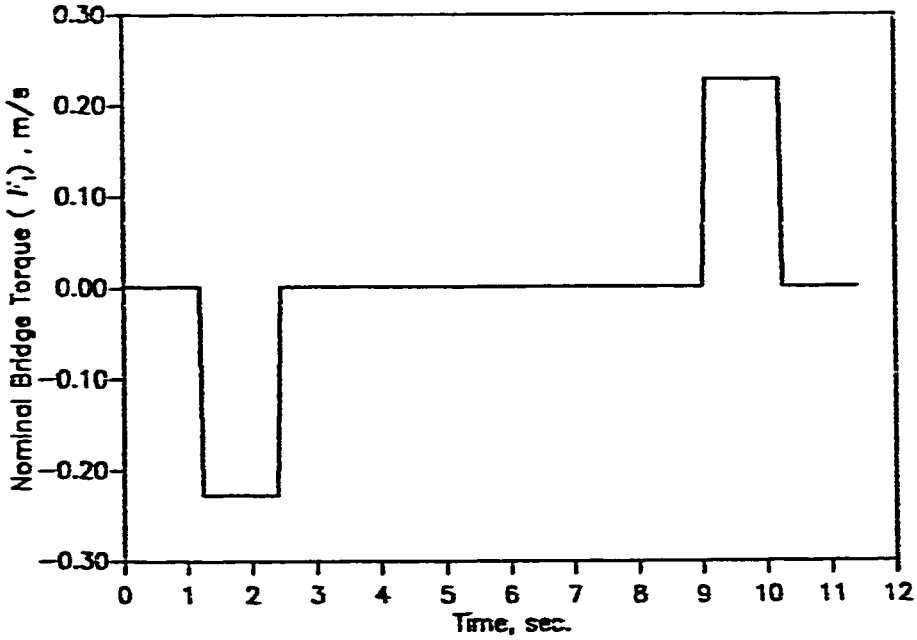
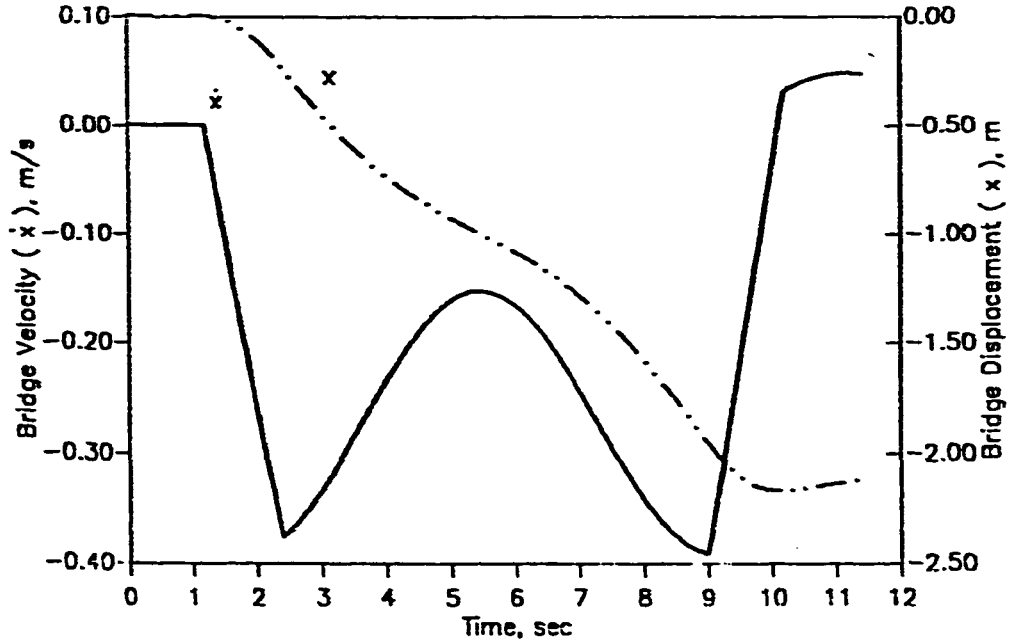


(b)

Figure 5.11: Trolley Motion-Case 3.
(a) Nominal Trolley Torque
(b) Trolley Velocity and Displacement



(a)

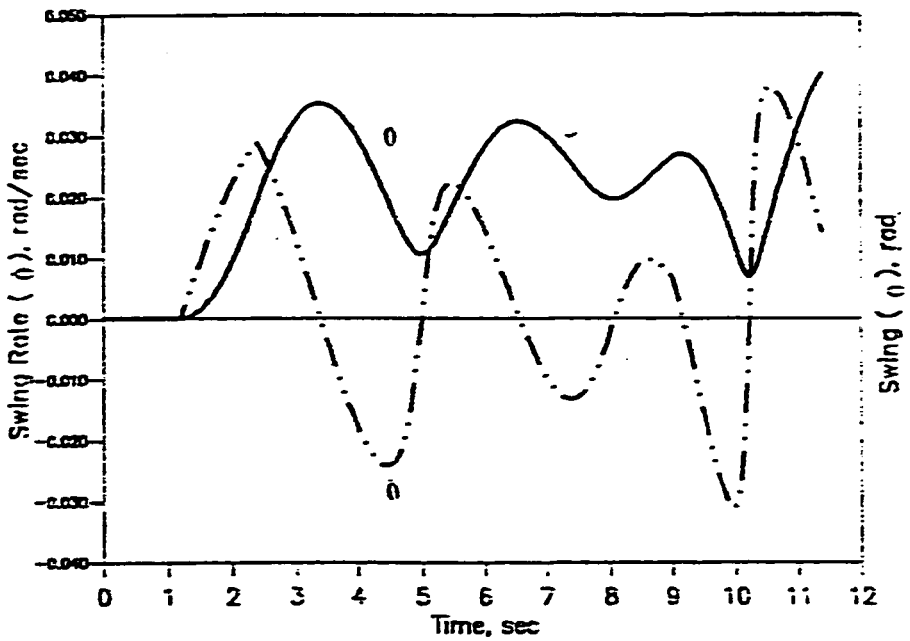


(b)

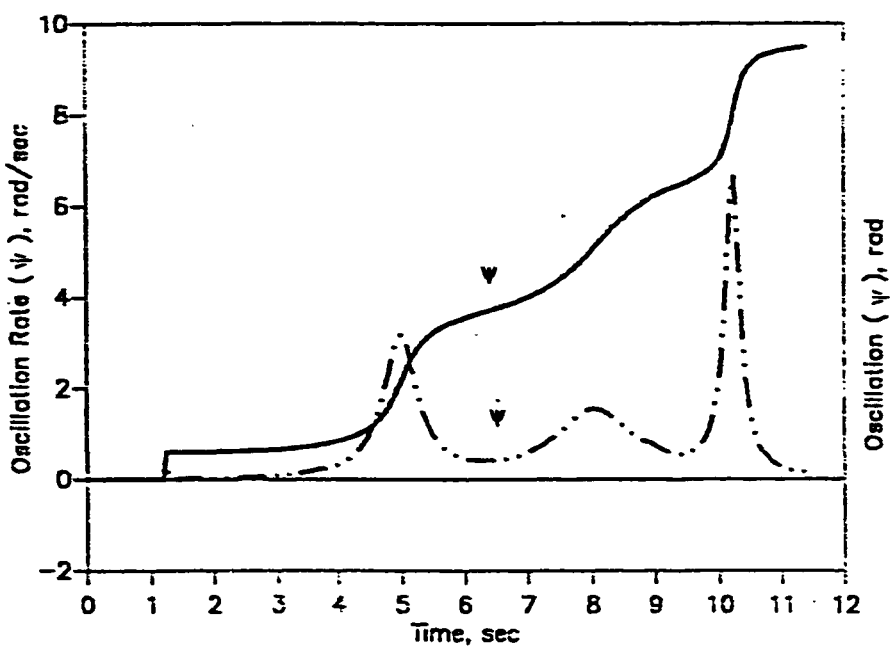
Figure 5.12: Bridge Motion-Case 3.
(a) Nominal Bridge Torque
(b) Bridge Velocity and Displacement

tion ψ which remains constant for about 4 sec but after that it starts increasing at a very fast rate and at the final position its value becomes very large. The high rates of the oscillations are due to the particular dynamics of the system and an understanding about these high rates of oscillations can be obtained by observing the equation (3.8). From this equation it can be observed that if only trolley is moving then the nominal oscillation angle is zero, but if at the same time the bridge also starts its motion with the same acceleration, as that of trolley, then the nominal oscillation angle increases abruptly to 0.7 radian. Similar observations can be obtained by using different values of accelerations.

Another important observation of these simulation results, as can be seen in Figs. 5.11 and 5.12, is the effect of the swings and oscillations on the motion of the trolley and the bridge, particularly in the third and the last steps. During these steps no input torques are applied to the trolley and bridge, but due to swings and oscillations the velocities of trolley and bridge are changing continuously.



(a)



(b)

Figure 5.13: Load Sways - Case 3.
(a) Swing Angle and Swing Rate
(b) Oscillation and Rate of Oscillation

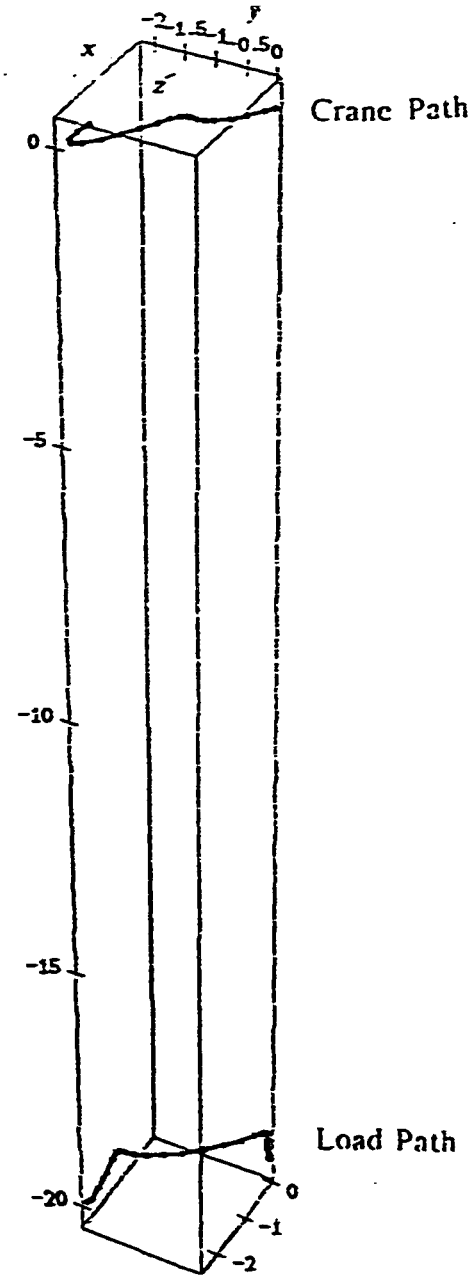


Figure 5.14: Crane and Load Paths - Case 3

5.4 CASE 4

In this case the performance of the overhead crane is simulated after applying the optimized control scheme. As discussed in Chapter 4, the control inputs are given as

$$
\begin{aligned}
u_1 &= c_1 a_1(t) + c_2 x_5(t) \\
u_2 &= c_3 a_2(t) + c_4 x_4(t) \\
u_3 &= c_5 a_3(t)
\end{aligned}
\tag{5.1}
$$

where $a_1(t)$, $a_2(t)$, and $a_3(t)$ are as defined below.

$$
a_1(t) = \begin{cases}
0.0 & t \le T_2 \\
\frac{1}{2}\left[1-\cos\left(\frac{10\pi(t-T_2)}{T_M}\right)\right] & T_2 \le t \le T_2 + \frac{T_M}{10} \\
1.0 & T_2 + \frac{T_M}{10} \le t \le T_2 + \frac{9\,T_M}{10} \\
\frac{1}{2}\left[1+\cos\left(\frac{10\pi(t-T_2) - \frac{9\,T_M}{10}}{T_M}\right)\right] & T_2 + \frac{9\,T_M}{10} \le t \le T_{M1} \\
-\frac{1}{2}\left[1-\cos\left(\frac{10\pi(t-T_{M1})}{T_M}\right)\right] & T_{M1} \le t \le T_{M1} + \frac{T_M}{10} \\
-1.0 & T_{M1} + \frac{T_M}{10} \le t \le T_{M1} + \frac{9\,T_M}{10} \\
-\frac{1}{2}\left[1+\cos\left(\frac{10\pi(t-T_{M1}) - \frac{9\,T_M}{10}}{T_M}\right)\right] & T_{M1} + \frac{9\,T_M}{10} \le t \le T_3 \\
0.0 & t \ge T_3
\end{cases}
\tag{5.2}
$$

$$a_2(t) = a_1(t)$$

where

$$T_2 = 4.0\ sec.$$

$$T_{M1} = \frac{t_f}{2} \qquad (t_f \text{ is optimized final time})$$

$$T_M = T_{M1} - T_2 \tag{5.3}$$

$$T_3 = t_f - 4$$

and

$$a_3(t) = \begin{cases} -\frac{1}{2}\left[1-\cos\left(\frac{10\pi t}{T_1}\right)\right] & 0 \le t \le \frac{T_1}{10} \\ -1.0 & \frac{T_1}{10} \le t \le \frac{9\,T_1}{10} \\ -\frac{1}{2}\left[1+\cos\left(\frac{10\pi(t-\frac{9\,T_1}{10})}{T_1}\right)\right] & \frac{9\,T_1}{10} \le t \le T_1 \\ \frac{1}{2}\left[1-\cos\left(\frac{10\pi(t-T_1)}{T_2-T_1}\right)\right] & T_1 \le t \le T_1 + \frac{T_2-T_1}{10} \\ 1.0 & T_1 + \frac{T_2-T_1}{10} \le t \le T_1 + 9\frac{(T_2-T_1)}{10} \\ \frac{1}{2}\left[1+\cos\left(\frac{10\pi\left((t-T_1)-\frac{9(T_2-T_1)}{10}\right)}{(T_2-T_1)}\right)\right] & T_1 + 9\frac{(T_2-T_1)}{10} \le t \le T_2 \\ 0.0 & \frac{T_2}{10} \le t \le T_3 \end{cases} \tag{5.4}$$

For $t \ge T_3$ the same cycle as for $t \le T_2$ is repeated with the sign switched.

The functions $a_1(t)$, $a_2(t)$ and $a_3(t)$ are selected in this form, instead of step inputs, to avoid the problem encountered by the numerical integration method. These functions are arbitrary. Their selection depend upon the desired transport route and is based upon practical experience.

The following control- and state variable- constraints have been selected from references [2] and [3] and are given as

CONTROL-CONSTRAINTS

$$-0.30 \le u_1 \le 0.30 \ m/s^2$$

$$-0.30 \le u_2 \le 0.30 \ m/s^2$$

$$-0.50 \le u_3 \le 0.50 \ m/s^2$$

STATE VARIABLES-CONSTRAINTS

$$-2.0 \le x_2 \le 2.0 \ m/s$$

$$-1.0 \le x_4 \le 1.0 \ m/s$$

$$-0.5 \le x_{10} \le 0.5 \ m/s$$

and the final constraints are

$$19.97 \le x_1(t_f) \le 20.03 \ m$$

$$x_2(t_f) = 0.0 \ m/s$$

$$9.97 \le x_3(t_f) \le 10.03 \ m$$

$$x_4(t_f) = 0.0 \ m/s$$

$$-0.001 \le x_5(t_f) \le 0.001 \ rad$$

$$-0.001 \le x_6(t_f) \le 0.001 \ rad/s$$

$$-0.06 \le x_7(t_f) \le 0.06 \ rad$$

$$-0.06 \le x_8(t_f) \le 0.06 \ rad/s$$

$$14.97 \le x_9(t_f) \le 15.03 \ m$$

$$x_{10}(t_f) = 0.0 \ m/s$$

Using the optimization package, discussed in chapter 4, the optimized coefficient, $c_1, \ldots, c_4$ are calculated as

$$c_1 = 0.01052$$

$$c_2 = 0.01427$$

$$c_3 = 0.00528$$

$$c_4 = 0.00010$$

$$t_f = 99.95$$

It should be noted that hoisting motion does not produce any swing. Therefore the coefficient c_5 is taken to be 0.01, in equation (5.1) for simulating the results.

The simulation results of Figs. 5.15 - 5.27 show that the computed optimized control scheme works well. It minimizes the load swing, from the beginning and keeps it very close to zero during the transfer and at the end of the transfer as well. The state variables feedback effects can also be noticed on the control input, particularly at time equal to 96 s, where the swing angle θ starts increasing, but due to the appropriate control action it decreases.

It can also be noticed that the oscillations ψ are not as small as the swings are, but they are still in acceptable range.

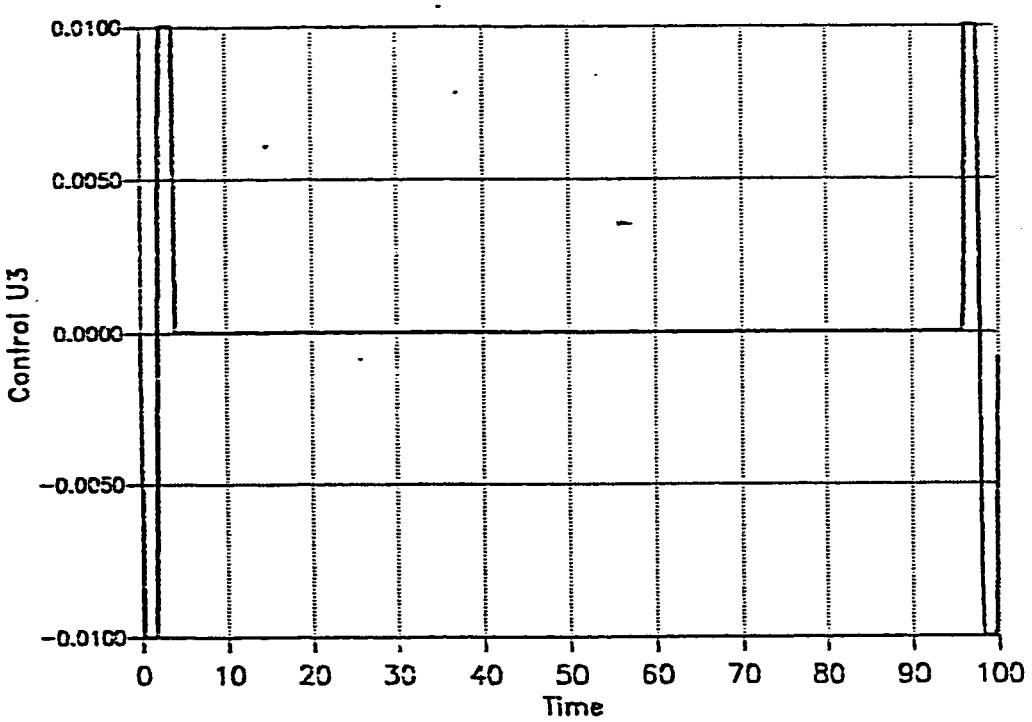


Figure 5.15: Hoist Acceleration - Case 4

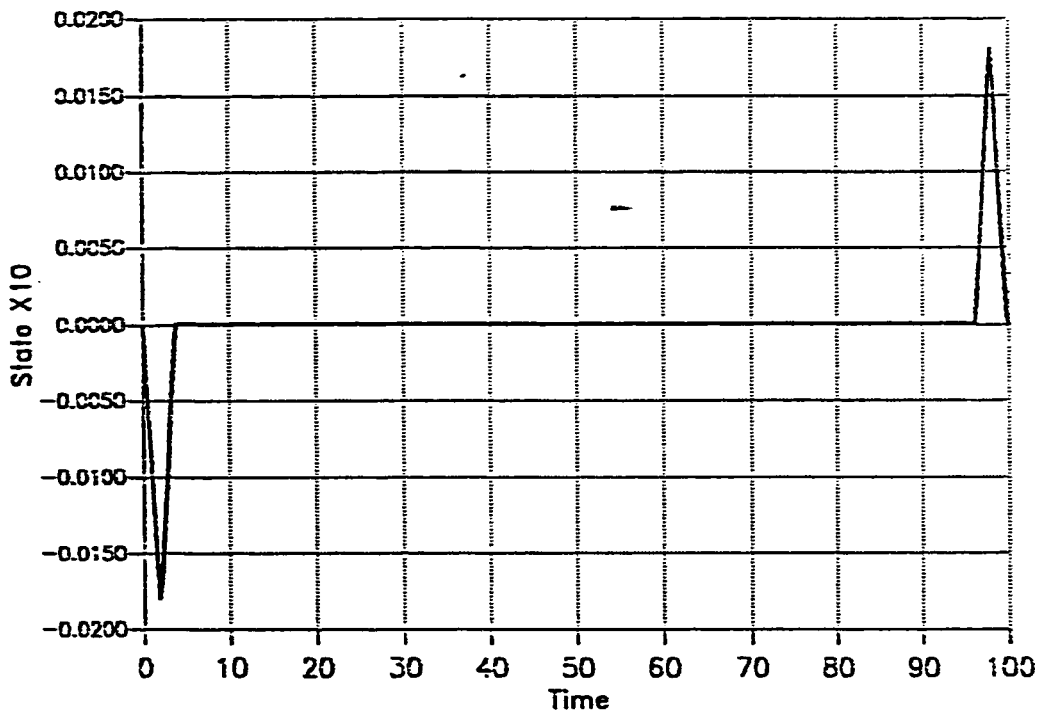


Figure 5.16: Hoist Velocity - Case 4

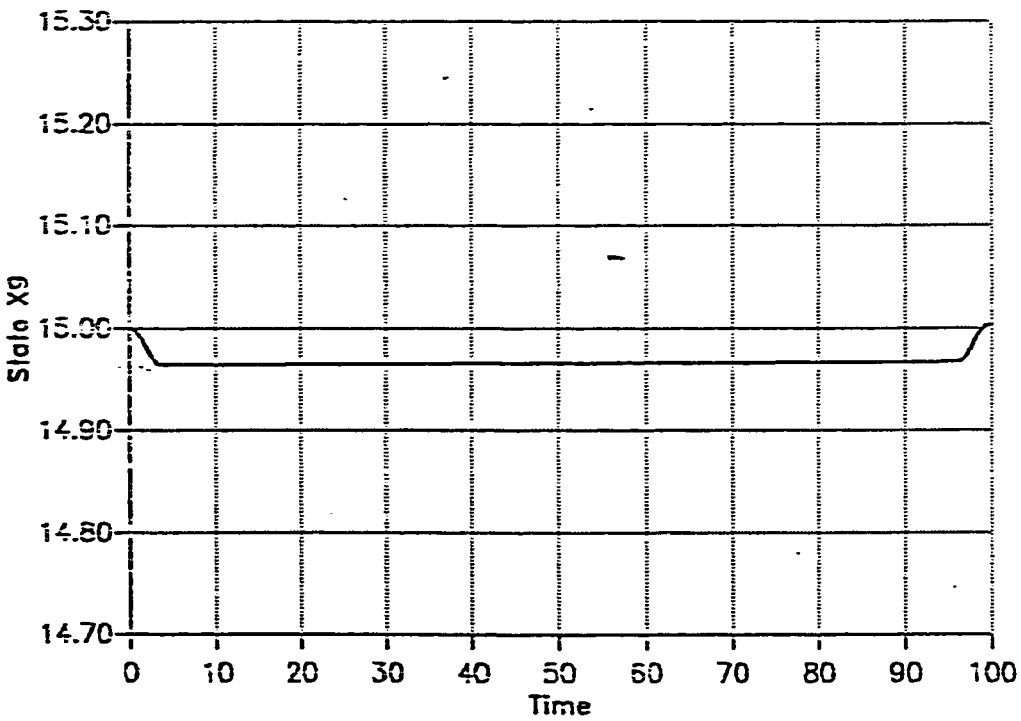


Figure 5.17: Cable Length - Case 4

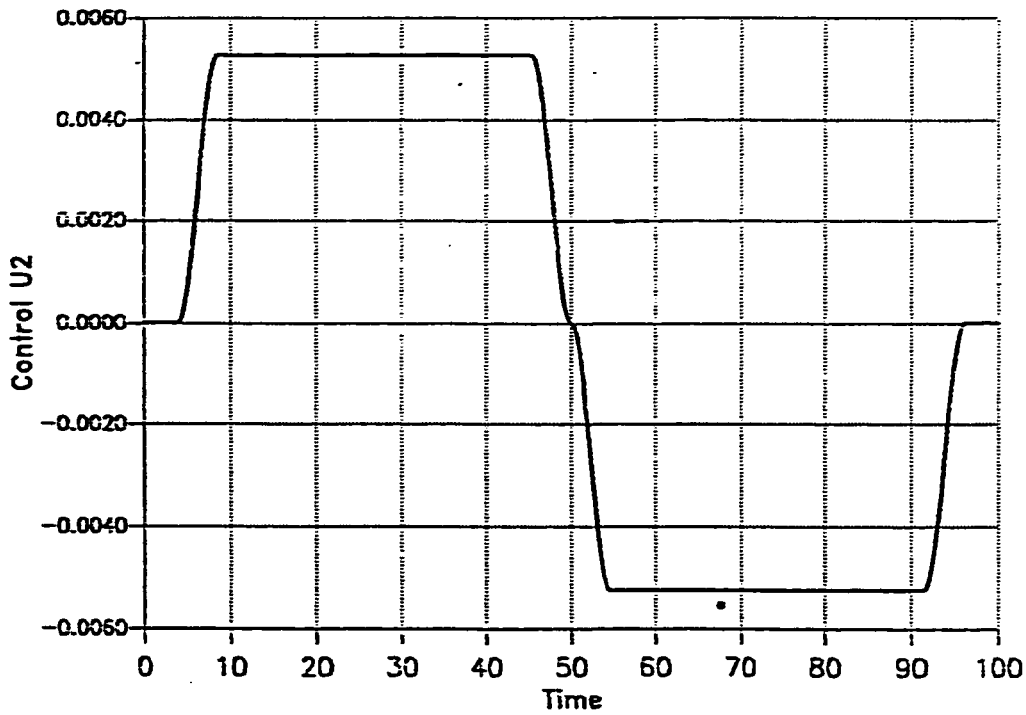


Figure 5.18: Trolley Acceleration - Case 4

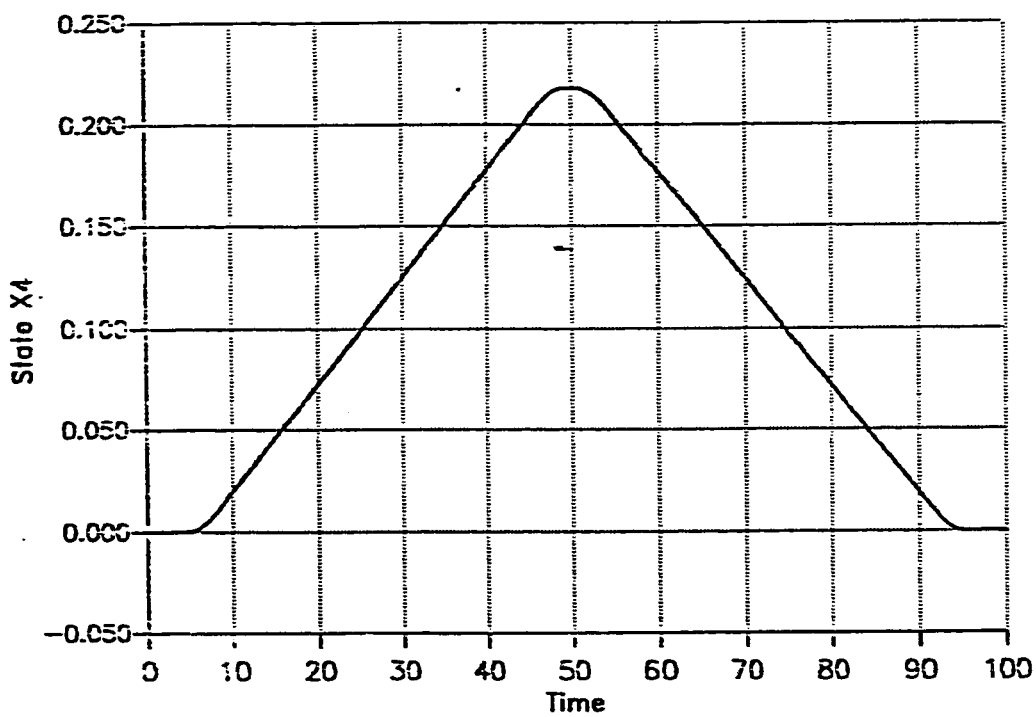


Figure 5.19: Trolley Velocity - Case 4

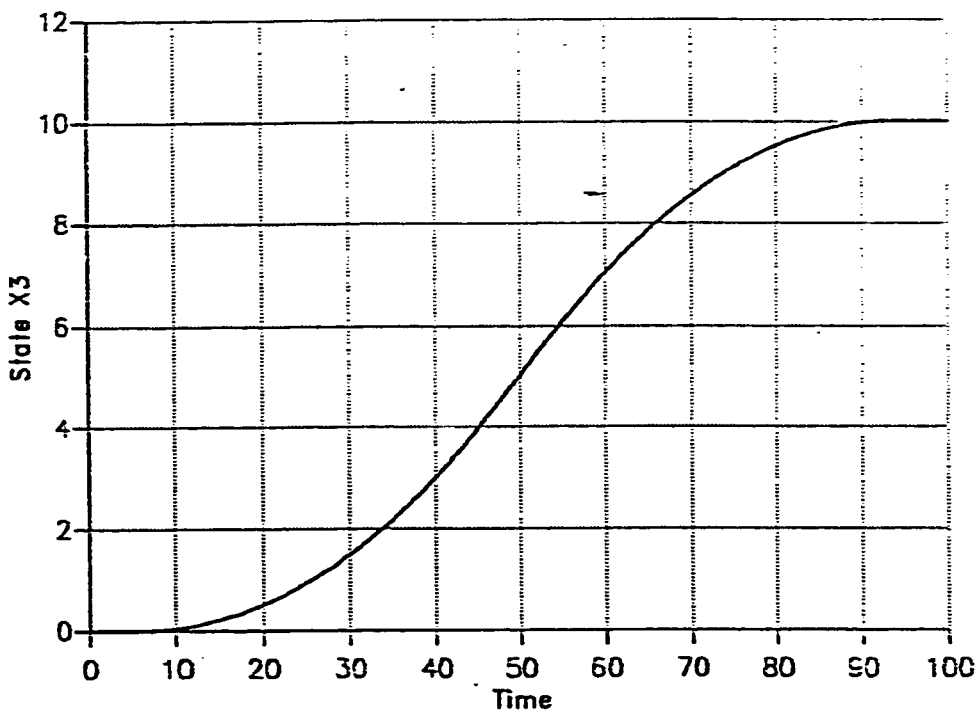


Figure 5.20: Trolley Displacement - Case 4

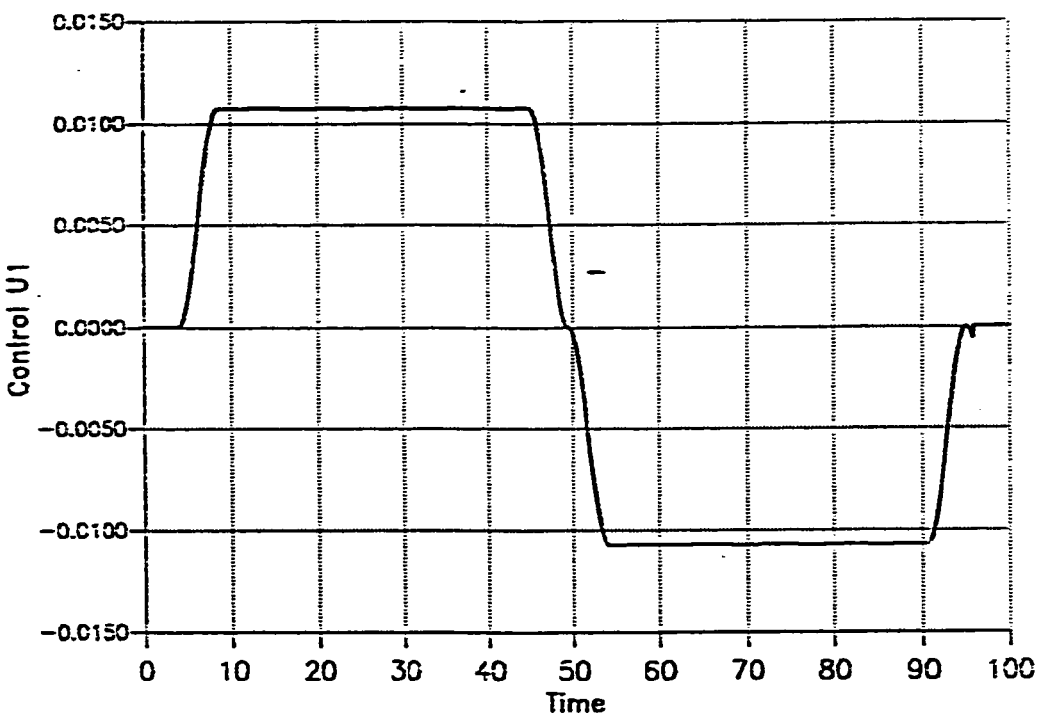


Figure 5.21: Bridge Acceleration - Case 4

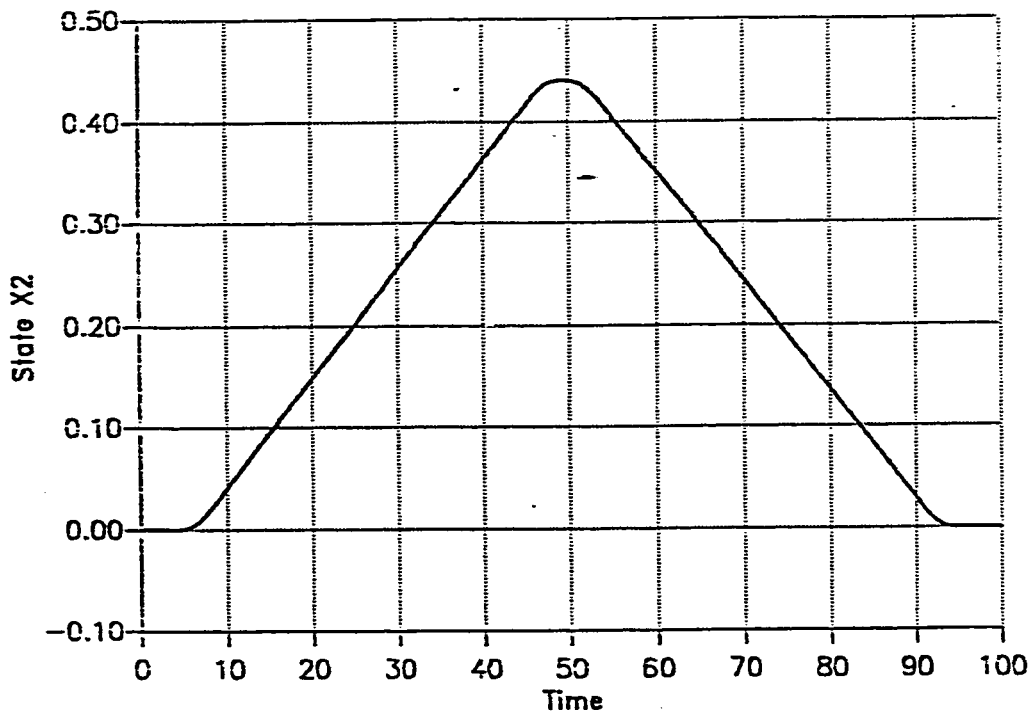


Figure 5.22: Bridge Velocity -- Case 4

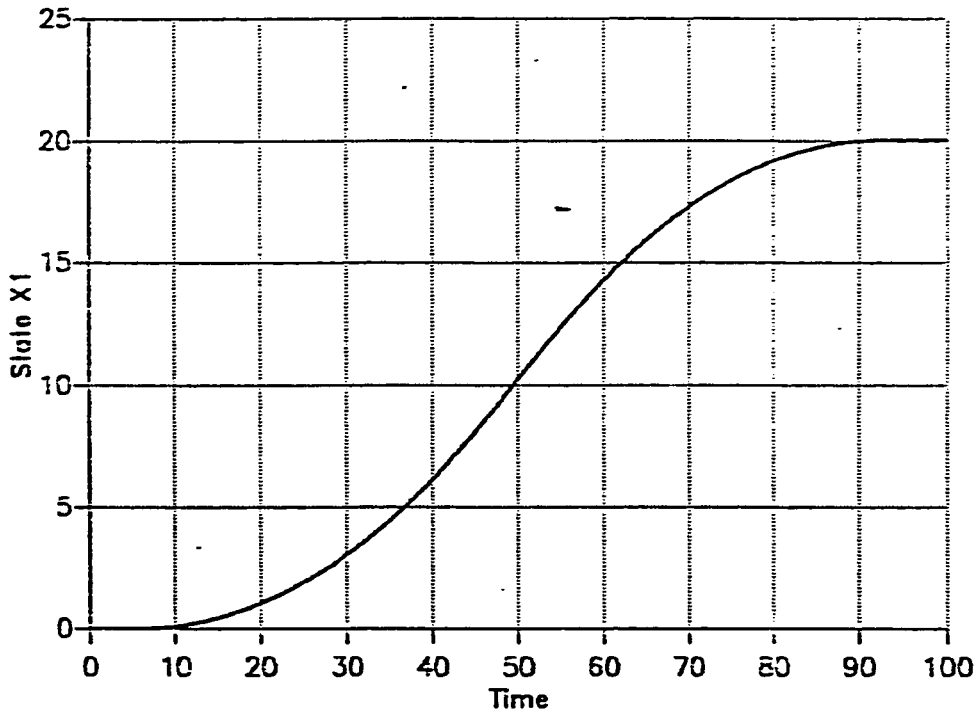


Figure 5.23: Bridge Displacement - Case 4

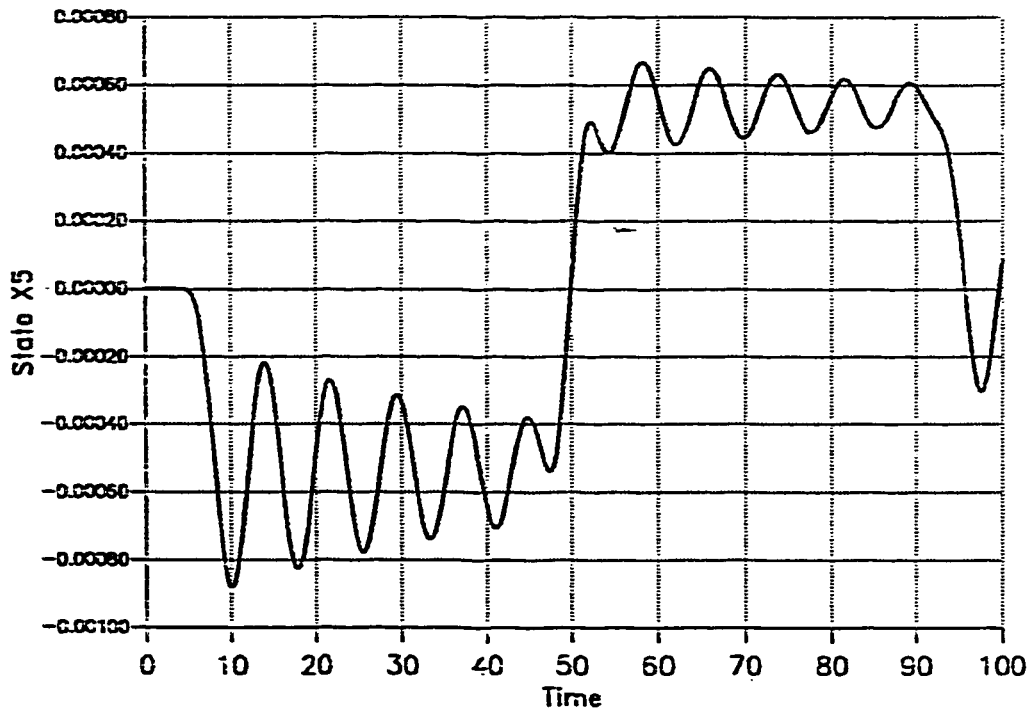


Figure 5.24: Swing Angle - Case 4

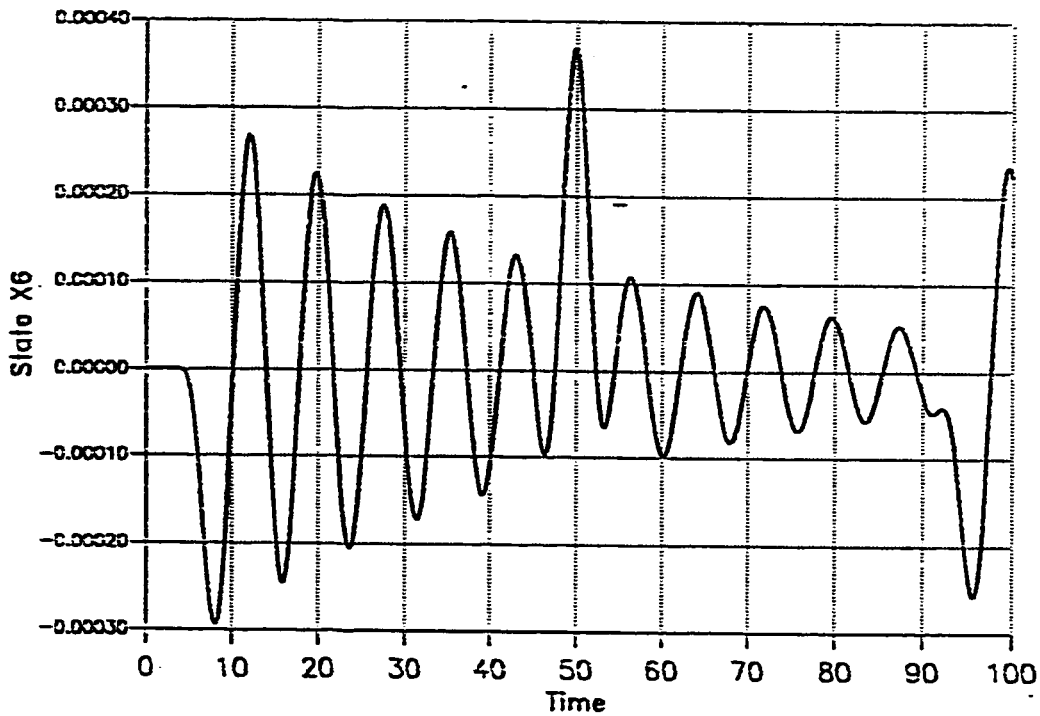


Figure 5.25: Swing Rate - Case 4

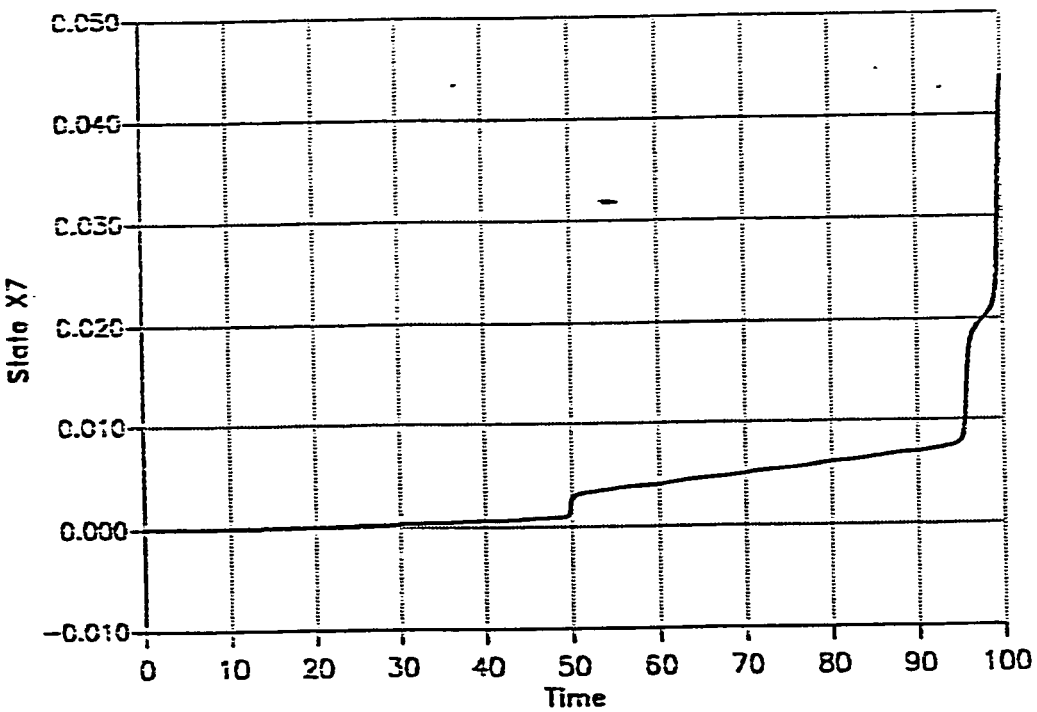


Figure 5.26: Oscillation - Case 4

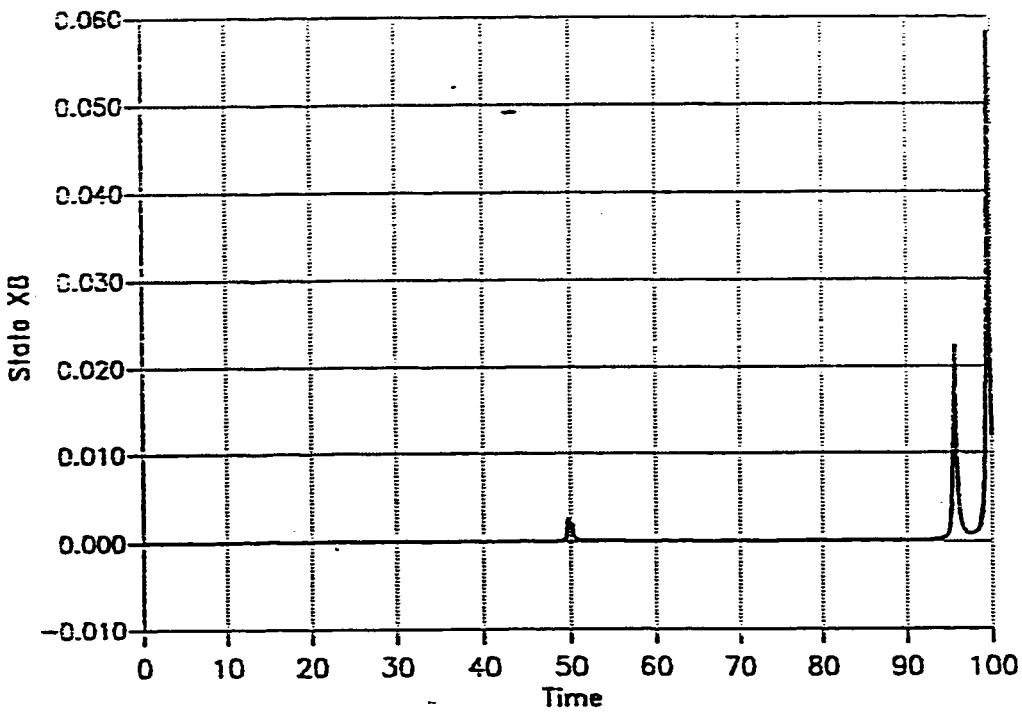


Figure 5.27: Rate of Oscillation - Case 4

Chapter 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 CONCLUSIONS

Various dynamical models for overhead cranes are presented. The most general case of crane motions i.e. simultaneous travelling of the bridge, traversing of the trolley, and the load hoisting is considered. The general nonlinear model is derived by using the Lagrange's equation of motion. Two different models of industrial importance namely: acceleration-control model, and torque-control model are derived from the general nonlinear model. The nominal operating conditions are discussed, and a linearization technique is used to obtain the linearized models by expanding the nonlinear model about these nominal operating conditions using the Taylor's series.

The nonlinear acceleration-control model is used to develop an optimized control law for controlling the motion of the crane. By this, an object can be transported to a desired destination in a state of rest in the minimum possible time, while minimizing the load swing during the transport. A detailed description of an optimized feedback control method including the performance index selection criteria, the control-constraints, and the state-variable constraints is provided. The control law uses only those states which are of prime importance and can be measured easily. A numerical optimization technique, which is used for solving the given problem, is also discussed.

Simulation results for cases 1 to 3 show the behavior of the nonlinear acceleration-control and torque-control models. They also show the absolute need, in practice, of controlling the crane motion to damp the load swing during the transfer and at the final destination as well and the simulation results of case 4 show the effectiveness of the optimized control scheme.

6.2 RECOMMENDATIONS

Practical implementation of control schemes is usually carried out by manipulating the input stator voltages of the driving motors. An overall model that incorporates the motors' electrical and mechanical dynamics and can accept the stator voltage as inputs is, therefore, highly desirable. This forms a basis for future research in this area.

Another potential research area is the testing of the developed modelling and control techniques to an actual overhead crane. Also, the practical implementation of crane control schemes need to be realized by hardware components that are supported by the appropriate software.

A more realistic model can be developed in future by relaxing the assumptions made in this study, i.e. a model which takes into account the structure flexibility, the dissipative effects, and the actual load (rigid body of irregular geometry) considerations.

The present work did not consider the aspect of energy consumption. An optimum control scheme that is also able to minimize the energy consumed by the driving motor is another topic for further research.

Appendix A

Description of Nonlinear Torque-Control Model Matrices and Functions

The matrices [A] and [B] introduced in equation (2.16) and the functions $f_1(\cdot), \ldots, f_5(\cdot)$ introduced in equations (2.17) are shown as follows: (Note that, due to limitations of Mathematica, the symbols θ, ψ, l, $\dot{\theta}$ and $\dot{\psi}$ are replaced by t, s, r, p, and q respectively.)

```
[A] = {{1, 0, b1*l*Cos[t]*Sin[s], b1*l*Cos[s]*Sin[t],
         b1*Sin[s]*Sin[t]},
       {0, 1, b2*l*Cos[s]*Cos[t], -(b2*l*Sin[s]*Sin[t]),
        b2*Cos[s]*Sin[t]},
       {(Cos[t]*Sin[s])/l, (Cos[s]*Cos[t])/l, 1, 0, 0},
       {Cos[s]/(l*Sin[t]), -(Sin[s]/(l*Sin[t])), 0, 1, 0},
       {b3*Sin[s]*Sin[t], b3*Cos[s]*Sin[t], 0, 0, 1}}
```

```
[B] = {{F1 - 2*b1*l*p*q*Cos[s]*Cos[t] -
        2*b1*p*r*Cos[t]*Sin[s] -
        2*b1*q*r*Cos[s]*Sin[t] +
        b1*l*(p^2 + q^2)*Sin[s]*Sin[t]},
       {F2 - 2*b2*p*r*Cos[s]*Cos[t] +
        2*b2*l*p*q*Cos[t]*Sin[s] +
        b2*l*(p^2 + q^2)*Cos[s]*Sin[t] +
        2*b2*q*r*Sin[s]*Sin[t]},
       {(-2*p*r)/l - (g*Sin[t])/l + q^2*Cos[t]*Sin[t]},
       {(-2*q*r)/l - (2*p*q*Cos[t])/Sin[t]},
       {F3 + b3*l*p^2 + b3*g*Cos[t] + b3*l*q^2*Sin[t]^2}}
```

```
f1 =
 {(((-(b1*l*Cos[t]*Sin[s]) + b1*b2*l*Cos[s]^2*Cos[t]*Sin[s] +
 b1*b2*l*Cos[t]*Sin[s]^3)*
 ((-2*p*r)/l - (g*Sin[t])/l + q^2*Cos[t]*Sin[t]))
+ ((-(b1*Sin[s]*Sin[t]) + b1*b2*Cos[s]^2*Sin[s]*Sin[t] +
 b1*b2*Sin[s]^3*Sin[t])*(F3 + b3*l*p^2 + b3*g*Cos[t] +
 b3*l*q^2*Sin[t]^2))
+ ((F1 - 2*b1*l*p*q*Cos[s]*Cos[t] - 2*b1*p*r*Cos[t]*Sin[s] -
 2*b1*q*r*Cos[s]*Sin[t] + b1*l*(p^2 + q^2)*Sin[s]*Sin[t])*
 (1 - b2*Cos[s]^2*Cos[t]^2 - b2*Sin[s]^2
                          - b2*b3*Cos[s]^2*Sin[t]^2))
+ ((F2 - 2*b2*p*r*Cos[s]*Cos[t] + 2*b2*l*p*q*Cos[t]*Sin[s] +
 b2*l*(p^2 + q^2)*Cos[s]*Sin[t] + 2*b2*q*r*Sin[s]*Sin[t])*
 (-(b1*Cos[s]*Sin[s]) + b1*Cos[s]*Cos[t]^2*Sin[s] +
 b1*b3*Cos[s]*Sin[s]*Sin[t]^2))
+ ((-2*q*r)/l - (2*p*q*Cos[t])/Sin[t])*
 (-(b1*l*Cos[s]*Sin[t]) + b1*b2*l*Cos[s]^3*Cos[t]^2*Sin[t] +
  b1*b2*l*Cos[s]*Cos[t]^2*Sin[s]^2*Sin[t] +
  b1*b2*b3*l*Cos[s]^3*Sin[t]^3
                  + b1*b2*b3*l*Cos[s]*Sin[s]^2*Sin[t]^3)} /
 (1 - b1*Cos[s]^2 - b2*Cos[s]^2*Cos[t]^2
    + b1*b2*Cos[s]^4*Cos[t]^2 -
      b2*Sin[s]^2 - b1*Cos[t]^2*Sin[s]^2 +
      2*b1*b2*Cos[s]^2*Cos[t]^2*Sin[s]^2 +
      b1*b2*Cos[t]^2*Sin[s]^4 -
      b2*b3*Cos[s]^2*Sin[t]^2 + b1*b2*b3*Cos[s]^4*Sin[t]^2 -
      b1*b3*Sin[s]^2*Sin[t]^2 +
      2*b1*b2*b3*Cos[s]^2*Sin[s]^2*Sin[t]^2 +
      b1*b2*b3*Sin[s]^4*Sin[t]^2)
```

```
f2 =
  {(((-(b2*l*Cos[s]*Cos[t]) + b1*b2*l*Cos[s]^3*Cos[t] +
  b1*b2*l*Cos[s]*Cos[t]*Sin[s]^2)*
  ((-2*p*r)/l - (g*Sin[t])/l + q^2*Cos[t]*Sin[t]))
 + ((-(b2*Cos[s]*Sin[t]) + b1*b2*Cos[s]^3*Sin[t] +
   b1*b2*Cos[s]*Sin[s]^2*Sin[t])*
   (F3 + b3*l*p^2 + b3*g*Cos[t] + b3*l*q^2*Sin[t]^2))
 + ((F1 - 2*b1*l*p*q*Cos[s]*Cos[t] - 2*b1*p*r*Cos[t]*Sin[s] -
   2*b1*q*r*Cos[s]*Sin[t] + b1*l*(p^2 + q^2)*Sin[s]*Sin[t])*
   (-(b2*Cos[s]*Sin[s]) + b2*Cos[s]*Cos[t]^2*Sin[s] +
   b2*b3*Cos[s]*Sin[s]*Sin[t]^2))
 + ((F2 - 2*b2*p*r*Cos[s]*Cos[t] + 2*b2*l*p*q*Cos[t]*Sin[s] +
   b2*l*(p^2 + q^2)*Cos[s]*Sin[t] + 2*b2*q*r*Sin[s]*Sin[t])*
   (1 - b1*Cos[s]^2 - b1*Cos[t]^2*Sin[s]^2 -
                                   b1*b3*Sin[s]^2*Sin[t]^2))
 + (((-2*q*r)/l - (2*p*q*Cos[t])/Sin[t])*
   (b2*l*Sin[s]*Sin[t] -
    b1*b2*l*Cos[s]^2*Cos[t]^2*Sin[s]*Sin[t] -
    b1*b2*l*Cos[t]^2*Sin[s]^3*Sin[t] -
    b1*b2*b3*l*Cos[s]^2*Sin[s]*Sin[t]^3 -
                          b1*b2*b3*l*Sin[s]^3*Sin[t]^3))} /
   (1 - b1*Cos[s]^2 - b2*Cos[s]^2*Cos[t]^2 +
    b1*b2*Cos[s]^4*Cos[t]^2 -
    b2*Sin[s]^2 - b1*Cos[t]^2*Sin[s]^2 +
    2*b1*b2*Cos[s]^2*Cos[t]^2*Sin[s]^2 +
    b1*b2*Cos[t]^2*Sin[s]^4 -
    b2*b3*Cos[s]^2*Sin[t]^2 + b1*b2*b3*Cos[s]^4*Sin[t]^2 -
    b1*b3*Sin[s]^2*Sin[t]^2 +
    2*b1*b2*b3*Cos[s]^2*Sin[s]^2*Sin[t]^2 +
    b1*b2*b3*Sin[s]^4*Sin[t]^2)
```

```
f3 =
  {((-((Cos[t]*Sin[s])/l) + (b2*Cos[s]^2*Cos[t]*Sin[s])/l +
   (b2*Cos[t]*Sin[s]^3)/l)*
   (F1 - 2*b1*l*p*q*Cos[s]*Cos[t] - 2*b1*p*r*Cos[t]*Sin[s] -
   2*b1*q*r*Cos[s]*Sin[t] + b1*l*(p^2 + q^2)*Sin[s]*Sin[t]))
  + ((-((Cos[s]*Cos[t])/l) + (b1*Cos[s]^3*Cos[t])/l +
   (b1*Cos[s]*Cos[t]*Sin[s]^2)/l)*
   (F2 - 2*b2*p*r*Cos[s]*Cos[t] + 2*b2*l*p*q*Cos[t]*Sin[s] +
   b2*l*(p^2 + q^2)*Cos[s]*Sin[t] + 2*b2*q*r*Sin[s]*Sin[t]))
  + (((-2*q*r)/l - (2*p*q*Cos[t])/Sin[t])*
    (b1*Cos[s]*Cos[t]*Sin[s]*Sin[t] -
     b2*Cos[s]*Cos[t]*Sin[s]*Sin[t]))
  + (((b2*Cos[s]^2*Cos[t]*Sin[t])/l -
     (b1*b2*Cos[s]^4*Cos[t]*Sin[t])/l +
     (b1*Cos[t]*Sin[s]^2*Sin[t])/l -
     (2*b1*b2*Cos[s]^2*Cos[t]*Sin[s]^2*Sin[t])/l -
     (b1*b2*Cos[t]*Sin[s]^4*Sin[t])/l)*
     (F3 + b3*l*p^2 + b3*g*Cos[t] + b3*l*q^2*Sin[t]^2))
  + (((-2*p*r)/l - (g*Sin[t])/l + q^2*Cos[t]*Sin[t])*
   (1 - b1*Cos[s]^2 - b2*Sin[s]^2 - b2*b3*Cos[s]^2*Sin[t]^2 +
   b1*b2*b3*Cos[s]^4*Sin[t]^2 - b1*b3*Sin[s]^2*Sin[t]^2 +
   2*b1*b2*b3*Cos[s]^2*Sin[s]^2*Sin[t]^2 +
   b1*b2*b3*Sin[s]^4*Sin[t]^2))} /
  (1 - b1*Cos[s]^2 - b2*Cos[s]^2*Cos[t]^2 +
   b1*b2*Cos[s]^4*Cos[t]^2 -
   b2*Sin[s]^2 - b1*Cos[t]^2*Sin[s]^2 +
   2*b1*b2*Cos[s]^2*Cos[t]^2*Sin[s]^2 +
   b1*b2*Cos[t]^2*Sin[s]^4 -
   b2*b3*Cos[s]^2*Sin[t]^2 + b1*b2*b3*Cos[s]^4*Sin[t]^2 -
   b1*b3*Sin[s]^2*Sin[t]^2 +
   2*b1*b2*b3*Cos[s]^2*Sin[s]^2*Sin[t]^2 +
   b1*b2*b3*Sin[s]^4*Sin[t]^2)
```

```
f4 =
  {(((b1*Cos[s]*Cos[t]*Sin[s])/Sin[t] -
     (b2*Cos[s]*Cos[t]*Sin[s])/Sin[t])*
    ((-2*p*r)/l - (g*Sin[t])/l + q^2*Cos[t]*Sin[t]))
  + ((F1 - 2*b1*l*p*q*Cos[s]*Cos[t] - 2*b1*p*r*Cos[t]*Sin[s] -
     2*b1*q*r*Cos[s]*Sin[t] + b1*l*(p^2 + q^2)*Sin[s]*Sin[t])*
    (-(Cos[s]/(l*Sin[t])) + (b2*Cos[s]^3*Cos[t]^2)/(l*Sin[t]) +
    (b2*Cos[s]*Cos[t]^2*Sin[s]^2)/(l*Sin[t]) +
    (b2*b3*Cos[s]^3*Sin[t])/l +
    (b2*b3*Cos[s]*Sin[s]^2*Sin[t])/l))
  + ((F2 - 2*b2*p*r*Cos[s]*Cos[t] + 2*b2*l*p*q*Cos[t]*Sin[s] +
     b2*l*(p^2 + q^2)*Cos[s]*Sin[t] + 2*b2*q*r*Sin[s]*Sin[t])*
    (Sin[s]/(l*Sin[t]) -
     (b1*Cos[s]^2*Cos[t]^2*Sin[s])/(l*Sin[t]) -
     (b1*Cos[t]^2*Sin[s]^3)/(l*Sin[t]) -
     (b1*b3*Cos[s]^2*Sin[s]*Sin[t])/l -
      (b1*b3*Sin[s]^3*Sin[t])/l))
  + (((b1*Cos[s]*Sin[s])/l - (b2*Cos[s]*Sin[s])/l)*
     (F3 + b3*l*p^2 + b3*g*Cos[t] + b3*l*q^2*Sin[t]^2))
  + (((-2*q*r)/l - (2*p*q*Cos[t])/Sin[t])*
     (1 - b2*Cos[s]^2*Cos[t]^2 - b1*Cos[t]^2*Sin[s]^2 -
     b2*b3*Cos[s]^2*Sin[t]^2 - b1*b3*Sin[s]^2*Sin[t]^2))} /
   (1 - b1*Cos[s]^2 - b2*Cos[s]^2*Cos[t]^2 +
     b1*b2*Cos[s]^4*Cos[t]^2 -
     b2*Sin[s]^2 - b1*Cos[t]^2*Sin[s]^2 +
     2*b1*b2*Cos[s]^2*Cos[t]^2*Sin[s]^2 +
     b1*b2*Cos[t]^2*Sin[s]^4 -
     b2*b3*Cos[s]^2*Sin[t]^2 + b1*b2*b3*Cos[s]^4*Sin[t]^2 -
     b1*b3*Sin[s]^2*Sin[t]^2 +
     2*b1*b2*b3*Cos[s]^2*Sin[s]^2*Sin[t]^2 +
     b1*b2*b3*Sin[s]^4*Sin[t]^2)
```

```
f5 =
  {(((F2 - 2*b2*p*r*Cos[s]*Cos[t] + 2*b2*l*p*q*Cos[t]*Sin[s] +
   b2*l*(p^2 + q^2)*Cos[s]*Sin[t] + 2*b2*q*r*Sin[s]*Sin[t])*
   (-(b3*Cos[s]*Sin[t]) + b1*b3*Cos[s]^3*Sin[t] +
   b1*b3*Cos[s]*Sin[s]^2*Sin[t]))
  + ((F1 - 2*b1*l*p*q*Cos[s]*Cos[t] - 2*b1*p*r*Cos[t]*Sin[s] -
   2*b1*q*r*Cos[s]*Sin[t] + b1*l*(p^2 + q^2)*Sin[s]*Sin[t])*
   (-(b3*Sin[s]*Sin[t]) + b2*b3*Cos[s]^2*Sin[s]*Sin[t] +
   b2*b3*Sin[s]^3*Sin[t]))
  + (((-2*p*r)/l - (g*Sin[t])/l + q^2*Cos[t]*Sin[t])*
    (b2*b3*l*Cos[s]^2*Cos[t]*Sin[t] -
    b1*b2*b3*l*Cos[s]^4*Cos[t]*Sin[t] +
    b1*b3*l*Cos[t]*Sin[s]^2*Sin[t] -
    2*b1*b2*b3*l*Cos[s]^2*Cos[t]*Sin[s]^2*Sin[t] -
    b1*b2*b3*l*Cos[t]*Sin[s]^4*Sin[t]))
  + ((1 - b1*Cos[s]^2 - b2*Cos[s]^2*Cos[t]^2 +
     b1*b2*Cos[s]^4*Cos[t]^2 -
     b2*Sin[s]^2 - b1*Cos[t]^2*Sin[s]^2 +
     2*b1*b2*Cos[s]^2*Cos[t]^2*Sin[s]^2 +
     b1*b2*Cos[t]^2*Sin[s]^4)*
     (F3 + b3*l*p^2 + b3*g*Cos[t] + b3*l*q^2*Sin[t]^2))
  + ((-2*q*r)/l - (2*p*q*Cos[t])/Sin[t])*
     (b1*b3*l*Cos[s]*Sin[s]*Sin[t]^2 -
          b2*b3*l*Cos[s]*Sin[s]*Sin[t]^2)} /
  (1 - b1*Cos[s]^2 - b2*Cos[s]^2*Cos[t]^2 +
    b1*b2*Cos[s]^4*Cos[t]^2 -
    b2*Sin[s]^2 - b1*Cos[t]^2*Sin[s]^2 +
    2*b1*b2*Cos[s]^2*Cos[t]^2*Sin[s]^2 +
    b1*b2*Cos[t]^2*Sin[s]^4 -
    b2*b3*Cos[s]^2*Sin[t]^2 + b1*b2*b3*Cos[s]^4*Sin[t]^2 -
    b1*b3*Sin[s]^2*Sin[t]^2 +
    2*b1*b2*b3*Cos[s]^2*Sin[s]^2*Sin[t]^2 +
    b1*b2*b3*Sin[s]^4*Sin[t]^2)
```

Appendix B

Description of Linear Torque-Control Model Matrices and Coefficients

The matrices $[C]$, $[E]$, $[F]$, and $[K]$, introduced in equation (3.20) and the coefficients $d_1, \ldots, d_{20}$ and $e_1, \ldots, e_{15}$ introduced in equations (3.21) are shown as follows: (Note that, due to limitations of Mathematica, the symbols $b_1, b_2, b_3, \theta_o, \psi_o, l_o, \dot{l}_o$ and $\ddot{l}_o$ are replaced by b1, b2, b3, to, so, lo, l1, and l2 respectively.

```
[C] = {{1, 0, b1*lo*Sin[so], b1*lo*to*Cos[so],
        b1*to*Sin[so]},
       {0, 1, b2*lo*Cos[so],
        -(b2*lo*to*Sin[so]), b2*to*Cos[so]},
       {Sin[so]/lo, Cos[so]/lo, 1, 0, 0},
       {Cos[so]/(lo*to), -(Sin[so]/(lo*to)), 0, 1, 0},
       {b3*to*Sin[so], b3*to*Cos[so], 0, 0, 1}}

[E] = {{0,0, - 2*b1*l1*Sin[so], -2*b1*l1*to*Cos[so],0},
      {0,0, - 2*b2*l1*Cos[so], - 2*b2*l1*to*Sin[so],0},
      {0,0,- (2*l1)/lo, 0,0},
      {0,0,0, - (2*l1)/lo,0},
      {0,0,0,0,0}}

[K] = {{F1}, {F2}, {0}, {0}, {F3}}

[F] = {{0,0,- b1*l2*Sin[so],- b1*l2*to*Cos[so],0},
      {0,0, - b2*l2*Cos[so] , b2*l2*to*Sin[so],0},
      {0,0,-(g)/lo, 0, 0},
      {0,0,-(g)/lo,0,0},
      {0,0,0,0,0}}
```

```
d1 = [-((b1*b2*b3*l2*to^2*Cos[so]^2*Sin[so]) -
      (b1*l2*Sin[so]*(1 - b2*Cos[so]^2 -
       b2*b3*to^2*Cos[so]^2 - b2*Sin[so]^2)) -
      (g*(-(b1*lo*to*Cos[so]) + b1*b2*lo*to*Cos[so]^3 +
       b1*b2*b3*lo*to^3*Cos[so]^3 +
       b1*b2*lo*to*Cos[so]*Sin[so]^2 +
       b1*b2*b3*lo*to^3*Cos[so]*Sin[so]^2))/lo] -
      (g*(-(b1*lo*Sin[so]) + b1*b2*lo*Cos[so]^2*Sin[so] +
       b1*b2*lo*Sin[so]^3))/lo] /
     (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
      b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
      b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
      b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
      2*b1*b2*Cos[so]^2*Sin[so]^2 +
      2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
      b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)),
```

```
d2= (b1*b2*b3*l2*to^3*Cos[so]*Sin[so]^2) -
     (b1*l2*to*Cos[so]*(1 - b2*Cos[so]^2 -
      b2*b3*to^2*Cos[so]^2 - b2*Sin[so]^2) /

     (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
      b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
      b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
      b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
      2*b1*b2*Cos[so]^2*Sin[so]^2 +
      2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
      b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)
```

```
d3= (-2*b1*b2*b3*l1*to^2*Cos[so]^2*Sin[so]) -
    (2*b1*l1*Sin[so]*(1 - b2*Cos[so]^2 -
     b2*b3*to^2*Cos[so]^2 - b2*Sin[so]^2)) -
    (2*l1*(-(b1*lo*Sin[so]) +
     b1*b2*lo*Cos[so]^2*Sin[so] +
     b1*b2*lo*Sin[so]^3))/lo] /
   (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
     b2*b3*to^2*Cos[so]^2 +
     b1*b2*Cos[so]^4 + b1*b2*b3*to^2*Cos[so]^4 -
     b1*Sin[so]^2 - b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
     2*b1*b2*Cos[so]^2*Sin[so]^2 +
     2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
     b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4))
```

```
d4= [(-2*b1*b2*b3*l1*to^3*Cos[so]*Sin[so]^2) -
     (2*b1*l1*to*Cos[so]*(1 - b2*Cos[so]^2 -
      b2*b3*to^2*Cos[so]^2 - b2*Sin[so]^2)) -
     (2*l1*(-(b1*lo*to*Cos[so]) + b1*b2*lo*to*Cos[so]^3 +
      b1*b2*b3*lo*to^3*Cos[so]^3 +
      b1*b2*lo*to*Cos[so]*Sin[so]^2 +
      b1*b2*b3*lo*to^3*Cos[so]*Sin[so]^2))/lo] /
    (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
     b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
     b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
     b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
     2*b1*b2*Cos[so]^2*Sin[so]^2 +
     2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
     b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)
```

```
d5= -((b1*b2*b3*l2*to^2*Cos[so]*Sin[so]^2) -
      (b2*l2*Cos[so]*(1 - b1*Cos[so]^2 -
       b1*Sin[so]^2 - b1*b3*to^2*Sin[so]^2)) -
      (g*(-(b2*lo*Cos[so]) + b1*b2*lo*Cos[so]^3 +
       b1*b2*lo*Cos[so]*Sin[so]^2))/lo -
      (g*(b2*lo*to*Sin[so] -
       b1*b2*lo*to*Cos[so]^2*Sin[so] -
       b1*b2*b3*lo*to^3*Cos[so]^2*Sin[so] -
       b1*b2*lo*to*Sin[so]^3 -
       b1*b2*b3*lo*to^3*Sin[so]^3))/lo) /
```

```
d6= -((b1*b2*b3*l2*to^3*Cos[so]^2*Sin[so]) +
     (b2*l2*to*Sin[so]*(1 - b1*Cos[so]^2 - b1*Sin[so]^2 -
      b1*b3*to^2*Sin[so]^2)) /
     (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
      b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
      b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
      b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
      2*b1*b2*Cos[so]^2*Sin[so]^2 +
      2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
      b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)
```

```
d7=[(-2*b1*b2*b3*l1*to^2*Cos[so]*Sin[so]^2) -
   (2*b2*l1*Cos[so]*(1 - b1*Cos[so]^2 - b1*Sin[so]^2 -
    b1*b3*to^2*Sin[so]^2)) -
   (2*l1*(-(b2*lo*Cos[so]) + b1*b2*lo*Cos[so]^3 +
     b1*b2*lo*Cos[so]*Sin[so]^2))/lo] /
 (1 - b1*Cos[so]^2 - b2*Cos[so]^2 - b2*b3*to^2*Cos[so]^2 +
   b1*b2*Cos[so]^4 + b1*b2*b3*to^2*Cos[so]^4 -
   b1*Sin[so]^2 - b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
   2*b1*b2*Cos[so]^2*Sin[so]^2 +
   2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
   b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4))
```

```
d8 = [(-2*b1*b2*b3*l1*to^3*Cos[so]^2*Sin[so]) -
     (2*b2*l1*to*Sin[so]*(1 - b1*Cos[so]^2 -
      b1*Sin[so]^2 -  b1*b3*to^2*Sin[so]^2)) -
      2*l1*(b2*lo*to*Sin[so] -
      b1*b2*lo*to*Cos[so]^2*Sin[so] -
      b1*b2*b3*lo*to^3*Cos[so]^2*Sin[so] -
      b1*b2*lo*to*Sin[so]^3 -
      b1*b2*b3*lo*to^3*Sin[so]^3))/ lo] /
 (1 - b1*Cos[so]^2 - b2*Cos[so]^2 - b2*b3*to^2*Cos[so]^2 +
  b1*b2*Cos[so]^4 + b1*b2*b3*to^2*Cos[so]^4 -
  b1*Sin[so]^2 - b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
  2*b1*b2*Cos[so]^2*Sin[so]^2 +
  2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 + b1*b2*Sin[so]^4 +
  b1*b2*b3*to^2*Sin[so]^4)
```

```
d9= [-(g*(b1*to*Cos[so]*Sin[so] -
      b2*to*Cos[so]*Sin[so]))/lo -
     (b2*l2*Cos[so]*(-(Cos[so]/lo) + (b1*Cos[so]^3)/lo +
     (b1*Cos[so]*Sin[so]^2)/lo))-
     (b1*l2*Sin[so]*(-(Sin[so]/lo) +
     (b2*Cos[so]^2*Sin[so])/lo + (b2*Sin[so]^3)/lo)) -
     (g*(1 - b1*Cos[so]^2 - b2*b3*to^2*Cos[so]^2 +
      b1*b2*b3*to^2*Cos[so]^4 - b2*Sin[so]^2 -
      b1*b3*to^2*Sin[so]^2 +
      2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
      b1*b2*b3*to^2*Sin[so]^4))/lo] /
     (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
      b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
      b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
      b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
      2*b1*b2*Cos[so]^2*Sin[so]^2 +
      2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
      b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)


d10= (b2*l2*to*Sin[so]*(-(Cos[so]/lo) +
     (b1*Cos[so]^3)/lo + (b1*Cos[so]*Sin[so]^2)/lo)) -
     (b1*l2*to*Cos[so]*(-(Sin[so]/lo) +
     (b2*Cos[so]^2*Sin[so])/lo + (b2*Sin[so]^3)/lo)) /

     (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
      b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
      b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
      b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
      2*b1*b2*Cos[so]^2*Sin[so]^2 +
      2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
      b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)


d11= [(-2*b2*l1*Cos[so]*(-(Cos[so]/lo) + (b1*Cos[so]^3)/lo +
     (b1*Cos[so]*Sin[so]^2)/lo)) -
     (2*b1*l1*Sin[so]*(-(Sin[so]/lo) +
     (b2*Cos[so]^2*Sin[so])/lo + (b2*Sin[so]^3)/lo)) -
     (2*l1*(1 - b1*Cos[so]^2 - b2*b3*to^2*Cos[so]^2 +
      b1*b2*b3*to^2*Cos[so]^4 - b2*Sin[so]^2 -
      b1*b3*to^2*Sin[so]^2 +
      2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
      b1*b2*b3*to^2*Sin[so]^4))/lo] /
  (1 - b1*Cos[so]^2 - b2*Cos[so]^2 - b2*b3*to^2*Cos[so]^2 +
   b1*b2*Cos[so]^4 + b1*b2*b3*to^2*Cos[so]^4 -
   b1*Sin[so]^2 - b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
   2*b1*b2*Cos[so]^2*Sin[so]^2 +
   2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 + b1*b2*Sin[so]^4 +
   b1*b2*b3*to^2*Sin[so]^4))
```

```
d12 = [(-2*l1*(b1*to*Cos[so]*Sin[so] -
        b2*to*Cos[so]*Sin[so]))/lo] -
       (2*b2*l1*to*Sin[so]*(-(Cos[so]/lo) -
       (b1*Cos[so]^3)/lo + (b1*Cos[so]*Sin[so]^2)/lo))
       (2*b1*l1*to*Cos[so]*(-(Sin[so]/lo) +
       (b2*Cos[so]^2*Sin[so])/lo +
       (b2*Sin[so]^3)/lo))  /
      (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
       b2*b3*to^2*Cos[so]^2 +
       b1*b2*Cos[so]^4 + b1*b2*b3*to^2*Cos[so]^4 -
       b1*Sin[so]^2 -  b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
       2*b1*b2*Cos[so]^2*Sin[so]^2 +
       2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 + b1*b2*Sin[so]^4 +
       b1*b2*b3*to^2*Sin[so]^4)
```

```
d13= -((g*((b1*Cos[so]*Sin[so])/to -
       (b2*Cos[so]*Sin[so])/to))/lo -
       (g*(1 - b2*Cos[so]^2 - b2*b3*to^2*Cos[so]^2 -
        b1*Sin[so]^2 - b1*b3*to^2*Sin[so]^2))/lo -
       (b1*l2*Sin[so]*(-(Cos[so]/(lo*to)) +
       (b2*Cos[so]^3)/(lo*to) + (b2*b3*to*Cos[so]^3)/lo +
       (b2*Cos[so]*Sin[so]^2)/(lo*to) +
       (b2*b3*to*Cos[so]*Sin[so]^2)/lo)) -
       (b2*l2*Cos[so]*(Sin[so]/(lo*to) -
       (b1*Cos[so]^2*Sin[so])/(lo*to) -
       (b1*b3*to*Cos[so]^2*Sin[so])/lo -
       (b1*Sin[so]^3)/(lo*to) -
       (b1*b3*to*Sin[so]^3)/lo)) /
      (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
       b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
       b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
       b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
       2*b1*b2*Cos[so]^2*Sin[so]^2 +
       2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
       b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)
```

```
d14= -((b1*l2*to*Cos[so]*(-(Cos[so]/(lo*to)) +
       (b2*Cos[so]^3)/(lo*to) + (b2*b3*to*Cos[so]^3)/lo +
       (b2*Cos[so]*Sin[so]^2)/(lo*to) +
       (b2*b3*to*Cos[so]*Sin[so]^2)/lo)) +
       (b2*l2*to*Sin[so]*(Sin[so]/(lo*to) -
       (b1*Cos[so]^2*Sin[so])/(lo*to) -
       (b1*b3*to*Cos[so]^2*Sin[so])/lo -
       (b1*Sin[so]^3)/(lo*to) - (b1*b3*to*Sin[so]^3)/lo)) /
      (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
       b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
       b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
       b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
       2*b1*b2*Cos[so]^2*Sin[so]^2 +
       2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
       b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)
```

```
d15 = [{(-2*l1*((b1*Cos[so]*Sin[so])/to -
       (b2*Cos[so]*Sin[so])/to))}/lo -
       (2*b1*l1*Sin[so]*(-(Cos[so]/(lo*to)) +
       (b2*Cos[so]^3)/(lo*to) + (b2*b3*to*Cos[so]^3)/lo +
       (b2*Cos[so]*Sin[so]^2)/(lo*to) +
       (b2*b3*to*Cos[so]*Sin[so]^2)/lo)) -
       (2*b2*l1*Cos[so]*(Sin[so]/(lo*to)
       - (b1*Cos[so]^2*Sin[so])/(lo*to) -
       (b1*b3*to*Cos[so]^2*Sin[so])/lo -
       (b1*Sin[so]^3)/(lo*to) -
       (b1*b3*to*Sin[so]^3)/lo))]   /
     (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
      b2*b3*to^2*Cos[so]^2 +  b1*b2*Cos[so]^4 +
      b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
      b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
      2*b1*b2*Cos[so]^2*Sin[so]^2 +
      2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
      b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)
```

```
d16 = [(-2*l1*(1 - b2*Cos[so]^2 - b2*b3*to^2*Cos[so]^2 -
       b1*Sin[so]^2 - b1*b3*to^2*Sin[so]^2))/lo -
      (2*b1*l1*to*Cos[so]*(-(Cos[so]/(lo*to)) +
      (b2*Cos[so]^3)/(lo*to) + (b2*b3*to*Cos[so]^3)/lo +
      (b2*Cos[so]*Sin[so]^2)/(lo*to) +
      (b2*b3*to*Cos[so]*Sin[so]^2)/lo)) -
      (2*b2*l1*to*Sin[so]*(Sin[so]/(lo*to) -
      (b1*Cos[so]^2*Sin[so])/(lo*to) -
      (b1*b3*to*Cos[so]^2*Sin[so])/lo -
      (b1*Sin[so]^3)/(lo*to) -
      (b1*b3*to*Sin[so]^3)/lo))]  /
  (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
  b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
  b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
  b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
  2*b1*b2*Cos[so]^2*Sin[so]^2 +
  2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
  b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)
```

```
d17= -((g*(b1*b3*lo*to^2*Cos[so]*Sin[so] -
       b2*b3*lo*to^2*Cos[so]*Sin[so]))/lo -
      (b2*l2*Cos[so]*(-(b3*to*Cos[so]) +
       b1*b3*to*Cos[so]^3 +
       b1*b3*to*Cos[so]*Sin[so]^2)) -
      (b1*l2*Sin[so]*(-(b3*to*Sin[so]) +
       b2*b3*to*Cos[so]^2*Sin[so] +
       b2*b3*to*Sin[so]^3)) - (g*(b2*b3*lo*to*Cos[so]^2 -
       b1*b2*b3*lo*to*Cos[so]^4 + b1*b3*lo*to*Sin[so]^2 -
       2*b1*b2*b3*lo*to*Cos[so]^2*Sin[so]^2 -
       b1*b2*b3*lo*to*Sin[so]^4)) /

      (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
       b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
       b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
       b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
       2*b1*b2*Cos[so]^2*Sin[so]^2 +
       2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
       b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)
```

```
d18= (b2*l2*to*Sin[so]*(-(b3*to*Cos[so]) +
      b1*b3*to*Cos[so]^3 + b1*b3*to*Cos[so]*Sin[so]^2)) -
     (b1*l2*to*Cos[so]*(-(b3*to*Sin[so]) +
      b2*b3*to*Cos[so]^2*Sin[so] + b2*b3*to*Sin[so]^3) /

     (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
      b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
      b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
      b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
      2*b1*b2*Cos[so]^2*Sin[so]^2 +
      2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
      b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)


d19 = [(-2*b2*l1*Cos[so]*(-(b3*to*Cos[so]) +
       b1*b3*to*Cos[so]^3 + b1*b3*to*Cos[so]*Sin[so]^2)) -
      (2*b1*l1*Sin[so]*(-(b3*to*Sin[so]) +
       b2*b3*to*Cos[so]^2*Sin[so] + b2*b3*to*Sin[so]^3)) -
      (2*l1*(b2*b3*lo*to*Cos[so]^2 -
       b1*b2*b3*lo*to*Cos[so]^4 + b1*b3*lo*to*Sin[so]^2 -
       2*b1*b2*b3*lo*to*Cos[so]^2*Sin[so]^2 -
       b1*b2*b3*lo*to*Sin[so]^4))/lo] /
      (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
       b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
       b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
       b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
       2*b1*b2*Cos[so]^2*Sin[so]^2 +
       2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
       b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4))


d20 = (-2*l1*(b1*b3*lo*to^2*Cos[so]*Sin[so] -
       b2*b3*lo*to^2*Cos[so]*Sin[so]))/lo -
      (2*b2*l1*to*Sin[so]*(-(b3*to*Cos[so]) +
      b1*b3*to*Cos[so]^3 + b1*b3*to*Cos[so]*Sin[so]^2)) -
     (2*b1*l1*to*Cos[so]*(-(b3*to*Sin[so]) +
      b2*b3*to*Cos[so]^2*Sin[so] + b2*b3*to*Sin[so]^3))  /
    (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
     b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
     b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
     b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
     2*b1*b2*Cos[so]^2*Sin[so]^2 +
     2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
     b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)
```

```
e1 = (1 - b2*Cos[so]^2 - b2*b3*to^2*Cos[so]^2 -
      b2*Sin[so]^2))  /
     (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
      b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
      b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
      b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
      2*b1*b2*Cos[so]^2*Sin[so]^2 +
      2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
      b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)
```

```
e2 = b1*b3*to^2*Cos[so]*Sin[so]) /
     (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
      b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
      b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
      b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
      2*b1*b2*Cos[so]^2*Sin[so]^2 +
      2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
      b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)
```

```
e3= (-(b1*to*Sin[so]) + b1*b2*to*Cos[so]^2*Sin[so] +
      b1*b2*to*Sin[so]^3))/
     (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
      b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
      b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
      b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
      2*b1*b2*Cos[so]^2*Sin[so]^2 +
      2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
      b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)
```

```
e4= b2*b3*to^2*Cos[so]*Sin[so]) /
    (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
     b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
     b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
     b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
     2*b1*b2*Cos[so]^2*Sin[so]^2 +
     2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
     b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)


e5= (1 - b1*Cos[so]^2 - b1*Sin[so]^2 -
     b1*b3*to^2*Sin[so]^2) /
    (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
     b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
     b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
     b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
     2*b1*b2*Cos[so]^2*Sin[so]^2 +
     2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
     b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)


e6= (-(b2*to*Cos[so]) + b1*b2*to*Cos[so]^3 +
     b1*b2*to*Cos[so]*Sin[so]^2) /
    (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
     b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
     b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
     b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
     2*b1*b2*Cos[so]^2*Sin[so]^2 +
     2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2  +
     b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)


e7= (-(Sin[so]/lo) + (b2*Cos[so]^2*Sin[so])/lo +
     (b2*Sin[so]^3)/lo) /
    (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
     b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
     b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
     b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
     2*b1*b2*Cos[so]^2*Sin[so]^2 +
     2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
     b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)
```

```
e8 = (-(Cos[so]/lo) + (b1*Cos[so]^3)/lo +
      (b1*Cos[so]*Sin[so]^2)/lo) /
     (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
      b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
      b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
      b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
      2*b1*b2*Cos[so]^2*Sin[so]^2 +
      2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
      b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)


e9= ((b2*to*Cos[so]^2)/lo - (b1*b2*to*Cos[so]^4)/lo +
      (b1*to*Sin[so]^2)/lo -
      (2*b1*b2*to*Cos[so]^2*Sin[so]^2)/lo -
      (b1*b2*to*Sin[so]^4)/lo) /
     (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
      b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
      b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
      b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
      2*b1*b2*Cos[so]^2*Sin[so]^2 +
      2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
      b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)


e10= (-(Cos[so]/(lo*to)) + (b2*Cos[so]^3)/(lo*to) +
      (b2*b3*to*Cos[so]^3)/lo +
      (b2*Cos[so]*Sin[so]^2)/(lo*to) +
      (b2*b3*to*Cos[so]*Sin[so]^2)/lo)) /
     (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
      b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
      b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
      b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
      2*b1*b2*Cos[so]^2*Sin[so]^2 +
      2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
      b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)


e11= (Sin[so]/(lo*to) - (b1*Cos[so]^2*Sin[so])/(lo*to) -
      (b1*b3*to*Cos[so]^2*Sin[so])/lo -
      (b1*Sin[so]^3)/(lo*to) - (b1*b3*to*Sin[so]^3)/lo)) /
     (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
      b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
      b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
      b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
      2*b1*b2*Cos[so]^2*Sin[so]^2 +
      2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
      b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)
```

```
e12= ((b1*Cos[so]*Sin[so])/lo -
      (b2*Cos[so]*Sin[so])/lo)) /
     (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
      b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
      b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
      b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
      2*b1*b2*Cos[so]^2*Sin[so]^2 +
      2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
      b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)


e13= (-(b3*to*Sin[so]) + b2*b3*to*Cos[so]^2*Sin[so] +
      b2*b3*to*Sin[so]^3) /
     (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
      b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
      b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
      b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
      2*b1*b2*Cos[so]^2*Sin[so]^2 +
      2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
      b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)


e14= (-(b3*to*Cos[so]) + b1*b3*to*Cos[so]^3 +
      b1*b3*to*Cos[so]*Sin[so]^2) /
     (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
      b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4 +
      b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
      b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
      2*b1*b2*Cos[so]^2*Sin[so]^2 +
      2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
      b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)


e15= (1 - b1*Cos[so]^2 - b2*Cos[so]^2 +
      b1*b2*Cos[so]^4 - b1*Sin[so]^2 -
      b2*Sin[so]^2 + 2*b1*b2*Cos[so]^2*Sin[so]^2 +
      b1*b2*Sin[so]^4) /
     (1 - b1*Cos[so]^2 - b2*Cos[so]^2 -
      b2*b3*to^2*Cos[so]^2 + b1*b2*Cos[so]^4  +
      b1*b2*b3*to^2*Cos[so]^4 - b1*Sin[so]^2 -
      b2*Sin[so]^2 - b1*b3*to^2*Sin[so]^2 +
      2*b1*b2*Cos[so]^2*Sin[so]^2 +
      2*b1*b2*b3*to^2*Cos[so]^2*Sin[so]^2 +
      b1*b2*Sin[so]^4 + b1*b2*b3*to^2*Sin[so]^4)
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