King Fahd University of Petroleum and Minerals

Mechanical Engineering Department

System Dynamics and Control

(ME 413)

Term Project

SALMAN AL-FIFI

ID # 991694

DATA ANALYSIS:

PART(1):

(1) k_i *reduces the steady state error to zero.*

PART(2):

As we can see from the attached experiment :
For part(a),when k_i is fixed, we note that as k_p increases thae maximum over shoot, number of oscillation, settling time, and rise time increases.

For part(b), where k_p is fixed, it's noticeable that when $k_i<1$, the system has no oscillation (underdamped). However, $k_i=1$ the system is in a critical region.But, when $k_i>1$, oscillations start to occur and get higher as k_i gets higher.

PART(3):

FINDING THE CLOSED LOOP TRANSFER FUNCTION:

By Mason's gain formula:

$$path \rightarrow p_1 = \left(\frac{k_p s + k_i}{s}\right) * \left(\frac{G_1}{Ts+1}\right)$$

$$loop \rightarrow l_1 = -\left(\frac{k_p s + k_i}{s}\right) * \left(\frac{G_1}{Ts+1}\right)$$

$$\Delta = 1 - \sum l_i = 1 + \left(\frac{k_p s + k_i}{s}\right) * \left(\frac{G_1}{Ts+1}\right)$$

$$\Delta_1 = 1$$

$$\frac{Y(s)}{V(s)} = \frac{\sum p_i l_i}{\Delta} = \frac{\left(\frac{k_p s + k_i}{s}\right) * \left(\frac{G_1}{Ts+1}\right)}{1 + \left(\frac{k_p s + k_i}{s}\right) * \left(\frac{G_1}{Ts+1}\right)} \xrightarrow{unifying\ .the\ .denumerato\ r} \frac{\frac{[k_p s + k_i]G_1}{s(Ts+1)}}{\frac{s(Ts+1) + [(k_p s + k_i)G_1]}{s(Ts+1)}}$$

$$\xrightarrow{s(Ts+1)will\ .cancel\ .with\ .s(Ts+1)of\ .the\ .numenator} \frac{[k_p s + k_i]G_1}{s(Ts+1) + [(k_p s + k_i)G_1]} \Rightarrow \frac{[k_p s + k_i]G_1}{Ts^2 + s + G_1 k_p s + G_1 k_i}$$

$$\Rightarrow \frac{[k_p s + k_i]G_1}{Ts^2 + (G_1 k_p + 1)s + G_1 k_i} \Rightarrow \frac{[k_p s + k_i]G_1}{s^2 + \frac{(G_1 k_p + 1)}{T}s + \frac{G_1 k_i}{T}} \xrightarrow{comparing\ .denumerato\ r.with} s^2 + 2\zeta\omega_n s + \omega_n^2$$

Since, we are given that T=1.5 & $G_1=1$:

$$\omega_n = \sqrt{\frac{k_i}{1.5}} \quad \text{and} \quad \xi = \frac{(G_1 k_p + 1)}{3\sqrt{\frac{k_i}{1.5}}}$$

GETTINING M_P, ω_n,& ξ FROM THE EXPERIMENTAL PLOTS:

We find M_P and the period T from plots then, we use the following formulas:

$$i\ \xi = \frac{|\ln(M_p)|}{\sqrt{\pi^2 + \ln(M_p)^2}} \quad \text{and} \quad \omega_n = \frac{\omega_d}{\sqrt{1-\xi^2}} \quad \text{also, } T = \frac{mesured.dis\tan ce.on.plot(mm)}{speed.of.paper(2\ mm/\sec)} = \frac{2\pi}{\omega_d}$$

EXPERIMENTAL PART:

Table 3:

Ki	Kp	T	Mp	Wd	zeta	Wn
3.7	0.9	4	0.538462	1.57	0.060378	1.57287
3.7	1.8	4	0.237288	1.57	0.120488	1.581522
3.7	2.8	4	0.190789	1.57	0.13133	1.583717
3.7	3.7	4.25	0.258621	1.477647	0.115603	1.487621

Table 4:

Ki	Kp	T	Mp	Wd	zeta	Wn
2.4	0.9	4.5	0.323529	1.395556	0.101271	1.402767
3.7	1.4	4	0.217391	1.57	0.125102	1.582432
6.6	2.2	3	0.5	2.093333	0.06697	2.098044
10	3.7	2.5	0.166667	2.512	0.136985	2.535906

SIMULATION PART:

TABLE: 5

k_i	K_p	ω_n	ξ
3.7	0.9	1.57	0.403
3.7	1.8	1.57	0.594
3.7	2.8	1.57	0.807
3.7	3.7	1.57	0.998

Comment:
Since ω_n is a function of k_i only and k_i is constant, its value is constant also. While, ξ is changing because it is a function of k_i and K_p. Moreover, also we can conclude that ξ is directly proportional to K_p as seen above.

TABLE: 6

k_i	K_p	ω_n	ξ
2.4	0.9	1.265	0.5
3.7	1.4	1.57	0.51
6.6	2.1	2.098	0.493
10	3.7	2.58	0.606

Comment:
Here since ξ is nearly constant, we can see that is directly proportional to k_i.

COMPARSION BETWEEN SIMULATION AND EXPERIMENTAL PART:

$$\%ERROR=\left|\frac{\omega_{n_{\exp}}-\omega_{n_{theory}}}{\omega_{n_{theory}}}\right|$$

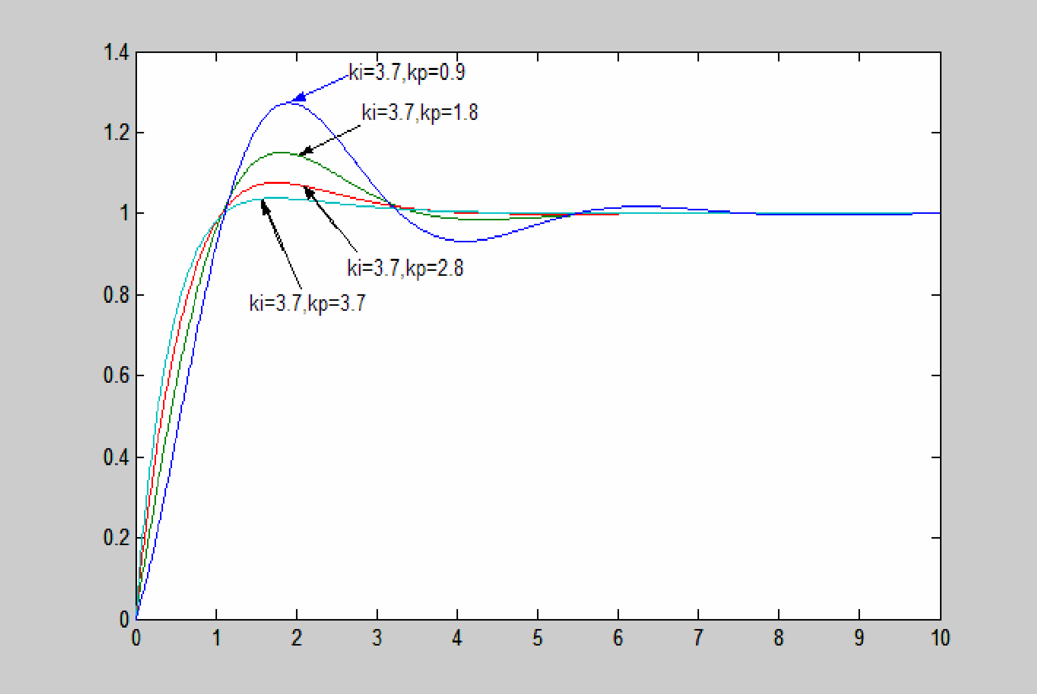
between table 3&5:
%error are %0,%0,%0, and %5.88 respectively.

Between table 4&6:
%error are %2,%0, %0.8 and %10.89 respectively.

MATLAB SIMULATION:

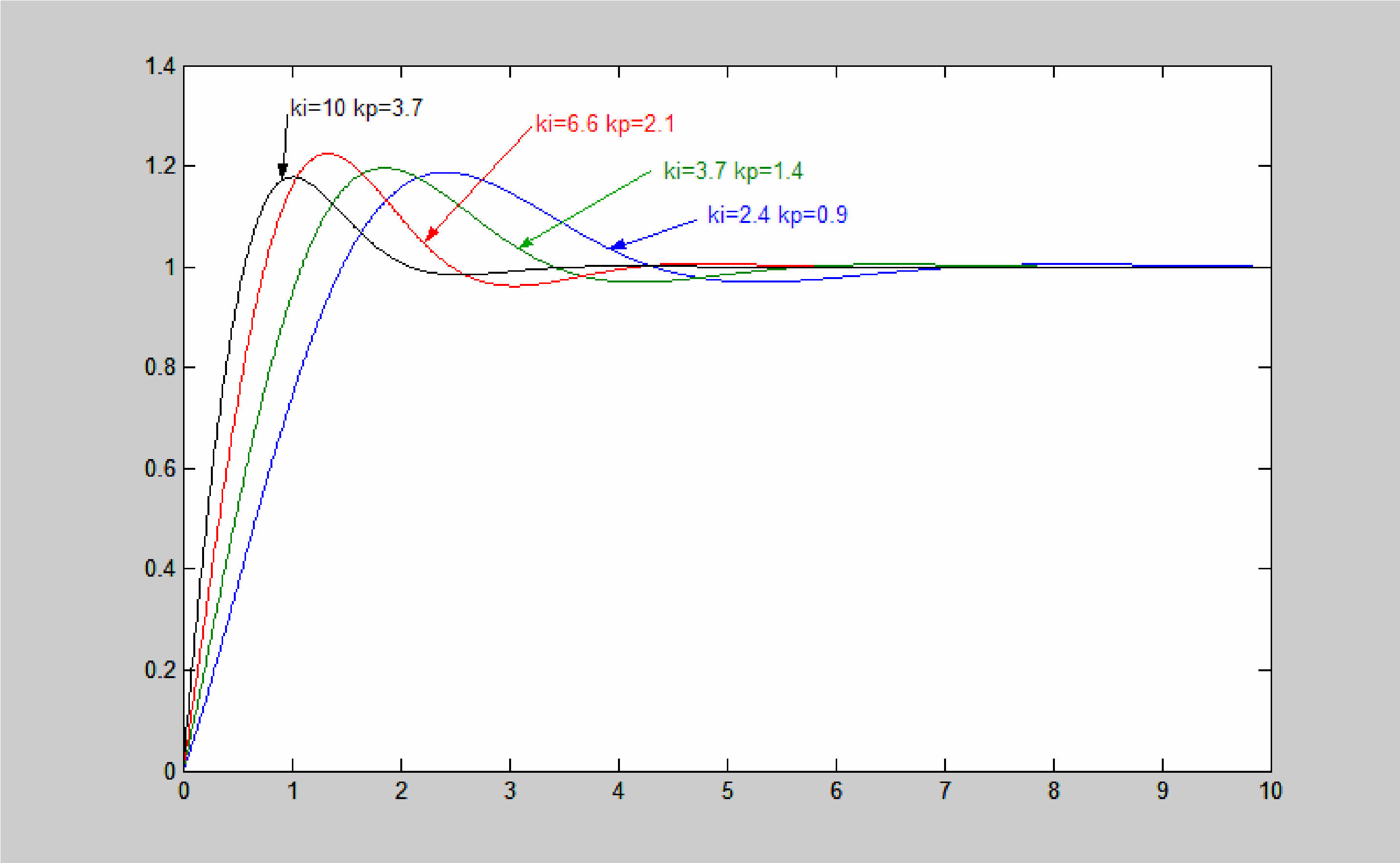
FOR TABLE 5:

```
num1=[0.9/1.5 3.7/1.5];
den1=[1 1.9/1.5 3.7/1.5];
num2=[1.2 3.7/1.5];
den2=[1 2.8/1.5 3.7/1.5];
num3=[2.8/1.5 3.7/1.5];
den3=[1 3.8/1.5 3.7/1.5];
num4=[3.7/1.5 3.7/1.5];
den4=[1 4.7/1.5 3.7/1.5];
t=0:0.01:10;
[c1,x1,t]=step(num1,den1,t);
[c2,x2,t]=step(num2,den2,t);
[c3,x3,t]=step(num3,den3,t);
[c4,x4,t]=step(num4,den4,t);
plot(t,c1, t,c2, t,c3, t,c4);
```



FOR TABLE 6:

```
num1=[0.6 1.6];
den1=[1 1.9/1.5 1.6];
num2=[1.4/1.5 3.7/1.5];
den2=[1 1.6 3.7/1.5];
num3=[1.4 4.4];
den3=[1 3.1/1.5 4.4];
num4=[3.7/1.5 10/1.5];
den4=[1 4.7/1.5 10/1.5];
t=0:0.01:10;
[c1,x1,t]=step(num1,den1,t);
[c2,x2,t]=step(num2,den2,t);
[c3,x3,t]=step(num3,den3,t);
[c4,x4,t]=step(num4,den4,t);
plot(t,c1, t,c2, t,c3, t,c4);
```


ki=10 kp=3.7
ki=6.6 kp=2.1
ki=3.7 kp=1.4
ki=2.4 kp=0.9
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
7
8
9
10