King Fahad University of Petroleum and Minerals

Aerospace Engineering Department

Fight Dynamics and Control

Aircraft Stability Derivatives and Flying Qualities

Done By:
SALMAN AL-FIFI
ID # 991694

Supervised By

Dr: Hanfy M.Omar

15/11/1424 H
7/1/2004 G

Objectives:

Based on the fighter aircraft (F104-A), the following is required:

- Estimating the longitudinal and lateral stability derivativesFinding longitudinal eigenvalues and compare them with those calculated by using phugoid and short period approximations.

- Finding the lateral eigenvalues and compare them with the approximation solutions.

- Plotting the time response of longitudinal and lateral motions.

- Designing a state feedback system if the flying qualities are not satisfied.

Background information:

Aircraft has many aspects that should be maintained properly in design, according to the type of the aircraft; all flying qualities would be different from one airplane to another based on the specific mission of that airplane. For example, the general aviation aircraft must be stable in all conditions to be more secure and comfortable for passengers. On the other hand, the fighter aircraft needs to be sometimes unstable to be more available for maneuvering. These differences occur because of stability derivatives that differ from one aircraft to another.

In this project, we will consider this issue and we will calculate these derivatives in a specific speed and then monitoring the results and see whether they give reasonable solutions and expected results.

In my own project I will consider the fighter aircraft **(F104-A)** to find what stated in the objectives and then come up with some comments and conclusions about the results.

Here is my aircraft with some coefficients that were given already but there are some other coefficients need to be calculated.

Aircraft: F104-A

C_L	C_D	C_{L_α}	C_{D_α}	C_{m_α}	$C_{L_{\dot\alpha}}$	$C_{m_{\dot\alpha}}$	C_{L_q}	C_{m_q}	C_{L_M}	C_{D_M}	C_{m_M}	$C_{L_{\delta_e}}$	$C_{m_{\delta_e}}$
0.735	0.263	3.44	0.45	−0.64	0.0	−1.6	0.0	−5.8	0.0	0.0	0.0	0.68	−1.46
0.2	0.055	2.0	0.38	−1.30	0.0	−2.0	0.0	−4.8	−0.2	0.0	−0.01	0.52	−0.10

C_{y_β}	C_{l_β}	C_{n_β}	C_{l_p}	C_{n_p}	C_{l_r}	C_{n_r}	$C_{l_{\delta_a}}$	$C_{n_{\delta_a}}$	$C_{y_{\delta_r}}$	$C_{l_{\delta_r}}$	$C_{n_{\delta_r}}$
−1.17	−0.175	0.50	−0.285	−0.14	0.265	−0.75	0.039	0.0042	0.208	0.045	−0.16
−1.0	−0.09	0.24	−0.27	−0.09	0.15	−0.65	0.017	0.0025	0.05	0.008	−0.04

Derivatives are per radian.

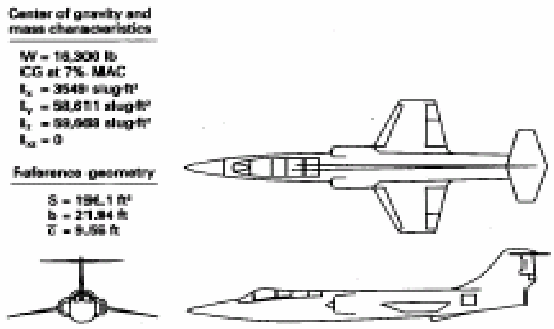

FIGURE B.2
Three-view sketch and stability data for the F-104-A fighter.

Figure (1)

I. **Estimating the longitudinal stability derivative**:

We use the following table to find the longitudinal coefficients:

TABLE 3.3
Equations for estimating the longitudinal stability coefficients

	X-force derivatives	Z-force derivatives	Pitching moment derivatives
u	$C_{X_u} = -[C_{D_u} + 2C_{D_0}] + C_{T_u}$	$C_{Z_u} = -\frac{M^2}{1-M^2}C_{L_0} - 2C_{L_0}$	$C_{m_u} = \frac{\partial C_m}{\partial M}M_0$
α	$C_{X_\alpha} = C_{L_0} - \frac{2C_{L_0}}{\pi e}\frac{C_{L_\alpha}}{AR}$	$C_{Z_\alpha} = -(C_{L_\alpha} + C_{D_0})$	$C_{m_\alpha} = C_{L_{\alpha w}}\left(\frac{X_{cg}}{\bar c} - \frac{X_{ac}}{\bar c}\right) + C_{m_{\alpha f}} - \eta V_H C_{L_{\alpha t}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$
$\dot\alpha$	0	$C_{Z_{\dot\alpha}} = -2\eta C_{L_{\alpha t}} V_H \frac{d\varepsilon}{d\alpha}$	$C_{m_{\dot\alpha}} = -2\eta C_{L_{\alpha t}} V_H \frac{l_t}{\bar c}\frac{d\varepsilon}{d\alpha}$
q	0	$C_{Z_q} = -2\eta C_{L_{\alpha t}} V_H$	$C_{m_q} = -2\eta C_{L_{\alpha t}} V_H \frac{l_t}{\bar c}$
δ_e	0	$C_{Z_{\delta_e}} = -C_{L_{\delta_e}} = -\frac{S_t}{S}\eta\frac{dC_{L_t}}{d\delta_e}$	$C_{m_{\delta_e}} = -\eta V_H \frac{dC_{L_t}}{d\delta_e}$

Subscript 0 indicates reference values and M is the Mach number.

- AR Aspect ratio
- C_{D_0} Reference drag coefficient
- C_{L_0} Reference lift coefficient
- C_{L_α} Airplane lift curve slope
- $C_{L_{\alpha w}}$ Wing lift curve slope
- $C_{L_{\alpha t}}$ Tail lift curve slope
- $\bar c$ Mean aerodynamic chord
- e Oswald's span efficiency factor
- l_t Distance from center of gravity to tail quarter chord
- V_H Horizontal tail volume ratio
- M Flight mach number
- S Wing area
- S_t Horizontal tail area
- $\frac{d\varepsilon}{d\alpha}$ Change in downwash due to a change in angle of attack
- η Efficiency factor of the horizontal tail

Figure (2)
Some of the coefficients are not available in the table.

By using the geometry used in figure 1 we can estimate some parameters to calculate the needed coefficients.

For the X-Force:

$$\boxed{C_{xu} = -3\, C_D = -0.526}$$

$C_{X\alpha} = C_L\, [1 - 2\, C_{L\alpha} / (\pi e\, AR)]$

Assuming that $e = 1$

$AR = b^2/s = 2.45$

So:

$$\boxed{C_{X\alpha} = 0.078/\text{rad}}$$

For the Z-Force:

$$C_{ZU} = -M^2/(1 - M^2)\, C_L - 2\, C_L$$

Where $M = 0.257$

And $C_L = 0.735$ (from appendix B)

$$\boxed{C_{ZU} = -1.52}$$

$C_{Z\alpha} = -(C_{L\alpha} + C_D)$

$$\boxed{C_{Z\alpha} = -3.7/\text{rad}}$$

$C_{Z\dot{\alpha}} = -2\, C_{L\alpha t}\, \eta\, V_H\, d\varepsilon/d_\alpha$

$d\varepsilon/d_\alpha = 2\, C_{L\alpha w} / (\pi\, AR) = 0.894$

But: $V_H = l_t S_t / SC$

Assuming that $S_t = 55.4\ \text{ft}^2$, $l_t = 21.94$ ft and $\eta = 1$

$V_H = 0.65$

And assuming that $C_{L\alpha t} = C_{L\alpha}/1.486 = 2.31$ /rad

So

$C_{Z\dot{\alpha}} = -2.68$ /rad

$C_{Zq} = -2\, C_{L\alpha t}\, \eta\, V_H$

$C_{Zq} = 3$/rad

$C_{Z\delta e} = -\, C_{L\alpha t}\, \eta\, \tau\, S_t / S_w$

From our text book (figure 2.21)

$\tau = 0.5$

So:

$C_{Z\delta e} = -0.33$/rad

Above calculations were for the coefficients that are not included in the appendix.
Here, all the longitudinal coefficients that w need are listed below in table1:

$C_{xu} = -0.526$ $C_{m\dot{\alpha}} = -1.6$ $C_{x\alpha} = 0.078$ $C_{mq} = -5.8$
$C_{zu} = -1.52$ $C_{Lq} = 0$
$C_{z\alpha} = -3.7$ $C_{z\dot{\alpha}} = -2.68$ (assume $S_t = 55.4\ ft^2$ and $l_t = 21.94$ ft)
$C_{zq} = 3$ $C_{z\delta e} = -0.33$ (assume $S_e = 13.85\ ft^2$, from the the figure 2.21 in our book $\tau = 0.5$)
$C_L = 0.735$ $C_{L\alpha} = 3.44$ $C_{m\alpha} = -0.64$ $C_D = 0.263$ $C_{D\alpha} = 0.45$ $C_{L\dot{\alpha}} = 0$

After finding the longitudinal coefficients we find the longitudinal stability derivatives by using the following equations in figure 3:

TABLE 3.5
Summary of longitudinal derivatives

$X_u = \frac{-(C_{D_u} + 2C_{D_0})QS}{m u_0} (s^{-1})$ $X_w = \frac{-(C_{D_\alpha} - C_{L_0})QS}{m u_0} (s^{-1})$

$Z_u = \frac{-(C_{L_u} + 2C_{L_0})QS}{m u_0} (s^{-1})$

$Z_w = \frac{-(C_{L_\alpha} + C_{D_0})QS}{m u_0} (s^{-1})$ $Z_{\dot{w}} = -C_{z_{\dot{\alpha}}} \frac{c}{2u_0} QS/(u_0 m)$

$Z_\alpha = u_0 Z_w$ (ft/s²) or (m/s²) $Z_{\dot{\alpha}} = u_0 Z_{\dot{w}}$ (ft/s) or (m/s)

$Z_q = -C_{z_q} \frac{c}{2u_0} QS/m$ (ft/s) or (m/s) $Z_{\delta_e} = -C_{z_{\delta_e}} QS/m$ (ft/s²)

Figure (3)

$m = w/g = 16300/32.2 = 506.2$ slug

Assuming the velocity $u_0 = 291.3$ ft/s at sea level

$Q = ½\ \rho\ u_0 = 1009.8\ lb/ft^2$

$QS/m = 391.2\ ft/s^2$

$QS/mu_0 = 1.34\ s^{-1}$

$C/2\ u_0 = 0.016$ s

$QSC/I_y = 32.27\ S^{-2}$

$(C/2\ u_0)\ QSC/\ I_y = .516\ S^{-1}$

By using the equations in figure 3,we find all needed longitudinal derivatives as follows in table 2:

$x_u = -\ 1.706$	$x_\alpha = 30.51$	$x_w = 0.383$
$z_\alpha = -\ 1447.44$	$z_{\alpha\cdot} = -\ 16.77$	$z_{\delta e} = -\ 129.1$

$z_u = -2.04$	$z_w = -4.98$	$z_{\dot{w}} = -0.058$
$M_\alpha = -20.65$	$M_{\dot{\alpha}} = -0.83$	
$M_q = -2.99$	$M_u = 0$	$M_{\delta e} = -47.11$
$M_{\dot{w}} = -0.00285$	$M_w = -0.071$	

Table (2)

II. **Estimating the lateral stability derivatives:**

We use the following table to find the lateral coefficients:

TABLE 3.4
Equations for estimating the lateral stability coefficients

	Y-force derivatives	Yawing moment derivatives	Rolling moment derivatives
β	$C_{y_\beta} = -\eta \frac{S_v}{S} C_{L_{\alpha_v}} \left(1 + \frac{d\sigma}{d\beta}\right)$	$C_{n_\beta} = C_{n_{\beta_{wf}}} + \eta_v V_v C_{L_{\alpha_v}} \left(1 + \frac{d\sigma}{d\beta}\right)$	$C_{l_\beta} = \left(\frac{C_{l_\beta}}{\Gamma}\right)\Gamma + \Delta C_{l_\beta}$ (see Figure 3.11)
p	$C_{y_p} = C_L \frac{AR + \cos\Lambda}{AR + 4\cos\Lambda} \tan\Lambda$	$C_{n_p} = -\frac{C_L}{8}$	$C_{l_p} = -\frac{C_{L_\alpha}}{12}\frac{1 + 3\lambda}{1 + \lambda}$
r	$C_{y_r} = -2\left(\frac{l_v}{b}\right)(C_{y_\beta})_{tail}$	$C_{n_r} = -2\eta_v V_v \left(\frac{l_v}{b}\right) C_{L_{\alpha_v}}$	$C_{l_r} = \frac{C_L}{4} - 2\frac{l_v}{b}\frac{z_v}{b} C_{y_{\beta_{tail}}}$
δ_a	0	$C_{n_{\delta_a}} = 2KC_{L_0}C_{l_{\delta_a}}$ (see Figure 3.12)	$C_{l_{\delta_a}} = \frac{2C_{L_{\alpha_w}}\tau}{Sb}\int_{y_1}^{y_2} cy\, dy$
δ_r	$C_{y_{\delta_r}} = \frac{S_v}{S}\tau C_{L_{\alpha_v}}$	$C_{n_{\delta_r}} = -V_v \eta_v \tau C_{L_{\alpha_v}}$	$C_{l_{\delta_r}} = \frac{S_v}{S}\left(\frac{z_v}{b}\right)\tau C_{L_{\alpha_v}}$

AR Aspect ratio
b Wingspan
C_{L_0} Reference lift coefficient
C_{L_α} Airplane lift curve slope
$C_{L_{\alpha_w}}$ Wing lift curve slope
$C_{L_{\alpha_v}}$ Tail lift curve slope
$\bar{c}$ Mean aerodynamic chord
K empirical factor
l_v Distance from center of gravity to vertical tail aerodynamic center
V_v Vertical tail volume ratio
S Wing area
S_v Vertical tail area
z_v Distance from center of pressure of vertical tail to fuselage centerline
Γ Wing dihedral angle
Λ Wing sweep angle
η_v Efficiency factor of the vertical tail
λ Taper ratio (tip chord/root chord)
$\frac{d\sigma}{d\beta}$ Change in sidewash angle with a change in sideslip angle

Figure (4)

Some of the lateral coefficients are not available in the appendix.
By using the geometry used in figure 1 we can estimate some parameters to calculate the needed coefficients.

$C_{YP} = C_L \tan \hat{} \, [AR + \cos \hat{}]/[AR + 4 \cos \hat{}]$

Where ^ is the swept angle
From table 1 ⟶ ^ = 0
So:

$C_{YP} = 0$

$C_{Yr} = - 2\,(l_v / b)$

Assuming that l_v = 15.5 ft

So:

$C_{Yr} = 1.65$

The rest of coefficients are available in the appendix as follows in table (3):

$C_{y\beta} = - 1.17$	$C_{yp} = 0$ (assume that the swept angle = 0)	
$C_{lp} = - 0285$	$C_{l\beta} = - 0.175$	$C_{n\beta} = 0.5$
$C_{np} = - 0.14$	$C_{yr} = 1.65$ (assume l_V = 15.5 ft)	
$C_{nr} = - 0.75$	$C_{lr} = 0.265$	$C_{y\delta a} = 0$
$C_{y\delta r} = 0.208$	$C_{n\delta a} = 0.0042$	$C_{n\delta r} = - 0.16$
$C_{l\delta a} = 0.039$	$C_{l\delta r} = 0.045$	

Table (3)

After finding the lateral coefficients we find the lateral stability derivatives by using the following equations in figure (5):

TABLE 3.6

Summary of lateral directional derivatives

$$Y_\beta = \frac{QSC_{y\beta}}{m}\ (\text{ft/s}^2)\ \text{or}\ (\text{m/s}^2) \qquad N_\beta = \frac{QSbC_{n\beta}}{I_z}\ (\text{s}^{-2}) \qquad L_\beta = \frac{QSbC_{l\beta}}{I_x}\ (\text{s}^{-2})$$

$$Y_p = \frac{QSbC_{yp}}{2mu_0}\ (\text{ft/s})\ (\text{m/s}) \qquad N_p = \frac{QSb^2C_{np}}{2I_zu_0}\ (\text{s}^{-1}) \qquad L_p = \frac{QSb^2C_{lp}}{2I_xu_0}\ (\text{s}^{-1})$$

$$Y_r = \frac{QSbC_{yr}}{2mu_0}\ (\text{ft/s})\ \text{or}\ (\text{m/s}) \qquad N_r = \frac{QSb^2C_{nr}}{2I_zu_0}\ (\text{s}^{-1}) \qquad L_r = \frac{QSb^2C_{lr}}{2I_xu_0}\ (\text{s}^{-1})$$

$$Y_{\delta_a} = \frac{QSC_{y\delta_a}}{m}\ (\text{ft/s}^2)\ \text{or}\ (\text{m/s}^2) \qquad Y_{\delta_r} = \frac{QSC_{y\delta_r}}{m}\ (\text{ft/s}^2)\ \text{or}\ (\text{m/s}^2)$$

$$N_{\delta_a} = \frac{QSbC_{n\delta_a}}{I_z}\ (\text{s}^{-2}) \qquad N_{\delta_r} = \frac{QSbC_{n\delta_r}}{I_z}\ (\text{s}^{-2})$$

$$L_{\delta_a} = \frac{QSbC_{l\delta_a}}{I_x}\ (\text{s}^{-2}) \qquad L_{\delta_r} = \frac{QSbC_{l\delta_r}}{I_x}\ (\text{s}^{-2})$$

Figure (5)

By using the equations in figure (5), we find all needed longitudinal derivatives as follows in table (4).

$Y_\beta = -457.7$	$Y_p = 0$	$L_p = -13.14$
$N_\beta = 36.41$	$L_\beta = -214.2$	$N_p = -0.384$
$Y_r = 24.3$	$N_r = -2.06$	$L_r = 12.2$
$Y_{\delta a} = 0$	$Y_{\delta r} = 81.37$	
$N_{\delta a} = 0.306$	$N_{\delta r} = -11.65$	
$L_{\delta a} = -47.74$	$L_{\delta r} = 55.09$	

Table (4)

- **Finding longitudinal eigenvalues and compare them with those calculated by using phugoid and short period approximations.**

We write the longitudinal derivatives in state space form:

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_w & 0 & -g \\ Z_u & Z_w & u_0 & 0 \\ M_u + M_{\dot{w}} Z_u & M_w + M_{\dot{w}} Z_w & M_q + M_{\dot{w}} u_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} 0 \\ Z_\delta \\ M_\delta \\ 0 \end{bmatrix} \begin{bmatrix} \Delta\delta \end{bmatrix}$$

To find the eigenvalues, we find the A-matrix and the B-matrix to be:

$$\begin{bmatrix} -1.571 & 0 & -0.917 & 0.111 \\ -214.2 & -13.14 & 12.2 & 0 \\ 36.41 & -0.384 & -2.06 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

A =

$$B = \begin{bmatrix} 0 & 0.279 \\ 47.74 & 55.09 \\ 0.306 & -11.65 \\ 0 & 0 \end{bmatrix}$$

By using programming:

The eigenvalues :

- <u>Exact method:</u>

  $\lambda_1 = -4.4093+4.09i$, $\lambda_2 = -4.4093-4.09i$ (short period)

  $\lambda_3 = -0.4541+0.2836$, $\lambda_4 = -0.4541 -0.2836$ (long period)

For short period:

$\eta = -4.4093$, which means that this mode is stable.

$\omega = 4.09$ rad/S

$T_{1/2} = 0.69/ \eta = 0.156$ S

$P = 2 \pi / \omega = 1.54$ S

$N_{1/2} = 0.11 \omega/ \eta = 0.1$ cycles

For long period:

$\eta = -0.4541$, which means that this mode is stable.

$\omega = 0.2836$ rad/S

$T_{1/2} = 0.69/ \eta = 1.52$ S

$P = 2 \pi / \omega = 22.2$ S

$N_{1/2} = 0.11 \omega/ \eta = 0.07$ cycles

- <u>Approximation method:</u>

For short period:

$\omega_{nsp} = [z_\alpha M_q / U_0 - M\alpha]^{0.5} = 5.96$ rad/s
$\zeta_{sp} = (M_q + M\dot{\alpha} + z_\alpha / U_0)/[2\,\omega_{nsp}] = 0.737$
$\lambda_{1,2} = -\zeta_{sp}\,\omega_{nsp} + i\,\omega_{nsp}\,(1-\zeta_{sp})^{0.5}$

$\lambda_1 = -4.39252+4.03i$, $\lambda_2 = -4.39252-4.03i$ $\eta = -4.39252$, which means that this mode is stable.
$\omega = 4.03$ rad/S
$T_{1/2} = 0.69/\eta = 0.157$ S
$P = 2\pi/\omega = 1.56$ S
$N_{1/2} = 0.11\,\omega/\eta = 0.1$ cycles

For long period:
$\omega_{np} = [-Z_u\, g / U_0]^{0.5} = 0.475$ rad/s
$\zeta_p = -X_u / (2\,\omega_{np}) = 0.743$
$\lambda_{1,2} = -\zeta_p\,\omega_{np} + i\,\omega_{np}\,(1-\zeta_p)^{0.5}$

$\lambda_3 = -0.353+0.318i$, $\lambda_4 = -0.353-0.318i$
$\eta = -0.353$, which means that this mode is stable.
$\omega = 0.318$ rad/S
$T_{1/2} = 0.69/\eta = 1.95$ S
$P = 2\pi/\omega = 19.76$ S $N_{1/2} = 0.11\,\omega/\eta = 0.09$ cycles

The differences between the exact method and the approximated method for longitudinal results are shown in table (5):

	Exact method	Approximated method	Difference
Short period	$T_{1/2} = 0.156$ $P = 1.54$	$T_{1/2} = 0.157$ $P = 1.56$	0.6 % 1.2 %
Long period	$T_{1/2} = 1.52$ $P = 22.2$	$T_{1/2} = 1.95$ $P = 19.76$	27 % 10 %

Table (5)

As we note in the percentage difference, the difference is a little small between the exact method and the approximated method, which gives us good evidence that we properly made our assumptions and we kept the right way in analysis.

- Finding the lateral eigenvalues and compare them with the approximation solutions.

We write the lateral derivatives in state space form:

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta}{u_0} & \frac{Y_p}{u_0} & -(1-\frac{Y_r}{u_0}) & \frac{g\cos\theta_0}{u_0} \\ L_\beta & L_p & L_r & 0 \\ N_\beta & N_p & N_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi \end{bmatrix} + \begin{bmatrix} 0 & \frac{Y_{\delta r}}{u_0} \\ L_{\delta a} & L_{\delta r} \\ N_{\delta a} & N_{\delta r} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{bmatrix}$$

To find the eigenvalues, we find the A-matrix and the B-matrix to be:

$$A = \begin{bmatrix} -1.571 & 0 & -0.917 & 0.111 \\ -214.2 & -13.14 & 12.2 & 0 \\ 36.41 & -0.384 & -2.06 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0.279 \\ 47.74 & 55.09 \\ 0.306 & -11.65 \\ 0 & 0 \end{bmatrix}$$

By using programming:
The eigenvalues :

- Exact method:

$\lambda_1 = 0.0006$ (spiral)

$\lambda_2 = -13.4014$ (roll)

$\lambda_3, = -1.6851 + 6.4051i$ and $\lambda_4 = -1.6851 - 6.4051i$ (Dutch roll)

For spiral:

$\eta = 0.0006$, which means that this mode is unstable.

$T_2 = 0.69/\ \eta = 1150$ S

For roll:

$\eta = -13.4014$, which means that this mode is stable.

$T_{1/2} = 0.69/\ \eta = 0.051$ S

For Dutch roll:

$\eta = -1.6851$, which means that this mode is stable.

$\omega = 6.4051$ rad/S

$T_{1/2} = 0.69/\ \eta = 0.409$ S

$P = 2\ \pi\ /\ \omega = 0.981$ S $N_{1/2} = 0.11\ \omega/\ \eta = 0.4$ cycles

- Approximation method:

For spiral:

$\lambda_{\ spiral} = [L_\beta\ N_r - L_r\ N_\beta]/\ L_\beta = 0.014$

$\eta = 0.014$, which means that this mode is unstable.

$T_2 = 0.69/\ \eta = 49.28$ S

For roll:

$\lambda_{\ roll} = L_P = -13.14$

$\eta = -13.14$, which means that this mode is stable.

$T_{1/2} = 0.69/\ \eta = 0.053$ S

For Dutch roll:

$\omega_{\ DR} = [(Y_\beta\ N_r\ - N_\beta\ Y_r + U_0\ N_\beta)/\ U_0]^{0.5} = 6.05$ rad/S

$\zeta_{DR} = -1/(2\ \omega_{\ DR})[\ (Y_\beta + U_0\ N_r)/\ U_0] = 0.3$

$\lambda_{DR} =-\ \zeta_{DR}\ \omega_{\ DR} + i\ \omega_{\ DR}\ (1-\ \zeta_p)^{0.5}$

$\lambda_{\ DR} = -\ 1.815+5.77i$ and $\lambda_4 = -\ 1.815-5.77i$

$\eta = -\ 1.815$, which means that this mode is stable.

$\omega = 5.77$ rad/S

$T_{1/2} = 0.69/\ \eta = 0.38$ S

$P = 2\ \pi\ /\ \omega = 1.09$ S $N_{1/2} = 0.11\ \omega/\ \eta = 0.35$ cycles

The differences between the exact method and the approximated method for lateral results are shown in table (6):

	Exact method			Approximated method		
	$T_{1/2}$	T_2	P	$T_{1/2}$	T_2	P
Spiral	-	1150	-	-	49.28	-
Roll	0.051	-	-	0.0535	-	-
Dutch roll	0.409	-	0.981	0.38	-	1.09

Table (6):

By looking to the values above, we note that the difference is a little small between the exact method and the approximated method for roll and Dutch roll modes, which gives us good evidence that we properly made our assumptions and we kept the right way in analysis. But for the spiral mode, the approximation method doesn't give good solutions.

- Plotting the time response of longitudinal motion for the initial condition: $\Delta\theta\ (0) = 0.1$, all other states are zeros.

By using programming:
The response of longitudinal motion (open loop) is:

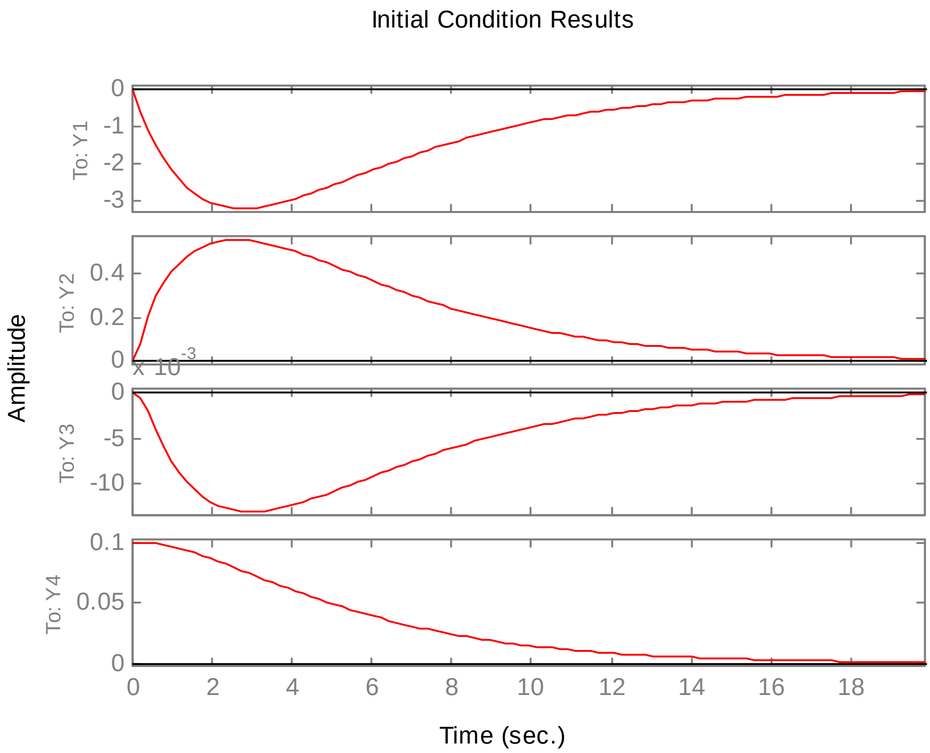

Figure (6)

By looking to the figure (6) above, we note that the aircraft is stable at this velocity $u_0 = 291.3$ ft/s, and the system needs little time to reach its steady state and quickly damped.
Actually I think that these result are satisfying pilots since the values of the undamped frequency and damping ratio are almost nice and lay in (level 2) of aircraft flying qualities.

- Plotting the time response of lateral motions for the initial condition: $\Delta\Phi(0) = 0.1$, all other states are zeros.

By using programming:
The response of lateral motion (open loop) is:

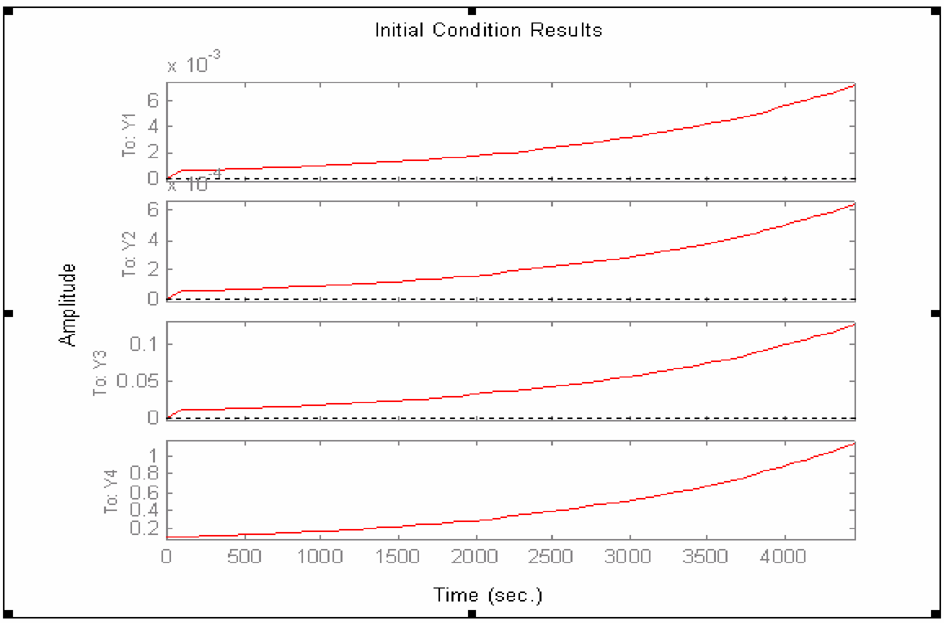

Figure (7)

By looking to the figure (7) above, we note that the aircraft is unstable at this velocity u_0 = 291.3 ft/s because of the spiral mode and that is expected because this aircraft is a fighter aircraft and needs to be unstable in some conditions to satisfy the maneuvering purposes.
Actually I think that these results are satisfying pilots since it has good flying qualities. (Level 2) aircraft qualities.

- ## Designing a state feedback system if the flying qualities are not satisfied:

According to the results that we got, no need to design a feedback system since we have good flying qualities.

For longitudinal flying qualities:

<u>In short period mode:</u>

ζ_{sp} = 0.737 (0.35< ζ_{sp} <1.3)

According to this value, this aircraft is categorized to be (level 1, category A) aircraft. (Table 4.10 in our textbook)

<u>In long period mode:</u>

ζ_p = 0.743 (ζ_p >0.04)

According to this value, this aircraft is categorized to be (level 1, category A) aircraft. (Table 4.10 in our textbook)

For lateral flying qualities:

<u>In spiral mode:</u>

T_2 = 49.28 S (min time to double is 12 S)

According to this value, this aircraft is categorized to be (level 1, category A) aircraft. (Table 5.4 in our textbook)

<u>In roll mode:</u>

$T_{1/2}$ = 0.053 S (the maximum is 1.4)

According to this value, this aircraft is categorized to be (level 1, category A) aircraft. (Table 5.5 in our textbook)

<u>In Dutch roll mode:</u>

ω_{DR} = 6.05 rad/S (minimum is 1)

$\zeta_{DR} = 0.3$ (minimum is 0.15)
According to this value, this aircraft is categorized to be (level 1, category B) aircraft. (Table 5.6 in our textbook)

- The above analysis shows that the flying qualities of this aircraft are very good since they lay in the level 1 aircraft. So, no need to design a feedback system.

Conclusion:

Actually by going through this project, I found that F104-A aircraft has very good flying qualities and its design was perfect. However, I got nice result while doing this project of how to deal with different issues and how to compromise between different parameters to get the best flying qualities and performance of any aircraft. Also I learnt the technique that is used to test the flying qualities of any aircraft. For me, I consider this project to be a summary for the whole course and good test to see whether students understand the course or not. So, thanks go to Dr: Hanafy Omar for giving such kind of project.

References:

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2. http:/www.mhcollege.com