CO-OPN/2 Applied to the Modeling of Cooperative Structured Editors*

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Abstract

In this paper we describe how to use the CO-OPN/2 (Concurrent Object-Oriented Petri Net) formalism, by developing a case study on groupware or, more specifically, on a cooperative editor of hierarchical diagrams. First of all, we present CO-OPN/2 and describe how some aspects specific to object orientation, such as the notions of class/object, of object reference and of the distinction between inheritance and sub-typing, are taken into account within the formalism. Afterwards, we show how these notions may be used for the modeling and the design of an editor of hierarchical Petri nets and discuss some points related to the concurrency. Finally, we show how flexible our modeling architecture is by giving some insight into to the design of other kinds of cooperative diagram editors.

Keywords: groupware, object-orientation, distributed systems, formal methods, Petri nets, algebraic specification, software engineering.

1 Introduction

The objective of this paper is to illustrate the modeling capabilities of the CO-OPN/2 specification language by means of a case study on groupware. The case study adopted consists of a Cooperative Diagram Editor (CDE) as has been proposed by R. Bastide, P. Palanque and C. Lakos. The aim of the case study has been to provide a common example for formal specification approaches that combine a formal model of concurrency with the object-oriented paradigm.

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CO-OPN/2 is a Petri net based object-oriented formal specification language devoted to the specification of concurrent systems. We presented in the previous workshop on "Object-Oriented Programming and Models of Concurrency" the syntax and the semantics of the CO-OPN/2 language and some principles concerning inheritance, sub-typing and refinement [BB95]. During this workshop many fine proposals for object-oriented extensions of Petri nets were proposed. Although these proposals all have the same objectives, more or less, it is difficult to precisely delineate their similarities and differences.

The presentation of a common case study can help to classify the various formal modeling approaches. However, it seems that several points should be clarified in order to understand each formalism well. A non-exhaustive list of questions, the answers to which can help to understand and distinguish the various approaches, follows.

1. What kind of data structures are provided ? How are they described ?
2. What is an object ?
3. How is a class considered ?
4. How do the objects interact with each other ?
5. How are the Petri nets and the object notion combined ?
6. What kind of concurrency is promoted (intra/ inter-object concurrency) ?
7. What is the underlying semantics of concurrency ?
8. How are the object identities managed ?
9. What do the terms inheritance and sub-typing mean ?

This paper is organized as follows. Section 2, the next section, briefly presents the inherent problems of cooperative systems. In Section 3, the principles of the CO-OPN/2 specification language, and the answers to the questions stated above are given. The actual case study is presented in Section 4. In Section 5 some general considerations about the modeling of the case study are given, while some aspects of its design are given in Section 6. The final section, Section 7, is devoted to a discussion of how this case study modeling might evolve, in order to permit the consideration of other diagram types.

2 Cooperative Software

This section is a brief presentation of the most important principles used in the design of cooperative software environments.

According to C. S. Ellis, a cooperative system is "a computer based system that supports groups of people engaged in a common task (or goal) and that provides an interface to a shared environment". Thus, in the cooperative system discussed here, we can say that the following computer science topics are involved:

- Distributed Systems: the design of a groupware application involves many users working on heterogeneous platforms which are linked by means of a computer network.
- Concurrency: the order in which the events in the cooperative system are related must be precisely studied so as to have a coherent and efficient shared environment.
- Networks and Telecommunications: the architecture and the functions of the cooperative systems are directly dependent upon concrete communication capabilities.
- Computer-Human Interaction: a multi-user interface design requires that new interaction modes be developed to permit complex synchronous, or asynchronous, group actions.

A basic notion of groupware is, above all, the shared context, which necessitates the description of a view of this context related to each user in interaction. Logically, this leads to concrete synchronism, or asynchronism, principles which are related to interaction time constraints. Other problems that must be solved by groupware are data granularity and strongly, or loosely, coupled works (which depend on the degree of cooperation between the users). Popular electronic mail or desktop conference systems are classic forms of groupware in the field of communication applications. For the shared editors we can quote systems such as Griffon [DQV92] or GroupDraw [GRWB92]), among others.

Several general multi-layer models have been given in order to have a precise framework for the development of groupware applications (see [Kar94] and [EW94]).

The architecture of synchronous groupware must be studied in order to build a robust, transparent and efficient system. The groupware must also allow for modifications of distributed data within some time limits that are satisfactory to the user and have no constraints upon the graphic interface. The two major approaches to groupware architectures are the centralized and the replicated architectures. The first has a central process which manages all the users' accesses along with the data integrity. The second introduces one process for each user and all these processes communicate together in order to preserve the integrity of the data which are replicated on each site.

We use a layered structure, in Section 5, for the specific cooperative editor model we wish to define, and we organize the system in a centralized way. It must be noted that the purpose of the Cooperative Distributed Editor modeling is to elaborate an initial abstract solution which respects the requirements.

3 CO-OPN/2 Principles

CO-OPN [BG91] (Concurrent Object-Oriented Petri Nets) is a specification language designed for the specification and the modeling of large concurrent systems. The two underlying formalisms of CO-OPN are the algebraic specifications and the Petri nets which are combined in a way that is similar to the combination of algebraic nets [Rei91]. The first formalism is used to describe the data structures and the functional aspects of a system, while the second serves to model its operational and concurrent features. However, both these formalisms are not suitable to specify "in the large". To compensate for the lack of structuring capabilities in the Petri nets, the object paradigm has been adopted. Thus, a system is considered as being a collection of independent entities which interact and collaborate together in order to accomplish the various tasks of the system. With respect to the data structures, the algebraic specifications are composed of modules which are organized into a hierarchy.

In order to overcome some limitations of CO-OPN regarding the object orientation, a new version, named CO-OPN/2 [BB95], has been developed which introduces some notions peculiar to object-orientation such as the notions of class, inheritance, and sub-typing. For the sake of homogeneity regarding the notion of sub-typing, order-sorted algebraic specifications [GM89] have been adopted for the description of the data structures. In these, many sorted algebraic specifications form a special case.

3.1 Object and Class

An object is considered as an independent entity composed of an internal state which provides some services to the exterior. The only way to interact with an object is to ask a service of it; the internal state is then protected from inadvertent accesses. Our point of view is that this protection mechanism, known as encapsulation, forms the essence of object-orientation and there should be no way of violating it.

CO-OPN/2 defines an object as begin an encapsulated algebraic net in which the places compose the internal state and the transitions model the concurrent behavior of the object. A place consists of a multi-set of algebraic values which are described by means of order-sorted algebraic values. The transitions are divided into two groups: the parameterized transitions, also called the methods, and the internal transitions. The former correspond to the services provided, while the latter composes the internal behaviors of an object. Contrary to what occurs with the methods, the internal transitions are invisible to the exterior world and may be considered as being spontaneous events.

An important characteristic of the systems we want to consider is that of their potential dynamic evolution in terms of the number of objects they may contain. Thus, the dynamic creation of objects is a major objective. In order to describe these dynamic evolving systems, the objects are grouped into classes. A class describes all the components of a set of objects and can be viewed as serving as an object template. All the objects of one class manifest the same structure.

3.2 Object Interaction

In our approach, the interaction with an object is synchronous, although asynchronous communications may be simulated. Thus, when an object requires a service it asks to be synchronized with the method (parameterized transition) of the object provider. The synchronization policy is expressed by means of a synchronization expression, which may involve many partners, as long as with three synchronization operators (one for simultaneity, one for sequence, and one for alternative or non-determinism). For example, an object may simultaneously request two different services of two different partners, followed by a request the service from a third object. This mechanism generalizes other approaches such as the transition fusion or the Petri net systems [Kie89].

3.3 Concurrency

From an intuitive point of view, each object possesses its own behavior and concurrently evolves with the others. The Petri net model naturally introduces both inter-object and intra-object concurrency into CO-OPN/2 because the objects are not restricted to sequential processes.

The step semantics of CO-OPN/2 allows for the expression expression of true concurrency in comparison with interleaving semantics.

Nevertheless, the purpose of CO-OPN/2 is that of capturing the abstract concurrent behavior of each entity modeled, with the concurrency granularity not found in the objects but rather in the invocations of the methods. A set of method calls can be concurrently exercised on the same object.

3.4 Object Identity

Within the CO-OPN/2 framework, each class instance has an identity, which is also called an object identifier, that may be used as a reference. Moreover, a type is explicitly associated with each class. Thus, each object identifier belongs to at least one type. In fact, the object identifiers are order-sorted algebraic values established by a special order-sorted algebra, and provided in order to capture the notion of sub-typing. This object identifier algebra is constructed in order to reflect the sub-type relation which is established between the classes, i.e. two carrier sets are related by inclusion if, and only if, the two corresponding types are related by sub-typing.

Since object identifiers are algebraic values, it is possible to define data structures which are build upon object identifiers, e.g. a stack or a queue of object identifiers. Evidently, the places of algebraic nets may contain object identifiers.

3.5 Inheritance and Sub-typing

We believe that inheritance and sub-typing are two different notions which are used for two different purposes. Inheritance is considered as being a syntactic

mechanism which avoids the necessity of developing classes from scratch and mainly serves in re-use of parts of existing specifications. A class may inherit all the features of another and may also add some services or change the description of some already defined services.

The sub-typing relationship is based upon the strong version of the substitutability principle [Ame90, LW93]. This principle implies that, in any context, a class instance of a given type may be substituted with another instance of the sub-type while the behavior of the whole system remains unchanged. In other words, the instances of the sub-type have a strong semantic conformance relationship with the super-type definition. This conformance relationship is based, in CO-OPN/2, upon the bisimulation between the semantics of super-type and the semantics the sub-type restricted to the behavior of the super-type.

Both inheritance and sub-typing relationships must be explicitly given and both of the hierarchies generated by these relationships do not necessarily coincide. In other words, two classes related by inheritance are not necessarily related by sub-typing. Identifying both inheritance and sub-typing hierarchies leads to several problems as stated by Snyder [Sny86] and America [Ame87].

3.6 Answers to the Questions

Before proceeding to the treatment of the case study, we now furnish answers to the questions presented in the introduction. These short answers summarize the information contained in the preceding subsections and are destinated to give an overview of CO-OPN/2.

1. What kind of data structures are provided ? How are they described ?

   The data structures consist of order-sorted algebraic values which are described by means of hierarchical order-sorted algebraic specifications.

2. What is an object ?

   An object is an encapsulated entity which provide some services to the outside. An object is represented by a Petri net in which the tokens are algebraic values. The places (multi-set of algebraic values) of the net represent the state of the object. Two kinds of transitions are provided, the internal transitions or spontaneous events and the parameterized transition or methods.

3. How is a class considered ?

   A class describes the structure of the objects, and is simply considered to be an object template.

4. How do the objects interact with each other ?

   An object interacts with some other objects by asking to be synchronized with them. The synchronization policy is expressed by means of a synchronization expression involving three operators (simultaneity, sequence, and alternative).

5. How the Petri nets and the object notion are combined ?

   In the previous workshop [Bas95] summarized this by "Objects inside Petri nets, Petri nets inside objects, unifying approach". Because of its use of object references, CO-OPN/2 may be classified in the "unifying approach" category.

6. What kind of concurrency is promoted (intra/inter-object concurrency) ?

   The Petri net model naturally implies the promotion of both intra/inter-object concurrency.

7. What is the underlying semantics of concurrency ?

   A true concurrency semantics (step semantics) has been adopted.

8. How are the object identities managed ?

   All the objects possess an identity or reference. These references are order-sorted algebraic values, established by a particular order-sorted algebra whose sub-sort relationship reflects the sub-typing relationship. Data structure of references may be constructed and may be contained in the places of an object.

9. What do the terms inheritance and sub-typing mean ?

   Inheritance is mainly considered to be a syntactic mechanism which serves in the re-use of parts of existing classes while sub-typing implies a strong semantic conformance relationship between the instances of the sub-type and the super-type instances. This conformance relationship is based on bisimulation. The inheritance process allows one to eliminate or re-define some previously defined components. Two classes related by inheritance are not necessarily related by sub-typing. Both of these relationships between classes must be explicitly expressed.

3.7 Other Approaches

The other approaches which may be compared to CO-OPN/2 may be categorized in the High Level Petri Net class of models. These include algebraic nets [Rei91], OPN [Lak95], Cooperative Nets [SB94], and CLOWN [BCC95] among others. All these models are based upon the Petri nets as well as upon some formalisms which vehicle the data structures aspects. Moreover, the object-oriented paradigm has been adopted by all these formalisms.

4 Cooperative Diagram Editor

We have chosen to adhere strictly to the original statement of the Cooperative Diagram Editor (CDE) case study, as has been suggested by Bastide Lakos and Palanque so as to correctly make a comparison between our modeling and other approaches. This section presents the original statement of the case study, equipped with some words that are highlighted by numbers which will be later clarified in Subsection 4.2. The details furnished by these later clarifications correspond to our specific interpretations of some freedom left by the authors of the case study.

4.1 Original Statement of the Case Study

The system to be studied is software allowing for cooperative editing of hierarchical diagrams. The diagrams may be the work products of some Object-Oriented Design methodology, Hardware Logic designs, *Petri Net diagrams*①, etc. (Note that if the diagrams happen to coincide with *the formalism you are proposing*②, be careful to distinguish clearly between the two.)

One key aspect of this problem is that the editor should cater for several users, working at different workstations, and cooperating in constructing the one diagram. In the Computer Supported Cooperative Work (CSCW) vocabulary, such a tool could be ranked amongst synchronous groupware (each user is informed in real time of the actions of the others) allowing for relaxed WYSIWIS (What You See Is What I See) : each user may have his own customized view of the diagram under design, viewing different parts of the drawing or examining it at a different level of detail.

A *second key aspect*③ of this problem is that the editor should cater for hierarchical diagrams, i.e. components of the diagram can be exploded to reveal subcomponents.

A simple *coordination protocol*④ is proposed to control the interactions between the various users:

1. (a) Users may join or leave the editing session at will, and may join with different levels of editing privileges. For example, a user may join the session merely to view the diagram, or perhaps to edit it as well (see below).
   (b) The current members of the editing session ought to be visible to all, together with their editing privileges.
2. (a) Graphical elements may be free or owned by a user.
   (b) Different levels of ownership should be supported, including ownership for deletion, encapsulation, modification, and inspection.
   (c) The ownership must be compatible with the user's editing privileges.
3. Ownership for deletion requires that no-one else has any ownership of the component – not even for inspection.
4. Ownership for encapsulation requires that only the owner can view the internal details of the component – all other users can only view the top level or *interface*⑤ to the component.
5. Ownership for modification allows the user to modify attributes, but not to delete the component.
6. Ownership for inspection only allows the user to view the *attributes*⑥.
7. Only ownership for encapsulation can persist between editing sessions. (Note that this ownership is tied to a particular user, not a particular workstation.) All other ownership must be surrendered between sessions.
8. Ownership for inspection is achieved simply by selecting the component.
9. Other forms of ownership (and release) are achieved by an appropriate command, having first selected the component.
10. The level of ownership of a component is visible to all other users, as is the identity of the owner.
11. The creator of an element owns it for deletion until it is explicitly released.

4.2 Specific Interpretations

As has been suggested in the statement of the case study, several kinds of cooperative editors may be considered and the designers may exercise a certain degree of freedom when making their interpretations. In order to present the informal specification with as much precision as possible, the following explain in details our specific interpretations of each highlighted words which appear the previous sub-section.

①,② We have decided to consider three kinds of CDE: hierarchical Petri nets similar to Petri nets systems [Kie89], SADT (Structured Analysis and Design Technique) [RS77] and CO-OPN/2 diagrams. The hierarchical Petri nets editor is used as basic case study and a complete modeling of this editor is presented. Nevertheless, both SADT and CO-OPN/2 diagrams editors are briefly presented as variants on the hierarchical Petri nets editor. We believe that both of these variant editors are relevant to the understanding of our presentation, seeing as the SADT diagrams have absolutely no connections to the Petri nets community, while CO-OPN/2 diagrams modeled by means of CO-OPN/2 introduce an elegant auto-reference exercise.

③ We have made the choice of representing a component by means of an object. In order to describe a hierarchical structure, two kinds of components have been introduced: the components which are hierarchical and the ones which are not. The former correspond to the nodes while the latter represent the leaves of the hierarchical structure.

④ The coordination protocol is only used to coordinate the diagram accesses of the users and must be distinguished from the synchronization protocol which, in turn, must fulfill the cooperative requirement which allow all users to act upon the same document. Moreover, it must ensure the integrity of the document, for example as described in Section 2.

⑤ With regard to the notion of interface, we have introduce the notion of anchor, the notion of link along with the notion of interface. An anchor of a component is a location to which another component may be attached. A link of a component is used to link two anchors, one of which is a being part of the component and the other part of another component. These two notions form the concept of the interface of a component corresponding to the set of sub-components which may be linked to any anchor of the component itself. Thus, the interface corresponds to a set of components. For example, the interface of the transition of the hierarchical Petri nets, which are under consideration consists of the input and output places.

⑥ The notion of attribute is present but we do not provide the specification associated with it. For example, the date of creation as well as the name of the creator of a component could be regarded as being attributes. Note that position and rotation are not considered as being attributes because they are integral parts of the component.

4.3 Example of an Editing Session

Here is given an example of a hierarchical Petri net which could be edited by means of our cooperative diagram editor. Figure 1 depicts a two level Petri net involving some transitions, some places, some arcs, and one token. The figure shows that both the transitions and the places have been placed in a hierarchical relationship (a transition is associated with a sub-net and a place may contain some tokens). For instance, hierarchical transition t (level 0) is associated with a sub-net (level 1) composed of the component labeled p_i^t, t_i^t, a_j^t for $i = 1, 2$ and $j = 1, \ldots, 4$ and the place p_1 (level 0) contains one token labeled r (level 1) which is also displayed at level 0. The anchors are represented by small black squares placed at the border of each component. Recall that an anchor is a location to which another object may be linked and that a component is linked to another component by giving both the anchors and the identity the other component.

The three names 'Nicolas', 'Olivier' and 'Didier' represent the three users involved in the editing session. 'Nicolas' and 'Olivier' edit the Level 0 while 'Didier' acts upon the Level 1.

5 Modeling Structure

The purpose of this case study is to provide a CO-OPN/2 model of a cooperative editor of hierarchical diagrams. Among the two kinds of architectures which were introduced in Section 2 (centralized and replicated), the centralized architecture has been chosen because it encompasses all the requirements of the case study. Moreover, the design of applications seems to

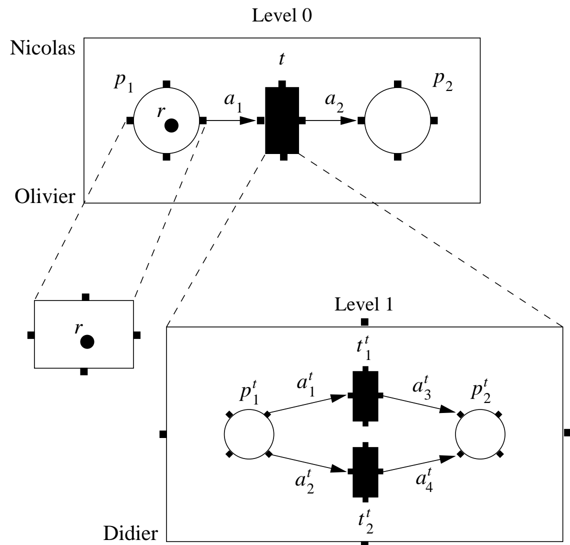

Figure 1: An Example of Editing Session.

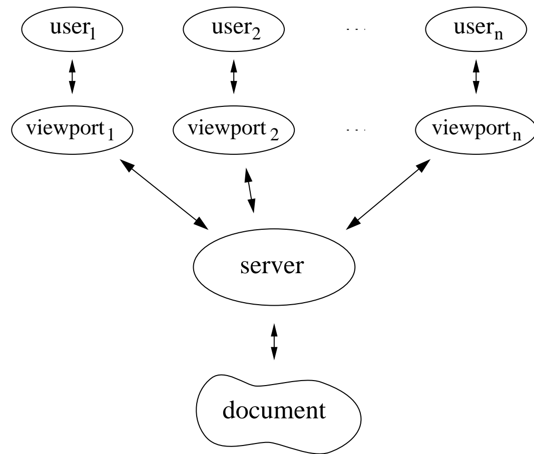

Figure 2: Overview of a CDE.

be easier with respect to centralized architectures in contrast to those which are replicated.

We use a layered abstract model of a cooperative software upon which to base our specification of a CDE. This model is a simplification of the one that has been proposed by Karsenty [Kar94] and has already been presented for the elaboration of a general cooperative system modeling which uses CO-OPN/2 [BBG96b].

We consider a Graphical Interface Layer (the viewports), a Centralized Synchronization Layer (the server) and an Abstract Document Representation Layer (the diagram document). This structure and its elements are described in this section.

5.1 Structure of a CDE

Four kinds of entities are involved in a centralized architecture of a CDE: users, viewports, a server and the document. Figure 2 depicts an overview of this structure in which one may see all four kinds of entities as well as the interaction between them, represented with the arrows.

All the users collaborate in the edition of a part of a hierarchical diagram by means of a viewport which provides some basic functions which permit that aspects of the document be modified. Viewports serve to collect the actions of its logged-in user, to transmit these actions to the server, and to display the document which pertains to the information sent by the server. The role of the server is to collect and handle the actions of the users, to update the document in accordance with the required coordination protocol, to inform every concerned viewport of the actions of the users and to allow for a simultaneous access to the document. Thus, when several users collaborate in the editing of one document, all of them see each others modifications.

5.2 Document Representation

As is required, a document has an organization that is hierarchical. In order to represent a tree structure, two kinds of components must be introduced : hierarchical components and atomic components. The former kind of components may include some sub-components and correspond to the nodes of the hierarchical document, while the latter kind of components represent the leaves of the structure and thus may not include any sub-components. A document is, in itself, a hierarchical component which contains all the components of the document. Since each component is represented by means of an object, i.e. an entity which possesses some attributes and provides some services, a document is essentially a collection of objects which are organized into a hierarchy.

5.3 Three Levels of Entities

In the interest of establishing generalizations and progressively refining our discussion, our modeling is organized in three main entity levels as is shown in Figure 3. Each level is, in fact, the result of a further step in the development of the specification which progressively enriches the simple specification of a single user editor. One may see the four entities which compose an editor presented horizontally, i.e. `User`,

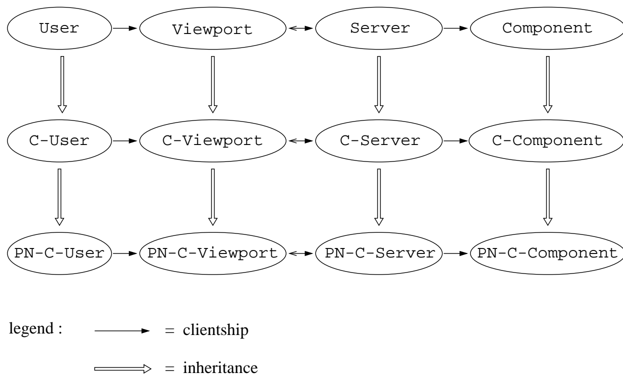

Figure 3: The Three Levels of Entities.

Viewport, Server, and Component. Vertically, the top level corresponds to classic editors, which does not permit any cooperation between more than one user. Thus, only one user at a time may edit a diagram. At the second level (C-User, C-Server, C-Viewport, C-Component), the notion of cooperation as well as the notions of attribute, ownership and interface are introduced. This second level, in fact, complies with all the proposed case study. The third level, depicted in Figure 3 shows an example of a concrete cooperative hierarchical Petri nets editor. It is composed of a PN-C-Server, PN-C-Viewport and its PN-C-Components. The effective components of a Petri nets, i.e. the arcs, the places, and the transitions, are not represented in this figure. Since they compose the document, they should be connected with the PN-C-Component entity. In the next sub-section these entities, which compose the document, are explained.

This structure makes possible to build various cooperative editors. At the third level of the Figure 3 we illustrate a cooperative Petri net editor but other kinds of CDE such as SADT diagrams or even CO-OPN/2 diagrams editor, could equally have been derived. These two variant editors will be briefly presented in Section 7.

5.4 Structure of the Classes

The classes which compose our modeling of a CDE, as well as the relationship between the classes, clearly arise from Figure 3. Thus, each oval of the figure corresponds to a class and the relationships between the classes are represented by the arrows. Three kinds of relationships between classes are especially relevant here : clientship, inheritance, and sub-typing.

Clientship

The clientship relationship between two classes indicates that an instance of one class requires for some services of an instance of another class by means of message passing or, in our case, of method synchronization. A clientship relationship is graphically represented by means of thin arrows. In Figure 3, for example, one may see, at the top level, that the User class uses the Viewport class because a user simply submits some requests to the viewport, e.g. to select a component or add a new component to the diagram, and no information are send back from the viewport to the user. A similar argument holds for the two other levels. Regarding the other entities (viewports, servers and components), the clientship relationship is symmetric. In the case of a cooperative editor, if we consider, for example, the user's action of adding a new sub-component to a given component will generate the following interaction between the other entities. First the viewport transmits the request to the server which asks for the creation of the new component and summons the concerned component to incorporate the new component. Then, the component involved communicates its new aspect to the server which must subsequently compute and send the information which must be transmitted to each viewport.

Inheritance

As mentioned in Section 3.5, we believe that inheritance and sub-typing are two different notions which are used for two different purposes. Inheritance is mainly considered to be a syntactic mechanism for reusing a part of an existing class while sub-typing pertains to the behavior of the instances and is a semantics concern.

In Figure 3, it clearly appears that the thick vertical arrows represent inheritance relationships. The User, C-User, Server, Viewport and Component classes have been built from scratch while the C-Server, C-Viewport and C-Component classes reuse or inherit from the initial classes and add what is needed in order to allow for cooperative edition. In a similar manner, the classes located at the lower level inherit from the higher level and add services required by a cooperative editor of hierarchical Petri nets, e.g services which are mainly related to the creation of new places, tokens, arcs or transitions. Note that the User and C-User classes have to be developed from scratch. Although both seem similar, too may services have to be redefined in the C-User class and inheritance does not furnish any benefit.

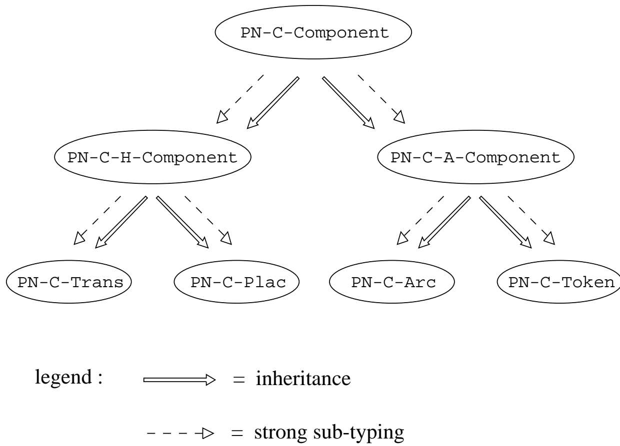

Figure 4: The Classes Related by Sub-typing.

Sub-typing

Recall that, in CO-OPN/2, sub-typing is based on the strong version of the substitutability principle. This principle implies that, in any context, a class instance of a given type may be substituted by another instance of the sub-type while the behavior of the whole system remains unchanged.

With respect to our modeling of a CDE, we shown that inheritance could be used without necessary imply a sub-typing relationship (c.f. Figure 3). Now, we introduce, in Figure 4, the classes which related, on the one hand, by sub-typing with the dashed arrows and, on the other hand, by inheritance. The classes at the lowest level represent the effective components involved in a cooperative hierarchical Petri net editor (i.e. transitions, places, arcs, and tokens), while the classes at the top and the second levels are used to classify and to make the distinction between the hierarchical components (i.e. the transitions and the places) and the atomic ones (i.e. the arcs and the tokens).

It is not surprising that the depicted classes in Figure 4 alone are related by sub-typing. In fact, we will see in the next section that contra-variant rule is violated between `Component`, `C-Component`, and `PN-Component`. A similar argument will be given for the viewport classes as well as for the server and user classes.

6 CO-OPN/2 and CDE Design

In this section we describe the CO-OPN/2 design of a cooperative hierarchical Petri net editor. In order to explain some relevant parts of the specification, we first describe the major syntactic aspects of the CO-OPN/2 language before introducing our design. Unfortunately, due to the size of most modules we do not provide a complete specification of these but rather a specification of the relevant parts. Nevertheless, the complete specification of the case study can be found in [BBG96a].

6.1 Syntactic Aspects of CO-OPN/2

A CO-OPN/2 specification consists of two kinds of modules: the algebraic abstract data type modules and the class modules. These two kinds of modules are composed of three parts : a header, which includes the information about inheritance and genericity; an interface, which describes what is accessible when another module uses it; and a body, which primarily conceals the properties of the operation, the behavior and the state of the objects.

When a non-generic class is developed from scratch, its header comes down to the keyword **Class**[1] followed by the name of the class. When a class is used only for classification, or when it is not completely implemented and the creation of some of its instances makes no sense, we preface the keyword **Class** by the keyword **Abstract**. In the **Interface** section, the field **Use** declares all the modules used by the current class. The **Type** field declares the name of the type of the instances which is used whenever an object identity has to be defined. This field has been introduced in order to avoid that the name of a module or a class and the name of its type are confused, especially in cases when inheritance or sub-typing are required. Both names are often very similar but address two different concepts. Usually classes are used to dynamically create new instances but it is also possible to declare static instances by means of the **Objects** field. All the services provided by the class instances are declared within the field **Methods**. Note that the mixfix and the applicative notation has been adopted for the profile of the methods. The final field **Creation** included in the interface section concerns the dynamic creation of the class instances. Within this field are listed the particular methods which create and initialize the objects; these methods may be used only once. A pre-defined creation method `create` is provided when the **Creation** field is empty or absent. The **Body** section includes some internal or spontaneous transitions declared under the **Transitions**

[1] Specification 1 may help the reader to understand the meaning of the various keywords introduced in this sub-section.

field as well as the attributes of the instances within the **Places** field. The **Initial** field describes the initial marking or the static initialization of each instance while the properties of the methods and internal transitions are described by means of behavioral axioms within the **Axioms** field. It is necessary to recall that a transition (parameterized transition or internal transition) may ask to be synchronized with other partners by means of a synchronization expression. The synchronization expressions takes place after the **with** keyword. The usual dot notation has been adopted and three synchronization operators has been provided: '`//`' for simultaneity '`..`' for sequence, '+' for alternative.

A behavioral axiom is established as follows

$$\mathit{Event}\ [\,\mathbf{with}\ \mathit{Sync}\,]\ ::\ [\,\mathit{Cond} \Rightarrow]\ \mathit{Pre} \rightarrow \mathit{Post}$$

in which *Cond* is an optional condition imposed upon the algebraic values involved in the axiom, *Event* is either an internal transition name or a method with parameters, and *Sync* is an optional synchronization expression. *Pre* and *Post*, respectively, correspond to what is consumed and what is produced at the different places within the net. Finally, the variables used within the **Axioms** field are grouped together under the **where** field.

6.2 Design of the Entities

After this short introduction of the CO-OPN/2 syntactic aspects, we can now describe the design of the various entities presented in Figure 3. Other concepts of the language will be explained as required.

Note that our design takes into account three levels of classes related by inheritance but not by sub-typing. Sub-typing is only present between the `PN-Component` class and the Petri net components classes. One may find all the entities and their relationships presented here depicted in Figure 3 or 4. However, for the sake of clarity, we have only provided the complete hierarchy of the classes which concern the components. In this paper, the `C-User`, `C-Viewport`, and `C-Server` classes are build from scratch and not derived from the top level as shown in Figure 3.

Users

The users involved in a CDE are modeled by means of the `C-User` class. This is a small and simple class, built from scratch, which encompasses some relevant aspects easy to explain. In order to give a global view of a CO-OPN/2 class, we provide the textual form of the `C-User` class in Specification 1 as well as its partial graphic outline in Figure 5.

```
Abstract Class C-User;
Interface
 Use Coord, Size, Angle,
     Privilege, Ownership;
 Type c-user;
 Methods
  join _ : privilege;
  leave;
  select  _ : coord;
  unselect;
  go-down;
  go-back;
  delete;
  move    _ : coord;
  resize  _ : size;
  rotate  _ : angle;
  own-for _ : ownership;
Body
 Use Unique, C-Viewport;
 Places
  idle   _ ,
  active _ : unique;
  cur-vp _ : c-viewport;
 Initial
  idle @;
 Axioms
  join pr with v.(self join pr) ::
   idle @ -> active @, cur-vp v;
  leave with v.leave ::
   active @, cur-vp v -> idle @;
  select p with v.select p ::
   active @, cur-vp v -> active @, cur-vp v;
  move p with v.move p ::
   active @, cur-vp v -> active @, cur-vp v;
  own-for os with v.own-for os ::
   active @, cur-vp v -> active @, cur-vp v;
  where
   p  : coord;      v  : c-viewport;
   pr : privilege;  os : ownership;
 ;; ... and more axioms ...
End C-User;
```

Specification 1: The Cooperative Users.

Within the **Use** field in the **Interface** section of Specification 1, one may see which algebraic abstract data types modules are used by the `C-User` class. These algebraic modules may be found in appendix. Nevertheless, we mention that the `Coord` ADT module defines the sort `coord`, which is pair of naturals and that the `Privilege` module defines both the `view` and `edit` user privileges as required. The four ownerships required by the proposal are defined in the ADT module `Ownership`. As for the `Unique` module, it only defines the generator '`@`' of sort `unique`, which plays the role of a black token.

Under the field **Type**, one finds the type name of the users.

Every name within the **Methods** field models an action that a user would like to accomplish by means of the viewport he is connected to. Among these meth-

ods we may mention the join and leave methods which correspond to the will of a user to join or to leave an editing session. The select method is used to select a component of the diagram at a given location, the go-down allows a hierarchical component, previously selected, to be visited, and the go-back rises again within in the hierarchy, delete removes a selected component. The move, resize, and rotate methods are used to modify aspects of selected components. The own-for method represents the user's will to own a component for a given ownership.

The Viewport and Unique modules are only used by in the **Body** section, this is why both of these modules are established within the **Body** section and not in the **Interface** section.

The instance variables are grouped under the **Places** field and are initialized as has been described in the **Initial** field (with the empty multi-set being the default value). These instance variables are involved in the behavioral axioms which describe the properties of the methods. For instance, the behavioral axiom

```
join pr with v.(self join pr) ::
  idle @ -> active @, cur-vp v;
```

indicates that the join method is asking to be synchronized with the join method for the privilege pr of an available existing viewport v. Recall that method synchronization (a method is a parameterized transition) corresponds to a unification of both formal and effective parameters and may occur if and only if pre- and post-conditions have been satisfied. In other words, the user must be idle (the value @ in the place idle can be consumed) and will become active (the value @ will be produced in the place active). Moreover, the free variable v will be unified with an available existing viewport and memorized by the current viewport attribute (cur-vp v). The self-reference **self** informs the viewport of the identity of the user who wants to be connected to it. The viewport will store this object identifier in order to know what connection has been established.

In a second example, we consider the axiom associated with the select method:

```
select p with v.select p ::
  active @, cur-vp v -> active @, cur-vp v;
```

The role of this action is the transmission of the user's will of selecting an existing component at a given position p to the current viewport v. The same remark holds for the other methods.

Figure 5 provides a partial graphic view of the C-User class. The inside of the ellipse includes the encapsulated elements, the solid arrows represent the

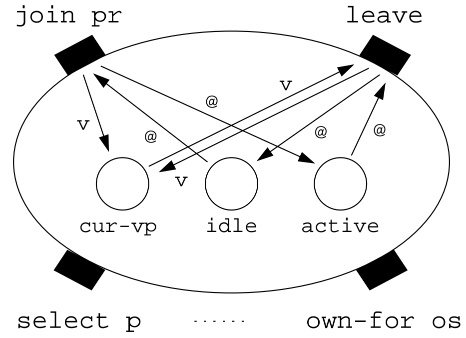

Figure 5: Outline of the C-User Class.

data flow, as described by the axioms, and the circles indicate the places. As for the rectangles, those which are white correspond to the internal or spontaneous transitions (which are not present in this class) and those which are dark indicate the methods. Note that for the sake of clarity not all the methods have been represented.

```
Class PN-C-User;
Inherit C-User;
 Rename c-user -> pn-c-user;
        C-Viewport -> PN-C-Viewport;
        c-viewport -> pn-c-viewport;
Interface
 Objects Didier, Nicolas, Olivier : c-user;
 Methods
  new-trans;
  new-plac;
  new-token;
  new-arc;
Body
 Axioms
  new-trans with v.new-trans ::
   cur-vp v -> cur-vp v;
  where
   v : pn-c-viewport;
 ;; ... and more axioms ...
End PN-C-User;
```

Specification 2: Cooperative Petri Net Users.

In order to obtain an effective class of users which may be involved in a hierarchical Petri net editor we make use of inheritance. In Specification 2 one may see the inheriting class PN-C-User which inherits from the class C-User as declared under the **Inherit** field. In this situation some renamings, declared under the **Rename** field, are necessary. As expected, the profiles and the properties of the new services provided by the inheriting class have been introduced within the **Methods** and **Axioms** fields, respectively. The new methods introduced in this class model the user's will to create the effective components involved in the hierarchical Petri net diagrams at a given location, i.e.

places, transitions, arcs, and tokens. Moreover, one observes that three static objects have been declared under the **Objects** field.

Viewport

A viewport has two main responsibilities. First a viewport has to transmit the user's actions to the server. Its second role is to receive the information of the server and to redisplay the document whenever it is necessary.

```
Abstract Class C-Viewport;
Interface
  Use Coord, Size, Angle, Picture,
      Privilege, Ownership, C-User;
  Type c-viewport;
  Methods
    _ join _ : c-user privilege;
    leave;
    select _ : coord;
    unselect;
    go-down;
    go-back;
    delete;
    move    _ : coord;
    resize  _ : size;
    rotate  _ : angle;
    own-for _ : ownership;
    display _ : picture;
    clear-screen;
Body
  Use C-Server, C-User;
  Places
    priv _ : privilege;
    cur-user _ : c-user;
  Axioms
    u join pr
      with the-server.(u login self) ::
      -> priv pr, cur-user u;
    leave with the-server.(u logout) ::
      cur-user u, priv pr -> ;
    select p with
      the-server.(u select p wth pr) ::
      cur-user u, priv pr ->
      cur-user u, priv pr;
    move p
      with the-server.(u move p wth pr) ::
      cur-user u, priv pr ->
      cur-user u, priv pr;
    own-for os
      with the-server.(u own-for os) ::
      cur-user u -> cur-user u;
    display pic  :: -> ;
    where
      p  : coord;
      pr : privilege;   os  : ownership;
      u  : c-user;      pic : picture;
  ;; ... and more axioms ...
End C-Viewport;
```

Specification 3: General Class of Viewports.

In Specification 3 we give a partial textual form of the C-Viewport class. One may see that the majority of the services concern the requests of the users. For example, the behavioral axiom of the _join_ method informs the server that the user u wishes to be logged-in with the privilege pr. Thus, the viewport stores both the parameters.

The select _ method asks the server to select a component which would be in location p, the current user and its privilege are also transmitted to the server.

The display method (abstractly modeled here) is activated by the server sends back the picture to be displayed on the screen.

We deliberately have not provided the specification of the effective class of the viewports involved in a hierarchical Petri net editor. We simply mention that the PN-C-Viewport class is based on the C-Viewport class and introduces the four services mandatory in the creation of the Petri net components.

```
Abstract Class Component;
Interface
 Use Coord, Size, Angle, String,
     Link, Set-Of-Links,
     Anchor, Set-Of-Anchors;
 Type component;
 Methods
  move      _ : coord;
  resize    _ : size;
  rotate    _ : angle;
  get-pos   _ : coord;
  put-label _ : string;
  add-link  _ ,
  del-link  _ : link;
  get-links _ : set-of-links;
  add-anchor  _ ,
  del-anchor  _ : anchor;
  get-anchors _ : set-of-anchors;
Body
 Places
  position  _ : coord;
  dimension _ : size;
  rotation  _ : angle;
  label     _ : string;
  links   _ : set-of-links;
  anchors _ : set-of-anchors;
 Axioms
  move p    :: position p' -> position p;
  resize s  :: size s'     -> size s;
  rotate r  :: rotation r' -> rotation r;
  get-pos p :: position p  -> position p;
  put-label lb  :: label lb' -> label lb;
  add-link l :: links lks -> links lks + l;
  del-link l :: links lks -> links lks-l;
  get-links lks :: links lks -> links lks;
  where
   p, p' : coord;      s, s'   : size;
   r, r' : angle;      lb, lb' : string;
   l : link;           lks : set-of-links;
 ;; ... and more axioms ...
End Component;
```

Specification 4: General Class of Components.

Components

A document is composed of components organized into a hierarchy . Some of these components are said to be

hierarchical in the sense that they may contain sub-components and some are said to be atomic because they represent the leaves of the tree structure of the document and, consequently, cannot contain any sub-components. In fact a document is a hierarchical component in itself.

The `Component` class given in Specification 4 models the simplest components of our class structure. This kind of components does not include any notion of cooperation specific to the case study. The component state consists of six places, four of them concern the graphic aspects of the component itself, i.e. `position`, `dimension`, `rotation` and `label`. The `anchors` and `links` places contain, respectively, the set of the locations at which another component may be attached, and which components are linked to (a link is a triple of two anchors and one component).

The notions of cooperation between several users required by the case study are introduced in the `C-Component` class which inherits from the previous class as illustrated in Specification 5. The particular `authorized-for` place associates a set of users with each ownership. The associations user-ownership are surrendered by the `surrender` service as required. The `authorized-for?` method is used by the server to determine if a given user owns the component for a given ownership, while the `own-for` method allows a user to modify its ownerships. The behavioral axiom of the `own-for` event ensures that nobody owns the component for `deletion` and adds the user `u` to the users who already own the component for `modification`.

The `PN-C-Component` class in Specification 6 models the components in relation with a Petri net editor. As usual, this class inherits from the `C-Component` with some necessary renamings. From this class we have derived the `PC-C-H-Component` class, related to the hierarchical Petri net components in Specification 7, and the `PN-C-A-Component` class given in appendix.

The `PN-C-H-Component` class introduces three new instance variables as well as some associated methods. The `parent` place represents the identity of the ancestor component, while the `children` place corresponds to the set of sub-components. As for the `disp-depth` instance variable, it corresponds to how many levels have to be display by the `component-pic` method which returns the picture of the component. Thus, some components may be opaque or not (e.g. the places are not opaque since their tokens must be displayed). Opaqueness is represented by the value 0.

```
Abstract Class C-Component;
Inherit Component;
 Rename component -> c-component;
        Link -> C-Link;
        link -> c-link;
Interface
 Use Attributes,Ownership,Picture,C-User;
 Methods
  get-attrib _, put-attrib _ : attributes;
  _ own-for _ ,
  _ authorized-for? _ : c-user ownership;
  surrender;
  component-pic _ : picture;
Body
 Use Set-Of-C-Users;
 Places
  attrib _ : attributes;
  _ authorized-for _ : set-of-c-users
                       ownership;
 Initial
  [] authorized-for deletion;
  [] authorized-for inspection;
  [] authorized-for modification;
  [] authorized-for encapsulation;
 Axioms
  u own-for deletion ::
   (((users1 = []) or (users1 = []+u)) and
    ((users2 = []) or (users2 = []+u)) and
    ((users3 = []) or (users3 = []+u)) and
    ((users4 = []) or (users4 = []+u))) =>
   users1 authorized-for deletion,
   users2 authorized-for inspection,
   users3 authorized-for modification,
   users4 authorized-for encapsulation
   ->
   ([] + u) authorized-for deletion,
   users2   authorized-for inspection,
   users3   authorized-for modification,
   users4   authorized-for encapsulation;
  u own-for os ::
   not (os = deletion) =>
    [] authorized-for deletion,
    users authorized-for os
    ->
    [] authorized-for deletion,
    (users + u) authorized-for os;
  u authorized-for? os ::
   (users + u) authorized-for os
   ->
   (users + u) authorized-for os;
  where
   os : ownership;   u : c-user;
   users, users1, users2,
   users3, users4 : set-of-c-users;
End C-component;
```

Specification 5: Cooperative Components.

```
Abstract Class PN-C-Component;
Inherit C-Component;
  Rename
    c-component -> pn-c-component;
    C-Link -> PN-C-Link;
    c-link -> pn-c-link;
    C-User -> PN-C-User;
    c-user -> pn-c-user;
    Set-Of-C-Users -> Set-Of-PN-C-Users;
    set-of-c-users -> set-of-pn-c-users;
End PN-C-Component;
```

Specification 6: Cooperative Petri Net Components.

```
Abstract Class PN-C-H-Component;
Inherit PN-C-Component;
 Rename pn-c-component -> pn-c-h-component;
Interface
 Use Coord, Picture, PN-C-Component;
 Subtype pn-c-h-component < pn-c-component;
 Methods
  add-child _ ,
  del-child _ : pn-c-component;
  get-child _ at _ : pn-c-component coord;
  get-parent _ : pn-c-component;
Body
 Use Depth, Set-Of-PN-C-Components;
 Places
  parent   _ : pn-c-component;
  children _ : set-of-pn-c-components;
  disp-depth _ : depth;
 Axioms
  add-child c ::
   children compnts -> children compnts+c;
  del-child c ::
   children compnts+c -> children compnts;
  get-child c at p
   with c.(get-pos p) ::
   children compnts+c -> children compnts+c;
  get-parent c : parent c -> parent c;
  where
   p : coord;    c : pn-c-component;
   compnts : set-of-pn-c-components;
End PN-C-H-Components;
```

Specification 7: Hierarchical Petri Net Components.

The four effective Petri net components, i.e. the transitions, places, arcs, and tokens, are constructed from both `PN-C-H-Component` and `PN-C-A-Component` classes. We only have provided, in Specification 8, the `PN-C-Trans` class which describes the class of the hierarchical transitions involved in the hierarchical Petri net editor. The sub-typing relationship, explicitly declared within the **`Subtype`** field, follows the hierarchy of Figure 4 in Section 5.4.

The **`Initial`** field in Specification 8 defines the static initialization of the instance variables. For instance, one may see the dimension of the transitions and the position of their four anchors. The dynamic creation and initialization is given by the `create-trans` method within the **`Creation`** field and by its behavioral axiom which, in this case, just takes charge of the initialization of the `position` instance variable.

The `component-pic` method, abstractly modeled here, is used to return the graphic aspects of the transition (just a rectangle) which will be transmitted to the appropriate viewports.

Server

The server is a crucial element which must :

- manage the identification of the users,

```
Class PN-C-Trans;
Inherit PN-C-H-Component;
 Rename pn-c-h-component -> pn-c-trans;
 Redefine component-pic _ : picture;
Interface
 Use PN-C-H-Component;
 Subtype pn-c-trans < pn-c-h-component;
 Creation create-trans _ : coord;
Body
 Initial
  dimension <50 100>;
  rotation 0;
  anchors []+<25 0>+<0 50>+<49 50>+<25 99>;
  links [];
  disp-depth 0;
 Axioms
  component-pic pic :: -> ;
  create-trans p :: -> position p;
  where
   p   : coord;
   pic : picture;
End PN-C-Trans;
```

Specification 8: Hierarchical Petri Net Transitions.

- save and restore a document,
- provide all the services required by the users,
- cope with the accesses to the shared document in accordance with the coordination protocol,
- allow the users to simultaneously access to the document,
- send to the viewports the relevant information to be displayed.

A partial textual form of the `C-Server` class which allows several users to edit simultaneously the same document is given in Specification 9.

In a centralized architecture only one server is present. Thus, we define in the `C-Server` interface within the **`Object`** field a static instance of this class `the-server`. Thus at system initialization this instance will be created and each class using the class server will be able to use this server instance.

In order to give an insight of the behavior of such a server, we explain in detail the `_move_wth_` method which may also be observed in all the classes presented in this paper. Moreover, in Figure 6, we have provided a graphic outline of the `C-Server` class stressing the `move` method behavior. The dashed arrows represent the synchronization expressions of the behavioral axioms.

The set of `triple-uvc` contained in the instance variable `assoc` associates each logged-in user with the viewport he is connected to, and the current component he is currently acting upon.

The behavioral axiom of the `move` method indicates that the user `u` wants to move the selected com-

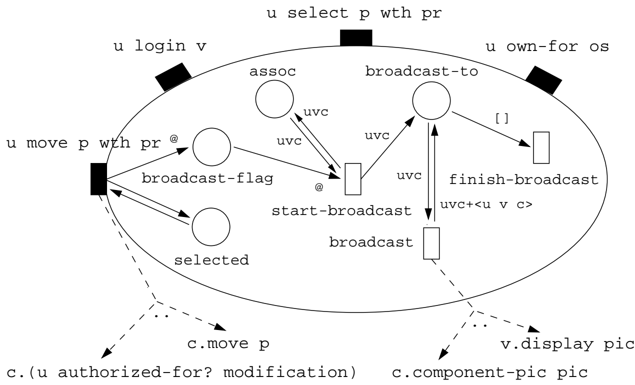

Figure 6: Outline of the Cooperative Server.

ponent according to its privilege `pr` to location `p`. To accomplish this task the server has to determine the already selected component and be synchronized, sequentially, with it. An additional necessary verification is to be sure that the user possesses the `edit` privilege. The synchronization expression first ensures that the user `u` has the authorization for the modification of the component `c` and then asks the component to move itself to the position `p`. The production of a black token `@` in the place `broadcast-flag` triggers off the transition `start-broadcast` to be fired.

As one can see in Figure 6, the `start-broadcast` internal transition copies the `assoc` place into the `broadcast-to` which will be able to send the information to be displayed to each viewport by means of two synchronization requests. The first synchronization request gets the graphic aspect of the expanded current component, while the second synchronization sends the picture to the appropriate viewport `v`. This process is ended when all triples of the `broadcast-to` place have been removed and only the empty set resource is left. Finally, the `finish-broadcast` transition consumes the empty set.

Here, it is necessary to stress that the internal transitions are invisible to the outside world and are considered as being spontaneous events. Thus, for the outside world the events which compose the `move` method seem to be performed simultaneously, whereas they are actually serialized.

Finally, from the abstract `C-Server` class we derive the `PN-C-Server` which describes the effective server involved in the hierarchical Petri net editor. The `PN-C-Server` class is presented in Specification 10. This class introduces the missing services related to the creation of the effective Petri net components.

```
Abstract Class C-Server;
Interface
 Use Coord, Size, Angle, Privilege,
     Ownership, C-User, C-Viewport;
 Type c-server;
 Object the-server : c-server;
 Methods
  _ login _ : c-user c-viewport;
  _ select _ wth _ ,
  _ move   _ wth _ : c-user coord
                     privilege;
  _ own-for _ : c-user ownership;
  save;   restore;   new-document;
Body
 Use Unique, C-Component,
  Set-Of-C-Components, C-Triple-uvc,
  Set-Of-C-Triple-uvc, Picture;
 Places
  document _ : c-component;
  broadcast-flag _ : unique;
  broadcast-to   _ : set-of-c-triple-uvc;
  assoc _ : set-of-c-triple-uvc;
  _ selected _ : c-user set-of-c-components;
 Initial
  broadcast-to [];
  assoc [];
 Transitions
  start-broadcast;
  finish-broadcast;
  broadcast;
 Axioms
  u login v ::
   assoc uvc, document doc ->
   assoc (uvc + <u v doc>),
   u selected [], document doc;
  u select p wth pr
   with c.get-child c' at p ::
   (pr = edit) or (pr = view) =>
   assoc (uvc + <u v c>),
   u selected compnts
   ->
   assoc (uvc + <u v c>),
   u selected (compnts + c'),
   broadcast-flag @;
  u move p wth pr
   with c'.(u authorized-for? modification)
        .. c'.move p ::
   pr = edit =>
   u selected ([] + c')
   ->
   u selected [], broadcast-flag @;
  u own-for os with c'.(u own-for os) ::
   u selected ([] + c') -> u selected [];
  start-broadcast ::
   broadcast-flag @, assoc uvc
   ->
   assoc uvc, broadcast-to uvc;
  broadcast
   with c.component-pic pic
        .. v.display pic ::
   broadcast-to (uvc + <u v c>)
   ->
   broadcast-to uvc;
  finish-broadcast :: broadcast-to [] -> ;
  where
   p   : coord;
   pr  : privilege;   os  : ownerskip;
   pic : picture;
   uvc : set-of-c-triple-uvc;
   c, c', doc : c-component;
   compnts : set-of-c-components;
   u : c-user;    v : c-viewport;
End C-Server;
```

Specification 9: Cooperative Server Class.

```
Class PN-C-Server;
Inherit C-Server;
 Rename
 c-server -> pn-c-server;
 C-User   -> PN-C-User;
 c-user   -> pn-c-user;
 Set-Of-C-Triple-uvc->Set-Of-PN-C-Triple-uvc;
 set-of-c-triple-uvc->set-of-pn-c-triple-uvc;
Interface
 Methods
  _ new-trans-wth _ : pn-c-user privilege;
  _ new-plac-wth  _ : pn-c-user privilege;
  _ new-token-wth _ : pn-c-user privilege;
  _ new-arc-wth   _ : pn-c-user privilege;
Body
 Axioms
  u new-trans-wth pr
   with c.(u authorized-for? modification)
        .. c'.create-trans <0 0>
        .. c.add-child c' ::
   pr = edit =>
   assoc (uvc + <u v c>)
   ->
   assoc (uvc + <u v c>), broadcast-flag @;
  where
   uvc : set-of-triple-pn-c-uvc;
   u : pn-c-user;   v : pn-c-viewport;
   c : pn-c-component;
 ;; ... and more axioms ...
End PN-C-Server;
```

Specification 10: Petri Net Server Class.

Within the **Axioms** field, one may see how a new Petri net transition is created at a position p for an user u with the privilege pr. First the ownership for modification of the current component is verified, then (sequentially) the creation of the transition in itself is requested followed the addition of the new transition to the current component.

6.3 Behavioral Properties of the Classes

In Figure 7 we have provided a snapshot of a system of objects taking part in the development of a hierarchical Petri net, in accordance with the example given in Section 4.3 and illustrated in Figure 1. Two kinds of relationships are represented, the solid arrows correspond to the synchronizations that can be performed, and the dashed arrows represent the link between the connected components.

The server object is a key element of the whole system. All the modifications concerning any component are transmitted to the server which dispatches the result of the modifications to the appropriate viewports. In CO-OPN/2, the concurrency is naturally managed by the places which are multi-sets of algebraic values and build states concurrently accessible by the object methods. Despite this modeling power, it is sometimes difficult to model concurrent accesses when operations on global multi-set states have to be considered. It is, for instance, the case for the children place of the PN-C-H-Component class which is a set of references modeled by means of an algebraic abstract data type and not a multi-set. This modeling choice prevents from accessing simultaneously some of the sub-components of a node.

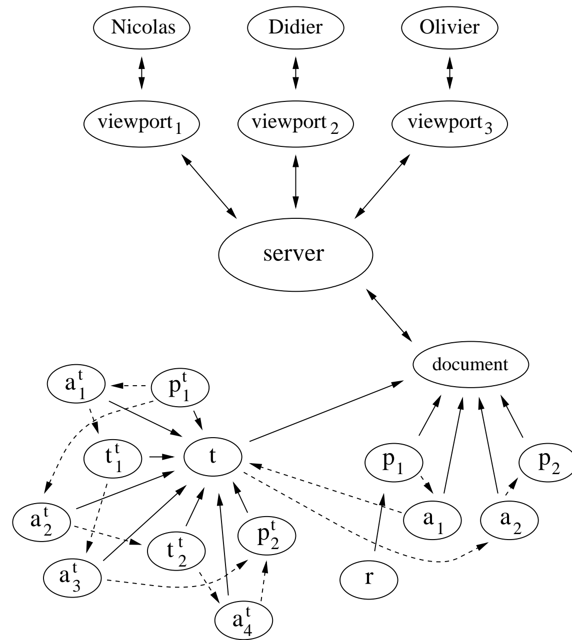

Figure 7: Class Instances of the Example.

The server component allows intra-concurrency. Indeed, most of the services (not login and logout) of the server may be simultaneously activated by two different users for two different components, for instance the _move_wth_ method. Moreover, some services may be requested by two different users for the same component, for instance the rotation and the shifting of a component. We did not have optimized the broadcast performed after a modification, for example a simultaneous modification of two components will produce two simultaneous request for the display service of the viewports (synchronously performed after the component modification).

7 Modeling of Other Editors

In this section we give one direction for the evolution of our modeling of the hierarchical Petri net editor in order to obtain other kinds of cooperative editors.

The class structure presented in this paper is flexible enough to take into account other cooperative editors such as an editor of SADT diagrams as well as an editor of CO-OPN/2 diagrams. The mandatory

modifications only concern the classes derived from second level of Figure 3. Thus, the lowest level of Figure 3 as well as the component hierarchy must be redefined according to the type of the diagram. Here follows two suggestions concerning the sub-typing relationship between the components of SADT diagrams and CO-OPN/2 diagrams.

SADT

SADT diagrams are quite simple, they consist of two kinds of components the boxes which may be arbitrarily nested, and the arrows. Thus, Figure 8 depicts the component hierarchy which could be adopted for the design of a cooperative editor of SADT diagrams.

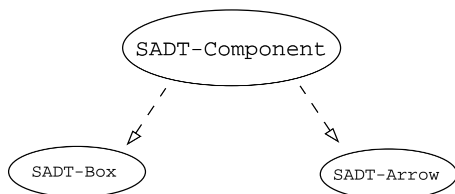

Figure 8: Sub-typing Hierarchy for SADT Components.

CO-OPN/2

A CO-OPN/2 specification is a set of interconnected classes which consist of an encapsulated Petri net equipped with some methods or parameterized transitions. Thus, CO-OPN/2 formalism defines only two levels of hierarchy with regard to the component nesting: the net level and the class level. At the net level, the CO-OPN/2 components are the places, the internal transitions, the methods and the arrows that represent the control flow. At the class level, the components are the classes and the arrows which express the synchronization requests. In Figure 9 we establish a possible component hierarchy for the a CO-OPN/2 diagram editor.

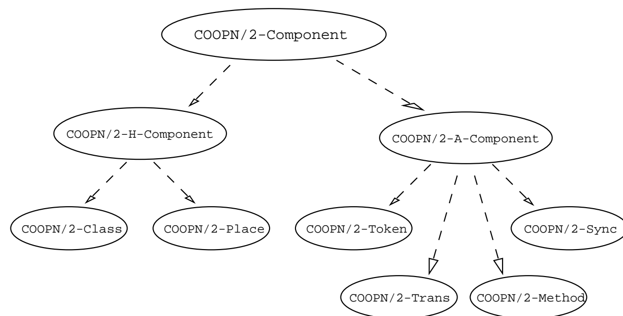

Figure 9: Sub-typing Hierarchy for CO-OPN/2.

8 Conclusion

In this paper we modeled a case study on groupware or, more specifically, on a cooperative editor of hierarchical diagrams. The aim of this case study was to provide a general example in order to compare various approaches which combined a formal model of concurrency and the object-oriented paradigm. First, we presented the CO-OPN/2 specification language by giving some explanations of the specific notions of object-orientation. As for the proposed case study, we showed how the CO-OPN/2 formalism is adapted for the modeling of cooperative applications in which concurrency is a major objective.

The distinction established between inheritance and sub-typing has allowed us to provide a flexible structure of classes and, at the same time, enabled us to treat the components which compose the diagrams in a consistent and regular way. However, it has appeared that a notion of sub-typing in which some semantic constraints could be relaxed, would increase the regularity of the treatments. The formalization of such a sub-typing relationship is one of our present research activities.

In order to provide a complete development system based on CO-OPN/2, we are working on the development of tools adapted to our formalism, and also studying the other phases of the life cycle such as the analysis and the test of object oriented software as well as the techniques of incremental implementation.

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A Full CO-OPN/2 Specification

```
1  ;; COOPN/2 specification of a Cooperative Editor of Hierarchical Diagrams
2  ;; -----------------------------------------------------------------------
3  ;; Contains all the classes and ADT modules involved in the case study except the very
4  ;; basic abstract data types such as the booleans, uniques, naturals, and strings;
5  ;; === Component class
6  ;; This class provides the basic elements of the graphic components involved in the editor.
7  ;; It will be used so as to build more complex components.  The services given by this class
8  ;; are classified into three categories:
9  ;; 1- the services in relation with the graphical aspects of the component (move, resize,
10 ;;    rotate, get-pos, put-label, ret-pic);
11 ;; 2- the services which allow the user to attach a new component (add-link, del-link,
12 ;;    get-links);
13 ;; 3- the services which allow the user to add/remove a new position where components can be
14 ;;    attached (add-anchor, del-anchor, get-anchors).
15 Abstract Class Component;
16 Interface
17   Use  Coord, Size, Angle, String, Link, Set-Of-Links, Anchor, Set-Of-Anchors;
18   Type component;
19   Methods
20     move      _ : coord;                        ;; modify the component aspects
21     resize    _ : size;
22     rotate    _ : angle;
23     get-pos   _ : coord;                        ;; provide the position
24     put-label _ : string;                       ;; modifie the component's name
25     add-link  _ ,                               ;; add and remove a given link
26     del-link  _ : link;
27     get-links _ : set-of-links;                 ;; provide all component links
28     add-anchor  _ ,                             ;; add and remove a given anchor
29     del-anchor  _ : anchor;
30     get-anchors _ : set-of-anchors;             ;; provide all the anchors
31 Body
32   Places
33     position  _ : coord;                        ;; where the component is located
34     dimension _ : size;                         ;; how big is the component
35     rotation  _ : angle;                        ;; how the component is oriented
36     label     _ : string;                       ;; component's name
37     links   _ : set-of-links;                   ;; which components are linked with
38     anchors _ : set-of-anchors;                 ;; where the components may be linked
39   Axioms
40     move p       :: position p' -> position p;
41     resize s     :: size s'     -> size s;
42     rotate r     :: rotation r' -> rotation r;
43     get-pos p    :: position p  -> position p;
44     put-label lb :: label lb'   -> label lb;
45     add-link l    :: links lks -> links (lks + l);
46     del-link l    :: links lks -> links (lks - l);
47     get-links lks :: links lks -> links lks;
48     add-anchor a      :: anchors anchs -> anchors (anchs + a);
49     del-anchor a      :: anchors anchs -> anchors (anchs - a);
50     get-anchors anchs :: anchors anchs -> anchors anchs;
51     where
52       p, p' : coord;
53       s, s' : size;
54       r, r' : angle;
55       lb, lb' : string;
56       l : link;
57       a : anchor;
58       lks     : set-of-links;
59       anchs   : set-of-anchors;
60 End Component;
```

```
61  ;; === Cooperative components
62  ;; This class enriches the Component class with the cooperative aspects of the editor.
63  ;; Four ownerships are taken into account
64  ;;   deletion      : requires that no-one else has any ownership
65  ;;   inspection    : only the owner can view the attributes
66  ;;   modification  : only the owner can modify the attributes
67  ;;   encapsulation : only the owner can view the internal details,
68  ;;                   can persist between editing sessions,
69  ;; The method authorized-for? plays the role of a predicate.  The method can be fired
70  ;; iff the given user owns the component for the given ownership.  This method uses
71  ;; the place authorised-for which associates a set of users for each ownership level.
72  ;; The method own-for is called whenever a user wants to own the component for a given
73  ;; ownership.  This method verifies the compatibility between the ownership and the
74  ;; privileges of the user.
75  Abstract Class C-Component;
76  Inherit Component;
77    Rename component -> c-component;
78           Link -> C-Link;
79           link -> c-link;
80           Set-Of-Links -> Set-Of-C-Links;
81           set-of-links -> set-of-c-links;
82  Interface
83    Use Attributes, Ownership, Picture, C-User;
84    Methods
85      get-attrib _ ,                             ;; provide and mofify the attributes
86      put-attrib _ : attributes;
87      _ own-for _ ,                              ;; own the component for an ownership
88      _ authorized-for? _ : c-user ownership;    ;; is the user authorized for an ownership ?
89      surrender;                                 ;; surrender all ownerships except encapsulation
90      component-pic _ : picture;                 ;; provide the picture of the component
91  Body
92    Use Set-Of-C-Users;
93    Places
94      attrib _ : attributes;                     ;; t-uple of attributes
95      _ authorized-for _ : set-of-c-users ownership;
96    Initial
97      [] authorized-for deletion;                ;; nobody owns the component for any ownership
98      [] authorized-for inspection;              ;; [] denotes the empty set
99      [] authorized-for modification;
100     [] authorized-for encapsulation;
101   Axioms
102     get-attrib t :: attrib t  -> attrib t;
103     put-attrib t :: attrib t' -> attrib t;
104     u own-for deletion ::
105       (((users1 = []) or (users1 = ([] + u))) and  ;; guaranty that nobody (expect u itself)
106        ((users2 = []) or (users2 = ([] + u))) and  ;; owns the component for any ownership
107        ((users3 = []) or (users3 = ([] + u))) and
108        ((users4 = []) or (users4 = ([] + u)))
109       ) =>
110         users1 authorized-for deletion,
111         users2 authorized-for inspection,
112         users3 authorized-for modification,
113         users4 authorized-for encapsulation
114         ->
115         ([] + u) authorized-for deletion,       ;; u is the only one who owns the component
116         users2   authorized-for inspection,     ;; for deletion and may be for inspection
117         users3   authorized-for modification,   ;; modification and encapsulation
118         users4   authorized-for encapsulation;
119     u own-for os ::                            ;; verify that nobody owns the component for
120       not (os = deletion) =>                   ;; deletion and add u as owner for os
121         [] authorized-for deletion, users authorized-for os
122         ->
123         [] authorized-for deletion, (users + u) authorized-for os;
124     u authorized-for? os ::                    ;; verify that the user u owns for ownership os
125         (users + u) authorized-for os -> (users + u) authorized-for os;
```

```
126      surrender ::                                ;; remove all the owners for each ownership
127        users1 authorized-for deletion,           ;; except for encapsulation
128        users2 authorized-for modification,
129        users3 authorized-for inspection
130        ->
131        [] authorized-for deletion,
132        [] authorized-for modification,
133        [] authorized-for inspection;
134      where
135        t  : attributes;
136        os : ownership;
137        u  : c-user;
138        users, users1, users2, users3, users4 : set-of-c-users;
139  End C-component;


140  ;; === Cooperative users
141  ;; A user may join of leave an editing session, and be connected to a viewport.
142  ;; A user performs some actions sending the appropriate message to a the viewport
143  ;; he is connected to.
144  Abstract Class C-User;
145  Interface
146    Use  Coord, Size, Angle, Privilege, Ownership;
147    Type c-user;
148    Methods                                       ;; the user's actions
149      join _ : privilege;                         ;; a user wants to join or leave an editing
150      leave;                                      ;; session with a given privilege.
151      select _ : coord;                           ;; select a component located in x,y
152      unselect;                                   ;; unselect all the selected components
153      go-down;                                    ;; visit the selected component
154      go-back;                                    ;; go back up in the hierarchy
155      delete;                                     ;; delete the selected component
156      move   _ : coord;                           ;; modify the selected component aspects
157      resize _ : size;
158      rotate _ : angle;
159      own-for _ : ownership;                      ;; modify the ownership of a sel. component
160  Body
161    Use Unique, C-Viewport;
162    Place
163      idle   _ ,                                  ;; the user is either idle xor active
164      active _ : unique;
165      cur-vp _ : c-viewport;                      ;; current viewport he is connected to
166    Initial
167      idle @;                                     ;; default user's state
168    Axioms
169      join pr with v.(self join pr) :: idle @ -> active @, cur-vp v;
170      leave with v.leave :: active @, cur-vp v -> idle @;
171      select p with v.select p :: active @, cur-vp v -> active @, cur-vp v;
172      unselect with v.unselect :: active @, cur-vp v -> active @, cur-vp v;
173      go-down with v.go-down :: active @, cur-vp v -> active @, cur-vp v;
174      go-back with v.go-back :: active @, cur-vp v -> active @, cur-vp v;
175      delete with v.delete :: active @, cur-vp v -> active @, cur-vp v;
176      move p with v.move p :: active @, cur-vp v -> active @, cur-vp v;
177      resize s with v.resize s :: active @, cur-vp v -> active @, cur-vp v;
178      rotate r with v.rotate r :: active @, cur-vp v -> active @, cur-vp v;
179      own-for os with v.own-for os :: active @, cur-vp v -> active @, cur-vp v;
180      where
181        p  : coord;
182        s  : size;
```

```
183       r  : angle;
184       v  : c-viewport;
185       pr : privilege;
186       os : ownership;
187 End C-User;


188 ;; === Cooperative viewport

189 ;; A viewport transmits each action of the user connected  to the server and collects
190 ;; the results.  For each user's action, the server sends back to the viewport the picture
191 ;; to be displayed by the viewport.

192 Abstract Class C-Viewport;
193 Interface
194   Use  Coord, Size, Angle, Picture, Privilege, Ownership, C-User;
195   Type c-viewport;
196   Methods
197     _ join _ : c-user privilege;              ;; a user asks to join/leave an editing
198     leave;                                    ;; session with a given privilege
199     select _ : coord;                         ;; select a component located in x,y
200     unselect;                                 ;; unselect all the selected components

201     go-down;                                  ;; visit the selected component
202     go-back;                                  ;; go back up in the hierarchy

203     delete;                                   ;; delete the selected component

204     move   _ : coord;                         ;; modify the selected component aspects
205     resize _ : size;
206     rotate _ : angle;

207     own-for _ : ownership;                    ;; modify the ownership of the sel. component

208     display _ : picture;                      ;; display the picture returned by the server
209     clear-screen;                             ;; used whenever a user join or leave
210 Body
211   Use C-Server, C-User;
212   Places
213     priv _ : privilege;                       ;; privileges of the current user
214     cur-user _ : c-user;                      ;; who is currently logged in
215   Axioms
216     u join pr with the-server.(u login self) :: -> priv pr, cur-user u;

217     leave with the-server.(u logout) :: cur-user u, priv pr -> ;

218     select p with the-server.(u select p wth pr) :: cur-user u, priv pr -> cur-user u, priv pr;

219     unselect with the-server.(u unselect) :: cur-user u -> cur-user u;

220     go-down with the-server.(u go-down-wth pr) :: cur-user u, priv pr -> cur-user u, priv pr;

221     go-back with the-server.(u go-back) :: cur-user u -> cur-user u;

222     delete with the-server.(u delete-wth pr) :: cur-user u, priv pr -> cur-user u, priv pr;

223     move p with the-server.(u move p wth pr) :: cur-user u, priv pr -> cur-user u, priv pr;

224     resize s with the-server.(u resize s wth pr) :: cur-user u, priv pr -> cur-user u, priv pr;

225     rotate r with the-server.(u rotate r wth pr) :: cur-user u, priv pr -> cur-user u, priv pr;

226     own-for os with the-server.(u own-for os) :: cur-user u -> cur-user u;

227     display pic  :: -> ;                      ;; abstractly defined here
228     clear-screen :: -> ;
229     where
230       p  : coord;
231       s  : size;
232       r  : angle;
233       pr : privilege;
234       os : ownership;
235       u  : c-user;
236       pic : picture;
237 End C-Viewport;


238 ;; === Cooperative server
```

```
239 ;; The server collects the users' actions sent by the viewports and performs them.
240 ;; It mainly solves the accesses to the various components by means of a simple
241 ;; protocol and broadcast to each viewport currently connected the results of the
242 ;; users actions, i.e. send a different picture to each viewport.
243 ;; The place assoc makes an association between each loggeg-on user, the viewport he is
244 ;; connected to, and the component he is currently working on. This place consists of
245 ;; a set which will be sequentially consulted during the broadcast which will be
246 ;; triggered off when a token ill be present in the broadcast-flag place.
247 ;; The selected place associates each logged-on user with thier set of selected components.
248 Abstract Class C-Server;
249 Interface
250   Use  Coord, Size, Angle, Privilege, Ownership, C-User, C-Viewport;
251   Type c-server;
252   Object the-server : c-server;                ;; static class instance
253   Methods
254     _ login _ : c-user c-viewport;             ;; same services as the c-user and c-viewport
255     _ logout  : c-user;                        ;; except that the user who ask the service
256                                                ;; is sent by the viewport as well as ist
257     _ select _ wth _ : c-user coord privilege;  ;; privilege when necessary
258     _ unselect : c-user;
259     _ go-down-wth _ : c-user privilege;
260     _ go-back : c-user;
261     _ delete-wth _ : c-user privilege;
262     _ move   _ wth _ : c-user coord privilege;
263     _ resize _ wth _ : c-user size  privilege;
264     _ rotate _ wth _ : c-user angle privilege;
265     _ own-for _ : c-user ownership;
266     save;                                      ;; save the document iff nobody use it
267     restore;                                   ;; load the document previously saved
268     new-document;                              ;; create a new document, abstractly defined
269 Body
270   Use Unique, C-Component, Set-Of-C-Components, C-Triple-uvc, Set-Of-C-Triple-uvc, Picture;
271   Places
272     document _ : c-component;                  ;; the document currently edited
273     broadcast-flag _ : unique;                 ;; trigger the broadcast
274     broadcast-to   _ : set-of-c-triple-uvc;    ;; auxiliary set of connected users/viewport
275     assoc _ : set-of-c-triple-uvc;
276     _ selected _ : c-user set-of-c-components;  ;; the set of selected components of each user
277   Initial
278     broadcast-to [];                           ;; nobody is connected yet
279     assoc [];                                  ;; nobody is connected yet
280   Transitions
281     start-broadcast;                           ;; internal transitions so as to broadcast
282     finish-broadcast;                          ;; the information to each connected viewport
283     broadcast;
284   Axioms
285     u login v ::
286         assoc uvc, document doc
287         ->
288         assoc (uvc + <u v doc>), u selected [], document doc;
289     u logout :: assoc (uvc + <u v c>), u selected compnts -> assoc uvc;
290     u select p wth pr with c.get-child c' at p ::
291       (pr = edit) or (pr = view) =>
292         assoc (uvc + <u v c>), u selected compnts
293         ->
294         assoc (uvc + <u v c>), u selected (compnts + c'), broadcast-flag @;
295     u unselect :: u selected compnts -> u selected [], broadcast-flag @;
296     u go-down-wth pr with c'.(u authorized-for? encapsulation) ::
297       ((pr = edit) or (pr = view)) =>
298         assoc (uvc + <u v c>), u selected ([] + c')
299         ->
300         assoc (uvc + <u v c'>), u selected [], broadcast-flag @;
```

```
301      u go-back with c.get-parent c' ::
302          assoc (uvc + <u v c>), u selected compnts
303          ->
304          assoc (uvc + <u v c'>), u selected [], broadcast-flag @;
305      u delete-wth pr with c.(u authorized-for? deletion) .. c.del-child c' ::
306        pr = edit =>
307          assoc (uvc + <u v c>), u selected []+c'
308          ->
309          assoc (uvc + <u v c>), u selected [], broadcast-flag @;
310      u move p wth pr with c.(u authorized-for? modification) .. c.move p ::
311        pr = edit =>
312          u selected ([] + c) -> u selected [], broadcast-flag @;
313      u resize s wth pr with c.(u authorized-for? modification) .. c.resize s ::
314        pr = edit =>
315          u selected ([] + c) -> u selected [], broadcast-flag @;
316      u rotate r wth pr with c.(u authorized-for? modification) .. c.rotate r ::
317        pr = edit =>
318          u selected ([] + c) -> u selected [], broadcast-flag @;
319      u own-for os with c.(u own-for os) :: u selected ([] + c) -> u selected [];
320      start-broadcast ::
321          broadcast-flag @, assoc uvc -> assoc uvc, broadcast-to uvc;
322      broadcast with c.component-pic pic .. v.display pic ::
323          broadcast-to (uvc + <u v c>) -> broadcast-to set-ucv;
324      finish-broadcast :: broadcast-to [] -> ;
325      save with doc.surrender :: document doc, assoc [] -> assoc [];
326      restore :: -> document doc;
327      new-document :: -> document doc;
328      where
329        p   : coord;
330        s   : size;
331        r   : angle;
332        pr  : privilege;
333        os  : ownerskip;
334        pic : picture;
335        uvc : set-of-c-triple-uvc;
336        c, c', doc : c-component;
337        compnts : set-of-c-components;
338        u : c-user;
339        v : c-viewport;
340  End C-Server;
341  ;; === Petri nets cooperative components
342  ;; This class forms the root of the sub-typing hierarchy of the concrete components
343  ;; involved in the editor.
344  Abstract Class PN-C-Component;
345  Inherit C-Component;
346    Rename
347      c-component -> pn-c-component;
348      C-Link -> PN-C-Link;
349      c-link -> pn-c-link;
350      C-User -> PN-C-User;
351      c-user -> pn-c-user;
352      Set-Of-C-Users -> Set-Of-PN-C-Users;
353      set-of-c-users -> set-of-pn-c-users;
354  End PN-C-Component;
355  ;; === Hierarchical Petri nets cooperative components
356  ;; This class introduces all the elements necessary for the hierarchical components
357  ;; such as the instance variable parent and children and the associated methods.
358  ;; Moreover the instance variable disp-depth is introduced, it is used by the component-pic
359  ;; so as to make some components not opaque such as the place in which the token are displayed.
360  ;; Opaqueness correspond to the value 0. (e.g. 0 for the transitions, 1 for the places).
361  ;; The concrete classes in relation with the hierarchical components of the editor
```

```
362 ;; will be derived from this class (transition, place).
363 Abstract Class PN-C-H-Component;
364 Inherit PN-C-Component;
365   Rename pn-c-component -> pn-c-h-component;
366 Interface
367   Use Coord, Picture, PN-C-Component;
368   Subtype pn-c-h-component < pn-c-component;
369   Methods
370     add-child _ ,                          ;; add and remove a child (sub-component)
371     del-child _ : pn-c-component;
372     get-child _ at _ : pn-c-component coord;    ;; return the child located in position x,y
373     get-parent _ : pn-c-component;             ;; return the parent of the component
374 Body
375   Use Depth, Set-Of-PN-C-Components;
376   Places
377     parent   _ : pn-c-component;               ;; who is the ancestor
378     children _ : set-of-pn-c-components;       ;; who are the children (sub-components)
379     disp-depth _ : depth;                      ;; how many levels have to be displayed
380   Axioms
381     add-child c :: children compnts -> children (compnts + c);
382     del-child c :: children (compnts + c) -> children compnts;
383     get-child c at p with c.(get-pos p) :: children (compnts + c) -> children (compnts + c);
384     get-parent c : parent c -> parent c;
385     where
386       p   : coord;
387       pic : picture;
388       c   : pn-c-component;
389       compnts : set-of-pn-c-components;
390 End PN-C-H-Components;

391 ;; === Atomic Petri nets cooperative components
392 ;; The concrete classes in relation with the atomic components of the editor
393 ;; will be derived from this class (arc, token).
394 Abstract Class PN-C-A-Component;
395 Inherit PN-C-Component;
396   Rename pn-c-component -> pn-c-a-component;
397 End PN-C-A-Component;

398 ;; === Petri net transitions
399 ;; A transition is a hierarchical component graphically represented by a rectangle
400 ;; equiped with four anchors.
401 Class PN-C-Trans;
402 Inherit PN-C-H-Component;
403   Rename pn-c-h-component -> pn-c-trans;
404   Redefine component-pic _ : picture;
405 Interface
406   Use PN-C-H-Component;
407   Subtype pn-c-trans < pn-c-h-component;        ;; a transition is a hierarchical component
408   Creation
409     create-trans _ : coord;                    ;; creation of a transition located in x,y
410 Body
411   Initial
412     dimension <50 100>;                        ;; horizontal and vertical sizes
413     rotation 0;                                ;; no rotation
414     anchors []+<25 0>+<0 50>+<49 50>+<25 99>;   ;; 4 anchors (midle of each side of the rect.)
415     links [];                                  ;; no link
416     disp-depth 0;                              ;; a transition is opaque
417   Axioms
418     component-pic pic :: -> ;                  ;; abstractly defined here;
419     create-trans p :: -> position p;           ;; initialize the last instance variable
420     where
```

```
421        p   : coord;
422        pic : picture;
423  End PN-C-Trans;


424  ;; === Petri net places
425  ;; A place is a hierarchical component graphically represented by a circle
426  ;; equiped with four anchors.
427  Class PN-C-Plac;
428  Inherit PN-C-H-Component;
429    Rename pn-c-h-component -> pn-c-plac;
430    Redefine component-pic _ : picture;
431  Interface
432    Use PN-C-H-Component;
433    Subtype pn-c-plac < pn-c-h-component;         ;; a place is a hierarchical component
434    Creation
435      create-plac _ : coord;                      ;; creation of a place at a given coordinate
436  Body
437    Initial
438      dimension <100 100>;                        ;; horizontal and vertical sizes
439      rotation 0;                                 ;; no rotation
440      anchors []+<50 0>+<0 50>+<99 50>+<50 99>;   ;; four anchors (east,west,north,south)
441      links [];                                   ;; no link
442      disp-depth 1;                               ;; sub-components (tokens) must be visible
443    Axioms
444      component-pic pic :: -> ;                   ;; abstractly defined here;
445      create-plac p :: -> position p;             ;; initialize the last instance variable
446      where
447        p   : coord;
448        pic : picture;
449  End PN-C-Plac;


450  ;; === Petri net tokens
451  ;; A token is an atomic component graphically represented by a disc.
452  Class PN-C-Token;
453  Inherit PN-C-A-Component;
454    Rename pn-c-a-component -> pn-c-token;
455    Redefine component-pic _ : picture;
456  Interface
457    Use PN-C-A-Component;
458    Subtype pn-c-token < pn-c-a-component;        ;; a token is an atomic component
459    Creation
460      create-token-at _ : coord;                  ;; creation of a token at a given coordinate
461  Body
462    Initial
463      dimension <5 5>;                            ;; horizontal and vertical sizes
464      rotation 0;                                 ;; no rotation
465      anchors [];                                 ;; no anchor
466      links [];                                   ;; no link
467    Axioms
468      component-pic pic :: -> ;                   ;; abstractly defined here;
469      create-token p :: -> position p;            ;; initialize the last instance variable
470      where
471        p   : coord;
472        pic : picture;
473  End PN-C-Token;


474  ;; === Petri net arc
475  ;; An arc is an atomic component graphically represented by an arrow.
476  Class PN-C-Arc;
477  Inherit PN-C-A-Component;
478    Rename pn-c-a-component -> pn-c-arc;
```

```
479   Redefine component-pic _ : picture;
480 Interface
481   Use PN-C-A-Component;
482   Subtype pn-c-arc < pn-c-a-component;          ;; an arc is an atomic component
483   Creation
484     create-arc _ : coord;                       ;; creation of an arc at a given coordinate
485 Body                                            ;; rotation and length
486   Initial
487     dimension <200 20>;                         ;; horizontal and vertical sizes
488     rotation 0;                                 ;; no rotation
489     anchors []+<0 10>+<199 10>;                 ;; no anchor
490     links [];                                   ;; no link
491   Axioms
492     component-pic pic :: -> ;                   ;; abstractly defined here
493     create-arc p :: -> position p;              ;; initialize the last instance variable
494     where
495       p    : coord;
496       pic  : picture;
497 End PN-C-Arc;


498 ;; === Petri nets cooperative users
499 ;;
500 ;; From the cooperative user class, this class introduces the necessary services
501 ;; for the creation of the four Petri nets components.
502 Class PN-C-User;
503 Inherit C-User;
504   Rename
505     c-user -> pn-c-user;
506     C-Viewport -> PN-C-Viewport;
507     c-viewport -> pn-c-viewport;
508 Interface
509   Objects Didier, Nicolas, Olivier : c-user;    ;; here three users statically created
510   Methods
511     new-trans;                                  ;; creation of the components
512     new-plac;
513     new-token;
514     new-arc;
515 Body
516   Axioms
517     new-trans with v.new-trans :: cur-vp v -> cur-vp v;
518     new-plac  with v.new-plac  :: cur-vp v -> cur-vp v;
519     new-token with v.new-token :: cur-vp v -> cur-vp v;
520     new-arc   with v.new-trans :: cur-vp v -> cur-vp v;
521     where
522       v : pn-c-viewport;
523 End PN-C-User;


524 ;; === Petri nets cooperative viewport
525 ;;
526 ;; From the cooperative viewport class, this class introduces the necessary services
527 ;; for the creation of the four Petri nets components.
528 ;; Each service calls the-server, unselect all selected components, and selects the
529 ;; new component.
530 Class PN-C-Viewport;
531 Inherit C-Viewport;
532   Rename
533     c-viewport -> pn-c-viewport;
534     C-User   -> PN-C-User;
535     c-user   -> pn-c-user;
536     C-Server -> PN-C-Server;
537     c-server -> c-server;
538 Interface
539   Methods
540     new-trans;                                  ;; creation of the components
541     new-plac;
```

```
542      new-token;
543      new-arc;
544  Body
545    Axioms
546      new-trans with the-server.(u new-trans-wth pr) ::
547        cur-user u, priv pr -> cur-user u, priv pr;
548      new-plac  with the-server.(u new-plac-wth pr) ::
549        cur-user u, priv pr -> cur-user u, priv pr;
550      new-token with the-server.(u new-token-wth pr) ::
551        cur-user u, priv pr -> cur-user u, priv pr;
552      new-arc   with the-server.(u new-arc-wth pr) ::
553        cur-user u, priv pr -> cur-user u, priv pr;
554      where
555        pr : privilege;
556        u  : pn-c-user;
557  End PN-C-Viewport;


558  ;; === Petri net cooperative server
559  ;; From the cooperative server class, this class introduces the necessary services
560  ;; for the creation of the four Petri nets components.
561  ;; Each service verify that the current component is own for modification, create the
562  ;; new component and adds it to the children of the current component.  Finally the
563  ;; broadcast is triggered off.
564  Class PN-C-Server;
565  Inherit C-Server;
566    Rename
567      c-server -> pn-c-server;
568      Set-Of-C-Component -> Set-Of-C-Component;
569      set-of-c-component -> set-of-c-component;
570      Set-Of-C-Triple-uvc -> Set-Of-PN-C-Triple-uvc;
571      set-of-c-triple-uvc -> set-of-pn-c-triple-uvc;
572  Interface
573    Methods
574      _ new-trans-wth _ : pn-c-user privilege;
575      _ new-plac-wth  _ : pn-c-user privilege;
576      _ new-token-wth _ : pn-c-user privilege;
577      _ new-arc-wth   _ : pn-c-user privilege;
578  Body
579    Axioms
580      u new-trans-wth pr
581        with c.(u authorized-for? modification) .. c'.create-trans <0 0> .. c.add-child c'
582        pr = edit =>
583          assoc (uvc + <u v c>) -> assoc (uvc + <u v c>), broadcast-flag @;
584      u new-plac-wth pr
585        with c.(u authorized-for? modification) .. c'.create-plac <0 0> .. c.add-child c'
586        pr = edit =>
587          assoc (uvc + <u v c>) -> assoc (uvc + <u v c>), broadcast-flag @;
588      u new-token-wth pr
589        with c.(u authorized-for? modification) .. c'.create-token <0 0> .. c.add-child c'
590        pr = edit =>
591          assoc (uvc + <u v c>) -> assoc (uvc + <u v c>), broadcast-flag @;
592      u new-arc-wth pr
593        with c.(u authorized-for? modification) .. c'.create-arc <0 0> .. c.add-child c'
594        pr = edit =>
595          assoc (uvc + <u v c>) -> assoc (uvc + <u v c>), broadcast-flag @;
596      where
597        uvc : set-of-triple-pn-c-uvc;
598        u : pn-c-user;
599        v : pn-c-viewport;
600        c : pn-c-component;
601  End PN-C-Server;


602  ;; *** Cooperative Editor of Hierarchical Diagrams
```

```
603
604 ;; abstract data types part

605 ;; A coordinate is a pair of naturals
606 Adt Coord as CartesianProduct2(Natural, Natural);
607   Morphism
608     elem1 -> natural;
609     elem2 -> natural;
610     _ = _ in ComparableElem1 -> _ = _ in Natural;
611     _ = _ in ComparableElem2 -> _ = _ in Natural;
612   Rename
613     cp2   -> coord;
614     fst _ -> xcoord _ ;
615     snd _ -> ycoord _ ;
616 End Coord;

617 ;; The depth is a natural number
618 Adt Depth as Natural;
619 End Depth;
620

621 ;; An anchor is a coordinate which represent where a component can be attached.
622 Adt Anchor as CartesianProduct2(Natural, Natural);
623   Morphism
624     elem1 -> natural;
625     elem2 -> natural;
626     _ = _ in ComparableElem1 -> _ = _ in Natural;
627     _ = _ in ComparableElem2 -> _ = _ in Natural;
628   Rename
629     cp2 -> anchor;
630 End Anchor;

631 ;; A link attaches a current component with another component by means two anchors.
632 ;; The first anchor is the anchor of the current component.
633 ;; The component is the component with which the current component is attached
634 ;; The second anchor is the anchor of the component with which the current component is attached.
635 Adt Link as CartesianProduct3(Anchor, Component, Anchor);
636   Morphism
637     elem1 -> anchor;
638     elem2 -> component;
639     elem3 -> anchor;
640     _ = _ in ComparableElem1 -> _ = _ in Anchor;
641     _ = _ in ComparableElem2 -> _ = _ in Component;
642     _ = _ in ComparableElem3 -> _ = _ in Anchor;
643   Rename
644     cp2 -> link;
645 End Link;

646 ;; A triple uvc makes an association between a user and the viewport he is connected with
647 ;; and the current component he is working with.
648 Adt Triple-uvc as CartesianProduct3(User, Viewport, Component);
649   Morphism
650     elem1 -> user;
651     elem2 -> viewport;
652     elem3 -> component;
653     _ = _ in ComparableElem1 -> _ = _ in User;
654     _ = _ in ComparableElem2 -> _ = _ in Viewport;
655     _ = _ in ComparableElem3 -> _ = _ in Component;
656   Rename
657     cp2 -> triple-uvc;
658 End Triple-uvc;

659 ;; The four required owerships levels are defined.
660 Adt Ownership;
```

```
661 Interface
662   Use  Boolean;
663   Sort ownership;
664   Generators
665     deletion,
666     inspection,
667     modification,
668     encapsulation : -> ownership;
669   Operation
670     _ = _ : ownership ownership -> boolean;
671 Body
672   Axioms
673     (deletion = deletion)           = true;
674     (deletion = encapsulation)      = false;
675     (deletion = modification)       = false;
676     (deletion = inspection)         = false;
677
678     (inspection = deletion)         = false;
679     (inspection = encapsulation)    = false;
680     (inspection = modification)     = false;
681     (inspection = inspection)       = true;
682     (modification = deletion)       = false;
683     (modification = encapsulation)  = false;
684     (modification = modification)   = true;
685     (modification = inspection)     = false;
686
687     (encapsulation = deletion)      = false;
688     (encapsulation = encapsulation) = true;
689     (encapsulation = modification)  = false;
690     (encapsulation = inspection)    = false;
691 End Ownership;

692 ;; Both the required privileges of a user are defined.
693 ;; A user may merely view a diagram or be able to edit (modify) it
694 Adt Privilege;
695 Interface
696   Use  Boolean;
697   Sort privilege;
698   Generators
699     view, edit : -> privilege;
700   Operation
701     _ = _ : privilege privilege -> boolean;
702 Body
703   Axioms
704     (edit = edit) = true;
705     (edit = view) = false;
706     (view = edit) = false;
707     (view = view) = true;
708 End Privilege;

709 ;; Actually the attributes consist of a t-uple
710 ;; This ADT must be more specific
711 Adt Attributes;
712 Interface
713   Sort attributes;
714 End Attributes;

715 ;; Actually a pictures consists of a matrix of pixels
716 ;; This ADT must be more specific
717 Adt Picture;
718 Interface
719   Sort picture;
720 End Picture;

721 ;; Abstract data type which are in relation with the cooperative aspects
```

```
722 Adt C-Link as CartesianProduct3(Anchor, C-Component, Anchor);
723   Morphism
724     elem1 -> anchor;
725     elem2 -> c-component;
726     elem3 -> anchor;
727     _ = _ in ComparableElem1 -> _ = _ in Anchor;
728     _ = _ in ComparableElem2 -> _ = _ in C-Component;
729     _ = _ in ComparableElem3 -> _ = _ in Anchor;
730   Rename
731     cp2 -> c-link;
732 End C-Link;

733 ;; A triple uvc makes an association between a user and the viewport he is connected with
734 ;; and the current component he is working with.
735 Adt C-Triple-uvc as CartesianProduct3(C-User, C-Viewport, C-Component);
736   Morphism
737     elem1 -> c-user;
738     elem2 -> c-viewport;
739     elem3 -> c-component;
740     _ = _ in ComparableElem1 -> _ = _ in C-User;
741     _ = _ in ComparableElem2 -> _ = _ in C-Viewport;
742     _ = _ in ComparableElem3 -> _ = _ in C-Component;
743   Rename
744     cp2 -> c-triple-uvc;
745 End C-Triple-uvc;

746 Adt PN-C-Link as CartesianProduct3(Anchor, PN-C-Component, Anchor);
747   Morphism
748     elem1 -> anchor;
749     elem2 -> pn-c-component;
750     elem3 -> anchor;
751     _ = _ in ComparableElem1 -> _ = _ in Anchor;
752     _ = _ in ComparableElem2 -> _ = _ in PN-C-Component;
753     _ = _ in ComparableElem3 -> _ = _ in Anchor;
754   Rename
755     cp2 -> pn-c-link;
756 End PN-C-Link;

757 ;; A triple uvc makes an association between a user and the viewport he is connected with
758 ;; and the current component he is working with.
759 Adt PN-C-Triple-uvc as CartesianProduct3(PN-C-User, PN-C-Viewport, PN-C-Component);
760   Morphism
761     elem1 -> pn-c-user;
762     elem2 -> pn-c-viewport;
763     elem3 -> pn-c-component;
764     _ = _ in ComparableElem1 -> _ = _ in PN-C-User;
765     _ = _ in ComparableElem2 -> _ = _ in PN-C-Viewport;
766     _ = _ in ComparableElem3 -> _ = _ in PN-C-Component;
767   Rename
768     cp2 -> pn-c-triple-uvc;
769 End PN-C-Triple-uvc;

770 ;; Various set used in the specification
771 Adt Set-Of-Anchors as Set(Anchor);
772   Morphism
773     elem  -> anchor;
774     _ = _ in ComparableElem -> _ = _ in Anchor;
775   Rename
776     set -> set-of-anchors;
777 End Set-Of-Anchors;

778 Adt Set-Of-Components as Set(Component);
779   Morphism
780     elem -> component;
781     _ = _ in ComparableElem -> _ = _ in Component;
782   Rename
783     set -> set-of-components;
784 End Set-Of-Components;
```

```
785 Adt Set-Of-C-Components as Set(C-Component);
786   Morphism
787     elem -> c-component;
788     _ = _ in ComparableElem -> _ = _ in C-Component;
789   Rename
790     set -> set-of-c-components;
791 End Set-Of-C-Components;

792 Adt Set-Of-PN-C-Components as Set(PN-C-Component);
793   Morphism
794     elem -> pn-c-component;
795     _ = _ in ComparableElem -> _ = _ in PN-C-Component;
796   Rename
797     set -> set-of-pn-c-components;
798 End Set-Of-PN-C-Components;

799 Adt Set-Of-Links as Set(Link);
800   Morphism
801     elem -> link;
802     _ = _ in ComparableElem -> _ = _ in Link;
803   Rename
804     set -> set-of-links;
805 End Set-Of-Links;

806 Adt Set-Of-C-Links as Set(C-Link);
807   Morphism
808     elem -> c-link;
809     _ = _ in ComparableElem -> _ = _ in C-Link;
810   Rename
811     set -> set-of-c-links;
812 End Set-Of-C-Links;

813 Adt Set-Of-PN-C-Links as Set(PN-C-Link);
814   Morphism
815     elem -> pn-c-link;
816     _ = _ in ComparableElem -> _ = _ in PN-C-Link;
817   Rename
818     set -> set-of-pn-c-links;
819 End Set-Of-PN-C-Links;

820 Adt Set-Of-Triples-uvc as Set(Triple-uvc);
821   Morphism
822     elem -> triple-uvc;
823     _ = _ in ComparableElem -> _ = _ in Triple-uvc;
824   Rename
825     set -> set-of-triples-uvc;
826 End Set-Of-Triples-uvc;

827 Adt Set-Of-C-Triples-uvc as Set(C-Triple-uvc);
828   Morphism
829     elem -> c-triple-uvc;
830     _ = _ in ComparableElem -> _ = _ in C-Triple-uvc;
831   Rename
832     set -> set-of-c-triples-uvc;
833 End Set-Of-C-Triples-uvc;

834 Adt Set-Of-PN-C-Triples-uvc as Set(PN-C-Triple-uvc);
835   Morphism
836     elem -> pn-c-triple-uvc;
837     _ = _ in ComparableElem -> _ = _ in PN-C-Triple-uvc;
838   Rename
839     set -> set-of-pn-c-triples-uvc;
840 End Set-Of-PN-C-Triples-uvc;

841 Adt Set-Of-C-Users as Set(C-User);
842   Morphism
843     elem -> c-user;
844     _ = _ in ComparableElem -> _ = _ in C-User;
```

```
845   Rename
846     set -> set-of-c-users;
847 End Set-Of-C-Users;

848 Adt Set-Of-PN-C-Users as Set(PN-C-User);
849   Morphism
850     elem -> pn-c-user;
851     _ = _ in ComparableElem -> _ = _ in PN-C-User;
852   Rename
853     set -> set-of-pn-c-users;
854 End Set-Of-PN- C-Users;



855 ;; *** Structured basic abstract data types

856 ;; === Comparable elements
857 ;; An equal operation is defined as well as the usual equivalence relation properties
858 Parameter Adt ComparableElem;
859 Interface
860   Use  Boolean;
861   Sort elem;
862   Operation
863     _ = _ : elem elem -> boolean;
864 Body
865   Theorems
866     (x = x) = true;                                      ;; reflexivity
867     (x = y) = true => (y = x) = true;                    ;; symmetry
868     (x = y) = true & (y = z) = true => (x = z) = true;  ;; transitivity
869     where
870     x, y, z : elem;
871 End ComparableElem;
872 Parameter Adt ComparableElem1;
873 Interface
874   Use  Boolean;
875   Sort elem1;
876   Operation
877     _ = _ : elem1 elem1 -> boolean;
878 Body
879   Theorems
880     (x = x) = true;                                      ;; reflexivity
881     (x = y) = true => (y = x) = true;                    ;; symmetry
882     (x = y) = true & (y = z) = true => (x = z) = true;  ;; transitivity
883     where
884     x, y, z : elem1;
885 End ComparableElem1;
886 Parameter Adt ComparableElem2;
887 Interface
888   Use  Boolean;
889   Sort elem2;
890   Operation
891     _ = _ : elem2 elem2 -> boolean;
892 Body
893   Theorems
894     (x = x) = true;                                      ;; reflexivity
895     (x = y) = true => (y = x) = true;                    ;; symmetry
896     (x = y) = true & (y = z) = true => (x = z) = true;  ;; transitivity
897     where
898     x, y, z : elem2;
899 End ComparableElem2;
900
901 Parameter Adt ComparableElem3;
902 Interface
903   Use  Boolean;
904   Sort elem3;
905   Operation _ = _ : elem3 elem3 -> boolean;
906 Body
```

```
907   Theorems
908     (x = x) = true;                                     ;; reflexivity
909     (x = y) = true => (y = x) = true;                   ;; symmetry
910     (x = y) = true & (y = z) = true => (x = z) = true;  ;; transitivity
911     where
912     x, y, z : elem3;
913 End ComparableElem3;
914

915 ;; === Ordered elements
916 Parameter Adt OrderedElem;
917 Interface
918   Use  Boolean;
919   Sort elem;
920   Operations
921     _ < _ ,
922     _ = _ : elem elem -> boolean;
923 Body
924   Theorems
925     (x < y) = true & (y < x) = true => (y = x) = true;  ;; anti-symmetry
926     (x < y) = true & (y < z) = true => (x < z) = true;  ;; transitivity
927     (x = x) = true;                                     ;; reflexivity
928     (x = y) = true => (y = x) = true;                        ;; symmetry
929     (x = y) = true & (y = z) = true => (x = z) = true;  ;; transitivity
930     where
931     x, y, z : elem;
932 End OrderedElem;

933 ;; === Generic sets of comparable elements
934 Generic Adt Set(ComparableElem);
935 Interface
936   Use  Natural, Boolean;
937   Sort set;
938   Generators
939     []      : -> set;                        ;; empty set
940     _ + _   : set elem -> set;               ;; add an element to a set
941   Operations
942     _ - _   : set elem -> set;               ;; remove an element to a set
943     _ + _   ,
944     _ - _   : set set  -> set;               ;; add/sub to sets
945     _ = _   ,                                ;; equality, inclusion, strict inclusion
946     _ <= _ ,
947     _ < _   ,
948     _ > _   ,
949     _ >= _ : set set -> boolean;
950     # _     : set -> natural;                ;; cardinality of a set
951     _ inside? _ : elem set -> boolean;       ;; belonging predicate
952 Body
953   Axioms
954     ((s + e1) + e2) = ((s + e2) + e1);       ;; commutativity of the insertion (generator)
955     ((s + e1) + e1) = (s + e1);              ;; no duplication in set (generator)
956     s - []  = s;
957     s - (s1 + e1)  = (s - e1) - s1;
958     [] + s  = s;
959     (s1 + e1) + s2 = (s1 + s2) + e1;
960     (e1 = e2) = true  => ((s + e1) - e2) = s;
961     (e1 = e2) = false => ((s + e1) - e2) = (s - e2) + e1;
962     (s1 = s2) = (s1 <= s2) and (s2 <= s1);
963     [] <= [] = false;
964     (s + e1) <= [] = false;
965     [] <= (s + e1) = true;
966     (e1 = e2) = true  =>  (s1 + e1) <= (s2 + e2)  = s1 <= s2;
967     (e1 = e2) = false => ((s1 + e1) <= (s2 + e2)) = (e1 inside? s2) and s1 <= (s2 + e2);
968
```

```
969     s1 < s2  = s1 <= s2 and not (s2 <= s1);
970     s1 >= s2 = s2 <= s1;
971     s1 > s2  = s2 < s1;
972     # [] = 0;
973     # (s + e1) = succ (# s);
974     e1 inside? [] = false;
975     (e1 inside? (s + e2)) = (e1 = e2) or (e1 inside? s);
976     where
977       e1, e2    : elem;
978       s, s1, s2 : set;
979 End Set;

980 ;; === Cartesian Products
981 Generic Adt CartesianProduct2(ComparableElem1, ComparableElem2);
982 Interface
983   Use  Boolean;
984   Sort cp2;
985   Generator
986     < _ _ > : elem1 elem2 -> cp2;
987   Operations
988     fst _ : cp2 -> elem1;                          ;; first projection
989     snd _ : cp2 -> elem2;                          ;; second projection
990     _ = _ : cp2 cp2 -> boolean;
991 Body
992   Axioms
993     fst (<x y>) = x;
994     snd (<x y>) = y;
995     (<x y> = <u v>) = (x = u) and (y = v);
996     where
997       x, u : elem1;
998       y, v : elem2;
999 End CartesianProduct2;

1000 Generic Adt CartesianProduct3(ComparableElem1, ComparableElem2, ComparableElem3);
1001 Interface
1002   Use  Boolean;
1003   Sort cp3;
1004   Generator
1005     < _ _ _ > : elem1 elem2 elem3 -> cp3;
1006   Operations
1007     fst _ : cp3 -> elem1;                          ;; first projection
1008     snd _ : cp3 -> elem2;                          ;; second projection
1009     thd _ : cp3 -> elem3;                          ;; third projection
1010     _ = _ : cp3 cp3 -> boolean;
1011 Body
1012   Axioms
1013     fst (<x y z>) = x;
1014     snd (<x y z>) = y;
1015     thd (<x y z>) = z;
1016     (<x y z> = <u v w>) = (x = u) and (y = v) and (z = w);
1017     where
1018       x, u : elem1;
1019       y, v : elem2;
1020       z, w : elem3;
1021 End CartesianProduct3;
```