THE FORMULATION OF A BUILDING THERMAL DESIGN OPTIMIZATION MODEL

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ABSTRACT

Building design is a decision making process where decisions are made on the selection of certain design variables in order to achieve certain objectives (i.e. economy, thermal comfort, visual comfort, aesthetics, etc.). Information on the relationships between the variables and the desired objectives are necessary for proper decision making. Architects have traditionally reached their design decisions based on past experience. However, total reliance upon individual experience may lead to incomplete and inaccurate results. Therefore, given today's complexities in building design as well as advances in computer technology, systematic approaches can be used as an aid to, not a replacement for, building designers in the decision making process.

The aim of this paper is to describe the formulation of an optimization model for the thermal design of building envelopes. This requires defining building design variables, a criterion of optimality, constraints, and a suitable thermal simulation model that can be integrated into the proper optimization technique.

1. INTRODUCTION

Whole building energy design is a concept based on the idea that optimum energy performance is not a simple addition of parts, but rather a complex, dynamic integration of parts, a balancing of tradeoffs which turn negatives into positives [1]. Recognizing that building design is very much dependent on the climatic conditions for each region leads to the need for integrating the building thermal design with the overall design process. This would help the designer to decide early in the design process on some of the design alternatives that will minimize both thermal discomfort in the occupied space and, therefore, the reliance upon backup heating and air-conditioning systems.

Proper design of buildings can reduce the reliance upon supplemental mechanical heating and air-conditioning systems to achieve thermal comfort. The requirements for such systems depend on the function and schedule, as well as the climate that influences the thermal performance of the building and its design. The function and schedule of the building are operational parameters over which architectural designers have little control. The climate, however, can only be modified by the designer through proper selection and integration of the building physical components throughout the design process. As a transition space through which interaction between indoor and outdoor environment takes place, as shown in Figure 1, the building envelope is a determining factor in the consumption of energy in most buildings and the selection of its components can significantly impact the thermal performance.

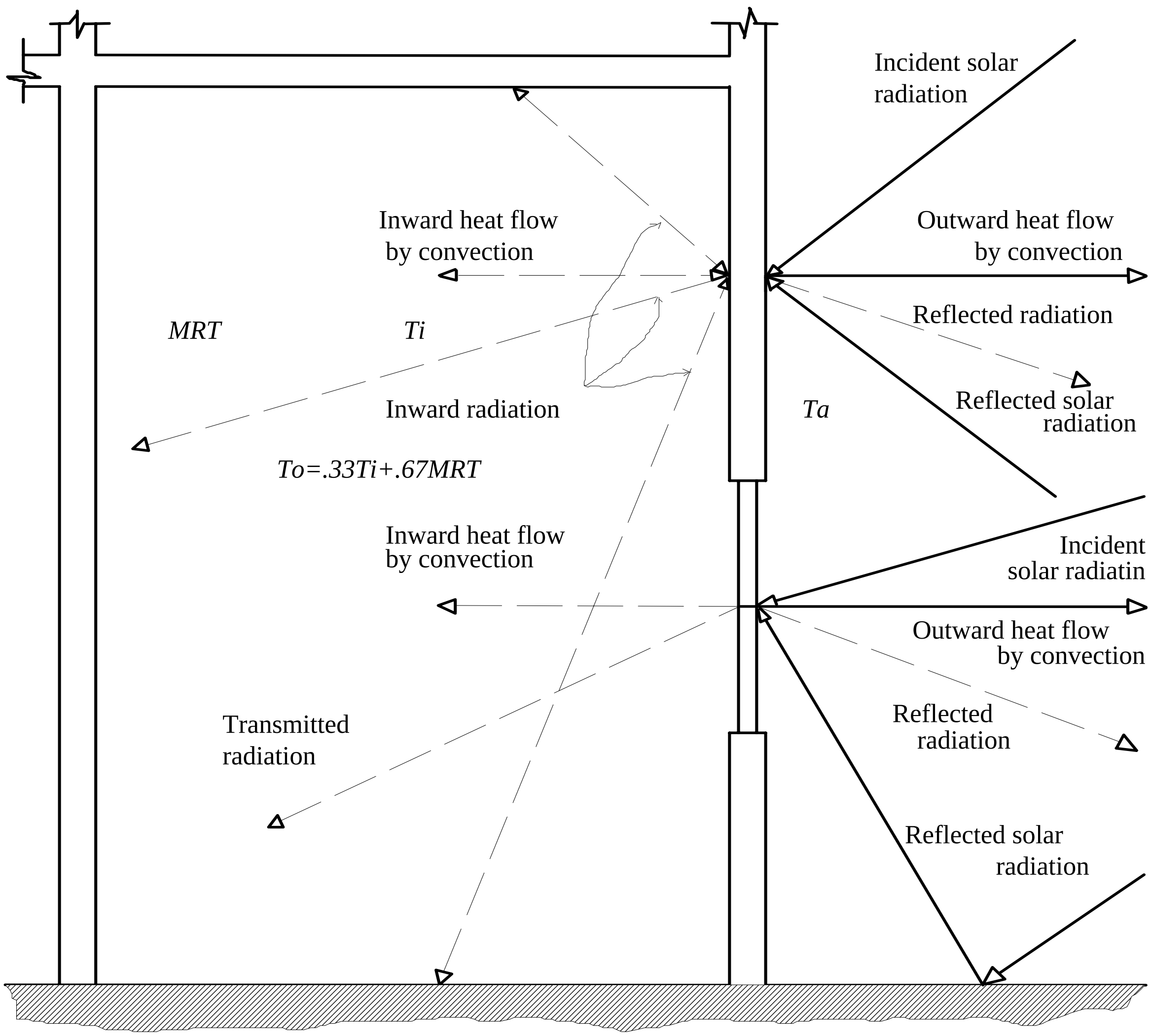

Figure 1: Heat transfer through the building envelope.

An integrated approach for the environmental design of buildings can be achieved by employing optimization techniques to their environmental performance. However, integration of all building environmental parameters can be a difficult and complex problem, and an optimum thermal performance of buildings, for example, can be achieved by coupling a proper optimization technique to the thermal performance analysis of buildings.

Coupling a proper optimization technique to the thermal performance analysis of buildings accounts for the interaction between different design variables and helps not only to optimize energy use in buildings, but also to provide the designer with quantitative guidance on the likely best combination of building design variables for different climates. This paper focuses on the formulation of a thermal design optimization model for the physical components of building envelopes to minimize the energy required to achieve thermal comfort in air-conditioned buildings, or to minimize thermal discomfort in the absence of mechanical heating and air-conditioning systems.

2. SYSTEMS APPROACH AND BUILDING DESIGN

The complexity of problems associated with contemporary buildings and the many variables and interrelations that link them cannot adequately be penetrated by a series of implicit evaluations. Such approaches tend to produce deficient buildings in too many aspects [2].

Technological advances in building structural systems and materials, heating, air-conditioning, lighting and other human comfort-designed systems as well as human needs and requirements for new spatial arrangements and new building types and the associated costs- all lead to the necessary integration of building technology and aesthetics with the function of buildings.

Building design is a process that should be thought of in a whole as an interrelation of parts, acting together to achieve desired objectives and is more than just simple addition of these parts. Therefore, total integration of these parts and their associated variables is necessary for better and efficient solutions to architectural problems.

In order to reach such efficient solutions, there is a need for adequate information to guide building designers in the selection and handling of alternatives. This requires a clear definition of ill-defined architectural problems which designers presently lack.

A systems approach helps in reaching decisions that are optimum for the system as a whole through the division of complex systems into smaller and more manageable components that are logically linked to achieve defined objectives using logical and systematic procedures that can be explained and repeated. In building design a similar concern can be realized in designing buildings effectively to behave in a way that achieves the desired objectives for which they are intended.

Applying a systems approach implies the implementation of optimization techniques. Therefore, in optimization the best solution is sought that satisfies objectives from among a field of feasible solutions under the restriction of certain constraints. Optimization utilizes mathematical techniques to systematically model and analyze decision problems which is basically the focus of the field of Operations Research.

In optimization, decisions are made on certain quantitative measures to get the best course of action possible for a decision problem. To decide on how to design, build, regulate, or operate a physical or economics system using a systems approach requires three main elements[3,4]:

1. Selection alternatives from which a selection is made (variables).
2. Accurate and quantitative knowledge of the system variables interaction (constraints).
3. A single measure of system effectiveness (objective function)

Although optimization models require a great deal of mathematics, creativity and imagination in formulating the problem and its implementation are as important for successful and effective utilization in practice. In order to make decisions using a systems approach, it is necessary to understand processes and to be able to control them. In order to understand processes, their inputs and outputs must be identified and the associated properties must be specified with a proper relationship that links them together. Criteria are also necessary to compare output to objectives which help in controlling the process [2].

Optimization does not require prior knowledge of the solution to the problem as is the case in simulation. However, optimization models have the disadvantage of difficult and sometimes impractical formulation of the problem into a mathematical model, especially ill-defined problems such as those encountered in architectural design where systematic approaches are not traditional practice.

Even though the use of mathematical models in building design is relatively new, application of optimization techniques in different building design problems has taken place over the past 30 years. Such applications range from spatial allocation problems as well as site developments and land use to the design of structural and mechanical systems in buildings with different degrees of success.

The most common architectural problem for which early application of optimization techniques took place is that of spatial arrangement in buildings. Many optimization models were developed to aid designers in the layout of spaces [5] and allocation of activities within spaces for small and multi-story buildings [6,7] The basic objective for these models is to minimize the total communication cost between spaces and the allocation of their activities.

For the thermal design of buildings, most of the efforts were directed to the development of simulation models [8]. However, the speed of today's computers and the availability of suitable energy simulation programs facilitates the integration of simulation models and optimization techniques to the thermal design of buildings for decision making purposes.

Traditional practice has been followed in choosing the capital and operating cost as the criterion of optimization. Wilson and Templeman [9] described a model for determining the thermal design of an office building with minimum initial and operating costs. They used the total discounted cost of the entire heating and insulation process as the criterion of optimality. Based on that and applying geometric programming optimization technique, they developed a computer model that gives the designer an idea about the heating plant capacity and the optimum insulation along with the optimum cost. They assumed that the structure of the building has been designed including the internal and external configurations. The sizes and thermal properties of wall, floor and partition materials as well as the general desired thermal performance of the building and type of heating fuel used are also assumed to be known [9]. These assumptions make their model of limited help in providing building designers with prescriptive information that are mostly needed in the early phases of the design process.

D'Cruz, Radford and Gero [10,11] developed an optimization model for early decision making of the design of parallelepiped open plan office buildings based on thermal load, daylight availability, net usable area and capital cost as the building performance multi-criteria of optimality. They used dynamic programming for building optimization over design variables of window geometry, wall and roof construction, building orientation, massing, floor area and building shape.

Tradeoff diagrams for the physical environment design in buildings were developed by Radford and Gero [12]. They produced a visual solution in terms of tradeoff diagrams for the peak summer internal environmental temperature and the daylight factor criteria in the space.

Different optimization techniques were also utilized to optimize the use of insulation over the components of passive as well as air-conditioned buildings based on technical as well as economical considerations. The common objective is to maximize net energy savings from using the proper amount and distribution of insulation over the building envelope [13].

Based on thermal discomfort as the criterion of optimality, Gupta [14] and Gupta et al. [15] described a model that uses a sequential simplex type of search procedure to optimize the thermal performance of buildings under periodic indoor and outdoor design conditions using typical outdoor weather cycle for summer in Australian cities over several design variables.

3. THE BUILDING AS A THERMAL SYSTEM

In building design process decisions are made on the shape, orientation and selection of the physical components of the building and their arrangements to achieve certain objectives. These decisions are usually limited by certain constraints some of which are outside the control of the designer. The framework of input, process and output approach is influenced by many factors in the building design process:

Inputs:

- Design know-how (professional and technical)
- Climatic conditions
- Energy sources

Objectives:

- Human needs
- Social needs
- Environmental objectives
- Technical objectives

Constraints:

- Cost
- Technology
- Human characteristics
- Physical environment
- Aesthetics
- Practicality
- Regulatory (codes, municipal req., ...etc.)

The inputs may come from the relation between the design subsystems (structural, mechanical, electrical,...etc.) or from outside the system as pre-specified values by the designer.

Design constraints range from those imposed by the client to those related to municipal requirements as well as site restrictions - all of which have to be considered in the optimization process. The structure of the design process of an optimum building thermal design follows the basic framework of the systems approach as shown in Figure 2.

3.1 System Design Variables

Identification of design variables that could affect thermal performance of buildings is necessary in understanding the inputs and outputs and the formulation of building thermal design as an optimization problem. Proper integration of these variables can help to minimize energy requirements to achieve thermal comfort in an air-conditioned building as well as minimizing thermal discomfort in the occupied space in the absence of mechanical heating and air-conditioning systems. Design variables with significant impact on buildings' thermal performance vary over a range of design parameters including siting, building shape, glazing, wall and roof construction, massing, infiltration and operational parameters. Each parameter may be represented by one or more design variable(s). A summary of the important building thermal design variables considered in the optimization is shown in Table 1.

3.2 Objectives

The ultimate goal of building design is to provide occupants with a comfortable environment. In order to determine an optimum thermal design performance based on occupant comfort, it is necessary to establish a relationship between thermal comfort and the factors that have an impact on the thermal performance of buildings. The relationship can then be used to select an optimum combination of building design parameters that achieve the desired objectives. In order to control the design process in a systematic approach, it is necessary to formulate a criterion that can be used to compare the process outputs to objectives. Building thermal design can be optimized with the objective of minimizing building capital and operating cost, minimizing thermal load or minimizing thermal discomfort in the occupied space.

Cost optimization requires the distributing system and plant characteristics to be included as design variables. However, energy cost can fluctuate and might not be a good criterion to base the design decisions upon, especially in the early stages of the design process. Also, some important design parameters, such as building orientation can have significant thermal contribution while not costing

anything. Therefore, for early decision making, an integral view of the building environmental performance based on criteria other than cost might be more desirable. Buildings thermal design is normally optimized to minimize energy requirements to achieve thermal comfort in the air-conditioned space. However, for unconditioned buildings minimum thermal discomfort can be the objective in the absence of environmental control systems.

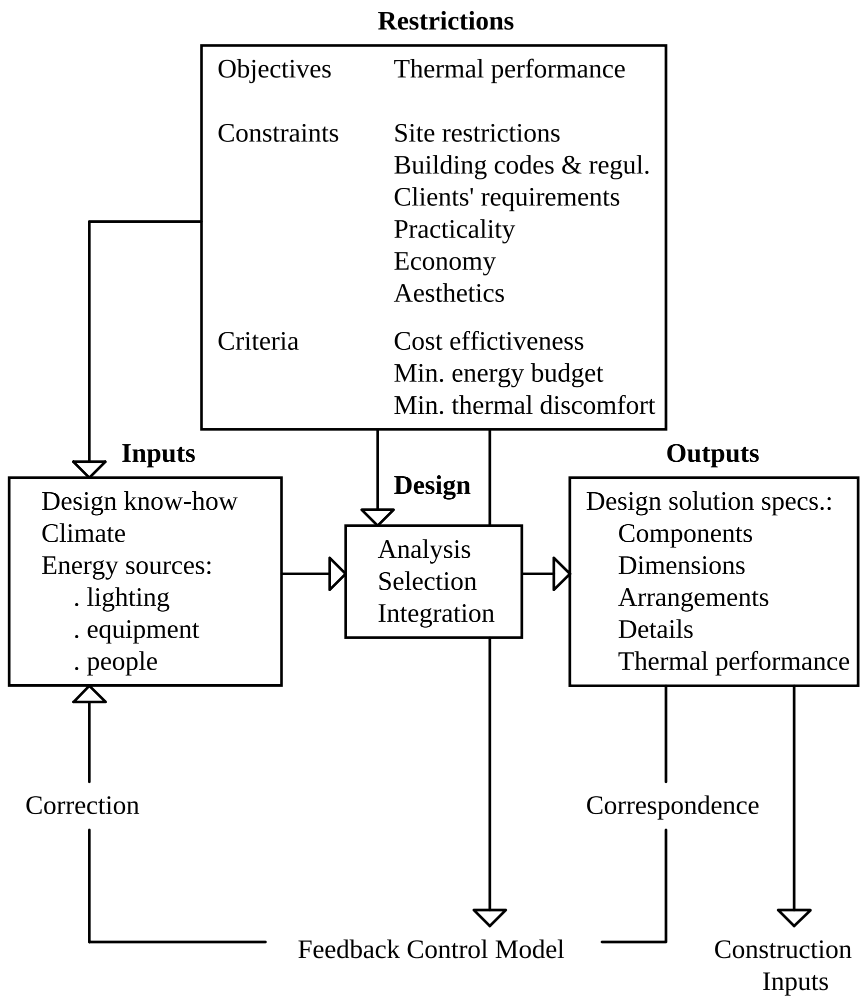

Figure 2: Structure of building thermal design optimization process.

For the purpose of the proposed model, two separate objective functions will be considered for both types of buildings. For the former case of air-conditioned buildings, especially those with high internal loads regardless of the outdoor environment, optimization based on the criterion of thermal discomfort might give unrealistic results when the operational parameters of the building are considered. Therefore, optimization based on the objective of minimum annual source energy utilization kWh/m^2.yr (MBtu/sq. ft.yr) will be considered. Such criterion can be used for any type or size of air-conditioned buildings which is already incorporated in the ENERCALC [16,17] hourly energy simulation program that will be used in the proposed optimization model in the form:

***Objective Function 1*:**

$$\textbf{Min} \quad Q_s = \sum_{i=1}^{n} (Q_{gas,i} + Q_{elec.,i}) / FA \qquad (1)$$

where

n	=	number of hours of the year, 8760 hrs;
Q_s	=	annual source energy utilization index, kWh/m^2 yr (Btu/sq. ft.yr);
$Q_{elec.}$	=	electric energy (Btu);
Q_{gas}	=	gas energy (Btu);
FA	=	building gross floor area, m^2 (ft^2);

source line energy (Btu) = 10,500.KWh.

TABLE 1. Building thermal design optimization variables

Parameter	Variable	Comment
Siting	Latitude, deg.	Pre-specified
	Longitude, deg.	Pre-specified
	Elevation above sea level, m (ft.)	Pre-specified
	Climatic conditions	Pre-specified
Building shape	Gross floor area, m^2 (ft^2)	Pre-specified
	Building height, m (ft.)	Pre-specified
	Aspect ratio	
Orientation	Relation to north, deg.	
Glazing	Glass area / wall area, %	
	Shading coefficient	
	U-Value, W/m^2.C (Btu/hr-F-ft$^{2)}$	
	Emittance	
Wall construction	U-Value, W/m^2.C (Btu/hr-F-ft$^{2)}$	
	Surface absorptance	
Roof construction	U-Value, W/m^2.C (Btu/hr-F-ft$^{2)}$	
Mass	Time lag of the envelope mass, hr	
	Internal mass, kg/m^2 (lb/ft^2) of floor	
Infiltration	Air changes per hour, ach/hr	
Operational	Lighting, W/m^2 (W/ft^2)	
	Equipment, W/m^2 (W/ft^2)	Pre-specified
	People, W/person (Btu/person)	
	Schedule of use	
	Function	

However, for the latter case of unconditioned buildings, especially skin load dominated type, where interaction with the outdoor environment is more influential on their thermal performance, optimization based on the occupants thermal discomfort as the criterion of optimality, similar to that followed by Gupta [14], is found to be more desirable since an environment with minimum thermal discomfort conditions is expected to yield optimum thermal loads. Both objectives will be integrated into the optimization model as two separate options. For thermal comfort evaluation in unconditioned building, an environmental index needs to be used as discussed next.

3.2.1 Thermal comfort There has been a great deal of research on human sensation and thermal comfort. As a result of such extensive research, six major factors of human thermal comfort response have been identified. These factors of dry-bulb temperature, mean radiant temperature, air velocity, relative humidity, activity level and clothing can be classified into two major groups:

- Personal or physical factors (clothing and metabolism), most of which are under human control.
- Measurable environmental factors which can be controlled-to a certain extent-by the building designer (air and surface temperatures, air motion and relative humidity).

In typical indoor clothing, most people perform light, primarily sedentary activity [18], and for the purpose of this research, acceptable thermal environment is based on the assumption of typical indoor conditions. Therefore, we are interested in those measurable environmental factors that can be controlled by the designer within which thermal comfort can be achieved given such specified typical conditions of personal factors.

ANSI/ASHRAE Standard 55-1992 [18] specifies an acceptable relative humidity range of 30 and 60 percent, and at low activity levels the influence of humidity on the recommended ASHRAE summer and winter comfort zones is minor. Changes in humidity levels can be offset by changing space temperature where an increase of 10% in relative humidity can be offset by a decrease of only 0.5 F (0.3 C) in air temperature.

Temperature, on the other hand, is the most important environmental parameter with respect to thermal comfort. The use of an index temperature that accounts for both dry-bulb and mean radiant temperatures may be useful in evaluating thermal comfort in a space. Operative temperature is numerically a weighted average temperature that integrates the influence of both air and mean radiant temperatures based on their respective convective and radiative heat transfer coefficients and is expressed as:

$$T_o = \frac{h_c T_a + h_r T_r}{h_c + h_r} \tag{2}$$

or

$$T_o = a T_a + (1 - a) T_r \tag{3}$$

where

T_o = operative temperature;

T_a = air temperature;

T_r = mean radiant temperature (*MRT*);

h_c = convective heat transfer coefficient;
h_r = radiative heat transfer coefficient;
$a \leq 1$.

This weighted average temperature is considered to be a sufficient criterion for thermal comfort evaluation for unconditioned buildings provided that air velocity and relative humidity are within acceptable limits given typical indoor clothing and light activity level. Based on this criterion of thermal comfort, the objective function has been selected to minimize the discomfort degree hours in the occupied space subject to constraints on the variables that are under the control of the designer and can be formulated as:

Objective Function 2:

$$Min\ \ DDHS = \sum_{i=1}^{n} [(T_{oi} - T_{cu})^+ + (T_{cl} - T_{oi})^+] \qquad (4)$$

where

n = number of hours of the year, 8760 hrs;
T_{oi} = calculated comfort operative temperature at the i^{th} hour, ^oC (^oF);
T_{cu} = comfort operative temperature upper limit, ^oC (^oF);
T_{cl} = comfort operative temperature lower limit, ^oC (^oF);
DDH= discomfort degree hours;
+ = only positive values are summed.

The objective is to minimize the area between the curves and the boundaries of the comfort zone for the occupied space operative temperature profile (as illustrated in Figure 3) by proper integration of the previously discussed design variables through the use of a proper optimization technique.

3.3 Constraints

The choice and range of variations of design variables are governed by many factors. These governing factors include site restrictions, building codes and municipal regulations, clients' requirements, practicality, economy and aesthetics. Any one or more of these variables could be limited within a certain range by the designer to meet any of the above requirements.

The designer is expected to have some knowledge about the building site, the building codes and local municipal regulations and client requirements from which certain constraints can be established on the variables for the control of the optimization. Such constraints include limits on the glazing area, dimensions of the building and thermal properties of its envelope components.

For this model, controlling maximum and minimum values are specified in advance for each of the 15 design variables as follows:

$$U_{r\min} \le U_r \le U_{r\max}$$
$$U_{w\min} \le U_w \le U_{w\max}$$
$$a_{w\min} \le a_w \le a_{w\max}$$
$$TL_{\min} \le TL \le TL_{\max}$$
$$U_{g\min} \le U_g \le U_{g\max}$$
$$SC_{\min} \le SC \le SC_{\max}$$
$$e_{g\min} \le e_g \le e_{g\max}$$
$$p_{\min_i} \le p_i \le p_{\max_i}$$
$$ach_{\min} \le ach \le ach_{\max}$$
$$psf_{\min} \le psf \le psf_{\max}$$
$$1 \le AR \le AR_{\max}$$
$$0^o \le orientation \le 360^o$$

Where

Ur = roof thermal transmittance, $W/m^2.C$ (Btu/hr F sq.ft);
Uw = wall thermal transmittance, $W/m^2.C$ (Btu/hr.F.sq.ft);
a_w = wall absorptance;
TL = time lag, hr;
Ug = glass thermal transmittance, $W/m^2.C$ (Btu/hr.F.sq.ft);
SC = shading coefficient of the window;
e_g = glass emittance;
p_i = percentage of glass area to wall area, Ag/Aw; $i=1,..,4$ for all four walls;
ach = air changes per hour, ach/hr;
psf = internal mass, kg/m^2 (lb/sq. ft) of floor;
AR = building aspect ratio, length of north wall/east wall.

4. OPTIMIZATION TECHNIQUE

In optimization, decisions are made on the best solution that satisfies specific objectives from among a range of feasible solutions. The application of optimization techniques to architectural design is relatively new and requires careful formulation of the problem. The choice of a proper optimization technique is not easy for such ill-defined problems. Although there is a wide range of optimization techniques, not all of them are suitable for applications to building design problems. Many architectural problems require non-linear relationships with non-differentiable objective functions. Therefore, search methods of optimization, where the directions of minimization are determined from successive evaluations of the objective function, were found to be suitable for these types of problems. Examples of such methods include direct search of Hooke and Jeeves [19] and Flexible Polyhedron Search by Nelder and Mead [20,21].

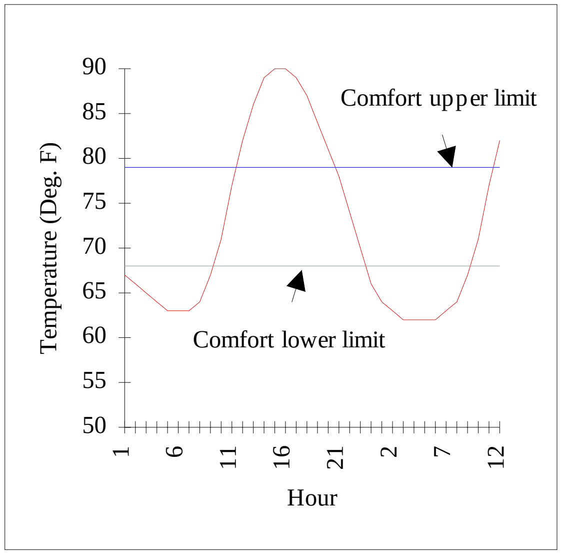

Figure 3: Temperature-time profile with comfort limits superimposed.

Since the Flexible Polyhedron Search technique is more efficient and can deal with curving valleys and ridges, it will be implemented in the proposed model for the thermal optimization of building design. This method minimizes the objective function of *n* independent variables using (*n+1*) vertices of a flexible polyhedron. The highest and lowest values of the objective function are then determined where the highest value is projected through the centroid of the remaining vertices and replaced by a better value. Then, the process continues and the polyhedron is adjusted systematically in the direction of improving objective values by the four operations of reflection, expansion, contraction and reduction until the search converges to the optimum.

The Nelder and Mead optimization technique is designed for unconstrained optimization type of problems. Therefore, provisions will be made to deal with the constraints imposed by the problem under consideration of the form $h_{\min_i} \le x_i \le h_{\max_i}$ as illustrated above.

5. THERMAL SIMULATION MODEL

Since the chosen objective functions can not be expressed directly in terms of the identified building design variables, a thermal simulation model needs to be integrated with the optimization technique of Nelder and Mead for building thermal performance evaluation and comparison of successive values of the objective function. There are many powerful energy simulation programs available. However, in addition to its availability and access to the source code for modifications, the ENERCALC program [16,17] was found to adequately represent the specified building thermal design parameters with accuracy while maintaining simplicity of simulation.

The program is suitable for evaluation of the two previously described objective functions. It calculates the annual source energy utilization for the building based on an hourly simulation for the 8,760 hours of the year. It also includes a space floating temperature option that gives hourly room temperatures. This will be utilized to calculate the mean radiant temperature (*MRT*) in the space for thermal comfort evaluation purposes.

The simulation program will then be used as a subroutine in the optimization model that is called whenever a new set of design variables are established to evaluate the objective functions of annual source energy utilization and annual Discomfort Degree Hours (*DDH*) at that point for comparison with previously performance tested results for air-conditioned and unconditioned buildings, respectively.

6. CONCLUSIONS

The formulation of an optimization model for the thermal design of building envelopes has been presented. The model is intended to help building designers decide early enough in the design process on the best design solution that will satisfy the objective of minimum energy requirements to achieve thermal comfort in the occupied space. The model is based on transient heat transfer analysis where an hourly energy simulation program is used for the evaluation of objective functions for accurate representation of the building thermal behavior. Attempts were made to make the model simple and flexible for future additions of energy related issues not being considered at this stage such as daylighting. More development and validation of the model, as well as results from implementing the model into the design of buildings at different climatic regions will be presented in subsequent papers.

ACKNOWLEDGMENTS

The author would like to thank King Fahd University of Petroleum and Minerals in Dhahran, Saudi Arabia for their support throughout this research.

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