COORDINATE SEARCH METHOD TO OPTIMIZE HEAT FLOW ON A CONSTRUCTAL DISC

BY

MUHAMMED ZAHID AYAR

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This thesis, written by

MUHAMMED ZAHID AYAR

under the direction of his Thesis Advisor and approved by his Thesis Committee, has been presented to and accepted by the Dean of the College of Graduate Studies, in partial fulfillment of the requirements for the degree of

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Thesis Committee

6/6/11

Dr. A. F. AHMED (Chairman)

06/6/2011

Dr. B. S. YILBAS (Co – Chairman)

6/6/2011

Dr. A. Z. SAHIN (Member)

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Department Chairman

Dean, College of Graduate Studies

13/6/11

Date

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Dedicated to my beloved mother and father.

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THESIS ABSTRACT

Name: MUHAMMED ZAHID AYAR

Title: COORDINATE SEARCH METHOD TO OPTIMIZE HEAT FLOW ON A CONSTRUCTAL DISC

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With the increasing accessibility of high performance computing power to engineers in the recent years, we need methods to utilize this power to help in the generation of optimal industrial designs. For this purpose, a constructal algorithm has been implemented. Constructal theory is a new way of searching for the optimum geometry of a flow system. The constructal principle says that the form of a system is a result of a struggle for better performance amongst the existence of constraints on the system. Different systems need different constructal configuration searches based on their performance objectives and constraints. In this work, the numerical optimization method called the Coordinate Search Method has been made use of in order to perform the constructal configuration search to find the optimal heat conducting channel design on a disc. This work serves as the first step in the development of a General Constructal Designer Tool where the input to the tool will be the description of the flow system and the tool will find the optimal flow configurations for the system. This interdisciplinary work makes use of the Coordinate Search Method, Constructal Theory, and Heat Transfer concepts.

Keywords: optimization, constructal theory.

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ملخّص الرسالة

الاسم: محمد زاهد ايار

عنوان الرسالة: طريقة البحث عن الإحداثيات لتحسين التدفق الحراري على قرص إنشائي

الختصص:علوم الحاسب الآلي و المعلومات

تأريخ التخّر ج: رجب 1432

مع تزايد إمكانية الحصول على قوة حوسبة عالية الأداء من قبل المهندسين في السنوات الأخيرة، نحن بحاجة إلى طرق للإستفادة من هذه القوة الحسابية للمساعدة في توليد نماذج صناعية مثلى. و لهذا الغرض، تم تنفيذ خوارزمية النظرية الإنشائية (constructal).تعتبر هذه النظرية طريقة جديدة للبحث عن الهندسة الأمثل لأنظمة التدفق. يقول المبدأ الإنشائي ان شكل النظام هو نتيجة لمجهود إيجاد أداء أفضل باعتبر وجود قيود على النظام. تحتاج أنظمة مختلفة عمليات بحث إنشائية مختلفة بالإستناد على أهداف أدائها والقيود. في هذا الرسالة، تم إستخدام عملية تحسين رقمية تسمى طريقة البحث عن الإحداثيات للقيام بعمليات بحث إنشائية لإيجاد أفضل تصميم لتدفق الحرارة على قرص. و يعتبر هذا البحث بمثابة الخطوة الأولى في تطويرأداة عامة للتصميم الإنشائي حيث تتكون مدخلات الأداة من توصيف نظام التدفق و تقوم الأداة فيما بعد بالبحث عن تكوينات التدفق الأمثل للنظام. يعتمد هذا العمل المتعدد التخصصات على طريقة البحث عن الإحداثيات، النظرية الإنشائية و مفاهيم نقل الحرارة.

درجة الماجستير في العلم

جامعة الملك فهد للبترول ولمعادن

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Chapter 1

Introduction

In Section 1.1, an overview of numerical optimization is provided for the reader. Then later on in Section 1.2, an introduction into Constructal Theory is provided, which is an optimization paradigm dealing mainly with the geometry of a flow system. After that in Section 1.3, a background on heat transfer is given. In Section 1.4, a description of how Constructal Theory works with the Coordinate Search Method is discussed and shown how this is part of a Constructal Search as defined in the literature [13]. After that in the Sections 1.5, 1.6, 1.7, and 1.8, the motivations of this thesis work, the scope of the work, the contributions, and an overview of the thesis is provided in that respective order.

1.1 Optimization Background

1.1.1 Introduction to Optimization

Optimization is the process of choosing the best element from a set of available alternatives [4]. To understand optimization, the simplest possible case that we can consider is where we are seeking to minimize or maximize a function by systematically selecting input values for the function from a given set. This minimum or maximum value of the function, depending on the optimization problem, is the best element in the set of the output of the function.

The optimization process described in the preceding paragraph is one of the simplest possible optimization procedure. A large part of the field of Applied Mathematics deals with generalizing optimization theory and techniques into different formulations that can be used to solve a large variety of optimization problems.

1.1.2 Mathematical Formulation of Optimization

In optimization, mathematically we either minimize or maximize an objective function that is subjected to constraints on it's variables. The following is a listing of the conventional notation:

- x - vector of variables of the objective function, also called parameters or unknowns

- f - objective function, scalar function of x that is minimized or maximized
- c_i - constraint functions, which are scalar functions of x. These constraint functions specify some equations and inequalities that the unknowns x needs to satisfy.

With the notation above, we can write the optimization problem as follows:

$$\min_{x \in R^n} f(x) \qquad \text{subject to} \quad \begin{aligned} c_i(x) &= 0, \quad i \in \mathcal{E}, \\ c_i(x) &\geq 0, \quad i \in \mathcal{I}, \end{aligned} \tag{1.1}$$

$\mathcal{E}$ is the set of equality constraints and $\mathcal{I}$ is the set of inequality constraints.

An example optimization problem is to:

$$\min (x_1 + 3)^2 + (x_2 - 4)^2 \qquad \text{subject to} \quad \begin{aligned} x_1 - x_2^2 &\leq 0, \\ x_1 + x_2 &\leq 3. \end{aligned} \tag{1.2}$$

We can write the problem using conventional notation:

$$f(x) = (x_1 + 3)^2 + (x_2 - 4)^2, x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{1.3}$$

1.1.3 History of Optimization

The history of optimization starts with the history of man ever trying to improve any of the processes that he is using to get more of his objective. Starting from simple optimization methods such as making sharper tools to increase amount of hunted

animals, crop rotation techniques to optimize harvest, and methods of star tracking to optimize time of arrival for sea travels, optimization methods served people in many fields. With time, the methods in various fields have improved gradually but with the advent of digital computers in the 1950s, optimization methods have developed much more rapidly [8].

Some historical examples of optimization include Gauss' steepest descent technique. Leonid Kantorovich came up with the linear programming mathematical technique to help him with the task of optimizing the production in the plywood industry for the Soviet government in 1939. Linear programming was also used by George B. Dantzig to mechanize the planning process of a time-staged deployment, training, and logistical supply program in the late 1940s [2]. In 1947, Dantzig published the Simplex optimization algorithm. John von Neumann developed the theory of duality within the field of linear programming that helps in the analysis of the objective function.

The optimization process involves finding the best or optimal solution to a problem. For some problems, the best solution would be a maximum of a value such as maximum return on investment. For other problems the optimal value would be the minimum of a value such as minimizing the expenses in a factory operation.

In the 1630, Fermat developed a way to find the minima or maxima of a function by finding the points where the derivative is equal to zero [6]. Fermat's Theorem states that the optima This method of finding optOptimization algorithms that have

been developed based on this method of finding optima at points of the function where the derivative is zero are called derivative-based optimization algorithms. However, they perform properly in cases where the variables in the parameter set are independent and will fail to perform in cases where the variables depend on each other.

Optimization is the process by which better versions of things are produced. Optimization can be applied in different fields including stock market investments by optimizing the portfolio with the objectives of maximizing gains and minimizing risks, improving the manufacturing process by minimizing transportation costs and maximizing profits, and optimizing heat flow in thermodynamic structures to mention a few. There are different optimization algorithms that have been developed over time to suit different types of problems. To optimize a given system, design, or process, one has to choose a suitable optimization algorithm for the problem as well as adapt the problem to the chosen optimization algorithm.

Optimization algorithms begin by initially guessing a solution and then they iteratively improve their estimate until a solution is reached. Convergence is the term used to describe the event that a solution to the optimization problem has been reached. A wide variety of optimization algorithms have been developed within the past 60 years to address different types of problems [30]. Good optimization algorithms should have the following three important qualities namely:

1. Robustness - The algorithm should perform satisfactorily for a wide variety

of problems of the same class for all reasonable input values.

2. Efficiency - The algorithm shouldn't need too much computation time or storage.

3. Accuracy - The algorithm should precisely find a solution without being too sensitive to errors in the data or rounding errors during computation.

1.1.4 Sub-fields of Optimization

The field of optimization can be divided [4] into the following major sub-fields:

convex programming - this is used when the objective function is convex and the constraints form a convex set. According to [30]:

- A convex set is one in which the straight line between any two points in the set is also completely in the set.
- Formally, a set is convex with the following condition: for any two points x , $y \in$ S, $\alpha\ x + (1\text{-}\alpha)y \in$ S for all $\alpha \in [0,1]$.
- A function is convex when its domain S is a convex set and for any two points x & y, following is true: f($\alpha\ x + (1\text{-}\alpha)y) \leq \alpha$ f(x) + (1-α)f(y), for all $\alpha \in [0,1]$.
- linear programming - optimizing a linear objective function with a set of linear constraints. This set is called a polyhedron. If it is bounded, it is

called a polytope.

- second order cone programming
- semidefinite programming
- conic programming
- geometric programming

integer programming - optimization problem in which some or all variables are restricted to be integers.

quadratic programming - the objective function can have quadratic terms while the constraints still have to be linear

nonlinear programming - either the objective function or the constraints have a non-linear component. It is possible that a non-linear program can be convex but it may not be. The difficulty of solving the problem is affected more by the convexity of the program rather than the linearity.

stochastic programming - some of the constraints or parameters depend on random variables.

robust programming - Instead of using random variables like stochastic programming, robust programming attempts to control the uncertainty of the program by taking into account inaccuracies of the input data.

combinatorial optimization - the set of feasible solutions is discrete or can be reduced to a discrete set. Some examples are:

- traveling salesman problem
- minimum spanning tree problem
- vehicle routing problem
- eight queens puzzle
- knapsack problem
- cutting stock problem

- has applications in :

- AI
- machine learning
- mathematics
- software engineering

infinite-dimensional optimization

metaheuristics

constraint satisfaction - constraint programming is included in this sub-field

disjunctive programming fields dealing with dynamic contexts:

- calculus of variations
- optimal control
- dynamic programming
- mathematical programming with equilibrium constraints

multi-objective optimization - for example, for a structural design, we can have two objectives such as minimizing weight and maximizing rigidity. However, these objectives may be conflicting and therefore we need to make a trade-off between them. Out of all the possible designs for such a structure, one of the designs can be very light but not rigid, one can be very rigid but very heavy. The set of all possible trade-offs is called the Pareto set.

A design is Pareto optimal if it is not dominated by other designs. A design is dominated by another design only if it is worse in all respects to the other design. to be a Pareto optimal design, the design should be better than one design in at least one aspect.

multi-modal optimization - evolutionary multi-model optimization is included in this sub-field.

1.1.5 Optimization Algorithms

Over the years, a great number of optimization algorithms have been developed. The algorithms can be grouped as follows [5]:

- methods calling functions:
  - golden section search
  - interpolation methods
  - line search
  - successive parabolic interpolation
- methods calling gradients:
  - convergence (trust region, wolfe conditions)
  - gauss-newton
  - gradientn
  - levenberg-Marquardt
  - Conjugate gradient
  - Quasi-Newton (BFGS, L-BFGS)
- methods calling Hessians:
  - Newton's method
  - Sequential quadratic programming
- non-linear programming:
  - Barrier methods

  - Penalty methods
  - Augmented Lagrangian methods
  - Sequential quadratic programming
  - Successive linear programming

- convex minimization:
  - cutting-plane method
  - interior point method
  - reduced gradient (Frank-Wolfe)
  - Subgradient method
  - Semidefinite programming

- linear programming:
  - ellipsoid method
  - interior point method
  - Karmarkar's algorithm
  - Simplex algorithm

- combinatorial algorithms:
  - approximation algorithm

  - dynamic programming
  - greedy algorithm
  - integer programming (branch & bound or cut)

- meta-heuristics:
  - evolutionary algorithm
  - hill climbing
  - local search
  - simulated annealing
  - tabu search

To optimize situations in which there exists any dependency among the variables in the parameter set, derivative-free optimization algorithms have been developed. One such derivative-free algorithm is the Coordinate Search algorithm which is used in this work to optimize the configuration of heat conducting channels on a disc. The Coordinate Search algorithm freezes the changing of all variables except one and optimizes that variable by a line search to go closer to the solution. After that variable has been optimized, it proceeds to the next variable and freezes the other variables. This process is repeated until all the variables are optimized in this manner.

1.1.6 Applications of Optimization

Optimization has applications in mechanical engineering, economics, operations research, and many other fields. To get a brief idea of the different fields where optimization can be applied the following list is provided as some of the applications of non-linear programming optimization [30]:

- chemical equilibrium and process control
- gasoline blending
- oil extraction, blending, and distribution
- forest thinning and harvest scheduling
- economic equilibration of supply and demand interactions under various market behavioural phenomena
- pipe network design for reliable water distribution systems
- electric utility capacity expansion planning and load management
- production and inventory control in manufacturing
- least squares estimation of statistical parameters and data fitting
- design of aircraft, ships, bridges, and other structures

1.2 Constructal Theory

A new optimization methodology called Constructal Theory is being used by increasing numbers of people for optimizing thermodynamic architectures. This method is based on the constructal law stated by Adrian Bejan in 1996 [1]: "For a finite-size (flow) system to persist in time (to live), its configuration must evolve such that it provides easier access to the imposed currents that flow through it." A tree-shaped architecture in a given structure is maintained by the flows that move throughout that structure. These flows may be chemical, fluid, heat, or other types of flows. In all these different types of flows, there is one principle of how the structures of these flows are generated. This principle is called the constructal law and it is deterministic. The principle is the optimal distribution of imperfection.

Constructal theory describes the design of things we see around us in different fields such as biology, engineering, and geology. For example let us consider a river drainage basin, which consists of an area (the plain) to point (river mouth) flow pattern. According to the constructal law, successive configurations of smaller global flow resistances are required to be generated with respect to time. As a result, a balancing of resistances is needed between the seepage along the hill slopes and the main channel flows. The resistances (imperfections) cannot be eliminated from the system, however, they can be distributed in the system so as to make the global effect minimal, and make the river basin the least imperfect it can be. With

time, the river basin changes to become an equilibrium flow-access configuration.

The value of this way of thinking is that it points the mind to the configuration of a given thing, whether that is a river basin, a cooling system for an electronics package, or a lung within a body. The unknown in design is the configuration. The constructal law guides our thinking in an effort to discover the configuration.

For the river basin example, the constructal law finds the tree configuration with balances between different flow resistances like the Darcy (seepage) flows and the main channel flows. The tree flow is the theoretically effective method to provide flow access between one point (source/sink) and an infinity of points (area/volume). The tree structure provides us with a complexity that allows for multiple-length scales that are distributed non-uniformly across the given area/volume.

The tree shape and the multiple-length scale features are provided in any configuration that involves a flow between a point and an area/volume. This could be the flow trees in electronics, city traffic, vascularized tissues, or lightning. The principle used to generate and discover this tree configuration is universal.

The tree is not the only configuration that is provided by the constructal law. With fluid flow between two points, a straight tube is the configuration that can be used. Examples of round tubes in the natural world and from engineering include blood vessels, volcanic shafts, subterranean rivers, and plumbing.

The configuration generation phenomenon, called design, has scientific principles which are now becoming known. As a result, it becomes possible to learn to expect

where opportunities lie for discovering better configurations in different systems in all kinds of engineering and scientific applications. This strategy reduces the effort and time required in looking for the optimal design and comes as a benefit of learning the generation of design as a scientific topic.

The constructal paradigm had its beginnings with Adrian Bejan in 1996 when he was trying to solve the problem of minimizing the heat resistance between a heat generating volume and one point [33]. He found that a tree network with every feature being a result rather than an assumption was the optimal solution for his problem. From this result, he made the conclusion that every natural tree structure is also the result of performance optimization from volume to point flow.

The constructal method deals with the generation of the architecture of flows [33]. To generate a tree-shaped architecture, you start with small pieces and build up to larger assemblies or "constructs". To construct an optimal structure, you optimize it at every level of scale, starting from its smallest part. According to Constructal theory, a system's shape and internal flow structure doesn't develop by chance, but by a struggle to perform better.

While designing and optimizing thermodynamic systems [14], Adrian Bejan discovered a deterministic principle to generate geometric form in systems. He found that form is a result of a struggle for better performance to meet the objective of the flow system amongst its' existing constraints. Some of the discovered optimized geometries are:

- tree-shaped flows
- round tubes
- river cross sections

Some geometries have regularly spaced internal channels such as:

- compact heat exchangers
- cracked solids
- honeybee swarms

There can be different flow architectures such as:

- flow between volume/area and point (and vice versa), like:
  - river
  - respiratory system
  - circulatory system
- flow between point and point through which the optimal geometry of the flow would be a straight round tube.

According to Constructal Theory, the macroscopic visible structure of a flow is derived from the method of flow and not the contents. The flow structure can be in the form of channels, streets, ducts, and fins. The flow structure will be visible only if flow has at least 2 regimes of flow with different resistivities (high and low).

high resistivity flow - Darcy flow, viscous diffusion - covers most of space

low resistivity flow - laminar and turbulent flow - streams n ducts

The geometric balance between flow regimes can cause eddies and Benard convection.

The strategy in Constructal theory is to start with an elemental volume at the micro-scale and assemble and optimize at every step of the optimization of the system towards the macro-scale. The constructal theory also provides a new time arrow in physics which states that flow structures evolve toward easier flowing structures. In the case of volume/area to point flows, the smallest volume is known and fixed. For example, for a disc flow environment, the elemental volume would be a sector of the circle. The chief unknown of the architecture would be the shape of this sector. The optimization of the sector can be done by minimizing the global thermal resistance of the sector.

When a flow system is complex, the currents and resistances are many and diverse. Higher performance can be achieved by balancing each resistance against the rest. The distribution and redistribution of resistances is done by means of changes to the architecture. Flow systems should be free to change. Morphing is the result of collisions between the global objective and the global constraints of the system. When the structure of a macroscopic flow system is complex, it can have many tree-shaped paths for the flows within it. The interstices of the flow system

(spaces between the smallest flow branches) can be filled by elemental systems. The structure can have multiple scales of length, time, and force.

Constructal theory is comprised of many lessons learned in terms of finding the optimal geometry for a given flow system. These lessons are made use of in order to more quickly find the optimal geometry for a given flow system. Some the lessons are:

- single duct with large cross-section offers smaller flow resistance than 2 ducts with smaller cross-section connected in parallel
- lowest resistance belongs to shortest duct
- duct whose cross-section doesn't vary longitudinally offers lower resistance than one with variable cross-section
- for point-to-volume/area flow, the best architecture is a tree
- straight duct is the best geometry for point-point flows
- the optimal size step between branching levels is $D_i + 1/Di = 2^{(} - 1/3)$. Starting with Murray's study of blood vessels, many studies have shown this.
- The are smaller numbers of branches when:
  - point flow has access to all directions around that point
  - point flow is toward one direction only

- We can improve tree geometry by allowing it more freedom to morph.

Lessons are available for some cases but not for all, therefore for new flow systems, new constructal configuration searches need to be performed to find the optimal flow geometry.

Constructal theory is different than traditional design which works by building a model (which is an assumed macro-configuration) and works on optimizing it. If time and money permits, 1 or 2 alternatives may be produced in the traditional design methodology. The "constructal strategy liberates designer from straight-jacket of modeling (assumption of certain macroscopic structure)". The "physical configuration is the chief unknown in design". The "configuration of complex system will eventually fit inside specified constrained macroscopic volume".

The architecture of a flow system is a set of geometric variables that can be optimized at each assembly level. The geometry isn't random but results from principle - which is the optimal distribution of imperfections within the system. For this reason, tree structures can be found everywhere in nature and engineering. Good geometry makes a system achieve the highest performance therefore geometry is important.

1.3 Heat Transfer Background

To make sure that the reader is familiar with the heat transfer terminology that will be used throughout this thesis, a concise background is provided in this section which will go over heat conduction concepts and different heat conducting structures that can be used for cooling purposes.

Heat transfer refers to the flow of energy between systems.

To understand heat transfer, let us imagine a gas in which there exists molecules that are moving by translation, vibration, and rotation. These movements of translation, vibration and rotation of the molecules are due to the energy that is going through those particles that are making up the gas matter. The temperature of the gas is associated with this energy of the constituents of the gas matter. When two particles collide in the gas environment, a transfer of energy from the more energetic to the lesser energetic particle takes place. This is called heat conduction and it always takes place from a region of higher temperature (with more energy) to a region of lower temperature. So whenever we have a temperature gradient, energy transfer or heat conduction takes place towards the direction of decreasing temperature. Whenever we have a net transfer of energy by random molecular motion towards any direction, this process is called a diffusion of energy. Diffusion is defined as the random movement of particles from areas of higher to lower concentrations. We can conclude that heat transfer or heat is moving energy due to a temperature gradient.

There are different types of heat transfer that can occur:

1. Heat conduction - occurs between two stationary mediums

2. Heat convection - occurs between mediums that are moving relative to each other such as a moving fluid over a surface

3. Thermal radiation - is the energy that is given out by all objects that have a temperature

4. Phase-Change transfer - is the energy generated as a result of a phase-change in a given matter

Heat conduction can occur through the movement of free electrons on a given matter. Heat conduction can also take place with the lattice vibrations of a matter [3]. The rate of heat conduction decreases as the density of the matter decreases. In general, metallic matters are the best heat conductors because they have metallic bonds rather than covalent or ionic bonds between its atoms.

It is possible to quantify heat transfer processes by using rate equations which measure the amount of energy transferred per unit of time. For heat conduction, we use the Fourier equation to calculate the amount of heat transferred. Fourier's heat conduction equation is a phenomenological law which means that it is based on observations only and not derived from existing theory [25]. To take an example, let us imagine a cylinder rod that is insulated on its surface and end faces are

maintained at different temperatures $T1 > T2$. Due to the temperature difference, heat will flow from the first end along the positive x direction to the second end. In this experiment, we are able to measure the heat transfer rate q_x and want to find its relation with the following variables:

1. $\triangle T$ - this is the temperature difference between end 1 and end of the cylinder rod
2. $\triangle x$ - this is the rod length
3. A - cross-sectional area

To find if the variables above are directly or indirectly proportional to the known heat flow rate q_x, first, we can imagine a case where for the cylinder rod, we hold the $\triangle T$ and $\triangle x$ constant and allow variation in q_x and A. For this case, we find that as the heat flow rate q_x increases, the cross-sectional area A will also increase. If we hold $\triangle T$ and A constant and allow $\triangle x$ to vary, we see that by increasing q_x, such as by increasing the temperature $T1$ at the first end, the temperature at $T2$ may rise but it won't rise enough to maintain the previous $\triangle T$, so to keep the previous $\triangle T$ constant, we need to reduce $\triangle x$. This shows that $\triangle x$ and q_x are inversely proportional. Then if we hold A and $\triangle x$ constant, we see that increasing $\triangle T$ will also increase q_x. Hence we have the following proportionality between the variables:

$$q_x \propto A\frac{\triangle T}{\triangle x}$$

Even by changing the material of the rod, the above proportionality still holds. For the same values of A, $\triangle T$, and $\triangle x$, the value of q_x changes by changing the material. For example, for plastic, the q_x value will be lower than that for metal even when we keep A, $\triangle T$, and $\triangle x$ equal for the two materials. We can convert the proportionality above into an equation by introducing a coefficient that will reflect the behavior of different materials:

$$q_x = kA\frac{\triangle T}{\triangle x}$$

In the equation above, k is the coefficient for thermal conductivity and an important property of any material. By taking the limit as $\triangle x \to 0$, the heat rate equation becomes:

$$q_x = -kA\frac{dT}{dx} \tag{1.4}$$

The heat flux equation becomes:

$$q''_x = \frac{q_x}{A} = -k\frac{dT}{dx} \tag{1.5}$$

The minus sign is put into the equation because heat rate increases going towards the place in the material where the temperate decreases. Since the heat flux is a directional quantity, we can use vector notation to write a more general equation of heat flux:

$$q'' = -k\nabla T = -k\left(i\frac{\partial T}{\partial x} + j\frac{\partial T}{\partial y} + k\frac{\partial T}{\partial z}\right) = iq''_x + jq''_y + kq''_z \tag{1.6}$$

1.4 Constructal Theory with the Coordinate Search Method

When we have a multi-variable function that has independent variables, gradient-based approaches can be used to find the optimum value (minimum or maximum) of that function.

However when we have a function whose variables are dependent on each other, we cannot use partial differential equations. Often the solution of multi-variable optimization problems is desired to be found with a gradient-free algorithm. This can be the cases when gradient evaluations are difficult and/or gradients do not exist.

The Coordinate Search Method is one such gradient-free algorithm. It's advantages are that it is:

- very simple to implement
- relatively robust
- not requiring of any gradients

Some of the disadvantages of this algorithm are:

- may take a lot of calculations
- converges more slowly than the steepest descent method

- can get stuck needing special tricks to move along

The Coordinate Search Method is also called:

- Cyclic coordinate method - CCM
- Alternating variables method
- Coordinate descent method

As mentioned in Section 1.2, for some flow systems for which the optimal geometry is not known, a new constructal configuration search needs to be done. Rather than performing an exhaustive search that would take a long time, the coordinate search method is to be applied to find the optimal configuration which is based on a set of possibly dependent variables that make up the architecture or configuration of a flow system.

1.5 Motivation of the Work

The approach to the optimum design is based on the constructal law, which considers minimizing the heat losses (or heat resistance) in the system. A flow system has four components: objective, behavior (which can be modeled by equations such as Bernoulli's equation or others depending on the types of flows), constraints (such as system size, available material for flow channels, etc.), and the geometry of the

flow system. For a flow system, given the first three components, the constructal design method aims to produce the optimal geometry for that flow system.

For a given set of conditions for a known flow system, a constructal configuration for the geometry is expected to make the flow system perform optimally. For a different set of conditions for the same system, a different constructal configuration is required. Some flow systems that have Y-shaped junctions with laminar flow have known constructal configurations such as having bifurcated sub-branches having a width of half of the parent branch. These known optimal relations between the different parts of a geometry have been discovered. However the constructal configuration found for one set of conditions may not by the optimal configuration for the same flow system under a different set of conditions. A new constructal configuration needs to be found for the new set of conditions.

In this work, we developed an easy way to specify conditions for a heat flow system on an insulated disc and a way to search for the constructal configuration for that system. We have automated this process on one of the simplest cases, so as to successfully complete it. This is the first step in the development of an artificially intelligent constructal design tool whose function is to produce optimal configurations for any given flow system. The user will provide the objective of a flow system (such as to maximize heat flow) and the constraints of the system (such as the size and amount of material used for flow channels) and the Constructal Design Tool will identify Insha'Allah the equations for modeling the behavior of

the system (such as heat flow equations in our case) and perform a search for the constructal configuration. This search may also utilize the parallel hardware so as to reduce the computation time if possible.

1.6 Scope of the Work

The scope of this work is limited to producing constructal configurations for heat flow coming from the center of an insulated disc and flowing across conducting channels towards the circumference of the disc. The configuration can have only one bifurcation level and each branch can be split up to five ends. This serves as a foundation to develop the artificially intelligent Constructal Design Tool that should recognize the types of flows running through a described system and draw, discretize, and apply the appropriate equations on the discretized mesh in the process of finding the optimal configuration for the system whose description comes from the user of the tool.

1.7 Contributions

Constructal theory can be utilized for geometric optimization to find the optimal design for any given flow system. However, there was no tool currently available that easily allows one to produce designs that are based on constructal theory and consequently see the heat distribution on those designs. With this thesis, a tool was

developed that can produce an optimally configured disc (with one bifurcation level) when provided a volume constraint for the heat conducting channels and a range of acceptable number of branches. The coordinate search optimization method was tailored to solve the optimization of the configuration of the heat conducting channels on the disc.

1.8 Thesis Overview

The thesis is organized as follows:

1. Chapter 2 provides a literature survey on papers that are related to the topics covered in the thesis, namely Constructal Theory and optimization methods, with emphasis on the Coordinate Search method.

2. Chapter 3 talks about how the Coordinate Search Method works and discusses how it was implemented to solve the objective of the thesis, namely to find an optimal configuration of heat conducting channels on a disc.

3. Within Chapter 4, the results achieved by performing an exhaustive search through the search space are discussed as well as the results and performance of the Coordinate Search Method. A sample set of screenshots of the results are also provided in this chapter.

4. Chapter 5 concludes the thesis with a summary of the thesis work, highlighting

the benefit of using the Coordinate Search method for constructal configuration searches. The benefit of bifurcating flow channels is also pointed out. A section on the future possibilities of extension of this thesis work discusses the design of a more general Constructal Design Tool that can be used to optimize any flow geometry in general.

Chapter 2

Literature Review

The literature review will cover recent papers for topics related with Constructal Theory and with optimization and the Coordinate Search Method in particular. The first section of the review talks about papers in which ideas from Constructal Theory were used to optimize systems with different optimization objectives. The second section deals with papers that reported their performance analysis on Constructal systems as well as papers that talk about the application of Constructal Theory to optimize a variety of different types of structures. The third section reviews a paper that praises Constructal Theory and compares it to other theories in science. A paper that argues against Constructal Theory is also review in this section. The next section reviews papers that talk about an assortment of different optimization methods available today. The sixth section discusses papers dealing with optimization tools that can be used to assist in the optimization process such

as algorithm configuration tools. The seventh section reviews papers that talk about the Coordinate Search method, and the last section covers some papers that discuss optimization for large-scale problems with large search spaces.

2.1 Optimization Objectives

In constructal design method of flow systems, there may be many different objectives to aim for in order to optimize a given system. Chen et al. [17] have followed the constructal design method to minimize entransy dissipation for volume to point heat conduction. They showed that the constructs based on entransy dissipation could decrease the mean temperature difference better than the constructs based on the minimization of the maximum temperature difference and could improve thermal conductivity greatly.

Chen et al. [16] in another study, have aimed to minimize thermal resistance to optimize a heat exchanger. They have found that there is no one-to-one correspondence between the minimum entropy generation rate and the maximum heat transfer rate therefore; the minimum entropy generation principle can't be used for heat exchanger couple optimization.

In another paper, Chen et al. [15] have minimized the constructal entransy dissipation of an electromagnet. They have concluded that the optimized constructs based on the minimization of entransy dissipation are the same as the constructs

that are based on the minimization of the maximum temperature difference when the density of the thermal current in the high conductive link is linear with the length of the construct. They were different only when the density of the thermal current was not linear with the length of the construct. Their results showed that the solenoid (electromagnet) that was optimized based on the minimization of entransy dissipation was considerably larger than the one optimized only from the electromagnetic point of view.

Rocha et al. [34] have done a constructal design for cooling a disc-shaped area by conduction. They have employed a strategy of optimally placed inserts of highly conductive material based on the principle of the minimization of global resistance.

Ghodoossi [19] studied the entropy generation rate in uniform heat generating areas that were cooled by conducting paths and used this as a criterion for rating the heat flow performance of constructal designs. He reported that the heat flow performance did not necessarily increase with the increase of the internal complexity of the heat generating area.

2.2 Performance Analysis and Optimum Design of Structures

There have been many papers published with regards to analyzing the performance of a variety of different structures based on constructal theory and talking about the

process of optimizing those structures. Kundu and Bhanja [27] have analyzed the performance and optimization of a constructal T-shaped fin that was subjected to a variable thermal conductivity and convective heat transfer coefficient. They have found that the present analytical model for thermal performance always predicts an under value for fin performance compared to published results; whereas the analytical model established by the authors determines an optimum heat transfer rate and produces an over value.

Arslanturk [10] has studied the optimum design of space radiators that have a temperature-dependent thermal conductivity. He has found that the performance of a radiator is significantly affected by variable thermal conductivity if it is the case that there are large temperature differences.

Wei et al. [36] studied the constructal optimization for discrete variable cross-section path for a flow between an area and a point. They have found that the minimum of the maximum thermal resistance can be obtained by assembling the cross-section conducting path based on constructal theory. They have also found that the optimized minimum thermal resistance that is based on a variable cross-section conducting path is smaller than the one that is based on a constant cross-section conducting path. Also, when the optimum number of lower-order constructs that are used to assemble higher-order constructs was fixed, the constructal method based on discrete variable cross-section conducting paths further reduced the thermal resistivity.

Chen et al. [16] have optimized a heat exchanger by minimizing thermal resistance. Raja et al. [32] have looked at the thermal performance of a multi-block heat exchanger that has been designed on the basis of constructal theory. The results of their study showed that the constructal heat exchangers that are both finned and unfinned have an effectiveness that is around 20% higher than conventional heat exchangers under similar conditions.

Chen et al. [15] have minimized the constructal entransy dissipation of an electromagnet. The results of their study showed that for a fixed G and φ, the minimum mean temperature difference decreased as the number of cooling disks n is increased. As n increased so did the length of the solenoid (electromagnet), and the radius and volume decreased. As a result, the solenoid that was optimized based on the minimization of entransy dissipation was considerably larger than the one optimized only from the electromagnetic point of view.

Daguenet-Frick et al. [18] have designed a constructal micro-channel network for flow-boiling in a disc-shaped body. They have observed that the highest temperature is found where the distance between two micro-channels is the largest (predominantly at the edges). They have shown that Murray's law is the best solution for characterizing successive diameter ratios for complex structures. From their work, they have concluded that increasing the number of channels will decrease the thermal resistance however complicated the structure may be. The use of a radial structure that has $2n_0$ central channels turned out to be more efficient than a

one pairing level design that had n_0 central channels. When the pumping power is low, the radial flow pattern provided the least thermal resistance. In the case of medium pumping power, having a design with pairing level displayed the lowest thermal resistance. For high pumping power, a two pairing level design showed the best solution. They have also concluded that complexity is not necessarily the best solution.

Rocha et al. [34] have done a constructal design for cooling a disc-shaped area by conduction. They have employed a hierarchical strategy to develop an optimal internal structure of a round heat-generating body that was cooled at its center with the help of optimally distributed inserts of highly conductive material placed based on the objective of the minimization of global resistance that was subject to global constraints (total volume and total volume of the highly conductive material).

Kuddusi and Egrican [26] analyzed fourteen different constructal theory applications that involved tree-shaped flow networks and have seen that the constructal designs did not necessarily improve flow performance with increases in the internal branching of the flow field.

Xu et al. [38] studied heat conduction in fractal tree-like branched networks. They have found that the effective thermal conductivity of the branched networks is always less than that of a single channel, and that the value for the thermal conductivity can go to zero in certain conditions. When the branching number N was fixed, the heat conduction rate reached a maximum at a given diameter ratio β_m.

This value for the diameter ratio corresponded to the fractal dimension $D_d = 2.0$. They have also found that the heat conduction in the branched networks was very different from Murray's law for both the laminar regime and the turbulent flow regime. Boichot et. al. [14] examined the process of tree-network structure generation for heat conduction by cellular automaton. They proposed a simple algorithm to solve the case of cooling a heat generating surface by means of conduction. Using simple assumptions and basic rules, the algorithm equalized thermal gradients between high and low conductive materials on the surface. This led to the emergence of a tree-network configuration. Comparisons with the analytical constructal theory showed that in all test cases, the cellular automaton produced the same result as the analytical theory.

2.3 Constructal Theory Praise and Criticism

There are many papers that praise Constructal Theory and how it works and that it describes a new physical law that governs the geometry of flow systems. There also those papers which put forward a view of Constructal Theory not in as favorable a light of the first type of papers. Pramanick and Das [31] sought the basis of analogies among physical theories and documented how Schmidt's criterion of fin design, the tangent law of conductive heat transport, and Fermat's principle of geometrical optics are special conditions of the method of synthetic constraint, which in turn is

a natural consequence of the constructal law.

Kuddusi and Egrican [26] have made a critical review of constructal theory. Among the fourteen different constructal theory applications that involve tree-shaped flow networks that they have analyzed, they have reported that the constructal designs did not necessarily improve flow performance if the internal branching of the flow field is increased. On the contrary, they wrote that the performance will predominantly be lowered if internal branching of the flow field is increased.

2.4 Optimization Methods Review

There are a plethora of optimization methods available that are being used to solve problems in many fields of science, engineering, and other fields such as investing. There are also different types of optimizations methods available that can be used for different purposes. Torczon and Trosset [35] have briefly surveyed the history of pattern search methods and have identified their common structure and pointed out the key features that the Nelder-Mead simplex algorithm lacks. They have given some practical suggestions to use pattern search algorithms in serial and distributed computing environments.

April et al. [9] have prepared a tutorial on using optimization methods such as the Tabu Search, Scatter Search, Mixed Integer Programming, and Neural Networks together with simulation. They have shown that two possible applications for

these optimization techniques are project portfolio management and supply chain management.

Hansen et al. [20] have investigated particle swarm behavior on ill-conditioned functions and have found that the performance is very good. They have argued that invariance properties such as rotational invariance, and others, are desirable because of the improvements in the performance prediction. Auger et al. [12] compared the performances of the quasi-Newton BFGS algorithm, the NEWUOA derivative free optimizer, the Covariance Matrix Adaptation Evolution Strategy (CMA-ES), the Differential Evolution (DE) algorithm, and Particle Swarm Optimizers (PSO). The benchmark function used for comparing reflected important challenges faced in real-world problems. Performance depending on the conditioning of the problem and rotational invariance of algorithms was investigated.

Yuan et al. [40] compared the performance of three modern continuous optimizations algorithms, MADS, BOBYQA, and CMA-ES, with population-based iterated sampling and random sampling to automate algorithm configuration of numerical parameters. Their experiments showed that BOBYQA had the best performance for low-dimensional problems and CMA-ES seemed to be robust over all dimensions.

2.5 Optimization Tools

Along with the developed optimization methods that are being used, some optimization tools have also been developed to help in configuring the parameters that are being used by the optimization algorithm in order to find the optimal case among large sets of possibilities. Adenso-Diaz and Laguna [7] describe the development of CALIBRA, a tool that attempts to discover the best values for up to five search parameters for a given procedure. CALIBRA uses Taguchi's fractional factorial experimental designs together with a local search procedure. Search results are not guaranteed to be optimal. The authors have tested CALIBRA with six existing heuristic-based procedures and have found that CALIBRA was able to match or improve the performance compared to the performance of the procedures with parameters suggested by the procedure developers.

Hutter et al. [23] described an automatic framework for the algorithm configuration problem. They have reviewed a family of local-search-based algorithm configuration procedures and have presented new techniques to speed them up by adaptively limiting time spent on individual configuration evaluations. Using their automated algorithm configurations procedures, the authors have reported substantial and consistent performance improvements.

Audet and Orban [11] devised a general framework for identifying locally optimal algorithmic parameters. Their framework utilized a tailored surrogate function

to steer the search towards a local solution. They utilized the Mesh Adaptive Direct Search (MADS) algorithm for the non-smooth optimization of expensive functions. Hutter et al. [22] experimentally investigated model-based approaches to optimize the performance of parameterized randomized algorithms. They have found that the sequential parameter optimization (SPO) method offered the most robust performance. They have proposed a new version of SPO called SPO+ that was developed as a result of investigation into the design decisions of SPO and the performance consequences of each decision. The authors have demonstrated that SPO+ achieved state-of-the-art performance compared to other parameter optimization approaches.

2.6 Coordinate Search Method

Among the various optimization methods that exist today, the coordinate search method is an optimization method that has often been used in practice [30] to find the optimal case for different optimization problems. Huyer and Neumaier [24] presented a global optimization algorithm that is based on a multi-level coordinate search. They stated that this algorithm is guaranteed to converge if the function is continuous within the neighborhood of a global minimizer. Their test results showed that the MCS is competitive with existing algorithms for problems with reasonable finite bound constraints and that MCS did better most of the time for classical test problems from the Dixon and Szego set with bounded constraints. For

unconstrained problems of dimensions greater than or equal to 4, the MCS performs less than satisfactory.

Hong [21] proposed to solve the optimization-via-simulation problems with integer-ordered decision variables by means of the coordinate search algorithm. He showed that the generated solutions are guaranteed to converge to locally optimal solutions. He also compared the coordinate search algorithm to the COMPASS algorithm. Yu et al. [39] presented two methods for speeding up iterative reconstruction (IR) methods by substituting 1D line search with one-step updates using the coordinate descent optimization (another name for coordinate search method [30]). They have found that these two methods greatly reduced the IR computation with no loss of convergence speed.

Lin [28] proposed an efficient parallel processing multi-coordinate descent algorithm with line-search to solve large-scale unconstrained optimization problems with sparse structures. He has proved that this method will converge and stated that it is efficient as well. Torczon and Trosset [35] have briefly surveyed the history of pattern search methods (of which the coordinate search method is one [30]) and have identified their common structure and pointed out the key features that the Nelder-Mead simplex algorithm lacks. They have given some practical suggestions to use pattern search algorithms in serial and distributed computing environments.

2.7 Large-Scale Optimization

A lot of times, optimization algorithms can perform well by finding locally or globally optimal solutions on limited sets in reasonable time frames. Some optimization algorithms are designed to find optimal solutions in much larger sets. Lin [28] proposed an efficient parallel processing multi-coordinate descent algorithm with line-search to solve large-scale unconstrained optimization problems with sparse structures. He has proved that this method will converge and stated that it is efficient as well.

NESTEROV [29] proposed new methods to solve large-scale optimization problems. His technique is based on random partial updates of decision variables. His results indicated that this technique provides a high degree of efficiency for optimizing large-scale problems. Wetter [37] have developed an adaptive simulation precision control algorithm that can be used together with a family of derivate-free optimization algorithms to more robustly simulate thermal energy in entire buildings. There results show that their coarse approximation strategy reduces the computation time upto 77%.

Chapter 3

The Coordinate Search Method

Among the various optimization methods that exist today, the coordinate search optimization method was found to be the most suitable for our specific purpose of finding the best configuration of heat channels in our constructal design method search phase. This is because our specific case requires a derivative-free optimization method and the coordinate search method was one of the easiest to use and was sufficient for the task at hand. In this chapter, first an overview of the coordinate search method will be provided. Then a discussion of the Constructal Design Tool will be provided as well as a description of how the coordinate search method was practically applied in the thesis work to find the optimal configuration of heat conducting channels on a constructal disc.

3.1 Theory

The coordinate search method is a very intuitive way of finding the optimal solution to a multi-parameter problem in which the parameters are dependent on each other. Let us say that we have a 2 dimensional problem where the search parameters that we have are called x1 and x2. To find the optimal case with the coordinate search method, for the first iteration of this method, we will freeze all the parameters except for the first one, in this case, x1 and optimize x1. By optimizing x1, it means that we will go through all the possible values for x1 and for each value, we will look at the objective function, we will then keep track of the value of x1 that gives us the best value from the objective function among all the possible values for x1. Once we have found the optimal value for x1 with x2 frozen, now we will move on to the second iteration of the coordinate search method and freeze x1 with its optimal value and optimize x2. Once we have found the optimal value for x2 (based on the objective function), we will have then run n iterations, where n=2 in this case. We compare the minimum value of the objective function found so far with the minimum value found in the last n iterations (which is nothing) and note that things are improving so we move on to the next n iterations of the coordinate search. We continue the process for the next n iterations and if the minimum value is reduced even further, we move on to the next n iterations. If it is not reduced, or it increases, then we stop the coordinate search.

3.2 Constructal Design Tool

MATLAB has a Partial Differential Equation Toolbox (PDE Toolbox) available since many years. This useful toolbox allows one to draw a diagram of the system that they are analyzing and then the PDE Toolbox will automatically discretize your drawing into a mesh and solve a partial differential equation on the nodes of the mesh. Before solving the PDE equation on your mesh (such as the heat equation), the user can also specify the heat conductivity (k) and heat generation (Q) values for the different regions of the drawing of the system (such as the branches or the disc, for example).

Even though the PDE Toolbox is very helpful in being able to solve PDE Equations on arbitrary geometries, the drawing of the geometries themselves may take quite some time and meticulous effort especially if many components are involved in the system. For this purpose, a Constructal Design Tool was developed in the C language using the Open GL library to be able to quickly produce the required geometry and feed the x and y coordinates of the geometry as input into PDE Toolbox for the solution of the heat equation to be computed.

Initially, the Constructal Design Tool helped greatly in terms of speed of being able to generate new disc configurations and then solve the heat equation on them with PDE Toolbox to get the temperature distribution on the disc. However, for a lot of cases, the geometry that was inputted into the PDE Toolbox from the

Constructal Design Tool was producing errors and therefore, the heat equation wasn't able to be solved on those geometries. For the cases which did work, the time for computation was getting to be very high (see Table I and Table III for computation times).

3.3 MATLAB PDE Toolbox

The MATLAB (Matrix Labratory) software by Mathworks has a component called the Partial Differential Equation Toolbox that can be started from the MATLAB console with the 'pdetool' command.

The PDE Toolbox provides a graphical user interface that allows you to draw a 2D domain on which you can specify a partial differential equation to be solved on. The pdetool makes it an automatic process to discretize your 2D domain into a triangulated mesh. The mesh can also be refined to have more elements just with the click of a button.

The PDE Toolbox has a workflow that is used to solve one of the built-in partial differential equations on your 2D domain. You select the partial differential equation from a drop-down list. The available partial differential equations are:

- volumeConstraint = 0.0053
- discRadius = 1.000000

- disck = 0.010000
- discQ = 0.000000
- beamRadius = 0.100000
- beamk = 30.000000
- beamQ = 86.000000
- branchk = 30.000000
- branchQ = 0.000000

The workflow to solve a partial differential equation on a 2D domain is made up of 5 modes within the PDE Toolbox:

1. Draw Mode
2. Boundary Mode

3.4 Mathematical Formulation of the Heat Equation Solution

First Domain $[0 \leq x^2 + y^2 \leq r_0, where r_0 = ...]$

$$k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] + Q = 0$$

$Q = Q_0 f(r)$ where $Q_0 = constant$ and $r = x^2 + y^2$ and $0 \leq r \leq r_0$.

Boundary Conditions:

$$r = r_0 : T^-(r) = T^+(r)$$

and

$$k\frac{\partial T^-}{\partial r} = k\frac{\partial T^+}{\partial r}$$

where T^- represents temperature in domain 1 and T^+ corresponds to temperature in domain 2.

It should be noted that k varies along the circumference of the circle with radius r_0, i.e. $S_1 = \theta_1 * r_0$ and $S_2 = S_1 + S_{width}$ where θ_1 is the angle corresponding to two conducting roots and S_{width} is the width of the conducting roots (figure()).

$$S = S_1 : k_n = k_1$$

$$S = S_1 + S_{width} : k_n = k_2$$

where k_2 is the thermal conductivity of the disc and S is the length in circumference of the r_0 circle.

Second Domain

($r_{outer} \geq r \geq r_0$ where $r = \sqrt{x^2 + y^2}$)

Diffusion Equation:

$$k\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right]=0$$

Boundary Conditions:

The solution domain is shown in figure (). It can be observed two different materials are laid in the physical domain with different thermal conductivities. For each material in its geometric configuration, thermal conductivity is considered to be constant. In addition, temperature and heat flux continuity are incorporated as boundary conditions between each configuration corresponding to each material. $T1 = T2$ at boundaries of two configurations.

$$k_1\frac{\partial T_1}{\partial x}=k_2\frac{\partial T}{\partial x}$$

where T_1 and T_2 are temperatures at boundaries of the first and second configurations respectively.

3.5 Coordinate Search Implementation

The strategy used in optimization is based on the central temperature on the disc. The search algorithm for the optimization tool seeks for the configuration that results in the minimum temperature in the center of the disc.

The coordinate search method was selected as a numerical optimization method to find a optimal heat channel configuration on the disc. A MATLAB script was written incorporating the coordinate search method within it to perform the channel configuration search. The coordinate search method is one of the simplest derivative-free optimization methods available. The basic idea behind its operation is that it optimizes each search parameter one-by-one. It consists of a number of n iterations where n is the number of search parameters. During each iteration, only one parameter is optimized.

For the implementation of the coordinate search method in this thesis work, we had two 'while' loops, one nested inside the other within a MATLAB script. The outer 'while' loop represents the total number of runs where one run consists of n iterations. The inner 'while' loop represents one iteration. During each iteration, the MATLAB coordinate search script would call the Constructal Design Tool with the current values of the search parameters (# branches, # ends, and root length). In response, the Constructal Design Tool would generate another MATLAB script file containing the geometry information for a disc with the specified number of branches, ends, and root length. This new script would be generated in a syntax that is prepared to be used with the PDE Toolbox in MATLAB. This new script called 'disc' would be called by the original coordinate search script and as a result the temperature distributions on the disc would be calculated and the central temperature on the disc would be saved by the coordinate search method within its list

of cases along with the run-time of the computation for that case.

The central temperature on the disc is the measure used to rank the performance of each configuration. A lower central temperature means a better configuration as that indicates that more of the central heat was able to travel out of the center due to better heat channel configuration.

3.6 Algorithm Validation

To validate the correctness of our constructal algorithm that is used to generate the disc configurations, we have made a comparison with one of Dr. Bejan's works [34] and have found that the data agrees closely as shown in the figure below:

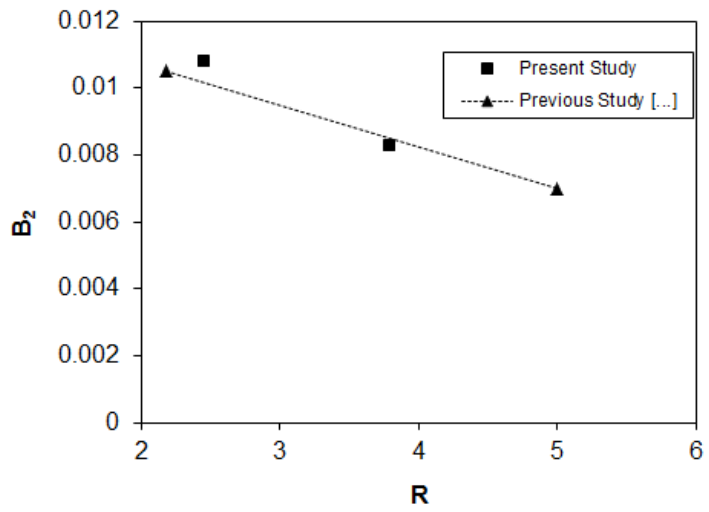


Figure 3.1 - Constructal geometry generation algorithm validation.

Chapter 4

Results and Discussion

The coordinate search method algorithm was developed and executed on a machine - which shall be referred to as 'the Quadro-plex' from now on - with the following specifications:

- 64-bit version of Windows 7 Professional
- 2 Quad-core Intel Xeon X5570 2.93 GHz 64-bit processors (each having 8MB Cache, 4 Cores, and 8 Threads)
- 64 GB of memory
- More than 2.5 TB of Hard Disk storage space
- 2 NVIDIA Quadro FX 5800 video cards (each with 4096 MB of GDDR3 RAM)
- Total of 480 GPU cores to allow upto 61,440 threads running simultaneously

- MATLAB 2010b software
- NVIDIA CUDA Toolkit 3.2
- 3 32” Sony Bravia LCD HDTVs assigned to it

Two types of searches were executed during the thesis, the first one being the traditional exhaustive search, which took 48 hours to complete and the second being the coordinate search which took 7.58 hours.

4.1 Results of the Exhaustive Search

As a result of the exhaustive search running on the Quadro-plex for 48 hours, 2,088 figures and diagrams have been produced representing the cases that have been analyzed successfully by the search algorithm. For each case that was analyzed, there were 3 files that were generated and recorded:

- Snapshot of the MESH diagram in the PDE Toolbox
- Snapshot of the 3D Temperature plot for the temperature distribution on the disc
- 3-D Temperature plot figure that can be loaded in MATLAB and rotated and viewed in 3-dimensions

There were many cases that could not be analyzed due to errors produced in the MATLAB PDE Toolbox. The number of possible cases to search through for the exhaustive search in our experiment can be calculated simply by multiplying together the number of discrete elements for each search parameter. In our configuration of the exhaustive search, we had 3 search parameters:

- # of branches - ranged from 2 to 27 (reduced from 200 due to time)
- # of ends - ranged from 1 to 5
- Root Length - ranged from 0.1 to 0.7 with increments of 0.1

Total # Cases = (27-2 +1) * (5-1 +1) * ((0.7 - 0.1)/0.1 + 1) = 910

However due to the very long time of computation (2 days) making the Quadroplex unusable for other work, and the sufficiency of the data produced for our purposes, the exhaustive search was stopped before it could analyze every single possible case. The last case that was analyzed was the case with 27 branches 4 ends and a root length of 0.70. So that means that cases with 27 branches with 5 ends have not been analyzed, which number 7 cases. That makes the total number of cases traversed by the exhaustive search: 910 - 7 = 903

From those 903 cases, 704 of them were able to proceed to the mesh generation phase of the process, and only 692 of them were analyzed successfully. We can call those errors that happen before the mesh is generated a mesh error. Those errors

that happen after mesh generation can be called post-mesh error. In this exhaustive search, we had:

- 903 total cases
- 199 cases with mesh errors
- 12 cases with post-mesh errors
- 211 cases with errors
- 692 successfully analyzed cases

The exhaustive search as it is implemented currently in the Constructal Design Tool (together with MATLAB integration) ran on the Quadro-plex, for a period of around 48 hours and has gone through 903 cases. Among those cases only 692 were successfully analyzed and 211 cases had some type of error and were therefore skipped from the analysis. The error rate for the current run of the exhaustive search with the Constructal Design Tool is 23.37%.

In our constructal search for our experiment, we had the following conditions:

- volumeConstraint = 0.0053
- discRadius = 1.000000
- disck = 0.010000
- discQ = 0.000000

- beamRadius = 0.100000
- beamk = 30.000000
- beamQ = 86.000000
- branchk = 30.000000
- branchQ = 0.000000

The rootWidth variable was automatically calculated as the other parameters were changed for each search case to make sure the volume constraint was upheld.

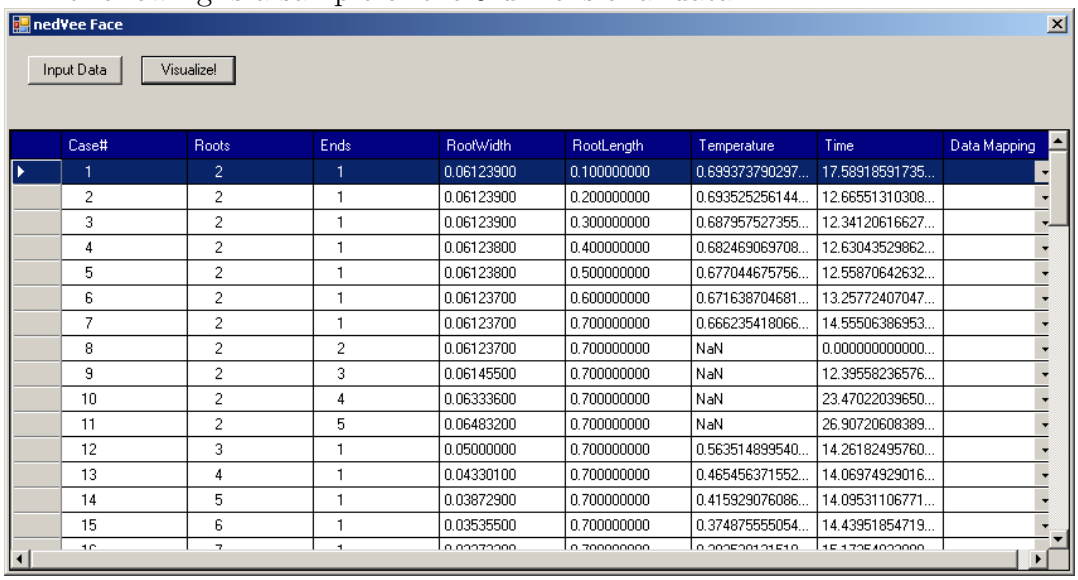
Cases are compared with each other based on the central temperature. Lower values for the central temperature indicate better channel configuration. This is because the low central temperature indicates that more of the heat from the center has flowed onto the disc indicating that the configuration of the channels was able to carry more heat compared to other configurations, hence making that configuration better for heat flow.

To better analyze the results of the simulation, a data visualization tool was developed to be able to visualize 5 dimensional data. The data plot will show the 5-dimensional plot in 3-dimensional space. In this case, we had the five dimensions of:

- Root Count
- Ends Per Root

- Root Width
- Root Length
- Temperature

The following is a sample of the 5-dimensional data:



Case#	Roots	Ends	RootWidth	RootLength	Temperature	Time	Data Mapping
1	2	1	0.06123900	0.100000000	0.699373790297...	17.58918591735...	
2	2	1	0.06123900	0.200000000	0.693525256144...	12.66551310308...	
3	2	1	0.06123900	0.300000000	0.687957527355...	12.34120616627...	
4	2	1	0.06123800	0.400000000	0.682469069708...	12.63043529862...	
5	2	1	0.06123800	0.500000000	0.677044675756...	12.55870642632...	
6	2	1	0.06123700	0.600000000	0.671638704681...	13.25772407047...	
7	2	1	0.06123700	0.700000000	0.666235418066...	14.55506386953...	
8	2	2	0.06123700	0.700000000	NaN	0.000000000000...	
9	2	3	0.06145500	0.700000000	NaN	12.39558236576...	
10	2	4	0.06333600	0.700000000	NaN	23.47022039650...	
11	2	5	0.06483200	0.700000000	NaN	26.90720608389...	
12	3	1	0.05000000	0.700000000	0.563514899540...	14.26182495760...	
13	4	1	0.04330100	0.700000000	0.465456371552...	14.06974929016...	
14	5	1	0.03872900	0.700000000	0.415929076086...	14.09531106771...	
15	6	1	0.03535500	0.700000000	0.374875555054...	14.43951854719...	

Figure 4.1 - A snapshot of the 5-dimensional data results.

The next series of snapshots show the visualization that was automatically produced by the visualization tool developed for multi-dimensional data:

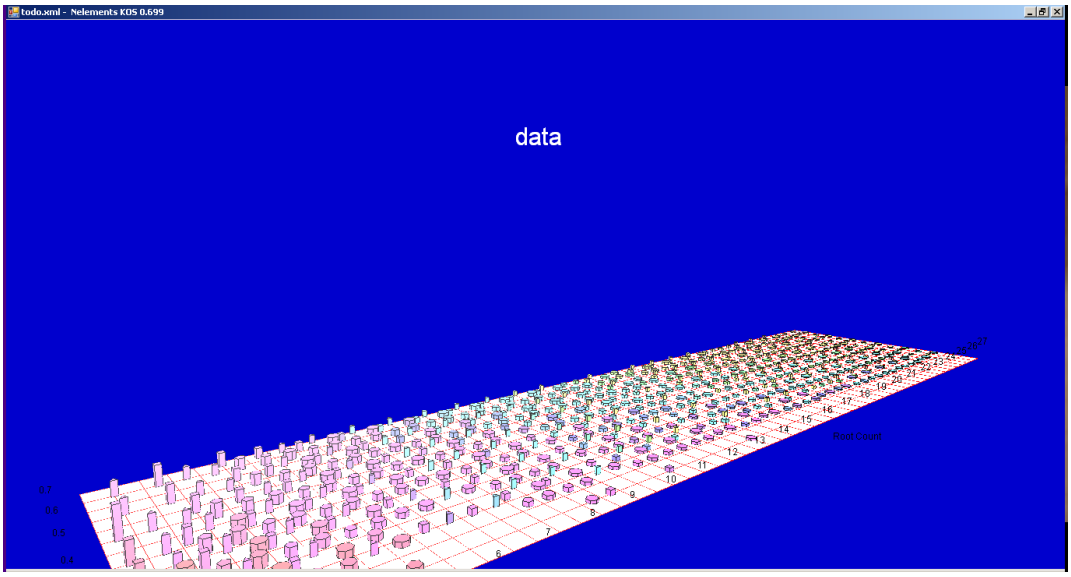


Figure 4.2 - A view of the 5 dimensional data visualization.

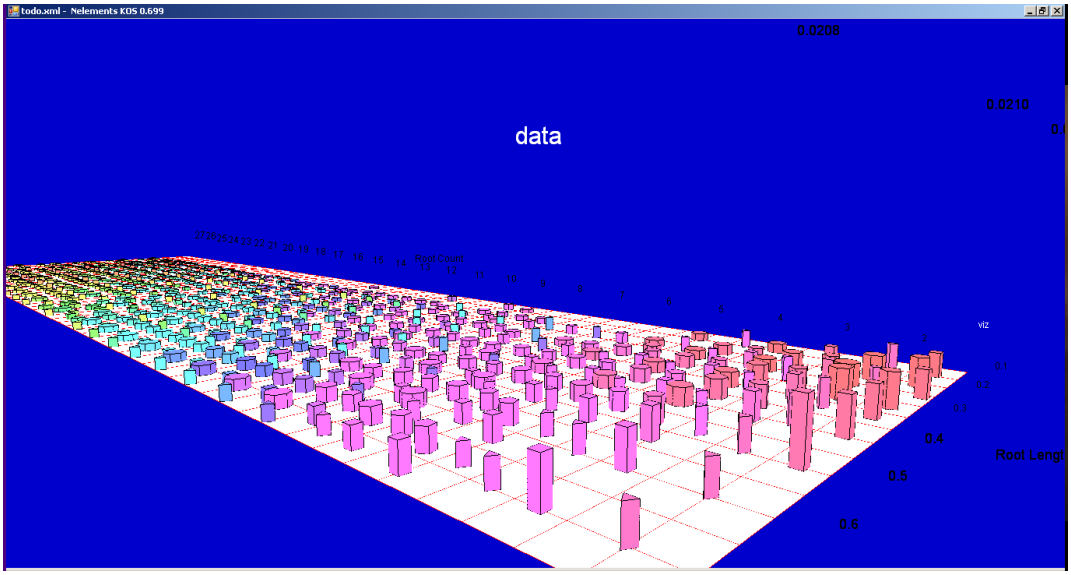


Figure 4.3 - Another view of the 5 dimensional data visualization.

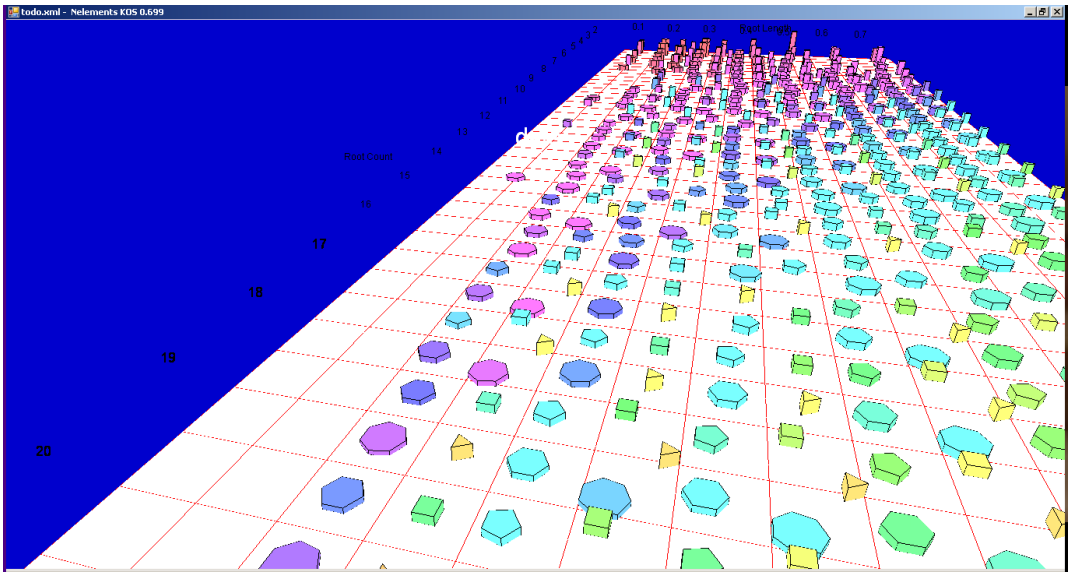


Figure 4.4 - An overview of the 5 dimensional data visualization.

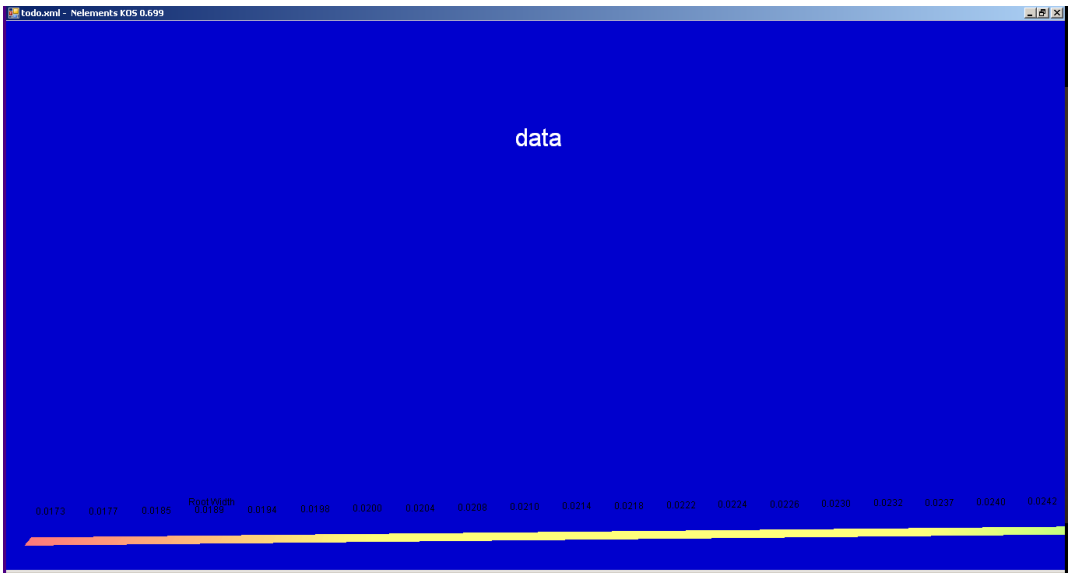


Figure 4.5 - Color gradient used to map the root width variable on the data visualization.

For the data visualization of the 5 variables from the data results, the first variable - root count is mapped to the x-axis of the grid. The root length is mapped

onto the z-axis. The grid is divided into data patches. Each data patch represents a specific root count and root length. Each data patch may contain a number of towers. These towers represents the different possible end count per root values which range from 1 to 5. If the end count per root value is 1, then we have a triangle shaped tower, where a triangle has 1 + 2 sides. If the end count per root value is 2, then we have a square shaped tower where 2 + 2 is the 4 sides of a square, and so on. The height of the tower represents the central temperature of the disc. For this reason, as we go farther along the x-axis, towards increasing the root count, we can see the tower heights decreasing meaning that with the increase in root counts, the central temperature decreases.

Last of all, we have the root width which is represented as the color of the tower. A legend is also provided as shown in Figure 4.5, which maps the color from a color gradient strip to a root width.

The following is a plot from the obtained data showing the '# of branches' on the disc as the x-axis and the 'central temperature' as the y-axis:

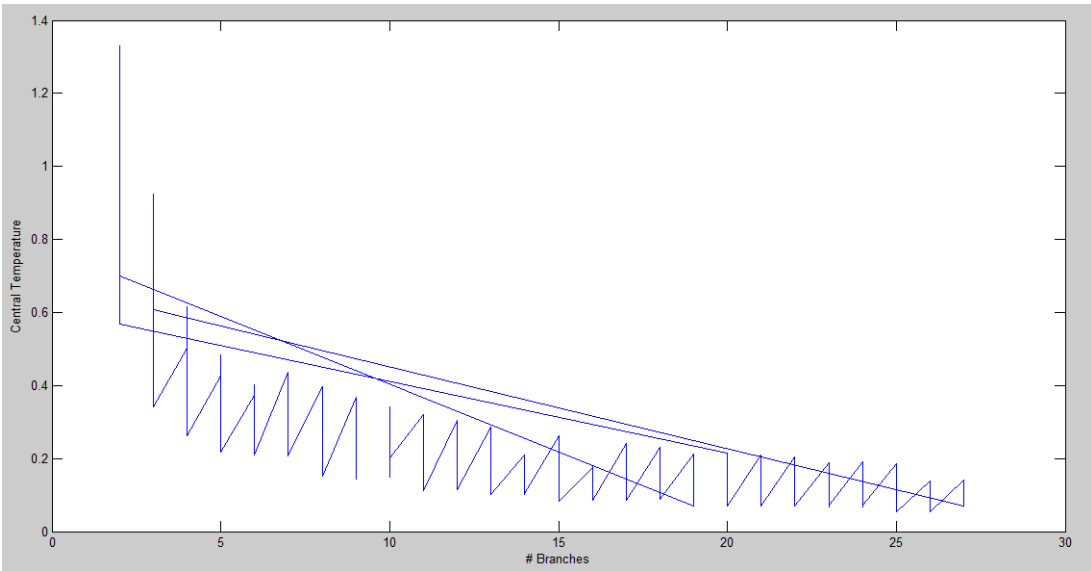


Figure 4.6 - Number of branches versus Central Temperature plot.

The figure shows that there is a general trend that the central temperature of the disc decreases with the increase of the number of branches on the disc. If one observes the data visualization produced by the visualization tool, you can also observe the same pattern of decreasing central temperature with the increase in the number of roots on the disc.

From among all the cases that have been searched with the exhaustive search, the best case that has been found was the 25 branch case with 5 ends and with a root length of 0.20. The central temperature of the disc in that case was 0.0532. The following is a snapshot of the mesh of the 25 branch 5 end case with a root length of 0.20:

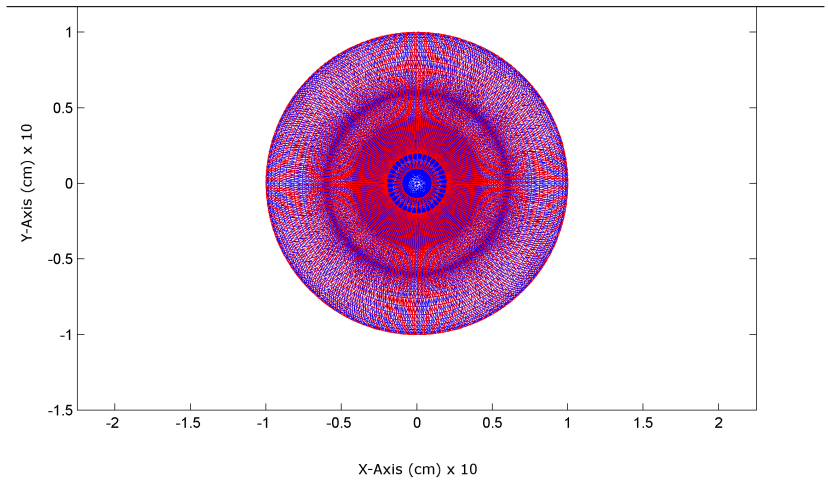


Figure 4.7 - Mesh plot of the 25 branch 5 end 0.20 root length case.

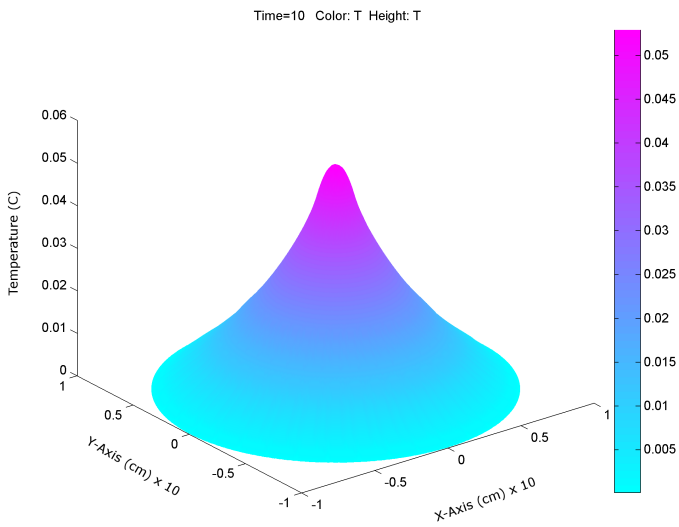


Figure 4.8 - Temperature plot of the 25 branch 5 end 0.20 root length case.

(CT=0.0532)

The snapshot above shows the 3-dimensional temperature plot of the best case. The plot shows that the temperature of the disc is highest at the center, where the

heat is being generated with a Q value of 86. Then by means of the 25 branches each with 5 ends, the heat generated from the center of the disc is being transported towards the circumference of the disc producing the temperature distribution seen above.

4.2 Results of the Coordinate Search

Whereas the exhaustive search needed to go over 640 cases to arrive at the correct (optimal) case within the search range, the coordinate search achieved this feat at its 46th case. However, because of the extra iteration that the coordinate search needed to perform to complete its run cycle, 21 more cases were analyzed to bring the total number of cases analyzed up to 71. The whole run-time of 7.58 hours for the coordinate search is a big time gain compared to the exhaustive search's time of 48 hours.

With the coordinate search method, we didn't have to traverse all the possible cases to arrive at the optimal case. We can see the small percentage of cases traversed with the coordinate search method by looking at the data visualization of the results of the search which shows only the data that was obtained during the search. One can note how most of the grid which was full during the exhaustive search is empty and the same optimal value has been reached:

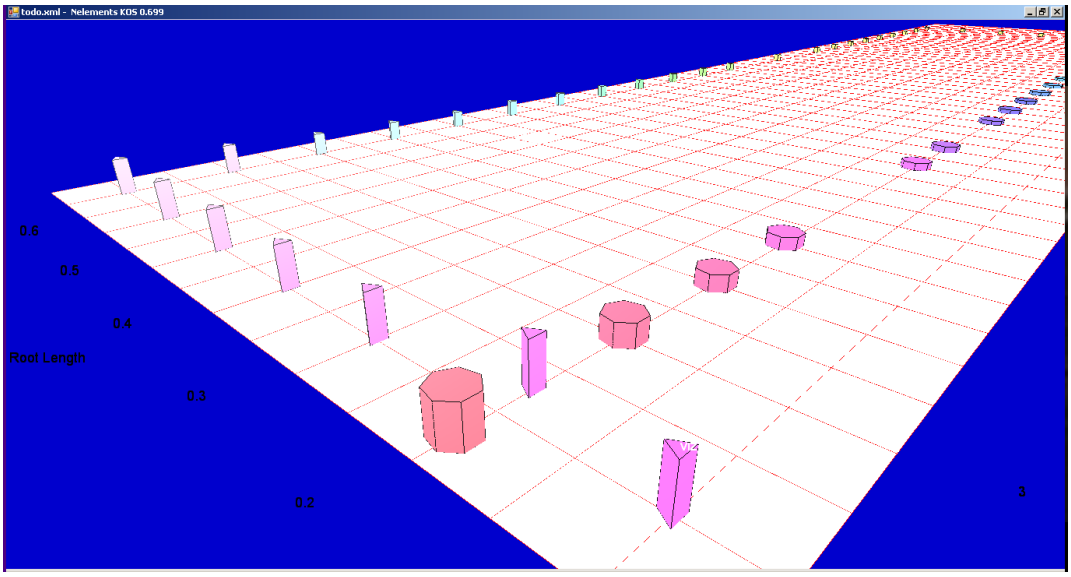


Figure 4.9 - View from lower-left corner of grid.

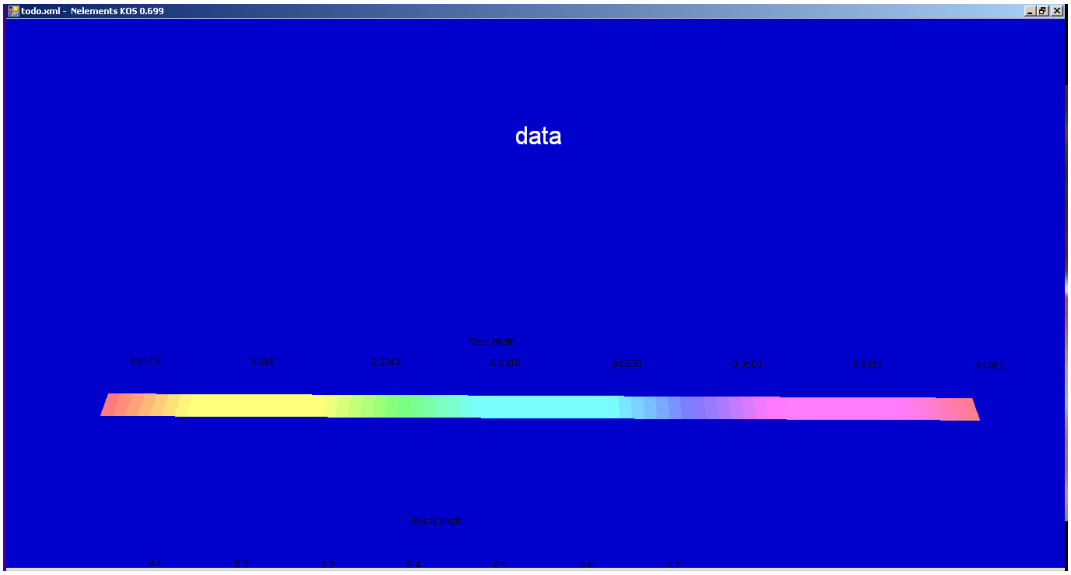


Figure 4.10 - Color gradient used to map the root width.

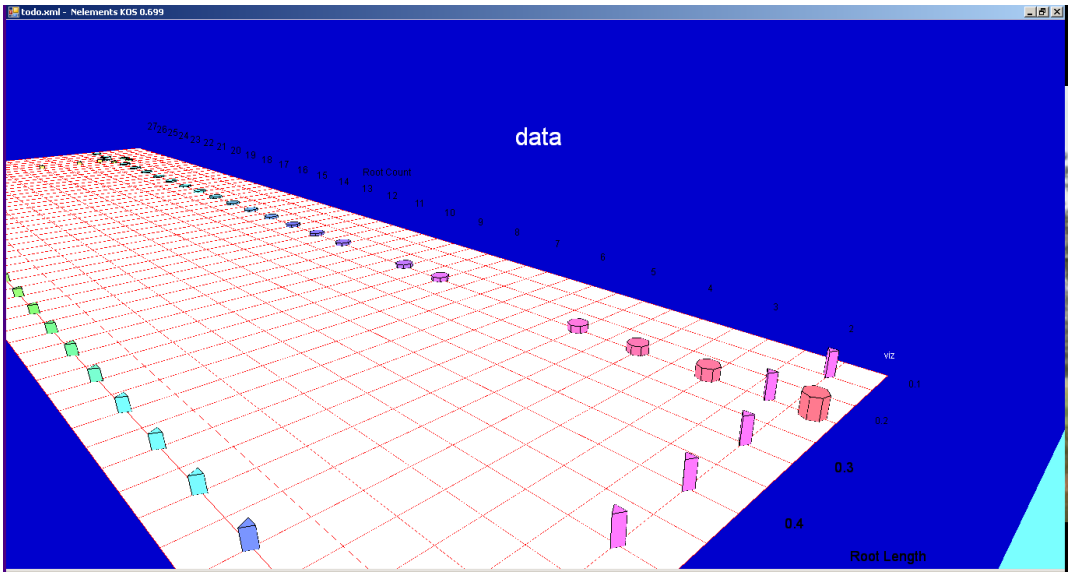


Figure 4.11 - View from the top left corner of grid.

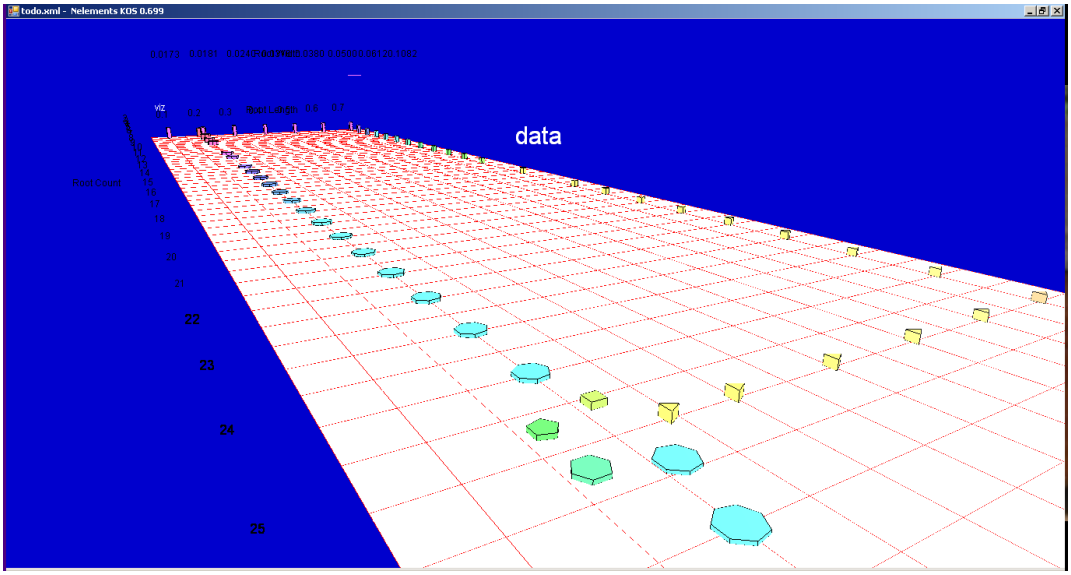


Figure 4.12 - Perspective from lower right corner of grid.

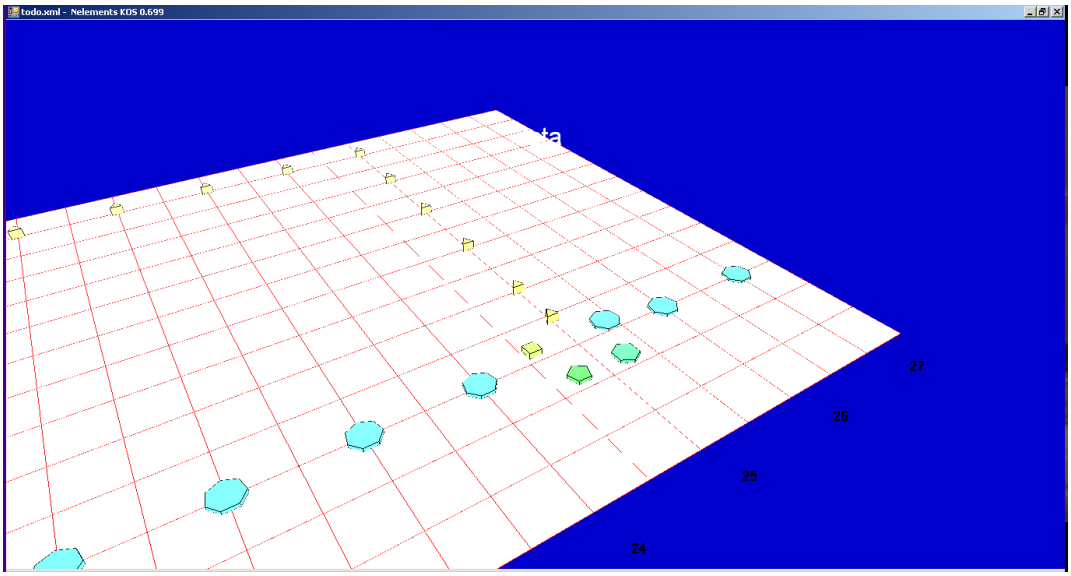


Figure 4.13 - View of lower right corner with optimal data patch in sight (25,.20).

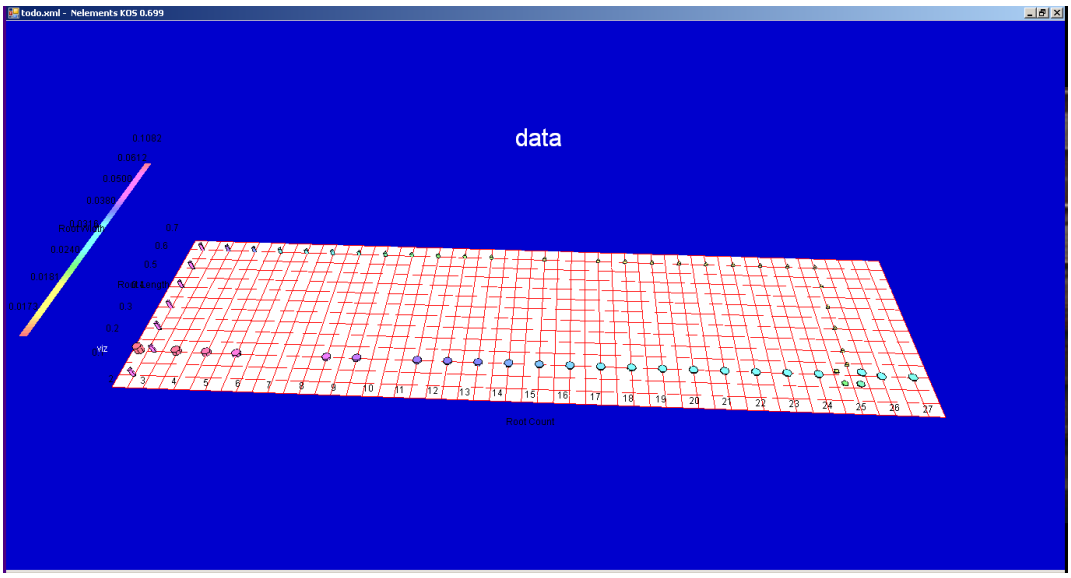


Figure 4.14 - Overall view of grid.

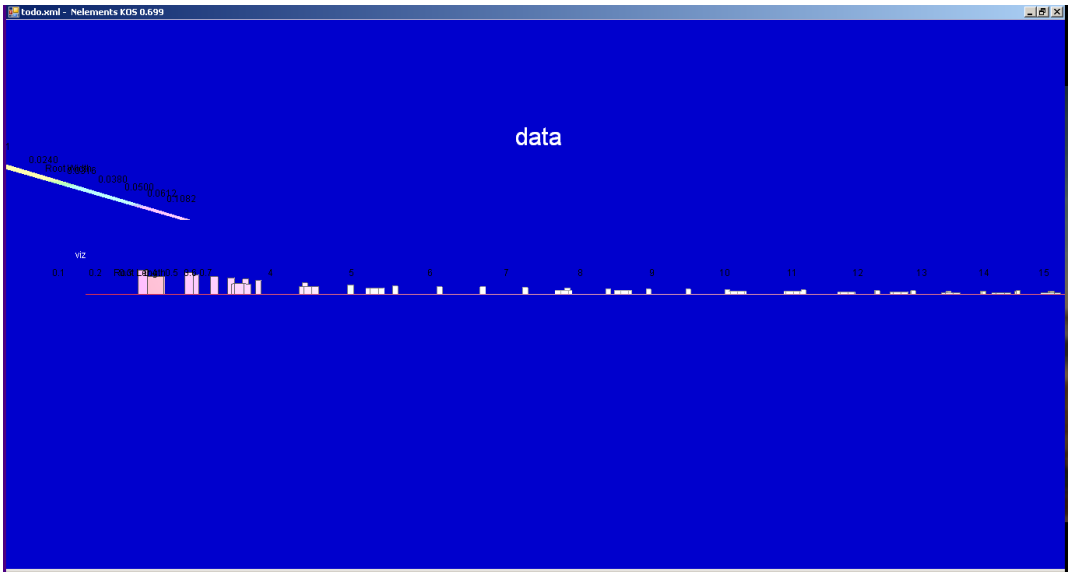


Figure 4.15 - Side view of grid to see heights of towers decreasing.

4.3 Sample Snapshots from the Search Results

Some sample snapshots of the cases analyzed during the searches are provided below:

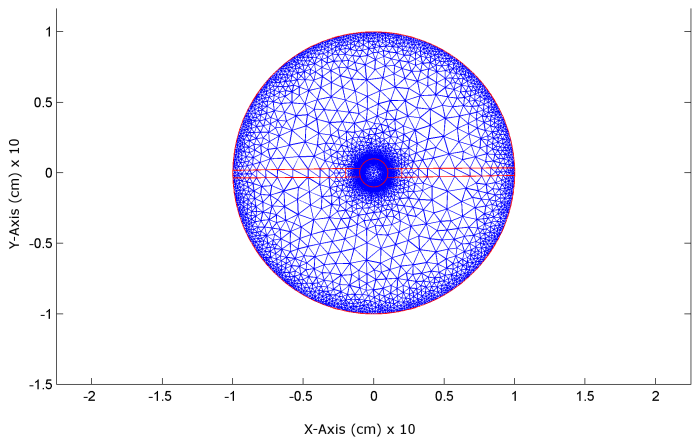


Figure 4.16 - Mesh plot of the 2 branch 1 end 0.20 root length case.

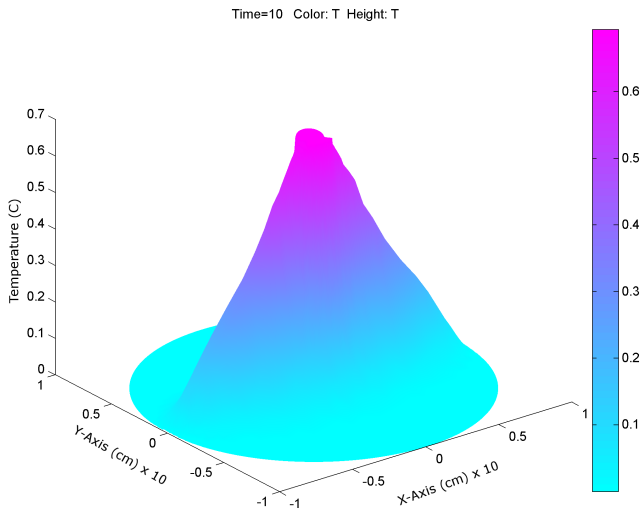


Figure 4.17 - Temperature plot of the 2 branch 1 end 0.20 root length case.

(CT=0.6935)

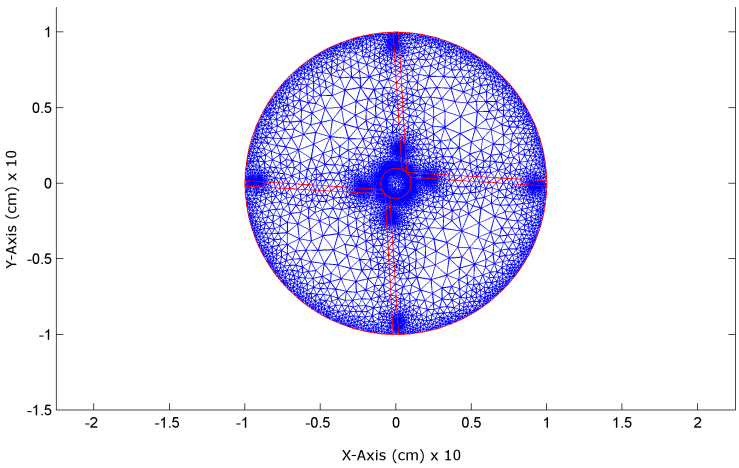


Figure 4.18 - Mesh plot of the 2 branch 2 end 0.10 root length case.

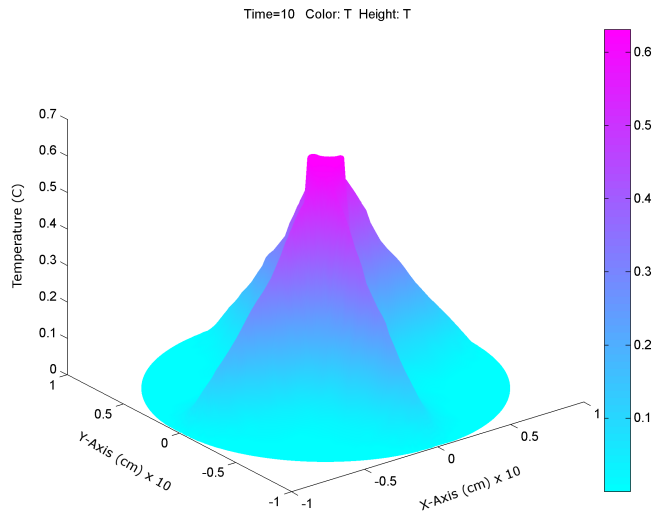


Figure 4.19 - Temperature plot of the 2 branch 2 end 0.10 root length case.

(CT=0.6313)

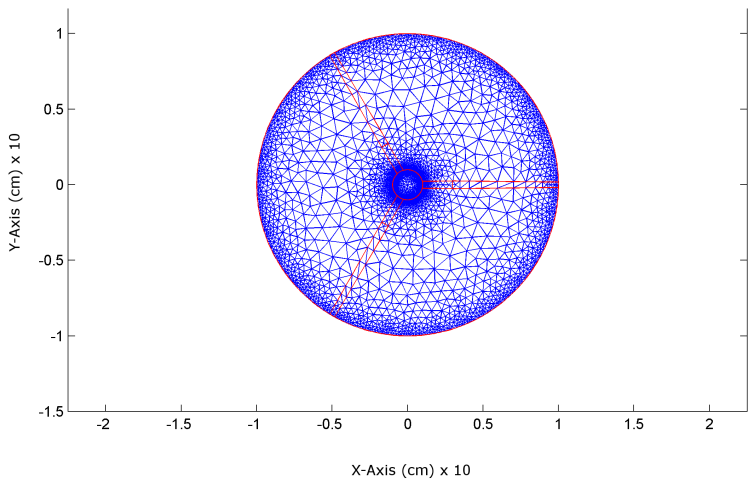


Figure 4.20 - Mesh plot of the 3 branch 1 end 0.30 root length case.

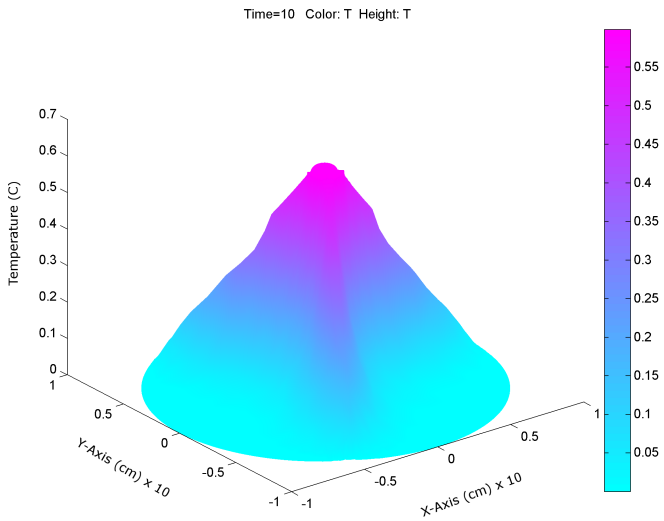


Figure 4.21 - Temperature plot of the 3 branch 1 end 0.30 root length case.

(CT=0.5991)

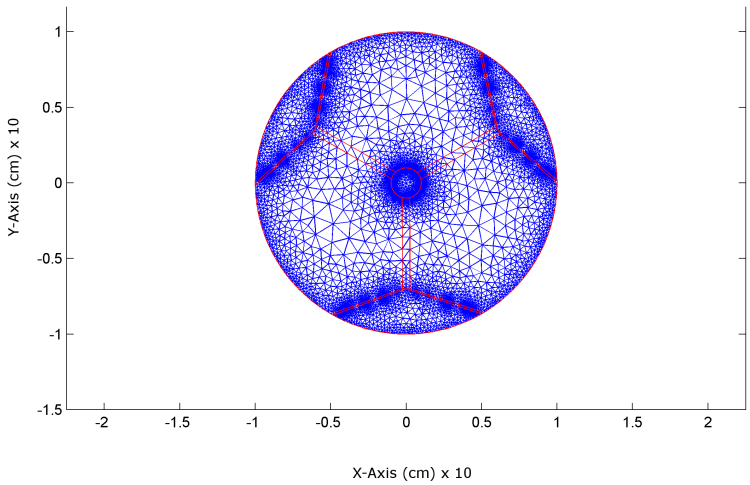


Figure 4.22 - Mesh plot of the 3 branch 2 end 0.70 root length case.

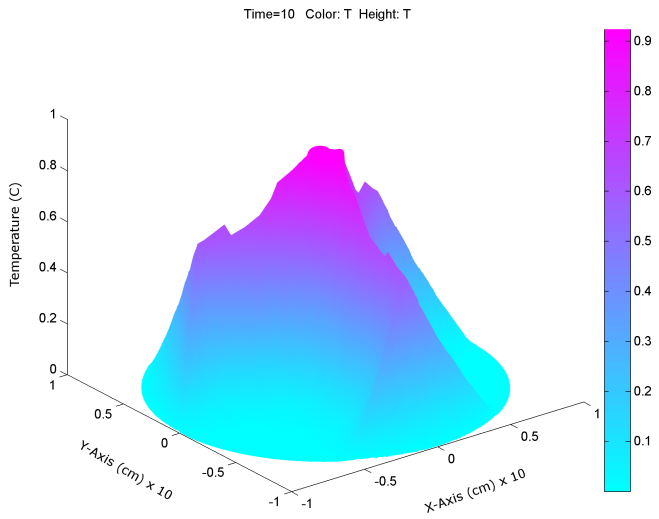


Figure 4.23 - Temperature plot of the 3 branch 2 end 0.70 root length case.

(CT=0.9239)

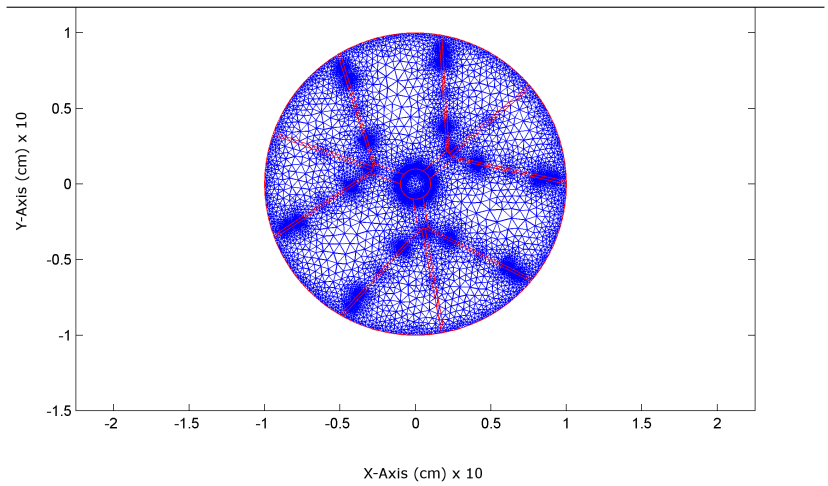


Figure 4.24 - Mesh plot of the 3 branch 3 end 0.30 root length case.

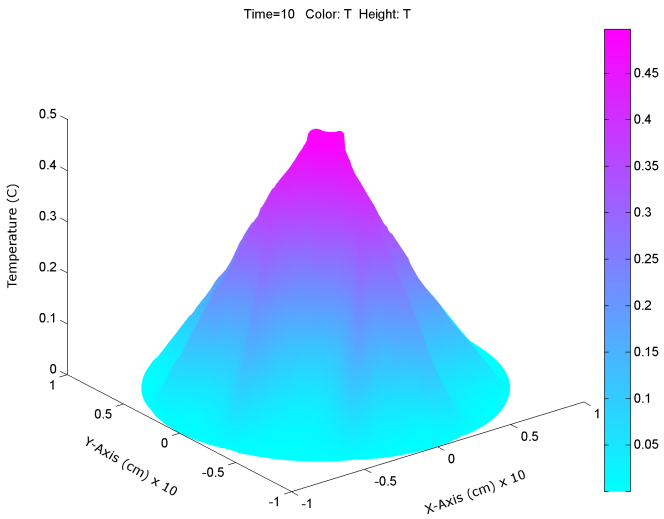


Figure 4.25 - Temperature plot of the 3 branch 3 end 0.30 root length case.

(CT=0.4975)

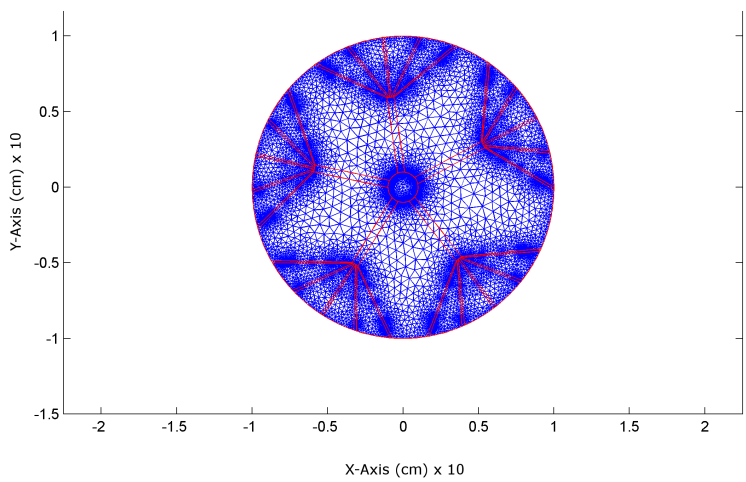


Figure 4.26 - Mesh plot of the 5 branch 5 end 0.60 root length case.

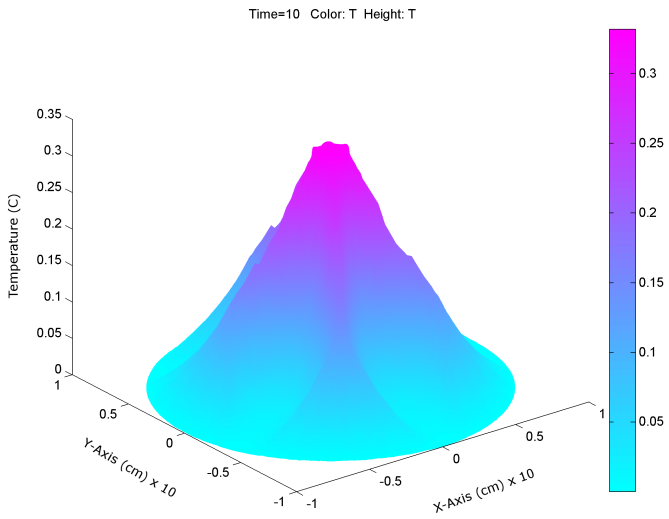


Figure 4.27 - Temperature plot of the 5 branch 5 end 0.60 root length case.

(CT=0.3321)

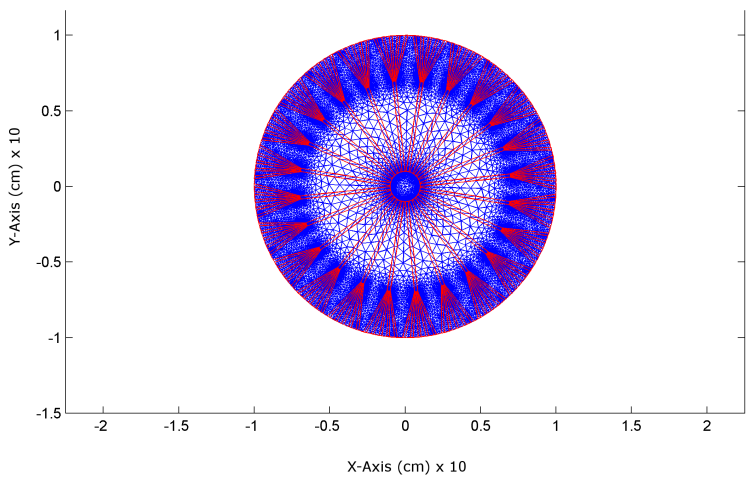


Figure 4.28 - Mesh plot of the 21 branch 5 end 0.70 root length case.

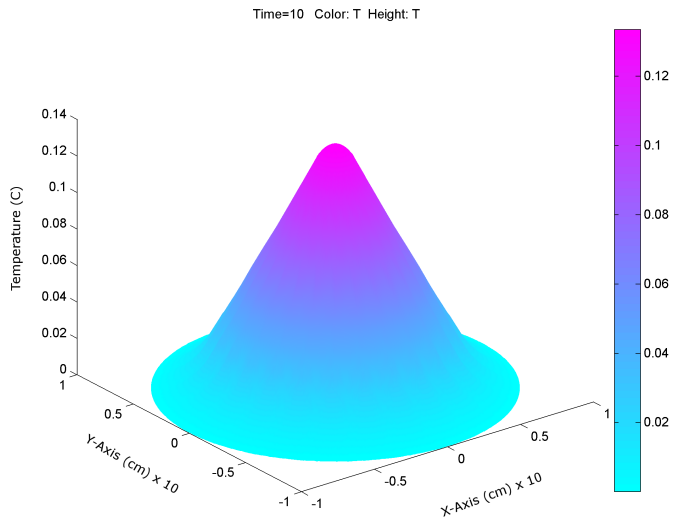


Figure 4.29 - Temperature plot of the 21 branch 5 end 0.70 root length case.

(CT=0.1338)

4.4 Observations of the Results

Some observations from the search results are that in some cases like the 26 branch 5 end .20 root length case and the 25 branch 5 end .20 root length case (difference of one in the branch number), the central temperature value is very close (0.0535 versus 0.0532). There is also a very close value for the central temperature between the cases with 9 branches 4 ends and .20 root length and the case with 9 branches 5 ends and .20 root length having central temperature values of 0.1601 and 0.1400 respectively. Another observation is that on some of the temperature plots there are spikes in temperature along some of the heat branches. One thing to investigate

would be whether making a bifurcation at that spike point would improve the heat flow of the disc for that case or not.

Cases with very uniform looking temperature plots such as the case with 8 branches 4 ends per branch and a root length of .20 seem to have a much better heat flow performance than even those cases that are close to it in terms of difference in number of ends per branch or number of branches. The case just mentioned (8 branches, 4 ends, and .20 root length) has a central temperature of 0.1740. Some cases that are close it geometrically but don't have a uniform looking temperature plot have much higher central temperature values like following cases:

- 8 branches 4 ends and .40 root length, CT = 0.2091
- 8 branches 5 ends and 0.50 root length, CT = 0.2111

If you're looking for .20 root length cases:

- 9 branches 1 end and 0.20 root length, CT = 0.3684
- 9 branches 2 ends and 0.20 root length, CT = 0.2422

It looks like for the cases in which errors occurred, the root length was at an extreme, either at 0.7 to one side or 0.1 / 0.2 on the other side.

Chapter 5

Conclusion

5.1 Thesis Work Summary

A new way of optimizing flow systems and generating the optimal geometry - the constructal design method - has been integrated together with an existing numerical optimization method - the coordinate search method to make the configuration search easier to perform and more automatic. As a result of this study, it was found that the coordinate search method greatly reduces the time and number of cases needed to be analyzed in order to find the optimal case among a range of possibilities in the search space. The exhaustive search took 48 hours to find an optimal solution whereas the coordinate search method took only 7.58 hours to achieve the same result of finding the optimal configuration.

The results of this study also show that at constant volume and area, bifurcation

of the channels of flow does indeed increase the heat flow on a disc from the center to its circumference. However, that doesn't necessarily mean arbitrarily increasing the number of branches of flow or bifurcating the existing branches will increase heat flow. A configuration search must be performed to find the optimal configuration of the number of branches and number of bifurcations per branch. For our experiment of finding the optimal heat flow configuration for a disc from its center to its circumference, we found the optimal configuration to be 25 branches with 5 ends each and a root length of .20 times the radius of the disc. To satisfy the volume constraint of 0.0053, the Constructal Disc Tool automatically calculated a root width of 0.0322.

The tool developed as a result of this study, is able to find the optimal design (within a range of possible designs) for heat flow from the center of a disc to its circumference under variable conditions. The elements making up the conditions are:

- The Volume Constraint
- The disc heat conductivity value (k)
- The disc heat generation value (Q)
- The radius of the central beam going through the disc (and providing heat to the disc)

- The beam heat conductivity value (k)
- The beam heat generation value (Q)
- The branches' heat conductivity value (k)
- The branches' heat generation value (Q)

All the conditions above can be changed and a new configuration search can be run by the Constructal Disc Tool to find the optimal branch configuration under those new conditions.

5.2 Limitations

This study was the first step into automating the constructal design method using a numerical optimization technology - the coordinate search method. We started with the simplest possible case just to get it working through the whole process successfully. Therefore, the Constructal Disc Tool can only optimize the design of geometry with the following limitations:

- The only optimization is for heat flow on a disc.
- The direction of heat flow should be from the center of the disc to its circumference.
- The designs will be limited to one level of bifurcation.

- The configuration search portion of the process took very long time:

  48 hours to go through 903 cases for exhaustive search

  7.58 hours to go through 71 cases for coordinate search

- For a case with 1 end per branch there should not be a difference in performance for different root lengths as the channel should be just one long piece. This issue needs to be resolved to make the simulations more realistic.

5.3 Future Work

5.3.1 General Constructal Design Tool

As a result of this introductory work, a general design has been developed to overcome the limitations of the current Constructal Disc Tool that incorporates the coordinate search method and that works with integration with the MATLAB 2010b software. The objective of the general Constructal Design Tool is to generate optimal designs for different flow systems, whether they consist of heat, electricity, thermal stress, mechanical stress, people, knowledge, gas, liquid, energy, network traffic, street traffic, car traffic, air traffic, or other types of flows. The current design that has come about as a result of this thesis work for the general Constructal Design Tool consists of 3 components:

- Artificial Intelligence component

- Geometry Generator component
- Visualization component

The usage of the general Constructal Design Tool is meant to be very user-friendly with only a minimum amount of input required by the user. The user should just provide a description of the constraints of the flow system that he or she has such as the 3-dimensional area into which the system needs to fit in or the volume constraint of flow channels which can represent the amount of material that can be used to produce the flow channels. The user can also optionally provide the objective(s) for the flow system at hand. Then after that the Constructal Design Tool should work to find the optimal configuration of the flow channels for the required system. A little more detail about the current design of how the Constructal Design Tool is supposed to work is provided below with its components:

Input to the tool:

- Constraints and description of the flow system
- Objective(s) of the flow system (This input is optional. If no objective(s) are provided, the AI component will parse the constraints and the description of the flow system to determine which flows are involved and automatically set objectives to optimize those flows).

Components of the Tool:

- AI component:
  - Determine which flows are involved in system (will have equations to deal with each type of flow, such as Bernoulli's equation for gases, Fourier equation for heat, etc.).
  - Determine from the description the basic structure of the system

    Is it a disc, a pipe, etc.?

    What are the directions of the flow(s)? (Disc center to circumference, etc.)
  - Generate list of checkpoints (things to check on the generated geometry to evaluate the performance of the flows based on the objective(s) of the system) (For example in the MS Thesis this was simply a check of the central temperature on the disc structure).
  - Working with the geometry generator, identify parameters that can be used in the geometry to generate more branches and more bifurcations and modify branch shape and bifurcation shapes. These will be the search parameters used in the configuration search (possibly using something like the coordinate search method) to find the optimal configuration.
- Geometry Generator:
  - From the information prepared by the AI component, generate the sim-

plest possible mesh that has the constraints of the system as well as the flows that have been identified. The basic structure can have straight round tubes for all the flows with no bifurcations.

  - Discretize the mesh to make it ready for equations to be applied to the discretized mesh nodes.

- Visualization component:

  - For the purposes of data analysis plots of different search parameters vs. each other can be produced.

  - Good looking 3-dimensional plots for temperature, stress, etc.

5.3.2 Performance Upgrade of the Constructal Design Tool

The constructal search performed with the coordinate search method took a very long time in spite of being optimized somewhat. Initially the MATLAB PDE Toolbox script that the Constructal Design Tool was producing was taking a long time to compute in the MATLAB PDE Toolbox. For that reason, the Constructal Design Tool was upgraded to skip some of the unnecessary steps the MATLAB PDE Toolbox was performing to compute the temperature distribution. Some substantial improvements were achieved as shown in the time comparisons below, however, complicated cases are still taking very long time to compute such as the 25 branch 5 end 0.5 root length case which took a little more than an hour to compute. The time

to compute for a case was taken using the tic and toc functions in MATLAB. The PDE Toolbox script was saved in a file called disc.m, so to time the performance of a given script, one would write it as follows in the MATLAB prompt:

tic; disc; toc;

This would cause the contents of the script within the disc.m file to execute and after that the elapsed time would be outputted to the MATLAB console. For each case, a different disc.m file is produced by the Constructal Design Tool that specifies precisely the geometry of the disc on which to perform the temperature distribution calculations.

# Branches	# Ends	Root Length	Version 0.8 w/ PDE Toolbox	Version 1.0 w/ PDE Toolbox (w/ vConstraint)	Speedup
2	1	0.5	50.60 seconds	13.33 seconds	379%
5	1	0.5	88.85 seconds	14.73 seconds	603%
10	1	0.5	151.35 seconds	15.51 seconds	976%
12	1	0.5	184.16 seconds	16.07 seconds	1146%
15	1	0.5	Error	16.00 seconds	NA
17	1	0.5	270.11 seconds	17.38 seconds	1554%
25	1	0.5	466.86 seconds	16.41 seconds	2845%

4	2	0.5	104.05 seconds	19.51 seconds	533%
5	2	0.5	119.27 seconds	21.74 seconds	548%
4	3	0.5	Error	32.49 seconds	NA
7	2	0.5	Error	23.15 seconds	NA
10	2	0.5	230.07 seconds	Error	NA
10	2	0.6	230.54 seconds	27.03 seconds	853%
25	2	0.5	728.54 s + Error	41.92 seconds	NA
25	5	0.5	Not supported	3714.57 seconds	NA

Table 5.1: Time Comparisons Between Two Versions of the Constructal Design Tool.

The initial script for the PDE Toolbox that was produced by the Constructal Design Tool was based on the data file produced by the PDE Toolbox when manually working with the PDE Toolbox and saving your work. We can call the sequence of steps in the script produced in this manner when manually working with the PDE Toolbox and saving your work as sequence A. Even though there has been a considerable increase in speed from sequence A, as indicated by the time comparisons above, the speed of the computation performed by the new sequence B is still very

slow and 48 hours of computation was only able to cover 903 cases.

To speed up the process two possibilities have been thought up. The first is to perform the computations in the C language instead of MATLAB so as to remove any unnecessary computations that are being done in MATLAB (such as checking if values are correct, or other operations). The second is to run the process on the parallel NVIDIA CUDA hardware through the C language and see if the process is speeded up with that method.

To have the whole process done within the Constructal Design Tool instead of working together with the MATLAB PDE Toolbox requires the implementation of the discretization process within the Constructal Design Tool. Discretization can be done in the case of the disc by making concentric circles starting from the center of the disc going towards the circumference. After that, lines can be drawn on the disc at regular theta values going from the center of the disc towards its circumference. This process would discretize the disc into a number of elements that are uniform on each radial level. By discretizing the disc in this manner, one can then proceed to have each discretized node in the center of each element and then solve for the temperature value of each element by producing a multitude of energy balance equations for each element. The figure below shows an example of a radially discretized mesh:

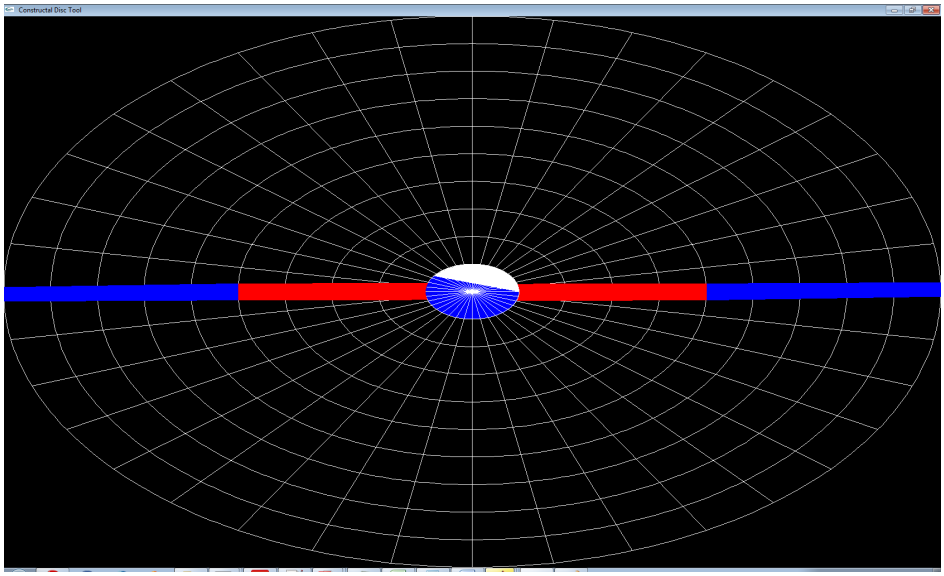


Figure 5.1 - Radial Mesh Discretization with CDT.

From the table of time comparisons above, it can be seen that the number of error cases has been reduced by the upgrade of the Constructal Design Tool to version 1 from version 0.8. It is likewise hoped that as more of the details of the whole calculation process is learned and implemented manually in the C language, without relying on the MATLAB PDE Toolbox, more of the errors (ideally all of them) will be removed in the process of calculating the temperature distribution for different configurations.

In addition to performance speedups, future versions of the Constructal Design Tool need to be able to generate configurations with multiple levels of bifurcations. Also, having a variable root length producing different heat flow results for a 1 end per branch case mentioned in section 5.2 needs to be resolved.

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Appendix A: Exhaustive Search Results Data

For this simulation, we had the following conditions:

Volume Constraint: 0.005300

Disc Radius: 1.000000

Disc k: 0.010000

Disc Q: 0.000000

Beam Radius: 0.100000

Beam k: 30.000000

Beam Q: 86.000000

Branch k: 30.000000

Branch Q: 0.000000

The following are the case results:

Case#	#Roots	#Ends	Root Width	Root Length	Central Temperature
1	2	1	0.0612	0.1	0.6994
2	2	1	0.0612	0.2	0.6935
3	2	1	0.0612	0.3	0.688
4	2	1	0.0612	0.4	0.6825
5	2	1	0.0612	0.5	0.677
6	2	1	0.0612	0.6	0.6716

7	2	1	0.0612	0.7	0.6662
8	2	2	0.0665	0.5	1.3296
9	2	2	0.0703	0.4	0.8733
10	2	2	0.0746	0.3	0.7684
11	2	2	0.0795	0.2	0.6912
12	2	2	0.0852	0.1	0.6313
13	2	3	0.0827	0.3	0.831
14	2	3	0.0915	0.2	0.7449
15	2	4	0.0887	0.3	0.7346
16	2	4	0.1008	0.2	0.639
17	2	5	0.1082	0.2	0.5677
18	3	1	0.05	0.2	0.6082
19	3	1	0.05	0.3	0.5991
20	3	1	0.05	0.4	0.5902
21	3	1	0.05	0.5	0.5812
22	3	1	0.05	0.6	0.5724
23	3	1	0.05	0.7	0.5635
24	3	2	0.0512	0.7	0.9239
25	3	2	0.0538	0.6	0.7805

26	3	2	0.0564	0.5	0.7017
27	3	2	0.0593	0.4	0.6442
28	3	2	0.0625	0.3	0.6037
29	3	2	0.0661	0.2	0.5744
30	3	2	0.0702	0.1	0.4118
31	3	3	0.0598	0.5	0.5911
32	3	3	0.0643	0.4	0.5391
33	3	3	0.0698	0.3	0.4975
34	3	3	0.0766	0.2	0.4577
35	3	4	0.0678	0.4	0.4772
36	3	4	0.0749	0.3	0.4305
37	3	4	0.0845	0.2	0.3885
38	3	5	0.0703	0.4	0.4331
39	3	5	0.0787	0.3	0.3863
40	3	5	0.0907	0.2	0.3416
41	4	1	0.0433	0.1	0.5016
42	4	1	0.0433	0.2	0.4952
43	4	1	0.0433	0.3	0.4891
44	4	1	0.0433	0.4	0.4832

45	4	1	0.0433	0.5	0.4772
46	4	1	0.0433	0.6	0.4713
47	4	1	0.0433	0.7	0.4655
48	4	2	0.0455	0.7	0.6163
49	4	2	0.0475	0.6	0.5475
50	4	2	0.05	0.5	0.5099
51	4	2	0.0521	0.4	0.481
52	4	2	0.0547	0.3	0.4597
53	4	2	0.0576	0.2	0.4438
54	4	3	0.0467	0.7	0.5106
55	4	3	0.05	0.6	0.477
56	4	3	0.0528	0.5	0.4398
57	4	3	0.0566	0.4	0.407
58	4	3	0.0612	0.3	0.3762
59	4	3	0.067	0.2	0.3511
60	4	4	0.055	0.5	0.3927
61	4	4	0.0598	0.4	0.3582
62	4	4	0.0658	0.3	0.3255
63	4	4	0.074	0.2	0.2961

64	4	4	0.0861	0.1	0.2695
65	4	5	0.0619	0.4	0.3271
66	4	5	0.0691	0.3	0.2931
67	4	5	0.0795	0.2	0.2618
68	5	1	0.0387	0.2	0.4249
69	5	1	0.0387	0.3	0.4232
70	5	1	0.0387	0.4	0.4214
71	5	1	0.0387	0.5	0.4196
72	5	1	0.0387	0.6	0.4178
73	5	1	0.0387	0.7	0.4159
74	5	2	0.0412	0.7	0.4827
75	5	2	0.043	0.6	0.4437
76	5	2	0.0449	0.5	0.4148
77	5	2	0.0469	0.4	0.3948
78	5	2	0.0491	0.3	0.3832
79	5	3	0.0424	0.7	0.4249
80	5	3	0.0449	0.6	0.3885
81	5	3	0.0478	0.5	0.3584
82	5	3	0.0511	0.4	0.3333

83	5	3	0.0602	0.3	0.2924
84	5	4	0.0433	0.7	0.3914
85	5	4	0.0462	0.6	0.3607
86	5	4	0.05	0.5	0.3259
87	5	4	0.0539	0.4	0.2948
88	5	4	0.0593	0.3	0.2709
89	5	4	0.0665	0.2	0.2478
90	5	4	0.0772	0.1	0.226
91	5	5	0.0472	0.6	0.3321
92	5	5	0.051	0.5	0.2987
93	5	5	0.0559	0.4	0.2698
94	5	5	0.0623	0.3	0.242
95	5	5	0.0715	0.2	0.2168
96	6	1	0.0354	0.2	0.3742
97	6	1	0.0354	0.3	0.3744
98	6	1	0.0354	0.4	0.3746
99	6	1	0.0354	0.5	0.3747
100	6	1	0.0354	0.6	0.3748
101	6	1	0.0354	0.7	0.3749

102	6	2	0.0379	0.7	0.4024
103	6	2	0.0395	0.6	0.3796
104	6	2	0.0411	0.5	0.3571
105	6	2	0.043	0.4	0.3457
106	6	2	0.045	0.3	0.3327
107	6	2	0.0473	0.2	0.3231
108	6	3	0.0391	0.7	0.3661
109	6	3	0.0413	0.6	0.3341
110	6	3	0.0439	0.5	0.3084
111	6	3	0.0469	0.4	0.2876
112	6	4	0.0399	0.7	0.3461
113	6	4	0.0425	0.6	0.3109
114	6	4	0.0456	0.5	0.2819
115	6	4	0.0494	0.4	0.2574
116	6	4	0.0543	0.3	0.2341
117	6	5	0.0404	0.7	0.3223
118	6	5	0.0433	0.6	0.2948
119	6	5	0.0468	0.5	0.2589
120	6	5	0.0512	0.4	0.2338

121	6	5	0.0571	0.3	0.2097
122	7	1	0.0327	0.2	0.4364
123	7	1	0.0327	0.3	0.4257
124	7	1	0.0327	0.4	0.4151
125	7	1	0.0327	0.5	0.4045
126	7	1	0.0327	0.6	0.394
127	7	1	0.0327	0.7	0.3835
128	7	2	0.0353	0.7	0.3539
129	7	2	0.0367	0.6	0.3289
130	7	2	0.0382	0.5	0.3179
131	7	2	0.0399	0.4	0.3085
132	7	2	0.0417	0.3	0.2969
133	7	2	0.0438	0.2	0.2884
134	7	3	0.0364	0.7	0.327
135	7	4	0.0372	0.7	0.2981
136	7	4	0.0396	0.6	0.2761
137	7	4	0.0425	0.5	0.2503
138	7	4	0.046	0.4	0.2286
139	7	4	0.05	0.3	0.2106

140	7	5	0.0376	0.7	0.2879
141	7	5	0.0403	0.6	0.2566
142	7	5	0.0435	0.5	0.2304
143	7	5	0.0476	0.4	0.208
144	8	1	0.0306	0.2	0.3975
145	8	1	0.0306	0.3	0.3875
146	8	1	0.0306	0.4	0.3775
147	8	1	0.0306	0.5	0.3677
148	8	1	0.0306	0.6	0.3579
149	8	1	0.0306	0.7	0.3482
150	8	2	0.0332	0.7	0.3139
151	8	2	0.0344	0.6	0.3011
152	8	2	0.0358	0.5	0.2912
153	8	2	0.0373	0.4	0.2779
154	8	2	0.0391	0.3	0.2709
155	8	2	0.041	0.2	0.2625
156	8	3	0.0342	0.7	0.2911
157	8	3	0.0361	0.6	0.2732
158	8	3	0.0383	0.5	0.2512

159	8	3	0.0408	0.4	0.2335
160	8	3	0.0439	0.3	0.2188
161	8	3	0.0478	0.2	0.206
162	8	4	0.0349	0.7	0.2757
163	8	4	0.0371	0.6	0.2478
164	8	4	0.0398	0.5	0.2299
165	8	4	0.043	0.4	0.2091
166	8	4	0.0472	0.3	0.1892
167	8	4	0.0529	0.2	0.174
168	8	5	0.0353	0.7	0.2661
169	8	5	0.0378	0.6	0.2361
170	8	5	0.0408	0.5	0.2111
171	8	5	0.0446	0.4	0.1899
172	8	5	0.05	0.3	0.1705
173	8	5	0.0568	0.2	0.1523
174	9	1	0.0289	0.2	0.3684
175	9	1	0.0289	0.3	0.3608
176	9	1	0.0289	0.4	0.3534
177	9	1	0.0289	0.5	0.346

178	9	1	0.0289	0.6	0.3387
179	9	1	0.0289	0.7	0.3315
180	9	2	0.0313	0.7	0.2937
181	9	2	0.0325	0.6	0.2818
182	9	2	0.0338	0.5	0.2656
183	9	2	0.0352	0.4	0.2581
184	9	2	0.0368	0.3	0.2481
185	9	2	0.0387	0.2	0.2422
186	9	2	0.0408	0.1	0.2359
187	9	3	0.0324	0.7	0.272
188	9	3	0.0341	0.6	0.2475
189	9	3	0.0362	0.5	0.2335
190	9	3	0.0386	0.4	0.2161
191	9	3	0.0415	0.3	0.2018
192	9	3	0.0451	0.2	0.1896
193	9	4	0.033	0.7	0.2595
194	9	4	0.0351	0.6	0.2319
195	9	4	0.0376	0.5	0.2095
196	9	4	0.0406	0.4	0.1908

197	9	4	0.0446	0.3	0.1747
198	9	4	0.05	0.2	0.1601
199	9	4	0.0577	0.1	0.1459
200	9	5	0.0334	0.7	0.2411
201	9	5	0.0357	0.6	0.2208
202	9	5	0.0385	0.5	0.1965
203	9	5	0.0421	0.4	0.1759
204	9	5	0.0468	0.3	0.1562
205	9	5	0.0536	0.2	0.14
206	10	1	0.0274	0.2	0.3423
207	10	1	0.0274	0.3	0.3351
208	10	1	0.0274	0.4	0.3281
209	10	1	0.0274	0.5	0.3211
210	10	1	0.0274	0.6	0.3142
211	10	1	0.0274	0.7	0.3073
212	10	2	0.0298	0.7	0.2689
213	10	2	0.0309	0.6	0.259
214	10	3	0.0307	0.7	0.2499
215	10	3	0.0324	0.6	0.2348

216	10	3	0.0343	0.5	0.2152
217	10	3	0.0366	0.4	0.1995
218	10	3	0.0394	0.3	0.1886
219	10	3	0.0428	0.2	0.1764
220	10	4	0.0313	0.7	0.238
221	10	4	0.0333	0.6	0.213
222	10	4	0.0357	0.5	0.1975
223	10	4	0.0386	0.4	0.1789
224	10	4	0.0423	0.3	0.1631
225	10	4	0.0474	0.2	0.1482
226	10	5	0.0317	0.7	0.2293
227	10	5	0.0339	0.6	0.2027
228	11	1	0.0261	0.2	0.3204
229	11	1	0.0261	0.3	0.3135
230	11	1	0.0261	0.4	0.3068
231	11	1	0.0261	0.5	0.3002
232	11	1	0.0261	0.6	0.2936
233	11	1	0.0261	0.7	0.287
234	11	2	0.0284	0.7	0.258

235	11	2	0.0295	0.6	0.2481
236	11	2	0.0306	0.5	0.2328
237	11	2	0.0319	0.4	0.2258
238	11	2	0.0334	0.3	0.2163
239	11	2	0.035	0.2	0.2106
240	11	2	0.0369	0.1	0.2047
241	11	3	0.0294	0.7	0.2204
242	11	3	0.031	0.6	0.1996
243	11	3	0.0328	0.5	0.1882
244	11	3	0.0349	0.4	0.1735
245	11	3	0.0376	0.3	0.1616
246	11	3	0.0409	0.2	0.1515
247	11	4	0.0299	0.7	0.2201
248	11	4	0.0318	0.6	0.2039
249	11	4	0.0341	0.5	0.1833
250	11	4	0.0368	0.4	0.1663
251	11	4	0.0404	0.3	0.1518
252	11	4	0.0452	0.2	0.1389
253	11	5	0.0303	0.7	0.195

254	11	5	0.0324	0.6	0.1786
255	11	5	0.0349	0.5	0.1581
256	11	5	0.0381	0.4	0.141
257	11	5	0.0424	0.3	0.1262
258	11	5	0.0485	0.2	0.1123
259	12	1	0.025	0.2	0.3037
260	12	1	0.025	0.3	0.2991
261	12	1	0.025	0.4	0.2945
262	12	1	0.025	0.5	0.29
263	12	1	0.025	0.6	0.2855
264	12	1	0.025	0.7	0.281
265	12	2	0.0273	0.7	0.2411
266	12	2	0.0283	0.6	0.2322
267	12	2	0.0293	0.5	0.2241
268	12	2	0.0306	0.4	0.2118
269	12	2	0.0319	0.3	0.2056
270	12	2	0.0335	0.2	0.1979
271	12	3	0.0282	0.7	0.2247
272	12	3	0.0297	0.6	0.2036

273	12	3	0.0314	0.5	0.192
274	12	3	0.0335	0.4	0.177
275	12	3	0.036	0.3	0.1668
276	12	3	0.0391	0.2	0.1554
277	12	4	0.0287	0.7	0.2141
278	12	4	0.0305	0.6	0.1905
279	12	4	0.0326	0.5	0.1763
280	12	4	0.0353	0.4	0.1588
281	12	4	0.0387	0.3	0.1441
282	12	4	0.0433	0.2	0.1312
283	12	5	0.029	0.7	0.2076
284	12	5	0.031	0.6	0.1821
285	12	5	0.0334	0.5	0.1612
286	12	5	0.0365	0.4	0.1437
287	12	5	0.0406	0.3	0.1286
288	12	5	0.0465	0.2	0.115
289	13	1	0.024	0.2	0.2872
290	13	1	0.024	0.3	0.2829
291	13	1	0.024	0.4	0.2785

292	13	1	0.024	0.5	0.2741
293	13	1	0.024	0.6	0.2698
294	13	1	0.024	0.7	0.2655
295	13	2	0.0262	0.7	0.226
296	13	2	0.0272	0.6	0.2182
297	13	2	0.0282	0.5	0.211
298	13	2	0.0294	0.4	0.204
299	13	2	0.0307	0.3	0.1941
300	13	2	0.0322	0.2	0.1883
301	13	3	0.0271	0.7	0.2108
302	13	3	0.0285	0.6	0.1982
303	13	3	0.0302	0.5	0.1806
304	13	3	0.0322	0.4	0.1701
305	13	3	0.0346	0.3	0.1573
306	13	3	0.0376	0.2	0.1467
307	13	4	0.0276	0.7	0.175
308	13	4	0.0293	0.6	0.1619
309	13	4	0.0314	0.5	0.1446
310	13	4	0.0339	0.4	0.1305

311	13	4	0.0372	0.3	0.1186
312	13	4	0.0416	0.2	0.1082
313	13	5	0.0279	0.7	0.1811
314	13	5	0.0298	0.6	0.1592
315	13	5	0.0322	0.5	0.1453
316	13	5	0.0351	0.4	0.1287
317	13	5	0.0447	0.2	0.1013
318	14	2	0.0253	0.7	0.2089
319	14	2	0.0262	0.6	0.1932
320	14	2	0.0272	0.5	0.1869
321	14	2	0.0283	0.4	0.1808
322	14	2	0.0296	0.3	0.1722
323	14	2	0.031	0.2	0.1672
324	14	3	0.0261	0.7	0.1862
325	14	3	0.0275	0.6	0.1753
326	14	3	0.0291	0.5	0.1647
327	14	3	0.031	0.4	0.1508
328	14	3	0.0333	0.3	0.1396
329	14	3	0.0362	0.2	0.1311

330	14	4	0.0266	0.7	0.1779
331	14	4	0.0283	0.6	0.1646
332	14	4	0.0302	0.5	0.147
333	14	4	0.0327	0.4	0.1355
334	14	4	0.0358	0.3	0.1221
335	14	4	0.0401	0.2	0.1099
336	14	4	0.0463	0.1	0.1003
337	14	5	0.0269	0.7	0.1841
338	14	5	0.0288	0.6	0.1682
339	14	5	0.031	0.5	0.1477
340	14	5	0.0338	0.4	0.1307
341	14	5	0.043	0.2	0.1035
342	15	1	0.0224	0.2	0.2623
343	15	1	0.0224	0.3	0.2602
344	15	1	0.0224	0.4	0.2581
345	15	1	0.0224	0.5	0.2559
346	15	1	0.0224	0.6	0.2538
347	15	1	0.0224	0.7	0.2516
348	15	2	0.0244	0.7	0.2122

349	15	2	0.0253	0.6	0.2039
350	15	2	0.0263	0.5	0.1899
351	15	2	0.0274	0.4	0.1837
352	15	2	0.0286	0.3	0.1775
353	15	2	0.03	0.2	0.1699
354	15	3	0.0252	0.7	0.1982
355	15	3	0.0266	0.6	0.178
356	15	3	0.0281	0.5	0.1673
357	15	3	0.03	0.4	0.1531
358	15	3	0.0322	0.3	0.1437
359	15	3	0.035	0.2	0.1332
360	15	4	0.0257	0.7	0.1897
361	15	4	0.0273	0.6	0.1671
362	15	4	0.0292	0.5	0.154
363	15	4	0.0316	0.4	0.1375
364	15	4	0.0346	0.3	0.1239
365	15	4	0.0387	0.2	0.1122
366	15	5	0.026	0.7	0.154
367	15	5	0.0278	0.6	0.1341

368	15	5	0.0327	0.4	0.1071
369	15	5	0.0364	0.3	0.0947
370	15	5	0.0416	0.2	0.0836
371	16	2	0.0265	0.4	0.1753
372	16	2	0.0277	0.3	0.1694
373	16	2	0.029	0.2	0.1635
374	16	3	0.0245	0.7	0.1779
375	16	3	0.0257	0.6	0.1667
376	16	3	0.0272	0.5	0.1505
377	16	3	0.029	0.4	0.1411
378	16	3	0.0312	0.3	0.1296
379	16	3	0.0339	0.2	0.1202
380	16	4	0.0249	0.7	0.1612
381	16	4	0.0265	0.6	0.1422
382	16	4	0.0283	0.5	0.1312
383	16	4	0.0335	0.3	0.106
384	16	4	0.0375	0.2	0.0962
385	16	5	0.0252	0.7	0.1563
386	16	5	0.0269	0.6	0.136

387	16	5	0.029	0.5	0.1236
388	16	5	0.0317	0.4	0.1085
389	16	5	0.0352	0.3	0.0959
390	16	5	0.0403	0.2	0.0847
391	17	1	0.021	0.2	0.2405
392	17	1	0.021	0.3	0.2385
393	17	1	0.021	0.4	0.2365
394	17	1	0.021	0.5	0.2344
395	17	1	0.021	0.6	0.2324
396	17	1	0.021	0.7	0.2303
397	17	2	0.023	0.7	0.1827
398	17	2	0.0238	0.6	0.1759
399	17	2	0.0247	0.5	0.1694
400	17	2	0.0257	0.4	0.1629
401	17	2	0.0269	0.3	0.1538
402	17	2	0.0282	0.2	0.1484
403	17	3	0.0237	0.7	0.1803
404	17	3	0.025	0.6	0.1689
405	17	3	0.0264	0.5	0.1525

406	17	3	0.0281	0.4	0.143
407	17	3	0.0302	0.3	0.1313
408	17	3	0.0329	0.2	0.1227
409	17	4	0.0242	0.7	0.1633
410	17	4	0.0257	0.6	0.1505
411	17	4	0.0275	0.5	0.1329
412	17	4	0.0364	0.2	0.098
413	17	5	0.0245	0.7	0.1584
414	17	5	0.0261	0.6	0.1378
415	17	5	0.0281	0.5	0.1252
416	17	5	0.0342	0.3	0.0971
417	17	5	0.0391	0.2	0.0861
418	18	1	0.0204	0.2	0.2312
419	18	1	0.0204	0.3	0.2292
420	18	1	0.0204	0.4	0.2272
421	18	1	0.0204	0.5	0.2252
422	18	1	0.0204	0.6	0.2232
423	18	1	0.0204	0.7	0.2212
424	18	2	0.0223	0.7	0.1951

425	18	2	0.0231	0.6	0.1782
426	18	2	0.024	0.5	0.1716
427	18	2	0.025	0.4	0.165
428	18	2	0.0261	0.3	0.1584
429	18	2	0.0274	0.2	0.1504
430	18	3	0.0231	0.7	0.1562
431	18	3	0.0243	0.6	0.1465
432	18	3	0.0257	0.5	0.137
433	18	3	0.0274	0.4	0.1242
434	18	3	0.0294	0.3	0.116
435	18	3	0.032	0.2	0.1068
436	18	4	0.0235	0.7	0.1654
437	18	4	0.025	0.6	0.1524
438	18	4	0.0267	0.5	0.1346
439	18	4	0.0288	0.4	0.1234
440	18	4	0.0354	0.2	0.0992
441	18	5	0.0238	0.7	0.1604
442	18	5	0.0254	0.6	0.1459
443	18	5	0.0274	0.5	0.1268

444	18	5	0.0332	0.3	0.0982
445	18	5	0.038	0.2	0.0871
446	19	1	0.0199	0.2	0.2123
447	19	1	0.0199	0.3	0.2104
448	19	1	0.0199	0.4	0.2086
449	19	1	0.0199	0.5	0.2067
450	19	1	0.0199	0.6	0.2049
451	19	1	0.0199	0.7	0.203
452	19	2	0.0217	0.7	0.1785
453	19	2	0.0225	0.6	0.1707
454	19	2	0.0234	0.5	0.1572
455	19	2	0.0243	0.4	0.1512
456	19	2	0.0254	0.3	0.1452
457	19	2	0.0266	0.2	0.138
458	19	3	0.0225	0.7	0.1671
459	19	3	0.0236	0.6	0.1483
460	19	3	0.025	0.5	0.1386
461	19	3	0.0266	0.4	0.1257
462	19	3	0.0286	0.3	0.1174

463	19	4	0.0229	0.7	0.1382
464	19	4	0.0243	0.6	0.1275
465	19	4	0.026	0.5	0.1169
466	19	4	0.0281	0.4	0.1035
467	19	4	0.0308	0.3	0.0928
468	19	4	0.0344	0.2	0.0839
469	19	5	0.0232	0.7	0.1235
470	19	5	0.0247	0.6	0.1125
471	19	5	0.0266	0.5	0.0981
472	19	5	0.0291	0.4	0.0885
473	19	5	0.0323	0.3	0.0779
474	19	5	0.037	0.2	0.0688
475	20	1	0.0194	0.2	0.2149
476	20	1	0.0194	0.3	0.213
477	20	1	0.0194	0.4	0.2111
478	20	1	0.0194	0.5	0.2092
479	20	1	0.0194	0.6	0.2073
480	20	1	0.0194	0.7	0.2054
481	20	2	0.0212	0.7	0.1805

482	20	2	0.0219	0.6	0.1727
483	20	2	0.0228	0.5	0.159
484	20	2	0.0237	0.4	0.153
485	20	2	0.0248	0.3	0.1469
486	20	2	0.026	0.2	0.1408
487	20	3	0.0219	0.7	0.169
488	20	3	0.023	0.6	0.1499
489	20	3	0.0244	0.5	0.1402
490	20	3	0.026	0.4	0.1306
491	20	3	0.0279	0.3	0.1187
492	20	4	0.0223	0.7	0.148
493	20	4	0.0237	0.6	0.1289
494	20	4	0.0253	0.5	0.1182
495	20	4	0.0274	0.4	0.1046
496	20	4	0.03	0.3	0.0938
497	20	4	0.0335	0.2	0.0848
498	20	5	0.0226	0.7	0.1324
499	20	5	0.0241	0.6	0.1138
500	20	5	0.026	0.5	0.1029

501	20	5	0.0283	0.4	0.0895
502	20	5	0.0315	0.3	0.0787
503	20	5	0.036	0.2	0.0696
504	21	1	0.0189	0.2	0.21
505	21	1	0.0189	0.3	0.2101
506	21	1	0.0189	0.4	0.2102
507	21	1	0.0189	0.5	0.2103
508	21	1	0.0189	0.6	0.2104
509	21	1	0.0189	0.7	0.2104
510	21	2	0.0214	0.6	0.1595
511	21	2	0.0222	0.5	0.1525
512	21	2	0.0231	0.4	0.1414
513	21	2	0.0242	0.3	0.1359
514	21	2	0.0253	0.2	0.1303
515	21	3	0.0214	0.7	0.1436
516	21	3	0.0225	0.6	0.1339
517	21	3	0.0238	0.5	0.1194
518	21	3	0.0253	0.4	0.1113
519	21	3	0.0272	0.3	0.1014

520	21	3	0.0296	0.2	0.0936
521	21	4	0.0218	0.7	0.1496
522	21	4	0.0231	0.6	0.1303
523	21	4	0.0247	0.5	0.1195
524	21	4	0.0267	0.4	0.1056
525	21	4	0.0293	0.3	0.0963
526	21	4	0.0327	0.2	0.0861
527	21	5	0.022	0.7	0.1338
528	21	5	0.0235	0.6	0.115
529	21	5	0.0253	0.5	0.104
530	21	5	0.0277	0.4	0.0904
531	21	5	0.0307	0.3	0.0795
532	21	5	0.0352	0.2	0.0703
533	22	1	0.0185	0.2	0.2034
534	22	1	0.0185	0.3	0.2034
535	22	1	0.0185	0.4	0.2035
536	22	1	0.0185	0.5	0.2036
537	22	1	0.0185	0.6	0.2036
538	22	1	0.0185	0.7	0.2037

539	22	2	0.0209	0.6	0.1612
540	22	2	0.0217	0.5	0.1541
541	22	2	0.0226	0.4	0.1471
542	22	2	0.0236	0.3	0.1373
543	22	2	0.0248	0.2	0.1316
544	22	3	0.0209	0.7	0.1451
545	22	3	0.022	0.6	0.1353
546	22	3	0.0233	0.5	0.1206
547	22	3	0.0248	0.4	0.1124
548	22	3	0.0266	0.3	0.1024
549	22	3	0.0289	0.2	0.0951
550	22	4	0.0213	0.7	0.1511
551	22	4	0.0226	0.6	0.1385
552	22	4	0.0242	0.5	0.1207
553	22	4	0.0261	0.4	0.1099
554	22	4	0.0286	0.3	0.0972
555	22	4	0.032	0.2	0.0869
556	22	5	0.0215	0.7	0.1352
557	22	5	0.023	0.6	0.1162

558	22	5	0.0248	0.5	0.105
559	22	5	0.027	0.4	0.0913
560	22	5	0.03	0.3	0.0802
561	22	5	0.0344	0.2	0.0709
562	23	1	0.0181	0.2	0.1895
563	23	1	0.0181	0.3	0.1895
564	23	1	0.0181	0.4	0.1896
565	23	1	0.0181	0.5	0.1896
566	23	1	0.0181	0.6	0.1897
567	23	1	0.0181	0.7	0.1897
568	23	2	0.0198	0.7	0.1507
569	23	2	0.0205	0.6	0.1443
570	23	2	0.0212	0.5	0.1381
571	23	2	0.0221	0.4	0.1318
572	23	2	0.0231	0.3	0.1232
573	23	2	0.0242	0.2	0.118
574	23	3	0.0204	0.7	0.1465
575	23	3	0.0215	0.6	0.1366
576	23	3	0.0227	0.5	0.1218

577	23	3	0.0242	0.4	0.1135
578	23	3	0.026	0.3	0.1052
579	23	3	0.0283	0.2	0.096
580	23	4	0.0208	0.7	0.1141
581	23	4	0.0221	0.6	0.1046
582	23	4	0.0236	0.5	0.0916
583	23	4	0.0255	0.4	0.0836
584	23	4	0.028	0.3	0.0745
585	23	4	0.0313	0.2	0.0673
586	23	5	0.0211	0.7	0.1365
587	23	5	0.0225	0.6	0.1236
588	23	5	0.0242	0.5	0.106
589	23	5	0.0264	0.4	0.0921
590	23	5	0.0294	0.3	0.0822
591	23	5	0.0336	0.2	0.0715
592	24	1	0.0177	0.2	0.1914
593	24	1	0.0177	0.3	0.1914
594	24	1	0.0177	0.4	0.1915
595	24	1	0.0177	0.5	0.1915

596	24	1	0.0177	0.6	0.1916
597	24	1	0.0177	0.7	0.1916
598	24	2	0.0193	0.7	0.1521
599	24	2	0.02	0.6	0.1457
600	24	2	0.0208	0.5	0.1394
601	24	2	0.0216	0.4	0.1331
602	24	2	0.0226	0.3	0.1268
603	24	2	0.0237	0.2	0.1193
604	24	3	0.02	0.7	0.1479
605	24	3	0.021	0.6	0.1379
606	24	3	0.0223	0.5	0.128
607	24	3	0.0237	0.4	0.1146
608	24	3	0.0255	0.3	0.1062
609	24	3	0.0277	0.2	0.0969
610	24	4	0.0204	0.7	0.1151
611	24	4	0.0216	0.6	0.1056
612	24	4	0.0231	0.5	0.0924
613	24	4	0.025	0.4	0.0843
614	24	4	0.0274	0.3	0.0752

615	24	4	0.0306	0.2	0.0678
616	24	5	0.0206	0.7	0.1378
617	24	5	0.022	0.6	0.1247
618	24	5	0.0237	0.5	0.107
619	24	5	0.0259	0.4	0.0957
620	24	5	0.0288	0.3	0.0829
621	24	5	0.0329	0.2	0.0724
622	25	1	0.0173	0.2	0.1861
623	25	1	0.0173	0.3	0.1862
624	25	1	0.0173	0.4	0.1862
625	25	1	0.0173	0.5	0.1862
626	25	1	0.0173	0.6	0.1862
627	25	1	0.0173	0.7	0.1862
628	25	2	0.019	0.7	0.1636
629	25	2	0.0196	0.6	0.147
630	25	2	0.0204	0.5	0.1407
631	25	2	0.0212	0.4	0.1343
632	25	2	0.0222	0.3	0.1279
633	25	2	0.0232	0.2	0.1203

634	25	3	0.0196	0.7	0.1211
635	25	3	0.0206	0.6	0.113
636	25	3	0.0218	0.5	0.105
637	25	3	0.0232	0.4	0.0943
638	25	3	0.025	0.3	0.0875
639	25	3	0.0271	0.2	0.0803
640	25	4	0.02	0.7	0.1161
641	25	4	0.0212	0.6	0.1065
642	25	4	0.0227	0.5	0.0932
643	25	4	0.0245	0.4	0.085
644	25	4	0.0268	0.3	0.0758
645	25	4	0.03	0.2	0.0684
646	25	5	0.0202	0.7	0.0956
647	25	5	0.0216	0.6	0.0868
648	25	5	0.0232	0.5	0.0752
649	25	5	0.0254	0.4	0.0677
650	25	5	0.0282	0.3	0.0596
651	25	5	0.0322	0.2	0.0532
652	26	2	0.0186	0.7	0.139

653	26	2	0.0193	0.6	0.1251
654	26	2	0.02	0.5	0.1197
655	26	2	0.0208	0.4	0.1144
656	26	2	0.0217	0.3	0.109
657	26	2	0.0228	0.2	0.1028
658	26	3	0.0192	0.7	0.1221
659	26	3	0.0202	0.6	0.1139
660	26	3	0.0214	0.5	0.1059
661	26	3	0.0228	0.4	0.095
662	26	3	0.0245	0.3	0.0882
663	26	3	0.0266	0.2	0.0809
664	26	4	0.0196	0.7	0.1171
665	26	4	0.0208	0.6	0.1074
666	26	4	0.0222	0.5	0.0979
667	26	4	0.024	0.4	0.0857
668	26	4	0.0263	0.3	0.0764
669	26	4	0.0294	0.2	0.0691
670	26	5	0.0198	0.7	0.0964
671	26	5	0.0228	0.5	0.0758

672	26	5	0.0249	0.4	0.0682
673	26	5	0.0276	0.3	0.06
674	26	5	0.0316	0.2	0.0535
675	27	2	0.0182	0.7	0.1402
676	27	2	0.0189	0.6	0.1332
677	27	2	0.0196	0.5	0.1207
678	27	2	0.0204	0.4	0.1153
679	27	2	0.0213	0.3	0.1099
680	27	2	0.0224	0.2	0.1045
681	27	3	0.0189	0.7	0.1315
682	27	3	0.0199	0.6	0.1149
683	27	3	0.021	0.5	0.1067
684	27	3	0.0224	0.4	0.0987
685	27	3	0.024	0.3	0.0889
686	27	3	0.0261	0.2	0.0816
687	27	4	0.0192	0.7	0.1181
688	27	4	0.0204	0.6	0.1083
689	27	4	0.0218	0.5	0.0987
690	27	4	0.0236	0.4	0.0864

691	27	4	0.0258	0.3	0.0781
692	27	4	0.0289	0.2	0.0696

Table 5.2: Exhaustive Search Results Data

After the search through different configurations, the best configuration was found to be: 651 25 5 0.0322 0.20 0.0532

Appendix B: Coordinate Search Results Data

For this simulation, we had the following conditions:

Volume Constraint: 0.005301

Disc Radius: 1.000000

Disc k: 0.010000

Disc Q: 0.000000

Beam Radius: 0.100000

Beam k: 30.000000

Beam Q: 86.000000

Branch k: 30.000000

Branch Q: 0.000000

The coordinate search took 27277.542583 seconds to complete.

The following are the case results:

Case#	#Roots	#Ends	Root Width	Root Length	Central Temp.	Compute Time
1	2	1	0.061239	0.1	0.699374	17.589186
2	2	1	0.061239	0.2	0.693525	12.665513
3	2	1	0.061239	0.3	0.687958	12.341206

4	2	1	0.061238	0.4	0.682469	12.630435
5	2	1	0.061238	0.5	0.677045	12.558706
6	2	1	0.061237	0.6	0.671639	13.257724
7	2	1	0.061237	0.7	0.666235	14.555064
8	2	2	0.061237	0.7	NaN	0
9	2	3	0.061455	0.7	NaN	12.395582
10	2	4	0.063336	0.7	NaN	23.47022
11	2	5	0.064832	0.7	NaN	26.907206
12	3	1	0.05	0.7	0.563515	14.261825
13	4	1	0.043301	0.7	0.465456	14.069749
14	5	1	0.038729	0.7	0.415929	14.095311
15	6	1	0.035355	0.7	0.374876	14.439519
16	7	1	0.032732	0.7	0.38352	15.172549
17	8	1	0.030618	0.7	0.348214	16.305175
18	9	1	0.028867	0.7	0.331481	16.195651
19	10	1	0.027386	0.7	0.307306	17.065546
20	11	1	0.026111	0.7	0.287017	16.90145
21	12	1	0.025	0.7	0.280992	17.542576
22	13	1	0.024019	0.7	0.265494	17.772066

23	14	1	0.024019	0.7	NaN	0
24	15	1	0.02236	0.7	0.251642	18.627744
25	16	1	0.02236	0.7	NaN	0
26	17	1	0.021004	0.7	0.230347	18.92728
27	18	1	0.020412	0.7	0.221245	18.026657
28	19	1	0.019868	0.7	0.203022	18.374195
29	20	1	0.019365	0.7	0.2054	18.301894
30	21	1	0.018898	0.7	0.210425	20.863744
31	22	1	0.018464	0.7	0.203691	21.32116
32	23	1	0.018058	0.7	0.189718	18.973469
33	24	1	0.017678	0.7	0.191571	20.059653
34	25	1	0.01732	0.7	0.186246	20.54592
35	26	1	0.01732	0.7	NaN	0
36	27	1	0.01732	0.7	NaN	0
37	25	1	0.01732	0.1	NaN	0
38	25	1	0.017321	0.2	0.186142	17.718645
39	25	1	0.017321	0.3	0.18616	20.640041
40	25	1	0.017321	0.4	0.186185	20.790829
41	25	1	0.017321	0.5	0.186226	21.337472

42	25	1	0.017321	0.6	0.186235	20.7554
43	25	2	0.023234	0.2	0.120326	26.627732
44	25	3	0.027136	0.2	0.08031	145.033356
45	25	4	0.030003	0.2	0.068363	772.796167
46	25	5	0.032231	0.2	0.053155	5495.194443
47	26	5	0.031605	0.2	0.053475	4473.304469
48	27	5	0.031014	0.2	0.053788	5415.325795
49	2	5	0.108159	0.2	0.567666	51.033828
50	3	5	0.090741	0.2	0.341646	58.216184
51	4	5	0.079465	0.2	0.261787	55.139366
52	5	5	0.071497	0.2	0.216837	56.963389
53	6	5	0.071497	0.2	NaN	0
54	7	5	0.071497	0.2	NaN	0
55	8	5	0.056821	0.2	0.152328	43.616108
56	9	5	0.053611	0.2	0.140025	64.937242
57	10	5	0.053611	0.2	NaN	0
58	11	5	0.048547	0.2	0.112269	98.810763
59	12	5	0.046481	0.2	0.114997	108.435831
60	13	5	0.044675	0.2	0.101327	209.140713

61	14	5	0.04305	0.2	0.10349	224.074423
62	15	5	0.041601	0.2	0.083559	438.816339
63	16	5	0.04028	0.2	0.084662	480.057816
64	17	5	0.039078	0.2	0.086098	462.948836
65	18	5	0.037977	0.2	0.087108	549.712673
66	19	5	0.03697	0.2	0.068803	598.554015
67	20	5	0.036034	0.2	0.06965	600.465566
68	21	5	0.035165	0.2	0.0703	602.766199
69	22	5	0.034357	0.2	0.070928	668.817366
70	23	5	0.033602	0.2	0.071529	679.71179
71	24	5	0.032894	0.2	0.072431	702.520804

Table 5.3: Coordinate Search Results Data

After the search through different configurations, the best configuration was found to be: 25 5 0.03223100 0.200000000 0.05315524066557129

The following cases were missed because of errors:

Case#	#Roots	#Ends	Root Width	Root Length	Central Temperature
8	2	2	0.061237	0.7	NaN

9	2	3	0.061455	0.7	NaN
10	2	4	0.063336	0.7	NaN
11	2	5	0.064832	0.7	NaN
23	14	1	0.024019	0.7	NaN
25	16	1	0.02236	0.7	NaN
35	26	1	0.01732	0.7	NaN
36	27	1	0.01732	0.7	NaN
37	25	1	0.01732	0.1	NaN
53	6	5	0.071497	0.2	NaN
54	7	5	0.071497	0.2	NaN
57	10	5	0.053611	0.2	NaN

Table 5.4: Error Cases in Coordinate Search

VITAE

Muhammed Zahid Ayar **Phone:** 050 698 3301

Software Engineer **e-mail:** zahid@ayar.org

Nationality: American

Present Address: P.O. Box 1356 KFUPM, Dhahran 31261 KSA

Permanent Address: 2339 15th street Troy, NY 12180 USA

Career Objective: Develop a global organization to serve the needs of people.

Work Experience:

KFUPM, *Research Assistant,* September 2009 - Present, KSA

AiT Telecom, *Software Engineer,* May 2006 - 2008, USA

Futureware IT, *Software Engineer,* September 2005 - March 2006, KSA

AYAR Software, *Web Developer,* July 2005 - August 2005, Turkey

KFUPM, *Instructor,* February 2005 - December 2005, KSA

Teknobil, *Software Engineer,* July 2004 - September 2004, Turkey

KFUPM, *Web Developer,* December 2003 - January 2006, KSA

AIT COMPUTERS, *Technician,* summers of 1998 and 1999, USA

Languages:

English Turkish Arabic

Education:

Master of Science in Computer Science, KFUPM, June 2011, KSA

Bachelor of Science in Software Engineering, KFUPM, January 2005, KSA