GAIN SCHEDULED FILTERING DESIGN FOR PARAMETER VARYING SYSTEMS

BY

ALAEDDIN KHALIL JARADAT

A Thesis Presented to the

DEANSHIP OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the

Requirements for the Degree of

MASTER OF SCIENCE

In

SYSTEMS ENGINEERING

JUNE 2010

KING FAHD UNIVERSITY OF PETROLEUM & MINIRALS
DHAHRAN, SAUDI ARABIA

DEANSHIP OF GRADUATE STUDIES

This thesis, written by ALAEDDIN KHALIL JARADAT under the direction of his thesis advisors and approved by his thesis committee, has been presented to and accepted by the Dean of Graduate Studies, in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE IN SYSTEMS ENGINEERING.

Thesis Committee

Prof. Magdi Mahmoud (Thesis Advisor)

Prof. Lahouari Cheded (Member)

Dr. Abedllwhed Al Saif (Member)

Prof. Fouad Al-Sunni
Department Chairman

Dr. Salam Zummo
Dean of Graduate Studies

13/10/10

Date

ACKNOWLEDGEMENTS

By the name of Allah, the Cherisher and Sustainer of the worlds. Peace and blessings of Allah be upon the Prophet.

I would like to thank my advisor, Prof. Magdi Mahmoud, for his help and guidance. I would also like to thank the committee members, Prof. Lahouari Cheded and Dr. Abedllwhed Al Saif, for their valuable comments and suggestions. Thanks are also due to the faculty members of Systems Engineering Department. I gratefully acknowledge the support and help of all fellow graduate students, especially Mr. Ali Elrayyah

Finally, I would like to submit my utmost gratitude to my parents, wife and family members for their support and prayers that have helped me throughout my study.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGERES

THESIS ABSTRACT

Name: Alaeddin Khalil Mosa Jaradat

Title: Gain Scheduled Filtering Design for Parameter Varying Systems

Major: Systems Engineering

Date: APRIL 2010

The Gain Scheduling (GS) design has become a popular method for non-linear system design, especially during the last decade. It has special features that make it easy to apply compared with other design methods for non-linear systems. Among those features, the most attractive is that GS employs linear design tools in the design stage. Recently, GS techniques have become one of the popular approaches to ‘linear parameter-varying’ (LPV) systems. In this Thesis, the problem of gain-scheduled $H\infty$ filter design for a class of parameter-varying systems with time-varying delays is discussed. An improved stability and $H\infty$-performance condition of the augmented system is developed. A sufficient existence condition of a desired gain-scheduled $H\infty$ filter is derived and expressed in terms of feasibility testing of linear matrix inequalities (LMIs) and explicit expressions of the filter gains are provided. Numerical simulations are presented to demonstrate the effectiveness of the proposed design method.

ملخص الرسالة

الاســـــــــــم: علاء الدين خليل موسى جرادات

عنوان الرسالة: تصميم تصفية من نوع الكسب المجدول للانظمة ذات العوامل المتغيرة .

التخصـــــص: هندسة النظم

تاريخ التخرج: يونيو 2010

لقد أصبح تصميم الكسب المجدول طريقة مشهورة للأنظمة غير الخطية, خصوصا في العقود الأخيرة. ان لهذا النوع من التصميم مزايا خاصة تجعله سهل التطبيق مقارنةً مع طرق تصميم أخرى. من بين هذه المزايا انه يستخدم أدوات التصميم الخطي لأنظمة غير خطية. حديثاً, أصبح تصميم الكسب المجدول طريقة مرتبطة بالأنظمة ذات العوامل المتغيرة.

في هذه الرسالة سيتم بناء تصميم لتصفية من نوع الكسب المجدول لنوع من الأنظمة ذات العوامل المتغيرة والمصحيبة بتأخير زمني, وسيتم تطوير معايير الإستقرار وشروط الأداء للنظام الخليط.

سيتم إشتقاق شرط وجود كفائي للمصفي المرغوب من نوع الكسب المجدول, وسيعرض باستخدام اختبار جدوى للمصفوفة الخطية الموجودة على شكل عدم المساواة. أخيراً, أمثلة محاكاة رقمية ستعرض لتشرح وتبين فعالية طريقة التصميم هذه.

CHAPTER 1

INTRODUCTION

1.1 Introduction

Gain Scheduling (GS) design has become a popular method for non-linear system design, especially during the last decade. It has special features that make it easy to apply compared with other design methods for non-linear systems. Among those features, the most attractive is that GS employs linear design tools in the design stage. See Leith, and Leithead [1.2], and Rugh and Shamma [1.3], for a survey on GS.

Many different design notions can be viewed as GS, such as switching gain values according to operating conditions, precompensating a non-linear gain with the inverse gain function, etc. The GS theory originates from the 1960s, and it has wide fields of applications, but the theoretical works were scarce up to 1990s. One reason for that is GS was considered "applied topic", and the hardware implementation of the GS techniques was classified as military secrets. After that, a considerable increase of interest in gain-scheduling in the literature with many new results obtained. Also new GS approaches were created like Liner Fractional Transformation GS, Fuzzy GS, and others.

A class of systems has a strong relation with GS, called Linear Parameter Varying (LPV) systems. This class of systems was invented by the American scientist Jeff S. Shamma in 1988. LPV systems take the following structure:

$$\begin{aligned} \dot{x} &= A(h(t))x + B(h(t))r \\ y &= C(h(t))x + D(h(t))r \end{aligned} \tag{1.1}$$

where the parameter, h(t), is an exogenous time-varying quantity (strictly independent of the state x of the system) which takes values in some allowable set. Under these conditions, an LPV system is simply a particular form of linear time-varying system.

Another feature may be added to system (1.1), which is a delay. Then the system will be called Linear Parameter Varying Time Delay (LPVTD) systems.

The Filtering problem is a common and known branch in Control theory. One of the most used approaches in this branch is H_∞ filtering. The purpose of H_∞ filtering problem is to design an estimator so as to guarantee that the resulting filtering error system is stable and the L_2- induced gain from the noise to the estimation error is less than a prescribed level. In this thesis, a LPVTD H_∞ Filter will be designed for LPVTD systems. The stability of augmented (error) system, which is composed of the LPVTD system and filter, will be studied and guaranteed depending on Lyapanov function and using the Linear Matrix Inequality (LMI) tools.

1.2 Historical comments

Very little was published on gain scheduling in the archival control research literature before 1990. One reason is that gain scheduling was considered an 'applied' topic, and scheduling techniques, particularly their hardware implementations, often were proprietary or classified as military secrets. A comment on two application areas: one commercial and one military, where gain scheduling has played an important role and where some documentation is available. Thus we confine remarks to some early examples of gain scheduling that are direct forerunners of current practice. Indeed there was great activity in these and other application areas in the period between the early work we describe below and the present. In any case, the market has now spoken: gain scheduling is an effective and economical method for nonlinear control design in practice. This is obvious from discussions with practitioners, and from the rapidly increasing, open publication of applications of gain scheduling in conference proceedings and journals. Because of cost considerations and performance requirements, it is not surprising that gain scheduling techniques were adopted in military applications relatively early, while commercial applications of gain scheduling began when digital computer control came on the scene. In other words, gain scheduling is an old idea, but before digital implementation of controllers, it was expensive and difficult to realize in hardware. This is reflected in the

quarter-century difference in the adoption of gain scheduling in the two application areas discussed. A phrase was said by Shamma in his research on gain scheduling that express the role of gain scheduling, "With little exaggeration we can say, for at least the last few decades: Machines that walk, swim, or fly are gain - scheduled". The LPV systems, as we mention in section 1.1, were invented by the American scientist Jeff S. Shamma in 1988. The Linear Fraction Transformation (LFT) synthesis was derived independently by Andy Packard, Greg Becker and Fen Wu (1997), and Pierre Apkarian and Pascal Gahinet. Noted that, similar ideas have been proposed by Lu and Doyle (1992,1995).

1.3 Literature Review

Wilson J. Rugh and Jeff S. Shamma [1.3] did a survey paper on gain scheduling in 2000, which is "Research on gain scheduling". In the same time, another survey paper about the same subject was done by D.J.Leith and WE.Leithead [1.2] titled by "Survey of Gain-Scheduling Analysis & Design".

A design of gain-scheduled H∞ filters for a class of parameter -varying discrete-time systems [1.7] was done by Shaosheng Zhou , Baoyong Zhang, Wei Xing Zheng. They utilize a parameter dependent Lyapunov function to establish a new characterization of asymptotic stability with H∞ norm bound for the linear parameter-varying (LPV) systems.

A gain-scheduled H∞ output feedback controller was designed for a class of parameter-varying systems with time-varying delays [1.8] by Zhang, Baoyong, Zhou, Shaosheng; Xu, Shengyuan in 2008. A new condition for the stability and H infinity performance analysis of the resulting closed-loop system is obtained.

An investigation of the problem of gain-scheduled guaranteed cost control for linear parameter-varying systems with time-varying state and input delays [1.9] was done by Junling Wang , Peng Shi, Junming Wang in 2008. Attention is focused on the design of guaranteed cost controllers such that the resulting closed-loop system is asymptotically stable and a parameter-dependent cost performance is also guaranteed. Based on a parameter-dependent Lyapunov function, a linear matrix inequality methodology is proposed for designing gain-scheduled state feedback controller, where the feedback gain depends on the scheduling parameters.

A new approach to the design of a gain scheduled linear parameter-varying (LPV) H_∞ controller, which places the closed-loop poles in the region that satisfies the specified dynamic response, for an n-joint rigid robotic manipulator [1.10] was done by Zhongwei Yu , Huitang Chen, Peng-yung Woo in 2002. The nonlinear time-varying robotic manipulator is modeled to be a LPV system with a convex polytopic structure with the use of the LPV convex decomposition technique in a filter introduced. State feedback controllers, which satisfy the H_∞ performance and the closed-loop pole-placement requirements, for each vertex of the convex polyhedron parameter space, are designed with the use of the linear matrix inequality (LMI) approach.

A paper, published in 2004 by Nguyen Thien Hoang [1.12], and others, deals with the design of gain-scheduled filters, whose state-space realization depends on real-time parameters of plants. The construction technique is based on nonlinear fractional transformation (NFT) representations of systems. This study leads to new linear matrix inequality (LMI) formulations.

An adaptive observer for single input-single output observable nonlinear systems, that can be transformed to a certain observable canonical form, was checked by Georges Bastin and Michel Gevers in1988 [1.13]. They provide sufficient conditions for stability of this observer. These conditions are in terms of the structure of the system and its canonical form, the boundedness of the parameter variations, and the sufficient richness of some signals. They presented applications to time-invariant bilinear systems, nonlinear systems in phase-variable form, a biotechnological process, and a robot manipulator.

The stability analysis and control of the Time delay systems has been examined extensively in the controls literature using both state-space and frequency domain methods (e.g., see Malek-Zavarei & Jamshidi, 1987 [1.14]; Watanabe, Nobuyama & Kojima, 1996 [1.15]; Dugard & Verriest, 1998 [1.16]).

CHAPTER 2

GAIN SCHEDULING TECHNIQUES

2.1 Introduction

Gain-scheduling (GS) is one of the most popular and less difficult approaches to control and filtering of nonlinear plants. It has special features that make it easy to apply compared with other design methods for non-linear plants. Among those features, the most attractive is that GS employs linear design tools in the design stage. Gain Scheduling has been widely and successfully applied in fields ranging from aerospace to process control [1.3]. Many of control and filtering methods are often described as “gain-scheduling” approaches. All of these are usually linked by a *divide and conquer* principle. This principle decomposes the nonlinear control or filtering design into a number of linear sub-problems. This divide and conquer approach is the source of much of the popularity of gain scheduling methods since it enables well established linear design methods to be applied to nonlinear problems. The classical gain-scheduling theory originates from the 1960s, and it has wide fields of applications, but the theoretical works were little up to 1990s. One reason for that is GS was considered as an "applied topic", and the hardware implementation of the GS techniques was classified as military secrets [1.3]. After that, a considerable increase interest in gain-scheduling in the literature with many new results obtained. Also a new GS approaches were created like Liner Fractional Transformation GS, Fuzzy GS, and others [2.10, and 2.11].

2.2 Classical Gain Scheduling

The classical GS based on measurements of operating conditions of the process is often a good way to compensate for variations in process parameters or known nonlinearities of the process [1.1].

2.2.1 The Concept of the Classical Gain Scheduling

The classical GS depends on finding auxiliary variables that correlate well with the changes in process dynamics. These variables called *scheduling variables*. It is then possible to reduce the effects of parameter variations simply by changing the parameters of the controller as functions of the scheduling variables. Gain scheduling can thus be viewed as a feedback control system in which the feedback gains are adjusted by using feed forward compensation (see Fig. 2.1). The concept of gain scheduling originated in connection with the development of flight control systems. In this application the Mach number and the dynamic pressure are measured by air data sensors and used as scheduling variables.

A main problem in the design of systems with gain scheduling is to find suitable scheduling variables. This is normally done on the basis of knowledge of the physics of a system. In process control the production rate can often be chosen as a scheduling variable, since time constants and time delays are often inversely proportional to production rate.

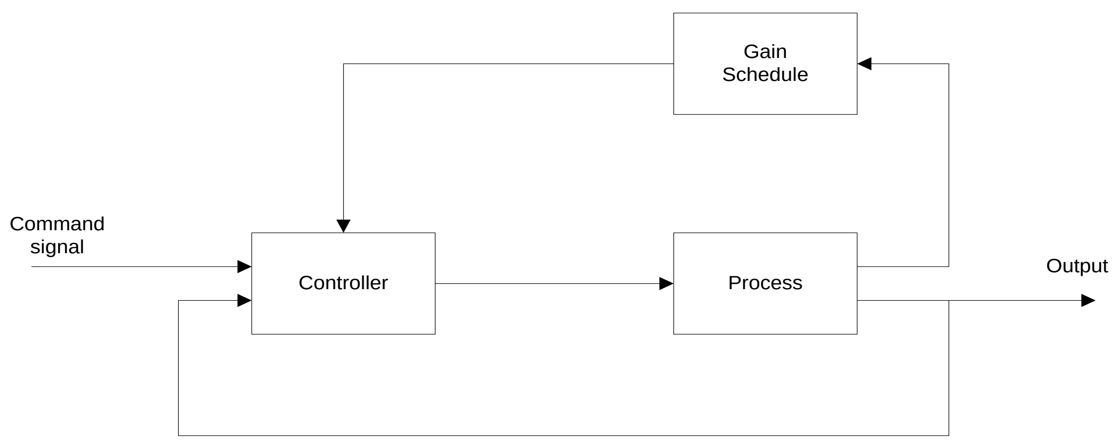


Fig. 2.1: Block diagram of a system in which influences of parameter variations are reduced by gain scheduling.

When scheduling variables have been determined, the controller parameters are calculated at a number of operating conditions by using some suitable design method. The controller is thus tuned or calibrated for each operating condition. The stability and performance of the system are typically evaluated by simulation. Particular attention is given to the

transition between different operating conditions. The number of entries in the scheduling tables is increased if necessary.

2.2.2 The Classical Gain Schedulers Design

It is difficult to give general rules for designing the classical gain-schedulers. The key question is to determine the variables that can be used as scheduling variables. It is clear that these auxiliary signals must reflect the operating conditions of the plant. Ideally, there should be simple expressions for how the controller parameters relate to the scheduling variables. It is thus necessary to have good insight into the dynamics of the process if gain scheduling is to be used.

Example 1: Gain Scheduler of A nonlinear Actuator

Consider the following nonlinear valve (F):

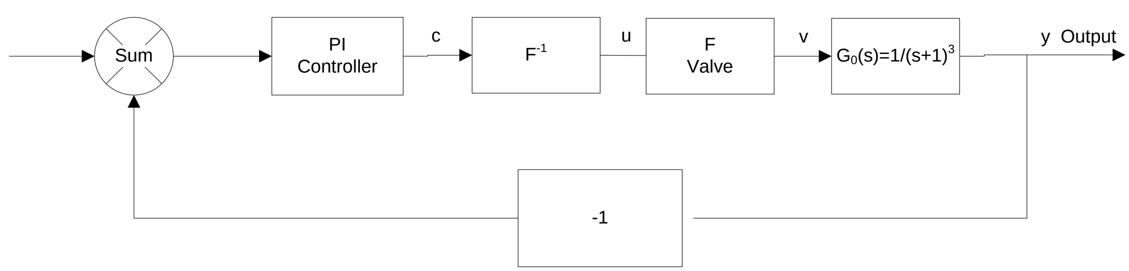


Fig. 2.2: The Nonlinear valve with approximated inverse compensation

The nonlinearity of the valve is assumed to be $v = f(u) = u^4 \quad ; \; u \geq 0$. To compensate for the nonlinearity, an approximation of the inverse of the valve characteristics ($\hat{f}^{-1}$) is used. A PI controller with (c) output is used. Then, the output of the controller is fed through this function before it is applied to the valve. This gives the relation

$$v = f(u) = f(\hat{f}^{-1}(c)) \tag{2.1}$$

The function $f(\hat{f}^{-1}(c))$ should have less variation in gain than f. If $\hat{f}^{-1}$ is the exact inverse, then v = c. Assuming that $f(u) = u^4$ is approximated by two lines: one connecting the points (0, 0) and (1.3, 3) and the other connecting (1.3, 3) and (2, 16) as shown in Fig. 2.3.

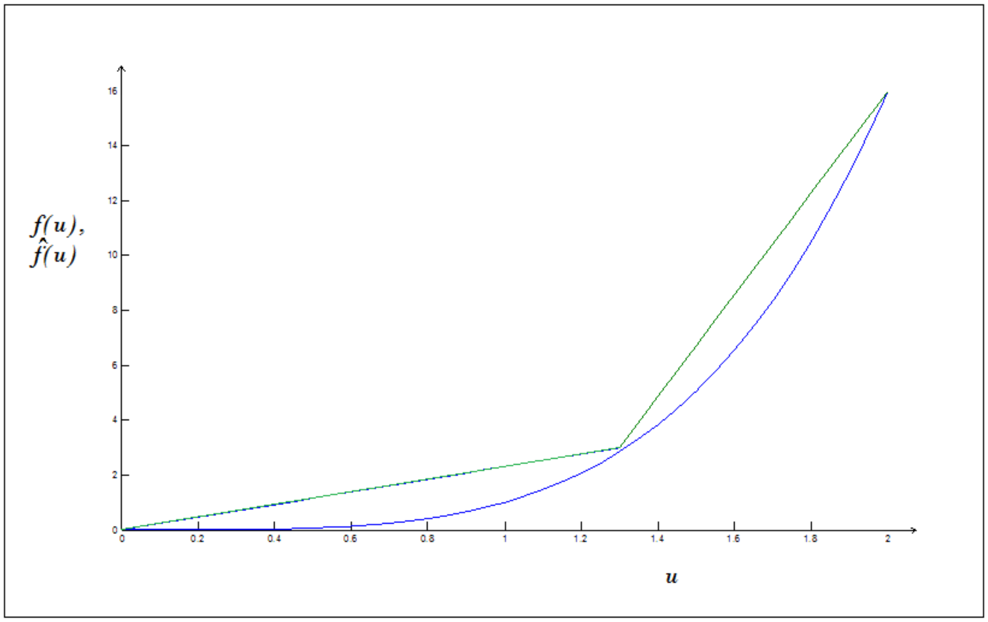


Fig. 2.3: Two lines approximation of $f(u) = u^4$

The closed-loop system was built using SIMULINK. The Simulink, and the m-file were used for this example are in Appendices 1, and 2. The result of the output appears in Fig. 2.4.

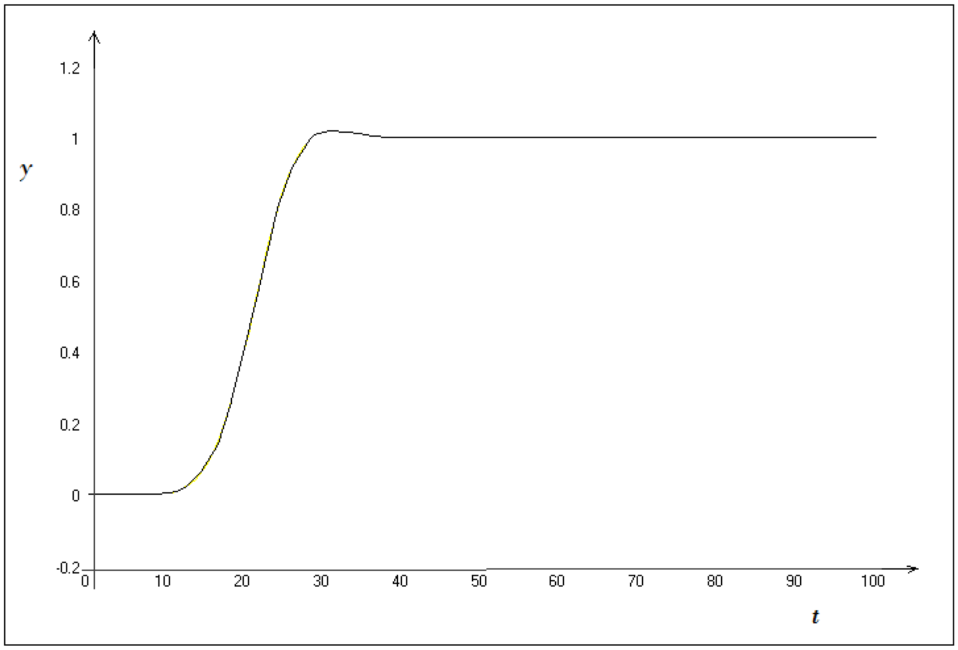


Fig. 2.4: The Output Result of Example 2.1

2.2.3 The Classical Gain Scheduling Drawbacks

One drawback of the Classical Gain Scheduling method is that the design may be time-consuming if it is not possible to use nonlinear transformations or auto-tuning. Another drawback is that the controller or the filter parameters are changed in open loop, without feedback from the performance of the closed-loop system. This makes the method impossible to use if the dynamics of the process or the disturbances are not known accurately enough.

2.3 Linear Fractional Transformation (LFT) Gain Scheduling

One of the main reasons to invent nonconventional gain scheduling technique is that these techniques exploit the behavior only in the vicinity of the equilibrium operating points (this generally imposes an inherent slow variation requirement on the system to ensure that the state remain close to equilibrium which is additional to any slow variation requirement associated with the change in linearised dynamics as the system moves from the vicinity of one equilibrium point to another). It is sometimes possible to obtain gain schedules by introducing nonlinear transformations in such a way that the transformed

system does not depend on the operating conditions (See Chapter 9 in [1.1]). Formulation of linear fractional transformation (LFT) models of systems involving nonlinear parameter variations in the state-space model is of interest for robust control system analysis and design.

2.3.1 Linear Fractional Construction

A Linear Fraction Representation (LFR) is a matrix-based way to describe a parameter varying functions. In this section, a linear fractional representation will be derived for a Linear Parameter Varying (LPV) system. The LPV system has the following structure

$$\begin{aligned} \dot{x}(t) &= A(h)x(t) + A_w(h)\omega(t) \\ y(t) &= B(h)x(t) + B_w(h)\omega(t) \\ z(t) &= L(h)x(t) + L_w(h)\omega(t) \end{aligned} \tag{2.2}$$

where

h=h(t): The time varying parameter

x:is the state

y:is the measured output

z:is the signal to be estimated

w:is the Noise signal

Considering all the system matrices are real-valued rational functions of h(t). Then, they will take the following LFT form:

$$\begin{bmatrix} A(h) & A_w(h) \\ B(h) & B_w(h) \\ L(h) & L_w(h) \end{bmatrix} = \begin{bmatrix} A & A_w \\ B & B_w \\ L & L_w \end{bmatrix} + \begin{bmatrix} A_\beta \\ B_\beta \\ L_\beta \end{bmatrix} \Omega(h)[I - D_\beta \Omega(h)]^{-1}[D \quad D_w] \tag{2.3}$$

where • $A, A_w, B, B_w, L, L_w, A_\beta, B_\beta, L_\beta, D, D_\beta$, and D_w are known constant matrices of appropiate dimensions

• $I - D_\beta \Omega(h)$: is non-singular for all h [Common Assumption for LFT]

• $\Omega(h) = \sum_{i=1}^{n} \upsilon_n(h)\Omega_n$; $s.t$ $\circ\ \upsilon_n(h) \geq 0$

$\circ \sum_{i=1}^{n} \upsilon_n(h) = 1$

With the notation of (2.3), the LPV system in (2.2) has the following equivalent description:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_w \omega(t) + A_\beta \beta(t) \\ y(t) &= Bx(t) + B_w \omega(t) + B_\beta \beta(t) \\ z(t) &= Lx(t) + L_w \omega(t) + L_\beta \beta(t) \\ \beta(t) &= \Omega(h)\alpha(t) \\ \alpha(t) &= Dx(t) + D_W \omega(t) + D_\beta \beta(t) \end{aligned} \tag{2.4}$$

where

$\alpha(t)$ and $\beta(t)$: are the non-linear parameters

The following section contain the theorem that describes how to build the LFT system (2.3) from the parameter system (2.2)

2.3.2 LFT Structure Theorem

For any rational matrix function $M(h) \in R^{n\times c}$, with no singularities, there exist nonnegative integers $r_1,\ldots, r_p$, and matrices $M(h) \in R^{n\times c}$, $L \in R^{n\times N}$, $K \in R^{N\times c}$, $D \in R^{N\times N}$, with $N = r_1+\ldots+r_p$, such that $M(h)$ has the following Linear Fraction Transformation (LFT) for all h :

$$M(h) = M + L\Omega(h)(I - D\Omega(h))^{-1} K \tag{2.5}$$

where

$$\Omega(h) = diag\ (h_1 I_{r1}, ..., h_L I_{rp})$$

$M, L, D, and\ K$: are known constant matrices of appropiate dimensions

Constructive Proof: It is based on a simple idea: first devise LFRs for simple (e.g. linear) functions, then use combination rules (such as multiplication, addition, etc.), to devise LFRs for arbitrary rational functions. To define the combination rules, we will start from two rational matrix-valued functions of $h \in \mathrm{R}^p$ that are described in the LFR format:

$$M_i(h) = M_i + L_i \Omega_i(h)(I - D_i \Omega_i(h))^{-1} K_i, \quad i = 1,2 \tag{2.6}$$

where

$$\Omega_i = \mathrm{diag}\ (h_1 I_{r^i 1}, ..., h_p I_{r^i p}),\ \mathrm{i} = 1,2.$$

In the sequel, we define $\tilde{\Omega} = \mathrm{diag}\ (\Omega_1, \Omega_2)$

LFT objects can be structured in one form of the following:

- *Addition.* The sum of $M_1(h)$ and $M_2(h)$ has the LFT

$$M(h) = M_1(h) + M_2(h) = M + L\tilde{\Omega}(h)(I - D\tilde{\Omega}(h))^{-1} K \qquad (2\ \ .)$$

with

$$\mathrm{M}(h) = \mathrm{M}_1 + \mathrm{M}_2, \quad \mathrm{L}=\begin{bmatrix} \mathrm{L}_1 & \mathrm{L}_2 \end{bmatrix}, \quad \mathrm{K}=\begin{bmatrix} \mathrm{K}_1 \\ \mathrm{K}_2 \end{bmatrix}, \quad \mathrm{D} = \mathrm{diag}\ (\mathrm{D}_1, \mathrm{D}_2)$$

Using row and column permutations, it is then possible to rewrite M(h) as in (2.4) (see the shuffling item below).

- *Multiplication.* The product of $M_1(h)$ and $M_2(h)$ is given by

$$\mathrm{M}(h) = \mathrm{M}_1(h)\mathrm{M}_2(h) = M + L\tilde{\Omega}(h)(I - D\tilde{\Omega}(h))^{-1}\ \mathrm{K} \tag{2.8}$$

where

$$\mathrm{M}=\mathrm{M}_1\mathrm{M}_2,\ \ \mathrm{L}=\begin{bmatrix}\mathrm{L}_1 & \mathrm{M}_1\mathrm{L}_2\end{bmatrix},\ \ \mathrm{K}=\begin{bmatrix}\mathrm{K}_1\mathrm{M}_2\\ \mathrm{K}_2\end{bmatrix},\ \ \mathrm{D}=\begin{bmatrix}\mathrm{D}_1 & \mathrm{K}_1\mathrm{L}_2\\ 0 & \mathrm{D}_2\end{bmatrix}$$

- *Stacking.* The combination of $M_1(h)$ and $M_2(h)$ is

$$\mathrm{M}(h)=[\mathrm{M}_1(h)\ \ \mathrm{M}_2(h)]=M+L\tilde{\Omega}(h)(I-D\tilde{\Omega}(h))^{-1}\ \mathrm{K} \tag{2.9}$$

with

$$\mathrm{M}=\begin{bmatrix}\mathrm{M}_1 & \mathrm{M}_2\end{bmatrix},\ \ \mathrm{L}=\begin{bmatrix}\mathrm{L}_1 & \mathrm{L}_2\end{bmatrix},\ \ \mathrm{K}=\mathrm{diag}\ (\mathrm{K}_1,\mathrm{K}_2),\ \ \mathrm{D}=\mathrm{diag}\ (\mathrm{D}_1,\mathrm{D}_2)$$

- *Shuffling.* Suppose we are given a matrix function

$$\mathrm{M}(h)=M_s+L_s\Omega_s(h)(I-D_s\Omega_s(h))^{-1}\ \mathrm{K}_s \tag{2.10}$$

with $\Omega_s(h)$ not necessarily in the required order (the variables may appear shuffled). Then, we can use row and column permutations to put the above representation back into the LFT format. Precisely, take a permutation matrix *E* such that for every *x*,

$$E^T\Omega_S(h)E=\Omega(h)=\mathrm{diag}\ (\mathrm{x}_1\mathrm{I}_{\mathrm{r}1},\ldots,x_nI_m)$$

In this case, M(h) has the LFT

$$\mathrm{M}(h)=M+L\Omega(h)(I-D\Omega(h))^{-1}\ \mathrm{K}$$

where

$$M=M_S,L=L_SE,K=E^TK_S\ \text{ and } \mathrm{D}=\mathrm{E}^{\mathrm{T}}\mathrm{D}_{\mathrm{S}}\mathrm{E}$$

- *Inversion.* If M is a square matrix with *M* (0) invertible, and has an LFT

$$\mathrm{M}(h)=M+L\Omega(h)(I-D\Delta(h))^{-1}\ \mathrm{K} \tag{2.11}$$

then its inverse can be written, for every h such that M(h) is invertible, as

$$\mathrm{M}(h)^{-1}=\mathrm{M}^{-1}\text{-}\mathrm{M}^{-1}L\Omega(h)(I-(D-KM^{-1}L)\Omega(h))^{-1}\ \mathrm{KM}^{-1}$$

2.4 Other Gain Scheduling Approaches: Neural/fuzzy GS

As it was described in section (2.3), one of the main reasons to invent a nonconventional gain scheduling techniques is that these techniques exploit the behavior only in the vicinity of the equilibrium operating points. However, in order to meet increasingly

stringent performance objectives, gain-scheduled techniques are frequently required to operate both during transitions between equilibrium operating points (which might transiently take the system far from equilibrium) and during sustained operation far from equilibrium. A number of approaches developed in the fuzzy-logic and neural network literatures attempt to relax restrictions to near equilibrium operation while remaining closely related to the classical gain-scheduling design philosophy. These design approaches typically involve the following steps:

1. The plant dynamics are formulated as a blended multiple model representation such as a Takagi-Sugeno model or local model network; that is, in the continuous time case, a system of the form

$$\dot{x} = \sum_i \bar{F}_i(x, r)\mu_i(\rho),\ y = \sum_i \bar{G}_i(x, r)\mu_i(\rho) \tag{2.12}$$

   Systems of this form are considered by, for example, Dong Ming Ge (2008) [2.6], Gawthrop (1995) [2.7] and Shorten et al. (1999) [2.8] and are closely related to the systems considered by Johansen & Foss (1993) [2.9], Johansen (1994) [2.10], and Kiriakidis (1999) [2.11]. The local models associated with the blended multiple model system,

$$\dot{x}_i = \sum_i \bar{F}_i(x, r),\ y = \sum_i \bar{G}_i(x, r) \tag{2.13}$$

   are blended via the scalar weighting (or validity) functions μ_i. The latter are typically normalized such that $\sum_i \mu_i(\rho) = 1$ with the quantity $\rho(x,r)$ embodying the dependence of the blending on the state and input.
2. The blended representation, (2.12), immediately suggests a natural divide and conquer type of design approach whereby a local controller/ filter is designed for each local model, (2.13).
3. The local controller /filter designs are then blended, using the weighting functions μ_i , to obtain a nonlinear controller with similar form to the plant, (2.12).

CHAPTER 3

Filtering of Linear Parameter Varying (LPV) Systems

3.1 Linear Parameter Varying (LPV) Systems

In 1988, Shamma started his work on linear parameter-varying (LPV) systems in the context of gain-scheduling. In 1991, Shamma again with Athans did another work on LPV systems. In these works, they consider the following structure of LPV systems:

$$\begin{aligned} \dot{x} &= A(h(t))x + B(h(t))r \\ y &= C(h(t))x + D(h(t))r \end{aligned} \qquad (3.1)$$

where the parameter, h(t), is an exogenous time-varying quantity (strictly independent of the state x of the system) which takes values in some allowable set. Under these conditions, an LPV system is simply a particular form of linear time-varying system. However, whilst a number of subsequent gain-scheduling approaches have taken LPV systems as their starting point, it is a priori far from clear that this class of systems is sufficiently rich to include a reasonable range of practical gain-scheduling applications. Linear time-varying representations of nonlinear systems are largely associated with series expansion linearization theory. Consider the following nonlinear system

$$\dot{x} = F(x,r),\ y = G(x,r) \qquad (3.2)$$

where

$$r \in \Re^m,\ y \in \Re^p, x \in \Re^n$$

It follows from series expansion linearization theory that the nonlinear system may be approximated by the linear time-varying system. However, for this approximation to be accurate there is a requirement that $|\delta x|$ and $|\delta r|$ be sufficiently small; that is, the series expansion linearization is only valid within a small neighborhood about the specific equilibrium operating point or trajectory $(\tilde{x}(t), \tilde{r}(t)\, \tilde{y}(t))$. In order to increase the size of the operating region within which a series expansion linearization-based LPV representation is valid, it is perhaps attractive to consider combining, in some sense, the series expansion linearization’s associated with a number of equilibrium points/trajectories. However, it is important to make a clear distinction between the LPV system, (3.1), and the family of linear systems associated with the equilibrium points/trajectories of a nonlinear system, (3.2). Clearly, the linearization family is a collection of dynamic systems whilst the LPV system is a single dynamic system. The state, input and output of a series expansion linearization are perturbation quantities which depend on the equilibrium point/trajectory considered. Hence, the members of the linearization family each have different state, input and output in general and cannot be directly combined to obtain an LPV system. Conventional series expansion theory does not support the reformulation of general nonlinear systems in LPV form without strong restrictions on the operating region. However, following a similar approach to Helmersson (1995b chapter 10), consider the nonlinear system

$$\begin{aligned} \dot{x} &= A(x,r)x + B(x,r)r \\ y &= C(x,r)x + D(x,r)r \end{aligned} \qquad (3.3)$$

where $r \in \Re^m$, $y \in \Re^p, x \in \Re^n$, and the input, and initial conditions of the state are restricted such that the solution x(t) is confined to some operating region $X \subset \Re^n$. It is immediately evident that the solutions to the nonlinear system, (3.3), are a subset of the solutions to the LPV system, (3.1), with $h \in X$. (Since the parameter (h) can vary arbitrarily in (3.1), the solutions to (3.3) are just the solutions to (3.1) associated with particular parameter trajectories). Hence, whilst it is generally not possible to reformulate a nonlinear system as an LPV system, it is possible to over-bound the nonlinear system, (3.3), by an LPV system in the sense that every solution to the nonlinear system is a

solution to the LPV system (but not vice versa). Of course some degree of conservativeness can be expected with such an approach. In addition, it is emphasized that support for arbitrary parameter variations in the LPV system is an essential feature of the over-bounding; for example, it is not possible to arbitrarily restrict the rate of variation of the parameter, $(\dot{h})$. Generalizing the definition of Shamma (1988), nonlinear systems, (3.3), are referred to here as quasi-LPV systems. Of course, it still remains to be established whether this class is sufficiently rich to include a reasonable range of gain-scheduling applications. In summary, these theoretical results do not support the representation of nonlinear dynamic systems in quasi-LPV form without, in general, considerable restrictions on the class of nonlinear systems considered or the allowable operating region. However, the restrictions on the operating region essentially arise from the limitations of series expansion linearization theory.

3.1.1 Practical Example of an LPV System: The Rotating Stall and Surge Problem

The rotating stall and surge are two instabilities that occur in jet engine compressors (Fig. 3.1) [3.16]. Rotating stall develops when there is a region of stagnant flow rotating around the circumference of the compressor causing undesired vibrations in the blades and reduced pressure rise of the compressor. The average flow is steady in time yet with a circumferentially nonuniform mass deficit as shown in Fig. 3.2. Surge is an axisymmetric oscillation of the flow through the compressor that can cause undesired vibrations in other components of the compression system and damage to the engine. The flow will be unsteady but circumferentially uniform, as depicted in Fig. 3.3.

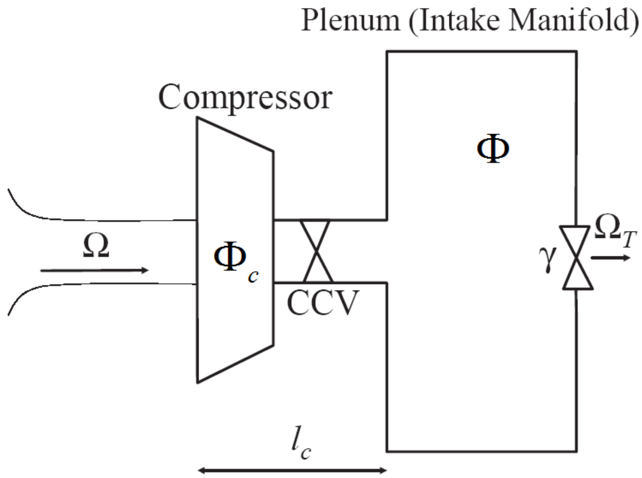


Fig. 3.1: Compression System

These phenomena represent critical operating conditions that limit the stability region at low mass flow in a compressor map. Physically, these aerodynamics flow instabilities may cause a rapid heating of the blades and increase the exit temperature of the compressor. Also, additional periodic loads, blade vibrations and fatigue are responsible for material durability reduction such as severe damages to the machine. In most cases this may lead to limit cycles in the compressor map. The essential differences between rotating stall and surge are that the average flow in pure rotating stall is steady in time, but the flow has circumferentially non-uniform mass deficit, while in pure surge the flow is unsteady but circumferentially uniform.

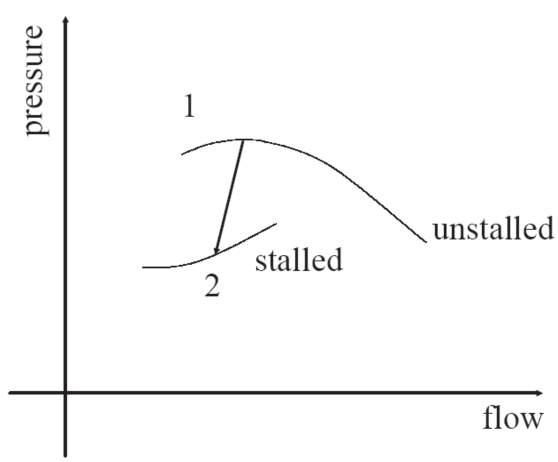


Fig. 3.2: Compressor map with stalled flow characteristic.

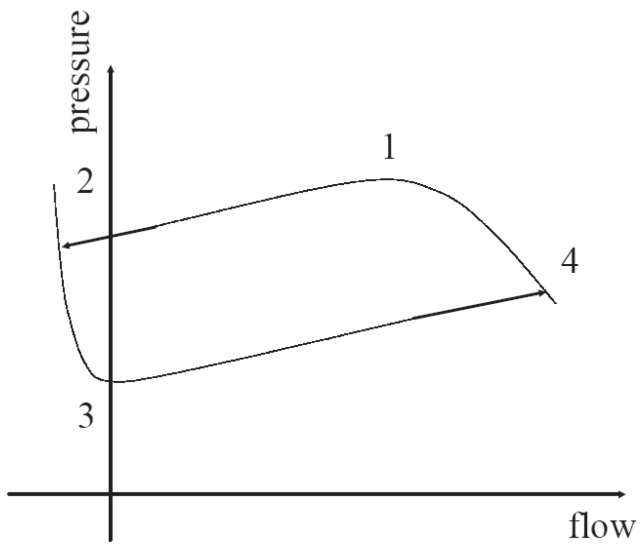


Fig. 3.3: Compressor map with deep Surge cycle.

Let us consider a simplified, lumped compression system model that describes rotating stall and surge instabilities, based on the Moore and Greitzer model [3.16]

$$
\begin{aligned}
\dot{\Omega} &= \frac{1}{l_c}[-\Phi+\Phi_c(\Omega)-3R(\Omega-1)] \\
\dot{\Phi} &= \frac{1}{4l_c B^2}(\Omega-\Omega_T) \qquad (3.4) \\
\dot{R} &= 3\mu R(2\Omega-\Omega^2-R)
\end{aligned}
$$

where Ω is the compressor mass flow, Φ is the pressure rise of the compressor and the R is the amplitude square of the first harmonic mode of the rotating stall disturbance, Ω_T is the input, the throttle mass flow. Hereafter, we set $l_c = 1$, $\mu = 0.6$, $B = 0.3$. A gain scheduler is needed to achieve the maximum compression efficiency while eliminating stall, assuming the state variables are available for feedback. In order to design this gain scheduler, we need to model the compression system dynamics into an LPV form. According to [3.1], we choose $\omega(\mathrm{k}) = \Omega(\mathrm{k}) - 2$, $\omega_{\mathrm{T}}(\mathrm{k}) = \Omega_{\mathrm{T}}(\mathrm{k}) - 2$, $\xi(k) = \Phi(k) - \Phi_e q(k)$ and the variable $p(\mathrm{k}) = \omega(\mathrm{k})$ as the scheduling variable. Then, we get the following LPV model, using the relaxation auxiliary variable $v(k)$:

$$x(k+1) = A(p(k))x(k) + B_1 u(k)$$
$$z(k) = C_1(p(k))x(k) \qquad (3 \;,$$

where the state variable $x(k)$, the input $u(k)$, and the dynamic matrices $A(p(k),\rho(k)), B_1, and\ C_1(p(k))$ are given by :

$$x(k) = [v(k)\ \ \xi(p(k)) - \xi_{eq}(p(k))\ \ R(k)]' \qquad (3 \;.$$

$$u(k) = \omega_{\mathrm{T}}(\mathrm{k}) \text{-} \omega_{\mathrm{T_{eq}}}(\mathrm{k}) \qquad (3.7)$$

$$A(p(k),\rho(k)) = \begin{bmatrix} 1 & -\frac{T}{l_c} & -\frac{3T}{l_c}(p+1) \\ 0 & 1-\frac{T}{l_c}(3p+\frac{3}{2}p^2) & -\frac{3T}{l_c}(3p+\frac{3}{2}p^2)(p+1) \\ 0 & 0 & 1-3\mu T(2p+p^2)-0.9\mu T\rho \end{bmatrix} \qquad (3.8)$$

$$B_1 = \begin{bmatrix} 0 \\ -\frac{T}{4l_c B^2} \\ 0 \end{bmatrix} \qquad (3.9)$$

$$C_1(p(k)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{T}{l_c} & -\frac{3T}{l_c}(p+1) \\ 0 & 0 & 1 \end{bmatrix} \qquad (3.10)$$

where $\rho = \rho(k)$ is a disturbance taking into account some nonlinear terms, and $p = p(\mathrm{k})$ is the scheduling parameter, and T is the sampling time.

3.1.2 Linear Parameter-Varying Time Delay (LPVTD) Systems

Time delays are often present in engineering systems due to measurement, transmission and transport lags, computational delays, or unmodeled inertias of system components. In

many engineering systems, the time delays are known functions of variable operating conditions or system parameters that can be measured in real-time. For example, the transport delay in an internal combustion engine is a known function of the engine speed. Similarly, linear parameter varying time delays (LPVTD) systems often appear in many manufacturing and chemical processes, biomedical systems and robotic systems where changes in the system dynamics result in variable time delay. The LPV analysis and control synthesis problems can be formulated as linear matrix inequality (LMI) constraints that can be solved using recently developed efficient interior-point optimization algorithms [3.2] and [3.3].

3.1.2.1 LPVTD Types

The LPV system can be state delayed, input delayed, or state and input delayed. The delay also may be constant or varying with time. For example, Mahmoud, 2004, consider the following LPVTD system of state delay type:

$$
\begin{aligned}
\dot{x}(t) &= A(h(t))x(t) + E(h(t))x(t-\tau(t)) + B(h(t))w(t) + G(h(t))u(t) \\
z(t) &= C(h(t))x(t) + D(h(t))w(t) + F(h(t))u(t) \\
y(t) &= L(h(t))x(t) + R(h(t))w(t)
\end{aligned}
\qquad (3.11)
$$

where τ is an unknown and bounded time-varying delay such that:

$$0 \le \tau(t) \le \tau^{+}, \;\; 1 \le \dot{\tau}(t) \le \tau^{*}$$

where the bounds τ^{+}, τ^{*} are known. Junling Wang, 2008, studied LPVTD with state and input delays. He considered the following structure:

$$
\begin{aligned}
\dot{x}(t) = {} & A(h(t))x(t) + A_h(h(t))x(t-\tau_1(t)) + B(h(t))u(t) \\
& + B_h(h(t))u(t-\tau_2(t))
\end{aligned}
\qquad (3.12)
$$

In this thesis, the system that will be considered is Linear Parameter Varying with state time varying delay as in (3.13).

$$
\begin{aligned}
\dot{x}(t) &= A(h(t))x(t) + A_d(h(t))x(t-\tau(t)) + A_w(h(t))w(t) \\
z(t) &= L(h(t))x(t) + L_d(h(t))x(t-\tau(t)) + L_w(h(t))w(t) \\
y(t) &= Bx(t) + B_d x(t-\tau(t)) + B_w w(t)
\end{aligned}
\qquad (3.13)
$$

where

h is the time varying parameter
x:is the state
y:is the measured output
z:is the signal to be estimated
w:is the Noise signal
d(t)= The time varing delay, which stisfies

$$0 \le d(t) \le \tau$$
$$\dot{d}(t) \le \mu < 1$$

3.2 Filtering of Linear Parameter Varying (LPV) Systems

Filters, estimators, or observers are names for the same tool that estimates the signals. Sometimes, there is no sensor can measure specific signal or the sensor is too much expensive. So, the best way for this type of problems is to build a filter that estimates this signal from other available signals. The most used technique in this field is H_∞ filtering. The purpose of H_∞ filtering problem is to design an estimator so as to guarantee that the resulting filtering error system is stable and the L2- induced gain from the noise to the estimation error is less than a prescribed level.

Very recently, the gain-scheduled H_∞ filtering problem was studied in [3.4] and in [3.5], [3.6] for continuous-time and discrete-time LPV systems, respectively. However, it's noted that most of the results that were derived in this type of problems are based on Lyapunov functions. To evaluate the published results on filtering of LPV systems, the following tables contain some useful information.

Paper	Model	Nature	Delay pattern
Bokor, 2000 [9]	Continuous	Non-delayed	-
Mahmoud, 2001 [10]	Continuous	Non-delayed	-
Barbosa, 2005 [11]	Continuous	Non-delayed	-
Daafouz, 2005 [12]	Discrete	Non-delayed	-
Velni, 2005 [13]	Continuous	Delayed	Constant
Sato, 2006 [14]	Continuous	Non-delayed	-
Wang, 2007 [15]	Discrete	Delayed	Time-varying
Zhou, 2007 [16]	Discrete	Non-delayed	-
Velni, 2008 [17]	Continuous	Delayed	Parametric delay

Table 3.1: Comparison-1 between the published papers of LPV Systems Filtering

Paper	Filtering Technique	Other Features
Bokor, 2000 [9]	LPV Filtering	Eigen-structure issue
Mahmoud, 2001 [10]	LPV H∞ Filtering	LMI Based
Barbosa, 2005 [11]	H2 Filtering	LMI Based
Daafouz, 2005 [12]	Polytopic Observering	Convex Optimization
Velni, 2005 [13]	LPVTD H∞ Filtering	LMI Based
Sato, 2006 [14]	LPV H2, and H∞ Filtering	LMI Based
Wang, 2007 [15]	LPV H∞ Filtering	PLMI Based
Zhou, 2007 [16]	LPV H∞ Filtering	LMI Based
Velni, 2008 [17]	Mixed H2/ H∞ Filtering	LMI Based with rate dependent varying parameters

Table 3.2: Comparison-2 between the published papers of LPV Systems Filtering

3.2.1 The H_∞ Filtering

The H_∞ filtering technique has been widely studied for the benefit of different time and frequency domain properties to the H_2 filtering technique. In the H_∞ setting, the exogenous input signal is assumed to be energy bounded rather than Gaussian. An H_∞ filter is designed such that the H_∞ norm of the system, which reflects the worst-case "gain" of the system, is minimized. The advantage of using an H_∞ filter in comparison with an H_2 filter is twofold. First, no statistical assumption on the input is needed. Second, the filter tends to be more robust when there exists additional uncertainty in the system, such as quantization errors, delays, and unmodeled dynamics [3.7].

There are two approaches to H_∞ filtering [3.8]:

1) Time Domain approach (see, e.g., [3.9], [3.10]);

2) Frequency Domain approach: It contains the following:

- Polynomial equation approach (see, e.g., [3.11], [3.12]);
- Interpolation approach (see, e.g., [3.13]).

The Frequency approach: polynomial and interpolation use transfer functions directly. They seem to be most suitable when specific frequency domain information, such as zeros, poles, bandwidth, etc., is available. In addition, frequency weighting on filtering errors and noise signals can be easily performed without worrying about the dimension increase of the weighted system, which may add the computational complexity. The main problem with the frequency approaches is that the formula is quite complicated, especially in the multivariable case. On other hand, the Time domain approach is a state-space approach. It is more popular due to the fact that solutions are expressed in simple formula as in Algebraic Riccati Equation (ARE) case, and that efficient numerical algorithms exist for solving these kinds of equations.

3.2.2 The LPVTD Filter

The filter that will be used to estimate the signal in the Linear Parameter Varying Time Delay (LPVTD) system (3.13) will be also Linear Parameter Varying Time Delay (LPVTD) filter with following structure:

$$\begin{aligned} \dot{x}_f(t) &= A_f(h(t))x_f(t) + A_{fd}(h(t))x_f(t-\tau(t)) + A_{fy}(h(t))y(t) \\ \hat{z}(t) &= L_f(h(t))x_f(t) + L_{fd}(h(t))x_f(t-\tau(t)) + L_{fy}(h(t))y(t) \end{aligned} \quad (3\ .\ 1$$

As shown in 3.14, The LPVTD Filter is in state space representation, which mean the approach that is used is time domain H_∞ filtering one. A filtering error system will be built from the systems (3.13) and (3.14). The stability of this system will be studied and guaranteed depending on Lyapunov function and using the Linear Matrix Inequality (LMI) tools.

CHAPTER 4

STRUCTURE AND STABILITY OF THE ERORR FILTERING SYSTEM

4.1 The LPV System and Filter Structures

The class of LPV systems, that will be studied here, has the following features:

- *It can be turned to Linear Fraction Transformation (LFT) system.*
- *It has the varying parameter h(t).*
- *It contains time varying delay in the state.*
- *It considers noise input w(t).*

4.1.1 The Time Varying Delayed LPV System Structure

The structure of the LPV system with the previous features is:

$$\begin{bmatrix} \dot{x}(t) \\ y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} A(h) & A_d(h) & A_w(h) \\ B & B_d & B_w \\ L(h) & L_d(h) & L_w(h) \end{bmatrix} \begin{bmatrix} x(t) \\ x_d(t) \\ w(t) \end{bmatrix} \tag{4.1}$$
$$x(t) = \varphi(t), t \in [-\tau, 0\ ]$$

where

h: is the time varying parameter
x:is the state
y:is the measured output
z:is the signal to be estimated
w:is the Noise signal
$x_d(t) = x(t - d(t))$
d(t)= The time varying delay, which stisfies

$$\begin{aligned} &0 \le d(t) \le \tau \\ &\dot{d}(t) \le \mu < 1 \end{aligned} \tag{4.2}$$

4.1.2 The LFT of the LPV System

The Liner Fraction Transformation (LFT) will be applied to the LPV system:

$$\begin{bmatrix} A(h) & A_d(h) & A_w(h) \\ L(h) & L_d(h) & L_w(h) \end{bmatrix} = \begin{bmatrix} A & A_d & A_w \\ L & L_d & L_w \end{bmatrix} + \begin{bmatrix} A_\beta \\ L_\beta \end{bmatrix} \Omega(h)[I - D_\beta \Omega(h)]^{-1}[D \quad D_d \quad D_w] \tag{4.3}$$

where

- $A, A_d, A_w, A_\beta, L, L_d, L_w, L_\beta, D, D_d, D_w$, and D_β are known constant matrices of appropiate dimensions
- $I - D_\beta \Omega(h)$:is non-singular for all h [Common Assumption for LFT]
- $\Omega(h) = \sum_{i=1}^{n} \upsilon_n(h)\Omega_n$; s. to ∘ $\upsilon_n(h) \ge 0$
  - ∘ $\sum_{i=1}^{n} \upsilon_n(h) = 1$
  - ∘ n :is the size of the gain schedule

Using the LFT in (4.3), an equivalent description of system (4.1) is given by:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x_d(t) + A_w \omega(t) + A_\beta \beta(t) \\ y(t) &= Bx(t) + B_d x_d(t) + B_w \omega(t) \\ z(t) &= Lx(t) + L_d x_d(t) + L_w \omega(t) + L_\beta \beta(t) \\ \beta(t) &= \Omega(h)\alpha(t) \\ \alpha(t) &= Dx(t) + D_d x_d(t) + D_W \omega(t) + D_\beta \beta(t) \end{aligned} \tag{4.4}$$

where

$\alpha(t)$ and $\beta(t)$:are non-linear parameters

4.1.3 The Time Varying Delayed LPV Filter Structure

For the purpose of estimating the signal (z), a delayed LPV Filter with the following general structure is used:

$$\begin{bmatrix} \dot{x}_f(t) \\ \hat{z}(t) \end{bmatrix} = \begin{bmatrix} A_f(h) & A_{fd}(h) & A_{fy}(h) \\ L_f(h) & L_{fd}(h) & L_{fy}(h) \end{bmatrix} \begin{bmatrix} x_f(t) \\ x_{fd}(t) \\ y(t) \end{bmatrix} \tag{4.5}$$

$wh\ r\!e\ \ x_{fd}(t) = x_f(t - d(t))$

4.1.4 The LFT of the LPV Filter

The Liner Fraction Transformation (LFT) will be applied on the LPV filter as follow:

$$\begin{bmatrix} A_f(h) & A_{fd}(h) & A_{fy}(h) \\ L_f(h) & L_{fd}(h) & L_{fy}(h) \end{bmatrix} = \begin{bmatrix} A_f & A_{fd} & A_{fy} \\ L_f & L_{fd} & L_{fy} \end{bmatrix} + \begin{bmatrix} A_{f\beta} \\ L_{f\beta} \end{bmatrix} [I - D_{f\beta}]^{-1} \begin{bmatrix} D_f & D_{fd} & D_{fy} \end{bmatrix} \tag{4.6}$$

where

$$A_f\ , A_{fd}\ , A_{fy}\ , A_{f\beta}, L_f\ , L_{fd}\ , L_{fy}\ , L_{f\beta}, D_f\ , D_{fd}\ , D_{fy}\ , \text{and}\ D_{f\beta}\ \text{are constant}$$
$$\text{matrices of appropiate dimensions to be determined}$$

Using the LFT in (4.6), an equivalent description of the filter in (4.5) is given by:

$$\begin{aligned} \dot{x}_f(t) &= A_f x_f(t) + A_{fd} x_{fd}(t) + A_{fy} y(t) + A_{f\beta} \beta_f(t) \\ \hat{z}(t) &= L_f x_f(t) + L_{fd} x_{fd}(t) + L_{fy} y(t) + L_{f\beta} \beta_f(t) \\ \beta_f(t) &= \alpha_f(t) \\ \alpha_f(t) &= D_f\ x_f(t) + D_{fd} x_{fd}(t) + D_{fy} y(t) + D_{f\beta} \beta_f(t) \end{aligned} \tag{4.7}$$

Substitute for y (t) from (4.4). Then (4.7) will be:

$$
\begin{aligned}
\dot{x}_f(t) &= A_f x_f(t) + A_{fd} x_{fd}(t) + A_{fy}[Bx(t) + B_d x_d(t) + B_w \omega(t)] + A_{f\beta}\beta_f(t) \\
\hat{z}(t) &= L_f x_f(t) + L_{fd} x_{fd}(t) + L_{fy}[Bx(t) + B_d x_d(t) + B_w \omega(t)] + L_{f\beta}\beta_f(t) \\
\beta_f(t) &= \alpha_f(t) \\
\alpha_f(t) &= D_f x_f(t) + D_{fd} x_{fd}(t) + D_{fy}[Bx(t) + B_d x_d(t) + B_w \omega(t)] + D_{f\beta}\beta_f(t)
\end{aligned}
\tag{4.8}
$$

4.1.5 The Filtering error system

The Filtering error dynamics is the augmented system of systems (4.4) and (4.8). The following layout shows the structure of the augmented system:

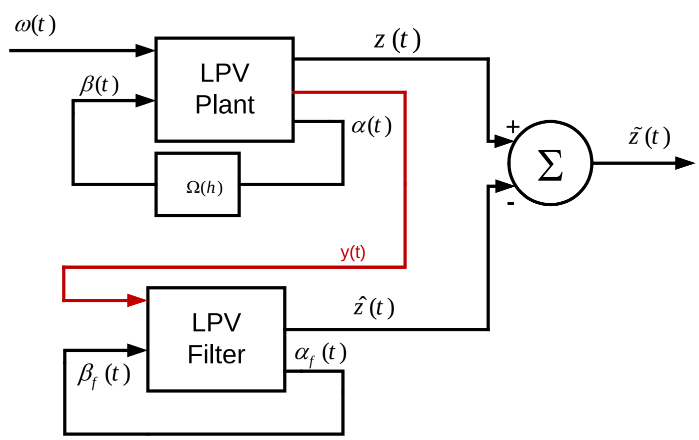


Fig. 4.1: The Augmented System

It is appear, in the previous figure, the relationship between the LPV plant, and the filter, where the output signal, in the red line, is the link between the plant and the filter. The mathematical structure of the augmented system can be described by:

$$
\begin{aligned}
\dot{X}_e(t) &= A_e X_e(t) + A_{ed} X_{ed}(t) + A_{ew}\omega(t) + A_{e\beta}\beta_e(t) \\
\tilde{z}(t) &= L_e X_e(t) + L_{ed} X_{ed}(t) + L_{ew}\omega(t) + L_{e\beta}\beta_e(t) \\
\beta_e(t) &= \Omega_e(h)\alpha_e(t) \\
\alpha_e(t) &= D_e X_e(t) + D_{ed} X_{ed}(t) + D_{ew}\omega(t) + D_{e\beta}\beta_e(t)
\end{aligned}
\tag{4.9}
$$

where

$$X_e(t) = [x^T(t) \quad x_f^T(t)]^T$$
$$X_{ed} = X_e(t-d(t)) = [x_d^T(t)\ x_{fd}^T(t)]^T$$
$$\beta_e(t) = [\beta^T \quad \beta_f^T]^T$$
$$\alpha_e(t) = [\alpha^T \quad \alpha_f^T]^T$$
$$\tilde{z}(t) = z(t) - \hat{z}(t)$$

$$A_e = \begin{bmatrix} A & 0 \\ A_{fy}B & A_f \end{bmatrix};\ A_{ed} = \begin{bmatrix} A_d & 0 \\ A_{fy}B_d & A_{fd} \end{bmatrix};\ A_{ew} = \begin{bmatrix} A_w \\ A_{fy}B_w \end{bmatrix}; A_{e\beta} = \begin{bmatrix} A_\beta & 0 \\ 0 & A_{f\beta} \end{bmatrix}$$
$$L_e = \begin{bmatrix} L - L_{fy}B & -L_f \end{bmatrix};\ L_{ed} = \begin{bmatrix} L_d - L_{fy}B_d & -L_{fd} \end{bmatrix};\ L_{ew} = L_w - L_{fy}B_w;\ L_{e\beta} = \begin{bmatrix} L_\beta & -L_{f\beta} \end{bmatrix}$$
$$D_e = \begin{bmatrix} D & 0 \\ D_{fy}B & D_f \end{bmatrix};\ D_{ed} = \begin{bmatrix} D_d & 0 \\ D_{fy}B_d & D_{fd} \end{bmatrix};\ D_{ew} = \begin{bmatrix} D_w \\ D_{fy}B_w \end{bmatrix}; D_{e\beta} = \begin{bmatrix} D_\beta & 0 \\ 0 & D_{f\beta} \end{bmatrix}$$
$$\Omega_e(h) = \begin{bmatrix} \Omega(h) & 0 \\ 0 & I \end{bmatrix}$$

4.2 The Stability and H∞ Performance Analysis

This section gives a characterization for the filtering error system with H∞ norm bound γ.

4.2.1 The Lyapunov System

Define the following lyapunov function:

$$V(x_t) = X_e^T(t)PX_e(t) + \int_{t-d(t)}^{t} X_e^T(s)QX_e(s)ds + \int_{\tau}^{0}\int_{t+\theta}^{t} \dot{X}_e^T(s)Z\dot{X}_e(s)ds\,d\theta \qquad (4.10)$$

where P, Q, and Z are positive weighting matrices.

Take the derivative of (4.10), we have:

$$\dot{V}(x_t) = X_e^T(t)P\dot{X}_e(t) + \dot{X}_e^T(t)PX_e(t) + X_e^T(t)QX_e(t) - (1-\dot{d}(t))X_{ed}^T QX_{ed}$$
$$+\tau\dot{X}_e^T(t)Z\dot{X}_e(t) - \int_{t-\tau}^{t} \dot{X}_e^T(s)Z\dot{X}_e(s)ds \qquad (4.11)$$

Apply the delay conditions (4.2) in (4.11), then the result as follows:

$$\dot{V}(x_t) \leq X_e^T(t)P\dot{X}_e(t) + \dot{X}_e^T(t)P\,X_e(t) + X_e^T(t)QX_e(t) - (1-\mu)X_{ed}^T QX_{ed}$$
$$+\tau\dot{X}_e^T(t)Z\dot{X}_e(t) - \int_{t-d(t)}^{t} \dot{X}_e^T(s)Z\dot{X}_e(s)ds \qquad (4.12)$$

From (4.9), $\dot{X}_e(t)$ has the following expression

$$\dot{X}_e(t) = A_e X_e(t) + A_{ed} X_{ed} + A_{e\beta}\Omega_e(h)\alpha_e(t) + A_{ew}\,\omega(t) \qquad (4.13)$$

Now, substitute (4.13) into the 1st and 2nd terms of the inequality (4.12)

Then, (4.12) will be:

$$\dot{V}(x_t) \leq \begin{bmatrix} X_e(t) \\ X_{ed}(t) \\ \dot{X}_e(t) \\ \alpha_e(t) \\ \omega(t) \end{bmatrix}^T \begin{bmatrix} Q + PA_e + A_e^T P & PA_{ed} & 0 & PA_{e\beta}\Omega_e(h) & PA_{ew} \\ * & -(1-\mu)Q & 0 & 0 & 0 \\ * & * & \tau Z & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix} \begin{bmatrix} X_e(t) \\ X_{ed}(t) \\ \dot{X}_e(t) \\ \alpha_e(t) \\ \omega(t) \end{bmatrix}$$
$$- \int_{t-d(t)}^{t} \dot{X}_e^T(s)Z\dot{X}_e(s)ds \qquad (4.14)$$

Now, consider the following semi-positive definite matrix (Xs):

$$\mathrm{X}_s = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix} \geq 0 \qquad (4.15)$$
$$\text{s.t } X_{21} = X_{12}^T,\; X_{31} = X_{13}^T,\; X_{32} = X_{23}^T$$

For any matrix like Xs, the following holds [See ref. 3.15]:

$$\tau\psi^T(\mathrm{t})\mathrm{X}_S\,\psi(\mathrm{t}) - \int_{t-d(t)}^{t} \psi^T(\mathrm{t})\mathrm{X}_S\,\psi(\mathrm{t})ds \geq 0 \qquad (4.16)$$

where

$$\psi(\mathrm{t}) = \begin{bmatrix} X_e^T(t) & X_{ed}^T & \dot{X}_e^T(t) \end{bmatrix}^T$$

In matrix form

$$\tau \begin{bmatrix} X_e(t) \\ X_{ed} \\ \dot{X}_e(t) \end{bmatrix}^T \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33} \end{bmatrix} \begin{bmatrix} X_e(t) \\ X_{ed} \\ \dot{X}_e(t) \end{bmatrix} - \int_{t-d(t)}^{t} \begin{bmatrix} X_e(t) \\ X_{ed} \\ \dot{X}_e(t) \end{bmatrix}^T \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33} \end{bmatrix} \begin{bmatrix} X_e(t) \\ X_{ed} \\ \dot{X}_e(t) \end{bmatrix} d \geq 0s \qquad (4.17)$$

Now, by adding the left side of (4.17) to the right side of (4.14), we have:

$$\dot{V}(x_t) \leq \begin{bmatrix} X_e(t) \\ X_{ed} \\ \dot{X}_e(t) \\ \alpha_e(t) \\ \omega(t) \end{bmatrix}^T \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} \\ * & \phi_{22} & \phi_{23} & 0 & 0 \\ * & * & \phi_{33} & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix} \begin{bmatrix} X_e(t) \\ X_{ed} \\ \dot{X}_e(t) \\ \alpha_e(t) \\ \omega(t) \end{bmatrix}$$

$$- \int_{t-d(t)}^{t} \begin{bmatrix} X_e(t) \\ X_{ed} \\ \dot{X}_e(t) \\ \dot{X}_e(s) \end{bmatrix}^T \begin{bmatrix} X_{11} & X_{12} & X_{13} & 0 \\ X_{12}^T & X_{22} & X_{23} & 0 \\ X_{13}^T & X_{23}^T & X_{33} & 0 \\ 0 & 0 & 0 & Z \end{bmatrix} \begin{bmatrix} X_e(t) \\ X_{ed} \\ \dot{X}_e(t) \\ \dot{X}_e(s) \end{bmatrix} ds \qquad (4.18)$$

where

$$\phi_{11} = Q + PA_e + A_e^T P + \tau X_{11} \qquad (4.19)$$

$$\phi_{12} = PA_{ed} + \tau X_{12} \qquad (4.20)$$

$$\phi_{13} = \tau X_{13} \qquad (4.21)$$

$$\phi_{14} = PA_{e\beta}\Omega_e(h) \qquad (4.22)$$

$$\phi_{15} = PA_{ew} \qquad (4.23)$$

$$\phi_{22} = -(1-\mu)Q + \tau X_{22} \qquad (4.24)$$

$$\phi_{23} = \tau X_{23} \qquad (4.25)$$

$$\phi_{33} = \tau Z + \tau X_{33} \qquad (4.26)$$

4.2.2 Asymptotic Stability

To grantee the asymptotic stability, the first matrix in inequality (4.18) which is,

$$\phi_1 = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} \\ * & \phi_{22} & \phi_{23} & 0 & 0 \\ * & * & \phi_{33} & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix} \tag{4.27}$$

should be negative definite, so the lyapunov function goes to zero as t → ∞. Now, matrix (4.27), which is Negative Semi Definite because it has zeros in the diagonal elements, should be turned to Negative Definite matrix.

In purpose of doing this task, the following free matrices E_1, E_2, E_3, and E_4 will be used such that [see ref. 1.5]:

$$\underline{X}_e \alpha_e(t) = \underline{X}_e \bar{X}_e + \underline{X}_e \bar{X}_{ed} + \underline{X}_e \bar{\alpha}_e + \underline{X}_e \bar{\omega} \tag{4.28}$$

$$\underline{X}_{ed} \alpha_e(t) = \underline{X}_{ed} \bar{X}_e + \underline{X}_{ed} \bar{X}_{ed} + \underline{X}_{ed} \bar{\alpha}_e + \underline{X}_{ed} \bar{\omega} \tag{4.29}$$

$$\underline{\alpha}_e \alpha_e(t) = \underline{\alpha}_e \bar{X}_e + \underline{\alpha}_e \bar{X}_{ed} + \underline{\alpha}_e \bar{\alpha}_e + \underline{\alpha}_e \bar{\omega} \tag{4.30}$$

$$\underline{\omega} \alpha_e(t) = \underline{\omega} \bar{X}_e + \underline{\omega} \bar{X}_{ed} + \underline{\omega} \bar{\alpha}_e + \underline{\omega} \bar{\omega} \tag{4.31}$$

where

$$\bar{X}_e = D_e X_e(t),\ \underline{X}_e = X_e^T(t) E_1, \bar{X}_{ed} = D_{ed} X_{ed}, \underline{X}_{ed} = X_{ed}^T(t) E_2$$
$$\bar{\alpha}_e = D_{e\beta} \Omega_e(h) \alpha_e(t), \underline{\alpha}_e = \alpha_e(t) E_3, \bar{\omega} = D_{ew}\, \omega(t), \underline{\omega} = \omega(t) E_4$$

From (4.28)-(4.31), we have

$$\psi_2^T \bar{\phi}_1 \psi_2 = 0 \tag{4.32}$$

where

$$\psi_2 = \begin{bmatrix} X_e(t) \\ X_{ed} \\ \dot{X}_e(t) \\ \alpha_e(t) \\ \omega(t) \end{bmatrix} \tag{4.33}$$

$$\bar{\phi}_1 = \begin{bmatrix} E_1 D_e & E_1 D_{ed} & 0 & E_1 D_{e\beta} \Omega_e(h) - E_1 & E_1 D_{ew} \\ E_2 D_e & E_2 D_{ed} & 0 & E_2 D_{e\beta} \Omega_e(h) - E_2 & E_2 D_{ew} \\ 0 & 0 & 0 & 0 & 0 \\ E_3 D_e & E_3 D_{ed} & 0 & E_3 D_{e\beta} \Omega_e(h) - E_3 & E_3 D_{ew} \\ E_4 D_e & E_4 D_{ed} & 0 & E_4 D_{e\beta} \Omega_e(h) - E_4 & E_4 D_{ew} \end{bmatrix} \tag{4.34}$$

Since there is no restriction on E_1, E_2, E_3, and E_4, we set

$$E_1 = D_e^T K/2, E_2 = D_{ed}^T K/2 \;, E_3 = (\Omega_e^T(h) D_{e\beta}^T + I)K/2, E_4 = D_{ew}^T K/2 \qquad (4.35) wh$$

ere K is a positive matrix, with suitable dimensions.

Now, let

$$\bar{\phi}_2 = \bar{\phi}_1 + \bar{\phi}_1^T \tag{4.36}$$

After substituting (4.35) into (4.34), $\bar{\phi}_2$ will be

$$\bar{\phi}_2 = \begin{bmatrix} D_e^T K D_e & D_e^T K D_{ed} & 0 & D_e^T K D_{e\beta} \Omega_e(h) & D_e^T K D_{ew} \\ * & D_{ed}^T K D_{ed} & 0 & D_{ed}^T K D_{e\beta} \Omega_e(h) & D_{ed}^T K D_{ew} \\ * & * & 0 & 0 & 0 \\ * & * & * & \Omega_e^T(h) D_{e\beta}^T K D_{e\beta} \Omega_e(h) - K & \Omega_e^T(h) D_{e\beta}^T K D_{ew} \\ * & * & * & * & D_{ew} K D_{ew} \end{bmatrix} \tag{4.37}$$

From (4.32) and (4.36), we have

$$\psi_2^T \bar{\phi}_2 \psi_2 = 0 \tag{4.38}$$

Now, let

$$\phi_2 = \phi_1 + \bar{\phi}_2 \tag{4.39}$$

Then, one can have the following

$$\dot{V}(x_t) \le \psi_2^T \phi_2 \psi_2 - \int_{t-d(t)}^{t} \psi_1^T \bar{X} \psi_1 ds \tag{4.40}$$

where

$$\psi_1 = \begin{bmatrix} X_e(t) \\ X_{ed} \\ \dot{X}_e(t) \\ \dot{X}_e(s) \end{bmatrix}, \quad \bar{X} = \begin{bmatrix} X_{11} & X_{12} & X_{13} & 0 \\ X_{12}^T & X_{22} & X_{23} & 0 \\ X_{13}^T & X_{23}^T & X_{33} & 0 \\ 0 & 0 & 0 & Z \end{bmatrix} \tag{4.41}$$

and

$$\phi_2 = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} \\ * & \phi_{22} & \phi_{23} & 0 & 0 \\ * & * & \phi_{33} & 0 & 0 \\ * & * & * & -K & 0 \\ * & * & * & * & 0 \end{bmatrix} + \begin{bmatrix} D_e^T K \\ D_{ed}^T K \\ 0 \\ \Omega_e^T(h) D_{e\beta}^T K \\ D_{ew}^T K \end{bmatrix} K^{-1} \begin{bmatrix} D_e^T K \\ D_{ed}^T K \\ 0 \\ \Omega_e^T(h) D_{e\beta}^T K \\ D_{ew}^T K \end{bmatrix}^T \tag{4.42}$$

Moreover, according to (4.9) and for arbitrary selected matrices Ti (i=1,2,3),[similar to ref. 1.5] we have:

$$2[X_e^T(t)T_1 - X_{ed}^T T_2 + \dot{X}_e^T(t) T_3] \times [\dot{X}_e(t) - A_e X_e(t) - A_{ed} X_{ed} - A_{e\beta} \beta_e(t)] = 0 \tag{4.43}$$

Notice that $\omega(t) = 0$, because we seek internal stability.

From (4.9), we have:

$$\beta_e(t)=\Omega_e(h)\alpha_e(t)$$

Then (4.43) will be:

$$2[X_e^T(t)T_1+X_{ed}^TT_2+\dot{X}_e^T(t)T_3+\alpha_e(t)T_4]\times[\dot{X}_e(t)-A_eX_e(t)-A_{ed}X_{ed}-A_{e\beta}\Omega_e(h)\alpha_e(t)]=0 \qquad (4.4$$

In matrix form

$$\begin{bmatrix} X_e(t) \\ X_{ed} \\ \dot{X}_e(t) \\ \alpha_e(t) \end{bmatrix}^T \begin{bmatrix} -T_1A_e-A_e^TT_1^T & -T_1A_{ed}-A_e^TT_2^T & T_1-A_e^TT_3^T & -T_1A_{e\beta}\Omega_e(h)-A_e^TT_4^T \\ * & -T_2A_{ed}-A_{ed}^TT_2^T & T_2-A_{ed}^TT_3^T & -T_2A_{e\beta}\Omega_e(h)-A_{ed}^TT_4^T \\ * & * & T_3+T_3^T & -T_3A_{e\beta}\Omega_e(h)+T_4^T \\ * & * & * & -T_4A_{e\beta}\Omega_e(h)-(T_4A_{e\beta}\Omega_e(h))^T \end{bmatrix} \begin{bmatrix} X_e(t) \\ X_{ed} \\ \dot{X}_e(t) \\ \alpha_e(t) \end{bmatrix}=0$$

The last zero quantity can be added to the first term of ϕ_2 in (4.42). Then, it will have the following shape:

$$\phi_2=\begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} \\ * & \phi_{22} & \phi_{23} & \phi_{24} & 0 \\ * & * & \phi_{33} & \phi_{34} & 0 \\ * & * & * & \phi_{44} & 0 \\ * & * & * & * & 0 \end{bmatrix}$$

where

$$\phi_{11}=Q+PA_e+A_e^TP+\tau X_{11}-T_1A_e-A_e^TT_1^T$$

$$\phi_{12}=PA_{ed}+\tau X_{12}-T_1A_{ed}-A_e^TT_2^T$$

$$\phi_{13}=\tau X_{13}+T_1-A_e^TT_3^T$$

$$\phi_{14}=PA_{e\beta}\Omega_e(h)-T_1A_{e\beta}\Omega_e(h)-A_e^TT_4^T$$

$$\phi_{15}=PA_{ew}$$

$$\phi_{44}=-K-T_4A_{e\beta}\Omega_e(h)-(T_4A_{e\beta}\Omega_e(h))^T$$

$$\phi_{22}=-(1-\mu)Q+\tau X_{22}-T_2A_{ed}-A_{ed}^TT_2^T$$

$$\phi_{23}=\tau X_{23}+T_2-A_{ed}^TT_3^T$$

$$\phi_{24}=-T_2A_{e\beta}\Omega_e(h)-A_{ed}^TT_4^T$$

$$\phi_{33}=\tau Z+\tau X_{33}+T_3+T_3^T$$

$$\phi_{34}=-T_3A_{e\beta}\Omega_e(h)+T_4^T$$

ϕ_2 still Negative Semi Definite, because of the zero in element (5,5). This zero will be get rid using H∞ analysis in the next section.

4.2.3 H∞ Performance Analysis

Here the H∞ performance will be analyzed with an H∞ noise attenuation level bound γ of the filtering error system (4.9) based on the Lyapunov function.

Consider the following index

$$g_{zw} = \int_0^{\tau} (\gamma^{-1}\tilde{z}^T \tilde{z} - \gamma\omega^T \omega) dt \tag{4.45}$$

Using the Leibniz-Newton formula, we can rewrite (4.45) as follows

$$g_{zw} = \int_0^{\tau} (\gamma^{-1}\tilde{z}^T \tilde{z} - \gamma\omega^T \omega + \dot{V}(x_t))d \; t + V(x_t)\Big|_{t=0} - V(x_\tau) \tag{4.46}$$

Since $V(x_\tau) > 0$ and $V(x_t)\Big|_{t=0} = 0$, we have [See ref. 1.5]

$$g_{zw} < \int_0^{\tau} (\gamma^{-1}\tilde{z}^T \tilde{z} - \gamma\omega^T \omega + \dot{V}(x_t))dt \tag{4.47}$$

Substituting the value of (z), then

$$\begin{aligned}\gamma^{-1}\tilde{z}^T \tilde{z} - \gamma\omega^T \omega = \gamma^{-1}&[L_e X_e(t) + L_{ed} X_{ed} + L_{ew}\omega(t) + L_{e\beta}\Omega_e(h)\alpha_e(t)]^T \\ &*[L_e X_e(t) + L_{ed} X_{ed} + L_{ew}\omega(t) + L_{e\beta}\Omega_e(h)\alpha_e(t)] - \gamma\omega^T\omega \\ &=: \psi_2^T \phi_{ZW} \psi_2\end{aligned} \tag{4.48}$$

where

$$\phi_{ZW} = \begin{bmatrix} \gamma^{-1}L_e^T L_e & \gamma^{-1}L_e^T L_{ed} & 0 & \gamma^{-1}L_e^T L_{e\beta}\Omega_e(h) & \gamma^{-1}L_e^T L_{ew} \\ * & \gamma^{-1}L_{ed}^T L_{ed} & 0 & \gamma^{-1}L_{ed}^T L_{e\beta}\Omega_e(h) & \gamma^{-1}L_{ed}^T L_{ew} \\ 0 & 0 & 0 & 0 & 0 \\ * & * & 0 & \gamma^{-1}\Omega_e^T(h)L_{e\beta}^T L_{e\beta}\Omega_e(h) & \gamma^{-1}\Omega_e^T(h)L_{e\beta}^T L_{ew} \\ * & * & 0 & * & \gamma^{-1}L_{ew}^T L_{ew} - \gamma I \end{bmatrix} \tag{4.49}$$

It follows from (4.39) and (4.48) that

$$\gamma^{-1}\tilde{z}^T \tilde{z} - \gamma\omega^T \omega + \dot{V}(x_t) \le \psi_2^T \phi_3 \psi_2 \tag{4.50}$$

where

$$\begin{aligned} \phi_3 &= \phi_2 + \phi_{ZW} \\ &= \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} \\ * & \phi_{22} & \phi_{23} & \phi_{24} & 0 \\ * & * & \phi_{33} & \phi_{34} & 0 \\ * & * & * & -K + \phi_{44} & 0 \\ * & * & * & * & -\gamma I \end{bmatrix} + \begin{bmatrix} D_e^T K \\ D_{ed}^T K \\ 0 \\ \Omega_e^T(h) D_{e\beta}^T K \\ D_{ew}^T K \end{bmatrix} K^{-1} \begin{bmatrix} D_e^T K \\ D_{ed}^T K \\ 0 \\ \Omega_e^T(h) D_{e\beta}^T K \\ D_{ew}^T K \end{bmatrix}^T \\ &+ \begin{bmatrix} L_e^T \\ L_{ed}^T \\ 0 \\ \Omega_e^T(h) L_{e\beta}^T \\ L_{ew}^T \end{bmatrix} (\gamma I)^{-1} \begin{bmatrix} L_e^T \\ L_{ed}^T \\ 0 \\ \Omega_e^T(h) L_{e\beta}^T \\ L_{ew}^T \end{bmatrix}^T \end{aligned} \tag{4.51}$$

The last derivations can be stated in the following theorem.

4.3 The Augmented System Stability Theorem

For given scalars $0 \le \tau, \mu < 1$, and $\gamma > 0$, the LPV-LFT filtering error system (4.9), and with time varying delay satisfying (4.2) is stable and with H_∞ norm bound γ, if there exist $P = P^T > 0, Q = Q^T > 0, Z = Z^T > 0$, and K>0 such that the following LMIs hold:

$$\begin{bmatrix} \phi_{11} & \phi_{12} & \tau X_{13}+T_1-A_e^T T_3^T & PA_{e\beta}\Omega_e(h)-T_1 A_{e\beta}\Omega_e(h)-A_e^T T_4^T & PA_{ew} & D_e^T K & L_e^T \\ * & \phi_{22} & \tau X_{23}+T_2-A_{ed}^T T_3^T & -T_2 A_{e\beta}\Omega_e(h)-A_{ed}^T T_4^T & 0 & D_{ed}^T K & L_{ed}^T \\ * & * & \tau Z+\tau X_{33}+T_3+T_3^T & -T_3 A_{e\beta}\Omega_e(h)+T_4^T & 0 & 0 & 0 \\ * & * & * & -K-T_4 A_{e\beta}\Omega_e(h)-(T_4 A_{e\beta}\Omega_e(h))^T & 0 & \Omega_e^T(h) D_{e\beta}^T K & \Omega_e^T(h) L_{e\beta}^T \\ * & * & * & * & -\gamma I & D_{ew}^T K & L_{ew}^T \\ * & * & * & * & * & -K & 0 \\ * & * & * & * & * & * & -\gamma I \end{bmatrix} < 0 \quad (4.52)$$

and

$$\begin{bmatrix} X_{11} & X_{12} & X_{13} & 0 \\ * & X_{22} & X_{23} & 0 \\ * & * & X_{33} & 0 \\ * & * & * & Z \end{bmatrix} \geq 0 \qquad (4.53)$$

where

$$\phi_{11} = Q + PA_e + A_e^T P + \tau X_{11} - T_1 A_e - A_e^T T_1^T$$
$$\phi_{12} = PA_{ed} + \tau X_{12} - T_1 A_{ed} - A_e^T T_2^T$$
$$\phi_{22} = -(1-\mu)Q + \tau X_{22} - T_2 A_{ed} - A_{ed}^T T_2^T$$

Proof

Invoke the Schur complement formula on (4.51). Then (4.52) will be hold

CHAPTER 5

THE FILTERING DESIGN PROBLEM

5.1 The Filter Design Theorem

Theorem: For the LPV system (4.1), with time varying delay satisfying (4.2), there exists LPV filter with time varying delay in the form of (4.5) such that the LPV-LFT filtering error system (4.9) is stable and with H_∞ norm bound γ, if there exist matrices $\bar{A}_f$, $\bar{A}_{f\beta}$, $\bar{D}_f$, $\bar{L}_f$, $\bar{D}_{f\beta}$, $\bar{L}_{f\beta}$, $P > 0, Q > 0, Z > 0, K > 0, and\ F > 0$ such that the following LMIs hold:

$$\begin{bmatrix} X_{111} & X_{112} & X_{121} & X_{122} & X_{131} & X_{132} \\ * & X_{113} & X_{122} & X_{123} & X_{132} & X_{133} \\ * & * & X_{221} & X_{222} & X_{231} & X_{232} \\ * & * & * & X_{223} & X_{232} & X_{233} \\ * & * & * & * & X_{331} & X_{332} \\ * & * & * & * & * & X_{333} \end{bmatrix} \geq 0 \tag{5.1}$$

$$\begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} & H_{15} & H_{16} & H_{17} & H_{18} \\ * & H_{22} & H_{23} & H_{24} & H_{25} & H_{26} & H_{27} & H_{28} \\ * & * & H_{33} & H_{34} & H_{35} & H_{36} & H_{37} & H_{38} \\ * & * & * & H_{44} & H_{45} & H_{46} & H_{47} & H_{48} \\ * & * & * & * & H_{55} & H_{56} & H_{57} & H_{58} \\ * & * & * & * & H_{65} & H_{66} & H_{67} & H_{68} \\ * & * & * & * & * & * & H_{77} & H_{78} \\ * & * & * & * & * & * & * & H_{88} \end{bmatrix} < 0 \tag{5.2}$$

where

$$H_{11} = \begin{bmatrix} \pi_{p1}A + A^T \pi_{p1} & A^T \pi_{p2} + B^T A_{fy}^T \\ \pi_{p2}^T A + A_{fy} B & \bar{A}_f + \bar{A}_f^T \end{bmatrix}$$

$$H_{12} = \begin{bmatrix} \pi_{p1}A_d + \tau X_{121} - A^T t_2^T & \tau X_{122} - B^T A_{fy}^T \\ \pi_{p2}^T A_d + A_{fy} B_d + \tau \bar{X}_{123} & A_{fd} + \tau \bar{X}_{124} - \bar{A}_f^T \end{bmatrix}, H_{13} = \begin{bmatrix} \tau X_{131} + t_1 - A^T t_3^T & \tau X_{132} - B^T A_{fy}^T \\ \tau \bar{X}_{133} & \tau \bar{X}_{134} + J_{p2} - \bar{A}_f^T \end{bmatrix}$$

$$H_{14} = \begin{bmatrix} \pi_{p1}A_\beta \Omega(h) - A^T t_4^T & \pi_{p1}A_\beta \Omega(h) - A^T t_4^T \\ \pi_{p2}^T A_\beta \Omega(h) & \pi_{p2}^T A_\beta \Omega(h) + \bar{A}_{f\beta} \end{bmatrix}, H_{15} = \begin{bmatrix} \pi_{p1}A_w \\ \pi_{p2}^T A_w + A_{fy} B_w \end{bmatrix}$$

$$H_{16} = \begin{bmatrix} D^T \pi_{k1} + B^T D_{fy}^T & D^T \pi_{k2} \\ \bar{D}_f & 0 \end{bmatrix}, H_{17} = \begin{bmatrix} L^T - B^T L_{fy}^T \\ -\bar{L}_f \end{bmatrix}, H_{18} = \begin{bmatrix} I & 0 \\ 0 & J_{p2} \end{bmatrix}$$

$$H_{22} = \begin{bmatrix} -(1-\mu)Q_{11} + \tau X_{221} - t_2 A_d - A_d^T t_2^T & -(1-\mu)Q_{12} + \tau X_{222} - B_d^T A_{fy}^T \\ -(1-\mu)Q_{21} + \tau X_{223} - A_{fy} B_d & -(1-\mu)Q_{22} + \tau X_{224} - A_{fd} - A_{fd}^T \end{bmatrix}$$

$$H_{23}=\begin{bmatrix}\tau X_{231}+t_2-A_d^T t_3^T & \tau X_{232}-B_d^T A_{fy}^T\\ \tau X_{233} & \tau X_{234}+I-A_{fd}^T\end{bmatrix}, H_{24}=\begin{bmatrix}-t_2 A_\beta \Omega(h)-A_d^T t_4^T & -t_2 A_\beta \Omega(h)-A_d^T t_4^T\\ 0 & -\bar{A}_{f\beta}\end{bmatrix}$$

$$H_{25}=\begin{bmatrix}0\\0\end{bmatrix} H_{26}=\begin{bmatrix}D_d^T\pi_{k1}+B_d^T D_{fy}^T & D_d^T\pi_{k2}\\ D_{fd}^T & 0\end{bmatrix}, H_{27}=\begin{bmatrix}L_d^T-B_d^T L_{fy}^T\\ -L_{fd}^T\end{bmatrix}, H_{28}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$H_{33}=\begin{bmatrix}\tau Z_{11}+\tau X_{331}+t_3+t_3^T & \tau Z_{12}+\tau X_{332}\\ \tau Z_{21}+\tau X_{333} & \tau Z_{22}+\tau X_{334}+2I\end{bmatrix}, H_{34}=\begin{bmatrix}-t_3 A_\beta \Omega(h)+t_4^T & -t_3 A_\beta \Omega(h)+t_4^T\\ 0 & -\bar{A}_{f\beta}\end{bmatrix}$$

$$H_{35}=H_{37}=\begin{bmatrix}0\\0\end{bmatrix},\ H_{36}=H_{38}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$H_{44}=\begin{bmatrix}-\pi_{k1}-t_4 A_\beta \Omega^T(h)-\Omega(h)A_\beta^T t_4^T & -\pi_{k2}-t_4 A_\beta \Omega^T(h)-\Omega(h)A_\beta^T t_4^T\\ -\pi_{k1}-J_{k2}-t_4 A_\beta \Omega^T(h)-\Omega(h)A_\beta^T t_4^T & -\pi_{k2}-t_4 A_\beta \Omega^T(h)-\Omega(h)A_\beta^T t_4^T\end{bmatrix}$$

$$H_{45}=\begin{bmatrix}0\\0\end{bmatrix}, H_{46}=\begin{bmatrix}\Omega^T(h)D_\beta^T\pi_{k1} & \Omega^T(h)D_\beta^T\pi_{k2}\\ \Omega^T(h)D_\beta^T\pi_{k1}+\bar{D}_{f\beta} & 0\end{bmatrix}, H_{47}=\begin{bmatrix}\Omega^T(h)L_\beta^T\\ \Omega^T(h)L_\beta^T-\bar{L}_{f\beta}\end{bmatrix}, H_{48}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$H_{55}=-gamma,\ H_{56}=\begin{bmatrix}D_w^T\pi_{k1}+B_w^T D_{f\beta}^T & D_w^T\pi_{k2}\end{bmatrix}, H_{57}=L_w^T-B_w^T L_{fy}^T,\ H_{58}=\begin{bmatrix}0 & 0\end{bmatrix}$$

$$H_{66}=\begin{bmatrix}-\pi_{k1} & -\pi_{k2}\\ -\pi_{k1}-J_{k2} & -\pi_{k2}\end{bmatrix}, H_{67}=\begin{bmatrix}0\\0\end{bmatrix}, H_{68}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$H_{77}=-gamma, H_{78}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix},\ H_{88}=\begin{bmatrix}-F_1 & -F_2\\ -F_2 & -F_3\end{bmatrix}$$

Such that the following filter parameters are given by:

$$
\begin{aligned}
A_f &= \bar{A}_f J_{p2}^{-T} \\
A_{f\beta} &= \bar{A}_{f\beta} J_{k2}^{-T} \\
D_f &= \bar{D}_f^T J_{p2}^{-T} \\
L_f &= \bar{L}_f J_{p2}^{-T} \\
D_{f\beta} &= \bar{D}_{f\beta}^T J_{k2}^{-T} \\
L_{f\beta} &= \bar{L}_{f\beta}^T J_{k2}^{-T}
\end{aligned}
$$

Proof

Define $F^{-1} = Q + \tau X_{11}$ and $P_t = P - T_1$. Then, substitute that in LMI (4.52) to get:

$$
\begin{bmatrix}
P_t A_e + A_e^T P_t^T + F^{-1} & \phi_{12} & \tau X_{13} + T_1 - A_e^T T_3^T & P_t A_{e\beta}\Omega_e(h) - A_e^T T_4^T & PA_{ew} & D_e^T K & L_e^T \\
* & \phi_{22} & \tau X_{23} + T_2 - A_{ed}^T T_3^T & -T_2 A_{e\beta}\Omega_e(h) - A_{ed}^T T_4^T & 0 & D_{ed}^T K & L_{ed}^T \\
* & * & \tau Z + \tau X_{33} + T_3 + T_3^T & -T_3 A_{e\beta}\Omega_e(h) + T_4^T & 0 & 0 & 0 \\
* & * & * & -K - T_4 A_{e\beta}\Omega_e(h) - (T_4 A_{e\beta}\Omega_e(h))^T & 0 & \Omega_e^T(h) D_{e\beta}^T K & \Omega_e^T(h) L_{e\beta}^T \\
* & * & * & * & -\gamma I & D_{ew}^T K & L_{ew}^T \\
* & * & * & * & * & -K & 0 \\
* & * & * & * & * & * & -\gamma I
\end{bmatrix} < 0 \quad (5.3)
$$

where

$$
\begin{aligned}
\phi_{12} &= P_t A_{ed} + \tau X_{12} - A_e^T T_2^T \\
\phi_{22} &= -(1-\mu)Q + \tau X_{22} - T_2 A_{ed} - A_{ed}^T T_2^T
\end{aligned}
$$

Apply Schur complement on (5.3)

$$\begin{bmatrix} P_t A_e + A_e^T P_t^T & \phi_{12} & \tau X_{13} + T_1 - A_e^T T_3^T & P_t A_{e\beta}\Omega_e(h) - A_e^T T_4^T & PA_{ew} & D_e^T K & L_e^T & I \\ * & \phi_{22} & \tau X_{23} + T_2 - A_{ed}^T T_3^T & -T_2 A_{e\beta}\Omega_e(h) - A_{ed}^T T_4^T & 0 & D_{ed}^T K & L_{ed}^T & 0 \\ * & * & \tau Z + \tau X_{33} + T_3 + T_3^T & -T_3 A_{e\beta}\Omega_e(h) + T_4^T & 0 & 0 & 0 & 0 \\ * & * & * & -K - T_4 A_{e\beta}\Omega_e(h) - (T_4 A_{e\beta}\Omega_e(h))^T & 0 & \Omega_e^T(h) D_{e\beta}^T K & \Omega_e^T(h) L_{e\beta}^T & 0 \\ * & * & * & * & -\gamma I & D_{ew}^T K & L_{ew}^T & 0 \\ * & * & * & * & * & -K & 0 & 0 \\ * & * & * & * & * & * & -\gamma I & 0 \\ * & * & * & * & * & * & * & -F \end{bmatrix} < 0 \quad (5.4)$$

5.2 The Congruent Transformation

The congruent transformation will be applied to (5.4) with $diag(J_p, I, I, J_k, I, J_k, I, I)$

where

$$J_p = \begin{bmatrix} I & 0 \\ 0 & J_{p2}^T \end{bmatrix};\ J_k = \begin{bmatrix} I & I \\ 0 & J_{k2}^T \end{bmatrix} \qquad (5.5)$$

Then, we have

$$\begin{bmatrix} J_p^T(P_t A_e + A_e^T P_t^T)J_p & J_p^T \phi_{12} & J_p^T(\tau X_{13} + T_1 - A_e^T T_3^T) & J_p^T(P_t A_{e\beta}\Omega_e(h) - A_e^T T_4^T)J_k & J_p^T PA_{ew} & J_p^T D_e^T K J_k & J_p^T L_e^T & J_p^T \\ * & \phi_{22} & \tau X_{23} + T_2 - A_{ed}^T T_3^T & (-T_2 A_{e\beta}\Omega_e(h) - A_{ed}^T T_4^T)J_k & 0 & D_{ed}^T K J_k & L_{ed}^T & 0 \\ * & * & \tau Z + \tau X_{33} + T_3 + T_3^T & (-T_3 A_{e\beta}\Omega_e(h) + T_4^T)J_k & 0 & 0 & 0 & 0 \\ * & * & * & J_k^T[-K - T_4 A_{e\beta}\Omega_e(h) - (T_4 A_{e\beta}\Omega_e(h))^T]J_k & 0 & J_k^T \Omega_e^T(h) D_{e\beta}^T K J_k & J_k^T \Omega_e^T(h) L_{e\beta}^T & 0 \\ * & * & * & * & -\gamma I & D_{ew}^T K J_k & L_{ew}^T & 0 \\ * & * & * & * & * & -J_k^T K J_k & 0 & 0 \\ * & * & * & * & * & * & -\gamma I & 0 \\ * & * & * & * & * & * & * & -F \end{bmatrix} < 0 \quad (5.6)$$

Define

$$P_t = J_p^{-T} \pi_p \qquad (5.7)$$

$$K = \pi_k J_k^{-1} \qquad (5.8)$$

where

$$\pi_p = \begin{bmatrix} \pi_{p1} & 0 \\ \pi_{p2}^T & I \end{bmatrix}; \pi_k = \begin{bmatrix} \pi_{k1} & \pi_{k2} \\ I & 0 \end{bmatrix} \tag{5.9}$$

5.3 Matrix (5.6) Disassembling

To disassemble the matrix (5.6), we substitute for some of its elements directly. On other hand, the other elements need some manipulation.

First of all, we have to define the following:

$$T_1 = \begin{bmatrix} t_1 & 0 \\ 0 & I \end{bmatrix}, T_2 = \begin{bmatrix} t_2 & 0 \\ 0 & I \end{bmatrix}, T_3 = \begin{bmatrix} t_3 & 0 \\ 0 & I \end{bmatrix}, T_4 = \begin{bmatrix} t_4 & 0 \\ 0 & 0 \end{bmatrix}$$

Manipulating term (1, 1)

$$\begin{aligned} J_p^T (P_t A_e + A_e^T P_t^T) J_p &= \pi_p A_e J_p + J_p^T A_e^T \pi_p^T \\ &= \begin{bmatrix} \pi_{p1} & 0 \\ \pi_{p2}^T & I \end{bmatrix} \begin{bmatrix} A & 0 \\ A_{fy} B & A_f \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & J_{p2}^T \end{bmatrix} + \left(\begin{bmatrix} \pi_{p1} & 0 \\ \pi_{p2}^T & I \end{bmatrix} \begin{bmatrix} A & 0 \\ A_{fy} B & A_f \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & J_{p2}^T \end{bmatrix} \right)^T \\ &= \begin{bmatrix} \pi_{p1} A & 0 \\ \pi_{p2}^T A + A_{fy} B & A_f \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & J_{p2}^T \end{bmatrix} + \left(\begin{bmatrix} \pi_{p1} A & 0 \\ \pi_{p2}^T A + A_{fy} B & A_f \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & J_{p2}^T \end{bmatrix} \right)^T \\ &= \begin{bmatrix} \pi_{p1} A & 0 \\ \pi_{p2}^T A + A_{fy} B & A_f J_{p2}^T \end{bmatrix} + \left(\begin{bmatrix} \pi_{p1} A & 0 \\ \pi_{p2}^T A + A_{fy} B & A_f J_{p2}^T \end{bmatrix} \right)^T \\ &= \begin{bmatrix} \pi_{p1} A + A^T \pi_{p1} & A^T \pi_{p2} + B^T A_{fy}^T \\ \pi_{p2}^T A + A_{fy} B & A_f J_{p2}^T + J_{p2} A_f^T \end{bmatrix} \end{aligned}$$

Define $\bar{A}_f = A_f J_{p2}^T$

$$Term\,(1,1) = \begin{bmatrix} \pi_{p1} A + A^T \pi_{p1} & A^T \pi_{p2} + B^T A_{fy}^T \\ \pi_{p2}^T A + A_{fy} B & \bar{A}_f + \bar{A}_f^T \end{bmatrix}$$

Manipulating term (1, 2)

$$J_p^T (P_t A_{ed} + \tau X_{12} - A_e^T T_2^T) = \pi_p A_{ed} + \tau J_p^T X_{12} - J_p^T A_e^T T_2^T$$

$$= \begin{bmatrix} \pi_{p1} & 0 \\ \pi_{p2}^T & I \end{bmatrix} \begin{bmatrix} A_d & 0 \\ A_{fy} B_d & A_{fd} \end{bmatrix} + \tau \begin{bmatrix} I & 0 \\ 0 & J_{p2}^T \end{bmatrix}^T \begin{bmatrix} X_{121} & X_{122} \\ X_{123} & X_{124} \end{bmatrix} - \begin{bmatrix} I & 0 \\ 0 & J_{p2}^T \end{bmatrix}^T \begin{bmatrix} A & 0 \\ A_{fy} B & A_f \end{bmatrix}^T \begin{bmatrix} t_2 & 0 \\ 0 & I \end{bmatrix}^T$$

$$= \begin{bmatrix} \pi_{p1} A_d & 0 \\ \pi_{p2}^T A_d + A_{fy} B_d & A_{fd} \end{bmatrix} + \tau \begin{bmatrix} X_{121} & X_{122} \\ J_{p2} X_{123} & J_{p2} X_{124} \end{bmatrix} - \begin{bmatrix} A^T t_2^T & B^T A_{fy}^T \\ 0 & J_{p2} A_f^T \end{bmatrix}$$

$$= \begin{bmatrix} \pi_{p1} A_d & 0 \\ \pi_{p2}^T A_d + A_{fy} B_d & A_{fd} \end{bmatrix} + \tau \begin{bmatrix} X_{121} & X_{122} \\ J_{p2} X_{123} & J_{p2} X_{124} \end{bmatrix} - \begin{bmatrix} A^T t_2^T & B^T A_{fy}^T \\ 0 & \bar{A}_f^T \end{bmatrix}$$

Define

$$\bar{X}_{123} = J_{p2} X_{123}$$
$$\bar{X}_{124} = J_{p2} X_{124}$$

Then

$$Term(1,2) = \begin{bmatrix} \pi_{p1} A_d & 0 \\ \pi_{p2}^T A_d + A_{fy} B_d & A_{fd} \end{bmatrix} + \tau \begin{bmatrix} X_{121} & X_{122} \\ \bar{X}_{123} & \bar{X}_{124} \end{bmatrix} - \begin{bmatrix} A^T t_2^T & B^T A_{fy}^T \\ 0 & \bar{A}_f^T \end{bmatrix}$$

$$= \begin{bmatrix} \pi_{p1} A_d + \tau X_{121} - A^T t_2^T & \tau X_{122} - B^T A_{fy}^T \\ \pi_{p2}^T A_d + A_{fy} B_d + \tau \bar{X}_{123} & A_{fd} + \tau \bar{X}_{124} - \bar{A}_f^T \end{bmatrix}$$

Manipulating term (1, 3)

Define

$$\bar{X}_{133} = J_{p2} X_{133}$$
$$\bar{X}_{134} = J_{p2} X_{134}$$

Then, substitute them in the follow

$$J_p^T(\tau X_{13}+T_1-A_e^T T_3^T)=\begin{bmatrix}\tau X_{131} & \tau X_{132}\\ \tau \bar{X}_{133} & \tau \bar{X}_{134}\end{bmatrix}+\begin{bmatrix}t_1 & 0\\ 0 & J_{p2}\end{bmatrix}-\begin{bmatrix}A^T t_3^T & B^T A_{fy}^T\\ 0 & \bar{A}_f^T\end{bmatrix}$$

$$=\begin{bmatrix}\tau X_{131}+t_1-A^T t_3^T & \tau X_{132}-B^T A_{fy}^T\\ \tau \bar{X}_{133} & \tau \bar{X}_{134}+J_{p2}-\bar{A}_f^T\end{bmatrix}$$

Manipulating term (1, 4)

$$J_p^T PA_{e\beta}\Omega_e(h)J_k=\pi_p A_{e\beta}\Omega_e(h)J_k-J_p^T A_e^T T_4^T J_k$$

$$=\begin{bmatrix}\pi_{p1} & 0\\ \pi_{p2}^T & I\end{bmatrix}\begin{bmatrix}A_\beta & 0\\ 0 & A_{f\beta}\end{bmatrix}\begin{bmatrix}\Omega(h) & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}I & I\\ 0 & J_{k2}^T\end{bmatrix}-\begin{bmatrix}I & 0\\ 0 & J_{p2}^T\end{bmatrix}^T\begin{bmatrix}A & 0\\ A_{fy}B & A_f\end{bmatrix}^T\begin{bmatrix}t_4 & 0\\ 0 & 0\end{bmatrix}^T\begin{bmatrix}I & I\\ 0 & J_{k2}^T\end{bmatrix}$$

$$=\begin{bmatrix}\pi_{p1}A_\beta & 0\\ \pi_{p2}^T A_\beta & A_{f\beta}\end{bmatrix}\begin{bmatrix}\Omega(h) & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}I & I\\ 0 & J_{k2}^T\end{bmatrix}-\begin{bmatrix}A^T t_4^T & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}I & I\\ 0 & J_{k2}^T\end{bmatrix}$$

$$=\begin{bmatrix}\pi_{p1}A_\beta\Omega(h) & 0\\ \pi_{p2}^T A_\beta\Omega(h) & A_{f\beta}\end{bmatrix}\begin{bmatrix}I & I\\ 0 & J_{k2}^T\end{bmatrix}-\begin{bmatrix}A^T t_4^T & A^T t_4^T\\ 0 & 0\end{bmatrix}$$

$$=\begin{bmatrix}\pi_{p1}A_\beta\Omega(h) & \pi_{p1}A_\beta\Omega(h)\\ \pi_{p2}^T A_\beta\Omega(h) & \pi_{p2}^T A_\beta\Omega(h)+A_{f\beta}J_{k2}^T\end{bmatrix}-\begin{bmatrix}A^T t_4^T & A^T t_4^T\\ 0 & 0\end{bmatrix}$$

$$=\begin{bmatrix}\pi_{p1}A_\beta\Omega(h)-A^T t_4^T & \pi_{p1}A_\beta\Omega(h)-A^T t_4^T\\ \pi_{p2}^T A_\beta\Omega(h) & \pi_{p2}^T A_\beta\Omega(h)+A_{f\beta}J_{k2}^T\end{bmatrix}$$

Define

$$\bar{A}_{f\ \beta} = A_{f\ \beta} J_{k2}^{T}$$

Then

$$Term(1,4) = \begin{bmatrix} \pi_{p1} A_{\beta} \Omega(h) - A^{T} t_{4}^{T} & \pi_{p1} A_{\beta} \Omega(h) - A^{T} t_{4}^{T} \\ \pi_{p2}^{T} A_{\beta} \Omega(h) & \pi_{p2}^{T} A_{\beta} \Omega(h) + \bar{A}_{f\ \beta} \end{bmatrix}$$

Manipulating term (1,5)

$$\begin{aligned} J_{p}^{T} P A_{ew} &= \pi_{p} A_{ew} \\ &= \begin{bmatrix} \pi_{p1} & 0 \\ \pi_{p2}^{T} & I \end{bmatrix} \begin{bmatrix} A_{w} \\ A_{fy} B_{w} \end{bmatrix} \\ &= \begin{bmatrix} \pi_{p1} A_{w} \\ \pi_{p2}^{T} A_{w} + A_{fy} B_{w} \end{bmatrix} \end{aligned}$$

Manipulating term (1,6)

$$\begin{aligned} J_{p}^{T} D_{e}^{T} K J_{k} &= J_{p}^{T} D_{e}^{T} \pi_{k} \\ &= \begin{bmatrix} I & 0 \\ 0 & J_{p2} \end{bmatrix} \begin{bmatrix} D^{T} & B^{T} D_{fy}^{T} \\ 0 & D_{f}^{T} \end{bmatrix} \begin{bmatrix} \pi_{k1} & \pi_{k2} \\ I & 0 \end{bmatrix} \\ &= \begin{bmatrix} D^{T} & B^{T} D_{fy}^{T} \\ 0 & J_{p2} D_{f}^{T} \end{bmatrix} \begin{bmatrix} \pi_{k1} & \pi_{k2} \\ I & 0 \end{bmatrix} \\ &= \begin{bmatrix} D^{T} \pi_{k1} + B^{T} D_{fy}^{T} & D^{T} \pi_{k2} \\ J_{p2} D_{f}^{T} & 0 \end{bmatrix} \end{aligned}$$

Define

$$\bar{D}_f = J_{p2} D_f^T$$

Then

$$Term(1,6) = \begin{bmatrix} D^T \pi_{k1} + B^T D_{fy}^T & D^T \pi_{k2} \\ \bar{D}_f & 0 \end{bmatrix}$$

Manipulating term (1,7)

$$J_p^T L_e^T = \begin{bmatrix} I & 0 \\ 0 & J_{p2} \end{bmatrix} \begin{bmatrix} L - L_{fy} B & -L_f \end{bmatrix}^T$$

$$= \begin{bmatrix} L^T - B^T L_{fy}^T \\ -J_{p2} L_f^T \end{bmatrix}$$

Define

$$\bar{L}_f^T = J_{p2} L_f^T$$

Then

$$Term(1,7) = \begin{bmatrix} L^T - B^T L_{fy}^T \\ -\bar{L}_f \end{bmatrix}$$

Manipulating term (2,2)

$$-(1-\mu)Q + \tau X_{22} - T_2 A_{ed} - A_{ed}^T T_2^T = -(1-\mu)\begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} + \tau \begin{bmatrix} X_{221} & X_{222} \\ X_{223} & X_{224} \end{bmatrix} - \begin{bmatrix} t_2 A_d & 0 \\ A_{fy} B_d & A_{fd} \end{bmatrix} - \begin{bmatrix} A_d^T t_2^T & B_d^T A_{fy}^T \\ 0 & A_{fd}^T \end{bmatrix}$$

$$= \begin{bmatrix} -(1-\mu)Q_{11} + \tau X_{221} - t_2 A_d - A_d^T t_2^T & -(1-\mu)Q_{12} + \tau X_{222} - B_d^T A_{fy}^T \\ -(1-\mu)Q_{21} + \tau X_{223} - A_{fy} B_d & -(1-\mu)Q_{22} + \tau X_{224} - A_{fd} - A_{fd}^T \end{bmatrix}$$

Manipulating term (2,3)

$$\tau X_{23}+T_2-A_{ed}^T T_3^T=\tau\begin{bmatrix}X_{231} & X_{232}\\ X_{233} & X_{234}\end{bmatrix}+\begin{bmatrix}t_2 & 0\\ 0 & I\end{bmatrix}-\begin{bmatrix}A_d^T t_3^T & B_d^T A_{fy}^T\\ 0 & A_{fd}^T\end{bmatrix}$$

$$=\begin{bmatrix}\tau X_{231}+t_2-A_d^T t_3^T & \tau X_{232}-B_d^T A_{fy}^T\\ \tau X_{233} & \tau X_{234}+I-A_{fd}^T\end{bmatrix}$$

Manipulating term (2,4)

$$(-T_2 A_{e\beta}\Omega_e(h)-A_{ed}^T T_4^T)J_k=-\begin{bmatrix}t_2 & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}A_\beta & 0\\ 0 & A_{f\beta}\end{bmatrix}\begin{bmatrix}\Omega(h) & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}I & I\\ 0 & J_{k2}^T\end{bmatrix}-\begin{bmatrix}A_d^T & B_d^T A_{fy}^T\\ 0 & A_{fd}^T\end{bmatrix}\begin{bmatrix}t_4^T & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}I & I\\ 0 & J_{k2}^T\end{bmatrix}$$

$$=\begin{bmatrix}-t_2 A_\beta\Omega(h) & -t_2 A_\beta\Omega(h)\\ 0 & -A_{f\beta}J_{k2}^T\end{bmatrix}-\begin{bmatrix}A_d^T t_4^T & A_d^T t_4^T\\ 0 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-t_2 A_\beta\Omega(h)-A_d^T t_4^T & -t_2 A_\beta\Omega(h)-A_d^T t_4^T\\ 0 & -A_{f\beta}J_{k2}^T\end{bmatrix}$$

but $A_{f\beta}J_{k2}^T=\bar{A}_{f\beta}$

then $$Term(2,4)=\begin{bmatrix}-t_2 A_\beta\Omega(h)-A_d^T t_4^T & -t_2 A_\beta\Omega(h)-A_d^T t_4^T\\ 0 & -\bar{A}_{f\beta}\end{bmatrix}$$

Manipulating term (2,5)

$$Term(2,5)=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

Manipulating term (2,6)

$$D_{ed}^{T}KJ_{k} = D_{ed}^{T}\pi_{k}$$
$$= \begin{bmatrix} D_{d}^{T} & B_{d}^{T}D_{fy}^{T} \\ 0 & D_{fd}^{T} \end{bmatrix}\begin{bmatrix} \pi_{k1} & \pi_{k2} \\ I & 0 \end{bmatrix}$$
$$= \begin{bmatrix} D_{d}^{T}\pi_{k1} + B_{d}^{T}D_{fy}^{T} & D_{d}^{T}\pi_{k2} \\ D_{fd}^{T} & 0 \end{bmatrix}$$

Manipulating term (2,7)

$$L_{ed}^{T} = \begin{bmatrix} L_{d}^{T} - B_{d}^{T}L_{fy}^{T} \\ -L_{fd}^{T} \end{bmatrix}$$

Manipulating term (3,3)

$$\tau Z + \tau X_{33} + T_{3} + T_{3}^{T} = \tau\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} + \tau\begin{bmatrix} X_{331} & X_{332} \\ X_{333} & X_{334} \end{bmatrix} + \begin{bmatrix} t_{3} & 0 \\ 0 & I \end{bmatrix} + \begin{bmatrix} t_{3}^{T} & 0 \\ 0 & I \end{bmatrix}$$
$$= \begin{bmatrix} \tau Z_{11} + \tau X_{331} + t_{3} + t_{3}^{T} & \tau Z_{12} + \tau X_{332} \\ \tau Z_{21} + \tau X_{333} & \tau Z_{22} + \tau X_{334} + 2I \end{bmatrix}$$

Manipulating term (3,4)

$$(-T_{3}A_{e\beta}\Omega_{e}(h) + T_{4}^{T})J_{k} = -\begin{bmatrix} t_{3} & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} A_{\beta} & 0 \\ 0 & A_{f\beta} \end{bmatrix}\begin{bmatrix} \Omega(h) & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} I & I \\ 0 & J_{k2}^{T} \end{bmatrix} + \begin{bmatrix} t_{4}^{T} & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} I & I \\ 0 & J_{k2}^{T} \end{bmatrix}$$
$$= \begin{bmatrix} -t_{3}A_{\beta}\Omega(h) & -t_{3}A_{\beta}\Omega(h) \\ 0 & -A_{f\beta}J_{k2}^{T} \end{bmatrix} + \begin{bmatrix} t_{4}^{T} & t_{4}^{T} \\ 0 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} -t_{3}A_{\beta}\Omega(h) + t_{4}^{T} & -t_{3}A_{\beta}\Omega(h) + t_{4}^{T} \\ 0 & -A_{f\beta}J_{k2}^{T} \end{bmatrix}$$

but $A_{f\beta}J_{k2}^{T} = \bar{A}_{f\beta}$

then $$Term(3,4)=\begin{bmatrix} -t_3A_\beta\Omega(h)+t_4^T & -t_3A_\beta\Omega(h)+t_4^T \\ 0 & -\bar{A}_{f\beta} \end{bmatrix}$$

Manipulating term (4,4)

$$J_k^T\,[-K-T_4A_{e\beta}\Omega_e(h)-(T_4A_{e\beta}\Omega_e(h))^T\,]J_k=-J_k^T\pi_k-J_k^TT_4A_{e\beta}\Omega_e(h)J_k-J_k^T\Omega_e^T(h)A_{e\beta}^TT_4^TJ_k$$

$$=-\begin{bmatrix} I & 0 \\ I & J_{k2} \end{bmatrix}\begin{bmatrix} \pi_{k1} & \pi_{k2} \\ I & 0 \end{bmatrix}-\begin{bmatrix} I & 0 \\ I & J_{k2} \end{bmatrix}\begin{bmatrix} t_4 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} A_\beta & 0 \\ 0 & A_{f\beta} \end{bmatrix}\begin{bmatrix} \Omega^T(h) & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} I & I \\ 0 & J_{k2}^T \end{bmatrix}-\begin{bmatrix} I & 0 \\ I & J_{k2} \end{bmatrix}\begin{bmatrix} \Omega(h) & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} A_\beta^T & 0 \\ 0 & A_{f\beta}^T \end{bmatrix}\begin{bmatrix} t_4^T & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} I & I \\ 0 & J_{k2}^T \end{bmatrix}$$

$$=-\begin{bmatrix} \pi_{k1} & \pi_{k2} \\ \pi_{k1}+J_{k2} & \pi_{k2} \end{bmatrix}-\begin{bmatrix} t_4A_\beta & 0 \\ t_4A_\beta & 0 \end{bmatrix}\begin{bmatrix} \Omega^T(h) & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} I & I \\ 0 & J_{k2}^T \end{bmatrix}-\begin{bmatrix} I & 0 \\ I & J_{k2} \end{bmatrix}\begin{bmatrix} \Omega(h) & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} A_\beta^Tt_4^T & A_\beta^Tt_4^T \\ 0 & 0 \end{bmatrix}$$

$$=-\begin{bmatrix} \pi_{k1} & \pi_{k2} \\ \pi_{k1}+J_{k2} & \pi_{k2} \end{bmatrix}-\begin{bmatrix} t_4A_\beta\Omega^T(h) & t_4A_\beta\Omega^T(h) \\ t_4A_\beta\Omega^T(h) & t_4A_\beta\Omega^T(h) \end{bmatrix}-\begin{bmatrix} \Omega(h)A_\beta^Tt_4^T & \Omega(h)A_\beta^Tt_4^T \\ \Omega(h)A_\beta^Tt_4^T & \Omega(h)A_\beta^Tt_4^T \end{bmatrix}$$

$$=\begin{bmatrix} -\pi_{k1}-t_4A_\beta\Omega^T(h)-\Omega(h)A_\beta^Tt_4^T & -\pi_{k2}-t_4A_\beta\Omega^T(h)-\Omega(h)A_\beta^Tt_4^T \\ -\pi_{k1}-J_{k2}-t_4A_\beta\Omega^T(h)-\Omega(h)A_\beta^Tt_4^T & -\pi_{k2}-t_4A_\beta\Omega^T(h)-\Omega(h)A_\beta^Tt_4^T \end{bmatrix}$$

Manipulating term (4,6)

$$J_k^T \Omega_e^T(h) D_{e\beta}^T K J_k = J_k^T \Omega_e^T(h) D_{e\beta}^T \pi_k$$

$$= \begin{bmatrix} I & 0 \\ I & J_{k2} \end{bmatrix} \begin{bmatrix} \Omega^T(h) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} D_\beta^T & 0 \\ 0 & D_{f\beta}^T \end{bmatrix} \begin{bmatrix} \pi_{k1} & \pi_{k2} \\ I & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \Omega^T(h) & 0 \\ \Omega^T(h) & J_{k2} \end{bmatrix} \begin{bmatrix} D_\beta^T & 0 \\ 0 & D_{f\beta}^T \end{bmatrix} \begin{bmatrix} \pi_{k1} & \pi_{k2} \\ I & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \Omega^T(h) D_\beta^T & 0 \\ \Omega^T(h) D_\beta^T & J_{k2} D_{f\beta}^T \end{bmatrix} \begin{bmatrix} \pi_{k1} & \pi_{k2} \\ I & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \Omega^T(h) D_\beta^T \pi_{k1} & \Omega^T(h) D_\beta^T \pi_{k2} \\ \Omega^T(h) D_\beta^T \pi_{k1} + J_{k2} D_{f\beta}^T & 0 \end{bmatrix}$$

Define

$$\bar{D}_{f\beta} = J_{k2} D_{f\beta}^T$$

Then

$$Term(4,6) = \begin{bmatrix} \Omega^T(h) D_\beta^T \pi_{k1} & \Omega^T(h) D_\beta^T \pi_{k2} \\ \Omega^T(h) D_\beta^T \pi_{k1} + \bar{D}_{f\beta} & 0 \end{bmatrix}$$

Manipulating term (4,7)

$$J_k^T \Omega_e^T(h) L_{e\beta}^T = \begin{bmatrix} I & 0 \\ I & J_{k2} \end{bmatrix} \begin{bmatrix} \Omega^T(h) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} L_\beta & -L_{f\beta} \end{bmatrix}^T$$

$$= \begin{bmatrix} \Omega^T(h) & 0 \\ \Omega^T(h) & J_{k2} \end{bmatrix} \begin{bmatrix} L_\beta^T \\ -L_{f\beta}^T \end{bmatrix}$$

$$= \begin{bmatrix} \Omega^T(h) L_\beta^T \\ \Omega^T(h) L_\beta^T - J_{k2} L_{f\beta}^T \end{bmatrix}$$

Define

$$\bar{L}_{f\beta} = J_{k2} L_{f\beta}^T$$

Then

$$Term(4,7) = \begin{bmatrix} \Omega^T(h) L_\beta^T \\ \Omega^T(h) L_\beta^T - \bar{L}_{f\beta} \end{bmatrix}$$

Manipulating term (5,6)

$$D_{ew}^T K J_k = D_{ew}^T \pi_k$$

$$= \begin{bmatrix} D_w^T & B_w^T D_{f\beta}^T \end{bmatrix} \begin{bmatrix} \pi_{k1} & \pi_{k2} \\ I & 0 \end{bmatrix}$$

$$= \begin{bmatrix} D_w^T \pi_{k1} + B_w^T D_{f\beta}^T & D_w^T \pi_{k2} \end{bmatrix}$$

Manipulating term (6,6)

$$
\begin{aligned}
-J_k^T KJ_k &= -J_k^T \pi_k \\
&= -\begin{bmatrix} I & 0 \\ I & J_{k2} \end{bmatrix}\begin{bmatrix} \pi_{k1} & \pi_{k2} \\ I & 0 \end{bmatrix} \\
&= \begin{bmatrix} -\pi_{k1} & -\pi_{k2} \\ -\pi_{k1} - J_{k2} & -\pi_{k2} \end{bmatrix}
\end{aligned}
$$

Combining these terms, with direct substitution for the rest, the result will be (5-2). ∇

5.4 Simulation Example-1

We take example-1 from BAOYONG ZHANG,2007 [1.5]. He considers the parameter-varying LFT delayed system (4.4), with the following parameter:

$$
\mathrm{A} = \begin{bmatrix} -2 & 1.099 \\ -1.635 & -3 \end{bmatrix};\ \mathrm{A_d} = \begin{bmatrix} -0.481 & 0.583 \\ -0.837 & 0.102 \end{bmatrix};\ \mathrm{A}_\beta = \begin{bmatrix} 0.012 & 0 \\ -0.08 & 0.255 \end{bmatrix}
$$

$$
\mathrm{L} = \begin{bmatrix} 1.013 & 0 \\ 0 & -0.998 \end{bmatrix};\ \mathrm{L_d} = \begin{bmatrix} 0.001 & 0 \\ 0 & -0.021 \end{bmatrix};\ \mathrm{L}_\beta = \begin{bmatrix} 0.653 & 0 \\ 0 & 0.442 \end{bmatrix}
$$

$$
\mathrm{D} = \begin{bmatrix} 0 & 0.498 \\ 0.099 & -0.001 \end{bmatrix};\ \mathrm{D_d} = \begin{bmatrix} -0.061 & 0.01 \\ 0 & .005 \end{bmatrix};\ \mathrm{D}_\beta = \begin{bmatrix} 0.042 & -0.07 \\ -0.021 & -0.1 \end{bmatrix}
$$

$$
\mathrm{B} = \begin{bmatrix} 1.001 & 1.894 \\ 0 & 0 \end{bmatrix};\ \mathrm{B_d} = \begin{bmatrix} 0 & 0 \end{bmatrix}
$$

$$
\tau = .09;\ \mu = .09;\ \gamma = 0.2913
$$

$$
\Omega_1 = [1\ 0;0\ 0];\ \upsilon_1 = 1 - |\sin(x_1)|
$$

$$
\Omega_2 = [0\ 0;0\ 1];\ \upsilon_2 = |\sin(x_1)|
$$

where x_1 is the first element of state x .

5.4.1 The LMI Solution of Example-1

Using the MATLAB LMI toolbox 7.6.0 (R2008a) to solve LMI (5.2) [See Appendix-3]. The filter has the following parameters

$$A_f = \begin{bmatrix} -2.5418 & 0.0673 \\ 0.0673 & -2.7178 \end{bmatrix};\ A_{fy} = \begin{bmatrix} 1.7439 & -0.3736 \\ -0.3736 & 0.2451 \end{bmatrix}; A_{f\beta} = \begin{bmatrix} -0.0007 & 0.0025 \\ 0.0027 & 0.0112 \end{bmatrix}$$

$$A_{fd} = \begin{bmatrix} 0.3023 & 0.0487 \\ 0.0487 & -0.1372 \end{bmatrix}; D_f = \begin{bmatrix} -0.0042 & -0.0010 \\ -0.0010 & -0.0767 \end{bmatrix}; D_{fy} = \begin{bmatrix} -0.0590 & -1.2271 \\ -1.2271 & 0.0444 \end{bmatrix}$$

$$D_{f\beta} = \begin{bmatrix} -0.0117 & -0.0089 \\ -0.0096 & -0.0141 \end{bmatrix}; D_{fd} = \begin{bmatrix} 0.0012 & -0.0127 \\ -0.0127 & -0.0054 \end{bmatrix}; L_f = \begin{bmatrix} 0.0085 & -0.0147 \\ -0.0147 & 0.0454 \end{bmatrix}$$

$$L_{fy} = \begin{bmatrix} 1.0307 & -0.0190 \\ -0.0190 & -0.4719 \end{bmatrix}; L_{fd} = \begin{bmatrix} 0.0001 & 0.0001 \\ 0.0001 & -0.0018 \end{bmatrix}; L_{f\beta} = \begin{bmatrix} 0.0064 & 0.0000 \\ 0.0000 & -0.0048 \end{bmatrix}$$

5.4.2 Simulation Results of Example-1

The plant system (4.4) and the error (The augmented) system (4.9) are implemented using SIMULINK. [See Appendix-4, and 5]. The results were as follows:

1- The States

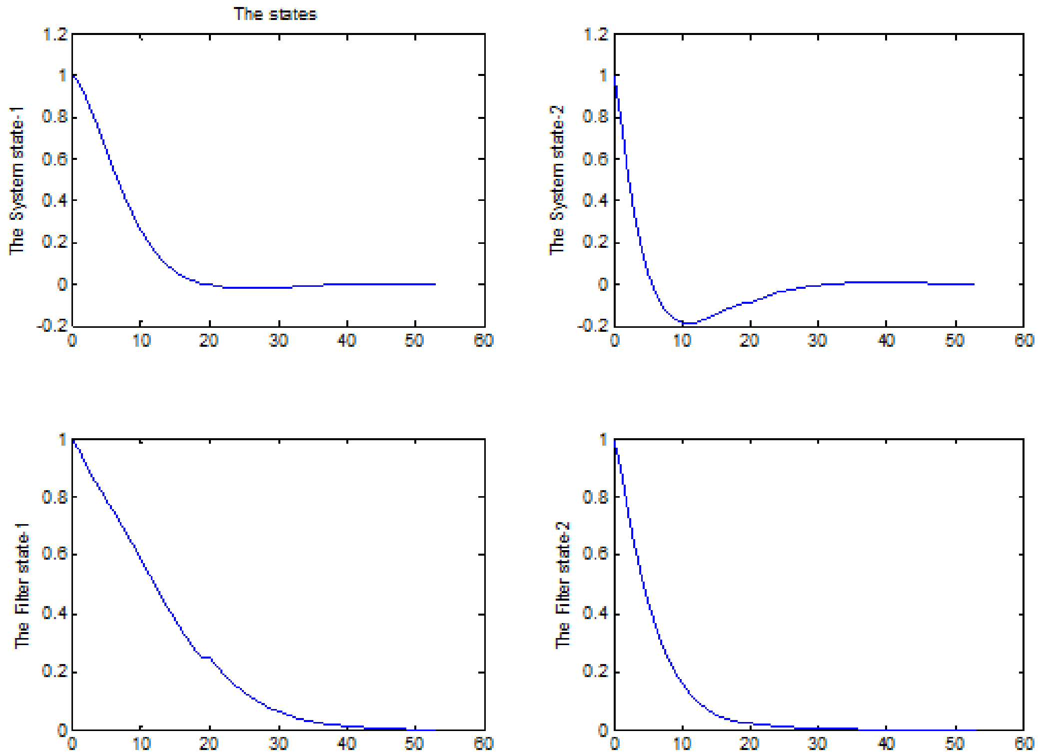


Fig. 5.1: The State Trajectories of the Simulation Example-1

2- The Outputs

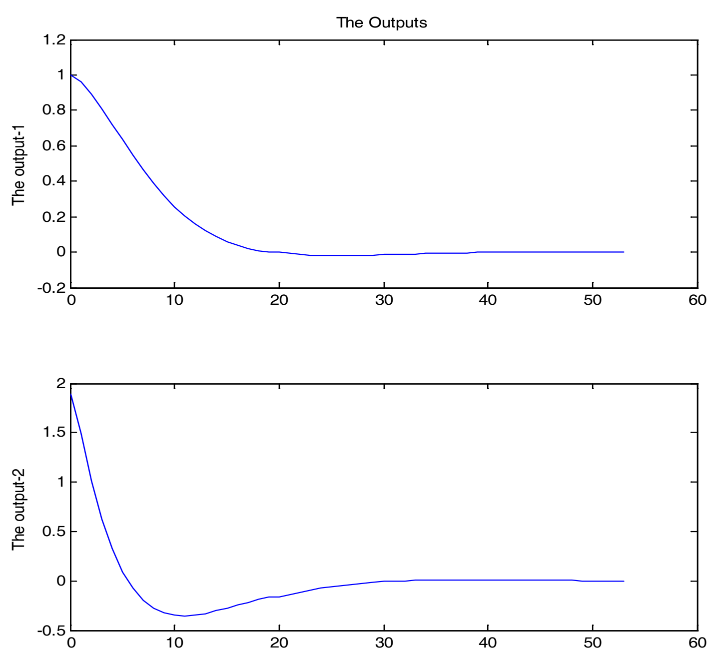


Fig. 5.2: The Output Trajectories of the Simulation Example-1

3- The first signal to be estimated, its estimate, & the error

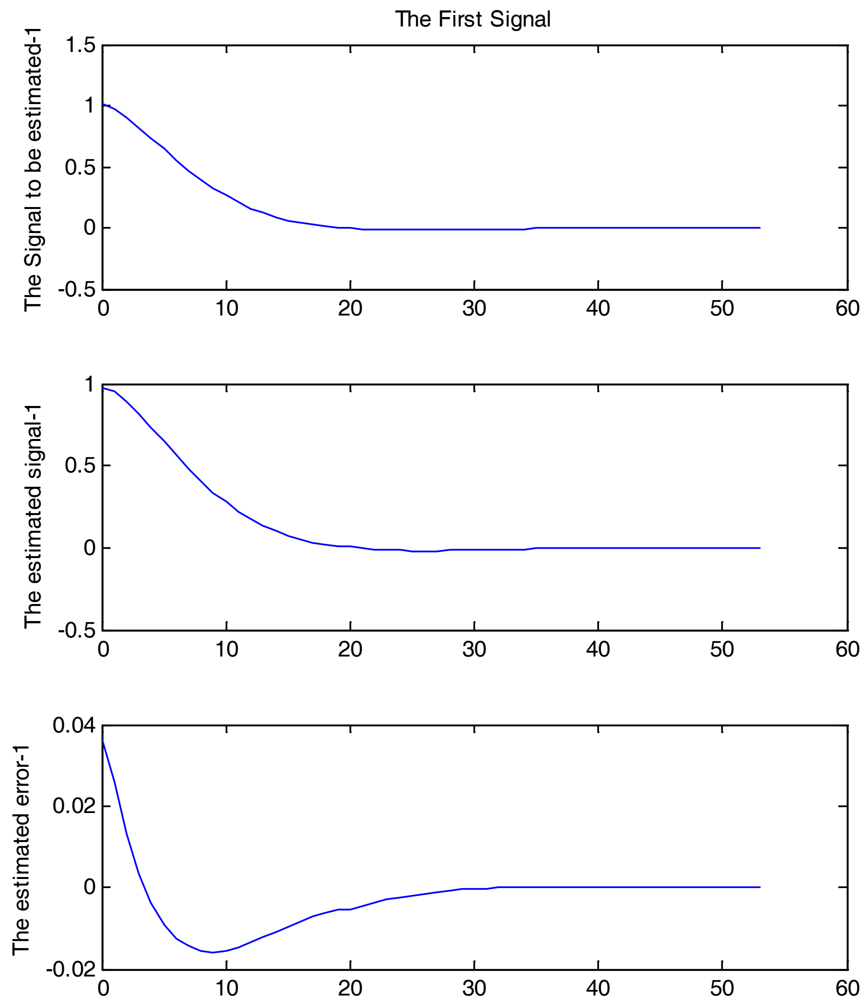


Fig. 5.3: The first signal to be estimated, its estimate, & the error Trajectories of the Simulation Example-1

4- The second signal to be estimated, its estimate, & the error

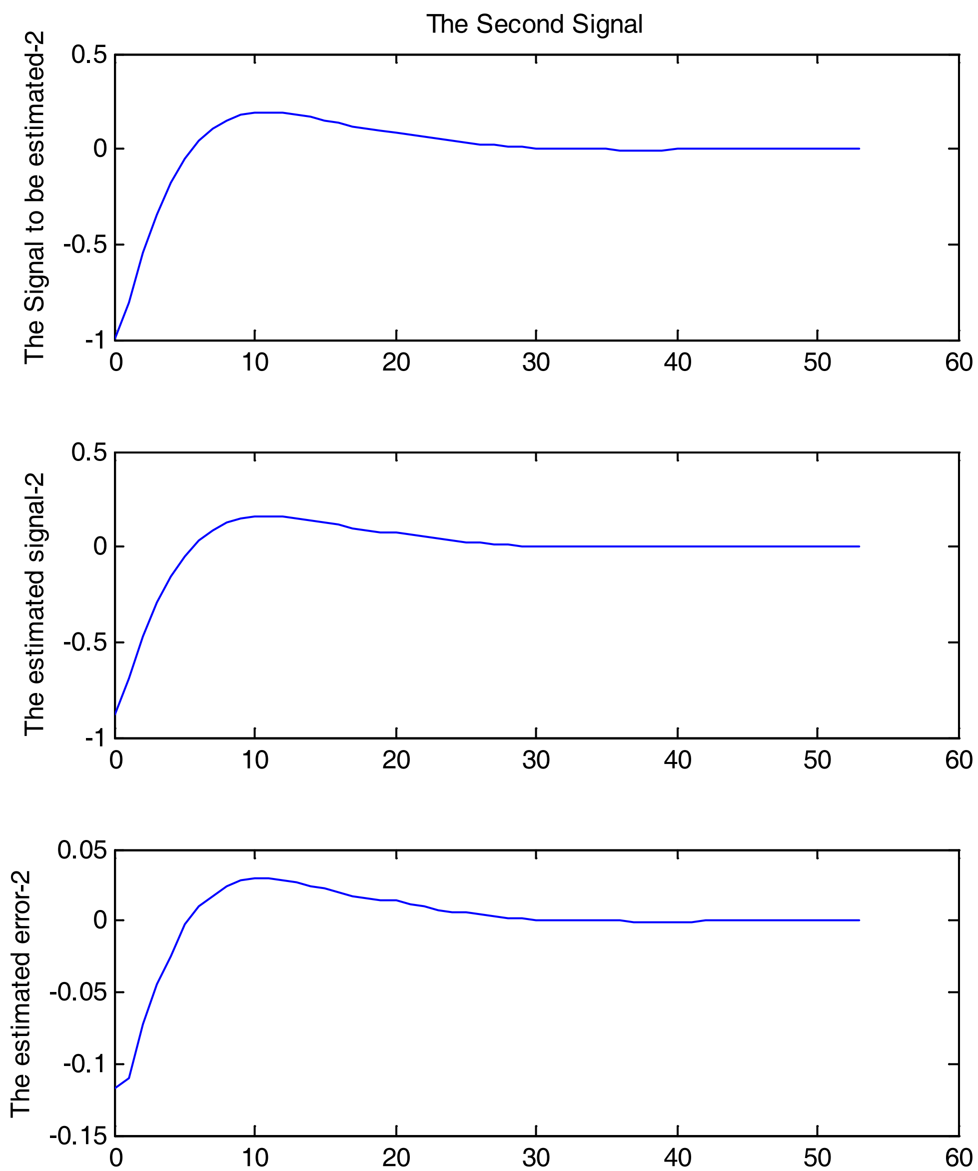


Fig. 5.4: The second signal to be estimated, its estimate, & the error Trajectories of the Simulation Example-1

5.5 Simulation Example-2

The second example is a fourth-order building of example-1. The fourth-order building is done as follows:

$$A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}$$

where

A: is 4x4 matrix and

A_1 : The original matrix of example-1 (2x2)

A_2=0.1*A_1

A_3=0.1*A_4

A_4=1.01A_1

Then the structure of the parameter-varying LFT delayed system (4.4) will be fourth order with the following parameter:

$$A = \begin{bmatrix} -2 & 1.099 & -0.2 & 0.11 \\ -1.635 & -3 & -0.164 & -0.3 \\ -0.202 & 0.111 & -2.02 & 1.11 \\ -0.165 & -0.303 & -1.651 & -3.03 \end{bmatrix}, A_d = \begin{bmatrix} -0.481 & 0.583 & -0.048 & 0.058 \\ -0.837 & 0.102 & -0.084 & 0.010 \\ -0.049 & 0.059 & -0.486 & 0.589 \\ -0.085 & 0.010 & -0.845 & 0.103 \end{bmatrix}$$

$$A_\beta = \begin{bmatrix} 0.012 & 0.000 & 0.001 & 0.000 \\ -0.080 & 0.255 & -0.008 & 0.026 \\ 0.001 & 0.000 & 0.012 & 0.000 \\ -0.008 & 0.026 & -0.081 & 0.258 \end{bmatrix}, L = \begin{bmatrix} 1.013 & 0.000 & 0.101 & 0.000 \\ 0.000 & -0.998 & 0.000 & -0.100 \\ 0.102 & 0.000 & 1.023 & 0.000 \\ 0.000 & -0.101 & 0.000 & -1.008 \end{bmatrix}$$

$$L_d = \begin{bmatrix} 0.001 & 0.000 & 0.000 & 0.000 \\ 0.000 & -0.021 & 0.000 & -0.002 \\ 0.000 & 0.000 & 0.001 & 0.000 \\ 0.000 & -0.002 & 0.000 & -0.021 \end{bmatrix}, L_\beta = \begin{bmatrix} 0.653 & 0.000 & 0.065 & 0.000 \\ 0.000 & 0.442 & 0.000 & 0.044 \\ 0.066 & 0.000 & 0.660 & 0.000 \\ 0.000 & 0.045 & 0.000 & 0.446 \end{bmatrix}$$

$$D = \begin{bmatrix} 0.000 & 0.498 & 0.000 & 0.050 \\ 0.099 & -0.001 & 0.010 & 0.000 \\ 0.000 & 0.050 & 0.000 & 0.503 \\ 0.010 & 0.000 & 0.100 & -0.001 \end{bmatrix}, D_d = \begin{bmatrix} -0.061 & 0.010 & -0.006 & 0.001 \\ 0.000 & 0.005 & 0.000 & 0.001 \\ -0.006 & 0.001 & -0.062 & 0.010 \\ 0.000 & 0.001 & 0.000 & 0.005 \end{bmatrix}$$

$$D_\beta = \begin{bmatrix} 0.042 & -0.070 & 0.004 & -0.007 \\ -0.021 & -0.100 & -0.002 & -0.010 \\ 0.004 & -0.007 & 0.042 & -0.071 \\ -0.002 & -0.010 & -0.021 & -0.101 \end{bmatrix}, B = \begin{bmatrix} 1.001 & 1.894 & 0.100 & 0.189 \\ 0.000 & 0.000 & 0.000 & 0.000 \\ 0.101 & 0.191 & 1.011 & 1.913 \\ 0.000 & 0.000 & 0.000 & 0.000 \end{bmatrix}$$

$$\tau = 0.09, \mu = 0.09, \gamma = 0.2913, \Omega = [1\,0\,0\,0; 0\,0\,0\,0; 0\,0\,0\,0; 0\,0\,0\,1]$$

and the other dynamics matrices of system (4.4) are zero with suitable dimensions.

5.5.1 The LMI Solution of example-2

MATLAB LMI toolbox 7.6.0 (R2008a) is used to solve LMI (5-2) that get the filter parameters. The augmented system (4.9) is simulated using SIMULINK

$$A_f = \begin{bmatrix} -3.5961 & 0.0142 & -0.0212 & 0.0113 \\ 0.0142 & -3.2858 & -0.0122 & 0.0062 \\ -0.0212 & -0.0122 & -3.5848 & 0.1593 \\ 0.0113 & 0.0062 & 0.1593 & -3.8165 \end{bmatrix}, A_{fd} = \begin{bmatrix} 0.2451 & -0.1126 & 0.0098 & -0.0141 \\ -0.1126 & -0.0775 & -0.0085 & -0.0033 \\ 0.0098 & -0.0085 & 0.3130 & 0.0744 \\ -0.0141 & -0.0033 & 0.0744 & -0.0761 \end{bmatrix}$$

$$A_{f\beta} = \begin{bmatrix} 0.0010 & 0.0038 & 0.0001 & 0.0004 \\ 0.0040 & 0.0002 & 0.0003 & 0.0018 \\ 0.0001 & 0.0003 & 0.0001 & 0.0011 \\ 0.0003 & 0.0019 & 0.0009 & 0.0105 \end{bmatrix}, A_{fy} = \begin{bmatrix} -0.0000 & 0.0002 & 0.0000 & 0.0000 \\ 0.0002 & -0.0954 & 0.0000 & -0.2185 \\ 0.0000 & 0.0000 & -0.0000 & -0.0003 \\ 0.0000 & -0.2185 & -0.0003 & 4.3582 \end{bmatrix}$$

$$D_f = \begin{bmatrix} -0.2742 & -0.0180 & -0.0216 & 0.0038 \\ -0.0180 & -0.0020 & -0.0068 & 0.0026 \\ -0.0216 & -0.0068 & -0.2738 & 0.0031 \\ 0.0038 & 0.0026 & 0.0031 & 0.0124 \end{bmatrix}, D_{fd} = \begin{bmatrix} -0.0346 & 0.0043 & -0.0030 & 0.0005 \\ 0.0043 & 0.0001 & 0.0005 & 0.0001 \\ -0.0030 & 0.0005 & -0.0432 & -0.0067 \\ 0.0005 & 0.0001 & -0.0067 & -0.0002 \end{bmatrix}$$

$$D_{f\beta} = \begin{bmatrix} -0.0155 & 0.0029 & -0.0004 & -0.0006 \\ 0.0028 & 0.0002 & 0.0001 & -0.0020 \\ -0.0003 & 0.0001 & -0.0011 & -0.0116 \\ -0.0007 & -0.0019 & -0.0122 & -0.0058 \end{bmatrix}, D_{fy} = \begin{bmatrix} -0.0001 & 0.0001 & 0.0000 & 0.0000 \\ 0.0001 & -1.2842 & 0.0000 & -0.0062 \\ 0.0000 & 0.0000 & -0.0001 & 0.0001 \\ 0.0000 & -0.0062 & 0.0001 & -1.3890 \end{bmatrix}$$

$$L_f = \begin{bmatrix} -0.1898 & 0.0019 & -0.0213 & 0.0029 \\ 0.0019 & 0.0010 & -0.0034 & 0.0070 \\ -0.0213 & -0.0034 & -0.1863 & -0.0046 \\ 0.0029 & 0.0070 & -0.0046 & 0.0479 \end{bmatrix}, L_{fd} = \begin{bmatrix} -0.0182 & 0.0022 & -0.0012 & 0.0004 \\ 0.0022 & 0.0001 & 0.0004 & 0.0002 \\ -0.0012 & 0.0004 & -0.0263 & -0.0046 \\ 0.0004 & 0.0002 & -0.0046 & 0.0003 \end{bmatrix}$$

$$L_{f\beta} = \begin{bmatrix} 0.0052 & -0.0001 & 0.0003 & -0.0000 \\ -0.0000 & 0.0000 & 0.0000 & -0.0002 \\ 0.0003 & -0.0000 & 0.0000 & 0.0001 \\ 0.0000 & -0.0002 & 0.0002 & -0.0032 \end{bmatrix}, L_{fy} = \begin{bmatrix} 0.0000 & -0.0001 & 0.0000 & 0.0000 \\ -0.0001 & 1.1075 & 0.0000 & -0.0191 \\ 0.0000 & 0.0000 & 0.0000 & -0.0001 \\ 0.0000 & -0.0191 & -0.0001 & 0.9627 \end{bmatrix}$$

5.5.2 Simulation Results of example-2

The error (The augmented) system (4.9) of example-2 is implemented using SIMULINK. The results were as follows:

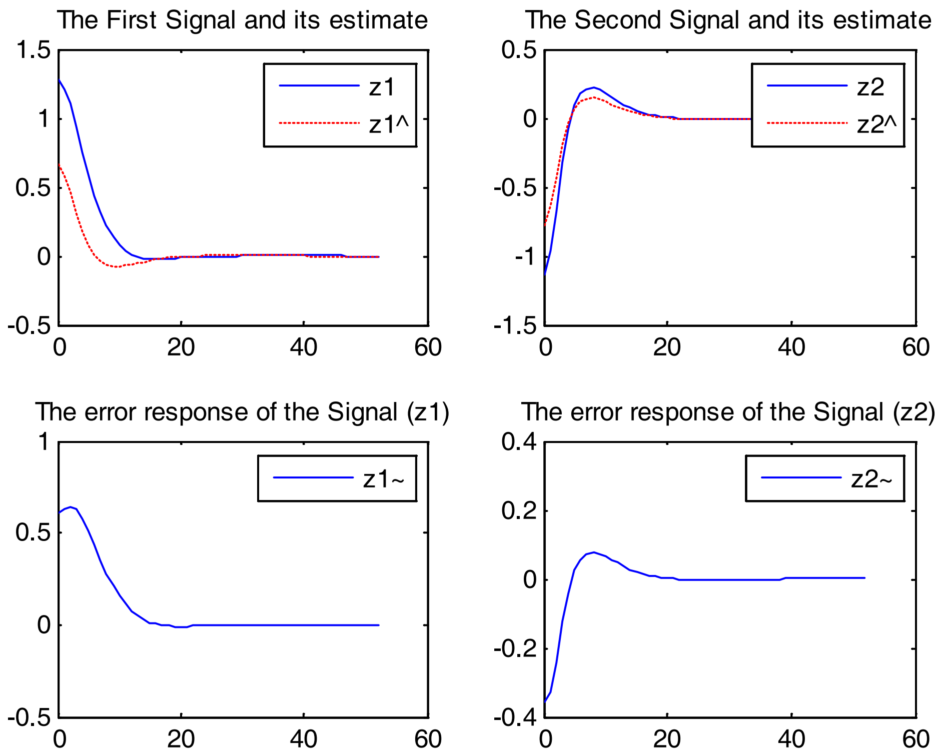


Fig. 5.5: The first and second signals to be estimated, their estimate, & the error Trajectories of the Simulation Example-2

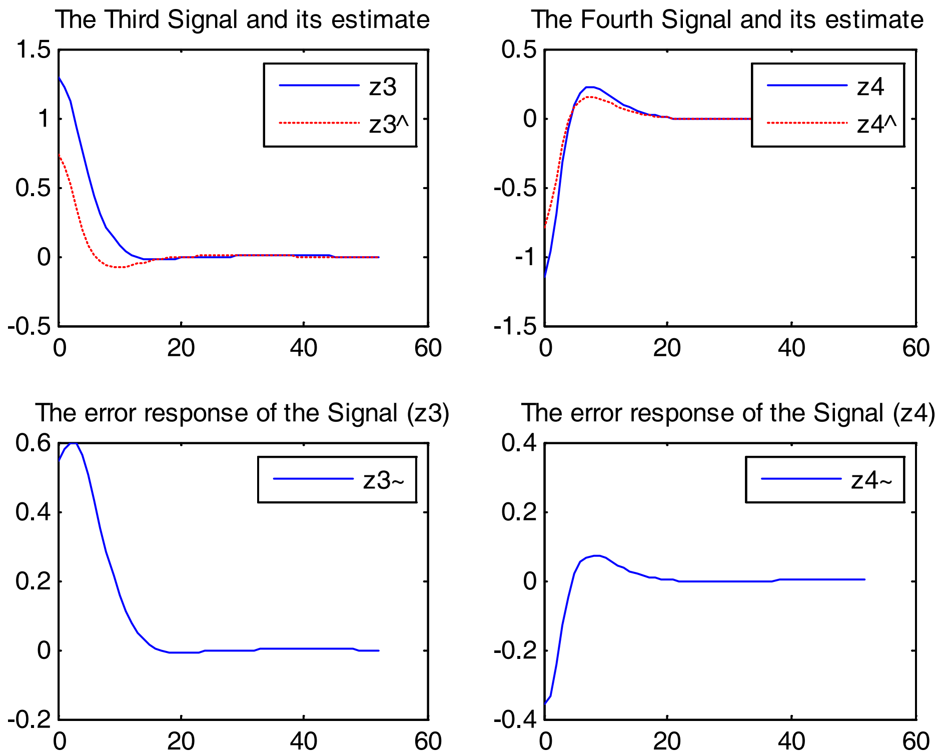


Fig. 5.6: The third and fourth signals to be estimated, their estimate, & the error Trajectories of the Simulation Example-2

5.6 Simulation Example-3

The third example is the example-1 of Velni and Grigoriadis [3.14], which is LPVTD system. In order to meet this example with our method structure, LFT will be applied, and then the resulting system (4.4) has the following state space matrices:

$$A=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}, A_\beta=\begin{bmatrix}0.2 & 0\\ 0.1 & 0\end{bmatrix}, A_d=\begin{bmatrix}0 & 0.1\\ -0.2 & -0.3\end{bmatrix}, A_w=\begin{bmatrix}0\\ -0.2\end{bmatrix}$$
$$D=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}, D_d=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}, D_w=\begin{bmatrix}-1\\ 0\end{bmatrix}, D_\beta=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$
$$L=\begin{bmatrix}0 & 1\\ 0.5 & 0\end{bmatrix}, L_\beta=\begin{bmatrix}0 & 0\\ 0.1 & 0\end{bmatrix}, L_d=\begin{bmatrix}0.5 & 0\\ 0 & 0\end{bmatrix}, L_w=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$
$$B=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, B_d=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, B_w=\begin{bmatrix}0 & 0.3\\ 0 & 0\end{bmatrix}$$

MATLAB LMI toolbox 7.6.0 (R2008a) is used to solve LMI (5.2) with the above matrices. The resulting filter parameters will be substituted in plant (4.4) to get the following results:

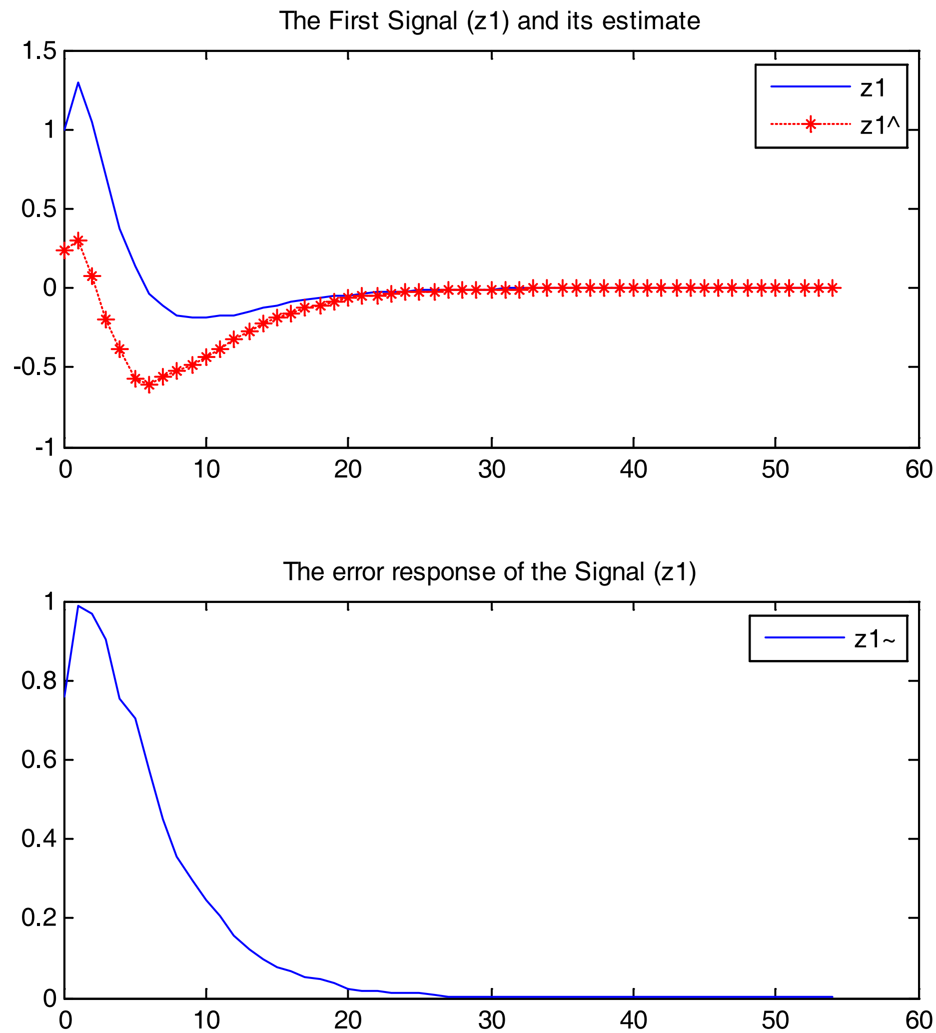


Fig. 5.7: The first signal to be estimated, its estimate, & the error Trajectory of the Simulation Example-3

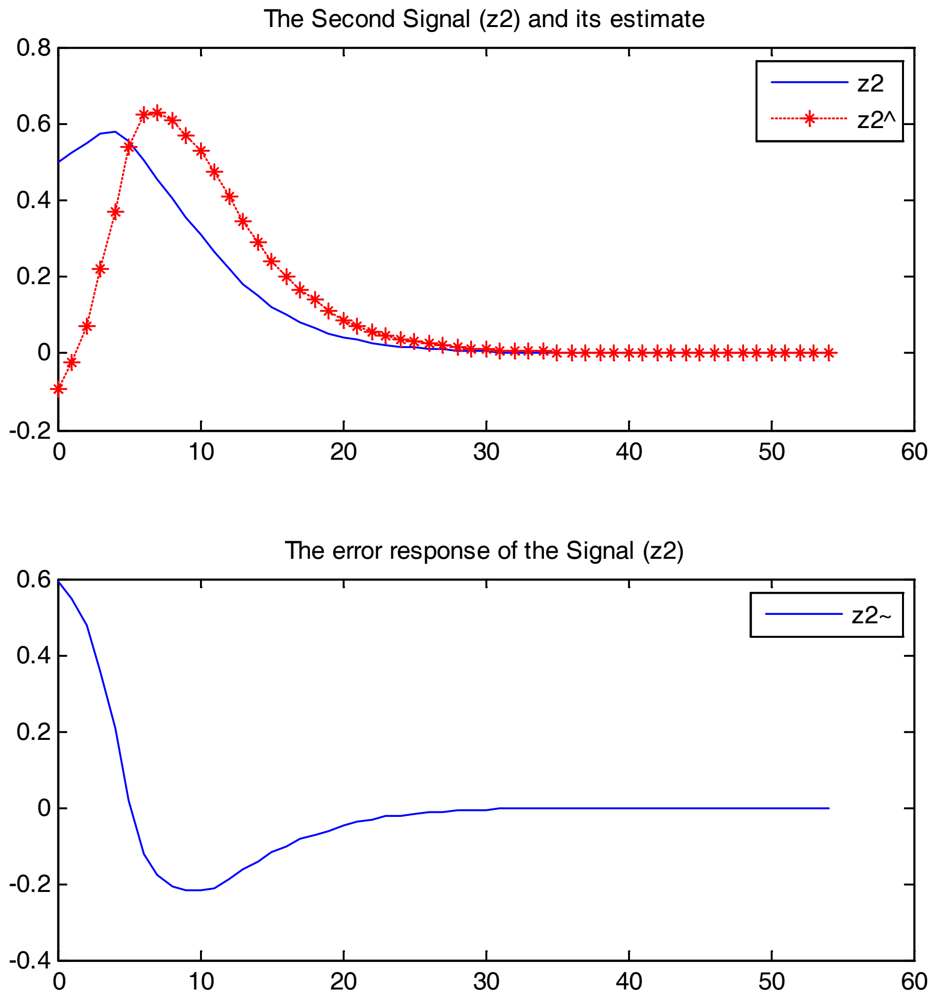


Fig. 5.8: The second signal to be estimated, its estimate, & the error Trajectory of the Simulation Example-3

CONCLUSION

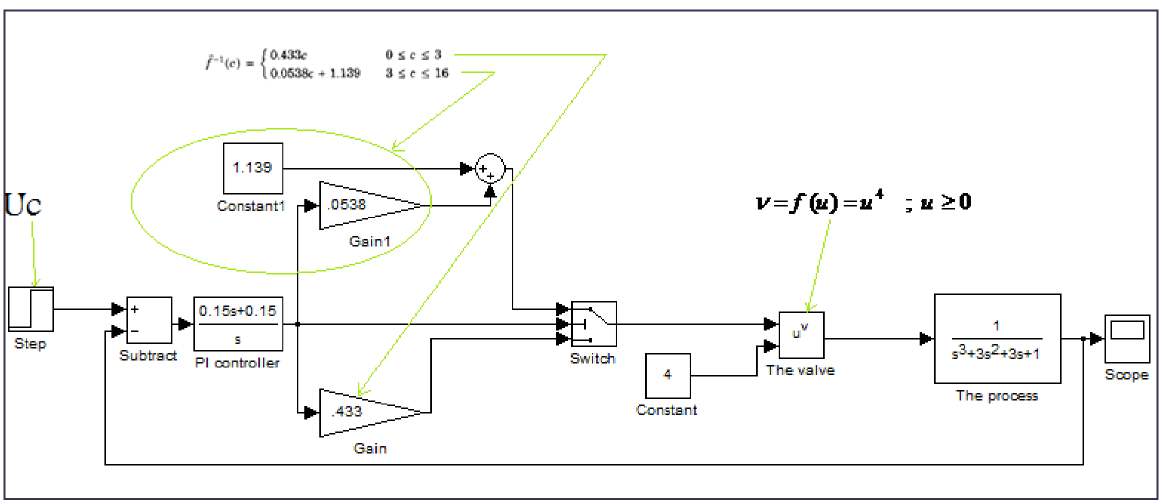
The problem, which is discussed in this Thesis, is gain-scheduled $H\infty$ filter design for a class of parameter-varying systems with time-varying delays. An improved stability and $H\infty$-performance condition of the augmented system was developed. A sufficient existence condition of a desired gain-scheduled $H\infty$ filter was derived and expressed in terms of feasibility testing of linear matrix inequalities (LMIs) and explicit expressions of the filter gains were provided. Three Numerical examples were presented to demonstrate the effectiveness of the proposed design method. The first two examples were 2nd order, while the third one was 4th order. The simulation of these examples was done using MATLAB. From the simulation results, it is evident that the magnitude of the estimation error is small and dies quickly thereby yielding good estimate of the state variables.

APPENDICES

Appendix 1: Example 2.1 m-file**:**

```
clc
u=0:.05:2.5;
v=u.^4;
plot(u,v);
axis([0 2 0 16]);
hold
x1=0:.05:1.3;
y1=3/1.3*x1;
x2=1.3:.05:2;
y2=13/.7*x2-21.1429;
plot(x1,y1,x2,y2);
```

Appendix 2: Example 2.1 Simulink-file**:**



Appendix 3: The LMI Solving m-file:

```
% Name :Alaeddin Khalil Mosa Jaradat
% Thesis of master degree in System Engineering
% Spring,2009
% Title:GAIN SCHEDULED FILTERING DESIGN FOR PARAMETER VARYING SYSTEM
%-----------------------------------------------------------------------
---

%----------------------- System Dynamics Matrices ----------------------
---
A=[-2 1.099;-1.635 -3];Ad=[-.481 .583;-.837 .102];AB=[.012 0;-.08 .255];
L=[1.013 0;0 -.998];Ld=[.001 0;0 -.021];LB=[.653 0;0 .442];
D=[0 .498;.099 -.001];Dd=[-.061 .01;0 .005];DB=[.042 -.07;-.021 -.1];
B=[1.001 0;0 1.894];Bd=[0 0];
Aw=[0 0;0 1];Bw=[0 0;0 0];Lw=[0 0 ; 0 0];Dw=[0 0 ; 0 0];
T=[.09 0;0 .09];Mu=.09;Mu1=[1-Mu 0;0 1-Mu];
ZZ=[0 0;0 0];
ZZ1=[0 0;0 0];
omega=[-1 0;0 1];
gamma=.2913*[1 0;0 1];
I=[1 0;0 1];
%F1=[.01 0;0 0.1];
%F2=[.001 0;0 0.001];
%F3=[.005 0;0 0.05];
Jp2=I;
%--------------------The Variables declaring commands ------------------
---
         setlmis([]);
  Af=lmivar(1,[2 1]);
  Afy=lmivar(1,[2 1]);
  AfB=lmivar(1,[2 1]);
  Afd=lmivar(1,[2 1]);
  Df=lmivar(1,[2 1]);
  Dfy=lmivar(1,[2 1]);
  DfB=lmivar(1,[2 1]);
  Dfd=lmivar(1,[2 1]);
  Lf=lmivar(1,[2 1]);
  Lfy=lmivar(1,[2 1]);
  Lfd=lmivar(1,[2 1]);
  LfB=lmivar(1,[2 1]);
  Jk2=lmivar(1,[2 1]);

  Pp1=lmivar(1,[2 1]);
  Pp2=lmivar(1,[2 1]);
  Pk1=lmivar(1,[2 1]);
  Pk2=lmivar(1,[2 1]);
  Q1=lmivar(1,[2 1]);
  Q2=lmivar(1,[2 1]);
  Q3=lmivar(1,[2 1]);
  F1=lmivar(1,[2 1]);
  F2=lmivar(1,[2 1]);
  F3=lmivar(1,[2 1]);
  Z1=lmivar(1,[2 1]);
  Z2=lmivar(1,[2 1]);
  Z3=lmivar(1,[2 1]);
```

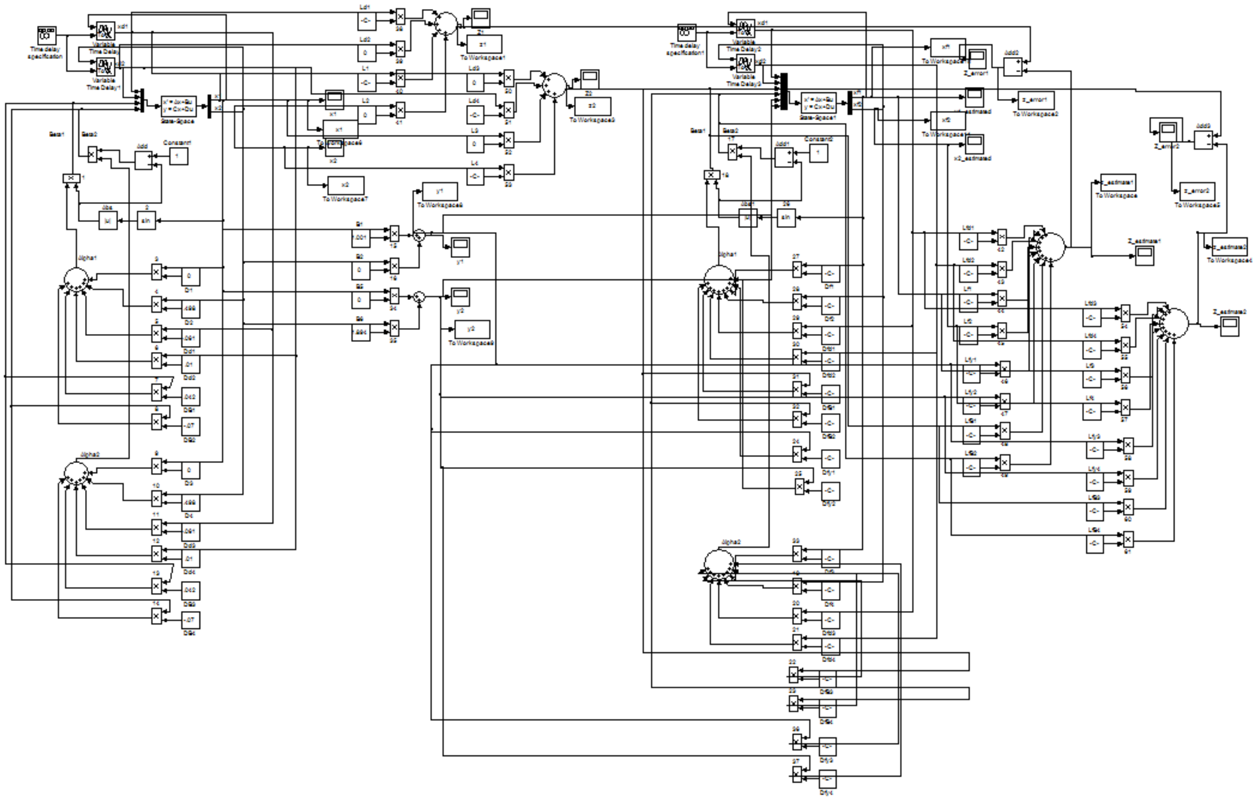
```
  X111=lmivar(1,[2 1]);
  X112=lmivar(1,[2 1]);
  X113=lmivar(1,[2 1]);
  X121=lmivar(1,[2 1]);
  X122=lmivar(1,[2 1]);
  X123=lmivar(1,[2 1]);
  X131=lmivar(1,[2 1]);
  X132=lmivar(1,[2 1]);
  X133=lmivar(1,[2 1]);
  X134=lmivar(1,[2 1]);
  X221=lmivar(1,[2 1]);
  X222=lmivar(1,[2 1]);
  X223=lmivar(1,[2 1]);
  X231=lmivar(1,[2 1]);
  X232=lmivar(1,[2 1]);
  X233=lmivar(1,[2 1]);
  X331=lmivar(1,[2 1]);
  X332=lmivar(1,[2 1]);
  X333=lmivar(1,[2 1]);
 %--------------- The Expression commands for the LMIs -----------------
---
   lmiterm([1 1 1 Pp1],1,A,'s');                        % LMI #1:
Pp1*A+A'*Pp1
  lmiterm([1 2 1 -Pp2],1,A);                            % LMI #1: Pp2'*A
  lmiterm([1 2 1 Afy],1,B);                             % LMI #1: Afy*B
  lmiterm([1 2 2 Af],1,1,'s');                          % LMI #1: Af+Af'
  lmiterm([1 3 1 -Pp1],Ad',1);                          % LMI #1: Ad'*Pp1'
  lmiterm([1 3 1 -X121],T,1);                           % LMI #1: T*X121'
  lmiterm([1 3 2 Pp2],Ad',1);                           % LMI #1: Ad'*Pp2
  lmiterm([1 3 2 -Afy],Bd',1);                          % LMI #1: Bd'*Afy'
  lmiterm([1 3 2 -X121],T,1);                           % LMI #1: T*X121'
  lmiterm([1 3 2 -X122],T,1);                           % LMI #1: T*X122'
  lmiterm([1 3 3 Q1],.5*(1-Mu),-1,'s');                 % LMI #1: -(1-Mu)*Q1
(NON SYMMETRIC?)
  lmiterm([1 3 3 X221],.5*T,1,'s');                     % LMI #1: T*X221 (NON
SYMMETRIC?)
  lmiterm([1 4 1 -X122],T,1);                           % LMI #1: T*X122'
  lmiterm([1 4 2 -Afd],1,1);                            % LMI #1: Afd'
  lmiterm([1 4 2 -X122],T,1);                           % LMI #1: T*X122'
  lmiterm([1 4 2 -X123],T,1);                           % LMI #1: T*X123'
  lmiterm([1 4 3 Q2],(1-Mu),-1);                        % LMI #1: -(1-Mu)*Q2
  lmiterm([1 4 3 X222],T,1);                            % LMI #1: T*X222
  lmiterm([1 4 4 Q3],.5*(1-Mu),-1,'s');                 % LMI #1: -(1-Mu)*Q3
(NON SYMMETRIC?)
  lmiterm([1 4 4 X223],.5*T,1,'s');                     % LMI #1: T*X223 (NON
SYMMETRIC?)
  lmiterm([1 5 1 -X131],T,1);                           % LMI #1: T*X131'
  lmiterm([1 5 2 -X133],T,1);                           % LMI #1: T*X133'
  lmiterm([1 5 3 -X231],T,1);                           % LMI #1: T*X231'
  lmiterm([1 5 4 -X232],T,1);                           % LMI #1: T*X232'
  lmiterm([1 5 5 Z1],.5*T,1,'s');                       % LMI #1: T*Z1 (NON
SYMMETRIC?)
  lmiterm([1 5 5 X331],.5*T,1,'s');                     % LMI #1: T*X331 (NON
SYMMETRIC?)
  lmiterm([1 6 1 -X132],T,1);                           % LMI #1: T*X132'
  lmiterm([1 6 2 -X133],T,1);                           % LMI #1: T*X133'
  lmiterm([1 6 3 -X232],T,1);                           % LMI #1: T*X232'
```

```
  lmiterm([1 6 4 -X233],T,1);                       % LMI #1: T*X233'
  lmiterm([1 6 5 Z2],T,1);                          % LMI #1: T*Z2
  lmiterm([1 6 5 X332],T,1);                        % LMI #1: T*X332
  lmiterm([1 6 6 Z3],.5*T,1,'s');                   % LMI #1: T*Z3 (NON
SYMMETRIC?)
  lmiterm([1 6 6 X333],.5*T,1,'s');                 % LMI #1: T*X333 (NON
SYMMETRIC?)
  lmiterm([1 7 1 -Pp1],omega'*AB',1);               % LMI #1:
omega'*AB'*Pp1'
  lmiterm([1 7 2 -Pp2],omega'*AB',1);               % LMI #1:
omega'*AB'*Pp2'
  lmiterm([1 7 3 0],ZZ);                            % LMI #1: ZZ
  lmiterm([1 7 4 0],ZZ);                            % LMI #1: ZZ
  lmiterm([1 7 5 0],ZZ);                            % LMI #1: ZZ
  lmiterm([1 7 6 0],ZZ);                            % LMI #1: ZZ
  lmiterm([1 7 7 Pp1],1,-1);                        % LMI #1: -Pp1
  lmiterm([1 8 1 -Pp1],omega'*AB',1);               % LMI #1:
omega'*AB'*Pp1'
  lmiterm([1 8 2 -Pp2],omega'*AB',1);               % LMI #1:
omega'*AB'*Pp2'
  lmiterm([1 8 2 -AfB],1,1);                        % LMI #1: AfB'
  lmiterm([1 8 3 0],ZZ);                            % LMI #1: ZZ
  lmiterm([1 8 4 0],ZZ);                            % LMI #1: ZZ
  lmiterm([1 8 5 0],ZZ);                            % LMI #1: ZZ
  lmiterm([1 8 6 0],ZZ);                            % LMI #1: ZZ
  lmiterm([1 8 7 Pp2],1,-1);                        % LMI #1: -Pp2
  lmiterm([1 8 8 Pp2],1,-1);                        % LMI #1: -Pp2
  lmiterm([1 9 1 -Pp1],Aw',1);                      % LMI #1: Aw'*Pp1'
  lmiterm([1 9 2 Pp2],Aw',1);                       % LMI #1: Aw'*Pp2
  lmiterm([1 9 2 -Afy],Bw',1);                      % LMI #1: Bw'*Afy'
  lmiterm([1 9 3 0],ZZ1);                           % LMI #1: ZZ1
  lmiterm([1 9 4 0],ZZ1);                           % LMI #1: ZZ1
  lmiterm([1 9 5 0],ZZ1);                           % LMI #1: ZZ1
  lmiterm([1 9 6 0],ZZ1);                           % LMI #1: ZZ1
  lmiterm([1 9 7 0],ZZ1);                           % LMI #1: ZZ1
  lmiterm([1 9 8 0],ZZ1);                           % LMI #1: ZZ1
  lmiterm([1 9 9 0],-gamma);                        % LMI #1: -gamma
  lmiterm([1 10 1 -Pk1],1,D);                       % LMI #1: Pk1'*D
  lmiterm([1 10 1 Dfy],1,B);                        % LMI #1: Dfy*B
  lmiterm([1 10 2 -Df],1,1);                        % LMI #1: Df'
  lmiterm([1 10 3 -Pk1],1,Dd);                      % LMI #1: Pk1'*Dd
  lmiterm([1 10 3 Dfy],1,Bd);                       % LMI #1: Dfy*Bd
  lmiterm([1 10 4 Dfd],1,1);                        % LMI #1: Dfd
  lmiterm([1 10 5 0],ZZ);                           % LMI #1: ZZ
  lmiterm([1 10 6 0],ZZ);                           % LMI #1: ZZ
  lmiterm([1 10 7 Pp1],1,DB*omega);                 % LMI #1: Pp1*DB*omega
  lmiterm([1 10 8 Pp1],1,DB*omega);                 % LMI #1: Pp1*DB*omega
  lmiterm([1 10 8 DfB],1,1);                        % LMI #1: DfB
  lmiterm([1 10 9 -Pk1],1,Dw);                      % LMI #1: Pk1'*Dw
  lmiterm([1 10 9 DfB],1,Bw);                       % LMI #1: DfB*Bw
  lmiterm([1 10 10 Pk1],1,-1);                      % LMI #1: -Pk1
  lmiterm([1 11 1 -Pk2],1,D);                       % LMI #1: Pk2'*D
  lmiterm([1 11 2 0],ZZ);                           % LMI #1: ZZ
  lmiterm([1 11 3 -Pk2],1,Dd);                      % LMI #1: Pk2'*Dd
  lmiterm([1 11 4 0],ZZ);                           % LMI #1: ZZ
  lmiterm([1 11 5 0],ZZ);                           % LMI #1: ZZ
  lmiterm([1 11 6 0],ZZ);                           % LMI #1: ZZ
```

```
  lmiterm([1 11 7 Pp2],1,DB*omega);                % LMI #1: Pp2*DB*omega
  lmiterm([1 11 8 0],ZZ);                          % LMI #1: ZZ
  lmiterm([1 11 9 -Pk2],1,Dw);                     % LMI #1: Pk2'*Dw
  lmiterm([1 11 10 Pk1],1,-1);                     % LMI #1: -Pk1
  lmiterm([1 11 10 Jk2],1,-1);                     % LMI #1: -Jk2
  lmiterm([1 11 11 Pk2],1,-1);                     % LMI #1: -Pk2
  lmiterm([1 12 1 Lfy],1,-B);                      % LMI #1: -Lfy*B
  lmiterm([1 12 1 0],L);                           % LMI #1: L
  lmiterm([1 12 2 Lf],1,-1);                       % LMI #1: -Lf
  lmiterm([1 12 3 Lfy],1,-Bd);                     % LMI #1: -Lfy*Bd
  lmiterm([1 12 3 0],Ld');                         % LMI #1: Ld'
  lmiterm([1 12 4 -Lfd],1,-1);                     % LMI #1: -Lfd'
  lmiterm([1 12 5 0],ZZ1);                         % LMI #1: ZZ1
  lmiterm([1 12 6 0],ZZ1);                         % LMI #1: ZZ1
  lmiterm([1 12 7 0],LB*omega);                    % LMI #1: LB*omega
  lmiterm([1 12 8 LfB],1,-Jk2');                   % LMI #1: -LfB*Jk2'
  lmiterm([1 12 8 0],LB*omega);                    % LMI #1: LB*omega
  lmiterm([1 12 9 Lfy],1,-Bw);                     % LMI #1: -Lfy*Bw
  lmiterm([1 12 9 0],Lw);                          % LMI #1: Lw
  lmiterm([1 12 10 0],ZZ1);                        % LMI #1: ZZ1
  lmiterm([1 12 11 0],ZZ1);                        % LMI #1: ZZ1
  lmiterm([1 12 12 0],-gamma);                     % LMI #1: -gamma
  lmiterm([1 13 1 0],I);                           % LMI #1: I
  lmiterm([1 13 2 0],ZZ);                          % LMI #1: ZZ
  lmiterm([1 13 3 0],ZZ);                          % LMI #1: ZZ
  lmiterm([1 13 4 0],ZZ);                          % LMI #1: ZZ
  lmiterm([1 13 5 0],ZZ);                          % LMI #1: ZZ
  lmiterm([1 13 6 0],ZZ);                          % LMI #1: ZZ
  lmiterm([1 13 7 0],ZZ);                          % LMI #1: ZZ
  lmiterm([1 13 8 0],ZZ);                          % LMI #1: ZZ
  lmiterm([1 13 9 0],ZZ);                          % LMI #1: ZZ
  lmiterm([1 13 10 0],ZZ);                         % LMI #1: ZZ
  lmiterm([1 13 11 0],ZZ);                         % LMI #1: ZZ
  lmiterm([1 13 12 0],ZZ);                         % LMI #1: ZZ
  lmiterm([1 13 13 F1],1,-1);                      % LMI #1: -F1
  lmiterm([1 14 1 0],ZZ);                          % LMI #1: ZZ
  lmiterm([1 14 2 0],Jp2');                        % LMI #1: Jp2'
  lmiterm([1 14 3 0],ZZ);                          % LMI #1: ZZ
  lmiterm([1 14 4 0],ZZ);                          % LMI #1: ZZ
  lmiterm([1 14 5 0],ZZ);                          % LMI #1: ZZ
  lmiterm([1 14 6 0],ZZ);                          % LMI #1: ZZ
  lmiterm([1 14 7 0],ZZ);                          % LMI #1: ZZ
  lmiterm([1 14 8 0],ZZ);                          % LMI #1: ZZ
  lmiterm([1 14 9 0],ZZ);                          % LMI #1: ZZ
  lmiterm([1 14 10 0],ZZ);                         % LMI #1: ZZ
  lmiterm([1 14 11 0],ZZ);                         % LMI #1: ZZ
  lmiterm([1 14 12 0],ZZ);                         % LMI #1: ZZ
  lmiterm([1 14 13 F2],1,-1);                      % LMI #1: -F2
  lmiterm([1 14 14 F3],1,-1);                      % LMI #1: -F3

  TH0=getlmis;
    %------------------------ The Variables Results --------------------
-----

[aa, bb]=feasp(TH0);
```

```
Af=dec2mat(TH0,bb,Af);
Afy=dec2mat(TH0,bb,Afy);
AfB=dec2mat(TH0,bb,AfB);
Afd=dec2mat(TH0,bb,Afd);
Df=dec2mat(TH0,bb,Df);
Dfy=dec2mat(TH0,bb,Dfy);
DfB=dec2mat(TH0,bb,DfB);
Dfd=dec2mat(TH0,bb,Dfd);

  Lf=dec2mat(TH0,bb,Lf);
  Lfy=dec2mat(TH0,bb,Lfy);
  Lfd=dec2mat(TH0,bb,Lfd);
  LfB=dec2mat(TH0,bb,LfB);
  Jk2=dec2mat(TH0,bb,Jk2);



Pp1=dec2mat(TH0,bb,Pp1);
Pp2=dec2mat(TH0,bb,Pp2);
Pk1=dec2mat(TH0,bb,Pk1);
Pk2=dec2mat(TH0,bb,Pk2);

Z1=dec2mat(TH0,bb,Z1);
Z2=dec2mat(TH0,bb,Z2);
Z3=dec2mat(TH0,bb,Z3);
Q1=dec2mat(TH0,bb,Q1);
Q2=dec2mat(TH0,bb,Q2);
Q3=dec2mat(TH0,bb,Q3);
F1=dec2mat(TH0,bb,F1);
F2=dec2mat(TH0,bb,F2);
F3=dec2mat(TH0,bb,F3);

 %-------------------


 AfB=AfB*inv(Jk2');
 Df=Df';
 DfB=DfB'*inv(Jk2');
 LfB=LfB'*inv(Jk2');


  evlmi = evallmi(TH0,bb);   %this is the resulting matrice from
        [lhs1,rhs1] = showlmi(evlmi,1)
      b1=eig(lhs1-rhs1)

       eig_max=max(b1)
 filter_parameters=[Af;Afy;AfB;Afd;Df;Dfy;DfB;Dfd;Lf;Lfy;Lfd;LfB]
```

Appendix 4: The Simulink Program :



Appendix 5: The Plotting m-file :

```
open('Thesis05s_L_B_changed.mdl') % To open Simulink
n=size(z1);
n1=n(1);
t=0:n1-1;

figure

subplot(2,2,1)
plot(t',x1)
title('The states')
ylabel('The System state-1')
subplot(2,2,2)
plot(t',x2)
ylabel('The System state-2')
subplot(2,2,3)
plot(t',xf1)
ylabel('The Filter state-1')
subplot(2,2,4)
plot(t',xf2)
ylabel('The Filter state-2')

figure
```

```
subplot(2,1,1)
plot(t',y1)
title('The Outputs')
ylabel('The output-1')
subplot(2,1,2)
plot(t',y2)
ylabel('The output-2 ')

figure

subplot(3,1,1)
plot(t',z1)
title('The First Signal')
ylabel('The Signal to be estimated-1')
subplot(3,1,2)
plot(t',z_estimate1)
ylabel('The estimated signal-1  ')
subplot(3,1,3)
plot(t',z_error1)
ylabel('The estimated error-1')

figure

subplot(3,1,1)
plot(t',z2)
title('The Second Signal')
ylabel('The Signal to be estimated-2')
subplot(3,1,2)
plot(t',z_estimate2)
ylabel('The estimated signal-2  ')
subplot(3,1,3)
plot(t',z_error2)
ylabel('The estimated error-2')
```

BIBLIOGRAPHY

[1.1] Astrom, K.J., Wittenmark, B., “Adaptive Control”, Addison Wesley, 1989.

[1.2] D. J. Leith, W. E. Leithead, “Survey of Gain-Scheduling Analysis & Design”, Int. J. Control, Vol. 73, No. 11, 2000, pp 1001-1025.

[1.3] W.J. Rugh, J.S. Shamma, “Research on gain scheduling”, Automatica 36, 2000, pp 1401–1425.

[1.4] Boyd, S., L. El Ghaoui, E. Feron and V. Balakrishnan, “Linear Matrix Inequalities in Control”, SIAM, Philadelphia, 1994.

[1.5] Baoyong Zhang, Shaosheng Zhou, and Shengyuan Xu, Gain-scheduled H∞-output feedback control for parameter-varying systems with delays, 2007

[1.6] K.H. Tu and J.S. Shamma. Nonlinear gain-scheduled control design using set-valued methods. In Proc. of American Control Conference, 1998.

[1.7] Shaosheng Zhou , Baoyong Zhang, Wei Xing Zheng. A parameter-dependent Lyapunov function based design approach for gain-scheduled H∞ filters of LPV systems, 2007.

[1.8] Zhang, Baoyong, Zhou, Shaosheng; Xu, Shengyuan. Gain-scheduled H∞ infinity -output feedback control for parameter-varying systems with delays, IMA Journal of Mathematical Control and Information, v 25, 2008.

[1.9] Junling Wang , Peng Shi, Junming Wang. Gain-scheduled guaranteed cost control for LPV systems with time-varying state and input delays, 3rd International Conference on Innovative Computing Information and Control (ICICIC), 2008.

[1.10] Zhongwei Yu , Huitang Chen, Peng-yung Woo. Gain Scheduled LPV H∞ Control Based on LMI Approach for a Robotic Manipulator, Journal of Robotic Systems, v 19, n 12, December, 2002.

[1.11] Symens, W. , Van Brussel, H., Swevers, J., Paijmans, B. Gain-scheduling control of machine tools with varying structural flexibilities, Proceedings of the International Conference on Noise and Vibration Engineering, ISMA, 2004.

[1.12] Nguyen Thien Hoang, Hoang Duong Tuan, Pierre Apkarian, and Shigeyuki Hosoe. Gain-Scheduled Filtering for time varying discrete systems , IEEE Transactions On Signal Processing, 2004.

[1.13] Georges Bastin and Michel Gevers. Stable Adaptive Observers for Nonlinear Time-Varying Systems, IEEE Transactions On Automatic Control, VOL. 33, NO. 7,1988

[1.14] Malek-Zavarei, M., & Jamshidi, M. (1987). Time delay systems: analysis, optimization and applications. Amsterdam: North-Holland.

[1.15] Watanabe, K., Nobuyama, E., & Kojima, A. (1996). Recent advances in control of time delay systems: A tutorial review. Proceedings of the 35th IEEE Conference on Decision and Control, Kobe, Japan (pp. 2083-2089).

[1.16] Dugard, L., & Verriest, E. I. (1998). Stability and control of time-delay systems. London: Springer.

[2.1] Junling Wang, Peng Shi, and Junming Wang, Gain-Scheduled Guaranteed Cost Control for LPV Systems with Time-Varying State and Input Delays, The 3rd International Conference on Innovative Computing Information and Control, 2008

[2.2] Andy Packard, and Michael Kantner, Gain Scheduling the LPV Way, Conference on Decision and Control, Japan, December 1996

[2.3] Sun-Li Wu and others, Gain-scheduled control of PVTOL aircraft dynamics with parameter-dependent disturbance, Journal of the Franklin Institute 345 (2008) 906–925

[2.4] Emmanuel Prempain, and Ian Postlethwaite, L2 and H2 performance analysis and gain-scheduling synthesis for parameter-dependent systems, Automatica 44 (2008) 2081–2089

[2.5] Li Wen-qiang and Zheng Zhi-qiang, Robust Gain-Scheduling Controller to LPV System Using Gap Metric, International Conference on Information and Automation, June 20 -23, 2008, China

[2.6] Dong Ming Ge and Xian Lin Huang, Fuzzy Logic Based Gain-Scheduled Techniques, World Congress on Intelligent Control and Automation, June 25 - 27, 2008, Chongqing, China

[2.7] GAWTHROP,P.J.,1995, Continuous-time local state local model networks. Proceedings of IEEE Conference on Systems, Man & Cybernetics, Vancouver, Canada, pp852-857.

[2.8] SHORTEN,R., MURRAY-SMITH,R.,BJORGAN,R.,GOLLEE,H.,1999, On the interpretation of local models in blended multiple model structure. International Journal of Control, 72.

[2.9] JOHANSEN, T.A., FOSS,B.A., 1993, Constructing NARMAX models using ARMAX models. International Journal of Control, 58, 1125-1153.

[2.10] JOHANSEN, T.A., 1994, Fuzzy Model Based Control: Stability, Robustness and Performance Issues. IEEE Transactions on Fuzzy Systems, 2, 221-232.

[2.11] KIRIAKIDIS,K., 1999, Nonlinear Control System Design via Fuzzy Modelling and LMIs. International Journal of Control, 72, 676-685.

[3.1] K.H. Tu and J.S. Shamma. Nonlinear gain-scheduled control design using set-valued methods. In Proc. of American Control Conference, 1998.

[3.2] Boyd, S. P., El Ghaoui, L., Feron, E., & Balakrishnan, V. (1994). Linear matrix inequalities in systems and control theory. Philadelphia, PA: SIAM.

[3.3] Vandenberghe, L., &Boyd, S. (1994). A primal-dual potential reduction method for problems involving matrix inequalities. Mathematical Programming, 69, 205-236.

[3.4] S. S. Zhou, B. Y. Zhang, and W. X. Zheng, “Gain-scheduled H∞ filtering of parameter-varying systems,” Int. J. Robust Nonlinear Control, vol. 16, pp. 387-411, 2006.

[3.5] N. Hoang, H. Tuan, P. Apkarian, and S. Hosoe, “Gain-scheduled filtering for time-varying discrete systems,” IEEE Trans. Signal Processing, vol. 52, no. 9, pp. 2464-2476, 2004.

[3.6] M. Sato, “Filter design for LPV systems using quadratically parameter dependent Lyapunov functions,” Automatica, vol. 42, pp. 2017-2023, 2006.

[3.7] Y. Theodor, U. Shaked, and C. E. de Souza, “A game theory approach to robust discrete-time H1-estimation,” IEEE Trans. Signal Processing, vol. 42, pp. 1486–1495, June 1994.

[3.8] Huaizhong Li and Minyue Fu, Senior Member, IEEE, “A Linear Matrix Inequality Approach to Robust H_∞ Filtering” IEEE Trans. Signal Processing, vol. 45, NO. 9, Sep. 1997

[3.9] D. S. Bernstein and W. H. Haddad, “Steady-state Kalman filtering with an H_∞ error bound,” Syst. Contr. Lett., vol. 12, pp. 9–16, 1989.

[3.10] K. M. Nagpal and P. P. Khargonekar, “Filtering and smoothing in an H_∞ setting,” IEEE Trans. Automat. Contr., vol. 36, pp. 152–166, 1991.

[3.11] M. J. Grimble, “H_∞ design of optimal linear filters,” in Linear Circuits, Systems, and Signal Processing: Theory and Application, C. I. Byrnes, C. F. Martin, and R. E. Saeks, Eds. Amsterdam: North-Holland, 1988, pp. 533–540.

[3.12] M. J. Grimble and A. E. Sayed, “Solution of the H_∞ optimal linear filtering problem for discrete-time systems,” IEEE Trans. Acoust., Speech, Signal Processing, vol. 38, pp. 1092–1104, July 1990.

[3.13] M. Fu, “Interpolation approach to H_∞ optimal estimation and its interconnection to loop transfer recovery,” Syst. Contr. Lett., vol. 17, pp. 29–36, 1991.

[3.14] Javad Mohammadpour Velni, Karolos M. Grigoriadis, " Rate-Dependent Mixed H_2/H_∞ Filter Design for Parameter-Dependent State Delayed LPV Systems", IEEE Transactions on Circuits and Systems, vol. 55, no. 7, AUGUST 2008.

[3.15] Yong He, Min Wu, Jin-Hua She, and Guo-Ping Liu, "Parameter-Dependent Lyapunov Functional for Stability of Time-Delay Systems with Polytopic-Type Uncertainties", IEEE Transactions on Automatic Control, Vol. 49, No. 5, 2004.

[3.16] Manfredi Maggiore, and Kevin Passino, "Output Feedback Control of Jet Engine Stall and Surge Using Pressure Measurements".

VITAE

- Alaeddin Khalil Jaradat.
- Nationality : Palestinian
- Born in Hebron, Palestine on May 22th, 1984.
- Received BSc from University of Jordan, Jordan in 2006.
- Joined King Fahd University of Petroleum & Minerals in October 2007.
- The research interests are focusing on Linear Parameter Varying (LPV) Systems, particularly Stability of Time Delayed LPV Systems.

- Present and Permanent Address : Quesemeh - Amman - Jordan
- Email: eng_alaajaradat@yahoo.com
- Tel: 00962788654861