FUZZY BASED SIC MULTIUSER DETECTORS FOR WIRELESS CHANNELS

BY

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This thesis written by **MOHAMMED AHSAN ALI** under the direction of his thesis advisor and approved by his thesis committee has been presented to and accepted by the Dean of Graduate Studies in partial fulfillment of the requirements for the degree of **MASTER OF SCIENCE IN ELECTRICAL ENGINEERING.**

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Dedicated to

All those who are much better than me….

More deserving than me…

But could not succeed due to lack of opportunities and misfortune!

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In the name of Allah, the Most Gracious and the Most Merciful

All praise be to Allah (SWT) for His kindest blessings upon me and all the members of my family. I feel sincerely privileged to glorify His Name through this small accomplishment, and I ask Him to accept it as an act of worship. I ask for His blessings, mercy and forgiveness. May the peace and blessings of Allah be upon His dearest prophet, Muhammad (Peace Be upon Him).

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ABSTRACT

Name: MOHAMMED AHSAN ALI

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In this thesis, the linear Successive Interference Cancellation (SIC) detector is studied in detail. It is observed that it is equivalent to matrix filtering. A brief study of Non-Linear SIC (NLSIC) detector and the weighted SIC is carried out. Convergence analysis is also studied. Based on the knowledge of these detectors, a new technique is proposed which adds a weighting factor to the conventional linear SIC detector. With the help of mathematical analysis and simulations, it is shown that the proposed weighted SIC converges to the decorrelator. Simulations are carried out to analyze the performance of the proposed weighted SIC detector. With proper choice of weights, it can be seen that the weighted SIC detector performs better than the conventional linear SIC detector. The fact that weighted SIC performs better is exploited and it is proposed that the weights can be estimated based on a Fuzzy Inference System (FIS). This method provides smoother tuning of the weights for the weighted SIC detector. The FIS estimated the weight based on the SNR of the received signals. A study about using different membership functions for the FIS is also carried out. A number of simulations are performed to show that the Fuzzy-Based SIC (FBWSIC) detector performs better than the SIC and the NLSIC, in terms of convergence behavior also.

ABSTRACT (ARABIC)

الاسم: محمد أحسن علي

عنوان الأطروحة: كشف إلغاء التداخلات الخطية المتعاقبة المعتمد على نظام الاستلال الضبابي للأوساط اللاسلكية

الدرجة العلمية: ماجستير في العلوم

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في هذه الأطروحة، سيتم دراسة كاشف لإلغاء التداخلات الخطية المتعاقبة (SIC) بالتفصيل. ولقد لوحظ أنه يعادل ما يسمى مصفوفة التصفية. وسيتم عمل دراسة موجزة للكاشفات غير الخطية (NLSIC) والكاشفات المعايرة. وسيتم دراسة التحليلات المتقاربة أيضا. وعلى أساس معرفة هذه الكاشفات، سيتم اقتراح أسلوب جديد يعمل على إضافة عامل ترجيح للكاشفات الخطية التقليدية. ومع بعض المساعدة من التحليل الرياضي والمحاكاة، فإنه يظهر أن المقترح يتطابق مع (decorrelator) وهو أحد أنواع الكاشفات. وسيتم تنفيذ عدد من عمليات المحاكاة لتحليل أداء الكاشف المقترح. ومن خلال الاختيار الصحيح للمعايير، يمكن أن يرى أن الكاشف المرجح أداؤه أفضل من الكاشف الخطي التقليدي. حقيقة أن الكاشف المرجح يمكن استغلاله للعمل بشكل أفضل، ويقترح أن الأوزان يمكن أن تقدر على أساس نظام الاستدلال الضبابي (FIS). هذا الأسلوب يوفر سلاسة في ضبط الأوزان للكاشف المرجح. ونظام الاستدلال الضبابي يقدر وزنها على أساس التناسب بين الإشارة و العوائق (SNR) للإشارات الواردة. وسيتم عمل دراسة حول استخدام وظائف مختلفة لنظام الاستدلال الضبابي. وهناك عدد من عمليات المحاكاة التي ستجرى لإظهار أن الكاشفات الخطية التي تعتمد على نظام الاستدلال الضبابي (FBWSIC) أداؤه أفضل من (SIC) و(NLSIC) من حيث السلوك المتقارب أيضا.

Chapter 1

Introduction

1.1 Motivation and Background

Ever since the beginning of wireless communications, there has been a continuous increase in its demand day by day. Since the beginning of wireless communication, it has attracted a lot of attention and motivation for research. If the research progressed at a fast pace for analog communications, the digital world required a much more extensive research for better performance and high-speed data and voice communications. From GSM to CDMA, the evolution has been radical and is still evolving further with a rapid pace. Also, these schemes are expected to allow subscribers to access many services at a time, keeping in view better transmission quality services. For example, data usually needs a bit error rate (BER) of better than 10^{-6} as compared with a threshold BER of 10^{-3} for voice [1]. Moreover, multimedia services require higher transmission and better

reception quality. These requirements pose real challenges to the researchers in the wireless field. In order to fulfill these requirements, major improvements in the field of wireless technology were necessary.

As mentioned earlier, research in the digital world progressed at a fast pace and new techniques were developed for multiuser detection. With the available frequency spectrum saturated, efficient use of the bandwidth became the problem of utmost concern. From amongst the pioneering techniques, Code division Multiple Access (CDMA) could serve the demand of limited bandwidth very well and emerged as the technology of choice for the wireless industry.

CDMA provides a number of attractive features over the other multiple access schemes – Time Division Multiple Access (TDMA) and Frequency Division Multiple Access (FDMA). It also has the potential to provide higher capacity than the TDMA and FDMA schemes [2], [3] . It has many features to meet the high capacity and other performance requirements (e.g. seamless communication) for the emerging personal communication services (PCS). Some of these features are spectrum sharing, rejection of multipath signal components or utilizing them for recombining [4], and having a frequency reuse factor of one, in the cellular case. Moreover, each user is allowed to share a wide frequency spectrum for transmission [5], [6].

Ideally, the users in a CDMA system are distinguished by assigning with a signature sequence assigned to a user that is orthogonal to those of other users. Considering cellular mobile systems, mobile units transmit at random times so that their signals arrive asynchronously at the base station. Due to asynchronous users and hostile wireless

channel effects, it is difficult, if not impossible, to maintain orthogonality between the users' signals. Actually, there is no known set of code sequences that remains completely orthogonal when used in an asynchronous system. Thus, mutual interference between the users' signals can severely degrade the performance.

The principle shortcomings of the CDMA systems are Multi-Access Interference (MAI) and Near-far effect. MAI is introduced in CDMA systems due to lack of orthogonality of users' signature sequences particularly in the presence of channel fading and multipath propagation. It is the MAI that places a performance limit on CDMA systems.

In mobile communication system, the receiver moves constantly causing the received signal to fade with certain statistics. The conventional matched filter detector fails to demodulate weak signals due to the presence of stronger interfering signal, even when the cross-correlation between the signals is relatively low. This problem is known as the Near-far effect [7].

These 'inherent' problems of the CDMA scheme have attracted a lot of attention in the recent times. Many schemes have been proposed which were significantly efficient at the time of their proposal. Many of these have also evolved and new techniques have been developed based on these techniques.

The main methods used to combat these problems are broadly classified as Transmitter oriented and Receiver oriented [8]. Power control and Synchronization with orthogonal

code sets can be considered as methods applied at the reverse link whereas Multiuser detection schemes can be considered as the methods to be applied at the forward link[1].

The Multi-User Detectors (MUDs) used at the transmitter are the Conventional detector, the Optimum detector, the Linear decorrelating detector, the MMSE linear multiuser detector, Decision-feedback detector, the Decorrelating decision-feedback detector and Multistage interference cancellation. The two main techniques in multistage interference cancellation are the Parallel Interference Cancellation (PIC) and the Successive Interference Cancellation (SIC) [9]. Other modifications of these techniques are the hybrid techniques. Some of these are discussed in [10].

The conventional detector is a filter which is matched to the signature waveform of the desired user. It is also known as the Matched Filter Detector. Due to the MAI from other users, the matched filter detector is not optimal. Moreover, its performance degrades significantly when the received power of the interfering users is much greater than the desired user. Thus, the conventional matched filter detector is not near-far resistant [7]. To mitigate the near-far effect, power control is suggested. Although ideal power control is very cumbersome to implement, strict power control is necessary to ensure that the users have almost identical signal power when they arrive at the base station.

The matched filter considers the MAI as simple additive white Gaussian noise (AWGN). This is a reasonable approach due to the fact that CDMA interference is comprised of contributions from independent interferers. However, the MAI is not AWGN since its

[1] In this thesis, the receiver is the mobile station and the transmitter is the base station.

structure is well defined through the cross-correlation matrix. This fact could be exploited to achieve better performance results [10].

The matched filter detector, described above, was believed to be the optimum detector until proved otherwise by Verdu. In the early 1980's, Verdu in his novel work [7], showed that the near-far problem is not 'inherent' to the CDMA system but in fact, due to the nature of the detection scheme. His solution to this problem was the optimum Maximum Likelihood (ML) detector. Although it showed huge capacity improvement over the matched filter detector, its complexity increases exponentially with the number of users and makes its implementation impractical [11].

Motivated by this fact, extensive research was carried out to find a sub-optimal detectors with significant performance improvement and tolerable complexity. Several sub-optimal detectors were proposed in the past decade. Unfortunately, they either still exhibit high computational complexity or provide small performance improvement over the single user detector. In CDMA reverse link, the users spreading sequences is known to the receiver at the base station. This fact was exploited by researchers to propose a set of new detectors which came to be known as sub-optimum detectors.

Sub-optimum multiuser detectors can be classified into two categories namely linear and non-linear [12]. In the linear case, a linear transformation is applied to the soft outputs[2] of the bank of the matched filters in order to produce a new set of decision variables with MAI partially or totally decoupled. In the non-linear case, a non-linear transformation is applied to the soft outputs of the bank of the matched filters in order to estimate the

[2] It is the output from the detectors before applying the decision functions.

signal and then regenerate and cancel the interference from the received signal and therefore getting a cleaner version of the desired signal. These terms are elaborated in detail in chapters 2 and 3.

The decorrelator detector is one of the earliest discussed sub-optimal linear multiuser detectors. It possesses many useful features such as near-far resistance and total decoupling of the MAI. However, its complexity increases exponentially by power of 3 i.e. K^3, where K is the number of users [10]. This renders the implementation practically especially for asynchronous systems and systems using long spreading codes when the number of users is large. Noise enhancement is also another disadvantage of the decorrelator.

The Minimum Mean Squared Error (MMSE) detector was another alternative. It takes the background noise into account and generally provides better bit error rate (BER) performance than the decorrelator. However, it faces the task of implementing matrix inversion. Also, it requires estimation of the received amplitudes.

On one side, these sub-optimal detectors were being developed, many other schemes with modifications to the previous methods were studied. On the other side, multi-stage detectors began to emerge as the technique of choice. These could tackle the problem of MAI much better than the conventional detectors. Two main approaches were used to implement multi-stage detectors: the Successive Interference Cancellation (SIC) detector and the Parallel Interference Cancellation (PIC) detector [9]. Furthermore, linear and non-linear decision functions could also be used on these detectors [10].

The SIC detectors approach to the problem of cancellation of MAI in a serial manner i.e. each user's contribution is estimated, regenerated and cancelled from the received signal, so that the remaining users see diminishing MAI after every stage of cancellation. This process is repeated in multistage manner. The SIC scheme can be used for both long code and short code systems [11]. A slight modification to the SIC scheme which enhances the performance is ordering of the users in a descending order of their signal strength [10].

The multistage SIC detector is less complex compared to the decorrelator. Also, it requires a minimal amount of additional hardware over the matched filter compared to the other sub-optimal detectors. On the other hand, it exhibits unacceptable and inevitable delay as it requires one bit delay per user. Moreover, the use of a hard-limiter decision function for data bit decision in the SIC iterations leads to error propagation, which causes a higher BER error floor. Partial cancellation and use of soft-decision are some of the few suggested solutions to the problem [11].

It is known that the SIC converges to the decorrelator [10]. Therefore, it has asymptotically the same characteristics as the decorrelator. Considerable amount of work has been done on the SIC earlier and are mentioned in Chapter 3. Different decision functions were tested for the non-linear SIC detectors, some of which include hard-limiter, null zone and hyperbolic tangent. A lot of research has been conducted to reduce the delay inherent to the SIC. Some weighted SIC detectors were also introduced [13].

1.2 Contributions

To enhance the performance of the successive interference cancelation detector, we have worked in the direction of weighted SIC detector. This thesis investigates the usage of Fuzzy Logic to estimate a weighting factor which is applied to the decision statistic i.e. as a soft-decision for the SIC detector. The contributions can be summarized as follows:

- In Chapter 5, we propose a new weighted SIC method. This was developed to add a weight to the ICU of the conventional SIC detector. Convergence issues regarding this proposed method are discussed. It is then proposed to estimate the weight of this detector using Fuzzy Logic. We show how to use the concept of Fuzzy Logic to estimate the weighting factor for the SIC detector and is applied. With the help of simulation results, we show that considerable enhancement over the SIC and the non-linear SIC detector is achieved. This solves the problem of 'error floor' in the SIC detection scheme.

1.3 Thesis Overview

This thesis consists of six chapters which deal mainly with the problems associated with the SIC detector. It also proposes some solutions of its shortcomings.

- The second chapter briefly studies the uplink synchronous discrete channel model, introduces the basic multiuser detectors and discusses both the advantages and disadvantages.
- The third chapter covers details of the SIC detector. It is described as matrix filtering. This description enables us to easily derive the probability of error for

the linear SIC detector. The non-linear and the weighted SIC detector are also discussed.

- In the fourth chapter, the Fuzzy Logic is studied in detail. It is shown how to form a fuzzy inference system (FIS) for a particular problem.
- The fifth chapter uses the background from the third and fourth chapter and develops a weighted SIC detector. It also discusses how to incorporate a fuzzy based weight in the SIC detector, thereby developing a Fuzzy Based Weighted SIC (FBWSIC) detector. With the help of simulations, it is shown that the latter shows considerable improvement over the SIC. Analyses of the convergence behavior of the weighted SIC and FBWSIC is also discussed.
- In chapter six, we conclude the thesis by summarizing the results obtained in chapter five and commenting on those results. Any possible future work is also discussed.

Chapter 2

CDMA and Multiuser Detection

This chapter establishes and reviews a mathematical model for the multiuser detection schemes on which this thesis is based. In section 2.1, the basic CDMA discrete model is discussed. In proceeding section, some of the existing multiuser schemes are briefly reviewed and their advantages and disadvantages are discussed.

2.1 CDMA model

Consider the case of an uplink CDMA channel. Here, K users transmit simultaneously over a synchronous additive white Gaussian noise (AWGN) channel. The modulation method used is Binary Phase Shift Keying (BPSK). The structure of the discrete uplink channel is shown in Figure 2-1. Each user is characterized by its own Pseudo-Noise (PN) code of length N-chips.

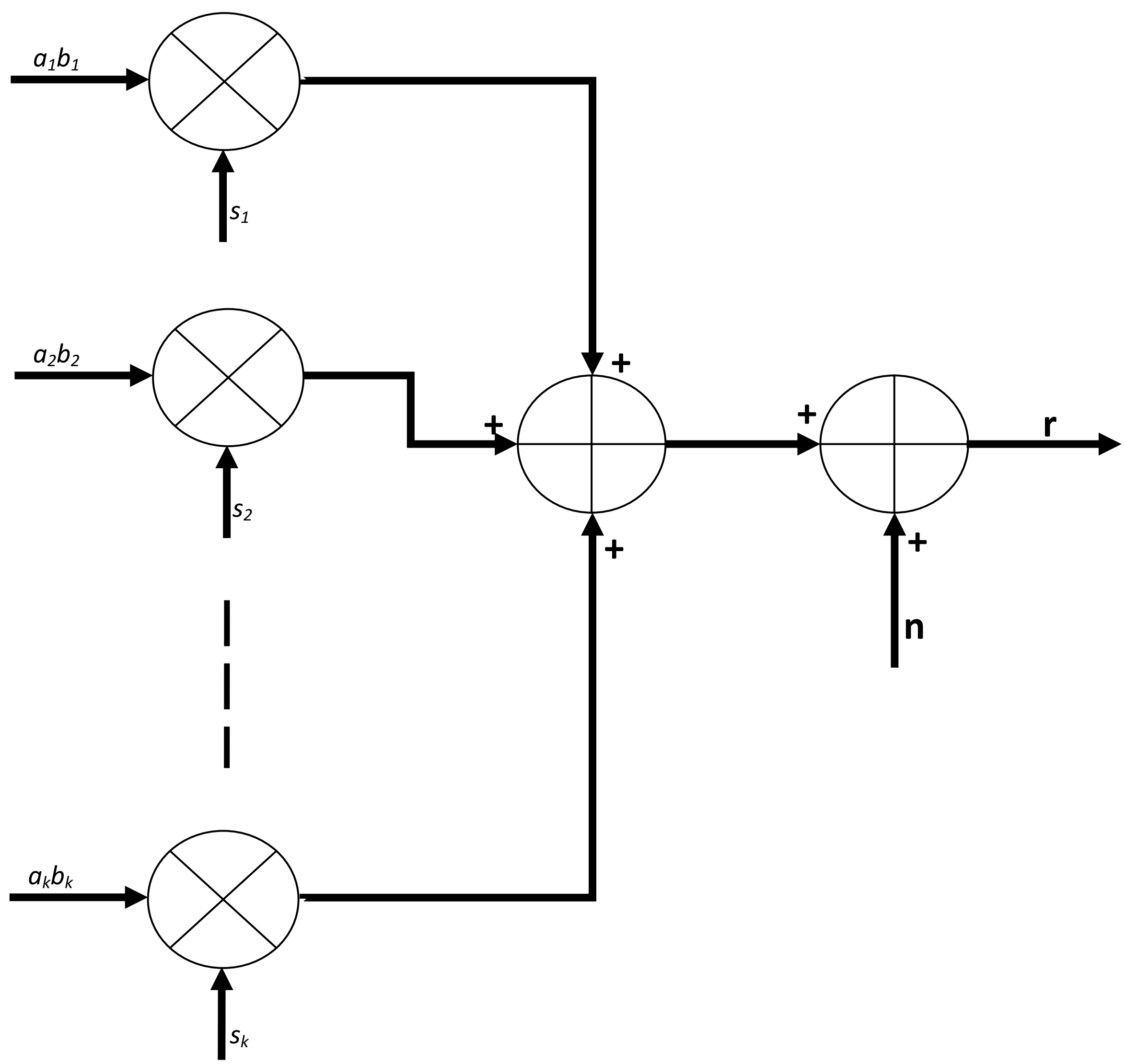


Figure 2-1 : Discrete synchronous CDMA uplink channel.

The received signal is expressed in a 'vector' form as[3]

$$\mathbf{r} = \mathbf{SAB} + \mathbf{n} \tag{2-1}$$

where:

$\mathbf{S} = (\boldsymbol{s_1}, \boldsymbol{s_2}, \boldsymbol{s_3}, \ldots., \boldsymbol{s_K}) \,\boldsymbol{\epsilon} \left\{\frac{\mathbf{-1}}{\sqrt{N}}, \frac{\mathbf{1}}{\sqrt{N}}\right\}^{N,K}$

$\mathbf{A} = diag(a_1, a_2, a_3, \ldots., a_K) \,\boldsymbol{\epsilon}\mathbb{R}^{K,K}$

$\mathbf{b} = (b_1, b_2, b_3, \ldots., b_K)^{\boldsymbol{T}} \,\boldsymbol{\epsilon} \,\{\mathbf{-1}, \mathbf{1}\}^K$

and

$\mathbf{n} = (n_1, n_2, \ldots. n_K)^T$

S is a matrix of size $N \times K$ of the spreading sequences $\boldsymbol{s}$ where $\boldsymbol{s_k}$ is the spreading sequence of the k^{th} user. **A** is the matrix of received amplitudes of size $K \times K$. **b** is a K-length vector of binary transmitted symbols. And **n** is a K-length vector of independently identically distributed additive white Gaussian distributed samples with zero-mean and variance $N_0/2$.

[3] For notational conveniences, **bold** case is used for matrices and vectors, and *italic* case for scalar quantities.

2.2 Multi-user detection

The output of the bank of matched filters in a multi-user detector can be expressed as discussed in the next section. Multiuser detectors can be classified as optimal and sub-optimal. The optimal detector is based on the Viterbi algorithm. It gives optimal theoretical performance. But the complexity grows exponentially with the number of users and therefore is not practical for CDMA systems. Sup-optimal detectors are easier to implement and can be classified as linear and non-linear.

2.2.1 The Matched Filter detector

One of the earliest and most basic Multiuser detection schemes is the matched filter detector. It is comprised of a bank of matched filters which are matched to the users' spreading sequences.

In a matrix-vector form, the matched filter can be expressed as

$$\mathbf{y_{MF}} = \mathbf{S^T S A B} + \mathbf{S^T n} = \mathbf{RAB} + \mathbf{z}, \tag{2-2}$$

where, $\mathbf{y}_{\mathrm{MF}}$ is the matrix of the outputs from the matched filters, $\mathbf{R}$ is the correlation matrix of the signature sequences i.e., $\mathbf{R} = \mathbf{S} \cdot \mathbf{S^T}$ and $\mathbf{z}$ is the noise vector where $\mathbf{z} = \mathbf{S^T n}$.

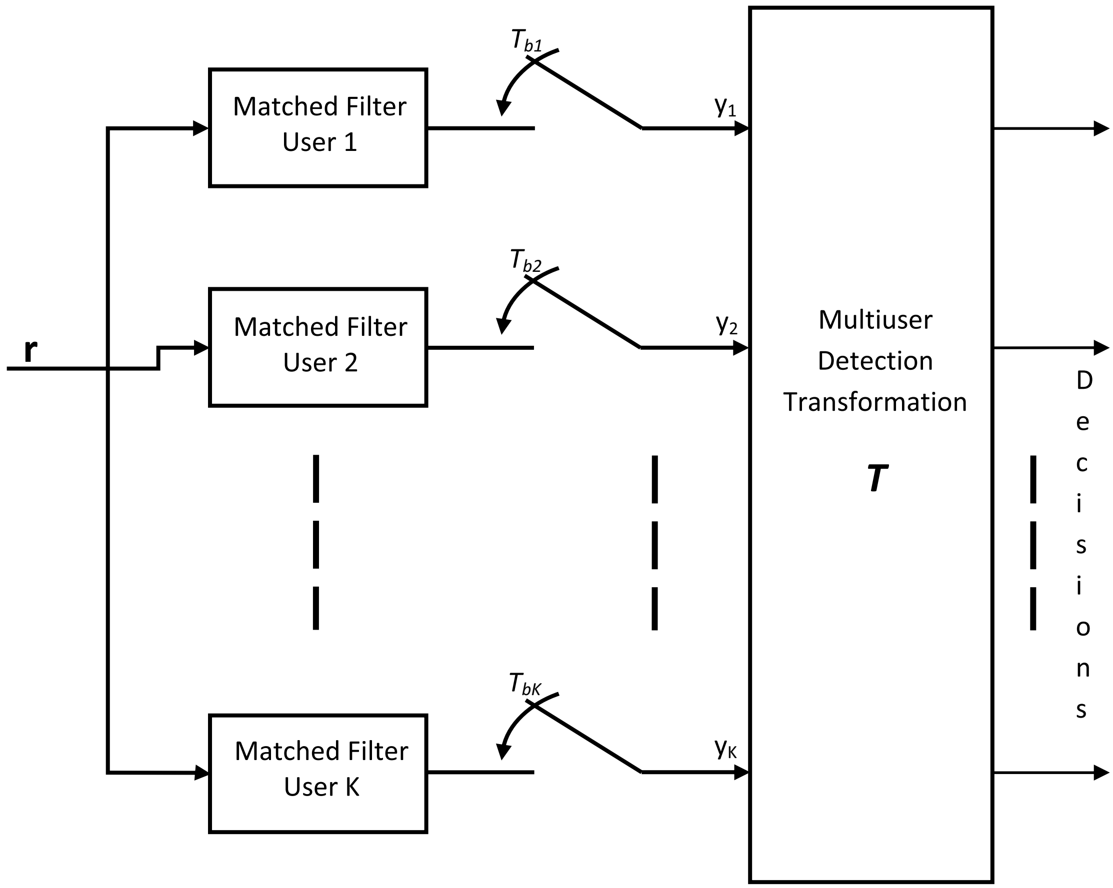


Figure 2-2 : Multiuser Detection.

In Figure 2-2, the outputs from the matched filters from each user are synchronized using the symbol interval T_b. The multiuser detection transformation $\boldsymbol{T}$ is of main interest. It determines the type of multi-user detector and is of considerable interest. For the matched filter detector, the decision function is applied to the vector $\mathbf{y}_{MF}$.

As mentioned earlier, MUDs can be classified as optimal and sub-optimal detectors. The optimal detector uses the Viterbi algorithm which gives optimal MUD performance. However, its complexity grows exponentially with the number of users and it does not remain feasible for the CDMA systems with many users. On the other hand, sub-optimal detectors are simpler to implement and are organized into two categories: Linear and Non-Linear [10].

As the name indicates, the linear detectors apply a linear transformation to the soft-outputs of the matched filter and output a new set of decision statistics. They are effective in reducing the effect of MAI on each user. Two of the most commonly used linear detectors are the Decorrelator and the Minimum Mean Squared Error (MMSE) detector.

2.2.2 The Decorrelator

In the decorrelator, the inverse of the correlation matrix is used. The transformation function $\boldsymbol{T}$ used here is $\boldsymbol{T} = \mathbf{R}^{-1}$. The users' decisions are made based on the equation

$$\hat{\mathbf{b}} = sign(\mathbf{R}^{-1}\mathbf{y}_{\mathbf{MF}}) = sign(\mathbf{AB} + \mathbf{R}^{-1}\mathbf{z}) \tag{2-3}$$

The decorrelator eliminates the MAI completely and provides unbiased estimates. It is often used as a benchmark for comparison. However, it enhances the noise in the system.

2.2.3 The MMSE Detector

The MMSE detector minimizes the mean squared error between the transmitted bits and the outputs from the transformation. It takes the background noise into account and uses the knowledge of the received signal powers. It implements a linear mapping, which minimizes $E[\ |\mathbf{b} - \boldsymbol{T}\mathbf{y}_{\mathrm{MF}}|^2\ ]$. This is the mean squared error between the actual data and the soft output of the matched filter. The users' decisions are based on the equation

$$\hat{\mathbf{b}} = sign((\mathbf{R} + \boldsymbol{\sigma}^{\mathbf{2}}\mathbf{A}^{-\mathbf{2}})^{-\mathbf{1}}\mathbf{y}_{\mathbf{MF}}) = sign\left((\mathbf{R} + \boldsymbol{\sigma}^{\mathbf{2}}\mathbf{A}^{-\mathbf{2}})^{-\mathbf{1}}(\mathbf{RAB} + \mathbf{n})\right) \qquad \text{(2-4)}$$

It attempts to balance the removal of the MAI and reduction of noise enhancement. However, it requires a prior knowledge of the estimation of received amplitudes.

Multistage detectors, on the other hand, could handle the problem of MAI and noise enhancement with ease. Successive Interference Cancellation and Parallel Interference Cancellation detectors are the two main multistage techniques. Hybrid and advanced techniques using combinations of the two are also used.

2.2.4 The Successive Interference Cancellation (SIC) detector

The SIC is a simple, but a powerful technique used for interference cancellation. At each stage, every user is decoded successively. The signal is then reconstructed and subtracted from the received signal. Initially, the users are ordered according to their signal powers. After a user is processed, the remaining users are again ordered according to their power. The procedure is performed iteratively as shown in Figure 2-3.

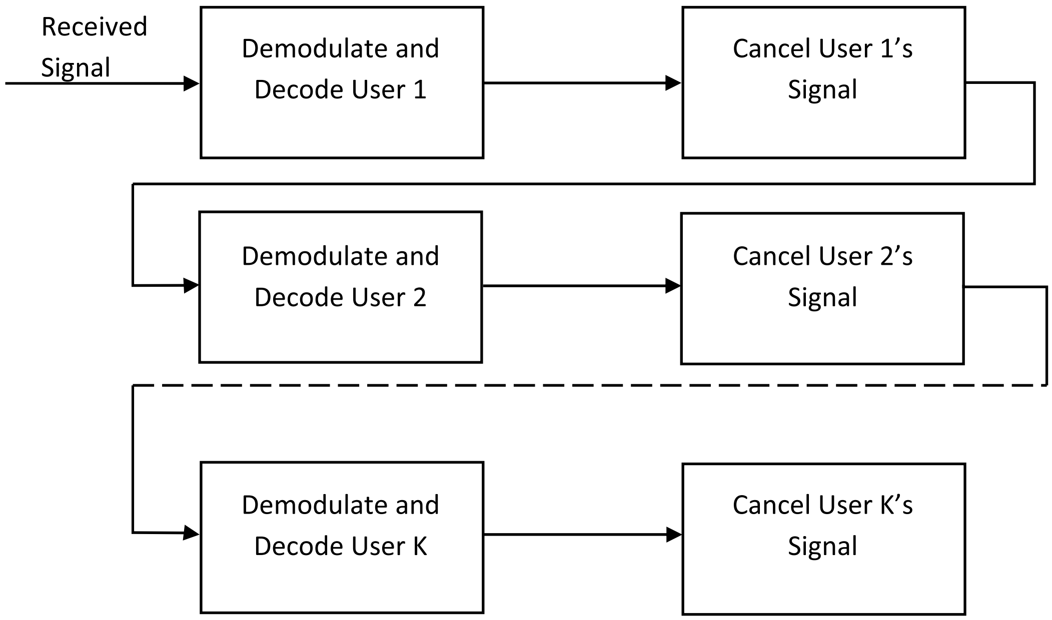


Figure 2-3 : Block Diagram of the SIC detector.

In general, a user decoded later benefits from cancellation of previous users and encounters improved Signal to Interference and Noise Ratio (SINR) [14]. Simply put, the strongest user will not benefit from any MAI cancellation and the weakest user will have a significant reduction in the MAI. However, a large delay is unavoidable since each user is decoded and cancelled successively. Some other disadvantages of the SIC include reordering the signals whenever the power profile changes and degradation of the performance if the strongest estimate is not reliable. The SIC detector is explained in detail in the next chapter, as it is the main topic of interest.

2.2.5 The Parallel Interference Cancellation (PIC) detector

To solve the problem of the delay in SIC, the PIC was suggested and was considered more attractive. It effectively estimates and subtracts out the MAI for each user **in parallel**. All the users are decoded and cancelled from the received signal simultaneously as shown in Figure 2-4. The processing power is distributed among multiple parallel demodulators and decoders. Thus the tradeoff between delay and complexity is well balanced. Moreover, the performance of parallel IC can approach successive IC with a small number of iterations [14].

For practical implementation, PIC and SIC schemes proved to be very successful and have attracted a lot of attention. These techniques rely on simple processing elements constructed around the matched filter concept.

2.3 Summary

In this chapter, a mathematical framework was developed for the signal and CDMA channel that will be used in the following chapters. Moreover, basic multiuser detection schemes were covered briefly which will be used extensively throughout this thesis.

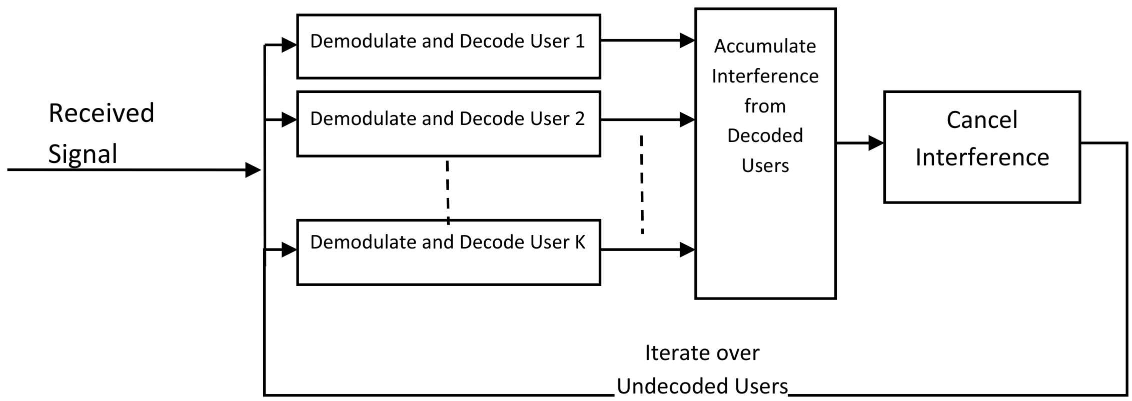


Figure 2-4 : Block Diagram of the PIC detector.

Chapter 3

Successive Interference Cancellation

The SIC detector takes a serial approach to cancel interference. Each stage detects, regenerates and cancels out one additional user interference from the received signal. It is a well established technique in multiuser detection. Although it has low complexity, many problems are inherent in its implementation. The main problem of concern is the large delay. In this chapter, we study the linear and non-linear SIC detectors. A brief overview of weighted SIC is also discussed. Sections 3.1 and 3.2 discuss the structure of the SIC and study the linear SIC in detail in terms of convergence behavior. Section 3.3 studies the decision functions for non-linear SIC. Weighted SIC is discussed in section 3.4.

3.1 Structure of the SIC detector

Many SIC structures have been proposed to date. The first structure based on the principle of interference cancellation was the multistage detector in [15]. A single-stage

nonlinear SIC was also proposed in [16]. Some other schemes were also proposed in [17], [18] and [19]. Further details on development of the SIC are found in [20].

A novel method was proposed in [10] which describes a simple Matrix-algebraic approach to the problem. It used the SIC model from [20] in which the linear SIC scheme corresponds to linear matrix filtering that can be performed directly on the received chip-matched filtered signal vector without explicitly performing the interference cancellation. The system model is described by the received signal as expressed in Section 2.1. The received signal is expressed by the equation

$$\mathbf{r} = \mathbf{SAB} + \mathbf{n},$$

where, the matrix $\mathbf{S}$ consists of the spreading codes of the *K* users, **A** is the channel gain matrix, $\mathbf{B}$ is the matrix of the data bits modulated using binary phase shift keying (BPSK) and $\mathbf{n}$ is the noise matrix which is AWGN with zero mean and variance $N_0/2$. The length of the spreading codes is *N*. Note that $\boldsymbol{s}_{\boldsymbol{k}}^{\boldsymbol{T}} \cdot \boldsymbol{s}_{\boldsymbol{k}} = 1$.

The received signal $\mathbf{r}$ is a matrix of dimension $N \times K$ bits. It is processed in individual bits. The structure of SIC consists of interference cancellation units (ICU) which are arranged in a multistage manner as shown in Figure 3-1. The function of a basic ICU is detailed in Figure 3-2.

At the input of the m^{th} stage, the ICU of the k^{th} user processes the received signal. It de-spreads the received signal $\mathbf{r} = \mathbf{e}_{1,1}$ to estimate the decision variable $\mathbf{y}_{m,k}$ of the k^{th} user at the m^{th} stage. The decision variable is defined as $\mathbf{y}_{m,k} = \boldsymbol{s}_{\boldsymbol{k}}^{\boldsymbol{T}}(\mathbf{e}_{m,k} + \mathbf{I}_{m,k})$. The term $\mathbf{I}_{m,k}$, the MAI due to the k^{th} user at the m^{th} stage, is obtained by spreading the residual

decision variable $\mathbf{y}_{m,k}$ using the spreading codes as $\mathbf{I}_{m,k} = \boldsymbol{s_k}\mathbf{y}_{m,k}$. It is then subtracted from the residual signal $\mathbf{e}_{m,k} + \mathbf{I}_{m-1,k}$ to get a cleaned version of the residual signal $\mathbf{e}_{m,k+1}$. The process is repeated in multistage as shown in Figure 3-1. More detail is provided in the following section.

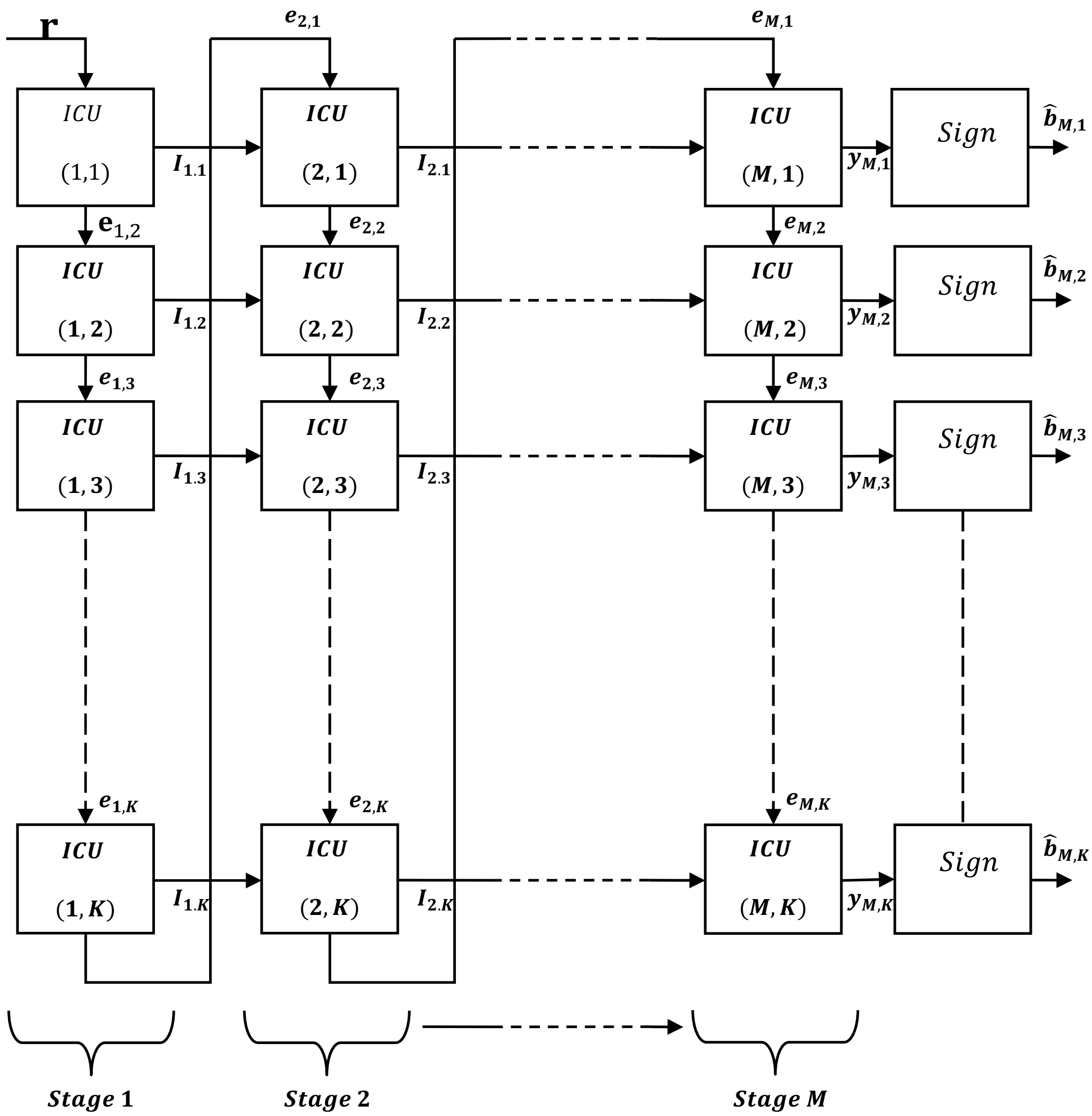


Figure 3-1 : Structure of the SIC in multistage.

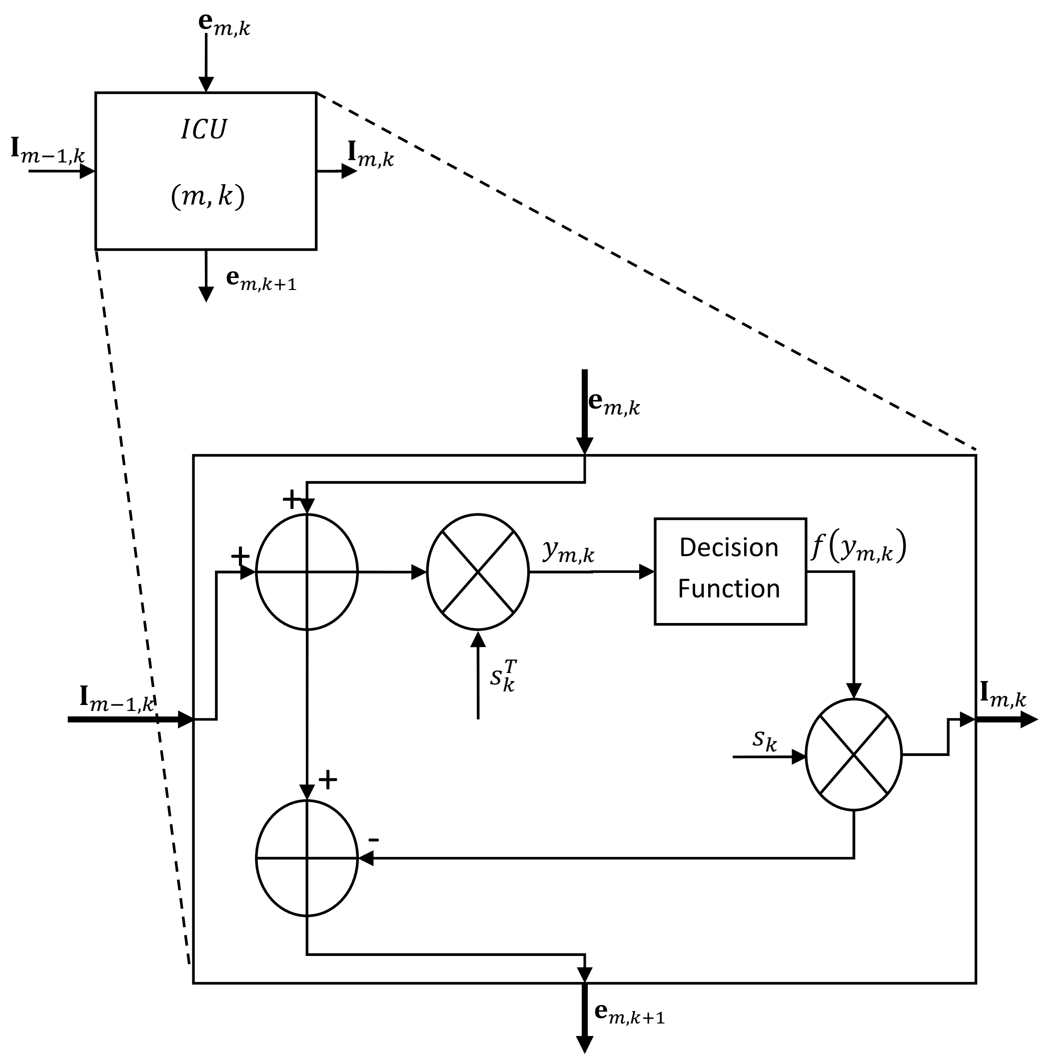


Figure 3-2 : Block diagram of an ICU.

3.2 Linear SIC detector

3.2.1 Structure

Consider the structure of the ICU of the SIC detector shown in Figure 3-2. For the linear case, the decision function $f(y_{m,k}) = y_{m,k}$. Therefore, $f(y_{1,1}) = \boldsymbol{y}_{1,1}$ and so on for each ICU. The residual signal at the input of the first ICU in the first stage is defined as $\boldsymbol{e}_{1,1} = \mathbf{r}$ and the corresponding decision variable is defined as $\boldsymbol{y}_{1,1} = \boldsymbol{s}_1^T(\boldsymbol{e}_{1,1} + \boldsymbol{I}_{0,1}) = \boldsymbol{s}_1^T\mathbf{r}$. Note that the MAI for the first stage for all the users is 0 i.e. $\boldsymbol{I}_{0,1} = \boldsymbol{I}_{0,2} = \cdots = \boldsymbol{I}_{0,K} = 0$.

For the second ICU in the first stage, the received signal vector is obtained by estimating the MAI due to the first user and then subtracting it from the received signal i.e. $\boldsymbol{e}_{1,2} = \boldsymbol{e}_{1,1} + \boldsymbol{I}_{o,1} - \boldsymbol{I}_{1,1} = \boldsymbol{e}_{1,1} - \boldsymbol{s}_1\boldsymbol{s}_1^T\mathbf{r} = (\boldsymbol{I} - \boldsymbol{s}_1\boldsymbol{s}_1^T)\mathbf{r}$. The consequent decision variable is defined as $\boldsymbol{y}_{1,2} = \boldsymbol{s}_2^T(\boldsymbol{e}_{1,2} + \boldsymbol{I}_{0,2}) = \boldsymbol{s}_2^T(\boldsymbol{I} - \boldsymbol{s}_1\boldsymbol{s}_1^T)\mathbf{r}$. Here, $\boldsymbol{I}$ is the identity matrix.

Similarly, we can define the decision statistics and the residual signal for all the users at different stages. The closed form of the residual signal at the output of the k^{th} user ICU at the first stage is expressed as

$$\boldsymbol{e}_{1,k} = \coprod_{j=k-1}^{1} \left(\boldsymbol{I} - \boldsymbol{s}_j\boldsymbol{s}_j^T\right)\mathbf{r} = \boldsymbol{\Phi}_{k-1}\mathbf{r}$$

where $\coprod$ indicates the product of matrices with decreasing indices. The corresponding decision variable is expressed as $\boldsymbol{y}_{1,k} = \boldsymbol{s}_k^T\boldsymbol{e}_{1,k} = \boldsymbol{s}_k^T\boldsymbol{\Phi}_{k-1}\mathbf{r}$. The residual signal at the output of the last ICU in the first stage is

$$\boldsymbol{e}_{1,K+1} = \coprod_{j=K}^{1} \left(\boldsymbol{I} - \boldsymbol{s}_j \boldsymbol{s}_j^T\right) \mathbf{r} = \boldsymbol{\Phi}_K \mathbf{r}$$

This residual signal from the last ICU of the first stage is used as the input for the first ICU in stage 2 as shown in Figure 3-1. It is expressed as $\boldsymbol{e}_{2,1} = \boldsymbol{e}_{1,K+1} = \boldsymbol{\Phi}_K \mathbf{r}$. The corresponding decision variable is expressed as $\boldsymbol{y}_{2,1} = \boldsymbol{s}_1^T\left(\boldsymbol{e}_{2,1} + \boldsymbol{I}_{1,1}\right) = \boldsymbol{s}_1^T \boldsymbol{\Phi}_K \mathbf{r} + \boldsymbol{s}_1^T \mathbf{r} = \boldsymbol{s}_1^T(\boldsymbol{\Phi}_K + \boldsymbol{I})\mathbf{r}$. The input to the second ICU in the second stage is $\boldsymbol{e}_{2,2} = \boldsymbol{e}_{2,1} + \boldsymbol{I}_{1,1} - \boldsymbol{I}_{2,1} = (\boldsymbol{I} - \boldsymbol{s}_1 \boldsymbol{s}_1^T)\boldsymbol{\Phi}_K \mathbf{r}$ and the decision variable is $\boldsymbol{y}_{2,2} = \boldsymbol{s}_2^T\left(\boldsymbol{e}_{2,2} + \boldsymbol{I}_{1,2}\right) = \boldsymbol{s}_2^T(\boldsymbol{I} - \boldsymbol{s}_1 \boldsymbol{s}_1^T)(\boldsymbol{\Phi}_K + \boldsymbol{I})\mathbf{r}$. In the same manner, we can find a general expression of the residual signal and the decision variable of the k^{th} user at the m^{th} stage defined respectively as:

$$\mathbf{e}_{m,k} = \boldsymbol{\Phi}_{K-1}(\boldsymbol{\Phi}_K)^{m-1}\mathbf{r}, \tag{3-1}$$

and

$$\mathbf{y}_{m,k} = \boldsymbol{s}_k^T \boldsymbol{\Phi}_{k-1} \sum_{i=0}^{m-1} (\boldsymbol{\Phi}_K)^{i}\, \mathbf{r} = \mathbf{g}_{m,k}^T \mathbf{r}, \tag{3-2}$$

where

$$\boldsymbol{\Phi}_K = \coprod_{j=k}^{1} \left(\boldsymbol{I} - \boldsymbol{s}_j \boldsymbol{s}_j^T\right) \tag{3-3}$$

The decision variables can be written in a matrix form as

$$\mathbf{y}_{\boldsymbol{m}} = \mathbf{G}_{\boldsymbol{m}}^{\boldsymbol{T}}\mathbf{r}, \tag{3-4}$$

where

$$\mathbf{G}_{\boldsymbol{m}} = \left[\mathbf{g}_{\boldsymbol{m},\mathbf{1}}, \mathbf{g}_{\boldsymbol{m},\mathbf{2}}, \cdots, \mathbf{g}_{\boldsymbol{m},\boldsymbol{g}}, \cdots \mathbf{g}_{\boldsymbol{m},\boldsymbol{K}}\right]. \tag{3-5}$$

Therefore, the SIC detector can be described as linear matrix filtering that can be performed directly on the received chip-matched filtered signal vector without explicitly performing the interference cancellation.

The Bit Error Rate (BER) or the probability of error of the SIC detector can be generalized for the k^{th} user and m^{th} stage as shown in [20]:

$$P_b(m,k) = \frac{1}{2^{K-1}} \sum_{\substack{\boldsymbol{all\,b} \\ b_k=\mathbf{1}}} Q\left(\frac{\sqrt{2}\mathbf{g}_{m,k}^{\boldsymbol{T}}\mathbf{SAB}}{\sqrt{N_0\mathbf{g}_{m,k}^{\boldsymbol{T}}\mathbf{g}_{m,k}}}\right) \tag{3-6}$$

where $Q(\cdot)$ is the Q-function.

3.2.2 Convergence Analysis

This section discusses the convergence analysis of the SIC detector. First, we will mathematically establish the result that the SIC converges to the decorrelating detector. Then with the help of simulations, we will show that the SIC converges to the decorrelator.

3.2.2.1 Mathematical analysis

We know from the previous section that $\boldsymbol{y}_{m+1,k} = \boldsymbol{s}_k^T \boldsymbol{e}_{m+1,k} + \boldsymbol{y}_{m,k}$. Here, the term $e_{m+1,k}$ can be rewritten in terms of the decision variables y as

$$\boldsymbol{e}_{\boldsymbol{m}+\boldsymbol{1},\boldsymbol{k}} = \mathbf{r} - \sum_{j=1}^{k-1} \boldsymbol{s}_j \boldsymbol{y}_{\boldsymbol{m}+\boldsymbol{1},\boldsymbol{j}} - \sum_{j=k}^{K} \boldsymbol{s}_j \boldsymbol{y}_{\boldsymbol{m},\boldsymbol{j}}$$

(3-7)

When the SIC reaches convergence, then at stage m, $\boldsymbol{y}_{m,k} = \boldsymbol{y}_{m+1,k}$. Then equation (3-7) becomes

$$\boldsymbol{s}_{\boldsymbol{k}}^{\boldsymbol{T}} \mathbf{r} = \boldsymbol{s}_{\boldsymbol{k}}^{\boldsymbol{T}} \left(\sum_{j=1}^{k-1} \boldsymbol{s}_j \boldsymbol{y}_{\boldsymbol{m}+\boldsymbol{1},\boldsymbol{j}} + \sum_{j=k}^{K} \boldsymbol{s}_j \boldsymbol{y}_{\boldsymbol{m},\boldsymbol{j}} \right)$$

(3-8)

But since $\boldsymbol{y}_{m,k} = \boldsymbol{y}_{m+1,k}$, equation (3-8) becomes

$$\boldsymbol{s}_{\boldsymbol{k}}^{\boldsymbol{T}} \mathbf{r} = \boldsymbol{s}_{\boldsymbol{k}}^{\boldsymbol{T}} \sum_{j=1}^{K} \boldsymbol{s}_j \boldsymbol{y}_{\boldsymbol{m},\boldsymbol{j}} = \boldsymbol{s}_{\boldsymbol{k}}^{\boldsymbol{T}} \mathbf{S} \mathbf{y}_{\mathbf{m}}$$

(3-9)

Thus for all $k = 1,2,\cdots,K$, we can conclude that $\mathbf{r} = \mathbf{S}\mathbf{y}_{\mathbf{m}}$ and consequently,

$$\mathbf{S}^{\mathbf{T}} \mathbf{r} = \mathbf{S}^{\mathbf{T}} \mathbf{S} \mathbf{y}_{\mathbf{m}} \quad (3\text{-}10)$$

Rearranging equation (3-10), we can write

$$\mathbf{y}_{\mathbf{m}} = (\mathbf{S}^{\mathbf{T}}\mathbf{S})^{-\mathbf{1}} \mathbf{S}^{\mathbf{T}} \mathbf{r} \quad (3\text{-}11)$$

and finally if $\mathbf{R} = \mathbf{S}^{\mathbf{T}} \cdot \mathbf{S}$ is not singular, then equation (3-11) becomes

$$\mathbf{y_m} = \mathbf{R}^{-1}\mathbf{S}^{\mathbf{T}}\mathbf{r} = \mathbf{R}^{-1}\mathbf{y_{MF}} = \mathbf{y_{DEC}} \quad (3\text{-}12)$$

which can be verified using equation (2-3). Thus, we can conclude from equation (3-12) that the SIC detector converges to the decorrelating detector. For further reference, the reader is referred to [10] and [20].

3.2.2.2 Analysis using simulations

We investigated the convergence behavior of the SIC detector using the MATLAB. The following parameters were used to perform the simulation[4]:

Number or users: K =20

Spreading Sequence length: L_c = 31 (Gold Codes)

We use a single path AWGN with perfect power control. Users are ordered according to their matched filter output and are ordered at every stage. Rayleigh fading channel is used. The simulation is performed at an SNR of 20dB. The simulation results are shown in Figure 3-3.

Because of the heavily loaded system i.e. 20 users, we can see that the convergence is at the 15th stage. For less number of users, the convergence is achieved earlier. Also note that the optimal performance is achieved before the convergence. Many factors are associated with this unexpected behavior of the SIC. Note that the BER does not generally decrease monotonically as the number of stages increase. In fact, the minimum overall BER is achieved prior to convergence. The optimum number of stages is

[4] These simulation parameters will be used throughout the thesis.

determined by system load, correlation between users and the noise level and does not have to be the same for different users [21]. Another reason for such behavior is stated in [20]. The author explains "At intermediate stages where the residual received vector gets close to estimating the noise vector correctly, it is therefore possible that the linear SIC performs better in terms of BER than at convergence. The potential improvement is however relatively insignificant and of theoretical interest only."

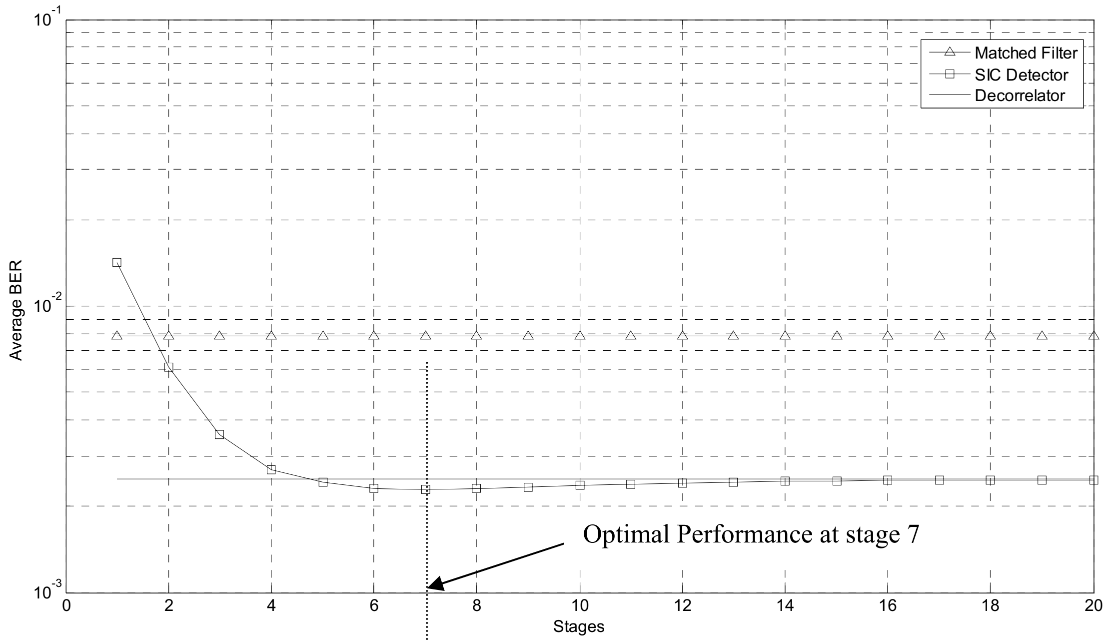


Figure 3-3 : Convergence behavior of the SIC detector.

3.3 Non-linear SIC detector

The non-linear SIC is similar to the linear SIC except that it differs in its ICU structure. It consists of a non-linear mapping function. Recall that in the linear SIC, the mapping function used is $f_x\left(y_{m,k}\right) = \boldsymbol{y}_{m,k}$. As can be inferred from Figure 3-2, we use a non-linear mapping or decision function for $f_x(\cdot)$. Some of the common decision functions used are the hyperbolic tangent, the hard-limiter, clipped soft decision, null zone and unit clipper [21]. Other possible decision functions which were studied in the literature can be referred from [11] and [23].

We have studied from literature and analyzed the hyperbolic tangent function. The decision function is expressed as $f_{TAN}\left(y_{m,k}\right) = tanh\left(\boldsymbol{y}_{m,k}\right)$. Simulations were carried out to investigate the NLSIC. The bit error rate plot is shown in Figure 3-4.

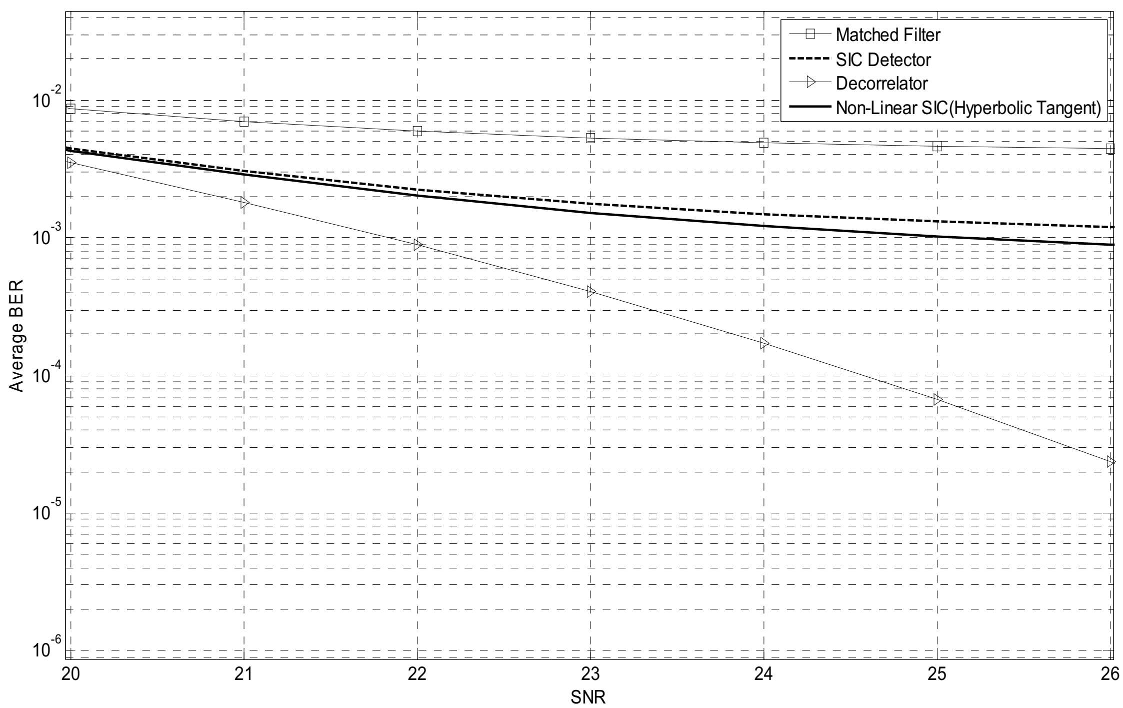


Figure 3-4 : BER analysis of the Non-linear SIC.

Note that the non-linear SIC shows slight improvement over the linear SIC detector.

3.4 Weighted SIC

A further advancement of the linear and non-linear SIC was the weighted SIC. Bentrcia, in his paper [13], proposed a technique of adding a weighting factor $\omega_{p,k}$ as shown below

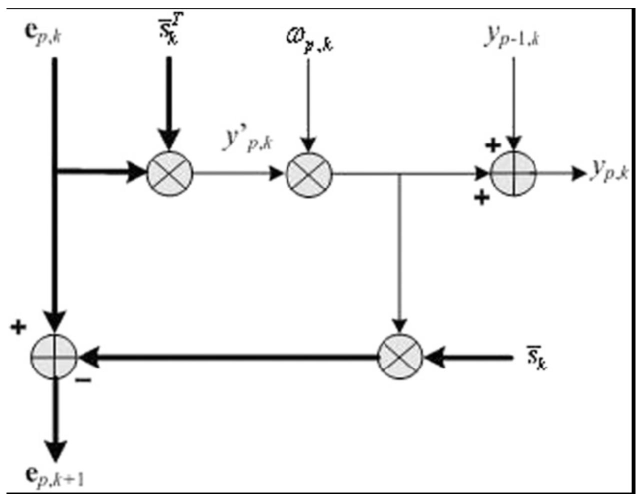


Figure 3-5 : Basic ICU of the weighted SIC detector [13].

For further information about the structure and derivation of the weighting factor of the weighted SIC, the reader is referred to [13].

3.5 Summary

In this chapter, we have studied the SIC detectors in detail. In section 3.2, we have studied and analyzed the linear SIC detector. Section 3.3 investigated the non-linear SIC detector and section 3.4 briefly discussed the weighted SIC detector. This chapter was a brief literature review of the SIC structures proposed to date. In our proposed method, we will introduce a weighted SIC method, different from that of section 3.4. It further uses a fuzzy-based estimated weight. More details are discussed in chapter 5.

Chapter 4

The Fuzzy Logic

Although the notion of fuzzy logic has been around for some time, it is still considered a "non-classic" approach to problem-solving. Fuzzy logic has been applied in different fields, such as automated control and decision making support. Some of the uses also include automated drug infusion, medical diagnosis support, handwritten digits recognition, fraud detection, automated sleep-states classification, automated navigation systems and industrial and consumer goods applications. Day to day consumer products such as cameras, washing machines and automobiles, are currently being sold with embedded fuzzy controllers.

The underlying idea in fuzzy logic applications is that people are capable of decision making using imprecise or uncertain knowledge, whereas traditional computer algorithms require precise information. Fuzzy Logic provides a means to represent human knowledge, i.e. knowledge with a "degree-of-truth", in a way that can be

processed with the use of computers. This chapter will provide an introduction to Fuzzy Logic, presenting some basic concepts, and show several applications, taken from the literature and from the speaker's own experience.

4.1 Introduction

The Fuzzy Logic, since its first introduction in 1965, faced a lot of opposition due to its nature. As the name suggests, the calculations are done based on 'fuzzy' or uncertain assumptions. But in essence, the fuzzy logic uses mathematical models and calculations. Surprisingly, the results obtained using the fuzzy-based systems are good, and in some cases, superior than their 'non-fuzzy' counterpart.

Fuzzy Logic was originated in 1965 by Lotfi A. Zadeh, Professor for Computer Science at the University of California in Berkeley [24], [25], [26]. Basically, Fuzzy Logic (FL) is a multivalued logic that allows intermediate values to be defined between conventional evaluations like true/false, yes/no, high/low, etc. Notions like rather tall or very fast can be formulated mathematically and processed by computers, in order to apply a more humane way of thinking in the programming of computers. A Fuzzy system is an alternative to traditional notions of set membership and logic that has its origins in ancient Greek philosophy.

To understand how fuzzy logic works, we need to elaborate the basics of a fuzzy logic system. Here we will focus on the fuzzy set theory and present the basic definitions and operations. Please be aware that the interpretation of fuzzy set theory in the following is just one of several possible; Zadeh and other authors have suggested alternative definitions also.

4.2 Fuzzy Set

The very basic notion of fuzzy systems is the fuzzy set. Fuzzy sets are a further development of the mathematical concept of a set. The concept of sets was first developed by the German mathematician Georg Cantor [30]. His theory of sets met much resistance during his lifetime, but nowadays most mathematicians believe it is possible to express most, if not all, of mathematics in the language of set theory. Many researchers are looking at the consequences of 'fuzzifying' set theory, and much mathematical literature is the result. For control engineers, fuzzy logic and fuzzy relations are the most important in order to understand how fuzzy rules work.

In classical mathematics, we are familiar with the concept of the 'conventional' set. A set is a collection of objects which can be treated as a whole. In the language of FL, this conventional set is known as the 'crisp set'. A crisp set can be specified by its elements (or members as known in the language of FL) which characterize a set completely. For example, the list of members $A = \{0,1,2,3\}$ specifies a finite set. Conventional or crisp sets are binary. An element either belongs to the set or doesn't.

Fuzzy sets, on the other hand have more than an either-or criterion for membership. Take for example the set of young people. A one year old baby will clearly be a member of the set, and a 100 years old person will not be a member of this set, but what about people at the age of 20, 30, or 40 years? Another example is a weather report regarding high temperatures, strong winds, or nice days. In other cases a criterion appears nonfuzzy, but is perceived as fuzzy: a speed limit of 60 kilometers per hour, a check-out time at 12 noon in a hotel, a 50 years old man. Zadeh proposed a 'grade of membership', such that

the transition from membership to non-membership is gradual rather than abrupt [23]. In other words, Fuzzy Sets can represent the degree to which a quality is possessed.

The grade of membership for all its members thus describes a fuzzy set. An item's grade of membership is normally a real number between 0 and 1, often denoted by the Greek letter μ. The higher the number, the higher the membership. Cantor's set is a special case where elements have full membership, i.e. $\mu = 1.$ Cantor's sets are the 'non-fuzzy' sets, today known as crisp sets, which avoids the dilemma.

Notice that there is no formal basis for how to determine the grade of membership. The membership for a 50 year old in the set 'young' depends on one's own view. The grade of membership is a precise, but subjective measure that depends on the context. The membership is spread over a wide range of values, which is known as the *Universe of Discourse*.

4.2.1 Universe of Discourse

Elements of a fuzzy set are taken from a Universe of Discourse or Universe for short. The universe contains all elements that can come into consideration. Even the universe depends on the context. For example, the set of 'young people' could have all the human beings in the world as its universe. Alternatively, it could be the numbers between 0 and 100. An advantage of using the universe is to suppress faulty measurement data. The universe can use numerical or non-numerical data. For example, the terms {bitter, sweet, sour….} are non-numerical data and are said to be taken form a *psychological continuum*.

To classify the numerical or non-numerical data over the whole universe, we use what are known as *Membership Functions*.

4.2.2 Membership Functions

In the universe of discourse, every element is a member of the fuzzy set to some grade. Some of the elements may also have a grade of zero. The set of elements that have a non-zero membership is called the *support* of the fuzzy set. The function that ties a number to each element x of the universe is called the *membership function* $\mu(x)$.

Membership Functions (MF) are of many shapes: bell-shaped, s-shaped, triangular or trapezoidal shaped. But the design of the MFs is not limited to these shapes. One can design a MF according to the specification of the problem. Also, these MFs can be continuous or discrete. For the continuous case, it is a mathematical function. In the discrete form the membership function and the universe are discrete points in a list. Sometimes it is more convenient with discrete representation.

Figure 4-1 shows a few examples of MFs. These were generated in MATLAB. Note the different shapes of the MFs. MF1 is of 'trapezoidal' shape, MF2 is 'triangular', MF3 is 'gauss-bell' and MF4 is of 'gaussian' shape. The X-axis ranges the whole *Universe of Discourse*. Also note that the Y-axis can have values between 0 and 1 i.e. a fuzzy set, A, is normalized to its largest membership value $a/\max(a)$.

The MFs should reflect the designer's knowledge about the problem. It provides smooth transition between member and nonmembers of a fuzzy set. They should also be simple to calculate. The author in [40] explains that the membership functions can be chosen by the user arbitrarily, based on the user's experience. Hence, the membership functions for

two users could be quite different depending upon their experiences, perspectives, cultures, etc. and in scientific terms, depending upon the problem.

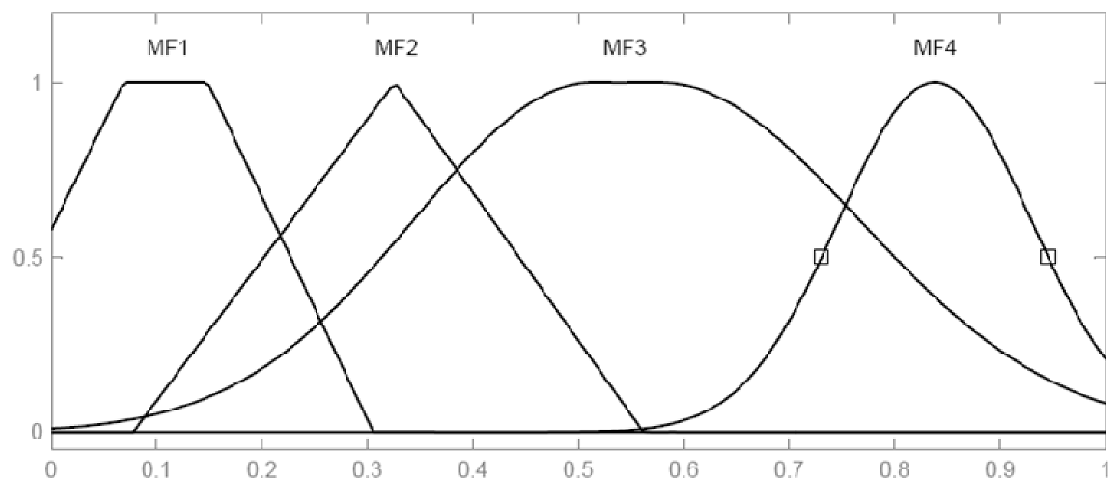


Figure 4-1 : Membership functions.

4.2.3 Operations on Fuzzy sets

Just like the operations in the set theory, there are similar operations in the Fuzzy set theory. We can introduce basic operations on fuzzy sets. Similar to the operations on crisp sets, we can also perform intersection, union and negation (complement) on fuzzy sets. In his very first paper about fuzzy sets [23], Zadeh suggested the minimum operator for the intersection and the maximum operator for the union of two fuzzy sets. It can be shown that these operators coincide with the union and intersection if we only consider the membership degrees 0 and 1. Figure 4-2 describes briefly how the operations on the fuzzy sets can be related to the operations on conventional sets.

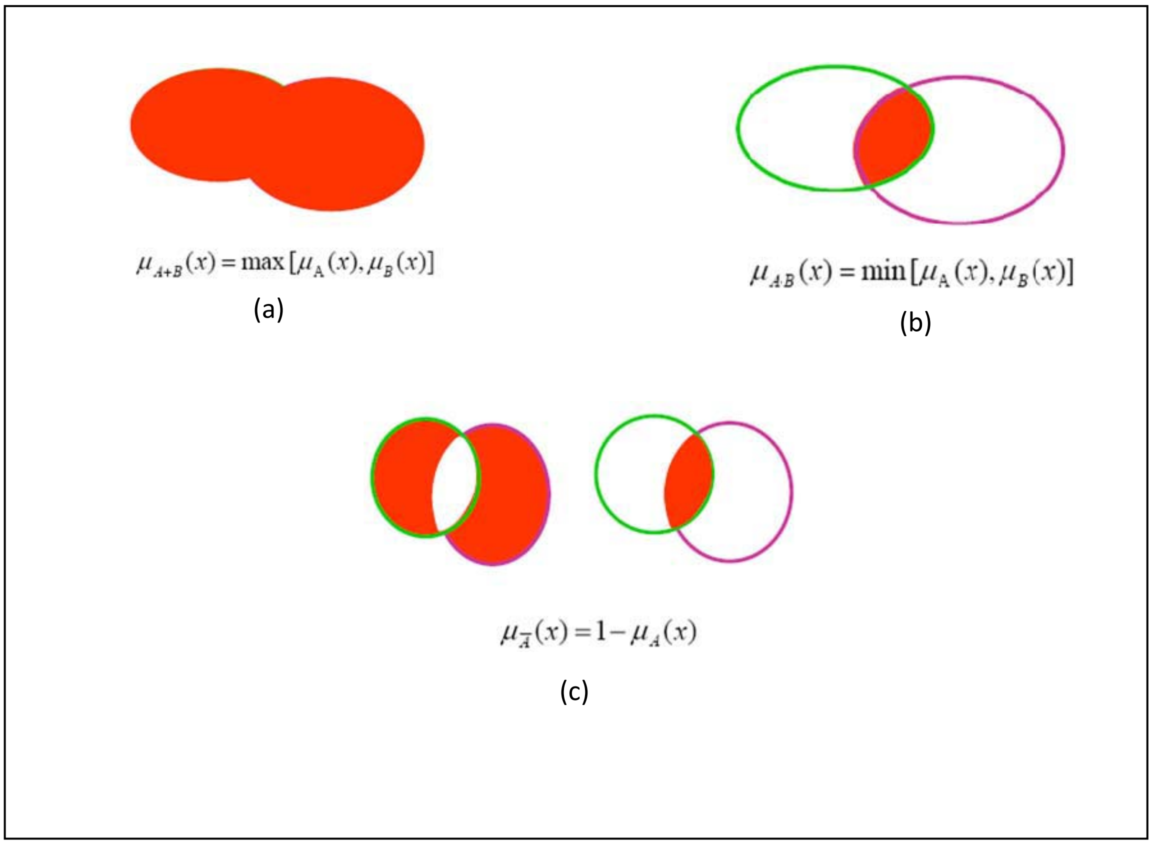


Figure 4-2 : Basic Operations on Fuzzy Sets: (a) Union, (b) Intersection and (c) Complement.

The membership function is obviously a crucial component of a fuzzy set. It is therefore natural to define operations on fuzzy sets by means of their membership functions. For example, if A is a fuzzy interval between 5 and 8 and B is a fuzzy number about 4, then, the above shown operations in terms of fuzzy MFs can be expresses as shown in Figure 4-3. The figures (a) and (b) describe two different sets in the form of MFs. The shaded part in figure (c) is the intersection and the shaded part in figure (d) is the union of A and B. The complement of A and B is shown in figure (d).

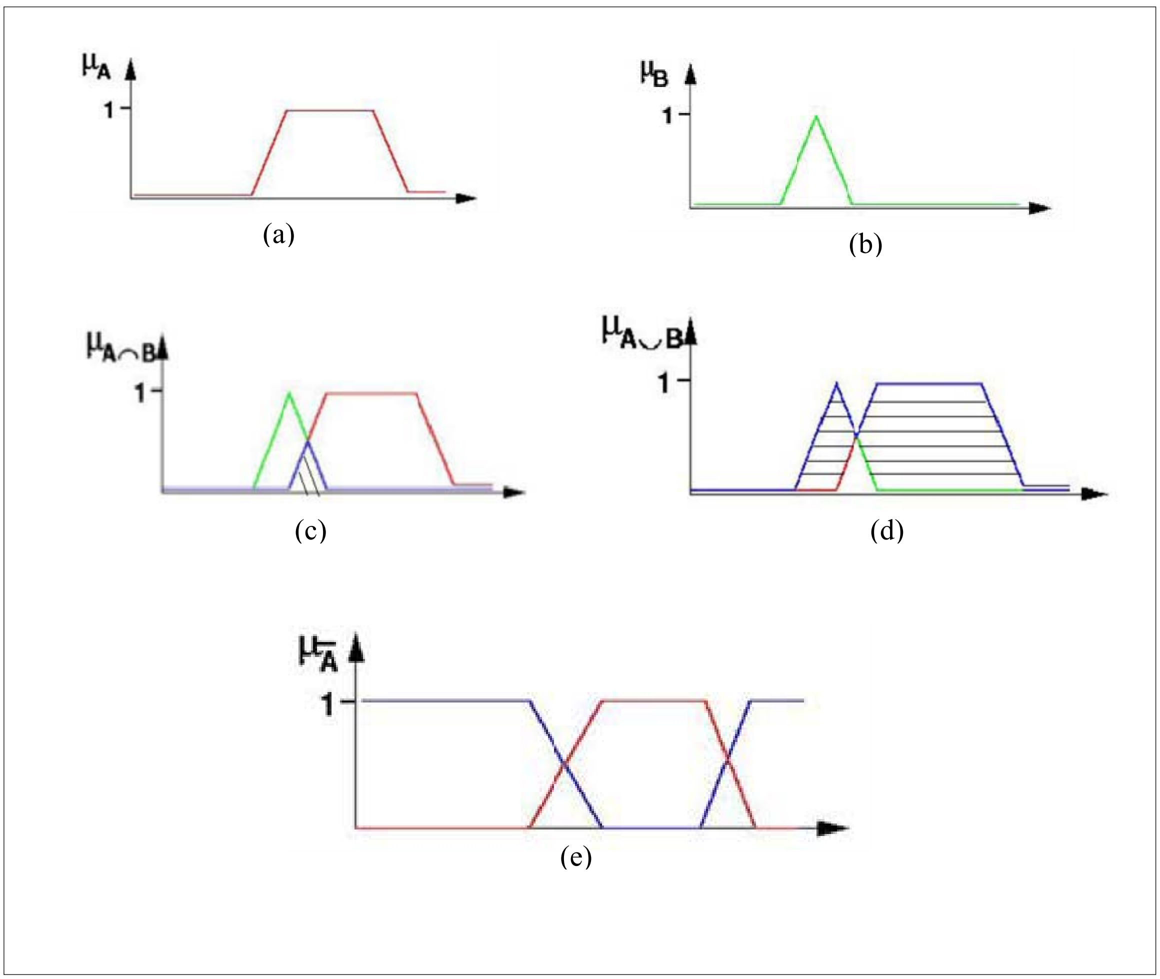


Figure 4-3 : Basic operations on Fuzzy MFs.

The operations in Figure 4-3 are the most basic operations of the fuzzy sets. The intersection operation corresponds to the minimum, the union corresponds to the maximum and the complement corresponds to the negation. Max and min are the most commonly used operators. Other operators are also used according to the problem. These operators are very helpful to predict the outcome of long linguistic sentences. The union is the fuzzy OR and the intersection is the fuzzy AND. An example, with the help of Figure 4-4, will elaborate the above mentioned points. Consider the case where we have 2 inputs, A and B, which have their own MFs. The universe of A spans over the range $\{0,1\}$ and the universe of B spans over $\{0,40\}$. Given $A = 0.4$ and $B = 20$, we can see from Figure 4-4 that degree of membership of $A = 0.7$ and $B = 0.9$. Applying min, we have $A \cap B = \min(A, B) = 0.7$.

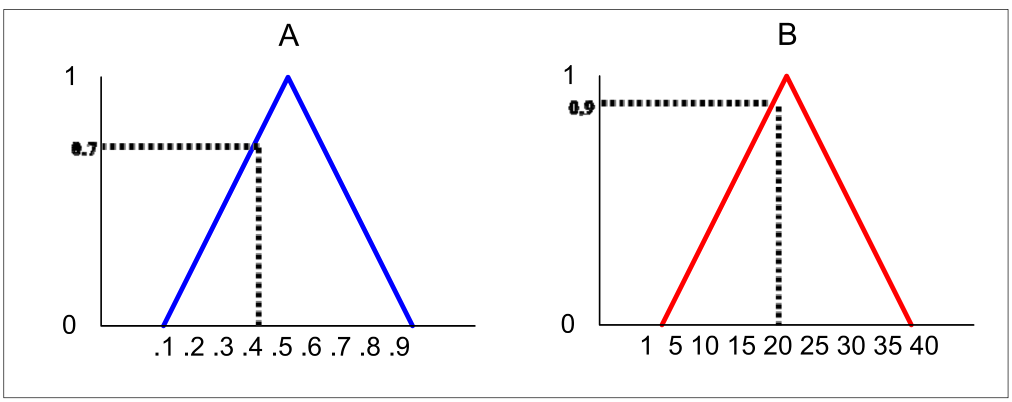


Figure 4-4 : Example of fuzzy operator.

These MFs are classified based on linguistic variables. The usage of fuzzy variables like {low, medium, high} or {very small, small, big, very big} to name the MFs is the essence of the fuzzy logic. The usage of such linguistic terms to define MFs gives the user enormous flexibility to design the system. An example is shown in the Figure 4-5.

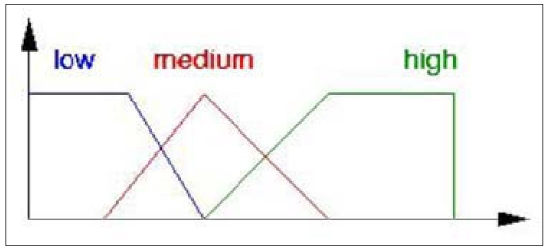


Figure 4-5 : MFs based on linguistic variables.

4.3 The 'Fuzzy' Logic

Logic started as the study of language in arguments and persuasion, and it may be used to judge the correctness of a chain of reasoning, in a mathematical proof for example. In conventional reasoning, a proposition is either true or false, but not both. In Fuzzy logic, a proposition may be true or false or have an *intermediate* value, such as maybe true. The sentence *X is tall* is an example of such a proposition in fuzzy logic. It depends on one's own view what the definition of *tall* is. It may be convenient to restrict the possible truth values to a discrete domain, say {0, 0.5, 1} for linguistic variables such as *false*, *maybe true* and *true*. Such is the case of a *multi-valued* logic. In practice a finer subdivision of the unit interval may be more appropriate.

4.3.1 Connectives

In daily conversation and mathematics, sentences are connected with the words like *and*, *or*, *if-then* and *if and only if*. In the 'fuzzy' language, these are called *connectives*. Connectives are used to map linguistic variables of real life to the fuzzy problem. A sentence which is modified by the word "not" is called the *negation* or *complement* of the original sentence. The word "and" is used to join two sentences to form the *conjunction* of the two sentences. Similarly, a sentence formed by connecting two sentences with the word "or" is called the *disjunction* of the two sentences.

Using two sentences (or conditions), we can construct one of the form “if sentence 1 is X *and* sentence 2 is Y then…”. This is called a *conditional* sentence. Note that we use the connective *and* between the two conditions. The sentence following “If” is the *antecedent*, and the sentence following “then” is the *consequent*. Usually, the fuzzy conditions are of the form

If <condition 1 is …> and <condition 2 is …> then <conclusion>

Condition 1 and condition 2 are linguistic terms in the form of MFs. The <conclusion> is also a fuzzy set, which represents the linguistic term expressing a flexible predicate, which characterizes the output behavior of the system if all conditions are satisfied. Once a given problem is formulated, the consequents are then implied to a numerical form from a linguistic sentence. Using the connectives, the conditions and the conclusions, the fuzzy *rules* are formulated. The rules are of the form

IF input 1 is X and input 2 is Y then output is Z

The rules are tabulated using all the conditions and variables in a *rule base*.

The rule base formation is not a matter of trivial concern. Lot of research has been done in the direction and many methods have been proposed which were very effective in rule base formation. Mendel and Wang proposed a method in their paper [32]. They proposed a five step method which is used to extract numerical data from linguistic fuzzy variables. Another modification of [32] was proposed in [33].

4.3.2 Inference

In order to draw conclusions from a rule base we need a mechanism that can produce an output from a collection of *if-then* rules. The verb *infers* means to conclude from evidence, deduce, or to have as a logical consequence. To understand the concept, it is useful to think of a function $y = f(x)$ where f is a given function, x is the independent variable, and y is the result.

The above mentioned components of a FL system can be better understood with the help of an example, as elaborated in the following section.

4.4 Example of a simple system using Fuzzy Logic

A system which works on the basis of Fuzzy Logic is called a Fuzzy Inference System (FIS). An FIS can broadly divided into 5 processes namely "*fuzzification* of the input variables, *application* of the fuzzy operator (AND or OR) in the antecedent, *implication* from the antecedent to the consequent, *aggregation* of the consequents across the rules, and *defuzzification*" [34]. Each of these steps is elaborated below for better understanding of the reader.

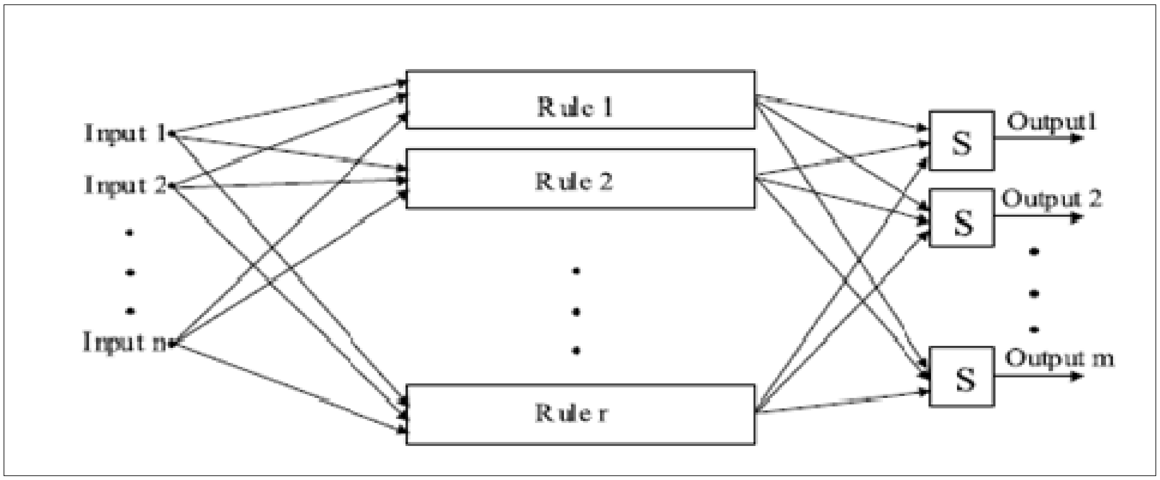
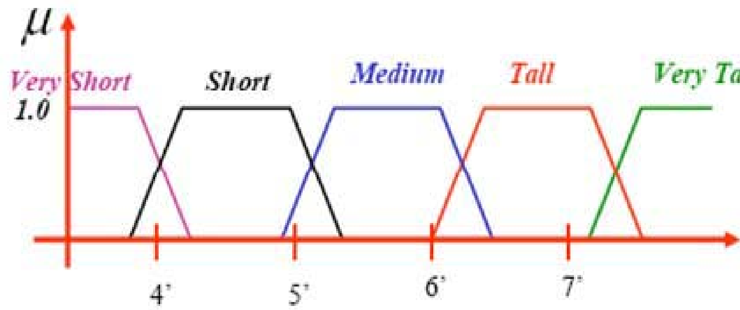
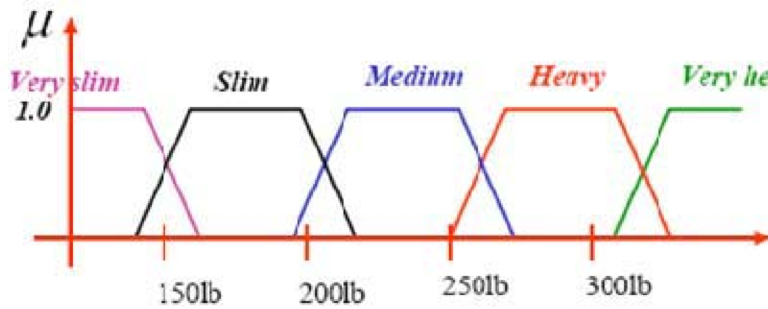


Figure 4-6 : Basic structure of an FIS.

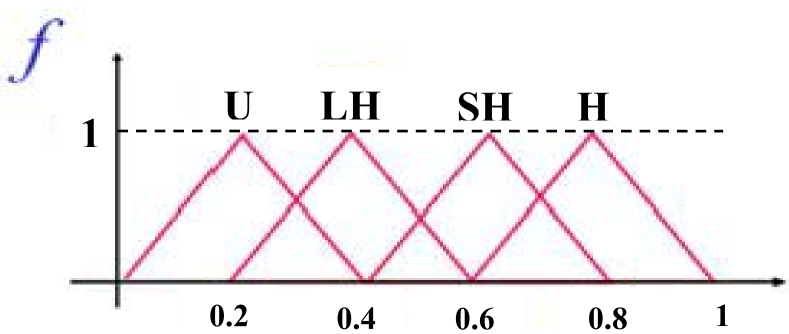
Fuzzification of the input variables: In this step, the input variables of the system are 'fuzzified'. Instead of fixed (crisp) numerical values, they are converted to vague or fuzzy values with the help of MF. A very basic example commonly used in day-to-day life is to find how healthy a person is based on height and weight [36]. We have two inputs: height and weight. Based on our experience or intuition, we classify the height (in ft) under the MFs {Very Short, Short, Medium, Tall, Very Tall} and the weight (in lbs) under the MFs {Very Slim, Slim, Medium, Heavy, Very Heavy}.



(a)



(b)



(c)

Figure 4-7 : Fuzzification of the inputs (a) Height (b) Weight and the output (c) Decision.

Here, the author has used five MFs for each input. Note that the shapes of the MFs trapezoidal but they can also be different based on user's own experience. The MFs of the output are triangular. Also note that the maximum value of a MF is 1.

Application of the fuzzy operator: Once the input values and the decision is *fuzzified*, they are related using the connectives AND and OR. Using the data, a rule base is formed. The rules reflect experts' decisions. They are tabulated as fuzzy words and can be grouped in subsets. Rules can be redundant and can be adjusted to match desired results.

We need to define a certain output to form the rules. In this case, the decision is how healthy a certain person is. It is classified as *f = {Unhealthy U, Less Healthy LH, Somewhat Healthy SH, Healthy H}.* The MF of the inputs and decision is shown in Figure 4-7. The rules are formed as mention in Section 4.3.1. A rule-base table is shown in Table 4-1.

Weight / Height	Very Slim	Slim	Medium	Heavy	Very Heavy
Very Short	H	SH	LH	U	U
Short	SH	H	SH	LH	U
Medium	LH	H	H	LH	U
Tall	U	SH	H	SH	U
Very Tall	U	LH	H	SH	U

Table 4-1 : Rule Base of 'Healthy' example.

The rows represent the height and the columns represent the weight. A rule can be read from the table as follows:

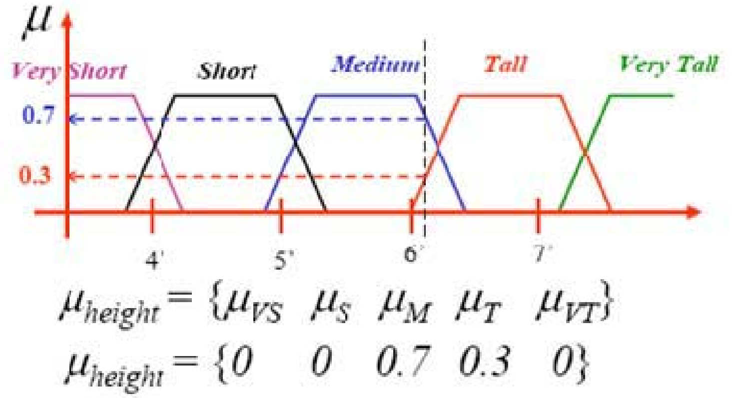
IF <Height is Medium> *AND* <Weight is Medium> *THEN* <Man is Healthy>

Implication from the antecedent to the consequent: Once the rule base is ready, we can compute the memberships of the inputs. Assume, in our example, that a person X has height of 6’1” and weight of 140 lbs. To calculate the membership graphically, we draw a perpendicular line at the 6’1” mark of the universe of discourse and extend the points of intersections of the MFs to the degree of membership. The same procedure is done for the MFs of weight also. It is illustrated in Figure 4-8.

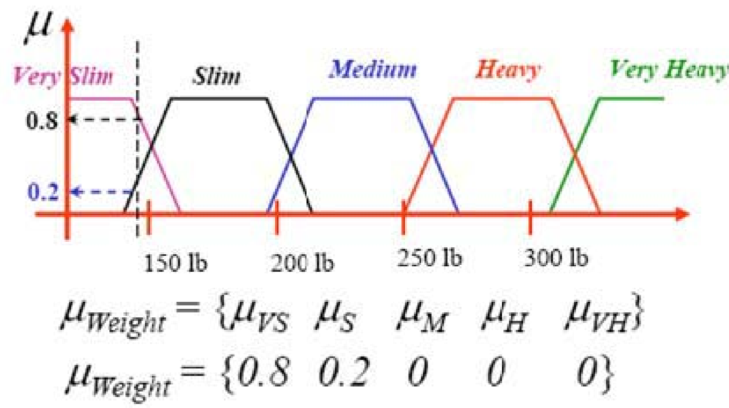
In (a) of Figure 4-8, we can see that the height 6’1” has memberships under two functions: Medium and Tall. It has a membership value of 0.3 in medium and a value of 0.7 in tall. In linguistic terms, this means that the person X (of height 6’1”) is 30% in tall and 70% in medium range. Hence the symbols $\mu_T = 0.3$ and $\mu_M = 0.7$. A similar procedure is done for the weight as shown in (b) of Figure 4-8.

Using this membership, it is then mapped to the rule base. The corresponding rows and columns are selected, the particular membership value is assigned and in the final step, the *min* operation is implied.

Aggregation of the consequents: once the membership has been implied in the rule base, aggregation of the outputs is required. The outputs are then scaled according to the calculated weights of the MFs.



(a)



(b)

Figure 4-8 : Example of degree of MF (a) for height and (b) for weight.

Defuzzification: this is the final step of an FIS in which the fuzzy outputs from the system are converted to a crisp or fixed value. The most commonly used methods of defuzzification are the maximum method and centroid calculation.

In the max method, the set with the largest membership value is selected. In our example $f = \{\mu_U = 0.3, \mu_{LH} = 0.7, \mu_{SH} = 0.2, \mu_H = 0.2\}$, we can see that $\mu_{LH} = 0.7$ is the maximum among all the decision. Thus, we choose the option Less Healthy.

The centroid calculation method is more calculation technique. It gives a crisp value output unlike the maximum method which gives a more fuzzified output of 'Less Healthy'. It uses the scaled fuzzified outputs and calculates the Final Decision (FD) using the formula

$$FD = \frac{\sum \mu \cdot D}{\sum \mu}$$

where μ is the value of the scaled membership and *D* is the membership. The above equation then gives the value for our example as

$$FD = \frac{0.3 \times 0.2 + 0.7 \times 04 + 0.2 \times 0.6 + 0.2 \times 0.8}{0.3 + 0.7 + 0.2 + 0.2} = 0.4429$$

This crisp value of 0.4429 falls under the membership of LH and SH group i.e. 75% LH and 25% SH as shown in the Figure 4-9.

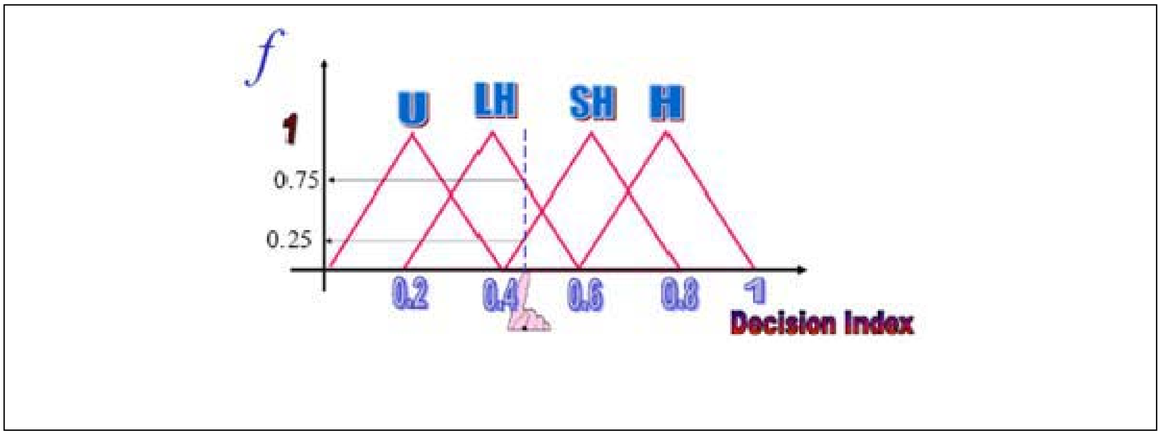


Figure 4-9 : Fuzzy Decision Index.

The centroid calculation method is widely used because it gives crisp outputs which are much preferred for scientific purposes.

4.5 Example of Interference Cancellation using fuzzy logic

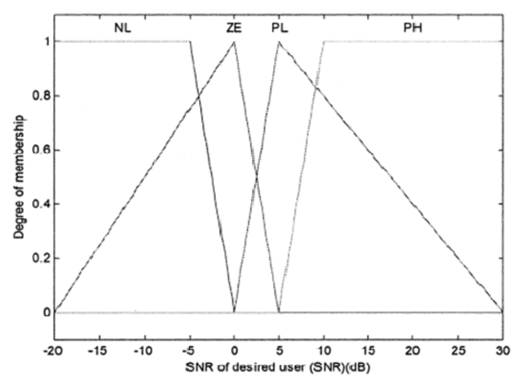
Another field which achieved considerable progress due to fuzzy logic was the field of Telecommunications. From hand-off strategies to estimating weights in multiuser detection, fuzzy has always found its place has been appreciated. In this section, we study an example from literature review which uses fuzzy logic to estimate a weighting factor for a PIC scheme.

The author in [41] proposed a multistage fuzzy-based partial PIC multiuser detector for multi-carrier direct-sequence code-division multiple-access (MC-CDMA) communication systems over frequency selective fading channels. The partial cancellation weight tries to reduce the cancellation error in PIC schemes due to the wrong interference estimations in the early stages and thus outperforms the conventional PIC under the heavy load for MC-CDMA systems. Therefore, the adaptive cancellation

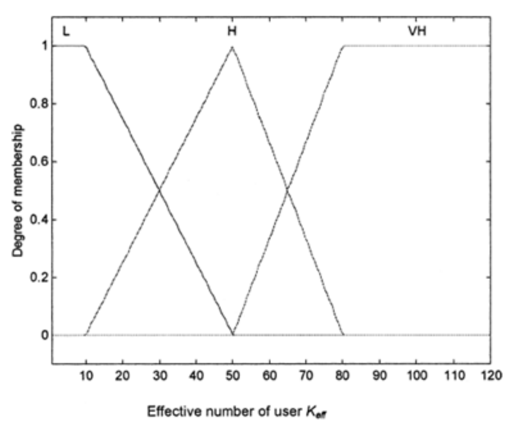
weights are inferred from a proposed multistage fuzzy inference system (FIS) to perform effective partial PIC multiuser detection.

In PIC multiuser detectors, the users' signals are processed in parallel at the same instant. Unlike the SIC, the PIC has one ICU which processes all the users in one stage.

The weight here is estimated using an FIS which has two inputs: the SNR and the effective number of users. The membership functions used for the two inputs are shown in Figure 4-10.



(a)



(b)

Figure 4-10 : MFs of the inputs (a) SNR and (b) Effective number of users.

The triangular MFs were used for both the inputs. As mentioned earlier, the shape of the MFs is arbitrary and can change from user to user. The SNRs are divided into four sets: Negative Low (NL), ZEro (ZE), Positive Low (PL) and Positive High (PH) based on the values of SNRs. Note that a MF can also range in the negative side also. The effective number of users K_{eff} is divided into three sets namely Low (L), High (H) and Very High (VH) depending on the number of effective users. This can also be changed according to the problem as the choice of the number of effective users can vary. A similar plot of the output is shown in Figure 4-11.

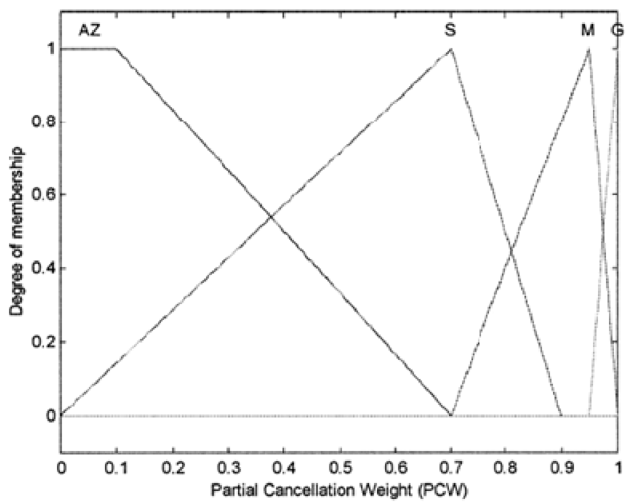


Figure 4-11 : MF plot of the output: Estimated weight.

The weights are divided into four MFs namely Almost Zero (AZ), Small (S), Medium (M), and Great(G).

The results obtained showed good improvements over the conventional PIC detector. Stage-wise comparison of these results by the author is shown in Figure 4-12.

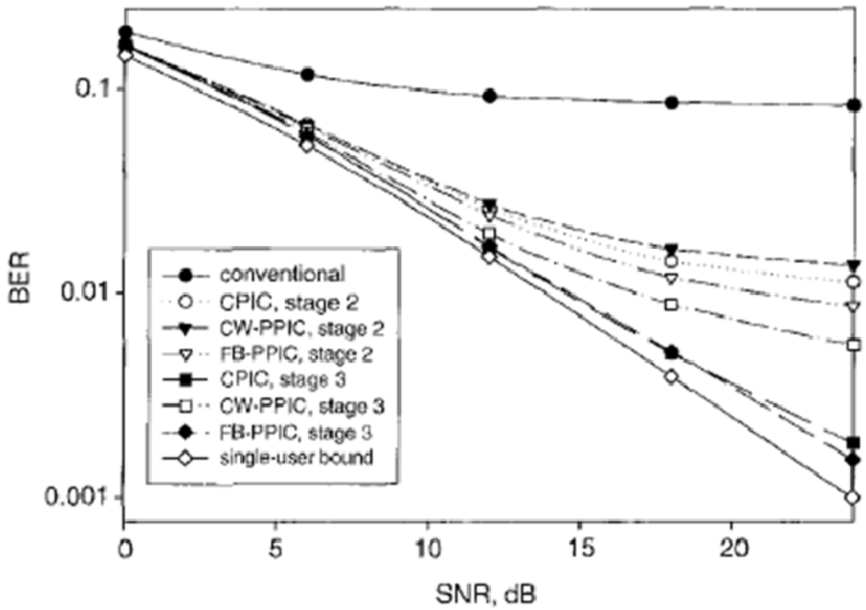


Figure 4-12 : Simulation results of fuzzy-based partial PIC.

4.6 Conclusion

Fuzzy Logic provides a different way to approach a control or classification problem. It focuses on what the system should do rather than trying to model how it works. One can concentrate on solving the problem rather than trying to model the system mathematically, if that is even possible. It provides a natural way to model some types of human expertise in a computer program.

On the other hand the fuzzy approach requires a sufficient expert knowledge for the formulation of the rule base, the combination of the sets and the defuzzification. It can deal with imprecision, and vagueness, but not uncertainty. It requires tuning of membership functions, using expertise or intuition.

In General, the employment of fuzzy logic might be helpful, for very complex processes, when there is no simple mathematical model (e.g. Inversion problems), for highly

nonlinear processes or if the processing of (linguistically formulated) expert knowledge is to be performed. According to literature the employment of fuzzy logic is not recommendable, if the conventional approach yields a satisfying result, an easily solvable and adequate mathematical model already exists, or the problem is not solvable.

Chapter 5

The Proposed Technique: Fuzzy Based Weighted SIC detector

In the last chapter, we have studied the fuzzy logic in detail and explained how an FIS works, elaborated with the help of an example. Although it started as a method in Control and Neural networks, Fuzzy Logic was adopted widely in many fields due to its simple and naïve approach of problem solving. Applications may be found in the areas of Environmental Protection, Economy, Image Processing, Power Systems, Social Sciences, Music, Hardware and in Medicine, like for example in support of diagnosis, in Medical Image Processing, in medical data mining, and in medical modeling [31].

In this chapter, we use the fuzzy logic to estimate a weighting factor for a weighted SIC detector. Section 5.1 discusses the structure of the weighted SIC detector. Section 5.2 introduces a method with which the weight of this detector is estimated using Fuzzy

Logic and how it is applied. Section 5.3 discusses the results obtained using the new technique followed by a conclusion.

5.1 The Proposed Weighted SIC detector

Consider the structure of SIC detector as shown in section 3.2.1. With a slight modification to the Linear SIC of section 3.2, we add a weighting factor to the decision statistic $\mathbf{y}$. A block diagram of the Weighted SIC (WSIC) structure is shown in Figure 5-1.

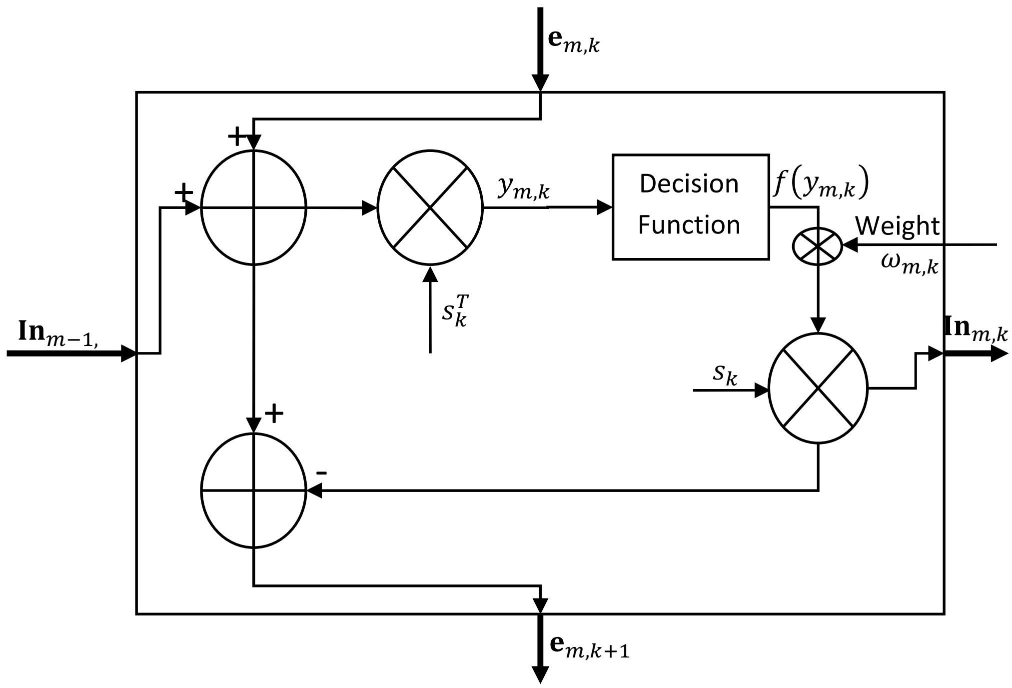


Figure 5-1 : Structure of the proposed weighted SIC detector.

The residual signal at the input of the first ICU in the first stage is defined as $\boldsymbol{e}_{1,1} = \mathbf{r}$ and the corresponding decision variable is defined as $\boldsymbol{y}_{1,1} = \omega_{1,1} \cdot \boldsymbol{s}_1^T\left(\boldsymbol{e}_{1,1} + \boldsymbol{In}_{0,1}\right) = \omega_{1,1} \cdot \boldsymbol{s}_1^T\mathbf{r}$. Note that the MAI for the first stage for all the users is 0 i.e. $\boldsymbol{In}_{0,1} = \boldsymbol{In}_{0,2} = \cdots = \boldsymbol{In}_{0,K} = 0$.

For the second ICU in the first stage, the received signal vector is obtained by estimating the MAI due to the first user and then subtracting it from the received signal i.e. $\boldsymbol{e}_{1,2} = \boldsymbol{e}_{1,1} + \boldsymbol{In}_{0,1} - \boldsymbol{In}_{1,1} = \boldsymbol{e}_{1,1} - \boldsymbol{s}_1\boldsymbol{s}_1^T\mathbf{r} = (\boldsymbol{I} - \boldsymbol{s}_1\boldsymbol{s}_1^T)\mathbf{r}$. The consequent decision variable is defined as $\boldsymbol{y}_{1,2} = \omega_{1,2} \cdot \boldsymbol{s}_2^T\left(\boldsymbol{e}_{1,2} + \boldsymbol{In}_{0,2}\right) = \omega_{1,2} \cdot \boldsymbol{s}_2^T(\boldsymbol{I} - \boldsymbol{s}_1\boldsymbol{s}_1^T)\mathbf{r}$. Here, $\boldsymbol{I}$ is the identity matrix.

Similarly, we can define the decision statistics and the residual signal for all the users at different stages. The closed form of the residual signal at the output of the k^{th} user ICU at the first stage is expressed as

$$\boldsymbol{e}_{1,k} = \coprod_{j=k-1}^{1}\left(\boldsymbol{I} - \boldsymbol{s}_j\boldsymbol{s}_j^T\right)\mathbf{r} = \boldsymbol{\Phi}_{k-1}\mathbf{r}$$

where $\coprod$ indicates the product of matrices with decreasing indices. The corresponding decision variable is expressed as $\boldsymbol{y}_{1,k} = \omega_{1,k} \cdot \boldsymbol{s}_k^T\boldsymbol{e}_{1,k} = \omega_{1,k} \cdot \boldsymbol{s}_k^T\boldsymbol{\Phi}_{k-1}\mathbf{r}$. The residual signal at the output of the last ICU in the first stage is

$$\boldsymbol{e}_{1,K+1} = \coprod_{j=K}^{1}\left(\boldsymbol{I} - \boldsymbol{s}_j\boldsymbol{s}_j^T\right)\mathbf{r} = \boldsymbol{\Phi}_K\mathbf{r}$$

This residual signal from the last ICU of the first stage is used as the input for the first ICU in stage 2. It is expressed as $\boldsymbol{e}_{2,1} = \boldsymbol{e}_{1,K+1} = \boldsymbol{\Phi}_K\mathbf{r}$. The corresponding decision

variable is expressed as $\boldsymbol{y}_{2,1} = \omega_{2,1} \cdot \boldsymbol{s}_1^T(\boldsymbol{e}_{2,1} + \boldsymbol{In}_{1,1}) = \boldsymbol{s}_1^T \mathbf{\Phi}_K \mathbf{r} + \boldsymbol{s}_1^T \mathbf{r} = \omega_{2,1} \cdot \boldsymbol{s}_1^T(\mathbf{\Phi}_K + \boldsymbol{I})\mathbf{r}$. The input to the second ICU in the second stage is $\boldsymbol{e}_{2,2} = \boldsymbol{e}_{2,1} + \boldsymbol{In}_{1,1} - \boldsymbol{In}_{2,1} = (\boldsymbol{I} - \boldsymbol{s}_1\boldsymbol{s}_1^T)\mathbf{\Phi}_K\mathbf{r}$ and the decision variable is $\boldsymbol{y}_{2,2} = \omega_{2,2} \cdot \boldsymbol{s}_2^T(\boldsymbol{e}_{2,2} + \boldsymbol{In}_{1,2}) = \omega_{2,2} \cdot \boldsymbol{s}_2^T(\boldsymbol{I} - \boldsymbol{s}_1\boldsymbol{s}_1^T)(\mathbf{\Phi}_K + \boldsymbol{I})\mathbf{r}$. In the same manner, we can find a general expression of the residual signal and the decision variable of the k^{th} user at the m^{th} stage defined respectively as:

$$\mathbf{e}_{\boldsymbol{m},\boldsymbol{k}} = \mathbf{\Phi}_{\boldsymbol{K}-\mathbf{1}}(\mathbf{\Phi}_{\boldsymbol{K}})^{\boldsymbol{m}-\mathbf{1}}\mathbf{r}, \tag{5-1}$$

and

$$\mathbf{y}_{\boldsymbol{m},\boldsymbol{k}} = \boldsymbol{\omega}_{\boldsymbol{m},\boldsymbol{k}} \cdot \boldsymbol{s}_{\boldsymbol{k}}^{\boldsymbol{T}}\mathbf{\Phi}_{\boldsymbol{k}-\mathbf{1}} \sum_{\boldsymbol{i}=\mathbf{0}}^{\boldsymbol{m}-\mathbf{1}} (\mathbf{\Phi}_{\boldsymbol{K}})^{\boldsymbol{i}}\,\mathbf{r} = \mathbf{f}_{\boldsymbol{m},\boldsymbol{k}}^{\boldsymbol{T}}\mathbf{r}, \tag{5-2}$$

where

$$\mathbf{\Phi}_{\boldsymbol{K}} = \prod_{\boldsymbol{j}=\boldsymbol{k}}^{\mathbf{1}} (\boldsymbol{I} - \boldsymbol{s}_{\boldsymbol{j}}\boldsymbol{s}_{\boldsymbol{j}}^{\boldsymbol{T}}). \tag{5-3}$$

The decision variables can be written in a matrix form as

$$\mathbf{y}_{\boldsymbol{m}} = \mathbf{F}_{\boldsymbol{m}}^{\boldsymbol{T}}\mathbf{r}, \tag{5-4}$$

where

$$\mathbf{F}_{\boldsymbol{m}} = [\mathbf{f}_{\boldsymbol{m},\mathbf{1}}, \mathbf{f}_{\boldsymbol{m},\mathbf{2}}, \cdots, \mathbf{f}_{\boldsymbol{m},\boldsymbol{g}}, \cdots \mathbf{f}_{\boldsymbol{m},\boldsymbol{K}}]. \tag{5-5}$$

Therefore, the proposed weighted SIC detector can also be described as linear matrix filtering that can be performed directly on the received chip-matched filtered signal vector without explicitly performing the interference cancellation.

5.1.1 Analysis of the proposed weighted SIC detector

The idea of weighted SIC detector implies the principle of partial cancellation of the users' signal from the received signal and is used to reduce the BER floor [11]. In the linear SIC detector, the received signal for the k^{th} user and m^{th} stage can be written as

$$\boldsymbol{e} = (\boldsymbol{s}_{\boldsymbol{1}}^{\boldsymbol{T}} a_1 b_1 + \boldsymbol{n}_{\boldsymbol{1}}) + \cdots + (\boldsymbol{s}_{\boldsymbol{j}}^{\boldsymbol{T}} a_j b_j + \boldsymbol{n}_{\boldsymbol{j}}) + \cdots + (\boldsymbol{s}_{\boldsymbol{k}}^{\boldsymbol{T}} a_k b_k + \boldsymbol{n}_{\boldsymbol{k}}), \tag{5-6}$$

where a_k is the received amplitude, b_k is the transmitted symbol and s_k is the signature sequence of the k^{th} user respectively. The AWGN for the k^{th} user is denoted by n_k. The users are ordered according to their received powers. As a result, the user with the highest power will be processed first. The MAI of the k^{th} user is then given by

$$\boldsymbol{MAI}_{\boldsymbol{k}} = \sum_{\substack{\boldsymbol{j}=\boldsymbol{1} \\ \boldsymbol{j}\neq\boldsymbol{k}}}^{\boldsymbol{K}} \boldsymbol{s}_{\boldsymbol{j}}^{\boldsymbol{T}} a_j b_j + \boldsymbol{n}_{\boldsymbol{j}}, \tag{5-7}$$

and the corresponding noise would be $\boldsymbol{n}_{\boldsymbol{k}}$. In short, it is required to estimate the decoupled signal $\boldsymbol{s}_k^T a_k b_k$ for the k^{th} user and estimate the data bits b_k.

In conventional linear SIC detectors, processing of multiuser interference removes from each ICU the interference produced by the corresponding user, accessing the channel in a serial order. Normally, SIC is performed in a multistage approach as demonstrated in Figure 2-3. Within each stage, the MAI signal is reconstructed based on the tentative decision from the previous ICU, and more reliable decision statistic is derived after the

subtraction of the corresponding estimated signal. The reconstructor can be implemented as discussed in Chapter 3. In particular, a multistage approach was suggested in [15], which, in a given stage, estimated a given user's bit under the assumption that the exact knowledge of the other users' bits needed to compute the multiuser interference could be replaced by *estimates* of these bits from the previous stage. Similar approach is used in this case for ICUs and is represented as

$$\boldsymbol{e}_{\boldsymbol{m},\boldsymbol{k}+\mathbf{1}} = \boldsymbol{e}_{\boldsymbol{m},\boldsymbol{k}} - \boldsymbol{s}_{\boldsymbol{k}}\boldsymbol{s}_{\boldsymbol{k}}^{\boldsymbol{T}}\boldsymbol{e}_{\boldsymbol{m},\boldsymbol{k}}. \tag{5-8}$$

When the estimate from the previous stage become more accurate, the performance of the multistage SIC will be better. However, the linear SIC cannot guarantee the performance will improve with more stages. Since the estimate made in the previous stage may be wrong, it can lead to a performance even worse than that without cancellation. As indicated in [39], if a wrong decision is subtracted, the increased interfering power would be four-fold.

In each ICU, the SIC detector tries to eliminate the interference caused by the user. This might not necessarily be the best philosophy. Rather, when the interference estimate is poor, it is preferable not to cancel the entire amount of estimated interference. As the cancellation progresses, the estimates improve and, thus, in the later stages of the iterative scheme, it becomes desirable to increase the weight of the interference being removed.

In the proposed weighted SIC scheme, a compromise is made between cancelling the interfering. The *estimated* signal $\hat{\boldsymbol{e}}_{m,k+1}$ is

$$\hat{\boldsymbol{e}}_{\boldsymbol{m},\boldsymbol{k}+\mathbf{1}} = (\boldsymbol{e}_{\boldsymbol{m},\boldsymbol{k}} - \boldsymbol{\omega}_{\boldsymbol{m},\boldsymbol{k}} \cdot \boldsymbol{s}_{\boldsymbol{k}}\boldsymbol{s}_{\boldsymbol{k}}^{\boldsymbol{T}}\boldsymbol{e}_{\boldsymbol{m},\boldsymbol{k}}) \tag{5-9}$$

Here, the users' interference is weighted and subtracted from the received signal. In other words, it is partially eliminated at each ICU. This technique can be very useful in the cases when the users' signals are ordered according to their powers. Note that when $\omega_{m,k} = 1$, we get the conventional linear SIC detector.

Simulations were carried out to verify the proposed analyses. A weight factor $\omega_{m,k} = 0.5$ was used. The simulation parameters used are the same as in Chapter 3. The result is shown in Figure 5-2. Similar results for weights 0.9 and 1 are shown in Figure 5-3 and Figure 5-4.

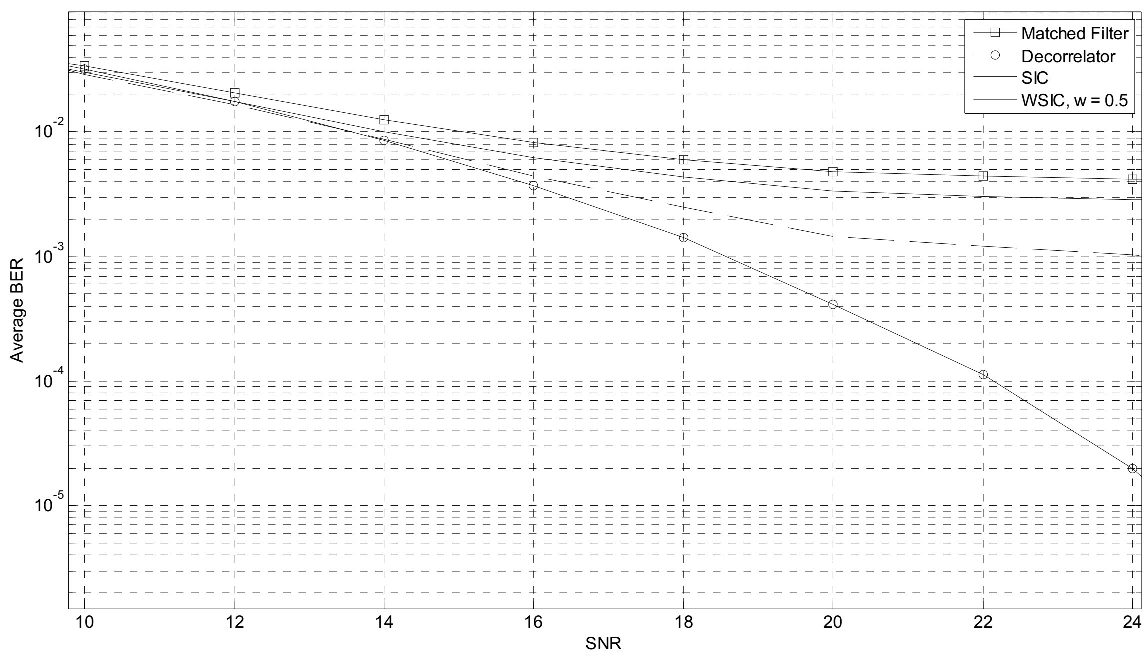


Figure 5-2 : Performance of WSIC vs SIC; 20 users, 4 stages weight = 0.5.

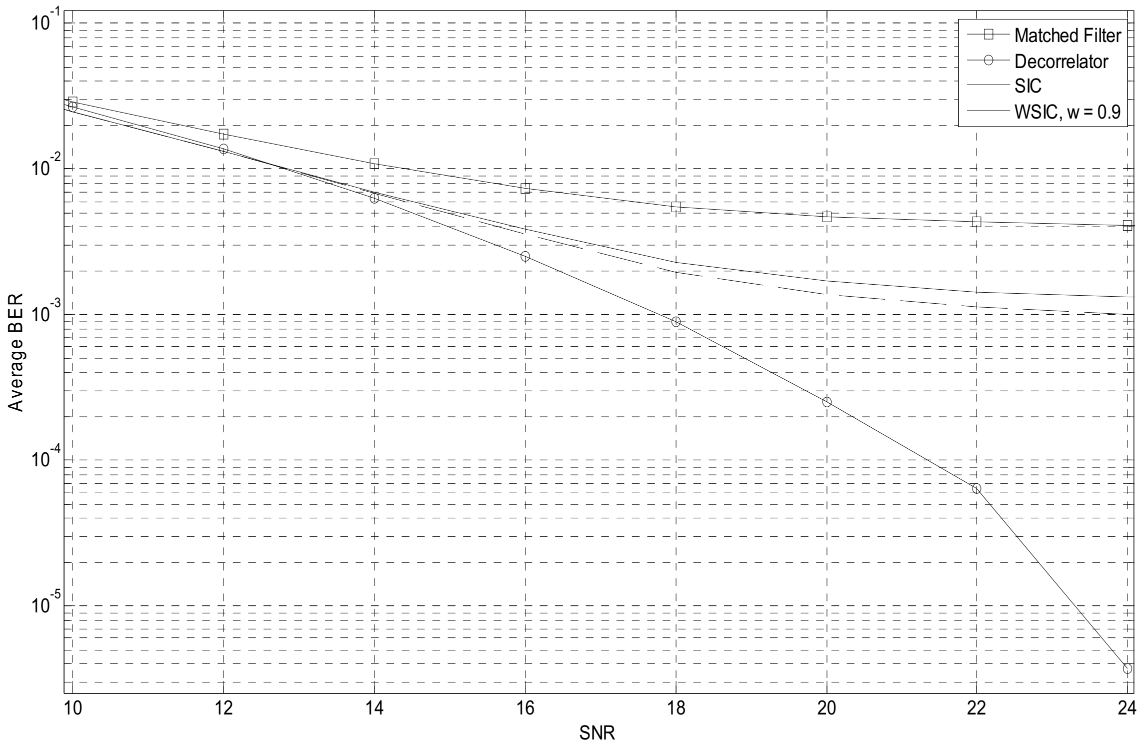


Figure 5-3 : Performance of WSIC vs SIC; 20 users, 4 stages weight = 0.9.

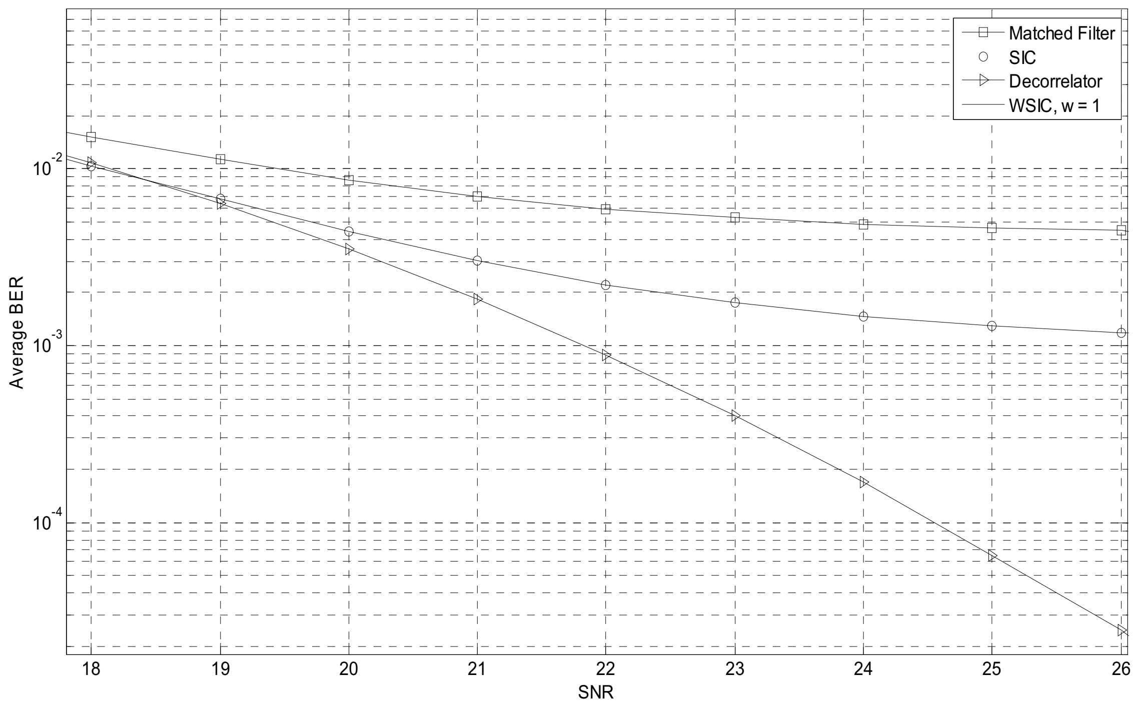


Figure 5-4 : Performance of WSIC vs SIC; 20 users, 4 stages weight = 1.

Note that equations (5-2), (5-3) and (5-4) are comparable to those of the conventional linear SIC detector in Chapter 3. Hence, the convergence properties and equations comply with the SIC detector. Convergence analysis of the proposed weighted SIC detector was carried out. As expected, it converges to the decorrelator and is shown in Figure 5-5.

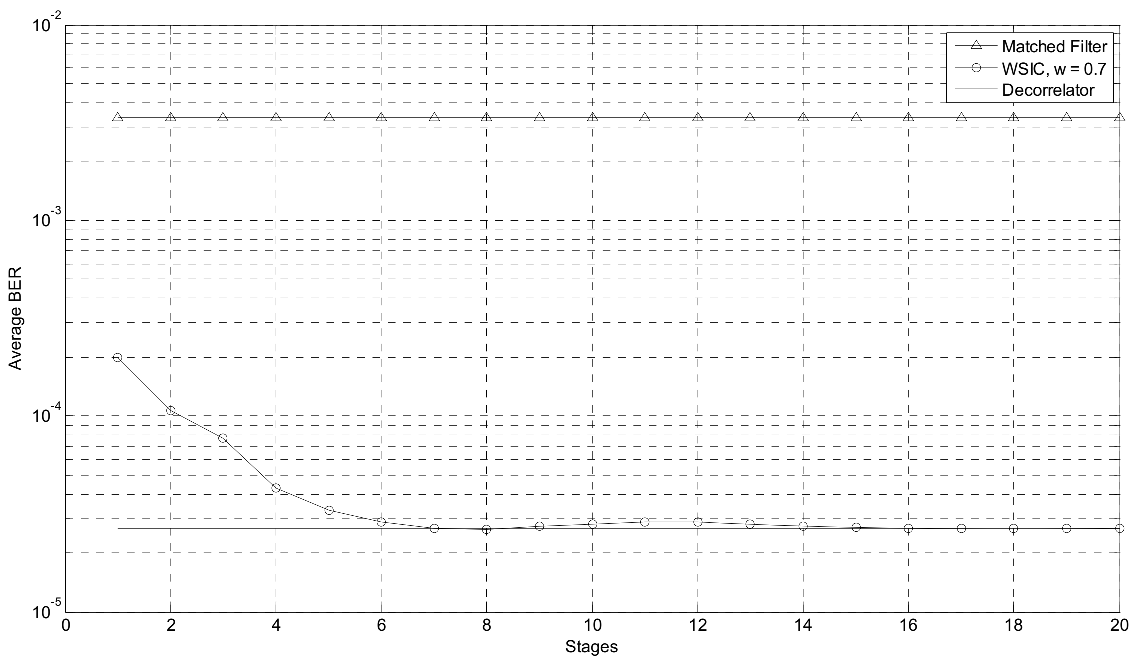


Figure 5-5 : Convergence behavior of WSIC, w=0.7, 20 users.

The weighted SIC in this case converges to the decorrelator at the 16th stage. Increasing the weight could enhance the performance. But this cannot be generalized due to the fact that weighting a wrong estimation of the signal could degrade the performance. Hence the fuzzy based weighted SIC detector was proposed. The behavior of weighted SIC seen in Figure 5-2, Figure 5-3 and Figure 5-4 can be utilized to enhance the performance of the SIC. If these weights are tuned according to the received powers of the users, the

results will improve the performance of the SIC detector. This is discussed in the following section.

5.2 The Proposed Fuzzy-Based Weighted SIC detector

The Fuzzy-Based Weighted SIC (FBWSIC) detector uses the idea of section 5.1 except that it uses another block in the basic ICU. This block is the FUZZY block that estimates the weight $\omega_{m,k}$ using fuzzy logic. A block diagram is shown in Figure 5-6. The weights used in the previous section were constant and same for all users at each stage. This section also introduces the concept of using different weights for different users and different stages.

Since the SNR changes for every ICU in every stage, using a fuzzy based inference system would estimate suitable weights rather than using constant weights. Note that fuzzy logic is not the only possible solution to estimate the weights. Some other methods like Particle Swarm, Genetic Algorithms or Neural Networks can also be used.

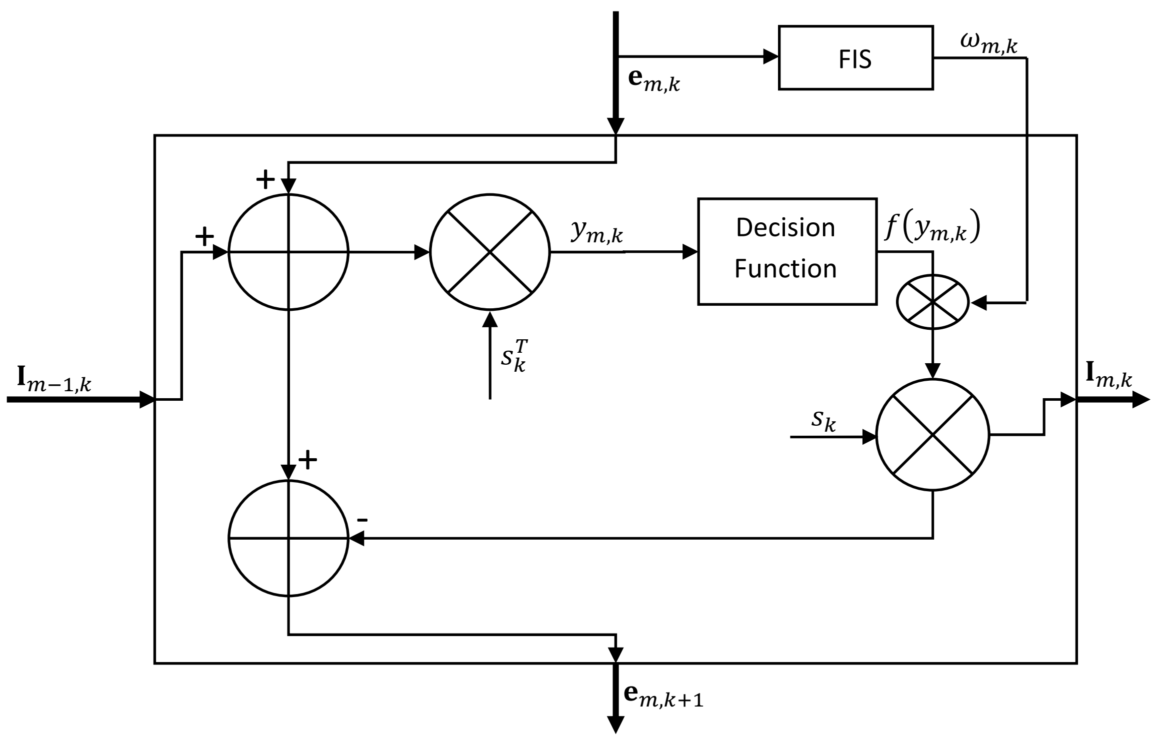


Figure 5-6 : ICU of the Fuzzy based Weighted SIC.

Using the model of Figure 5-6, an FIS was designed using two inputs and one output. The inputs were chosen as effective number of users K_{eff} and SNR. The output is the estimated weight $\omega_{m,k}$. A MATLAB block diagram is shown in Figure 5-7.

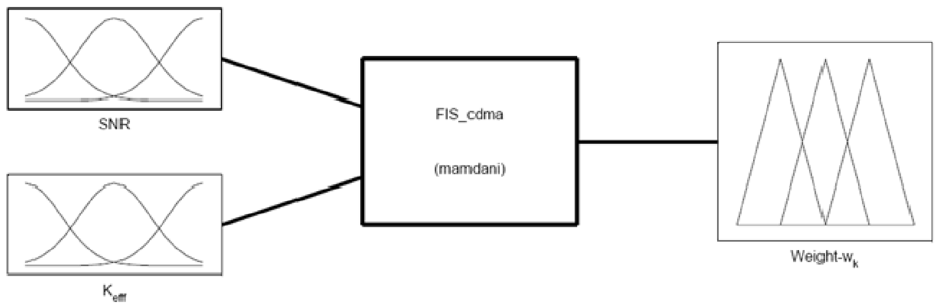


Figure 5-7 : MATLAB block of the FIS – Number of users and SNR.

The input SNR was classified into the MFs Negative Low (NL), ZEro (ZE), Positive Low (PL), Positive Medium (PM) and Positive High (PH) based on the knowledge of SNR from previous experiments. The input K_{eff} was classified into the MFs Very Few (VF), Few (F), MEDium (MED), Many (M) and Greater Many (GM). The output, estimated weight $\omega_{m,k}$, was divided into the MFs Almost Zero (AZ), Small (S), MEDium (MED), High (H), Very High (VH) and ONE. Gaussian MFs were used for both inputs and the output and can be seen in Figure 5-8.

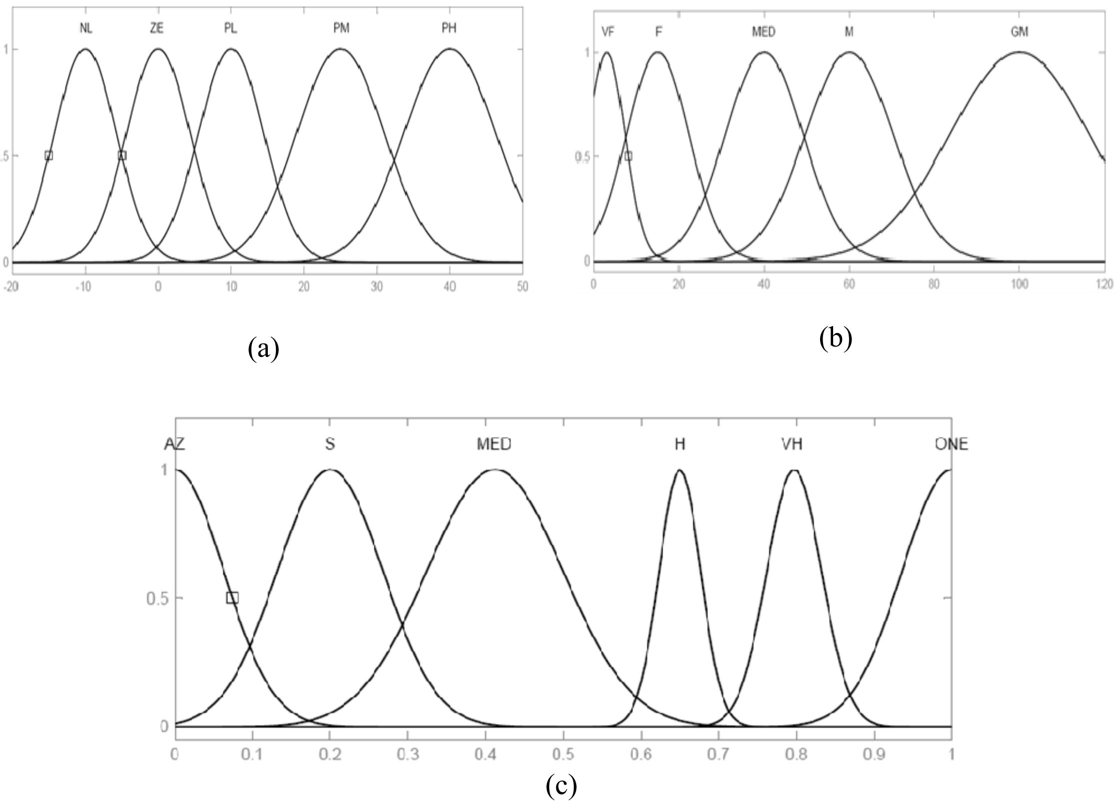


Figure 5-8 : MFs of the inputs (a) SNR and (b) K_{eff} and the output (c) weight.

The choice of the width of the MFs is arbitrary [40]. A rule base was formed using these linguistic variables and is shown in Table 5-1.

SNR / K_{eff}	**NL**	**ZE**	**PL**	**PM**	**PH**
VF	S	S	H	VH	VH
F	S	MED	H	VH	VH
MED	AZ	S	MED	H	H
M	AZ	S	S	MED	MED
GM	AZ	AZ	AZ	AZ	S

Table 5-1 : Rule Base of SNR vs weight.

Using these rules, simulations were carried out. It was observed that there was no performance improvement over the SIC detector. The result is shown in Figure 5-9.

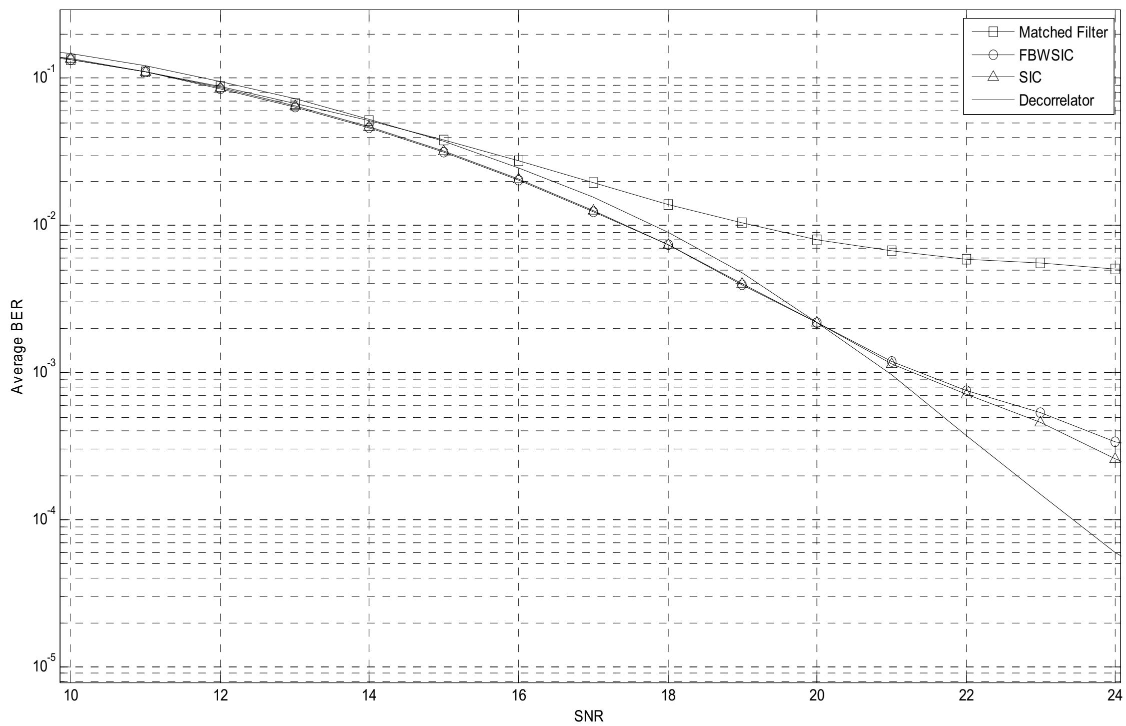


Figure 5-9 : BER plot of fuzzy based weighted SIC vs SIC, 20 users, 3 stages.

Other modifications of this FIS were also tried, using different widths of the MFs, varying the positions of the MFs and modifying the rules as well. The results were not better than the conventional linear SIC detector.

Another idea was to use the inputs as the SNR and the number of stages. MFs were designed and a rule base was formed. Simulations were carried out using more than one modifications of this FIS. This attempt did not prove to be successful.

Another technique was to use only one input and one output. The input in this case would be the SNR and the output would be the estimated weight. A MATLAB block diagram of how the FIS works is shown in Figure 5-10.

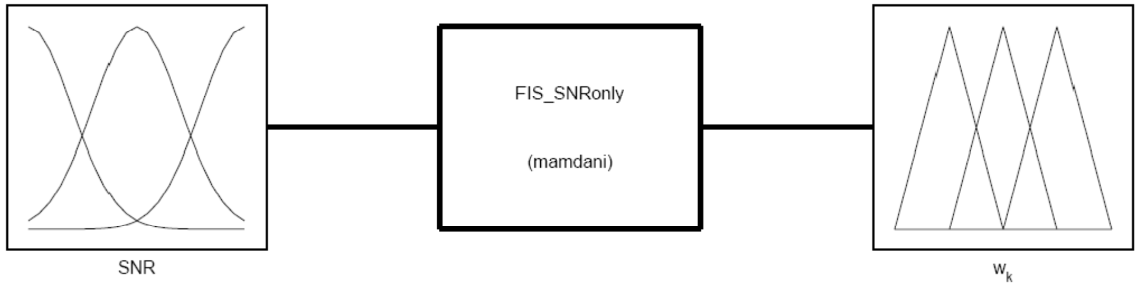


Figure 5-10 : MATLAB block of the FIS.

The block 'SNR' is the fuzzifier block for the input SNR and the block 'w' is the fuzzifier block for the output which is the estimated weight $\omega_{m,k}$. The fuzzifier block of the SNR is shown in Figure 5-11.

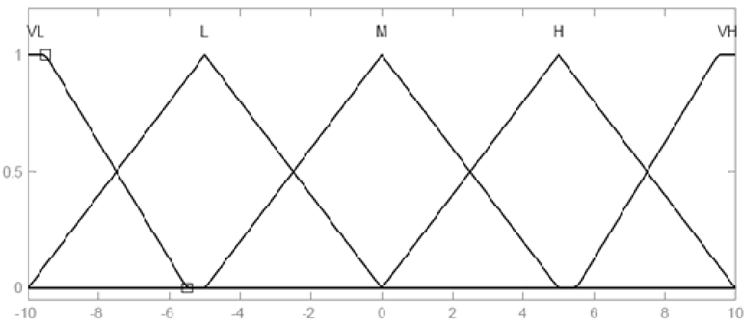


Figure 5-11 : Fuzzifier block of the input : SNR.

Here, we do the fuzzification of the input which is the SNR. Based on previous knowledge, we have classified the inputs into the MFs Very Low (VL), Low (L), Medium (M), High (H) and Very High (VH). The MFs L, M and H are triangular and VL and VH are of trapezoidal shape. Note that this choice of MFs is arbitrary.

Similarly, we have a fuzzifier block for the output which is the weight $\omega_{m,k}$. It is shown in Figure 5-12.

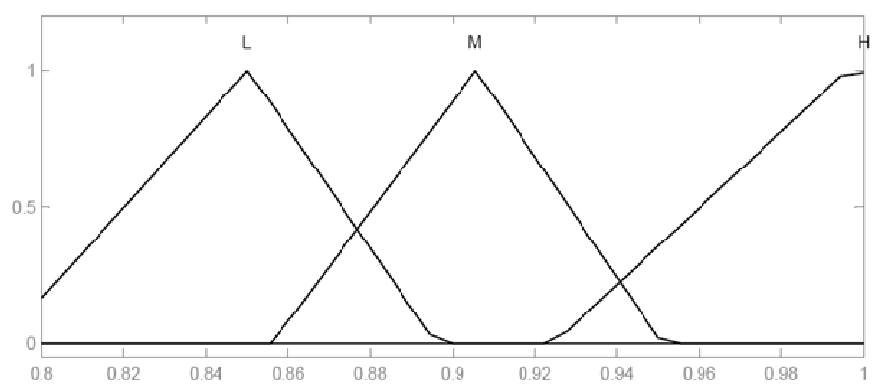


Figure 5-12 : Fuzzifier block of the output : estimated weight.

Here, we have classified the weights in three MFs Low (L), Medium (M) and High (H). The range, as seen in the figure was selected after many trials. Here, the MFs L and M are triangular and H is trapezoidal. Again, this choice is arbitrary.

Once the fuzzification has been performed, we need to define a set of rules which work on these MFs. Our rule base consists of 5 rules. These are stated below:

Rule 1: If SNR is VL then $\omega_{m,k}$ is H

Rule 2: If SNR is L then $\omega_{m,k}$ is M

Rule 3: If SNR is M then $\omega_{m,k}$ is L

Rule 4: If SNR is H then $\omega_{m,k}$ is M

Rule 5: If SNR is VH then $\omega_{m,k}$ is H

Once the rule base is ready, our FIS is ready to be implemented.

5.3 Simulation Results

Using this designed FIS, Monte-Carlo simulations for the BER vs SNR were carried out. The simulation parameters were the same as mentioned in 3.2.2.2. For the reader's convenience, these are reported here again:

We use a single path AWGN with perfect power control. Users are ordered according to their matched filter output and are ordered at every stage. Initially, the simulation was carried out using 10 users. The spreading codes used are the 31 length Gold codes. The result is shown in Figure 5-13.

Note that there is slight improvement over the SIC detector. Using the same parameters, more simulations were carried out, changing the number of users. The results show clear improvement over the conventional Linear SIC detector. These can be seen in the figures that follow.

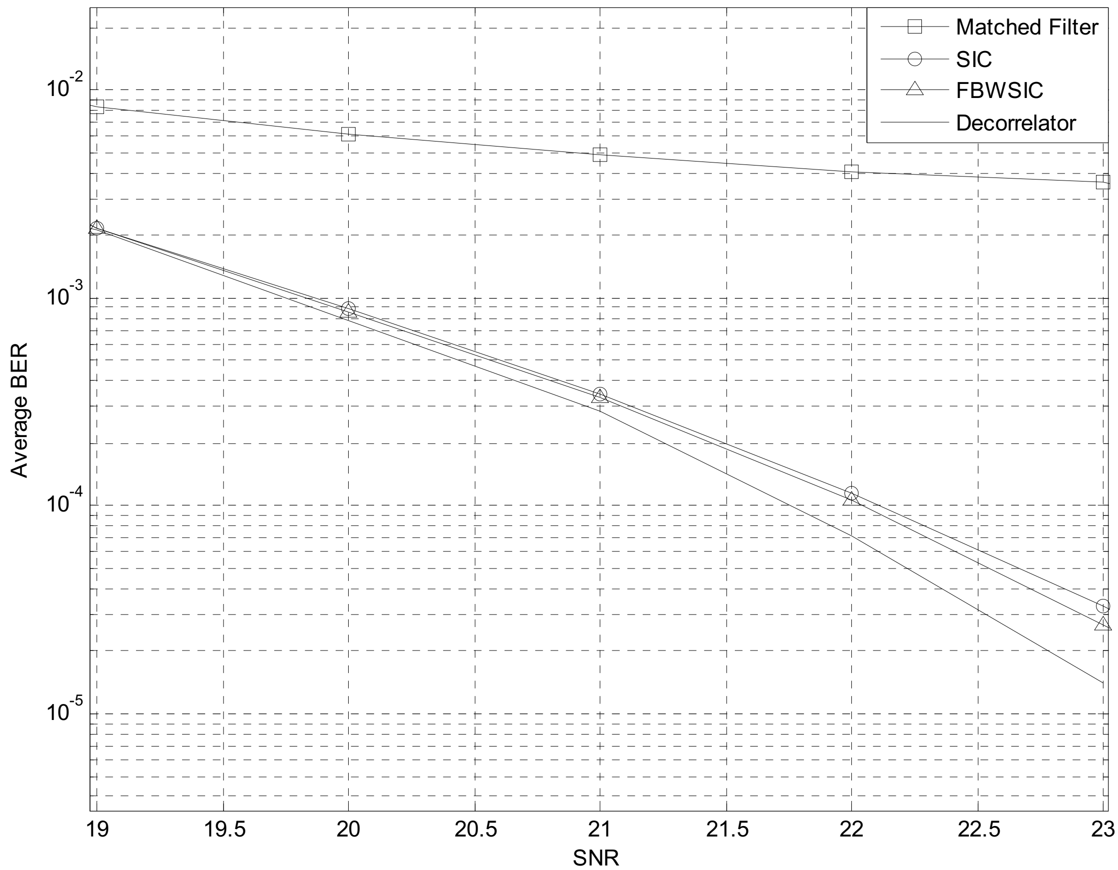


Figure 5-13 : BER vs SNR plot for Fuzzy Based Weighted SIC detector; 10 users and 2-stages.

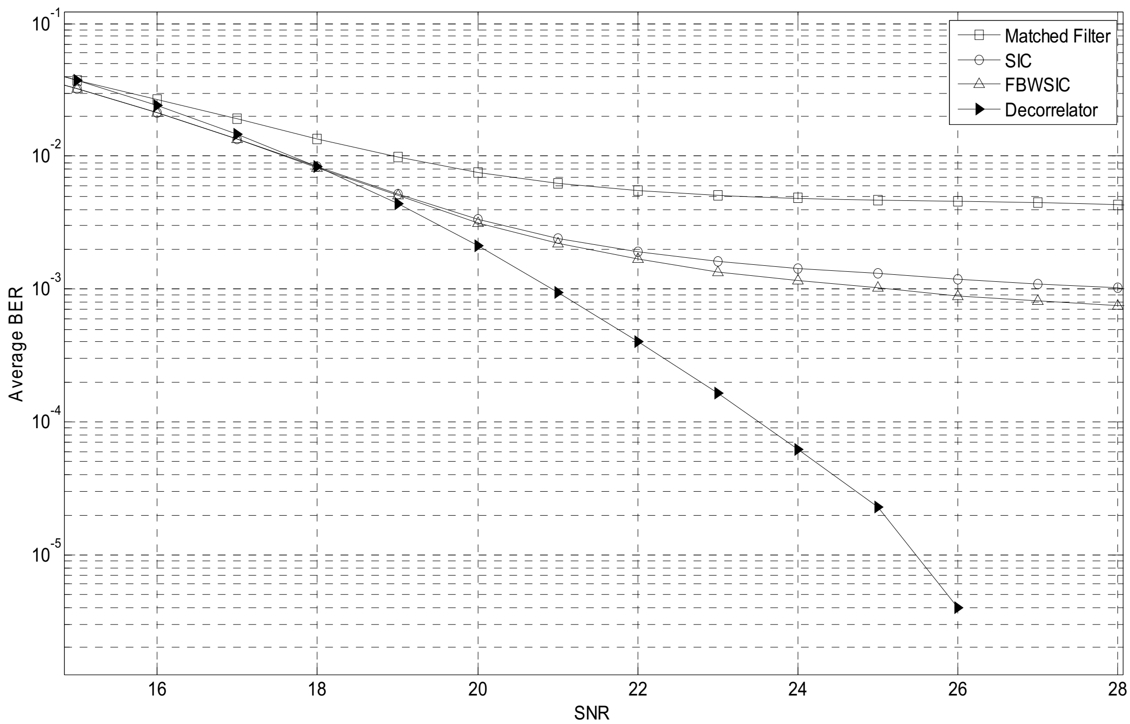


Figure 5-14 : BER vs SNR plot for Fuzzy Based Weighted SIC detector; 20 users and 2-stages.

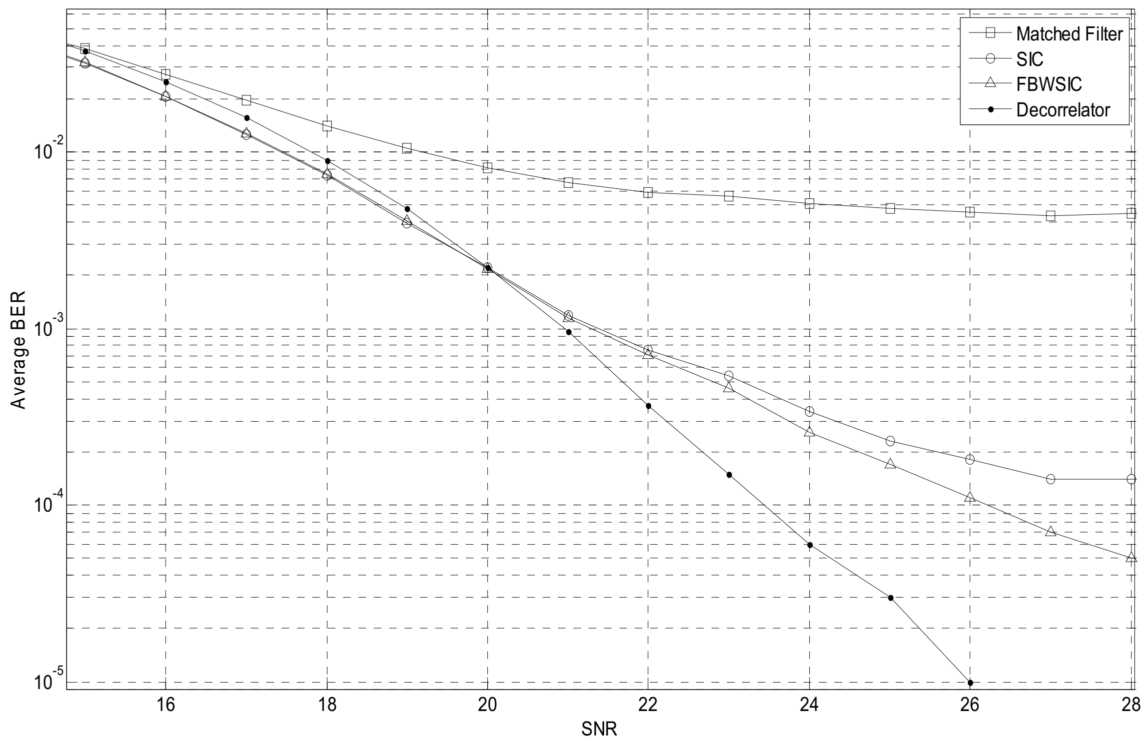


Figure 5-15 : BER vs SNR plot for Fuzzy Based Weighted SIC detector; 20 users and 3-stages.

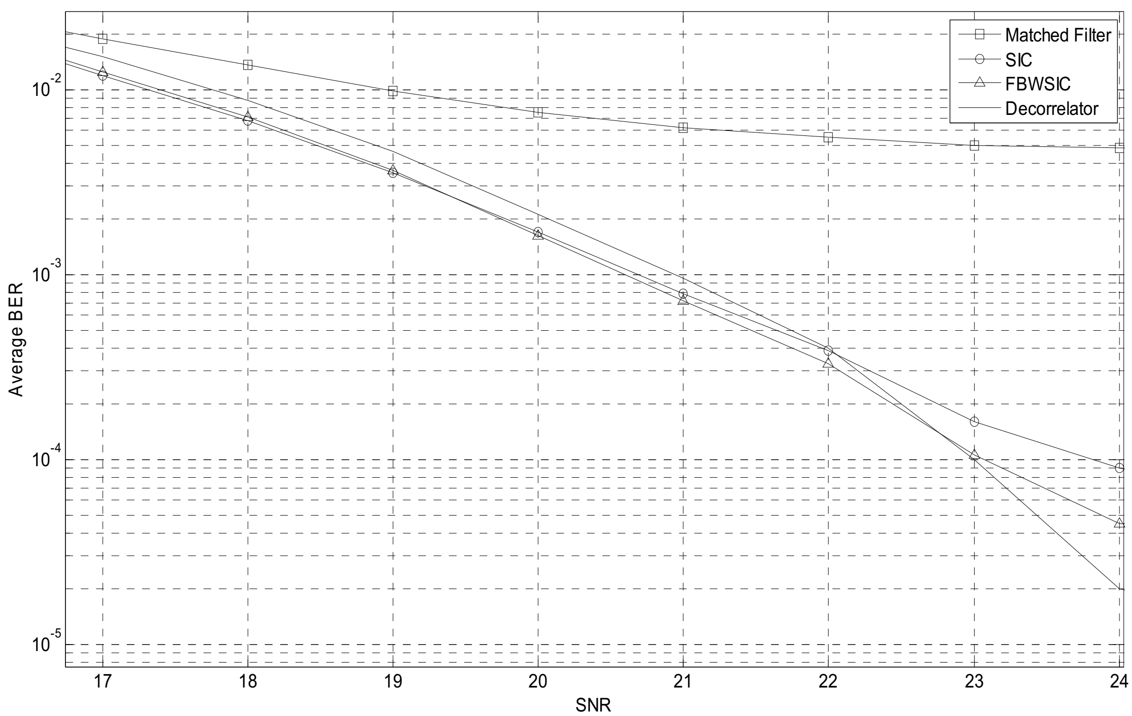


Figure 5-16 : BER vs SNR plot for Fuzzy Based Weighted SIC detector; 20 users and 4-stages.

The results discussed above show considerable improvement over the SIC. A few simulations were also carried out for the non-linear case. The hyperbolic tangent function was used in the decision function block in Figure 5-6. It was done for 20 users, 2-stages and 3-stages. The results are shown in Figure 5-17 and Figure 5-18.

It can be seen that the fuzzy based weighted SIC shows improvement over the SIC. Another simulation which compares the FBWSIC is the stage-wise comparison. It is shown in Figure 5-19.

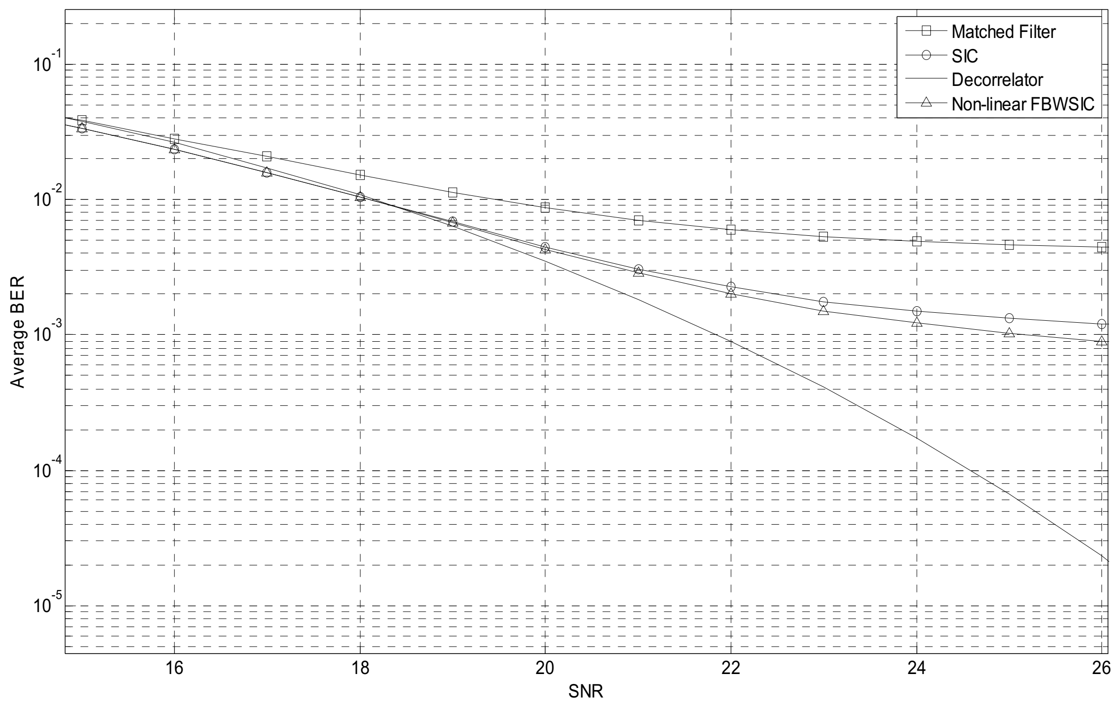


Figure 5-17 : BER vs SNR plot for Fuzzy Based Non-Linear Weighted SIC detector; 20 users and 2-stages.

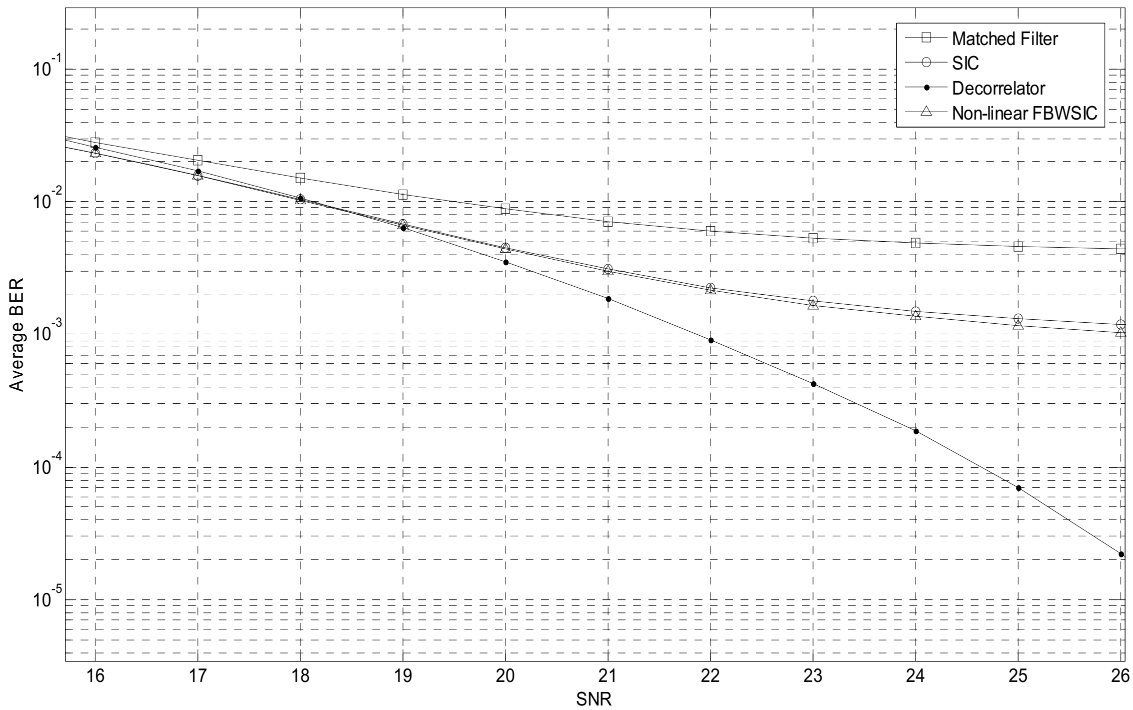


Figure 5-18 : BER vs SNR plot for Fuzzy Based Non-Linear Weighted SIC detector; 20 users and 3-stages.

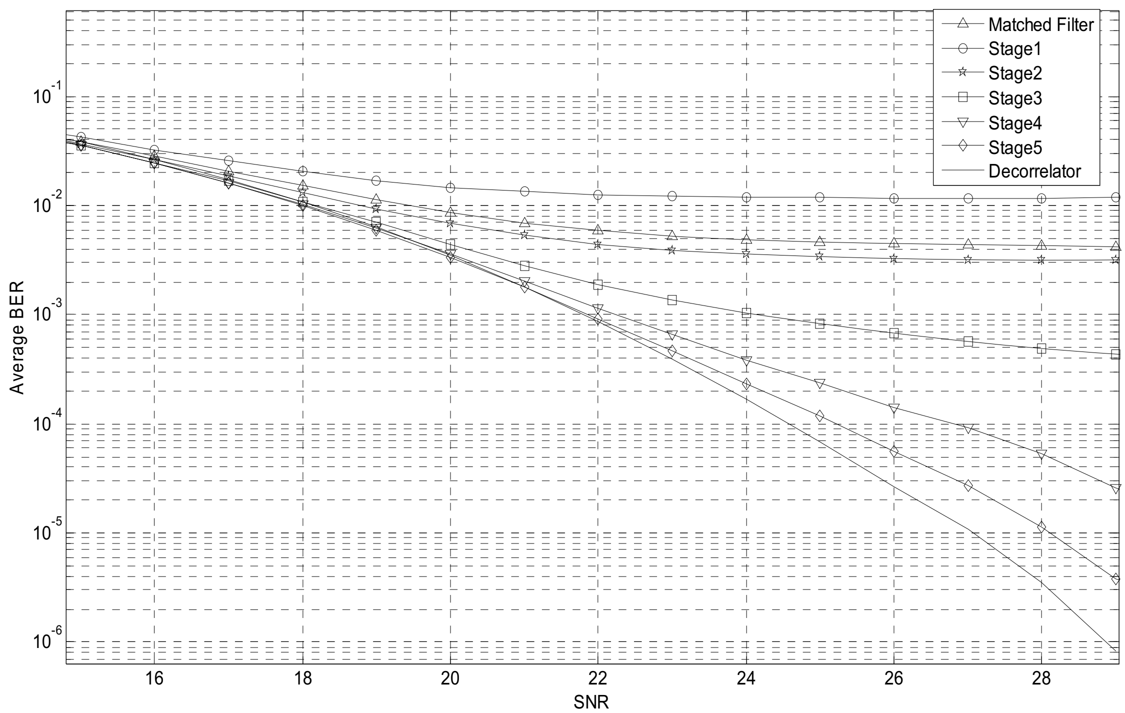


Figure 5-19 : A stage-wise comparison of the FBWNLSIC, 20 users.

Note that as we increase stages, the FBWSIC approaches the decorrelator. This behavior is expected because, as discussed in 3.2 and 5.1, the proposed structure of weighted SIC is same as the linear SIC and converges to the decorrelator. It is shown in Figure 5-20.

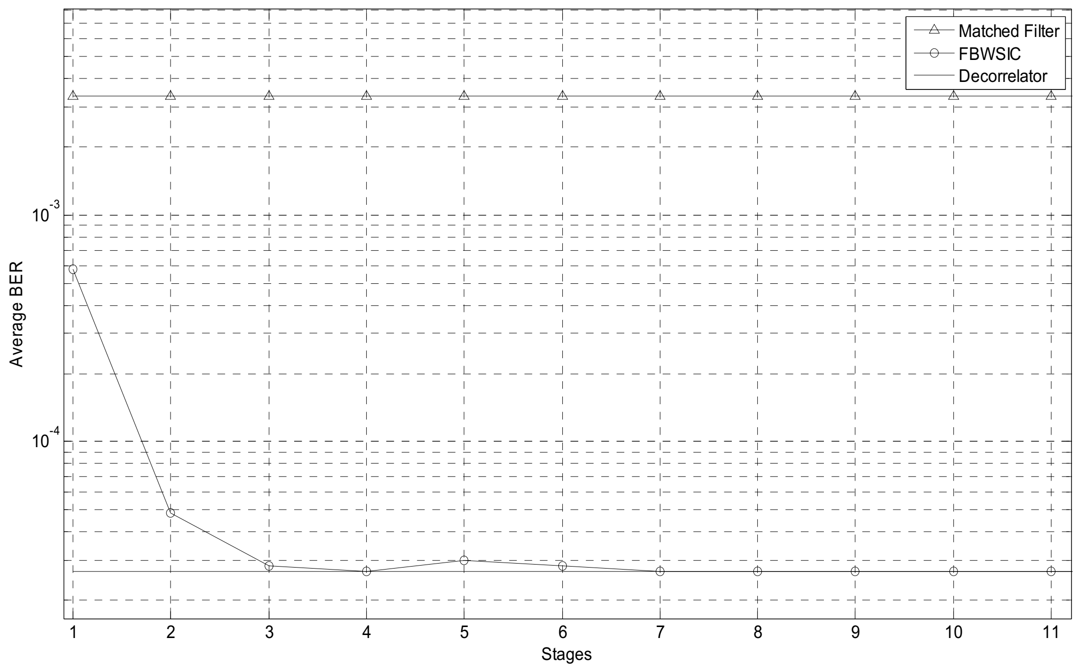


Figure 5-20 : Convergence of Fuzzy Based Weighted SIC, 15 users, 20dB.

The proposed technique converges to the decorrelator at the 7th stage. If we do the analysis at higher number of users, it is expected that the convergence is at later stages. This is verified in Figure 5-21.

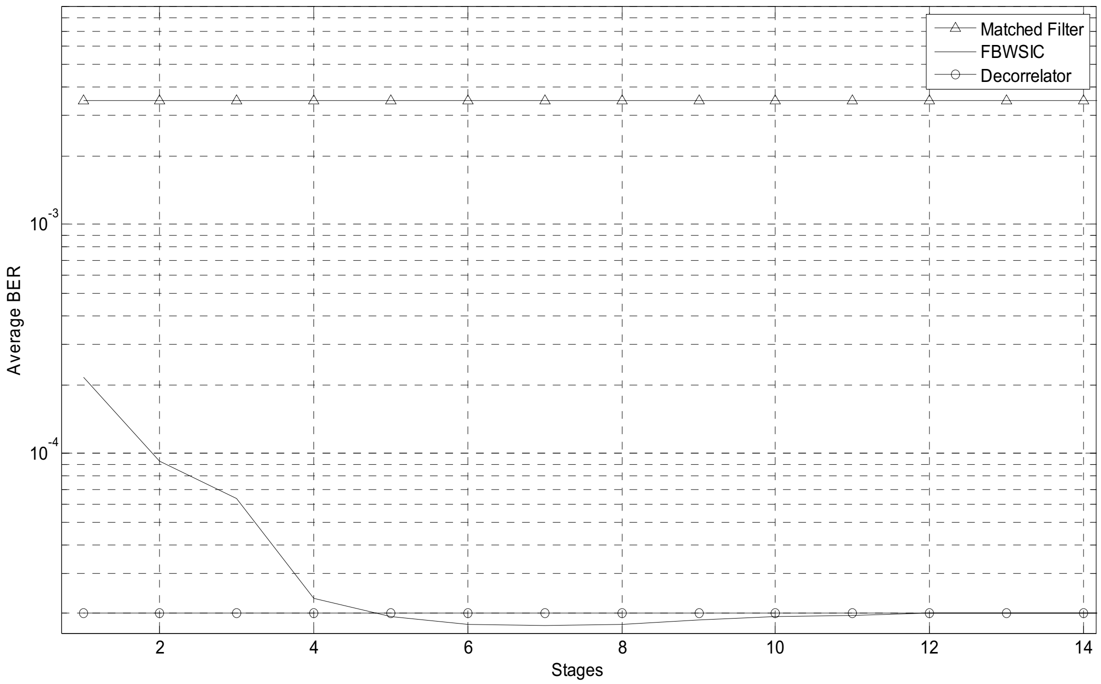


Figure 5-21 : Convergence of Fuzzy Based Weighted SIC, 20 users, 20dB.

Similar analysis was carried out to compare the convergence of the proposed technique and the conventional linear SIC detector. Result is shown in Figure 5-22. It can be seen that the proposed technique converges almost 2 stages before the conventional linear SIC converges to the decorrelator.

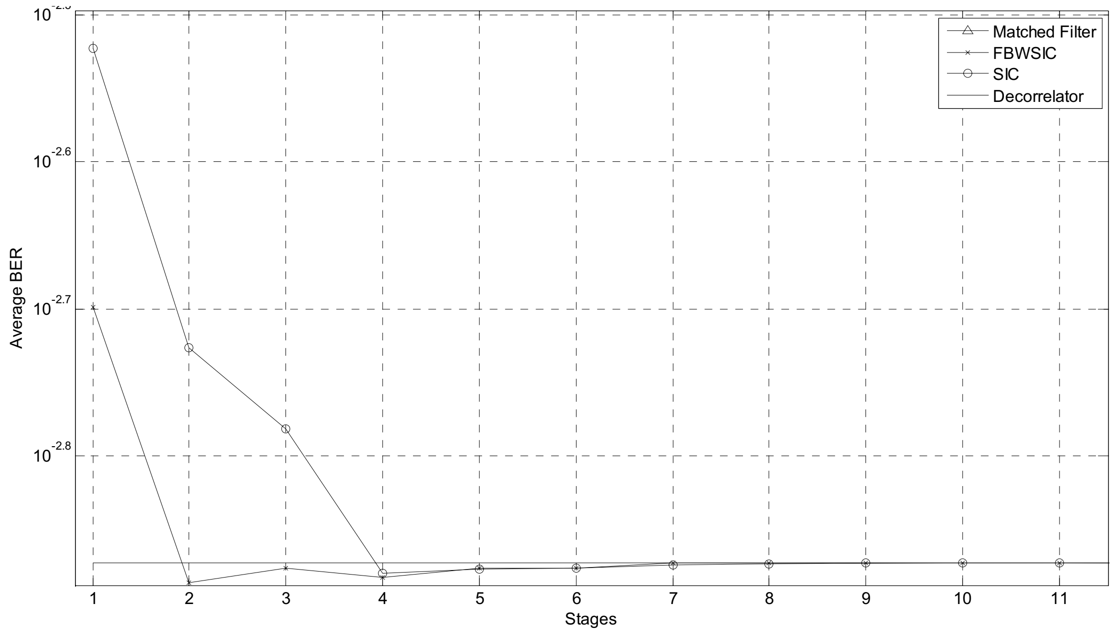


Figure 5-22 : Convergence of Fuzzy Based WSIC, 15 dB, 20 users.

5.4 Analysis of the Membership functions

In the first case, Gaussian membership functions were implemented in the FIS. In Figure 5-8(a), it can be seen that there are five MFs namely NL, ZE, PL, PM and PH based on the knowledge of SNR from previous experiments. This choice of the shape of the MFs was based on the fact that our distribution of SNR i.e. mean signal and noise variation was a Gaussian distribution. This was also noticed in [35]. The author used Gaussian MFs for the SNR as can be seen in Figure 5-23.

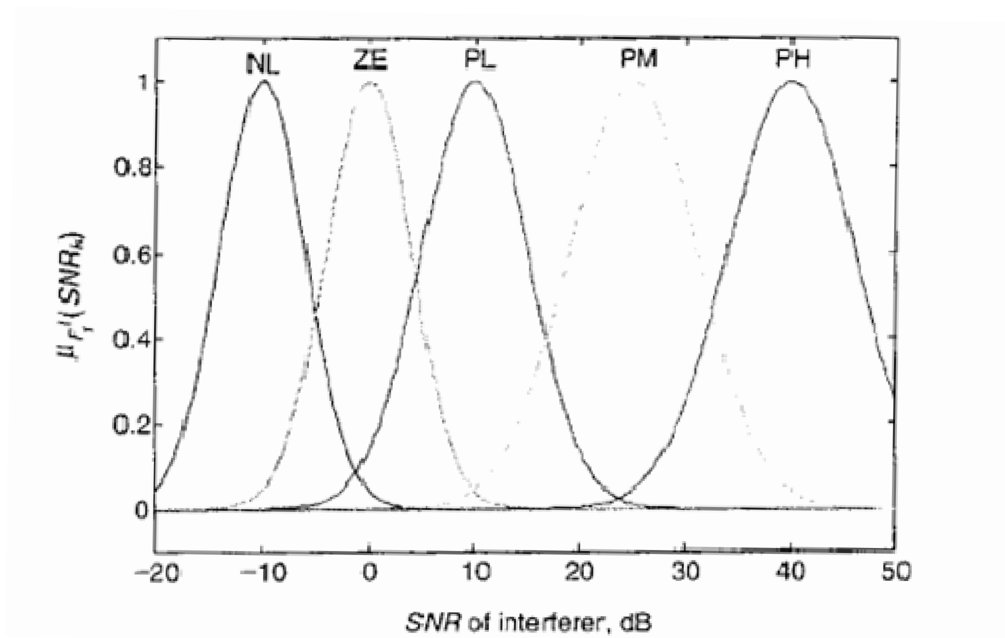


Figure 5-23 : Gaussian MF used in [35].

For low SNR near zero i.e. the MFs NL, ZE and PL, the MF is not as wide as it can be seen in the case of higher SNR. The MFs PM and PH, which include higher SNRs have wider variance. The centre value (mean) of this MF is the mean of the signal and the width (variance) of the MF is the variance of noise. A brief illustration is shown in Figure 5-24.

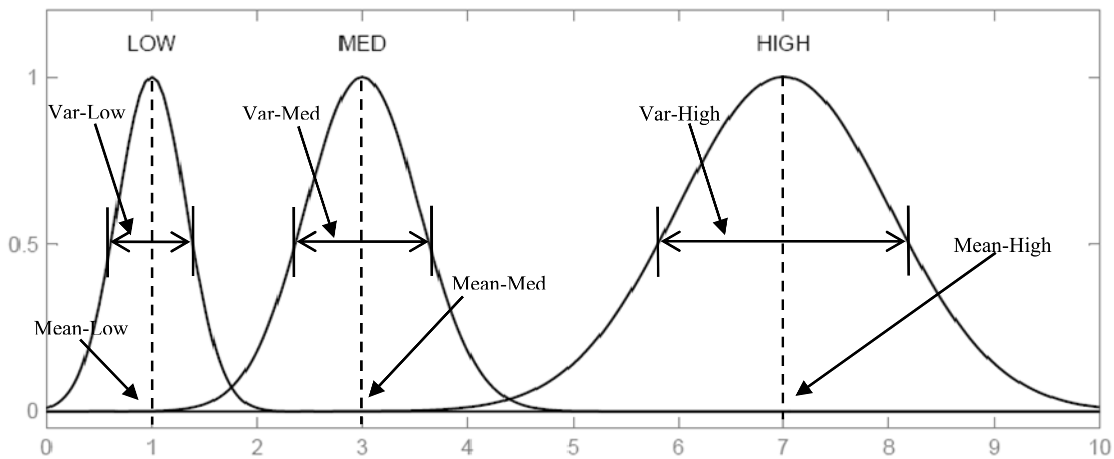


Figure 5-24 : Illustration of the Gaussian MFs.

When the SNR is low, the mean of the signal is low and has low noise variance. This is the case in the figure above when the mean of the signal is 1 i.e. in LOW. As the SNR increases, the mean of the signal increased and so does the variance of noise. This can be seen in the figure in the MF MED. For high SNRs, the mean of the signal is high and the variance of noise is also high, as shown in the MF HIGH.

Using this fact, the author in [35] designed the MFs. But the same author in another paper [41] performed the simulations using triangular MFs as shown in Figure 5-25.

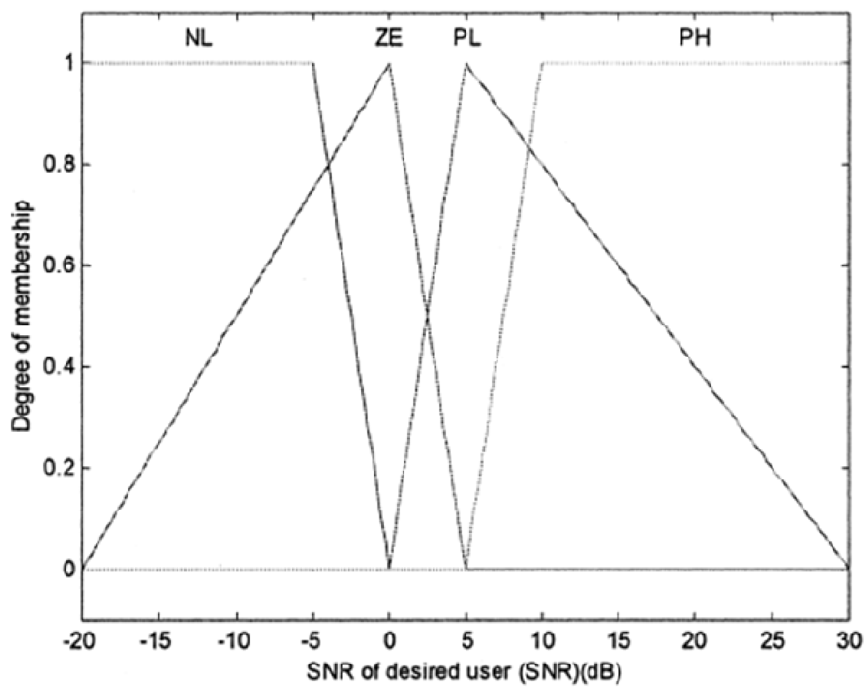


Figure 5-25 : Triangular MFs used in [41].

Interestingly, both these strategies showed good results. This anomaly motivated us to investigate the effects of different MFs on our system. Another FIS was designed using the MFs shown in Figure 5-11. To keep uniformity, the sizes of the MFs were not changed and only the shapes were modified to Gaussian as shown in Figure 5-26.

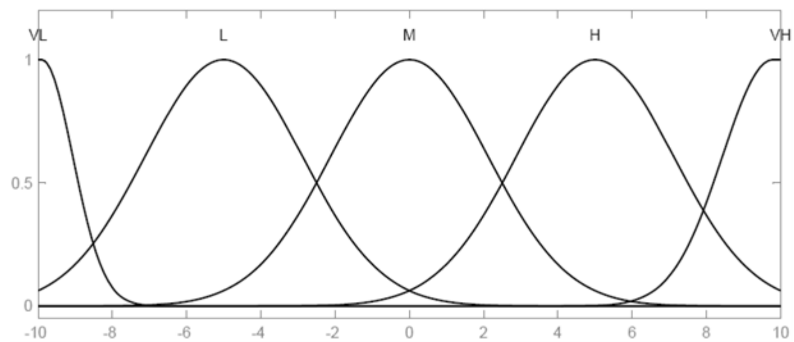


Figure 5-26 : Gaussian MFs for the system.

Simulations were carried out and the result is shown in Figure 5-27.

The way to select MF is to know how the variable's membership is behaving when its value is changed in the set. Gaussian and triangular MF's are both continuous, but in triangular there is exactly only one point, when a variable reaches 100% membership of the set, and other points are linearly decreasing in membership. In Gaussian, membership is assigned based on the shape of the curve. If we divided a variable into low, med and high, then in triangular MF, med is obtained only at one particular value with 100%. In Gaussian, this value will be approximately obtained at more than one point. Also in the med set, the way the membership is assigned to a variable is linear with respect to the value of the variable for triangular MF, whereas it is non-linear for Gaussian MF[5].

[5] Both Gaussian and triangular MF can be non-symmetric. So the choice mainly depends upon how the variable is divided into linguistic sets, and how the membership behaves in the set.

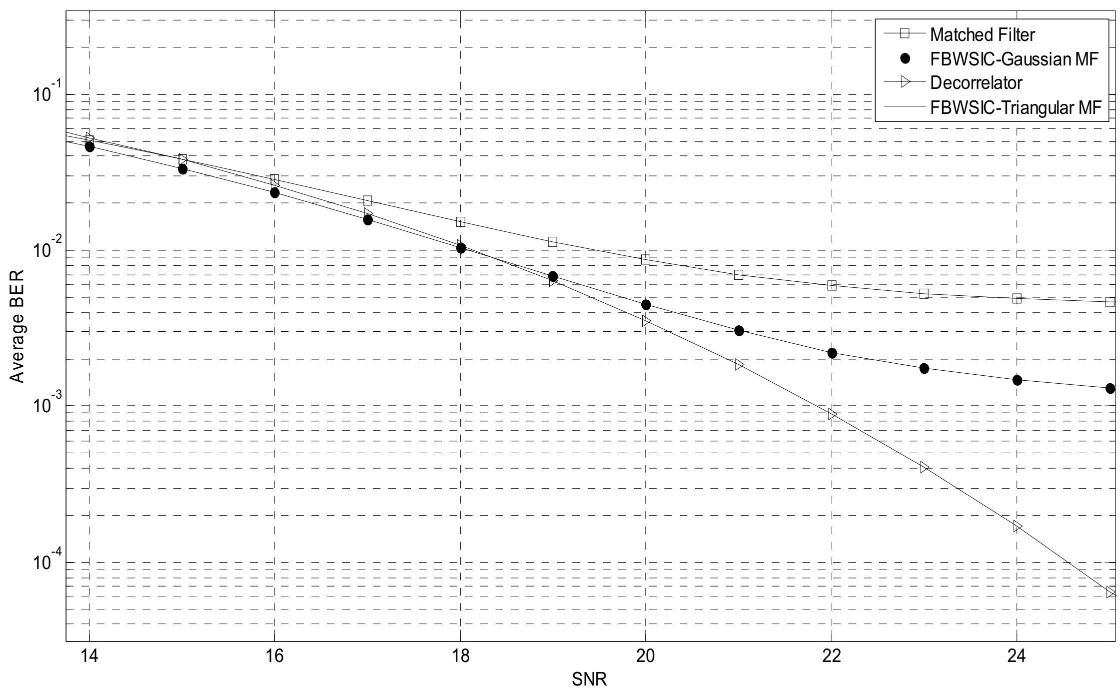


Figure 5-27 : Comparison of Triangular vs Gaussian MFs, 20 users, 3-stage.

It can be noticed that the results overlap i.e. there is no effect of using different MFs for our system. But this result cannot be generalized for other problems. Since our FIS has only one input, shape of MFs may not affect the system. This is an open question for interested researchers in the field and can be investigated.

5.5 Summary

In this chapter, we have proposed a new technique, a fuzzy-based weighted SIC detector which adds a weighting factor to the conventional linear SIC detector. This weight is estimated using a fuzzy inference system. Simulations were carried out and it was proved that the proposed technique, the Fuzzy-based Weighted SIC detector performs better than the linear SIC detector. It was also noted that the fuzzy-based non-linear SIC detector also performs better than the SIC.

Chapter 6

Conclusions and Future work

6.1 Conclusion

In this thesis, we have investigated an improvement to the conventional SIC detector. We started with an introduction about wireless technology and multiuser detection. A brief discussion about CDMA and multiuser detection was done in the following chapter. Next, we discussed in detail the Successive interference cancellation detectors. The linear, non-linear and weighted SIC structures were studied in detail.

The concept of Fuzzy Logic was introduced in chapter 4 and was elaborated in detail with the help of an example. In chapter 5, we introduce a new method, the proposed technique which introduces a weighting factor in the linear SIC structure. This weight is estimated using fuzzy logic.

A number of simulations were carried out to show that the proposed method performs better than its previous counterparts.

6.2 Future work

In order to enrich the work of this thesis and develop it further we suggest the following points:

- We have used triangular and trapezoidal membership functions for our FIS. This choice was arbitrary. Other types of MFs can be used and investigated.
- Other types of fading channels can be used and investigated.
- The input for our FIS is only the SNR. Any other method which involves more than one input can also be used. For example, number of users or number of stages can be used as the other input.
- We have used 31-length Gold codes. These codes have a correlation of 1 between the users. Other types of codes can be used.
- Out codes database consists of 33 codes. According to previous studies, we can use only 70% of the length of the codes i.e. a maximum of 20 users were used. Other codes which can support more users can be used.
- We have studied only the synchronous case. We suggest as well, investigating the performance of the previously developed detector in an asynchronous channel which is more realistic.

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Vitae

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