Process Targeting for Multi-stage Production System: A Network Approach

By

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In the name of Allah, the Most Beneficent, Most Merciful.

All praise is due to Allah, the source of all knowledge and strength. I acknowledge His infinite mercy and grace in making this work a success. And may His peace and blessings be upon his final messenger Muhammad (peace be upon him), a guidance and inspiration to our lives.

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ABSTRACT

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In recent industrial settings, products are often processed through multi-stage production systems that produce the same end product but at varying cost depending on quality. If the probabilities associated with its *scrap*, *rework* and *accept* states are known, we can better understand the nature of a production system and thus better capture the optimum target for a process.

This study develops a network-based model for determining the optimum process target levels within the framework of a multi-stage network production system. The proposed models are then illustrated through numerical examples, and sensitivity analysis is performed.

Key words: Production Process; Process Target; Production Planning.

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حديثا في البيئات الصناعية، غالبا ما تكون المنتجات المصنعة من خلال نظم متعددة مراحل الإنتاج التي تنتج نفس المنتج النهائي ولكن بمستويات متفاوتة من حيث التكلفة تبعا للنوعية. إذا كانت الاحتمالات المرتبطة باعادة التدوير وإعادة التصنيع وقبول المنتج معروفة ، يمكننا فهم أفضل لطبيعة نظام الإنتاج ، وبالتالي نكون أكثر قدرة على استيعاب الهدف الأمثل لهذه العملية.

هذه الدراسة تطور نظام شبكي لتحديد النماذج المثلى لعملية استهداف المستويات في إطار نظم الانتاج متعددة المراحل ومتعدد المنتجات. تم عمل نماذج مقترحة وتحليل الحساسية لايضاح أكثر لمبأ العمل للنظام المقترح.

CHAPTER ONE

BACKGROUND

The cost of quality has a significant factor on the cost of sales in typical companies. To tackle this issue, many companies have turned to improving the processes of achieving quality in order to reduce costs. A new perspective has led companies to reexamine the traditional assumptions and approaches used to achieve quality improvement. The classical approach of quality control, which focused on screening and correction of defects, is giving way to new methodologies that emphasize prevention. Unlike the classical approach, which assumes determined process settings (mean and/ or variance), the new approach views process settings as variables that can be controlled through investments in improved raw materials, worker training, and process capabilities. This new approach is called "process targeting".

Process targeting is one of the important problems in economics and quality control . In process targeting, it is assumed that process parameters or machine settings are variables, thus the objective of the problem is to find the optimum values of process parameters or machine settings that will achieve certain economical objectives. To effectively carry out this new approach, companies need methods to evaluate investments that are aimed at changing process settings.

The initial process targeting problem addressed is the "can filling problem". The first real attempt to tackle the can filling problem was in Springer [1951]. In general, the can

filling problem is described as follows: Consider a filling problem in which cans are produced continuously. The quality characteristic of interest is the net weight of the filled can. The value of this quality characteristic is a random variable X. A lower specification limit L exists for X. A can is accepted if $X \geq L$ and rejected otherwise. Accepted cans are sold at a fixed price a, while rejected cans are sold at a reduced price r, where $r < a$. In this problem, it was assumed that X follows a normal distribution with mean μ and standard deviation σ. Moreover, 100 percent inspection was used for product quality control and inspection is assumed to be error free. The objective of this problem is to find the optimal mean (target) μ so that the net income for the process is maximized. It is assumed that μ is a parameter that can be controlled by the filling machine setting.

Many research papers have addressed the process targeting problem. Each paper considers the problem with different assumptions. As a result, different models and solution methods exist in the literature. Despite the wide spectrum of variation of the process targeting problem that have been addressed, very few have considered the case where the product goes through two processes instead of one. Considering such a problem gives another dimension to the classical process targeting problem. Moreover, such a problem widely exists in multistage serial production systems, such as the electronics industry.

1.1. Key factors in process targeting problems

Process targeting problems are affected by many important factors. The main factors include quality characteristic distribution, product specification, process costs and market prices.

Quality characteristic distribution plays an important role in modeling the process targeting problem. In the literature, quality characteristic is assumed to be normally distributed with known variance. It is highly recommended to test the distribution using standard goodness of fit tests such as the Chi-square test and Kolmogorov test to ensure that this assumption is satisfied.

The specification limits on the quality characteristic determines the product acceptance criteria. The specification limits are usually determined by market and technical fitness and accurate information about them is necessary for realistic process targeting model.

There are many costs involved in the process targeting problem. These costs are production costs, material costs, inspection costs and rework costs. Knowledge about these costs is essential for obtaining realistic solutions for the process targeting problem.

Sometimes selling prices are included in the process targeting model and the objective in this case would be to maximize the expected profit. Therefore, market study is essential in developing the process targeting model. The market study determines the selling prices for all kind of items. Accepted items are sold at their regular market price a_1, while defective items are sold at a lower secondary market price a_2, where $a_1 > a_2$.

1.2. Quality control schemes

Various quality control schemes have evolved over time. These schemes include product control, process control and process capability analysis. Product control can be achieved by two techniques. The first technique is called acceptance sampling. This technique is concerned with inspection and decision making regarding products based on a sample

taken from the lot. The other technique is called 100% inspection in which a decision regarding products is made based on inspecting the whole lot.

Statistical Process Control (SPC) is a diagnostic tool that allows you to determine “assignable” versus “common” causes of variation. Common causes of variation are normal and affect every process while assignable causes of variation occur when something happens that is not usually part of the process. SPC allows you to identify when these assignable causes occur so that you can eliminate them and bring predictability, or “control” to a process without overreacting to normal variability. Control charts are one of the most effective SPC tools. Other SPC tools include Histogram, Check Sheet, Scatter Diagram and Pareto Chart.

Process capability analysis is an engineering study to estimate process capability. The estimate of process capability may be in the form of a probability distribution having a specified shape, center (mean), or spread (standard deviation).

1.3. Inspection

Inspection is often used to appraise the quality of purchased and manufactured items. Inspections can be divided into: 100 % inspections, sampling plans, and repeat inspections.

The first inspection scheme, 100 % Inspection, involves inspecting every product received. It is usually applicable in situations where the component is extremely critical and passing any defectives would result in an unacceptably high failure cost. The

disadvantages of this method are: it is expensive; it can not be used for destructive testing; it may cause a delay in the production schedule.

Sampling plans involve inspecting a sample of products drawn from a lot. The whole lot will be judged based on the sample. If the sample meets specifications, then we accept the whole lot otherwise lot will be rejected. This method has many advantages over the 100 % inspection. It costs less, involves less damage to the products, applicable to destructive testing, and involves fewer personnel.

1.4. Inspection error

The manufacture of quality products demands measurements that are both high precision and high accuracy because inspection is used to determine whether or not a product meets specifications. The inspection results are commonly used to influence the operation in making the current part or the production of the next part, thereby, correcting a potential quality problem before a product is completed. Hence, the accuracy and effectiveness of the inspection procedures and equipments are essential for precision manufacturing.

Unfortunately, there are always sources of errors in measuring equipments and measurement systems. The sources of errors that come from the measuring equipments include imperfect mechanical structure, errors in control systems, and environmental disturbances. As measurement error is defined as the discrepancy between actual and measured dimensions, it will be affected not only by the error resulted from the measuring equipment and the repeatability of the measurement, but also by the error resulted from the compound effect of machine errors, and the geometric characteristic of the measured surfaces. A variety of techniques have been developed to deal with machine

error modeling and compensation as well as uncertainty in inspection. Another source of error is the error coming from the sampling inspection. This type of error exists when the product's quality is controlled by sampling plan instead of 100% inspection.

Next, chapter two introduces the problem of process targeting, and provides a summary of previous work done in this field. In chapter three, the proposed network approach is first introduced after which few models of production systems are developed using that approach. In chapter four, numerical analysis is performed followed by an overall discussion for the study in chapter five.

CHAPTER TWO

INTRODUCTION

Automation is pervasive in most of complex manufacturing systems, nowadays, due to recent advances in relevant technologies. This trend is understandable since automated systems can perform rigorous procedures while providing consistent results and superior performance. In this context, the concern in product inspection is highly valued; product inspection is one of the major functions that ensure the quality of products and customer satisfaction. To achieve best product performance and consistent product, screening 100% inspection is becoming more attractive than traditional sampling techniques, recently. Highly automated inspection systems have found increasing applications in quality control processes. These systems are very useful in reducing error rates, inspection times, and inspection costs.

For a production process where products are produced continuously, screening limits are usually implemented based on a quality evaluation system that focuses primarily on the cost of nonconformance. Consider a certain quality characteristic, where products with a quality characteristic that either falls above an upper screening limit or below a lower screening limit are rejected, and a rejection cost is incurred. If the quality characteristic of a product is higher than the upper limit, the product can be reworked. Otherwise, the product is scrapped if the quality characteristic is below a lower limit. The proportion of rejected products largely depends on the levels and tolerance of screening limits. It is observed that the closer the upper and lower limits to one another, the higher the quality outcome. However, this may create higher rejection costs due to scrap and rework

procedures. Looser limits, on the other hand, reduce rejection costs while lowering the quality outcome (Phillips and Cho 2000) and increasing quality loss for customers.

Selecting the optimum process target is critically important since it affects the process defective rate, material cost, scrap or rework cost, and loss to the customer due to the deviation of the product's performance from an ideal target value (Phillips and Cho, 2000).

2.1. Literature review

The selection of appropriate process parameters i.e., mean and variance is of importance in reducing operational costs while maintaining high quality level in production processes. The selection of the appropriate process target has been studied extensively in the literature and is often referred to as the "filling or canning" problem. The initial work on this problem probably began with Springer (1951) who considered the problem of finding the optimal process mean for a canning process when both upper and lower control limits are specified. He assumed that the cost of producing under-limit and over-limit products is fixed.

Bettes (1962) studied the same problem as in Springer (1951) except that only the lower limit was specified. He found the optimal process mean and the upper limit for a fixed lower limit using an empirical method that depends on trial and error.

Hunter and Kartha (1977) addressed the problem of finding the optimal process mean with only a specified lower limit and in which under-filled items are sold at reduced

prices. They also assumed that conforming items are sold at a fixed price with a penalty cost due to excess in quality.

Nelson (1978) considered the same problem by Hunter and Kartha (1977). The objective of the paper was to find the best target value that will balance the give-away cost and the loss associated with rejected items so as to maximize net income. A four-cycle arithmetic graph is provided for determining the target value.

Nelson (1979) considered the same problem by Springer (1951). A nomograph is provided to set the process mean so that scrap cost is minimized.

Bisgaard et al. (1984) extended the model in W. Hunter and C. Kartha (1977) such that cans filled below specification limit are sold in a secondary market at a price proportional to the filled quantity. Carlsson (1984) included a more general income function for the same problem, and Arcelus and Banerjee (1985) extended the work assuming a linear shift in the process mean.

Golhar (1987) investigated the problem of selecting the optimum process mean in a canning process in which cans filled above the lower limit are sold at a fixed price, while the underfilled cans are emptied and refilled at a reprocessing cost. He determined, without measurement error, the optimum process mean that maximizes the expected profit per container.

Golhar and Pollock (1988) extended D. Golhar (1987) model to a process where both the process mean and the upper limit can be controlled. Underfilled and overfilled cans are

emptied and refilled. Simple approximate analytical expressions relating the optimal values to fundamental process parameters are given.

Rahim and Banerjee (1988) considered the problem of selecting the optimal production run for a process with random linear drifts. A cost function per unit of finished product is derived. A search algorithm and a graphical method were suggested to find the optimal production run.

Golhar (1988) considered the same problem stated in Golhar and Pollock (1988). A computer program is developed that calculates the desired optimal values.

Arcelus and Rahim (1990) presented an economic model which incorporates the joint control of both variable and attribute quality characteristics of a product. Items are acceptable if they meet the specifications for both types of characteristics at the same time. The objective is to simultaneously select the appropriate target values for the characteristics so as to maximize the expected income per lot.

Boucher and Jafari (1991) extended the line of research by evaluating the problem of finding the optimum target value under a sampling plan as opposed to 100% inspection. Both conditions when sampling results in destructive testing and nondestructive testing were examined.

Schmidt and Pfeifer (1991) extended the model by Golhar and Pollock (1988) to a capacitated (bottleneck) production process. A closed-form expression for the optimal

upper control limit is developed, and a one-way table and an approximating equation are provided for the optimal mean.

Al-Sultan (1994) extended the model of Boucher and Jafari (1991) to the case of two machines in series. He developed an algorithm for finding optimum target values for the two machines when a sampling plan is used.

Das (1995) determined the optimal process targets when lower specification limits were given by maximizing expected profits.

Chen and Chung (1996) and Hong and Elsayed (1999) studied the effects of inspection errors.

Usher et al. (1996) considered the process target problem in a situation where demand for a product did not exactly meet the capacity of a filling operation.

Liu and Taghavachari (1997) extended the model given by Schmidt and Pfeifer (1991) to the case where the amount of fill follows an arbitrary continuous distribution. The best process mean setting as well as the best upper specification limit are sought to maximize the expected profit per fill attempt. They found that the optimal upper limit is given by a very simple formula regardless of the shape of the distribution, while the optimal process mean is determined using a general condition.

Pollock and Golhar (1998) considered the canning process with constant demand and capacity constraint for the production process. They assumed that there is a penalty for producing a nonconforming cans.

Pakkala and Rahim (1999) presented a model for the most economical process target and production run.

Wen and Mergen (1999) described a method for setting the optimum process mean when a process was not capable of meeting specifications in the short term. They assumed that the process mean could be changed easily, and selected the process mean based on minimizing the costs of exceeding the upper specification limit and falling below the lower specification limit.

Al-Sultan and Pulak (2000) considered a manufacturing system with two machines in series. The manufactured product is assumed to have two attributes which are related to the processing of the product, by machine 1 and machine 2 respectively. Each attribute has a lower specification limit (LSL) set for it, and if the measured attribute for a certain product is less than its LSL, the product is recycled at a certain cost. A mathematical model is developed for finding the optimum setting point for each machine, and a numerical approach is suggested for solving this model.

The problem of jointly determining the process target and variance, as opposed to assuming a given variance, was studied by Rahim and Shaibu (2000), Rahim and Al-Sultan (2000), and Rahim et al. (2002).

Al-Fawzan and Rahim (2001) applied the Taguchi loss function to determine the optimal process target and variance.

Kim et al. (2000) proposed a model for determining the optimal process target while considering variance reduction and process capability.

Phillips and Cho (2000) proposed a model for the optimal process target under the situation in which a process distribution is skewed.

Teeravaraprug et al. (2000) developed a model for the most cost-effective process target using regression analysis for a case where empirical data concerning the costs associated with product performance were available.

Lee et al. (2001) considered determining the optimum target value of the quality characteristic and the screening limits for a correlated variable under single and two-stage screenings, with the assumption that the quality characteristic and the correlated variable were normally distributed.

Cho (2002) and Teeravaraprug and Cho (2002) studied the process target problem with the consideration of multiple quality characteristics using a quality loss function.

Chen et al. (2002) developed a model for determining the optimum process mean under a quality loss function by further modifying Wen and Mergen's cost model with both linear and quadratic asymmetrical quality loss functions of products within specifications. They also proposed a method of determining the optimum process mean for a poor process.

Chen and Chou (2002) determined the optimum process mean for a one-sided specification limit assuming that the quality characteristic followed a normal, lognormal, or exponential distribution.

Chen and Chou (2003a) developed a model for determining the optimum manufacturing target based on an asymmetric quality loss function assuming that the quality characteristic followed a uniform or triangular distribution. Chen and Chou (2003b) developed a similar model under bivariate quality characteristics with quadratic asymmetrical quality loss. Chen and Chou (2003c) modified a model by Phillips and Cho with linear quality loss for determining the optimum process mean under a given truncated beta distribution.

Chen (2003) considered the same optimal process mean problem, but for a larger-the-better Weibull quality characteristic by modifying a model by Cho and Leonard (1997), who considered a piecewise linear loss function. He determined the most economic target value of a process assuming a quadratic quality loss function under normally-distributed quality characteristics, with known mean and variance.

Tuffaha and Rahim (2004) studied the problem of process mean and production run under the quadratic loss function.

Most models for determining the optimum process target reported in the literature were derived assuming a single-stage production process, with a few exceptions, e.g. Al-Sultan and Pulak (2000). Furthermore, most of the process target models available in the literature have been developed using short-term probabilities of rework, scrap, etc, except for Bowling et al. (2004), which does not give a true representation of the system dynamics. Bowling et al. (2004) employs Markov principles to develop a model for optimum process target levels for multi-stage production system. However, Al-Zu'bi and Selim (2010) showed that the absorption probabilities are partially generated through that

approach. None the less, most complex modern manufacturing settings are not simply serial production systems which have not been analyzed in the literature. To address these issues, this study develops a model by employing a network approach (long-term probabilities) within for multi-stage network production systems.

2.2 Example Application from the industry

One example application for network production systems suggested in this study is in the Iron and Steel Industry. The example of interest is concerned with cast iron production.

Iron production is relatively unsophisticated. It mostly involves re-melting charges consisting of pig iron, steel scrap, foundry scrap, and ferroalloys to give the appropriate composition. The cupola, which resembles a small blast furnace, is the most common melting unit. Cold pig iron and scrap are charged from the top onto a bed of hot coke through which air is blown. Alternatively, a metallic charge is melted in a coreless induction furnace or in a small electric-arc furnace.

Cast iron is an alloy of iron that contains 2 to 4 percent carbon, along with varying amounts of silicon and manganese and traces of impurities such as sulfur and phosphorus. It is made by reducing iron ore in a blast furnace. The liquid iron is cast, or poured and hardened, into crude ingots called pigs, and the pigs are subsequently re-melted along with scrap and alloying elements in cupola furnaces and recast into molds for producing variety of products.

An illustrative production system is shown in Fig. 1 below. The first two stages in this system involve Metal Cutting. The cast iron pipe is inspected for its diameter's

conformance with the specifications. If the diameter is within the specification limits, the tube is passed for the next stage. Otherwise, the tube is reworked its diameter is smaller than the specification limits, and scrapped if its diameter is larger than the specification limits. The same applies for the next stage in the production process where inspection is performed for the conformance of the length of tube with the specification limits.

The third processing stage is heat treatment. Heat treatment is used to harden, soften, or modify other properties of materials that have different crystal structures at low and high temperatures. Upon inspection, the part is scrapped in case of non-conformance to specifications.

The final processing stage in this production system is plating. Plating is coating a metal or other material with a hard, nonporous metallic surface to improve durability and beauty. Such surfaces as gold, silver, stainless steel, palladium, copper, and nickel are formed by dipping an object into a solution containing the desired surface material, which is deposited by chemical or electrochemical action. Plating is done for decorative purposes, to increase the durability and corrosion-resistance, or for durability. The part is reworked or scrapped in case of non-conformance to specifications.

The four-stage network production system shown in Fig.1 consists of four processing stages; stages D (Diameter Inspection), stage L (Length Inspection), stage H (Heat Treatment), and stage P (Plating). This production system can produce four different products, P1 … P4, at different costs and selling prices. Product P1 is processed consecutively in stages D, L and P, product P2 in stages D, H and P, product P3 stages D and P, and product P4 in all four stages as illustrated below.

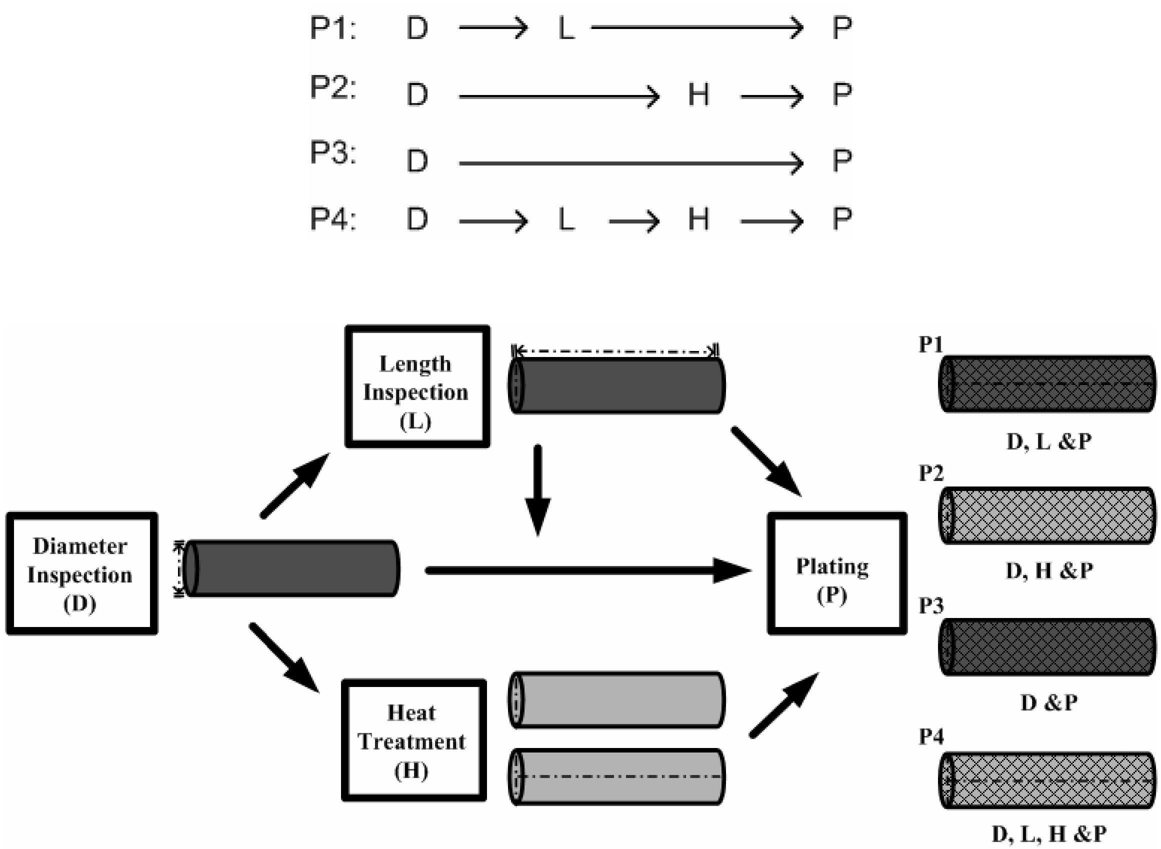

Fig. 1: Illustrative example application of network production system.

2.3. Motivation

Bowling et al. (2004) introduced a Markovian approach to study imperfect production systems where the output of a stage could be scraped, reworked, or accepted according to its conformance with defined specification limits of that stage. The approach is supposed to generate the absorption probabilities into scrap, rework, and accept states for each production stage. We show below that these probabilities are partially generated through that approach. To demonstrate our arguments we consider the two-stage production system shown in Fig. 2 below.

Bowling et al. (2004) introduced a transition probability matrix, P, to describe the transitions among four states. State 1 indicates processing an item at production stage 1, state 2 indicates processing an item at production stage 2, state 3 indicates that an item has been processed successfully at production stage 2, and state 4 indicates that an item is scraped. The "short-term" probabilities p11 and p22 are the rework probabilities associated with stages 1 and 2, respectively. p_{12} and p_{23} are the probabilities associated with accepting a product at stage 1 and 2, respectively. Finally, p_{14} and p_{24} are the probabilities of scrapping a product following stage 1 and 2, respectively. Matrix F shows the long-term absorption probabilities.

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} p_{11} & p_{12} & 0 & p_{14} \\ 0 & p_{22} & p_{23} & p_{24} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$F = \begin{bmatrix} f_{13} & f_{14} \\ f_{14} & f_{24} \end{bmatrix} = \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cc} 3 & 4 \end{array} \\ \begin{bmatrix} \dfrac{p_{12}p_{23}}{(1-P_{11})(1-P_{22})} & \dfrac{p_{14}}{(1-p_{11})} + \dfrac{p_{12}p_{24}}{(1-p_{11})(1-p_{22})} \\ \dfrac{p_{23}}{(1-p_{22})} & \dfrac{p_{24}}{(1-p_{22})} \end{bmatrix} \end{array}$$

The entries in F should be carefully interpreted. f_{23} is the long-term probability that an item is processed successfully through stage 2. However, f_{13} is the long-term probability that an item has been processed successfully through *both* stages 1 and 2.

To obtain the probability that an item has successfully passed from production stage 1 to production stage 2, one could lump states 2 and 3 and get the corresponding F matrix. However, a simple argument shows that this probability is given by $\frac{p_{12}}{1-p_{11}}$.

The term f_{24} is the long-term probability of an item being scraped after production stage 2. However, f_{14} is not only the long-term probability an item has been scraped after stage 1, but also that it has successfully passed stage 1 before being scrapped at stage 2.

Therefore, the expected profit per item for a two-stage serial production system can be expressed as follows:

$$E(PR) = [SP(1 - f_{14})] - \left[PC_1 + PC_2 \frac{p_{12}}{(1 - p_{11})}\right] - \left[SC_1 f_{14} + SC_2 \frac{p_{12}}{(1 - p_{11})} f_{24}\right]$$
$$- \left[RC_1 (m_{11} - 1) + RC_2 (m_{22} - 1) \frac{p_{12}}{(1 - p_{11})}\right]$$

where $E(PR)$ is the expected profit per item, SP_i is the selling price per item, PC_i is the processing cost per item associated with stage i, SC_i is the scrap cost per item associated with stage i, and RC_i rework cost per item associated with stage i.

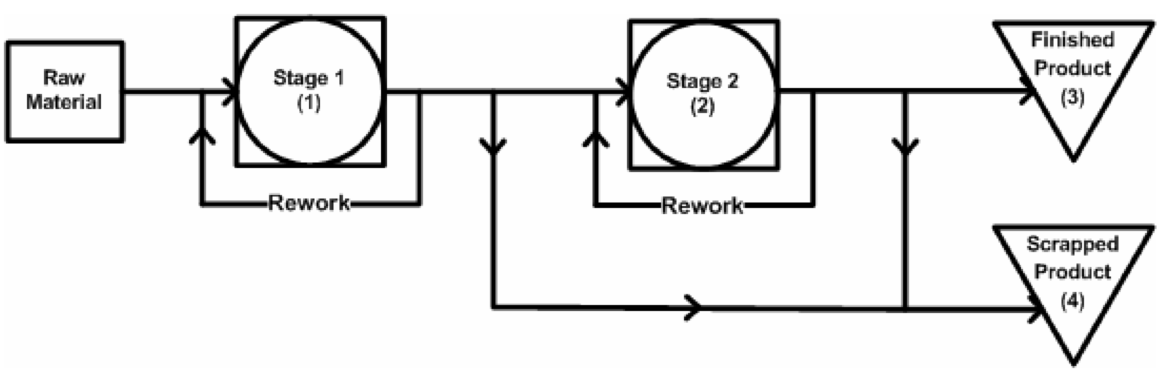

Fig. 2: Two-stage production system- Bowling et al. (2004).

CHAPTER THREE

MODELS DEVELOPMENT

Consider a multi-stage production system in which products are being produced continuously. Each stage is defined as having a single machine and a single inspection station. At each stage, the item is processed and the quality characteristic associated with the stage is examined at an inspection station. The item is then reworked, accepted or scrapped. Therefore, the expected profit per item can be expressed as follows:

$$E(PR) = E(BF) - E(PC) - E(SC) - E(RC) \qquad (1)$$

The purpose of this study is to develop a network model for determining the optimum process target value for each production stage. The study starts by developing the model for a serial single-stage and two-stage production systems. Then, a production system with a network of four stages is tackled raising the issue of single-entry for the raw material and single-exit for the finished products.

In this model the product has an entry stage (single entry point) for raw material after which the product is processed in one stage, two stages, three stages, or sent directly to the final exit stage.

The study then generalizes the model for *n*-stage production system. After that the study develops a model for production systems with multiple entry and exit points. In this

model raw material is passed to the system at multiple stages and products exit from multiple stages.

3.1. Notation and assumption

The developed model is based on the following assumptions:

1. Products are produced continuously.
2. All product items are subject to inspection.
3. When product performance falls below a lower specification limit or above an upper specification limit, a product is reworked or scrapped, respectively.
4. Each product requires the same inspection cost, which is included in the processing cost.
5. Quality characteristic, X_i, is a normally distributed random variable with mean μ_i and variance σ_i^2.
6. The process is under control.
7. The machine sequence is fixed i.e., products have to be processed at stage i before stage $j > i$.

The following is a summary of the notation used in this study:

***E*(PR)** expected profit per item

***E*(BF)** expected benefit per item

***E*(PC)** expected processing cost per item

***E*(SC)** expected scrap cost per item

***E*(RC)** expected rework cost per item

$\mathbf{SP}_i$ selling price per item of product Pi

$\mathbf{PC}_i$ processing cost per item associated with stage i

$\mathbf{SC}_i$ scrap cost per item associated with stage i

$\mathbf{RC}_i$ rework cost per item associated with stage i

$\mathbf{SPu}_i$ selling price per unit of raw material of product Pi

$\mathbf{PCu}_i$ processing cost per unit of raw material associated with stage i

$\mathbf{SCu}_i$ scrap cost per unit of raw material associated with stage i

$\mathbf{RCu}_i$ rework cost per unit of raw material associated with stage i

$\boldsymbol{n}$ number of stages

$\boldsymbol{X_i}$ quality characteristic associated with stage i

$\boldsymbol{\mu_i}$ process mean setting for machine i

$\boldsymbol{\sigma^2_i}$ process variance setting for machine i

$\boldsymbol{L_i}$ lower specification limit associated with stage i

$\boldsymbol{U_i}$ upper specification limit associated with stage i

$\boldsymbol{\Phi(x)}$ cumulative normal function

$\boldsymbol{p_{ij}}$ the probability of going from state i to state j in a single step

$\boldsymbol{\alpha}$ percentage of products passed from stage 1 to stage 2 after passing inspection at stage 1 in a four-stage network production system.

$\boldsymbol{\beta}$ percentage of products passed from stage 1 to stage 3 after passing inspection at stage 1 in a four-stage network production system.

$\boldsymbol{\gamma}$ percentage of products passed from stage 1 to stage 4 after passing inspection at stage 1 in a four-stage network production system.

$\boldsymbol{\theta_1}$ percentage of products passed from stage 2 to stage 3 after passing inspection at stage 2 in a four-stage network production system.

$\boldsymbol{\theta_2}$ percentage of products passed from stage 2 to stage 4 after passing inspection at stage 2 in a four-stage network production system.

$\boldsymbol{f_{ij}}$ percentage of products passed from stage i to stage j after passing inspection at stage i in a network production system.

Ri units of raw material source associated with stage i.

$\boldsymbol{f}_{k/h,i,j}$ is the distribution factor f_k when raw material source is Rh, Ri, or Rj.

ζ percentage of finished products coming out of stage 1 in a four-stage, multi-input, multi-output, network production system.

$\boldsymbol{\delta}_{1/h,i,j}$ percentage of products passed from stage 3 to stage 4 when raw material source is Rh, Ri, or Rj in a four-stage, multi-input, multi-output, network production system.

$\boldsymbol{\delta}_{2/h,i,j}$ percentage of finished products coming out of stage 3 when raw material source is Rh, Ri, or Rj in a four-stage, multi-input, multi-output, network production system.

$\boldsymbol{\theta}_{1/h,i}$ percentage of products passed from stage 2 to stage 4 when raw material source is Rh, or Ri in a four-stage, multi-input, multi-output, network production system.

$\boldsymbol{\theta}_{2/h,i}$ percentage of products passed from stage 2 to stage 3 when raw material source is Rh, or Ri in a four-stage, multi-input, multi-output, network production system.

3.2. Single-stage system

Consider a single-stage production system as shown in Fig. 3. The single-step network is shown in Fig. 4.

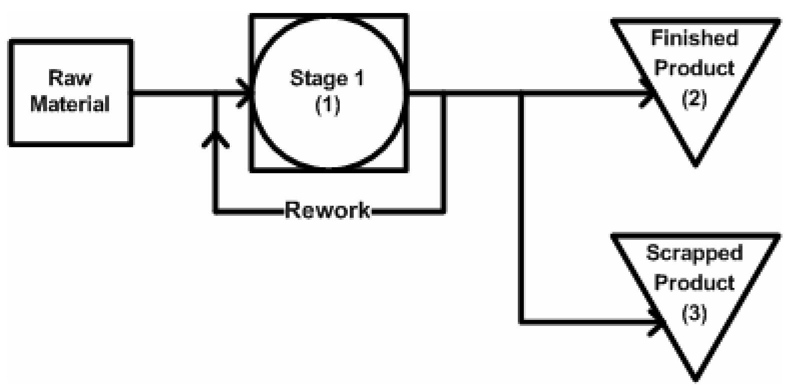

Fig. 3: A single-stage production system- Bowling et al. (2004).

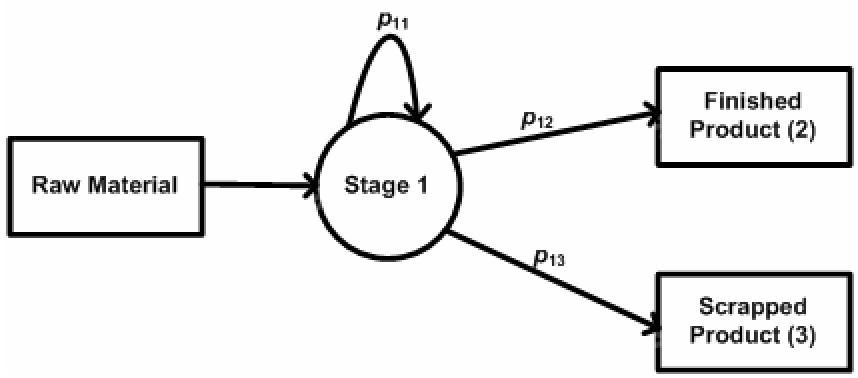

Fig. 4: Single-step network for a single-stage production system

where p_{11} is the probability of an item being reworked, p_{12} is the probability of an item being accepted, and p_{13} is the probability of an item being scrapped. Assuming a normally distributed quality characteristics as shown in Fig. 4, these probabilities can be expressed as follows:

$$p_{11} = \int_{U_1}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{1}{2}\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2} dx_1 = 1 - \Phi(U_1), \tag{2a}$$

$$p_{12} = \int_{L_1}^{U_1} \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{1}{2}\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2} dx_1 = \Phi(U_1) - \Phi(L_1), \tag{2b}$$

$$p_{13} = \int_{-\infty}^{L_1} \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{1}{2}\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2} dx_1 = \Phi(L_1). \tag{2c}$$

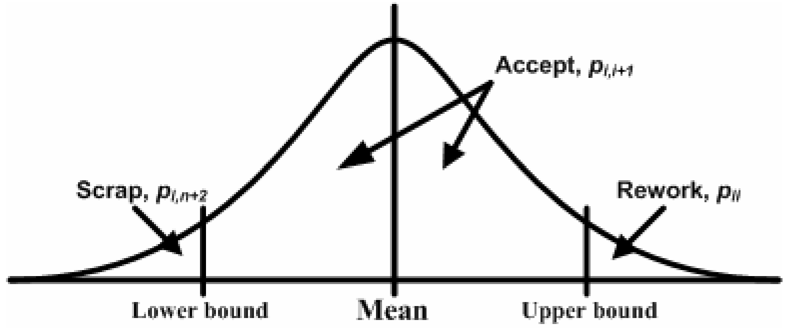

Fig. 5: Illustration of absorption probabilities for a normally-distributed quality characteristic-Bowling et al. (2004).

On the long-term, the behavior of the single-step network approaches that of the Primary network shown in Fig. 6. That is to say, eventually, products will end up in either of the two absorbing states i.e., states 2 and 3 after being reworked 0, 1, 2 … times. Therefore, the probabilities of being accepted and scraped, and the number of rework cycles can be expressed as follows:

$$\begin{aligned}
&p_{12} + p_{12}p_{11} + p_{12}p_{11}p_{11} + p_{12}p_{11}p_{11}p_{11} + \dots \\
&= p_{12}(1 + p_{11} + p_{11}{}^{2} + p_{11}{}^{3} + \dots) = p_{12}\sum_{k=0}^{\infty} p_{11}{}^{k} = \frac{p_{12}}{(1-p_{11})} \qquad (3a) \\
&p_{13} + p_{13}p_{11} + p_{13}p_{11}p_{11} + p_{13}p_{11}p_{11}p_{11} + \dots \\
&= p_{13}(1 + p_{11} + p_{11}{}^{2} + p_{11}{}^{3} + \dots) = p_{13}\sum_{k=0}^{\infty} p_{11}{}^{k} = \frac{p_{13}}{(1-p_{11})} \qquad (3b) \\
&\left(p_{12} + p_{13}\right)\sum_{r=1}^{\infty} r p_{11}{}^{r} = \left(1-p_{11}\right)\frac{p_{11}}{\left(1-p_{11}\right)^{2}} = \frac{p_{11}}{\left(1-p_{11}\right)} \qquad (3c)
\end{aligned}$$

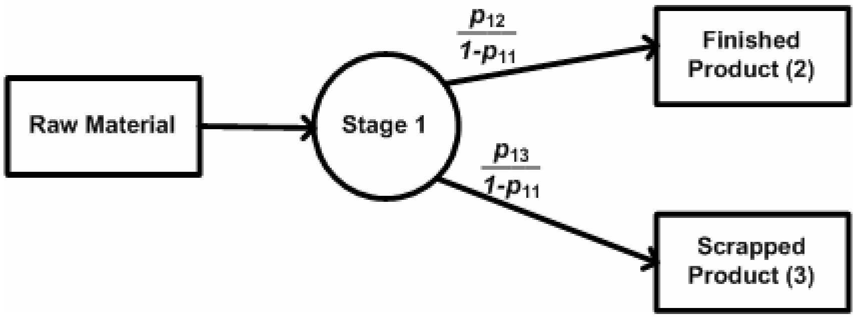

Fig. 6: Primary network for a single-stage production system

The expected profit can be obtained by using Eq. (1). As can be seen, Eq. (1) consists of the benefit, processing costs, scrap cost, and rework cost per item. The expected benefit is the selling price per item (SP_1) multiplied by the long-term percentage of accepted products. The benefit from selling product P_1 is the selling price per item for product P1, SP_1, multiplied by the long-term percentage of accepted products at stage 1 passed to stage 2.

The expected processing cost for a single-stage system is the expected processing cost per item at stage 1 (i.e., PC_1). The expected scrap cost per item is the scrap cost (SC_1) multiplied by the long-term percentage of scrapped products at stage 1. The expected rework cost per item is the rework cost (RC_1) multiplied by the long-term percentage of reworked products at stage 1.

Therefore, the expected profit per item for a single-stage production system can be expressed as follows:

$$E[PR] = SP_1 \frac{p_{12}}{(1-p_{11})} - PC_1 - SC_1 \frac{p_{13}}{(1-p_{11})} - RC_1 \frac{p_{11}}{(1-p_{11})} \qquad (4)$$

The equation can then be rewritten in terms of cumulative normal distribution as follows:

$$E[PR]=SP_1\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)-PC_1-SC_1\frac{\Phi(L_1)}{\Phi(U_1)}-RC_1\left(\frac{1}{\Phi(U_1)}\text{-}1\right) \tag{5}$$

The terms $\Phi(U_1)$ and $\Phi(L_1)$ are functions of the decision variables μ_1, the process mean for machine 1. Obviously, one would like to find the value of μ_1 that maximizes the expected profit. This can be performed numerically using a number of nonlinear optimization software packages.

3.3. Two-stage serial system

Consider a two-stage serial production system as shown in Fig. 7. The single-step network is shown in Fig. 8.

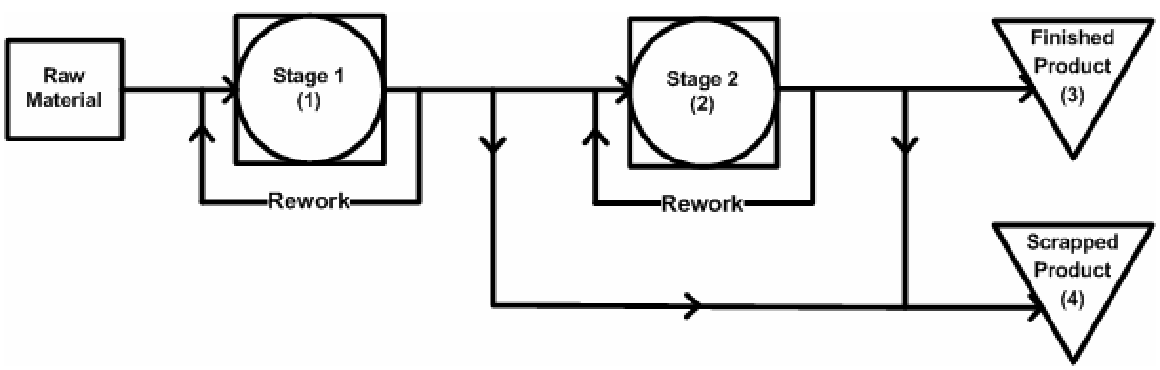

Fig. 7: A two-stage serial production system- Bowling et al. (2004).

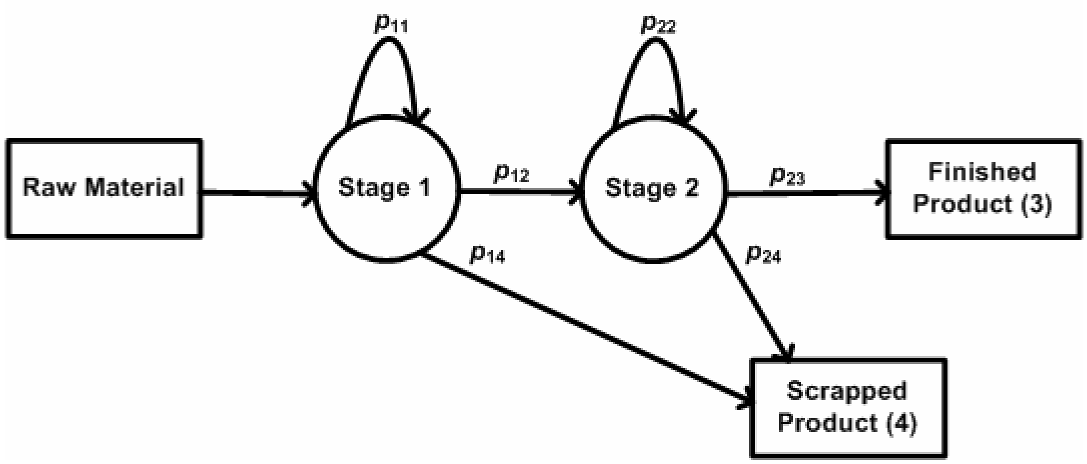

Fig. 8: Single-step network for two-stage serial production system

where $p_{i,i}$ is the rework probability associated with stage i, $p_{i,i+1}$ is the probability associated with accepting a product at stage i, and $p_{i,n+2}$ is the probability of scrapping a product at stage i, where n is the number of stages. Assuming a normally distributed quality characteristics as shown in Fig. 5, these probabilities can be expressed as follows:

$$p_{ii} = \int_{U_i}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{1}{2}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^2} dx_i = 1 - \Phi(U_i), \tag{6a}$$

$$p_{i,i+1} = \int_{L_i}^{U_i} \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{1}{2}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^2} dx_i = \Phi(U_i) - \Phi(L_i), \tag{6b}$$

$$p_{i,n+2} = \int_{-\infty}^{L_i} \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{1}{2}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^2} dx_i = \Phi(L_i). \tag{6c}$$

On the long-term, the behavior of the single-step network approaches that of the Primary network shown in Fig. 9. That is to say, eventually, products will end up in either of the

two absorbing states i.e., states n and $n+1$ after being reworked 0, 1, 2 … times. Therefore, the probabilities of being accepted, scraped and reworked can be expressed as follows:

$$\begin{aligned} & p_{i,i+1} + p_{i,i+1}p_{ii} + p_{i,i+1}p_{ii}\,p_{ii} + p_{i,i+1}p_{ii}\,p_{ii}\,p_{ii} + \ldots \\ & = p_{i,i+1}(1 + p_{ii} + p_{ii}{}^{2} + p_{ii}{}^{3} + \ldots) \\ & = \frac{p_{i,i+1}}{(1 - p_{ii})} \qquad (7a) \end{aligned}$$

$$\begin{aligned} & p_{i,n+2} + p_{i,n+2}p_{ii} + p_{i,n+2}p_{ii}\,p_{ii} + p_{i,n+2}p_{ii}\,p_{ii}\,p_{ii} + \ldots \\ & = p_{i,n+2}(1 + p_{ii} + p_{ii}{}^{2} + p_{ii}{}^{3} + \ldots) \\ & = \frac{p_{i,n+2}}{(1 - p_{ii})} \qquad (7b) \end{aligned}$$

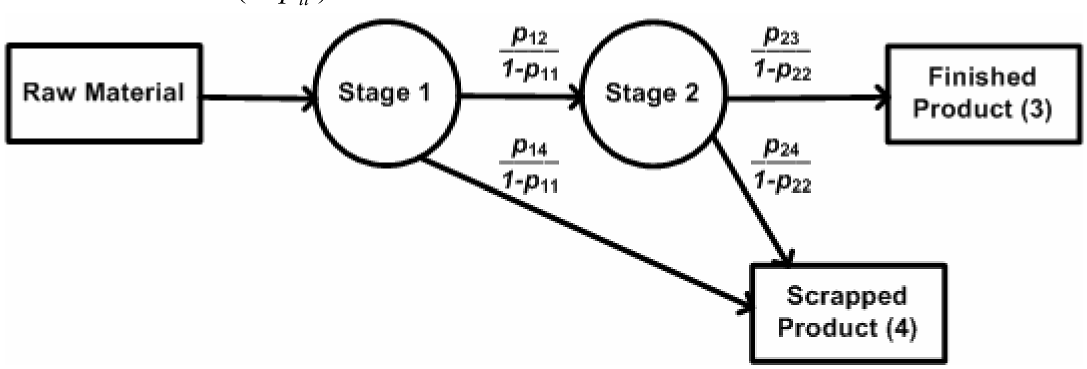

Fig. 9: Primary network for two-stage serial production system

The expected profit can be obtained by using Eq. (1). As can be seen, Eq. (1) consists of the benefit, processing costs, scrap cost, and rework cost per item. The expected benefit is the selling price per item (SP_1) multiplied by the long-term percentage of accepted products. The benefit from selling product P_1 is the selling price per item for product P1 (SP_1) multiplied by the long-term percentage of accepted products at stage 1 multiplied by the long-term percentage of accepted products at stage 2.

The expected processing cost for a two-stage serial system is the expected processing cost per item at stage 1 (PC_1) plus the expected processing cost per item at stage 2 (PC_2) multiplied by the long-term percentage of accepted products at stage 1.

The expected scrap cost per item is the scrap cost at stage 1 (SC_1) multiplied by the long-term percentage of scrapped products at stage 1 plus the scrap cost at stage 1 (SC_1) and the scrap cost at stage 2 (SC_2) multiplied by the long-term percentage of scrapped products at stage 2 multiplied by the long-term percentage of accepted products at stage 1.

The expected rework cost per item is the rework cost at stage 1 (RC_1) multiplied by the long-term percentage of reworked products at stage 1 plus the rework cost at stage 2 (RC_2) multiplied by the long-term percentage of reworked products at stage 2 multiplied by the long-term percentage of accepted products at stage 1.

Therefore, the expected profit per item for a two-stage serial production system can be expressed as follows:

$$\begin{aligned} E[PR] = & \left[SP_1 \frac{p_{12}}{(1-p_{11})} \frac{p_{23}}{(1-p_{22})} \right] \\ & - \left[PC_1 + PC_2 \frac{p_{12}}{(1-p_{11})} \right] \\ & - \left[SC_1 \frac{p_{14}}{(1-p_{11})} + (SC_1 + SC_2) \frac{p_{12}}{(1-p_{11})} \frac{p_{24}}{(1-p_{22})} \right] \\ & - \left[RC_1 \frac{p_{11}}{(1-p_{11})} + RC_2 \frac{p_{22}}{(1-p_{22})} \frac{p_{12}}{(1-p_{11})} \right] \quad (8) \end{aligned}$$

The equation can then be rewritten in terms of cumulative normal distribution as follows:

$$E[PR] = \left[SP_1 \left(1 - \frac{\Phi(L_1)}{\Phi(U_1)}\right)\left(1 - \frac{\Phi(L_2)}{\Phi(U_2)}\right) \right]$$
$$- \left[PC_1 + PC_2 \left(1 - \frac{\Phi(L_1)}{\Phi(U_1)}\right) \right]$$
$$- \left[SC_1 \frac{\Phi(L_1)}{\Phi(U_1)} + (SC_1 + SC_2)\left(1 - \frac{\Phi(L_1)}{\Phi(U_1)}\right)\frac{\Phi(L_2)}{\Phi(U_2)} \right]$$
$$- \left[RC_1 \left(\frac{1}{\Phi(U_1)} - 1\right) + RC_2 \left(\frac{1}{\Phi(U_2)} - 1\right)\left(1 - \frac{\Phi(L_1)}{\Phi(U_1)}\right) \right] \quad (9)$$

The terms $\Phi(U_1)$, $\Phi(L_1)$, $\Phi(U_2)$ and $\Phi(L_2)$ are function of the decision variables μ_1 and μ_2 which are the process mean for machines 1 and 2, respectively.

3.4. N-stage serial system

Consider n-stage production system as shown in Fig. 10. The single-step network is shown in the Fig. 11. The Primary network is shown in the Fig. 12.

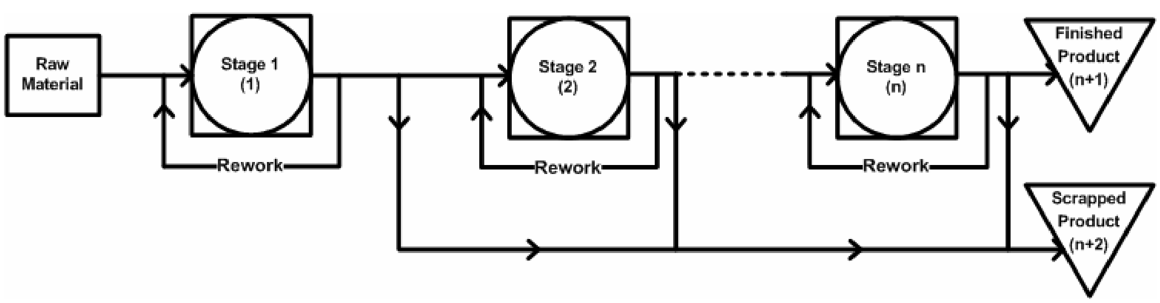

Fig. 10: N-stage serial production system- Bowling et al. (2004).

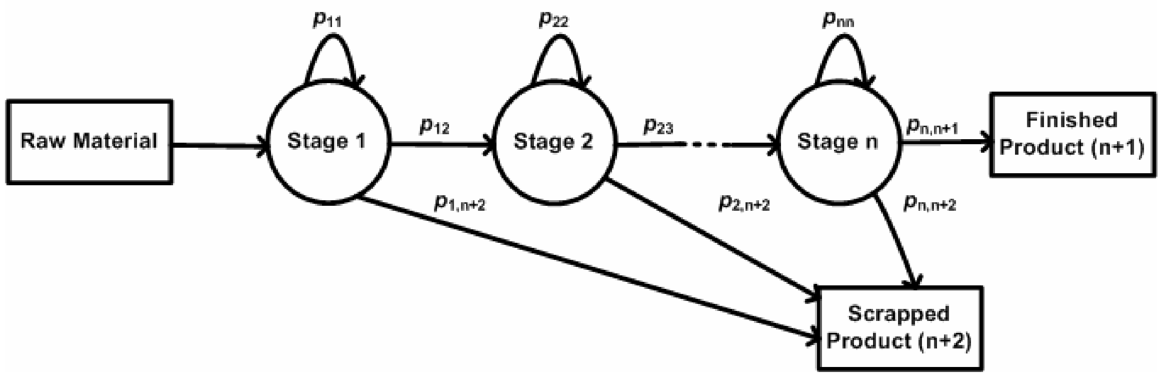

Fig. 11: Single-step network for n-stage serial production system

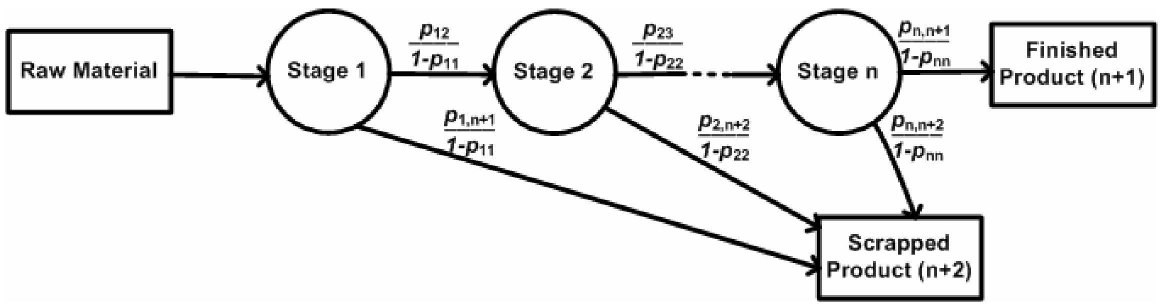

Fig. 12: Primary network for n-stage serial production system

The expected profit per item for n-stage serial production system can be expressed as follows:

$$
\begin{aligned}
E[PR] = & \left[SP_1 \prod_{i=1}^{n} \frac{p_{i,i+1}}{(1-p_{ii})} \right] \\
& - \left[PC_1 + \sum_{i=2}^{n} PC_i \prod_{j=1}^{i} \frac{p_{j,j+1}}{(1-p_{jj})} \right] \\
& - \left[SC_1 \frac{p_{1,n+2}}{(1-p_{11})} + \sum_{i=2}^{n} \left(\left(\sum_{k=1}^{i} SC_k \right) \frac{p_{i,n+2}}{(1-p_{ii})} \prod_{k=1}^{i-1} \left(\frac{p_{k,k+1}}{(1-p_{kk})} \right) \right) \right] \\
& - \left[RC_1 \frac{p_{11}}{(1-p_{11})} + \sum_{i=2}^{n} RC_i \frac{p_{ii}}{(1-p_{ii})} \prod_{k=1}^{i-1} \left(\frac{p_{k,k+1}}{(1-p_{kk})} \right) \right] \quad (10)
\end{aligned}
$$

3.5. Four-stage network system

Consider a four-stage network production system as shown in Fig. 13. The single-step network is shown in Fig. 14.

p_{ii} is the rework probability associated with stage i, $p_{i,i+1}$ is the probability associated with accepting a product at stage i, and $p_{i,n+2}$ is the probability of scrapping a product at stage i, where n is the number of stages.

$f_{ij} : p_{i,j}$ represents that $p_{i,j}$ is f_{ij} percent of $p_{i,i+1}$. f_{ij} is the percentage of production passed from stage i to stage j after passing inspection at stage i. Therefore, $\alpha : p_{12}$ represents that p_{12} is α percent of $p_{1,i+1}$, $\beta : p_{13}$ represents that p_{13} is β percent of $p_{1,i+1}$, $\gamma : p_{14}$ represents that p_{14} is γ percent of $p_{1,i+1}$, $\theta_1 : p_{24}$ represents that p_{24} is θ_1 percent of $p_{2,i+1}$, and $\theta_2 : p_{23}$ represents that p_{23} is θ_2 percent of $p_{2,i+1}$. Hence, $\alpha + \beta + \gamma = 1$ since $\alpha, \beta,$ and γ represent the percentages at which the production accepted at stage 1 is distributed amongst stages 2, 3, and 4, respectively. By the same token, $\theta_1 + \theta_2 = 1$ since θ_1 and θ_2 represent the percentages at which the production accepted at stage 2 is distributed amongst stages 4 and 3, respectively.

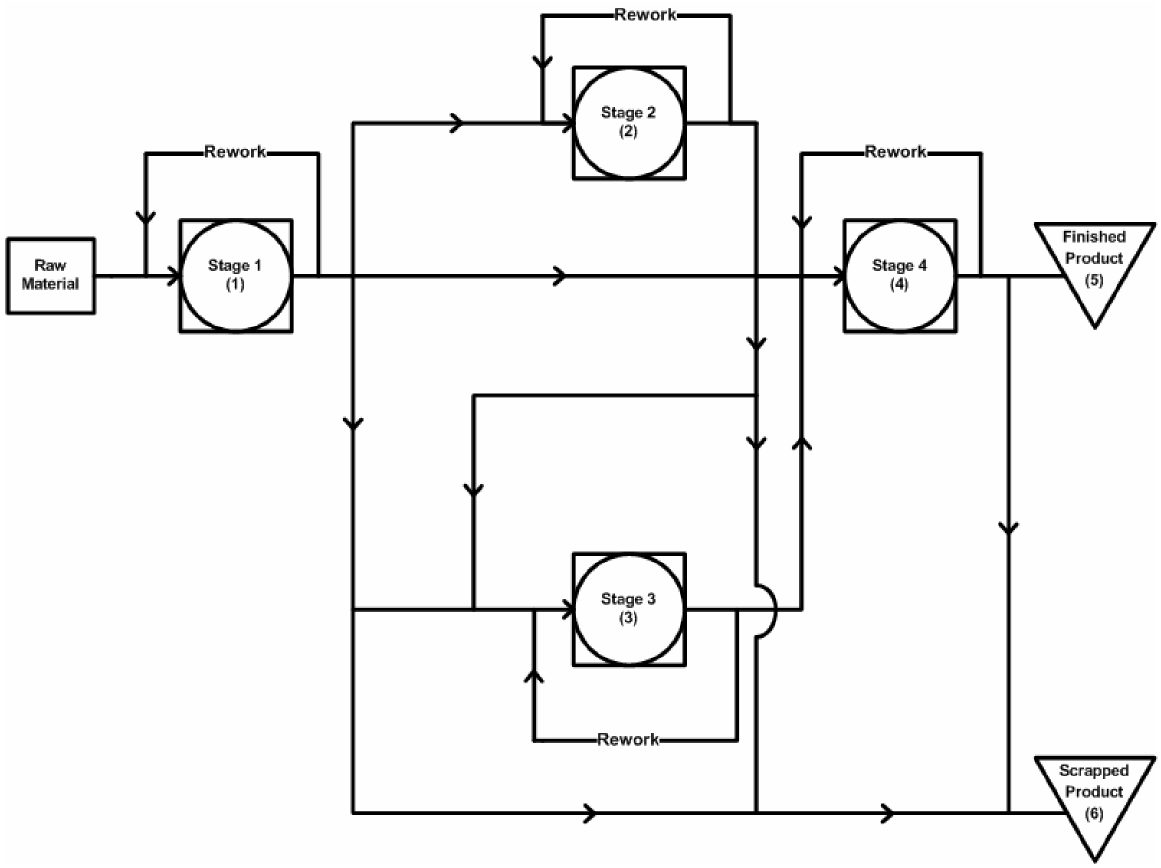

Fig. 13: A four-stage network production system

Assuming a normally distributed quality characteristics as shown in Fig. 5, these probabilities can be expressed as follows:

$$p_{ii} = \int_{U_i}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{1}{2}\left(\frac{x_i - \mu_i}{\sigma_i}\right)^2} dx_i = 1 - \Phi(U_i), \tag{11a}$$

$$p_{i,i+1} = \int_{L_i}^{U_i} \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{1}{2}\left(\frac{x_i - \mu_i}{\sigma_i}\right)^2} dx_i = \Phi(U_i) - \Phi(L_i), \tag{11b}$$

$$p_{i,n+2} = \int_{-\infty}^{L_i} \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{1}{2}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^2} dx_i = \Phi(L_i). \qquad (11c)$$

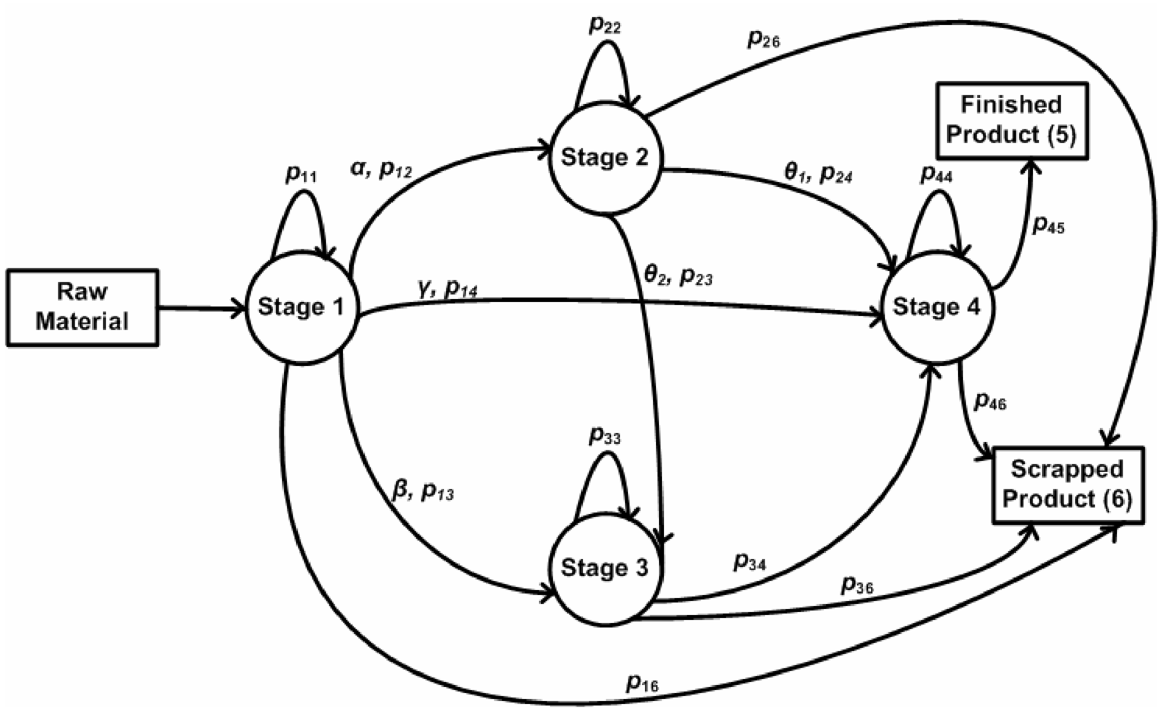

Fig. 14: Single-step network for four-stage network production system

On the long-term, the behavior of the single-step network approaches that of the Primary network shown in Fig. 15. That is to say, eventually, products will end up in either of the two absorbing states i.e., states n and n +1 after being reworked 0, 1, 2 … times. Therefore, the probabilities of being accepted and scraped, and the number of rework cycles can be expressed consecutively as $\frac{p_{i,i+1}}{(1 - p_{ii})}$, $\frac{p_{i,n+2}}{(1 - p_{ii})}$ & $\frac{p_{ii}}{(1 - p_{ii})}$.

The four-stage network production system can produce four different products (P1…P4) at different costs and selling prices. Product P1 is processed consecutively in stages 1, 2

and 4, P2 in stages 1, 3 and 4, P3 stages 1 and 4, and P4 in all four stages as represented next.

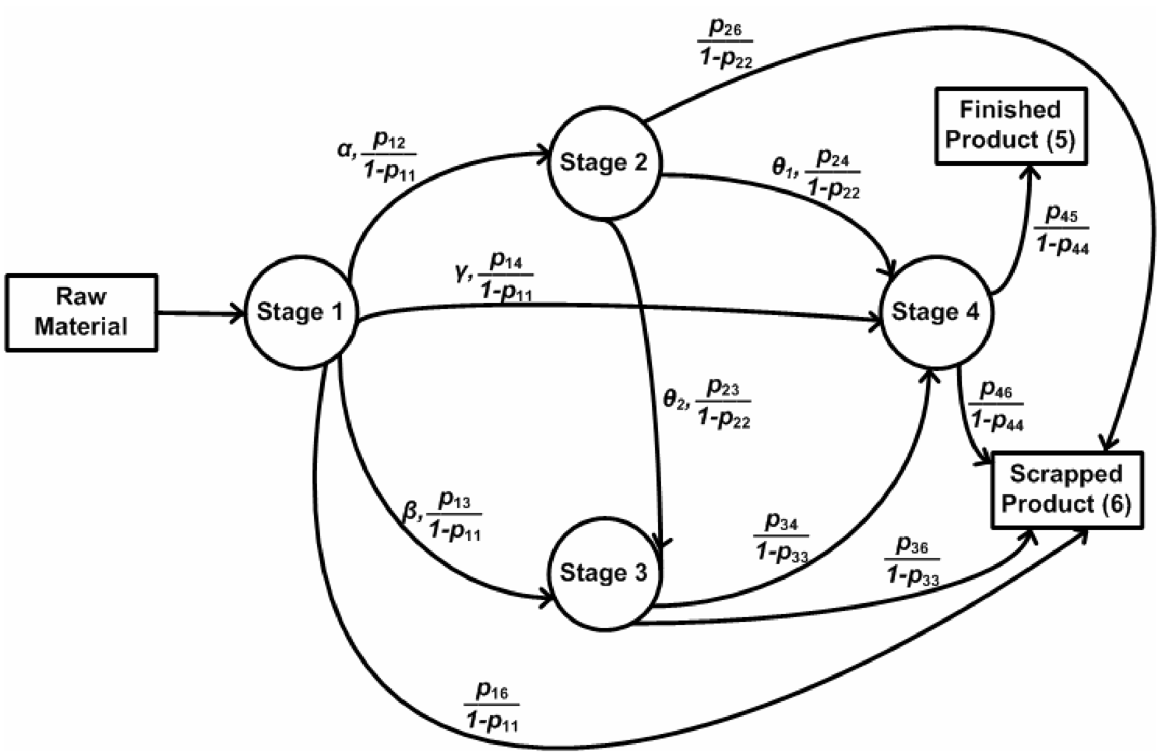

Fig. 15: Primary network for four-stage network production system

P1: $1 \longrightarrow 2 \longrightarrow 4$

P2: $1 \longrightarrow 3 \longrightarrow 4$

P3: $1 \longrightarrow 4$

P4: $1 \longrightarrow 2 \longrightarrow 3 \longrightarrow 4$

The percentage of products that survive the inspection criterion at one stage (i.e., products with performance within the lower and upper specification limits) are passed to the next stage(s) according to distribution factors α, β, γ, θ_1 and θ_2, which determine the

percentage output of each product as represented in Fig. 14. The percentage of the products P1 and P4 is set by α, β determines that of P2, and γ P3, hence, $\alpha + \beta + \gamma = 1$. Likewise, θ_1 and θ_2 set the percentages of products P1 and P4, respectively, hence, $\theta_1 + \theta_2 = 1$. These distribution factors, in a sense, allocate raw material, unfinished products and processing resources to products.

The expected profit can be obtained by using Eq. (1). As can be seen, Eq. (1) consists of the benefit, processing costs, scrap cost, and rework cost per item. The expected benefit is the selling price per item (SP_i) multiplied by the long-term percentage of accepted products. The benefit from selling product P_1, for example, is the selling price per item for product P1, SP_1, multiplied by the long by the portion of the long-term percentage of accepted products at stage 1 passed to stage 2, $\left(\frac{p_{12}}{1-p_{11}}\right)$, multiplied by the portion of the percentage of accepted products at stage 2 passed to stage 4, $\left(\frac{p_{24}}{1-p_{22}}\right)$, multiplied by the percentage of accepted products at stage 4, $\left(\frac{p_{45}}{1-p_{44}}\right)$. Likewise, the expected benefit for P2, P3 and P4 are formulated by performing a path-based analysis.

The expected processing cost for a four-stage network production system is the expected processing cost per item at stage 1, PC_1, plus the expected processing cost at stage 2 which is PC_2 multiplied by the portion of the long-term percentage of products accepted

at stage 1 passed to stage 2, $\left(\frac{p_{12}}{1-p_{11}}\right)$. Similarly, the analysis is continued to formulate the expected processing cost at stages 3 and 4.

The expected scrap cost per item is the scrap cost at stage 1, SC_1, multiplied by the long-term percentage of scrapped products at stage 1, $\left(\frac{p_{16}}{1-p_{11}}\right)$, plus (SC_1+SC_2) multiplied by the long-term percentage of scrapped products at stage 2 , $\left(\frac{p_{26}}{1-p_{22}}\right)$, multiplied by the long-term percentage of accepted products at stage 1 passed to stage 2, $\left(\frac{p_{12}}{1-p_{11}}\right)$. The expected scrap cost at stages 3 and 4 is formulated, similarly.

The expected rework cost per item is the rework cost at stage 1, RC_1, multiplied by the long-term percentage of reworked products at stage 1, $\left(\frac{p_{11}}{1-p_{11}}\right)$, plus the rework cost at stage 2, RC_2, multiplied by the long-term percentage of reworked products at stage 2 , $\left(\frac{p_{22}}{1-p_{22}}\right)$, multiplied by the long-term percentage of accepted products at stage 1 passed to stage 2, $\left(\frac{p_{12}}{1-p_{11}}\right)$. The expected rework cost at stages 3 and 4 is formulated, similarly.

Therefore, the expected profit per item for a four-stage network production system can be expressed as follows:

$$
\begin{aligned}
E[PR] = & \left[SP_1 \frac{p_{12}}{(1-p_{11})} \frac{p_{24}}{(1-p_{22})} \frac{p_{45}}{(1-p_{44})} + SP_2 \frac{p_{13}}{(1-p_{11})} \frac{p_{34}}{(1-p_{33})} \frac{p_{45}}{(1-p_{44})} \right] \\
& + \left[SP_3 \frac{p_{14}}{(1-p_{11})} \frac{p_{45}}{(1-p_{44})} + SP_4 \frac{p_{12}}{(1-p_{11})} \frac{p_{23}}{(1-p_{22})} \frac{p_{34}}{(1-p_{33})} \frac{p_{45}}{(1-p_{44})} \right] \\
& - \left[PC_1 + PC_2 \frac{p_{12}}{(1-p_{11})} + PC_3 \left(\frac{p_{13}}{(1-p_{11})} + \frac{p_{12}}{(1-p_{11})} \frac{p_{23}}{(1-p_{22})} \right) \right] \\
& - \left[PC_4 \left(\frac{p_{14}}{(1-p_{11})} + \frac{p_{12}}{(1-p_{11})} \frac{p_{24}}{(1-p_{22})} + \frac{p_{12}}{(1-p_{11})} \frac{p_{23}}{(1-p_{22})} \frac{p_{34}}{(1-p_{33})} + \frac{p_{13}}{(1-p_{11})} \frac{p_{34}}{(1-p_{33})} \right) \right] \\
& - \left[SC_1 \frac{p_{16}}{(1-p_{11})} + \left(SC_1 + SC_2 \right) \frac{p_{12}}{(1-p_{11})} \frac{p_{26}}{(1-p_{22})} + \left(SC_1 + SC_2 + SC_3 \right) \frac{p_{12}}{(1-p_{11})} \frac{p_{23}}{(1-p_{22})} \frac{p_{36}}{(1-p_{33})} \right] \\
& - \left[\left(SC_1 + SC_3 \right) \frac{p_{13}}{(1-p_{11})} \frac{p_{36}}{(1-p_{33})} + \left(SC_1 + SC_4 \right) \frac{p_{14}}{(1-p_{11})} \frac{p_{46}}{(1-p_{44})} + \left(SC_1 + SC_2 + SC_4 \right) \frac{p_{12}}{(1-p_{11})} \frac{p_{24}}{(1-p_{22})} \frac{p_{46}}{(1-p_{44})} \right] \\
& - \left[\left(SC_1 + SC_3 + SC_4 \right) \frac{p_{13}}{(1-p_{11})} \frac{p_{34}}{(1-p_{33})} \frac{p_{46}}{(1-p_{44})} + \left(SC_1 + SC_2 + SC_3 + SC_4 \right) \frac{p_{12}}{(1-p_{11})} \frac{p_{23}}{(1-p_{11})} \frac{p_{34}}{(1-p_{33})} \frac{p_{46}}{(1-p_{44})} \right] \\
& - \left[RC_1 \frac{p_{11}}{(1-p_{11})} + RC_2 \frac{p_{22}}{(1-p_{22})} \frac{p_{12}}{(1-p_{11})} + RC_3 \frac{p_{33}}{(1-p_{33})} \left(\frac{p_{13}}{(1-p_{11})} + \frac{p_{12}}{(1-p_{11})} \frac{p_{23}}{(1-p_{22})} \right) \right] \\
& - \left[RC_4 \frac{p_{44}}{(1-p_{44})} \left(\frac{p_{14}}{(1-p_{11})} + \frac{p_{12}}{(1-p_{11})} \frac{p_{24}}{(1-p_{22})} + \frac{p_{13}}{(1-p_{11})} \frac{p_{34}}{(1-p_{33})} + \frac{p_{12}}{(1-p_{11})} \frac{p_{23}}{(1-p_{22})} \frac{p_{34}}{(1-p_{33})} \right) \right] \qquad (12)
\end{aligned}
$$

The equation can then be rewritten in terms of cumulative normal distribution as shown in Eq. (13). The terms $\Phi(U_1)$, $\Phi(L_1)$, $\Phi(U_2)$, $\Phi(L_2)$, $\Phi(U_3)$, $\Phi(L_3)$, $\Phi(U_4)$ and $\Phi(L_4)$ are function of the decision variables μ_1, μ_2, μ_3 and μ_4 which are the process mean for machines 1, 2, 3 and 4, respectively. Obviously, one would like to find the value of μ_1, μ_2, μ_3 and μ_4 that maximizes the expected profit. This can be performed numerically using a number of nonlinear optimization software packages.

$$E[PR]=\left[SP_1\alpha\theta_1\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)\left(1-\frac{\Phi(L_2)}{\Phi(U_2)}\right)\left(1-\frac{\Phi(L_4)}{\Phi(U_4)}\right)+SP_2\beta\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)\left(1-\frac{\Phi(L_3)}{\Phi(U_3)}\right)\left(1-\frac{\Phi(L_4)}{\Phi(U_4)}\right)\right]$$
$$+\left[SP_3\gamma\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)\left(1-\frac{\Phi(L_4)}{\Phi(U_4)}\right)+SP_4\alpha\theta_2\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)\left(1-\frac{\Phi(L_2)}{\Phi(U_2)}\right)\left(1-\frac{\Phi(L_3)}{\Phi(U_3)}\right)\left(1-\frac{\Phi(L_4)}{\Phi(U_4)}\right)\right]$$
$$-\left[PC_1+PC_2\alpha\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)+PC_3\left(\beta\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)+\alpha\theta_2\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)\left(1-\frac{\Phi(L_2)}{\Phi(U_2)}\right)\right)\right]$$
$$-\left[PC_4\left(\gamma\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)+\alpha\theta_1\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)\left(1-\frac{\Phi(L_2)}{\Phi(U_2)}\right)+\alpha\theta_2\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)\left(1-\frac{\Phi(L_2)}{\Phi(U_2)}\right)\left(1-\frac{\Phi(L_3)}{\Phi(U_3)}\right)+\beta\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)\left(1-\frac{\Phi(L_3)}{\Phi(U_3)}\right)\right)\right]$$
$$-\left[SC_1\frac{\Phi(L_1)}{\Phi(U_1)}+(SC_1+SC_2)\alpha\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)\frac{\Phi(L_2)}{\Phi(U_2)}+(SC_1+SC_2+SC_3)\alpha\theta_2\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)\left(1-\frac{\Phi(L_2)}{\Phi(U_2)}\right)\frac{\Phi(L_3)}{\Phi(U_3)}\right]$$
$$-\left[(SC_1+SC_3)\beta\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)\frac{\Phi(L_3)}{\Phi(U_3)}+(SC_1+SC_4)\gamma\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)\frac{\Phi(L_4)}{\Phi(U_4)}\right]$$
$$-\left[(SC_1+SC_2+SC_4)\alpha\theta_1\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)\left(1-\frac{\Phi(L_2)}{\Phi(U_2)}\right)\frac{\Phi(L_4)}{\Phi(U_4)}\right]$$
$$-\left[(SC_1+SC_3+SC_4)\beta\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)\left(1-\frac{\Phi(L_3)}{\Phi(U_3)}\right)\frac{\Phi(L_4)}{\Phi(U_4)}\right]$$
$$-\left[(SC_1+SC_2+SC_3+SC_4)\alpha\theta_2\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)\left(1-\frac{\Phi(L_2)}{\Phi(U_2)}\right)\left(1-\frac{\Phi(L_3)}{\Phi(U_3)}\right)\frac{\Phi(L_4)}{\Phi(U_4)}\right]$$
$$-\left[RC_1\left(\frac{1}{\Phi(U_1)}-1\right)+RC_2\alpha\left(\frac{1}{\Phi(U_2)}-1\right)\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)\right]$$
$$-\left[RC_3\left(\frac{1}{\Phi(U_3)}-1\right)\left(\beta\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)+\alpha\theta_2\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)\left(1-\frac{\Phi(L_2)}{\Phi(U_2)}\right)\right)\right]$$
$$-\left[RC_4\left(\frac{1}{\Phi(U_4)}-1\right)\left(\gamma\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)+\alpha\theta_1\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)\left(1-\frac{\Phi(L_2)}{\Phi(U_2)}\right)+\beta\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)\left(1-\frac{\Phi(L_3)}{\Phi(U_3)}\right)\right)\right]$$
$$-\left[RC_4\left(\frac{1}{\Phi(U_4)}-1\right)\left(\alpha\theta_2\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)\left(1-\frac{\Phi(L_2)}{\Phi(U_2)}\right)\left(1-\frac{\Phi(L_3)}{\Phi(U_3)}\right)\right)\right] \tag{13}$$

3.6. Introducing a production planning aspect to the model

We started our analysis to formulate the expected profit function in Eq. (12) previously assuming that the distribution factors, α, β, γ, θ_1 and θ_2, are set according to the producer's preference, or experience. For example, the decision to set (α, β, γ, θ_1, θ_2) = (0.30, 0.20, 0.50 0.05, 0.25), means that 30% of the products that survive inspection at stage 1 are passed to stage 2, 20% to stage 3, and 50% to stage 4. Furthermore, 5% of the products that survive inspection at stage 2 are passed to stage 4, and 25% to stage 3. In

other words, 30% of the products that survive inspection at stage 1 are used to produce products P1 and P4, 20% to produce product P2, and 50% produce product P3.

However, when substituting back in Eq. (12) the optimum process mean for machines 1, 2, 3 and 4, μ_1*, μ_2*, μ_3* and μ_4*, respectively, and solving for the distribution factors, α, β, γ, θ_1 and θ_2, as the decision variables, the model reduces to the following equation ($c_1 \ldots c_8$ here are constants):

$$
\begin{array}{lc}
\textit{Maximize} & c_1\alpha + c_2\beta + c_3\gamma + c_4\alpha\theta_1 + c_5\alpha\theta_2 \\
\textit{subject to} & \alpha + \beta + \gamma = 1 \\
 & c_6\alpha = c_7\theta_1 + c_8\theta_2 \\
 & \alpha, \beta, \gamma, \theta_1, \theta_2 \geq 0
\end{array}
\tag{14}
$$

$c_1 \ldots c_5$ are obtained directly from substituting the optimum process mean for machines 1, 2, 3 and 4, μ_1*, μ_2*, μ_3* and μ_4*, respectively, in Eq. (12). $c_6 \ldots c_8$, however, are obtained as follows:

$$
\begin{aligned}
&\alpha \frac{p_{1,i+1}}{(1-p_{11})}\left(1-\frac{p_{26}}{(1-p_{22})}\right) = \left(\theta_1 + \theta_1\right)\frac{p_{2,i+1}}{(1-p_{22})} \\
\Rightarrow\ &\alpha\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)\left(1-\frac{\Phi(L_2)}{\Phi(U_2)}\right) = \left(\theta_1 + \theta_2\right)\left(1-\frac{\Phi(L_2)}{\Phi(U_2)}\right) \\
\Rightarrow\ &\alpha\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right) = \theta_1 + \theta_2
\end{aligned}
\tag{15}
$$

3.7. N-stage network system

The four-stage network production system is generalized as *n*-stage network production system as shown in the single-step network in Fig. 16.

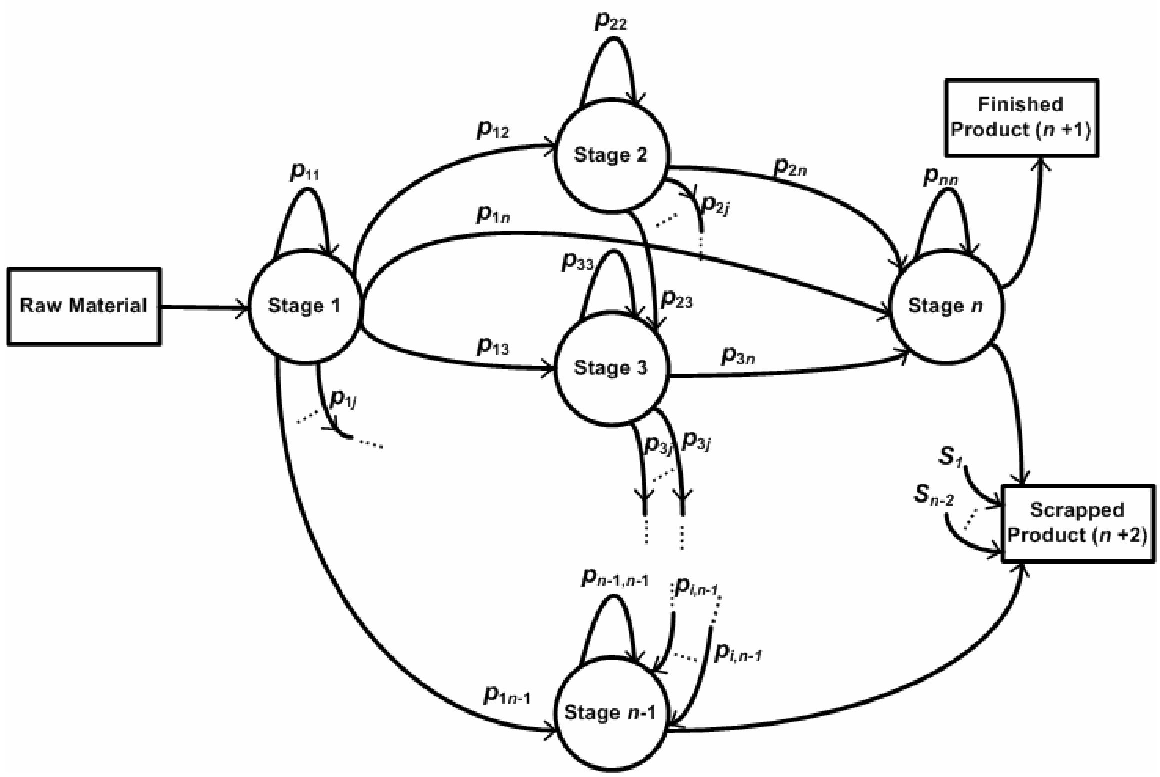

Fig. 16: Single-step network for *n*-stage network production system.

Next, we build the primary-network shown in Fig. 17, and formulate the objective function as before.

3.8. Multiple-input, multiple-output network system

Raw material or semi-processed products could also be made to enter the production process at any stage of the system, and finished products could exit the system from any stage, as well. (See Fig. 18 below for the corresponding single-step network)

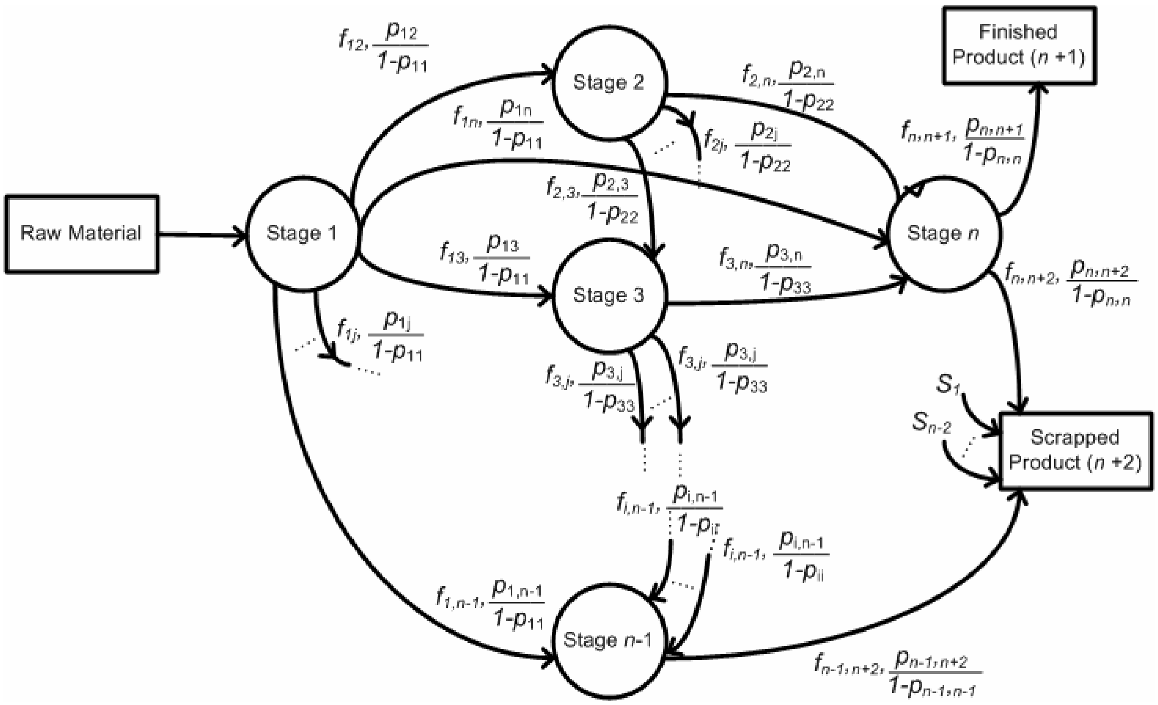

Fig. 17: Primary network for *n*-stage network production system.

3.9. Taking machine variance into account

In the context of a production process, the reduction in operational costs while maintaining a high quality level is a desirable ultimate goal for engineers and practitioners for many years. The selection of appropriate process parameters i.e., mean and variance is of major interest and importance in satisfying such a desirable goal.

In the previously developed models, optimum process variance, σ^2, is not targeted. A cost function corresponding to the cost of precision of the machine, for example, $f(\sigma) = e^{\sigma}$ could be added to the objective function to evaluate for optimum process target variance, in addition to process mean.

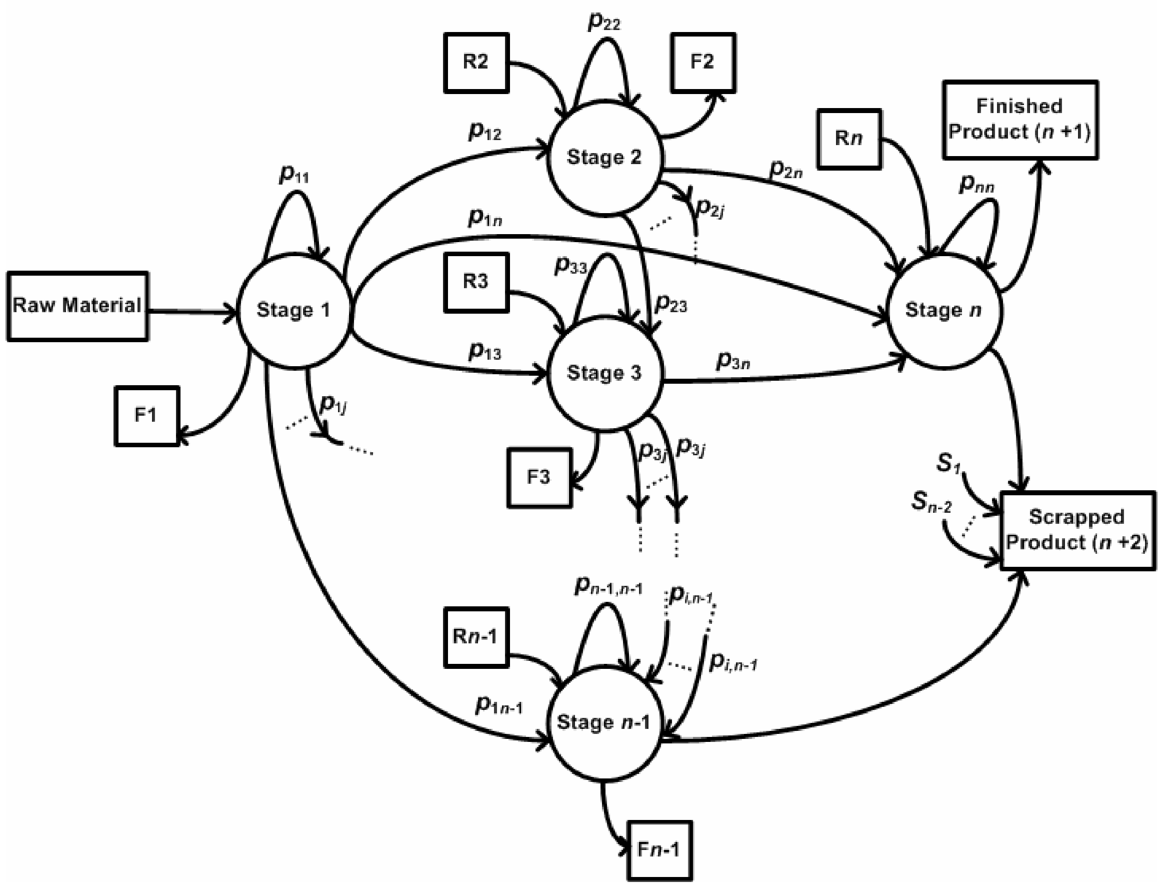

Fig. 18: Single-step network for multi-input, multi-output, *n*-stage network production system.

3.10 System operational aspects

One operational decision running such the production systems suggested earlier is how much raw material is needed to produce X amount of Product X?

Referring to our four-stage network production system shown in Fig. 15., to produce a certain quantity of product P1, A(P1), for example, we know that P1 runs through stages 1, 2 and 4 serially, with distribution factors α from stage 1 to stage 2, and θ_1 from stage 2 to stage 4. Therefore, the amount of raw materials, R, required in this case can be evaluates as follows:

$$A(P1) = R\frac{p_{12}}{(1-p_{11})}\frac{p_{24}}{(1-p_{22})}\frac{p_{45}}{(1-p_{44})} = R\alpha\theta_1\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)\left(1-\frac{\Phi(L_2)}{\Phi(U_2)}\right)\left(1-\frac{\Phi(L_4)}{\Phi(U_4)}\right)$$
$$\Rightarrow R = \frac{\alpha\theta_1}{A(P1)}\left(1-\frac{\Phi(L_1)}{\Phi(U_1)}\right)\left(1-\frac{\Phi(L_2)}{\Phi(U_2)}\right)\left(1-\frac{\Phi(L_4)}{\Phi(U_4)}\right) \quad ...(19)$$

3.11. Production costs computation revisited

Another approach to compute the production costs incurred in the system is to count the all possible outcome of the production process. Consider single-stage production system shown in Fig. 2. The product could be accepted after a single processing cycle for which only processing cost at stage 1 (i.e., PC_1) is incurred at probability $p_{i,i+1}$. By the same token, the product could be scrapped after a single processing cycle with probability $p_{i,n+2}$ incurring processing and scrap costs (PC_1 & SC_1).

Furthermore, the product could be accepted or scraped after 1, 2, 3 … rework cycles incurring the following costs, consecutively:

$$PC_1 p_{i,i+1}(p_{ii} + p_{ii}^2 + p_{ii}^3 + ...) + RC_1 p_{i,i+1}(p_{ii} + 2p_{ii}^2 + 3p_{ii}^3 + ...) \qquad (16a)$$
$$PC_1 p_{i,n+2}(p_{ii} + p_{ii}^2 + p_{ii}^3 + ...) + RC_1 p_{i,n+2}(p_{ii} + 2p_{ii}^2 + 3p_{ii}^3 + ...)$$
$$+SC_1 p_{i,n+2}(p_{ii} + p_{ii}^2 + p_{ii}^3 + ...) \qquad (16b)$$

And when we sum all the possible cases in which a product could be found, the outcome is similar to our previous analysis in Eq. (4).

On the other hand, another point worth elaborating is that scrap cost computation being a cumulative quantity from one stage to the next. That is to say, you might be tempted at first glance to formulate scrap cost for four-stage network production system, for example, as follows:

$$\left[SC_1 \frac{p_{16}}{(1-p_{11})} + SC_2 \frac{p_{12}}{(1-p_{11})} \frac{p_{26}}{(1-p_{22})} + SC_3 \frac{p_{36}}{(1-p_{33})} \left(\frac{p_{12}}{(1-p_{11})} \frac{p_{23}}{(1-p_{22})} + \frac{p_{13}}{(1-p_{11})} \right) \right]$$
$$\left[SC_4 \frac{p_{46}}{(1-p_{44})} \left(\frac{p_{14}}{(1-p_{11})} + \frac{p_{12}}{(1-p_{11})} \frac{p_{24}}{(1-p_{22})} + \frac{p_{13}}{(1-p_{11})} \frac{p_{34}}{(1-p_{33})} + \frac{p_{12}}{(1-p_{11})} \frac{p_{23}}{(1-p_{11})} \frac{p_{34}}{(1-p_{33})} \right) \right] \quad (17)$$

However, we believe that scrap cost is a cumulative cost. That is, if raw-material, or semi-processed material flaws out of a certain stage of the production system but gets scrapped at a later stage, this means that the former stage suffers an additional scrap cost; the cost of lost raw material and production resources. Thus, we formulate scrap cost for four-stage network production system as follows:

$$\begin{aligned}
&SC_1 \frac{p_{16}}{(1-p_{11})} + (SC_1 + SC_2) \frac{p_{12}}{(1-p_{11})} \frac{p_{26}}{(1-p_{22})} + (SC_1 + SC_3) \frac{p_{13}}{(1-p_{11})} \frac{p_{36}}{(1-p_{33})} \\
&+(SC_1 + SC_4) \frac{p_{14}}{(1-p_{11})} \frac{p_{46}}{(1-p_{44})} \\
&+(SC_1 + SC_2 + SC_4) \frac{p_{12}}{(1-p_{11})} \frac{p_{24}}{(1-p_{22})} \frac{p_{46}}{(1-p_{44})_1} \\
&+(SC_1 + SC_2 + SC_3) \frac{p_{12}}{(1-p_{11})} \frac{p_{23}}{(1-p_{22})} \frac{p_{36}}{(1-p_{33})} \\
&+(SC_1 + SC_3 + SC_4) \frac{p_{13}}{(1-p_{11})} \frac{p_{34}}{(1-p_{33})} \frac{p_{46}}{(1-p_{44})} \\
&+(SC_1 + SC_2 + SC_3 + SC_4) \frac{p_{12}}{(1-p_{11})} \frac{p_{23}}{(1-p_{11})} \frac{p_{34}}{(1-p_{33})} \frac{p_{46}}{(1-p_{44})} \qquad (18)
\end{aligned}$$

CHAPTER FOUR

NUMERICAL EXAMPLES

4.1. Two-stage serial system

Using the model developed earlier, we solved example 5.2 given in Bowling et al. (2004) for two-stage production system based on the same parameters; SP1 = 120, PC1 = 25, PC2 = 20, RC1 = 10, RC2 = 17, SC1 = 15, SC2 = 12, σ = 1.0, L1 = 8.0, L2 = 13.0, U1 = 12.0 and U2 = 17.0. Using exhaustive search, the expected profit is maximized at μ_1^* = 10.572 and μ_2^* = 15.5089 with an expected profit of 71.3575. Fig. 18 shows the expected profit as a function of the process means, μ_1 and μ_2.

4.2. Multi-stage serial system

Using the model developed earlier, we solved example 5.3 given in Bowling et al. (2004) for single-stage, two-stage, three-stage, four-stage, and five-stage production systems. Based on the same parameters, shown in Table 1, the optimum process means and expected profit for these cases are shown in Table 2.

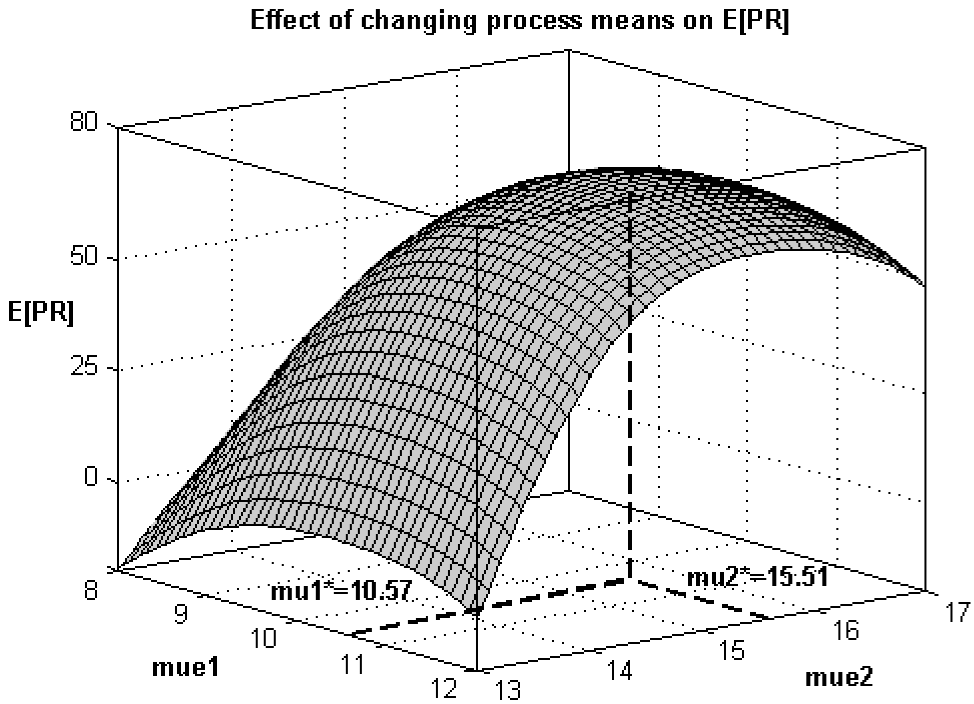

Fig. 19: Effect of changing process means on the expected profit for two-stage serial system.

Table 1: Data for a multi-stage serial production system- Bowling et al. (2004).

Parameter	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5
PC	25	20	12	15	4
RC	15	12	8	10	2
SC	10	17	5	12	3
σ	1.0	1.0	1.0	1.0	1.0
L	8	13	10	7	18
U	12	17	14	11	22

Table 2: Optimum process means and expected profit for a multi-stage serial production system

Parameter	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5
μ_1^*	10.5095	10.466	10.43	10.39	10.37
μ_2^*		15.591	15.57	15.54	15.53
μ_3^*			12.69	12.67	12.66
μ_4^*				9.66	9.65
μ_5^*					21.04
Expected profit	93.061531	71.411986	58.144561	41.752236	37. 218684

4.3. Four-stage network system

Consider a four-stage production system and the following parameters: SP1=120, SP2=125, SP3=115, SP4=140, PC1=30, PC2=25, PC3=20, PC4=15, RC1=10, RC2=17, RC3=20, RC4=23, SC1=15, SC2=12, SC3=9, SC4=6, σ = 1.0, α =0.20, β = 0.30, γ =0.50, θ_1 =0.40, θ_2 =0.60, L1=8, L2=13,L3=18, L4=23, U1=12, U2=17, U3=22, and U4=27. Using exhaustive search, the expected profit is maximized at μ_1^* = 10.6, μ_2^* = 15.7, μ_3^* = 20.2, μ_4^* = 25.4 with an expected profit of 57.7477.

4.4. Four-stage network system with variable distribution factors

Consider a four-stage production system and the following parameters: SP1=120, SP2=125, SP3=115, SP4=140, PC1=30, PC2=25, PC3=20, PC4=15, RC1=10, RC2=17,

RC3=20, RC4=23, SC1=15, SC2=12, SC3=9, SC4=6, σ = 1.0, α =0.30, β = 0.70, L1=8, L2=13,L3=18, L4=23, U1=12, U2=17, U3=22, and U4=27. The manufacturer is not producing P1 and P3, hence, γ =0, θ_1 =0, θ_2 =1.

Using exhaustive search, the expected profit is maximized at μ_1^* = 10.4, μ_2^* = 15.8, μ_3^* = 20.4, μ_4^* = 25.4 with an expected profit of 50.19529 using the initial distribution factors: α =0.30, and β = 0.70.

Then, when we use exhaustive search to solve for distribution factors by using the previously obtained process means, the expected profit is maximized at α =0, and β = 1 with an expected profit of 54.11505 (see Table 3). Using these distribution factors values, we use exhaustive search to maximize the expected profit at μ_1^* = 10.4, μ_2^* = 17, μ_3^* = 19.6, μ_4^* = 25.4 with an expected profit of 55.27775. This approach could lead to hitting a local optimum point; however, it is closer to optimum than none.

Table 3: Optimum process distribution factors, means and expected profit for a four-stage network production system with variable distribution factors

α →	0.001	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.999
β ←	**0.999**	**0.9**	**0.8**	**0.7**	**0.6**	**0.5**	**0.4**	**0.3**	**0.2**	**0.1**	0.001
EPR ←	**55.3482**	**53.4205**	**51.8369**	**50.3833**	**48.9998**	**47.6482**	**46.3413**	**45.0504**	**43.7808**	**42.5331**	41.312
μ_1 —	**10.5**	**10.5**	**10.5**	**10.5**	**10.5**	**10.5**	**10.5**	**10.5**	**10.5**	**10.5**	10.5
μ_2 ←	**17**	**16**	**15.8**	**15.8**	**15.7**	**15.6**	**15.6**	**15.6**	**15.6**	**15.5**	15.5
μ_3 →	**19.7**	**20**	**20.1**	**20.2**	**20.3**	**20.3**	**20.4**	**20.4**	**20.4**	**20.5**	20.5
μ_4 —	25.5	25.5	25.5	25.5	25.5	25.5	25.5	25.5	25.5	25.5	25.5

4.5. Four-stage, multi-input, multi-output network system

Consider a four-stage, multi-input, multi-output production system, as shown in Fig. 19 below. Product 1, P1, is supplied with raw material source 1, R1, and processed in stages 1, 2 and 4, consecutively. Product 2, P2, is supplied with raw material source 2, R2, and processed in stages 2, 3 and 4, consecutively. Product 3, P3, is supplied with raw material source 3, R3, and processed in stages 3 and 4, consecutively. Product 4, P4, is supplied with raw material source 2, R2, and processed in stages 2 and 4, consecutively. Production costs in this case are per unit of raw material, instead of per item as in the single-input, single-output production lines, since there are multiple sources for raw material of different prices. The notation used for distribution factors is modified to consider the source material such that, $f_{k/h,i,j}$ is the distribution factor f_k when raw material source is Rh, Ri, or Rj.

Consider the following parameters: SPu1(R1:1-2— 4)=120, SPu2(R2:2-3-4)=125, SPu3(R3:3-4)=115, SPu4(R2:2-4)=110, PCu1=30, PCu2=25, PCu3=20, PCu4=15, RCu1=10, RCu2=17, RCu3=20, RCu4=23, SCu1=15, SCu2=12, SCu3=9, SCu4=6, σ = 1.0, α =1, β = 0, γ =0, ζ =0, $\theta_{1/1}$ =1, $\theta_{1/2}$ =0.2, $\theta_{2/2}$ =0.8, $\theta_{3/1,2}$ =0, $\delta_{1/2}$ =1, $\delta_{1/3}$ =1, $\delta_{2/1,2,3}$ =0, L1=8, L2=13,L3=18, L4=23, U1=12, U2=17, U3=22, and U4=27. Using exhaustive search, the expected profit is maximized at μ_1^* = 10.4 , μ_2^* = 15.4, μ_3^* = 20.6, μ_4^* = 25.4 with an expected profit of 17874.

$$
\begin{aligned}
E[PR] = & \left[SPu_1 R_1 \frac{p_{12}}{(1-p_{11})} \frac{p_{24}}{(1-p_{22})} \frac{p_{45}}{(1-p_{44})} + SPu_2 R_2 \frac{p_{23}}{(1-p_{22})} \frac{p_{34}}{(1-p_{33})} \frac{p_{45}}{(1-p_{44})} \right] \\
& + \left[SPu_3 R_3 \frac{p_{34}}{(1-p_{33})} \frac{p_{45}}{(1-p_{44})} + SPu_4 R_2 \frac{p_{24}}{(1-p_{22})} \frac{p_{45}}{(1-p_{44})} \right] \\
& - \left[PCu_1 R_1 + PCu_2 \left(R_1 \frac{p_{12}}{(1-p_{11})} + R_2 \right) + PCu_3 \left(R_2 \frac{p_{23}}{(1-p_{22})} + R_3 \right) \right] \\
& - \left[PCu_4 \left(R_1 \frac{p_{12}}{(1-p_{11})} \frac{p_{24}}{(1-p_{22})} + R_2 \frac{p_{23}}{(1-p_{22})} \frac{p_{34}}{(1-p_{33})} + R_2 \frac{p_{24}}{(1-p_{22})} + R_3 \frac{p_{34}}{(1-p_{33})} \right) \right] \\
& - \left[SCu_1 R_1 \frac{p_{16}}{(1-p_{11})} + (SCu_1 + SCu_2) R_1 \frac{p_{12}}{(1-p_{11})} \frac{p_{26}}{(1-p_{22})} + (SCu_1 + SCu_2 + SCu_4) R_1 \frac{p_{12}}{(1-p_{11})} \frac{p_{24}}{(1-p_{22})} \frac{p_{46}}{(1-p_{44})} \right] \\
& - \left[SCu_2 R_2 \frac{p_{26}}{(1-p_{22})} + (SCu_2 + SCu_3) R_2 \frac{p_{23}}{(1-p_{22})} \frac{p_{36}}{(1-p_{33})} + (SCu_2 + SCu_3 + SCu_4) R_2 \frac{p_{23}}{(1-p_{22})} \frac{p_{34}}{(1-p_{33})} \frac{p_{46}}{(1-p_{44})} \right] \\
& - \left[SCu_3 R_3 \frac{p_{36}}{(1-p_{33})} + \left(SCu_3 + SCu_4 \right) R_3 \frac{p_{34}}{(1-p_{33})} \frac{p_{46}}{(1-p_{44})} + \left(SCu_2 + SCu_4 \right) R_2 \frac{p_{24}}{(1-p_{22})} \frac{p_{46}}{(1-p_{44})} \right] \\
& - \left[RCu_1 R_1 \frac{p_{11}}{(1-p_{11})} + RCu_2 R_1 \frac{p_{22}}{(1-p_{22})} \frac{p_{12}}{(1-p_{11})} + RCu_4 R_1 \frac{p_{44}}{(1-p_{44})} \frac{p_{12}}{(1-p_{11})} \frac{p_{24}}{(1-p_{22})} + RCu_2 R_2 \frac{p_{22}}{(1-p_{22})} + RCu_3 R_2 \frac{p_{33}}{(1-p_{33})} \frac{p_{23}}{(1-p_{22})} \right] \\
& - \left[RCu_4 R_2 \frac{p_{44}}{(1-p_{44})} \frac{p_{23}}{(1-p_{22})} \frac{p_{34}}{(1-p_{33})} + RCu_4 R_2 \frac{p_{44}}{(1-p_{44})} \frac{p_{24}}{(1-p_{22})} + RCu_3 R_3 \frac{p_{33}}{(1-p_{33})} + RCu_4 R_3 \frac{p_{44}}{(1-p_{44})} \frac{p_{34}}{(1-p_{33})} \right] \text{...(22)}
\end{aligned}
$$

The equation can then be rewritten in terms of cumulative normal distribution as follows:

$$
\begin{aligned}
E[PR] = & \left[SPu_1 R_1 \alpha \theta_{1/1} \left(1 - \frac{\Phi(L_1)}{\Phi(U_1)} \right) \left(1 - \frac{\Phi(L_2)}{\Phi(U_2)} \right) \left(1 - \frac{\Phi(L_4)}{\Phi(U_4)} \right) + SPu_2 R_2 \theta_{2/2} \delta_{1/2} \left(1 - \frac{\Phi(L_2)}{\Phi(U_2)} \right) \left(1 - \frac{\Phi(L_3)}{\Phi(U_3)} \right) \left(1 - \frac{\Phi(L_4)}{\Phi(U_4)} \right) \right] \\
& + \left[SPu_3 R_3 \delta_{1/3} \left(1 - \frac{\Phi(L_3)}{\Phi(U_3)} \right) \left(1 - \frac{\Phi(L_4)}{\Phi(U_4)} \right) + SPu_4 R_2 \theta_{1/2} \left(1 - \frac{\Phi(L_2)}{\Phi(U_2)} \right) \left(1 - \frac{\Phi(L_4)}{\Phi(U_4)} \right) \right] \\
& - \left[PCu_1 R_1 + PCu_2 \left(R_1 \alpha \left(1 - \frac{\Phi(L_1)}{\Phi(U_1)} \right) + R_2 \right) + PCu_3 \left(R_2 \theta_{2/2} \left(1 - \frac{\Phi(L_2)}{\Phi(U_2)} \right) + R_3 \right) \right] \\
& - \left[PCu_4 \left(R_1 \alpha \theta_{1/1} \left(1 - \frac{\Phi(L_1)}{\Phi(U_1)} \right) \left(1 - \frac{\Phi(L_2)}{\Phi(U_2)} \right) + R_2 \theta_{2/2} \delta_{1/2} \left(1 - \frac{\Phi(L_2)}{\Phi(U_2)} \right) \left(1 - \frac{\Phi(L_3)}{\Phi(U_3)} \right) + R_2 \theta_{1/2} \left(1 - \frac{\Phi(L_2)}{\Phi(U_2)} \right) + R_3 \delta_{1/3} \left(1 - \frac{\Phi(L_3)}{\Phi(U_3)} \right) \right) \right] \\
& - \left[SCu_1 R_1 \frac{\Phi(L_1)}{\Phi(U_1)} + (SCu_1 + SCu_2) R_1 \alpha \left(1 - \frac{\Phi(L_1)}{\Phi(U_1)} \right) \frac{\Phi(L_2)}{\Phi(U_2)} + (SCu_1 + SCu_2 + SCu_4) R_1 \alpha \theta_{1/1} \left(1 - \frac{\Phi(L_1)}{\Phi(U_1)} \right) \left(1 - \frac{\Phi(L_2)}{\Phi(U_2)} \right) \frac{\Phi(L_4)}{\Phi(U_4)} \right] \\
& - \left[SCu_2 R_2 \frac{\Phi(L_2)}{\Phi(U_2)} + (SCu_2 + SCu_3) R_2 \theta_{2/2} \left(1 - \frac{\Phi(L_2)}{\Phi(U_2)} \right) \frac{\Phi(L_3)}{\Phi(U_3)} + (SCu_2 + SCu_3 + SCu_4) R_2 \theta_{2/2} \delta_{1/2} \left(1 - \frac{\Phi(L_2)}{\Phi(U_2)} \right) \left(1 - \frac{\Phi(L_3)}{\Phi(U_3)} \right) \frac{\Phi(L_4)}{\Phi(U_4)} \right] \\
& - \left[SCu_3 R_3 \frac{\Phi(L_3)}{\Phi(U_3)} + \left(SCu_3 + SCu_4 \right) R_3 \delta_{1/2} \left(1 - \frac{\Phi(L_3)}{\Phi(U_3)} \right) \frac{\Phi(L_4)}{\Phi(U_4)} + \left(SCu_2 + SCu_4 \right) R_2 \theta_{1/2} \left(1 - \frac{\Phi(L_2)}{\Phi(U_2)} \right) \frac{\Phi(L_4)}{\Phi(U_4)} \right] \\
& - \left[RCu_1 R_1 \left(\frac{1}{\Phi(U_1)} - 1 \right) + RCu_2 R_1 \alpha \left(\frac{1}{\Phi(U_2)} - 1 \right) \left(1 - \frac{\Phi(L_1)}{\Phi(U_1)} \right) + RCu_4 R_1 \alpha \theta_{1/1} \left(\frac{1}{\Phi(U_4)} - 1 \right) \left(1 - \frac{\Phi(L_1)}{\Phi(U_1)} \right) \left(1 - \frac{\Phi(L_2)}{\Phi(U_2)} \right) \right] \\
& - \left[RCu_2 R_2 \left(\frac{1}{\Phi(U_2)} - 1 \right) + RCu_3 R_2 \theta_{2/2} \left(\frac{1}{\Phi(U_3)} - 1 \right) \left(1 - \frac{\Phi(L_2)}{\Phi(U_2)} \right) + RCu_4 R_2 \theta_{2/2} \delta_{1/2} \left(\frac{1}{\Phi(U_4)} - 1 \right) \left(1 - \frac{\Phi(L_2)}{\Phi(U_2)} \right) \left(1 - \frac{\Phi(L_3)}{\Phi(U_3)} \right) \right] \\
& - \left[RCu_4 R_2 \theta_{1/2} \left(\frac{1}{\Phi(U_4)} - 1 \right) \left(1 - \frac{\Phi(L_2)}{\Phi(U_2)} \right) + RCu_3 R_3 \left(\frac{1}{\Phi(U_3)} - 1 \right) + RCu_4 R_3 \delta_{1/3} \left(\frac{1}{\Phi(U_4)} - 1 \right) \left(1 - \frac{\Phi(L_3)}{\Phi(U_3)} \right) \right] \text{...(23)}
\end{aligned}
$$

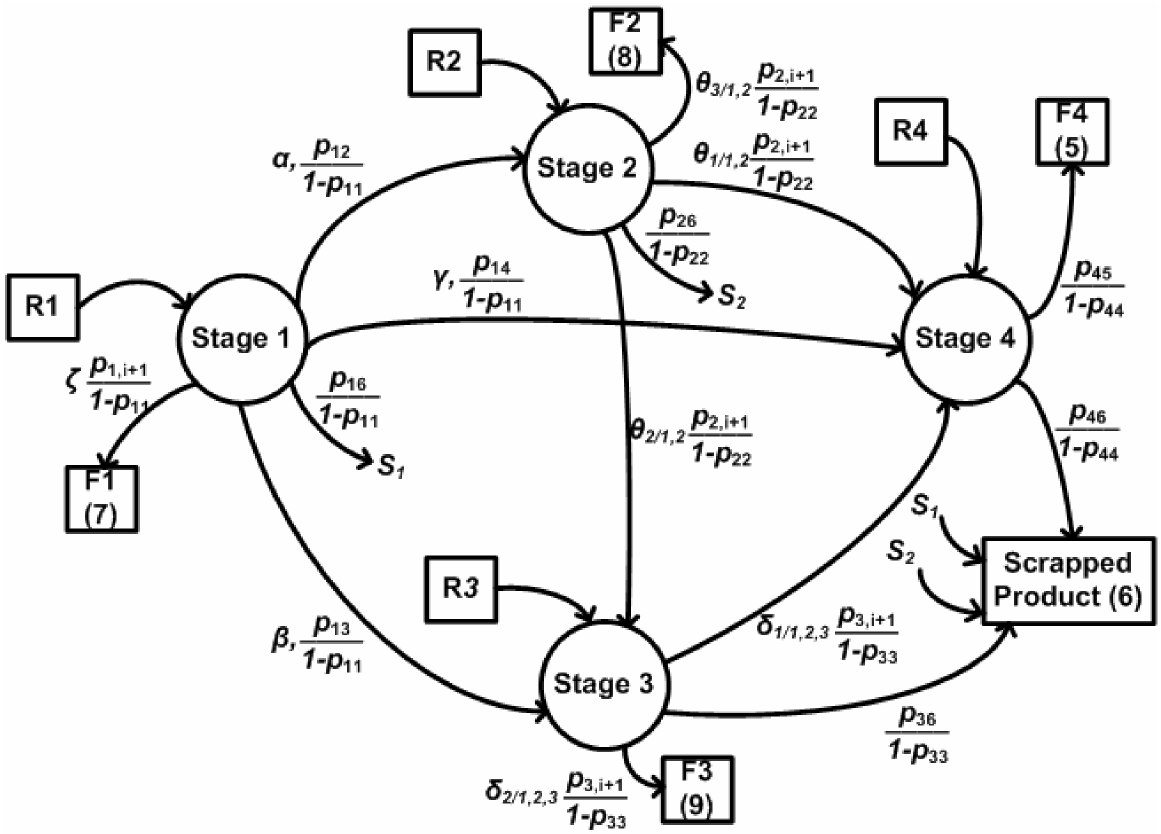

Fig. 20: Primary network for four-stage, multi-input, multi-output network production system.

CHAPTER FIVE

DISCUSSION

It is very beneficial to perform sensitivity analysis of the proposed model parameters to illustrate the possible impact of estimated parameters on the optimal process mean and the optimal expected profit. The rework and scrap cost were varied in the two-stage serial system and four-stage network system and their effects are shown in the following sections.

Tables 1 show the behavior of the optimum process mean and the optimum expected profit with the variation of the scrap and rework costs. Notice that in all cases the optimum process mean and expected profit is sensitive to changes in the rework and scrap cost values.

5.1. Sensitivity Analysis for Two-stage serial system

Table 1 shows the behaviors of the optimum process mean and the optimum expected profit with the variation of the scrap and rework costs for a two-stage serial production system. For cases 1–6, as scrap cost for stage 1 increases, the optimum means for both stages increase slightly. For cases 7–11, as scrap cost for stage 2 increases, the optimum mean for stage 1 remains relatively constant and that of stage 2 increases slightly. For cases 12–18, as rework cost for stage 1 increases, the optimum means for stage 1 decreases and that of stage 2 remains constant. For cases 19–23, as rework cost for stage 2 increases, the optimum mean for stage 1 remains relatively constant and that of stage 2

decreases. It is observed that the optimum expected profit decreases as scrap and rework costs increase for any of the stages.

Table 4: Sensitivity analysis for a two-stage serial production system

Cost parameter	Case #	Parameter value	Optimum process mean 1- exhaustive search	Optimum process mean 2- exhaustive search	Optimum expected profit- exhaustive search
SC1		7 11 15 19 23 27	10.55 10.56 10.57 10.58 10.59 10.60	15.50 15.50 15.51 15.52 15.52 15.53	71.4548 71.4057 71.3575 71.3101 71.2638 71.2182
SC2		4 8 12 16 20	10.57 10.57 10.57 10.57 10.57	15.50 15.50 15.51 15.52 15.52	71.41 71.3835 71.3575 71.3318 71.3067
RC1		2 6 10 14 15 18 19	10.95 10.69 10.57 10.49 10.48 10.43 10.42	15.51 15.51 15.51 15.51 15.51 15.51 15.51	72.2509 71.7293 71.3575 71.0526 70.9833 70.7885 70.7273
RC2		9 13 17 21 25	10.57 10.57 10.57 10.57 10.57	15.66 15.57 15.51 15.46 15.42	72.0259 71.6665 71.3575 71.0825 70.8328

5.2. Conclusion

In this study, the optimum process target (mean) levels for multi-stage network production system have been determined numerically using a network approach. The

study starts by developing a general model for the expected profit per item by taking into account processing, scrap, and rework costs. The general model for the expected profit for an *n*-stage serial production system was then presented. Further more, the model was developed for *n*-stage network production system and multi-input, multi-output network production system with an aspect of production planning. The effect of process standard deviation was also discussed. In addition, some operational aspects of running the production systems were discussed. By varying the cost parameters, such as scrap cost and rework cost, the sensitivity analysis showed the behavior of the optimum process target under different conditions.

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Master of Science - Systems Engineering

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Courses: Advanced Production System & Inventory Control; Stochastic Processes; Linear Programming; Modeling & System Identification; Distributed Comp. Control; Linear Control System; Computer Architecture; Computer Networks.

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University of Illinois at Urbana-Champaign, UIUC Aug, 2001 ~ May, 2002

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Research Assistant

Duties

- Conduct scientific research in Quality Control, Process Targeting, Production Planning, Inventory Control, Computer Architecture and Computer Networks.
- Assist in teaching courses
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McKinsey & Company 1st July ~ 15th Sep, 2008

Management Consulting

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Business Analyst - Summer Program

Duties

- Conducting business analysis in client engagement team setting.
- Working with subject experts to develop perspectives and insights for the client.
- Undertook Functional Knowledge Training: Mini - Basic Consulting Readiness, Corporate Finance, Organization, Strategy, Operations

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Duties

- Teach Computer and Electrical Eng. Labs: Digital Logic, Microprocessor and Electronics Circuits
- Train Cisco Academy Computer Networks courses: CCNA & Wireless LAN - Scored 100% in CCNA

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Quality and Testing Engineer

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- Testing electrical appliances for compliance with international safety and performance standards: IEC, ISO

e-Tech Systems April ~ July, 2004

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- Design, architect solutions
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- Conduct scientific research and experimentation in Electrical Eng: Inverter-fed induction machines, motors performance testing and motor tests

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English	**Excellent**
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