KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

Aerospace Engineering Department

AE 499: Fundamentals of Helicopters

TERM PROJECT

Second Semester 1425 H (032)

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Introduction

This project is about studying the performance of the Bell Helicopter model 407. The 407 is Sleek Fast Impressive it is the sports car in the air. Exceptional hot and high hover capability, this aircraft delivers the best speed, payload and range in its class. Seven Seats Maximum Cruise Speed 140 kn, 246 km/h.

The study includes finding out the hover ceiling, the maximum forward speed, the velocities of maximum endurance and range, the maximum endurance and range, maximum rate of climb, minimum descent rate, minimum descent angle, etc. also plotting the flapping response with the azimuth angle for different forward flight speeds.

Bell Helicopter model 407 Specifications

Total gross weight = 5250 lb.
Cruising speed is 120 knot = 203 ft/s

Fuel capacity = 127.8 US Gallon = 17.0844 ft^3
Fuel weight, $W_f = \rho V = 870$ lb (assume fuel is Kerosene ρ = 50.9 lb/ft^3)
Assume moment of inertia of helicopter is, I = 4000 lb-ft^2

Maximum engine power output capacity = 674 hp. (P_{av})
Engine power at forward level flight =630 hp. (P_{level})
Distance between axes of rotation of the main and tail rotors = 23.05 ft.

Main rotor diameter D_m = 35 ft (R_m = 17.5 ft).
Main rotor chord C_m = 10.75 in = 0.8958 ft.
Number of blades N_m = 4.
Twist, θ_{tw}= 13°.

$RPM_m = 413 \rightarrow \Omega_m = \left(\frac{413 \text{ Rev}}{\text{Min}}\right)\left(\frac{2\pi}{1\text{Rev}}\right)\left(\frac{1 \text{ Min}}{60 \text{ Sec}}\right) = 43.25$.

Tip speed, V_{Tm} = 757 ft/s.

Main rotor area, $A_m = \pi R_m^2 = \pi(17.5)^2 = 962.113 \text{ ft}^2$.

Main rotor solidity, $\sigma_m = \frac{N_m C_m}{\pi R_m} = \frac{4 \times 0.8958}{\pi \times 17.5} = 0.065$

Tail rotor diameter D_t = 5.4 ft (R_t = 2.7 ft).
Tail rotor chord C_t = 6.4 in = 0.5083 ft.
Number of blades N_t = 4.
Twist, θ_{tw} = 13°.

$RPM_t = 2500 \rightarrow \Omega_t = \left(\frac{2500 \text{ Rev}}{\text{Min}}\right)\left(\frac{2\pi}{1\text{Rev}}\right)\left(\frac{1 \text{ Min}}{60 \text{ Sec}}\right) = 261.8$.

Tip speed, V_{Tt} = 709 ft/s.

Tail rotor solidity, $\sigma_t = \frac{N_t C_t}{\pi R_t} = \frac{4 \times 0.5083}{\pi \times 2.7} = 0.24$.

Drag coefficient, $C_{do} = 0.008$.

Lift curve slope, a = 5.7.

Empirical correction factors, k = 1.15 and K = 4.7

Hover ceiling

Induced velocity at each altitude, $v_{im} = \sqrt{\dfrac{T_m}{2\rho A_m}}$, where T_m = W = 5250 lb.

Noting that the density changes with height (altitude)

$$\rho = \rho_{sea-level}\left[1 - \frac{0.00198h}{288.16}\right]^{4.2553}, \text{ and } \rho_{sea-level} = 0.00238 \text{ slug/ft}^3$$

Calculating the main rotor power at each altitude

$$P_m = kT_m v_{im} + \rho(\Omega_m R_m)^3 A_m\left[\frac{\sigma_m C_{do}}{8}\right]$$

Finding out the main rotor torque

$$Q_m = \frac{P_m}{\Omega_m}$$

Now, finding the tail rotor thrust

$$T_t = \frac{Q_m}{d}$$

Calculating the tail rotor power in the same way

$$P_t = kT_t v_{it} + \rho(\Omega_t R_t)^3 A_t\left[\frac{\sigma_t C_{do}}{8}\right]$$

Where, $v_{it} = \sqrt{\dfrac{T_t}{2\rho A_t}}$

The total power is addition of these two powers

$$P_{tot} = P_m + P_t$$

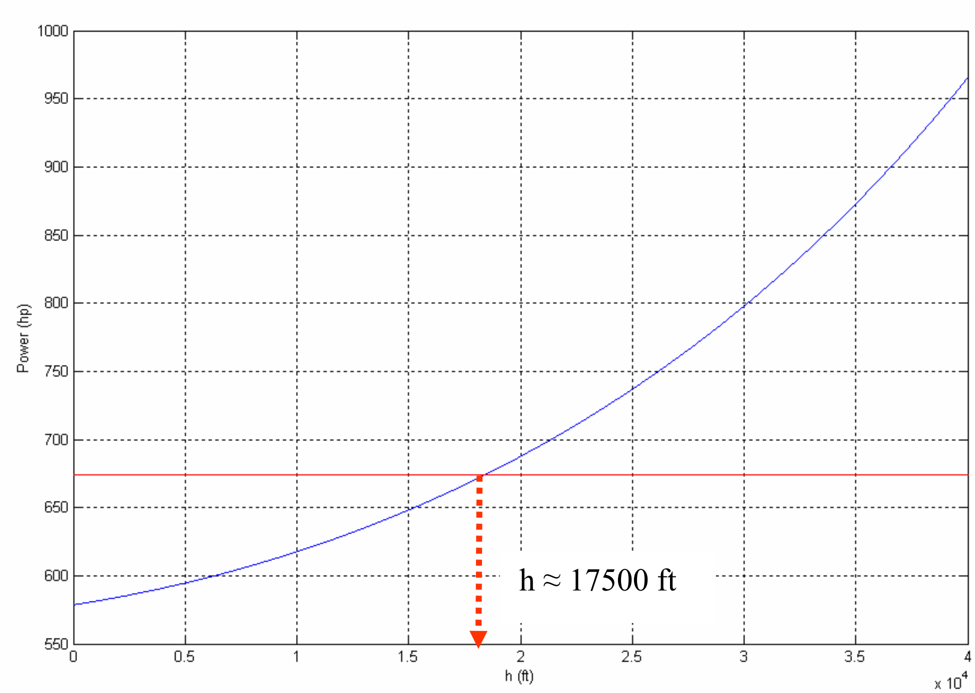
$$\text{Transmission Loss} = 100 \times \left(1 - \frac{630}{701}\right) = 10\ \%$$

To account for the transmission loss, the total power will be

$$P_{tot} = 1.1 \times (P_m + P_t)$$

So, we have to repeat this procedure for different altitude, which is done by Matlab (code attached in Appendix), and from the plot of the altitude (h) versus the power we can find the hover ceiling by finding the altitude which corresponds to the intersection of the power required and the power available (P_{av}). The hover ceiling will be at h = 17,500 ft.

Plot of the altitude (h) versus the power:



Maximum forward speed at sea level

At sea level, $\rho_{sea-level} = 0.00238$ slug/ft^3. Before we proceed we have to find the equivalent flat – plate area (f) from the Parasite Power equation, i.e.

$$C_{P_{parasite}} = \frac{1}{2}\frac{f}{A_m}\mu^3$$

Or, $f = \dfrac{2A_m C_{P_{parasite}}}{\mu^3}$

Let's assume that the helicopter is operating level forward flight at 200 ft/sec with supplied shaft power of 630 hp (P_{level}), then, the parasite power can be calculated by:

$$C_{P_{parasite}} = C_{P_{level}} - C_{P_{induced}} - C_{P_{profile}}, \text{ or}$$

Finding the level power coefficient:

$$C_{P_{level}} = \frac{P_{level}}{\rho A_m V_{Tm}^3} = \frac{630 * 550}{(0.00238)(962.113)(757)^3} = 3.48829\times10^{-4}$$

Finding the induced power coefficient:

$$C_T = \frac{W}{\rho A_m V_{Tm}^2} = \frac{5250}{(0.00238)(962.113)(757)^2} = 0.004$$

$$\mu = \frac{V_\infty}{V_{Tm}} = \frac{200}{757} = 0.264, \text{ and since } \mu > 0.2$$

$$C_{P_{induced}} = k\frac{C_T^2}{2\mu} = 1.15\frac{(0.004)^2}{2(0.264)} = 3.48485\times10^{-5}$$

Finding the profile power coefficient:

$$C_{P_{profile}} = \frac{\sigma_m C_{do}}{8}\left(1 + K\mu^2\right)$$

$$= \frac{(0.065)(0.008)}{8}\left(1 + (4.7)(0.264)^2\right) = 8.629213\times10^{-5}$$

The parasite power coefficient is:

$$C_{P_{parasite}} = 3.48829\times10^{-4} - 3.48485\times10^{-5} - 8.629213\times10^{-5}$$

$$= 2.276884\times10^{-4}$$

$$f = \frac{2A_m C_{P_{parasite}}}{\mu^3} = \frac{2(962.113)(2.276884 \times 10^{-4})}{(0.264)^3} = 23.81 \text{ ft}^2$$

Now let's proceed to find the maximum forward speed, the total power is found from:

$$C_P = k\lambda_i C_T + \frac{1}{2}\frac{f}{A_m}\mu^3 + \frac{\sigma_m C_{d_o}}{8}(1 + K\mu^2)$$

$$P = \rho A_m V_{Tm}^3 C_P$$

$$P = \rho A_m V_{Tm}^3 \left[k\lambda_i C_T + \frac{1}{2}\frac{f}{A_m}\mu^3 + \frac{\sigma_m C_{d_o}}{8}(1 + K\mu^2) \right]$$

Where, $\mu = \frac{V_\infty}{V_{Tm}}$

And, λ_i is found from $C_T = 2\lambda_i \sqrt{\mu^2 + (\mu + \lambda_i)^2}$

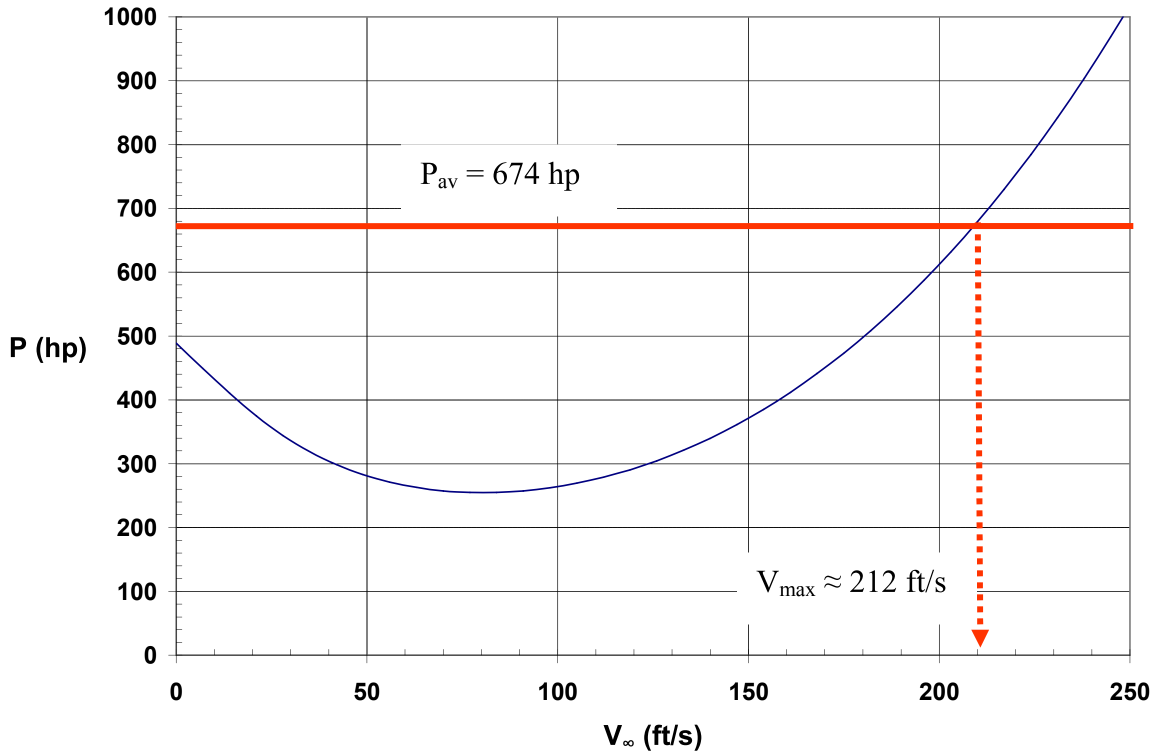
Or, $\lambda_i^4 + 2\mu\lambda_i^3 + 2\mu^2\lambda_i^2 - \frac{C_T^2}{4} = 0$

Now plotting the forward speed versus the power (see table for plot data), and from the intersection between the available power and the power curve, the maximum forward speed can be found which is, V_{max} = 212 ft/s

Table data for plot

V_∞	μ	λ_i	C_p	P	P (hp)
0	0.0000	0.044721	0.000271	268911	489
7	0.0092	0.039952	0.000249	247153	449
14	0.0185	0.035211	0.000227	225637	410
21	0.0277	0.030851	0.000207	206029	375
28	0.0370	0.027046	0.000190	189183	344
35	0.0462	0.023826	0.000176	175297	319
42	0.0555	0.021141	0.000165	164200	299
49	0.0647	0.018910	0.000157	155578	283
56	0.0740	0.017051	0.000150	149113	271
63	0.0832	0.015490	0.000146	144532	263
70	0.0925	0.014170	0.000143	141624	257
77	0.1017	0.013042	0.000141	140234	255
84	0.1110	0.012071	0.000141	140253	255
91	0.1202	0.011228	0.000143	141608	257
98	0.1295	0.010491	0.000145	144255	262
105	0.1387	0.009841	0.000149	148170	269
112	0.1480	0.009264	0.000154	153346	279
119	0.1572	0.008750	0.000161	159791	291
126	0.1664	0.008288	0.000169	167520	305
133	0.1757	0.007871	0.000178	176558	321
140	0.1849	0.007494	0.000188	186934	340
147	0.1942	0.007150	0.000200	198682	361
154	0.2034	0.006836	0.000213	211843	385
161	0.2127	0.006548	0.000228	226457	412
168	0.2219	0.006283	0.000244	242569	441
175	0.2312	0.006038	0.000262	260225	473
182	0.2404	0.005812	0.000281	279474	508
189	0.2497	0.005601	0.000302	300365	546
196	0.2589	0.005405	0.000325	322949	587
203	0.2682	0.005223	0.000350	347277	631
210	0.2774	0.005052	0.000376	373402	679
217	0.2867	0.004892	0.000404	401378	730
224	0.2959	0.004741	0.000434	431257	784
231	0.3052	0.004600	0.000466	463095	842
238	0.3144	0.004466	0.000500	496947	904
245	0.3236	0.004340	0.000536	532867	969
252	0.3329	0.004221	0.000575	570911	1038
259	0.3421	0.004109	0.000615	611135	1111

Plot of the forward speed versus the power



Speed For Maximum Endurance

Speed for maximum endurance corresponds to minimum power, that is:

$$
\begin{aligned}
V_{mp} &= \sqrt{\frac{W}{2\rho A_m}}\left(\frac{4k}{3f/A_m}\right)^{1/4} \\
&= \sqrt{\frac{5250}{2\times 0.00238\times 962.113}}\left(\frac{4\times 1.15}{3\times 23.81/962.113}\right)^{1/4} \\
&= 95 \text{ ft/s}
\end{aligned}
$$

Speed For Maximum Range

Speed for maximum range corresponds to minimum P/V, that is:

$$
\begin{aligned}
V_{y} &= \sqrt{\frac{W}{2\rho A_m}}\left(\frac{4k}{f/A_m}\right)^{1/4} \\
&= \sqrt{\frac{5250}{2\times 0.00238\times 962.113}}\left(\frac{4\times 1.15}{23.81/962.113}\right)^{1/4} \\
&= 125 \text{ ft/s}
\end{aligned}
$$

Maximum Endurance

Let's assume that $SFC = 0.40 \frac{lb}{hp \times s} = 2.064 \times 10^{-7} \frac{1}{ft}$

And the power at the cruise speed V_∞ = 203 ft/s is P = 588 hp (already calculated)

$$E = W_f \left[\frac{1}{P \times SFC} \right]_{W_{GTOW}} - \frac{W_f}{2}$$

$$= 870 \left[\frac{1}{(588)(550) \times 2.064 \times 10^{-7}} \right] - \frac{870}{2}$$

$$= 12599 \text{ s}$$

$$= 3.5 \text{ hr}$$

Maximum Range

Here $V_\infty = V_y$ = 125 ft/s

$$R = W_f \left[\frac{V_\infty}{P \times SFC} \right]_{W_{GTOW}} - \frac{W_f}{2}$$

$$= 870 \left[\frac{125}{(305)(550) \times 2.064 \times 10^{-7}} \right] - \frac{870}{2}$$

$$= 3140486.2 \text{ ft}$$

$$= 595 \text{ mile}$$

Maximum rate of climb

$$\max(V_c) = \frac{\max(P_{av} - P_{level-flight})}{T} = \frac{P_{av} - \min(P_{level})}{T}$$

As found previously the minimum power corresponds to V_{mp} = 95 ft/s, so find this power as follow:

$$P_{\min} = \rho A_m V_{Tm}^3 \left[k\lambda_i C_T + \frac{1}{2}\frac{f}{A_m}\mu^3 + \frac{\sigma_m C_{d_o}}{8}\left(1 + K\mu^2\right) \right]$$

Where,

$$\mu = \frac{V_{mp}}{V_{Tm}} = \frac{95}{757} = 0.1255 \;\; < 0.2$$

To find λ_i we use, $C_T = 2\lambda_i\sqrt{\mu^2 + \left(\mu + \lambda_i\right)^2}$

Or, $\lambda_i^4 + 2\mu\lambda_i^3 + 2\mu^2\lambda_i^2 - \frac{C_T^2}{4} = 0$

Using the value of $\mu = 0.1255$ and $C_T = 0.004$ to find out that $\lambda_i = 0.000127$

The minimum power is:

$P_{\min} = 97221.36$ Ibf-ft/s $= 171$ hp

$$\max(V_c) = \frac{P_{av} - \min(P_{level})}{T} = \frac{P_{av} - P_{\min}}{T \approx W} = \frac{\left(674 - 171\right) \times 550}{5250} = 52.7 \text{ ft/s}$$

Minimum descent rate

For minimum descent rate $C_p = 0$,

$$C_p = \frac{kC_w^2}{2\mu} + \frac{1}{2}\frac{f}{A}\mu^3 + \frac{\sigma C_{d_o}}{8}\left(1+K\mu^2\right) + C_w\lambda_c = 0$$

Solving for λ_c

$$\lambda_c = -\frac{kC_W}{2\mu} - \frac{1}{2C_W}\frac{f}{A}\mu^3 - \frac{\sigma C_{d_o}}{8C_W}\left(1+K\mu^2\right)$$

Finding minimum μ by setting $\frac{d\lambda_c}{d\mu} = 0$

$$\frac{d\lambda_c}{d\mu} = \frac{kC_W}{2\mu^2} - \frac{3}{2C_W}\frac{f}{A}\mu^2 - \frac{\sigma C_{d_o}}{4C_W}K\mu = 0$$

$\mu = 0.12155$, and $\lambda_{c,\min} = -0.04186$

To find minimum descent rate

$\lambda_c = \frac{V_D}{V_{Tm}}$, so that, $V_D = \lambda_{c.\min} V_{Tm} = -31.68$ ft/s

Minimum descent angle

To find minimum descent angle, we find minimum μ by setting $\frac{d\left(\lambda_c/\mu\right)}{d\mu} = 0$

$$\frac{\lambda_c}{\mu} = -\frac{kC_W}{2\mu^2} - \frac{1}{2C_W}\frac{f}{A}\mu^2 - \frac{\sigma C_{d_o}}{8C_W}\left(\frac{1}{\mu} + K\mu\right)$$

$$\frac{d\left(\lambda_c/\mu\right)}{d\mu} = \frac{kC_W}{2\mu^3} - \frac{1}{C_W}\frac{f}{A}\mu + \frac{\sigma C_{d_o}}{8C_W}\frac{1}{\mu^2} - \frac{\sigma C_{d_o}K}{8C_W} = 0$$

$\mu = 0.165516$, so that, $\alpha = \left|\frac{\lambda_c}{\mu}\right| = |-0.27952| = 0.27952 \text{ rad} = 16^0$

Flapping response

$$\overset{**}{\beta} + \gamma\left(1 + \frac{4}{3}\mu\sin(\psi)\right)\overset{*}{\beta} + \left[1 + \gamma\mu\cos(\psi)\left(\frac{1}{6} + \frac{\mu}{4}\sin(\psi)\right)\right] \quad \beta = \gamma f_h(\psi)$$

Where, $\gamma = \frac{\rho c a R^4}{I}$

And,

$$f_h(\psi) = \frac{1}{240}[30\theta_0 + 24\theta_{tw} - 40\lambda + 40\theta_{1s}\mu + 30\theta_0\mu^2 + 20\theta_{tw}\mu^2 + 15\theta_{1c}(2+\mu^2)\cos\psi -$$
$$10\mu(4\theta_{1s} + 3\theta_0\mu + 2\theta_{tw}\mu)\cos 2\psi - 15\theta_{1c}\mu^2\cos 3\psi + 30\theta_{1s}\sin\psi + 80\theta_0\mu\sin\psi$$
$$60\theta_{tw}\mu\sin\psi - 60\lambda\mu\sin\psi + 45\theta_{1s}\mu^2\sin\psi + 40\theta_{1c}\mu\sin 2\psi - 15\theta_{1s}\mu^2\sin 3\psi]$$

Proceeding with different forward speeds and the pilot input from the following graph:

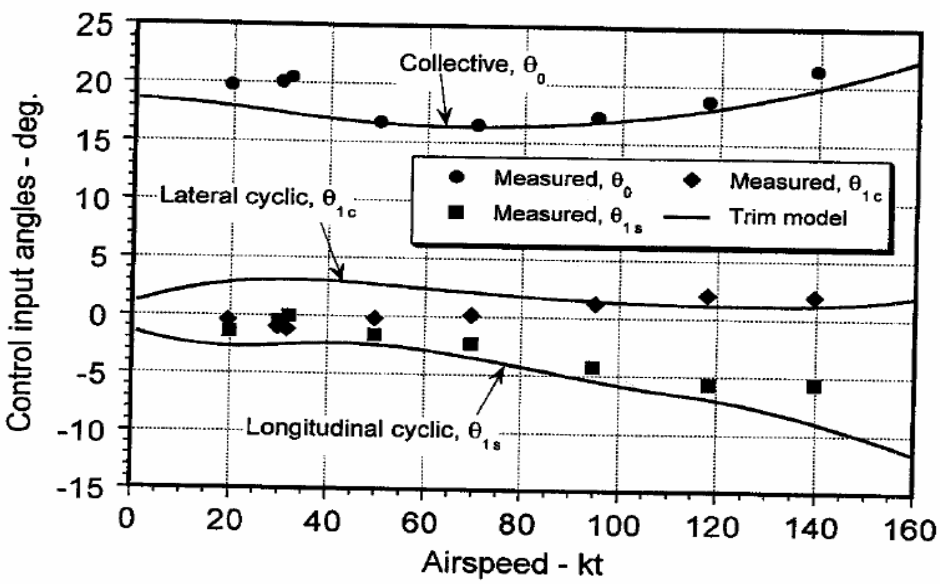
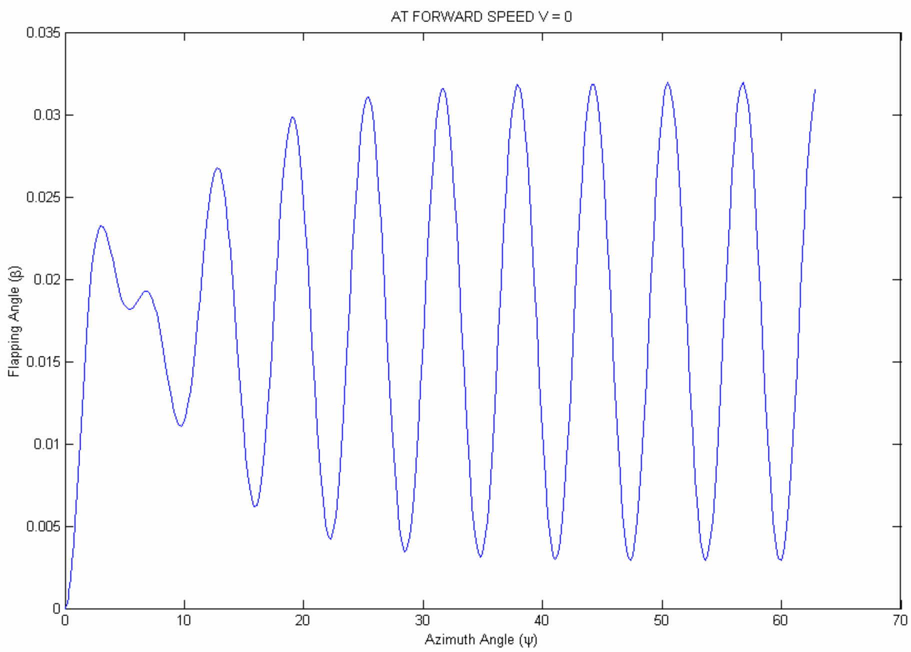


Figure 4.15 Representative variations in collective and cyclic pitch inputs to trim a rotor in forward flight. Propulsive trim calculation. Data source: Ballin (1987).

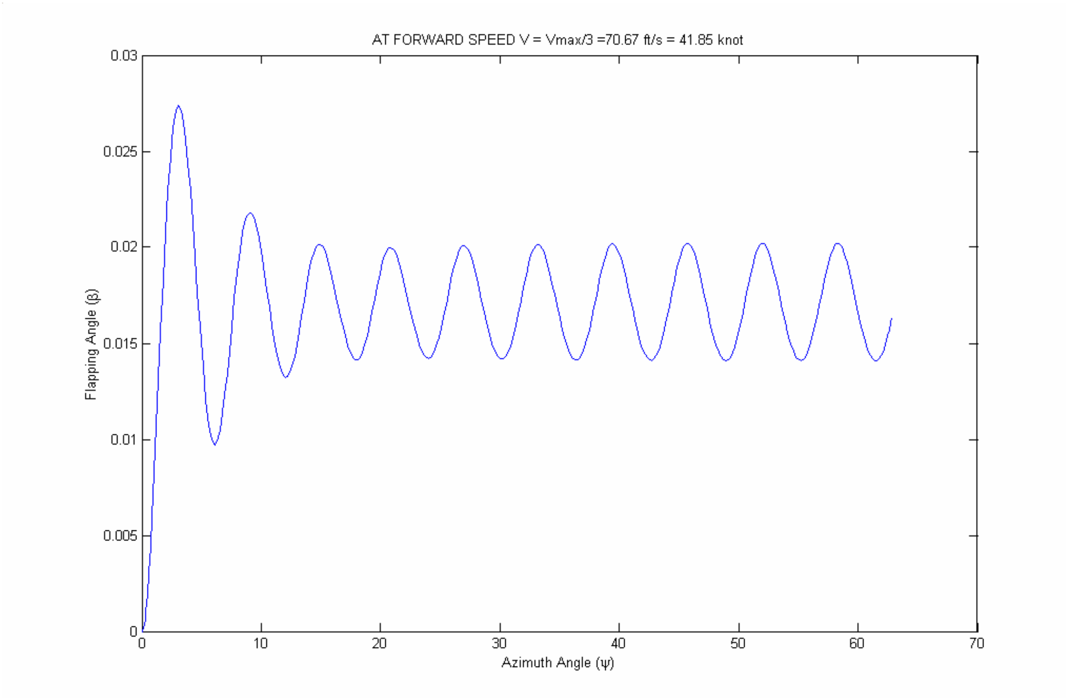
- ***At*** $V_\infty = 0$ ft/s

  $\theta_{twist} = 13^0 \Leftrightarrow \theta_0 = 18.5^0 \Leftrightarrow \theta_{1c} = 1^0 \Leftrightarrow \theta_{1s} = -1.5^0$



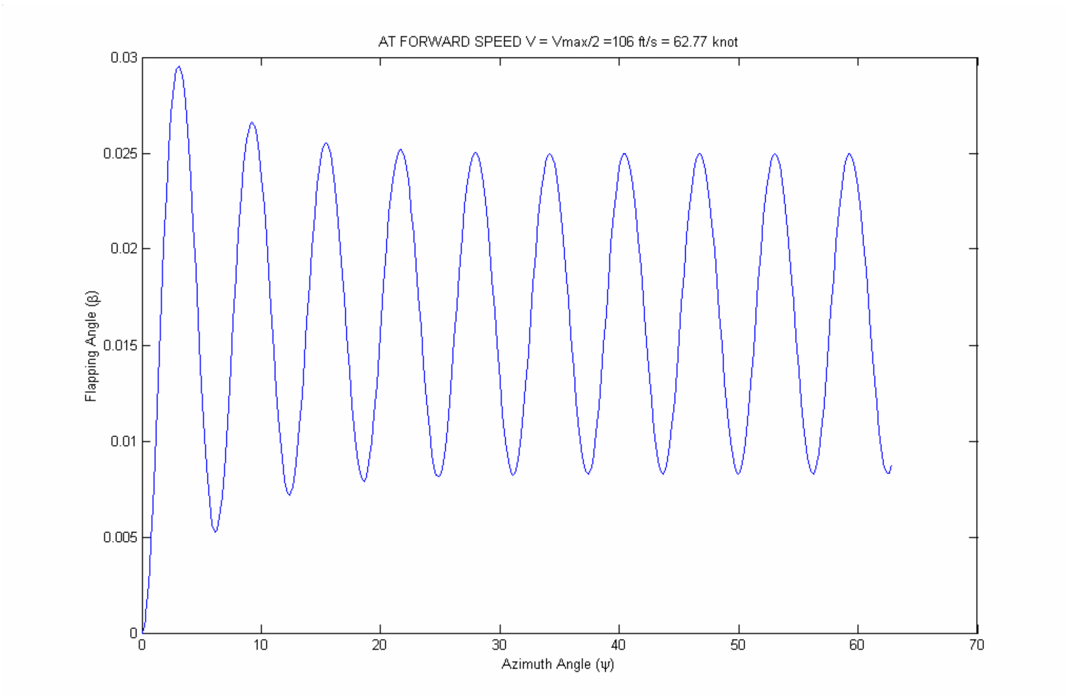
- ***At*** $V_\infty = \dfrac{V_{\max}}{3} = \dfrac{212}{3} = 70.67 \text{ ft/s} = 41.85 \text{ knot}$

  $\theta_{twist} = 13^0 \Leftrightarrow \theta_0 = 17^0 \Leftrightarrow \theta_{1c} = 3^0 \Leftrightarrow \theta_{1s} = -2.5^0$



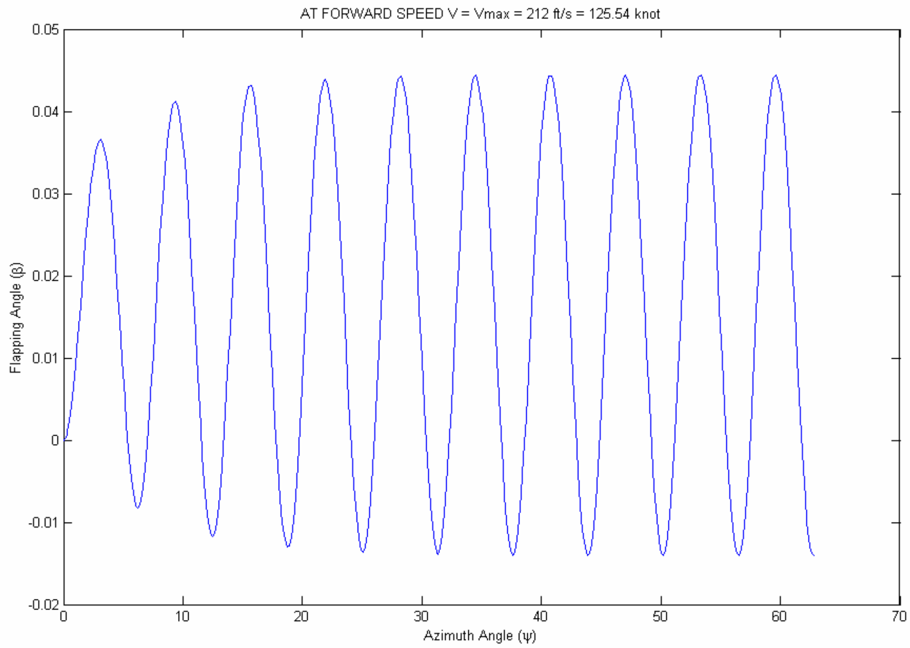
- $\boldsymbol{At}\ V_{\infty} = \dfrac{V_{\max}}{2} = \dfrac{212}{2} = 106 \text{ ft/s} = 62.77 \text{ knot}$

  $\theta_{twist} = 13^{0} \Leftrightarrow \theta_{0} = 16.5^{0} \Leftrightarrow \theta_{1c} = 2.5^{0} \Leftrightarrow \theta_{1s} = -3^{0}$



- $\boldsymbol{At}\ V_{\infty} = V_{\max} = 212 \text{ ft/s} = 125.54 \text{ knot}$

  $\theta_{twist} = 13^{0} \Leftrightarrow \theta_{0} = 18^{0} \Leftrightarrow \theta_{1c} = 1.5^{0} \Leftrightarrow \theta_{1s} = -6.5^{0}$



Comparison and Conclusion

At the end let's compare between the calculated results and the ones given by the Bell 407 helicopter company and find out the error percentage.

	Calculated results	Actual results	% Error
Hover ceiling	17,500 ft	16,050 ft	9 %
Maximum forward speed	212 ft/s	132 knot = 223 ft/s	5 %
Velocity of maximum endurance	95 ft/s	Not given	
Velocity of maximum range	125 ft/s	Not given	
Maximum endurance	3.5 hr	3.7 hr	5.4 %
Maximum range	595 mile	Not given	

Range at V_{∞} = 121 knot = 204 ft/s	460 mile	326 nm = 377 mile	22 %
Maximum rate of climb	52.7 ft/s	Not given	
Minimum descent rate	-31.67 ft/s	Not given	
Minimum descent angle	16^{o}	Not given	

Appendix

MatLab code for hover ceiling

```
clear all;close all;clc;
%data

W=5250; % Weight
Pav=674*550; % Available Power
k=1.15; % Power Correction factor
Cdo=0.008; % drag coefficient
% ***************   Main Rotor Data *********************
Rm=17.5; % Raduis
VT_m=757; % Tip speed
Omegam=VT_m/Rm; %Rotational Speed
Am=(pi*(Rm)^2); % Disk Area
Nm=4; % No of blades
Cm=0.8958;
%sigma=(Nm*Cm)/(pi*Rm);
% **************** Tail Rotor Data **********************
Rt=2.7; % Raduis
VT_t=709; % Tip speed
```

```
Omegat=VT_t/Rt; %Rotational Speed
At=(pi*(Rt)^2); % Disk Area
Nt=4; % No of blades
Ct=0.5083;
%************** Tail-Main Rotors Data ***********************
d=23.05; % distance between the rotors axes

h=0:10:40000; % Altitude range

% Begining of the loop

for i=1:length(h)     % counter

   rho=0.00238*(1-(0.00198*h(i)/288.16))^(4.2553); % Density
   % Main Rotor
   Tm=W; % Main rotor thrust
   Vm=(Tm/(2*rho*Am))^0.5; % induced velocity
   % The main rotor power

Pm(i)=k*Tm*Vm+(rho*(Rm*Omegam)^3*Am*(Nm*Cm/(pi*Rm))*(Cdo/8
));
   % tail rotor
   Q=Pm(i)/Omegam; % torque
   Tt=Q/d; % tail rotor thrust
   Vt=(Tt/(2*rho*At))^0.5; % induced velocity
   % the tail rotor power
   Pt(i)=k*Tt*Vt+(rho*(Rt*Omegat)^3*At*(Nt*Ct/(pi*Rt))*(Cdo/8));
   % T O T A L    P O W E R
   Ptot(i)=1.1*(Pm(i)+Pt(i));
end
% End of the loop

%plotting the results
plot(h,Ptot/550,h,Pav/550,'r');grid;xlabel('h (ft)');ylabel('Power (hp)');
```

MatLab code for flapping response (for $V_\infty = V_{max}$)

```
function dy=flapping(t,y)
dy=zeros(2,1);

w=5250;
rho=0.00238;
c=0.8958;
a=5.7;
R=17.5;
A=pi*R^2;
I=4000;
Vmax=212;
VT=757;

V_inf=Vmax;

jama=(rho*c*a*R^4)/I;

mu=V_inf/VT;

CT=w/(rho*A*VT^2);
```

```
lamda=0.005052;

theta_o=18*(pi/180);
theta_1c=1.5*(pi/180);
theta_1s=-6.5*(pi/180);
theta_tw=13*(pi/180);

a=jama*(1+(4/3)*mu*sin(t));
b=1+jama*mu*cos(t)*((1/6)+(mu/4)*sin(t));

f=(1/240)*(30*theta_o+24*theta_tw-
40*lamda+40*theta_1s*mu+30*theta_o*mu^2+20*theta_tw*mu^2+15*thet
a_1c*(2+mu^2)*cos(t)-
10*mu*(4*theta_1s+3*theta_o*mu+2*theta_tw*mu)*cos(2*t)-
15*theta_1c*mu^2*cos(3*t)+30*theta_1s*sin(t)+80*theta_o*mu*sin(t)+60
*theta_tw*mu*sin(t)-
60*lamda*mu*sin(t)+45*theta_1s*mu^2*sin(t)+40*theta_1c*mu*sin(2*t)-
15*theta_1s*mu^2*sin(3*t));

dy(1)=y(2);
dy(2)=-a*y(2)-b*y(1)+jama*f;


%[t,y]=ode45(@flapping,[0 20*pi],[0 0]);
%plot(t,y(:,1));
%title('AT FORWARD SPEED V = Vmax = 212 ft/s = 125.54 knot');
%xlabel('Azimuth Angle (\psi)');
%ylabel('Flapping Angle (\beta)');
```