A Modified Nodal Formulation Parametric Sensitivities for Power System Analysis

by

Jubran Ali Ahmed Refai

A Thesis Presented to the

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DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

ELECTRICAL ENGINEERING

May, 2000
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UMI
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PARAMETRIC SENSITIVITIES FOR
POWER SYSTEM ANALYSIS

BY

JUBRAN ALI AHMED REFAI

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MAY 2000
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DEDICATION

I dedicated this work to my parents, my wife and my children.

To my father and my mother for raising me in a manner that made it possible for me to achieve this degree. For their prayers, love and constant encouragement.

To my wife for her constant support, patience and understanding throughout the period of these studies.

To my children, Osama, Ali, Asma, Sarah and the new baby Yasser who gave me inspiration to proceed with my graduate studies.

JUBRAN ALI REFAI
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## Abbreviations

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<tr>
<td>MNF</td>
<td>Modified Nodal Formulation</td>
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<tr>
<td>PFC</td>
<td>Power Factor Correction</td>
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<td>NAF</td>
<td>Nodal Admittance Formulation</td>
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<td>THD</td>
<td>Total Harmonics Distortion</td>
</tr>
<tr>
<td>MNFSA</td>
<td>Modified Nodal Formulation Sensitivity Analysis</td>
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<td>FACTS</td>
<td>Flexible AC Transmission System</td>
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<td>OPF</td>
<td>Optimal Power Flow</td>
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<tr>
<td>EMS</td>
<td>Energy Management System</td>
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<td>KVL</td>
<td>Kirchhoff Voltage Law</td>
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<tr>
<td>KCL</td>
<td>Kirchhoff Current Law</td>
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<tr>
<td>VCCS</td>
<td>Voltage Controlled Current Source</td>
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<td>MNA</td>
<td>Modified Nodal Analysis</td>
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<td>PFC’s</td>
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خلاصة البحث

الاسم : جبران علي أحمد رفاعي

عنوان الدراسة: تحليل الحساسية (التأثير) في الأنظمة الكهربائية باستخدام الصيغة العقدية المطورة
التخصص : الهندسة الكهربائية

تاريخ الرسالة: صفر 1421 هجري

إن الهدف الرئيسي من أطرافحة هذا البحث هو تطوير أسلوب جديد لدراسة وتحليل الشبكات الكهربائية من خلال معرفة مدى حساسيتها (تأثيرها) لخواص مكوناتها الأساسية، بالاعتماد على الصيغة العقدية المطورة. أطلق على هذا الأسلوب اسم تحليل الحساسية (التأثير) باستخدام الصيغة العقدية المطورة. لقد تم صياغة هذا الأسلوب في قالب رياضي ومن ثم تم برمجته في الحاسب الآلي ليستعان به كأداة لتحليل ودراسة جودة التصميم المختلفة للشبكات الكهربائية.

يزود هذا البرنامج الدارس بمعايير لتقييم ودراسة الآثار المترتبة عن إضافة أو حذف أو تغيير أي مكونات الشبكة في الظواهر المختلفة لأنظمة الكهربائية. إن هذا الأسلوب يطبق في المساعدة على دراسة جودة الطاقة وتحديد ما في ارتفاع الفولتية الناتج عن الزرنين التوافقي في الشبكات الكهربائية. كما يكشف المشاكل مثل تحديد السبب الجوهري لظاهرة الزرنين والمساعدة على إيجاد الحلول المناسبة. إن الحالات التي تم دراستها قد أثبتت بأن هذا الأسلوب قادر على تحديد مثل هذه المشاكل وعوامل السببية لها وتفاعلها ببعدين الأنظمة السعودية والحيثية كما إن هذا الأسلوب يساعد في تحديد ومقارنة الحلول المختلفة.

درجة الماجستير في الهندسة الكهربائية
جامعة الملك فهد للبترول والمعادن
المملكة العربية السعودية - الظهران

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THESIS ABSTRACT

Name: JUBRAN ALI AHMED REFAI
Title: A MODIFIED NODAL FORMULATION PARAMETRIC SENSITIVITIES FOR POWER SYSTEM ANALYSIS
Major Field: ELECTRICAL ENGINEERING
Date of Degree: May 2000

The primary objective of this thesis research is to develop a Parametric Sensitivity approach based on the Modified Nodal Formulation (MNF) for Power System Analysis. This technique is called the Modified Nodal Formulation Sensitivity Analysis (MNFSA). This thesis developed the mathematical formulations for the MNFSA that was built into a computer program. The developed program offers a new tool to help in analyzing the quality of power system design by studying the effect of adding, removing and changing the characteristics of different power system equipment.

The MNFSA is applied to assist in identifying power quality issues related to harmonic over voltages and resonance in power systems. This application includes identifying the route cause of problems such as resonance phenomena and harmonic amplification and assets in finding solutions to these problems.

The case studies presented verified the MNFSA capabilities in identifying system problems, the parameters that are causing these problems, the interaction between the capacitive and reactive system parameters, and in devising solutions and mitigation methods.

MASTER OF SCIENCE DEGREE
KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
Dhahran, Saudi Arabia
May 2000
CHAPTER I

INTRODUCTION

1.1 General Background

One of the primary objectives of power system analysis is to study the impact of changes in power system parameters on system behavior. For example, an operation engineer may change the value of a system parameter, such as reactive power injection at a bus, in order to improve the voltage profile of certain busses or to avoid voltage collapse of the system. For the same reason, the engineer may also change the system topology by switching ON/OFF a component such as a capacitor bank or a transmission line. In every case it is important to know the effect of these changes on the system state of interest.

Studying the changes in system response is also important in the design process of power systems. In this case the interest is typically to examine design alternatives. If the design is to be optimized, the desired improvement to the system response should be achieved with minim investment.

Therefore, studying the effect of changes in power system parameters on the relevant aspects of the system response forms an important tool to both power system
designers and operation engineers [1-3]. This effect can be referred to, in general term, as power system parametric sensitivities.

Formulating the power system parametric sensitivities in an efficient algorithm will provide power systems engineers with a valuable tool to study certain power system phenomena and identifies effective means of dealing with these phenomena. In this research, a power system parametric sensitivity formulation based on the well-known Modified Nodal Formulation (MNF) [4,5] is proposed. The proposed Modified Nodal Formulation Sensitivity Analysis (MNFSA) is developed into a computer program and applied to study different power system issues.

A brief description to some of the issues that will be analyzed using the proposed formulation is outlined. The application and advantages of the sensitivity analysis and the MNF are also briefly discussed. The objectives, research work methodology and thesis contribution are detailed.

1.2 Application of Sensitivity Analysis in Engineering Problems

Sensitivity analysis is a mathematical process widely applied in engineering problems, medical, economics and many other applications to analyze models and data. The sensitivity analysis is used to validate parameter estimates obtained in model fitting along with their significance. If the sensitivity function is known, it will be easy to calculate the change in the system behavior from given parameter deviations and,
conversely, to calculate allowable parameter deviations from a given or pre-assigned system behavior. The latter problem is often referred to as the inverse sensitivity problem.

Historically, sensitivity analysis have provided a fundamental motivation for the use of feedback and are largely responsible for its development into what is called modern control theory, implying the principles of optimization and adaptation [6].

Sensitivity analysis is also used in conjunction with Taylor Series to serve as prediction tool by analyzing how a model output changes in response of variation to one or more of its parameters. Another useful application for sensitivities is to quickly quantify the effect of varying system parameters on system response. Sensitivity analysis also simplifies the analysis of data and models by identifying parameters that do not have significant impact on the system response. Those non-critical parameters can then be eliminated from the model without significant impact on the model accuracy.

To illustrate the importance of the sensitivity analysis, following are some situations in which calculating the effect of system parameters changes on the system response is required [7].

1.2.1 What if Analysis

It is often important to ask (What if?) questions such as What is the smallest acceptable value of Power Factor Correction Capacitor (PFC)? or What is the best location for this PFC?. Another typical example is the manufacturing tolerance. Equipment manufacturers usually guarantee compliance with specified requirement within tolerance allowance. For example, ANSI C57 [8] specifies manufacturing of
transformer impedance to be within ±7.5% of the specified value. In large power systems, and the advent of large number of scenarios, one of the basic computational needs is to be able, with ease, to simulate changes in the parameters of any system component.

1.2.2 Model Simplification

To calculate the response of a system, each component must be represented by a model. This process is still an art, and it is often uncertain as to whether the model being used is appropriate. Some of the questions to be answered are: For a given type of simulation; what system parameters that are of primary interest? And what are those parameters that can be made redundant? To answer these questions, it is necessary to compute the effect of the change or removal of each parameter on system response.

1.2.3 Normal Parametric Variations

In general, the value of a system parameter is a function of other factors such as temperature and frequency. Therefore, for most power systems the expected variations in these factors shall not cause system behavior to move outside specified limits.

1.2.4 Component Growth

In the case of some system components such as capacitors, aging is a factor that plays a role in changing its value. Therefore, it is important to account for the effect of aging factor on the system response. The knowledge of the effect of small changes in relevant system parameter on the system response can provide:
• An indication of those components which might usefully be adjusted to improve system behavior.

• A feeling for which may be the components principally responsible for the essential aspects of system behavior.

• An identification of any unduly sensitive parameter in the system.

In summary, calculation of sensitivity can provide very useful information on how system response reacts to parameter changes.

1.3 Basic Mathematical Concept of Sensitivity Theory

The mathematical problem to be solved in sensitivity theory is the calculation of the change in the system behavior due to the parameter variations [6]. Let the parameters of the system be represented by a vector \( \alpha = [\alpha_1 \alpha_2 \ldots]^T \). The mathematical model of a system relates the system parameters to a quantity characterizing its dynamic behavior (i.e. the state variables). In the following example, a system is described by a vector of differential equation as follows:

\[
\dot{x} = f(x, \alpha, t, u) \quad x(t_0) = x^0 \tag{1.1}
\]

Where: \( x \) represents the state vector

\( \alpha \) is the parameter vector

\( t \) is the time (for time domain analysis)

\( u \) is the excitation (input) vector

\( x^0 \) is the initial condition
As shown in Equation 1.1, the state vector \( x \) is function of the system parameters, that is

\[ x = x(\alpha). \] (1.2)

Equation 1.2 describes the relationship between the parameter vector and the state vector. Mathematically, this relationship should be unique, however, in practice this is not always correct.

Assume the parameter vector of the mathematical model to be \( \alpha_0 \), and the parameter vector of the actual system is \( \alpha_0 + \Delta \alpha \) of the actual parameter vector. In order to study the influence of the parameter deviations \( \Delta \alpha \) on the behavior of the system, the following is defined:

1. Define \( R_\alpha \) as the subspace of the parameter variations \( \Delta \alpha \) around \( \alpha_0 \), and
2. Define \( R_x \) as the corresponding subspace of the state vector

By these definitions the mapping \( \alpha \rightarrow x \) can be replaced by the mapping \( R_\alpha \rightarrow R_x \), as shown in Figure 1.1. \( R_x \) is uniquely determined by Equation 1.1, if \( R_\alpha \) is known. However, for a number of reasons, it is not reasonable to characterize the sensitivity in terms of Equation 1.1: first, since the direct solution of Equation 1.1 for all elements of \( R_\alpha \) requires an infinite number of solutions and depends on the definition of \( R_\alpha \) and, second, since the result for small parameter variations \( \| \Delta \alpha \| << \| \alpha_2 \| \) would be very inaccurate if approximations are applied for the evaluation of this equation. For example, this would be true in the case of numerical or analog computation.
Figure 1.1: Mapping of the parameters space into the state space

It is a common practice in sensitivity theory to define a so-called sensitivity function $S$ that, under certain continuity conditions, relates the elements of the set of the parameter deviations $\Delta \alpha$ to the elements of the set of the parameter-induced errors of the system function $\Delta x$ by the linear equation

$$\Delta x \approx S(\alpha_0) \Delta \alpha$$

Or, if $\Delta x$ and $\Delta \alpha$ are scalar,

$$S(\alpha_0) \approx \frac{\Delta x}{\Delta \alpha}$$

For infinitely small change in $\Delta \alpha$, the sensitivity function can be defined as [7]:

$$S_j = \frac{\partial x(\alpha)}{\partial \alpha_j}, \quad j=1,2,..k \quad (1.3)$$

Where $k$ is the number of system parameters.

Equation 1.3 calculates the change in the state vector as a result of small change in one of the system parameters $\alpha_j$. For a state variable $x_i$, the sensitivity can be written as

$$S_{ij} = \frac{\partial x_i(\alpha)}{\partial \alpha_j} \quad (1.4)$$
1.4 The Modified Nodal Formulation

The Modified Nodal Formulation (MNF) is a circuit analysis technique that models each circuit parameter with its own contribution as an individual entity. This technique can directly compute both branch current and node voltages as a primary state variable and consequently the matrices forming MNF equations may contain admittances and/or impedances. Having the ability to incorporate any node voltage and branch current in the MNF equations makes it possible to directly obtain sensitivities of any circuit state (voltage or current) to any circuit parameter. Another advantage of the MNF is the ability to model inductive elements as impedances and capacitive elements as admittances; this allows RLC circuits to be modeled as first order differential equations instead of integro-deferential equations.

The MNF can easily model circuit elements that cannot be represented in the Nodal Admittance Formulation (NAF), such as ideal voltage source, controlled voltage sources, short circuit. The MNF can easily model ideal voltage source, short circuit, and all four types of controlled sources. In addition to the modeling capabilities, each type of circuit elements in the MNF is defined by a stamp. Element stamp is a systematic approach used to easily adopt the MNF into a set of equations. Therefore, system equations are easily formulated by sequentially reading the circuit elements and augmenting element stamps into the system equations.
1.5 Harmonics and Power Quality Issues

One of the major emerging concerns in power systems is harmonics. Harmonic is a general term used to describe any oscillatory or repetitive types of phenomena. In Power Systems Analysis, however, the term Harmonics refers to a distortion in the current and voltage waveforms. Power system harmonics phenomenon is typically caused by the presence of nonlinear loads (Arc furnaces, Adjustable Speed Drives (ASDs), dc motor drives, and switch-mode power supplies). Non-linear loads draw distorted (non-sinusoidal) currents causing harmonic currents to flow in the power system. These harmonic currents can combine with system frequency response characteristics to cause harmonic voltage distortion.

These harmonic problems are compounded by the use of capacitor banks, which can result in resonance conditions magnifying the harmonic distortion levels. Capacitor banks are normally installed for power factor correction purposes to avoid utility demand charges and penalties, and to free up transformer capacity. The presence of resonance frequencies close to harmonic frequencies can cause propagation and amplification of these harmonic current into the power system.

Resonance point can be shifted as system parameters changes due to changes in system impedance. Since Power Systems are designed to operate at power frequency, the presence of voltages or currents at other frequencies will cause problems to many of the
power system equipment. These problems include malfunction of controls, capacitor failures, motor and transformer overheating, and increased system losses. Therefore, sensitivity analysis presented in this thesis, can be used as an effective tool to study harmonic propagation and amplification and to device adequate solution for harmonic problems in power systems. Such issues are discussed in the following sections.

1.5.1 Sources of Harmonics

Conventional loads such as motors, heaters, etc can be represented by a combination of passive circuit elements, namely constant inductance, capacitance and resistors. This means that when fed with a sinusoidal voltage they will draw a sinusoidal current of the same frequency. On the other hand, non-linear load draw non-sinusoidal current resulting in distortion to both voltage and current waveforms.

The degree by which this current/voltage deviates from a perfect sine wave is known as Total Harmonics Distortion (THD). Harmonic Distortion, as it applies to power systems analysis, is the measure by which a periodic non-sinusoidal waveform deviates from a pure Sine Wave. The term is applicable to voltage and current alike. Because there is no such thing as a perfect voltage or current, there are maximum distortion limits deemed as acceptable. When the harmonic distortion exceeds these limits, problems can be expected. The IEEE's 519 standard [7] specifies strict current distortion limits that can prove difficult for compliance by industrial and commercial facilities.
To describe harmonic behavior, terms such as Total Harmonic Distortion (THD), zero sequence and negative sequence harmonics, etc. are utilized. The most common sources of harmonics in power systems are as follows:

1.5.1.1 Power Electronics Devises: Following the development of semiconductor technology, new generation of non-linear loads such as personal computers, fax machines, photocopiers, variable speed drives, etc has been introduced. Nonlinear loads are devices, which do not exhibit constant impedance during the entire cycle of the applied sinusoidal voltage waveform. This means, when powered by a sinusoidal voltage, non-linear load do not draw a sinusoidal current. A typical example of these devises is the three-phase converter shown in Figure 1.2 used for ASD application. Figure 1.3 shows that the line current waveform is no longer sinusoidal.

1.5.1.2 Ferromagnetic Equipment: Ferromagnetism is the ability of certain ferrous materials to produce magnetic flux in the presence of a magnetic field. This phenomenon is the principle under which transformers and motors operate. The magnetizing characteristic of these devices is defined by a nonlinear relationship between the magnetic flux density (B) and the magnetic field intensity (H). This non-linearity is more pronounce in transformers than in motors due to the air gap present between stator and rotor. Since B is proportional to the voltage and H to the magnetizing current, the magnetizing current will not be sinusoidal when a sinusoidal voltage is applied. Fortunately, under normal operating conditions, this current account for only 0.5% to 1% of the full load current of a transformer. However, the presence of a large number of lightly loaded transformers, can lead to a significant amount of current distortion. If the
voltage increases significantly, during periods of light load, saturation of the core takes place. This dramatically increases the amount of distortion in the magnetizing current.
Figure 1.2 A: Three-Phase Power Converter
Figure 1.3: Line Current Three-Phase Converter and its Harmonic Spectrum.
1.5.1.3 Arcing Devices: Arcing devices deliver power through a gap of air or gas. Typical arcing devices are arc furnaces, fluorescent lighting, mercury, and sodium vapor lighting. The behavior of the current in an arc can be explained in three phases. The first phase, the current increase linearly as the applied gap-voltage is increased. This process continues till there are no additional electrons available to increase the flow of current. At this phase voltage will increase with no increase in the current flow. If the voltage continues to increase to the flash point, a luminous discharge occurs and the arc makes its appearance. An arcing device is basically a voltage clamp in series with a reactance that limits the current to a reasonable value.

1.5.2 Problems Associated With Harmonics Analysis

The widespread use of harmonic producing devises has increased the percentage of non-linear loads in power systems causing significant distortion to the current and voltage waveforms. The relationships and equations used to analyze balanced power system with ideal sinusoidal voltage source and linear passive impedances are therefore no longer applies to a system with harmonic problems. Therefore, it is important to use new tools and philosophy in order to analyze such behavior and mitigate their side effect on the power system. This has brought the need to include one additional dimension to traditional power analysis; the Harmonic Frequency Dimension. With a high density of these nonlinear devices, phenomena such as overloaded neutrals in spite of perfectly balanced phases, harmonic overload to transformers, frequent failure of power factor
correction capacitors are common occurrences that require a different approach in order to be understood.

The amplification and propagation of harmonics in power system are function of network impedance. Also, distortion to voltage waveform can result from harmonic current propagation throw the system. The presence of a system resonance at or near to the frequency of an existing harmonic source will result in the amplification of these harmonics. This amplification will result in further distortion to system voltages and increase the risk of equipment failure due to increase in harmonic levels. Changes in system operating condition such as capacitor switching, line switching, loading condition of transformers and induction motors will affect the behavior of harmonic propagation in the system. System parameters can also be altered in order to absorb harmonics at certain location or suppress harmonic amplification.

1.6 Research Objectives

The primary objective of this research is to develop a sensitivity analysis technique based on the well-known modified nodal formulation power systems analysis. The MNF offers superior capabilities that make it ideal for the proposed mathematical formulation.

The mathematical formulation used in the development of the technique is modular and general to permit the incorporation of the parameters of any power system element in the sensitivity analysis.
1.7 Research Methodology

To achieve the objectives of this research, the following work has been implemented:

- Propose a Power System Sensitivity Formulation based on the well-known MNF along with the required rigorous mathematical derivations.
- Device study systems that exhibit typical industrial power system concerns such as power quality and voltage security.
- Perform power system analysis on the study systems to identify voltage security and power quality concerns.
- Formulate the proposed Modified Nodal Formulation Sensitivity Analysis (MNFSA) algorithm into a computer program code and confirm the program results.
- Apply the MNFSA computer program to look into power quality problems (Harmonic and resonance analysis)
- Use the results of the MNFSA to study some power system concerns and alternative system designs to deal with such issues.
1.8 Research Contributions

The main contributions of this thesis research are as follows:

- Propose a new parametric sensitivity approach based on the modified nodal formulation to investigate power quality issues.
- Formulate the proposed sensitivity approach into a modular mathematical structure.
- Develop a power system analysis program based on the proposed approach. This program can be used as a tool to provide the following:
  1. Help power system engineers in understanding how parametric variation of any power system elements influence its response in order to evaluate the quality of alternative power system design having the same purpose. The tool can also provide an assessment mechanism to changes in power systems due to addition of new component such as power factor correction and harmonic filters etc
  2. Provide power systems sensitivity indices and measures to identify the route causes of power system problems and help in devising solutions
  3. Provide response gradients in optimization application.
1.9 Thesis Organization

This thesis is divided into six chapters. In Chapter "I", the basic mathematical background and the applications of the Sensitivity analysis in engineering problems is described. The advantages and modeling capabilities of the Modified Nodal Formulation and are briefly discussed. Description to some of the issues that are analyzed using the proposed formulation is outlined. The objectives, research work methodology and thesis contribution are detailed. Chapter "II" contains the review of literature on the application of sensitivity analysis and the MNF in power system analysis. The mathematical basis of the MNF is presented in Chapter "III". This is followed by detailed discussion to demonstrate the modeling capabilities of the MNF. A rigorous derivation of the proposed Modified Nodal Formulation Sensitivity Analysis (MNFSA) is detailed. Chapter (IV) presents the derivations of equations used in the development of the MNFSA program. The structure of the algorithm used to develop the code is then described. The different types of output results the program can produce are presented. Finally the program capabilities, applications and type of analysis that can be performed are also discussed. In Chapter (V), the MNFSA program is applied in evaluating several power systems and to demonstrate the ability of the proposed method in analyzing power system issues related to harmonic, harmonic amplification and power system resonance. Finally, in Chapter "VI" the conclusions and some of the recommendations for future work are outlined.
CHAPTER II

LITERATURE REVIEW

2.1 General Review

Parameter sensitivity analysis is a mathematical tool widely applied in engineering problems where mathematical models are used for the purposes of analysis and synthesis [6, 7, 11, 12, 13]. For example, sensitivity analysis has been used to perform exact frequency domain analysis for multiphase periodically switched network [14]. The method can be used to calculate sensitivity of system response to changes in any system parameters using the MNF. In order to be able to give a unique formulation of the mathematical problem, the mathematical model is usually assumed to be exact. This assumption is usually unrealistic since there is always a certain discrepancy between the actual system and its mathematical model. This is because mathematical models are often simplified or idealized intentionally in order to make the problem solvable and to accommodate uncertainty in the model parameters caused by manufacturing tolerance, temperature variation etc [7, 15]. If there are deviations between the real system and the mathematical model parameter and the solution is very sensitive to these parameters, the
results of the mathematical model may not be accurate. Therefore, it should be part of the solution to a practical problem to know the parameter sensitivity prior to its implementation and to reduce the sensitivity systematically if necessary [16]. Parameter sensitivity can be defined as the effect of parameter changes on the dynamics of a system such as time or frequency response, state or transfer function or any other quantity characterizing the dynamics of the system [6].

### 2.2 Application of Sensitivity Analysis in Power System

Sensitivity analysis has been widely used in power system analysis [17-33]. A general approach for power system sensitivity analysis was proposed in [17] to calculate the gradient of any system function with respect to system parameters. Following is a survey of areas in power system analysis where sensitivity analysis where primarily applied.

#### 2.2.1 Stability Analysis

##### 2.2.1.1 Voltage Stability: Sensitivity analysis has been widely used in voltage collapse and voltage stability analysis [18-28]. In [18] Sensitivity of reactive power generation to load changes to compute proximity to voltage collapse. Similar sensitivity approach has been used in [19] to compute the amount of reactive power needed to control voltage stability. In [20], the sensitivity of the minimum singular value of system Jacobian matrix where computed for the assessment of voltage instability.
Another method to determine voltage stability condition is presented in [21]. The method is based on calculating the reactive power margin available to any bus in the system. The reactive power margin is used to determine distance to voltage collapse and hence defines the state of system security to voltage collapse. The method is inherently sensitive to the limits of MVAR's resources reactive capability limits.

A method to diagnose midterm instability following large disturbance is presented in [22]. The proposed algorithm combines time domain simulation with sensitivity analysis to identify buses in the system at which load increase is most critical to voltage instability and determine the corresponding corrective actions such as load shedding and/or tap changer blocking on time scale. The approach has the limitations that active and reactive loads are modeled as voltage dependent type only and no frequency dependent load were considered in the load modeling.

In [23], a nodal sensitivity approach is used to obtain indices or measures for area or system voltage stability margin and required corrective action. The required corrective action is obtained by calculating real and reactive power generation shift factors using area sensitivity matrices. The sensitivity of the total reactive power injections at pilot area nodes (Q_{pilot}) to the total reactive generation of SVC controlled units (Q^{svc}) is used in obtaining indices for stability margins and corrective actions.

The sensitivity of loading margin to voltage collapse has been presented in [24]. The main idea of this research is to predict the voltage stability loading-margin of power system subject to changes in system parameters or control. System parameters of interest can be an impedance of a Flexible AC Transmission System (FACTS) controller or a
shunt impedance power factor correction etc. A linear and quadratic sensitivity approach has been proposed to predict loading margin as system parameters changes. One source of inaccuracy in this approach is the assumption that system equations remains the same as system parameters varied, which means the effect of generators reactive limits and other voltage regulation equipment can not easily be represented in the formulation. This work can be considered as an extension to the application of first order loading margin sensitivity that reported in [25].

In [26], similar sensitivity formulations to that developed in [24] were used to establish a contingency ranking for voltage collapse. The stability margin due to line outage is calculated using linear and quadratic sensitivity of loading margin to each contingency. The result is a computationally fast algorithm that can estimate loading margin for a nominal P-V curve much faster than the direct methods with reasonable accuracy. The algorithm has the same limitations presented in [24]. The method presented in [26] was used to determine the safe operating voltage stability margin of South England Power System voltage [27].

2.2.1.2 Dynamic Stability Analysis: The sensitivity analysis has also been used in small signal stability to deal with other highly non-linear power system phenomena such as interaction phenomena of FACTS devises in power systems [28]. The sensitivity analysis was used to create an index for limiting the interaction phenomena on the inter-area ties. The oscillation is reduced by optimally selecting the location of a Static Compensator (STACOM). The sensitivity analysis approach was based on small signal stability theory and eigenvalue analysis. A method for real time computation and control
of oscillations using sensitivity analysis was presented in [29]. The approach is based on computing eigenvalue sensitivity with respect to different system parameters, namely generator set points such as prime mover input power reference (Pc) and exciter reference voltage (Vref).

2.2.2 Optimal Power Flow and Contingency Analysis

Contingency analysis and Optimal Power Flow (OPF) are widely used by utilities as real-time tools to operate power systems at the most secured and efficient condition [30]. Because both tools require extensive computational efforts and the results are needed on real-time bases, sensitivity analysis are often used in such applications.

A method to compute network sensitivity factors for contingency analysis is discussed in [30]. This method provides a quick approach for studying hundreds of possible outages, which becomes very desirable when used on real-time basis so that corrective actions can be taken. The sensitivity factors used to approximate the change in line flows for changes in generation or on the network configuration. Two types of sensitivity factors derived from DC load flow are computed namely:

- Generation shift factors
- Line outage distribution factors

The generation shift factors are used to estimate changes in line flows due to generation or load changes while line outage shift factors are used to estimate line flow changes due to line outages. Sensitivity factors are also used to calculate changes in tie-line flow due to outages on external system [31]. This method is used to avoid modeling external system and thus results in reduction in the complexity of the system model. The
method also incorporates spare matrix inversion, which also increase the computational efficiency.

A sensitivity analysis approach to estimate the desired changes in the OPF solution due to small change in system operating conditions is presented in [32]. The approach is based on a least-square algorithm suitable for post-OPF. This algorithm was designed for a large utility power system Energy Management System (EMS). A generalized parametric algorithm for OPF is presented in [33]. The technique is based on two steps approach. In the first step is the optimal solution is found for arbitrarily chosen operating point. The sensitivity analysis is used to find the trajectories correspond to different loads. The approach shows the non-linearity of the OPF solution and its sensitivity to variation in parameters.

Sensitivity approach to determine a new state from an optimal state due to small change in system parameters such as system load or control-action weighting a factor is presented in [34]. The method is used to avoid a complete OPF solution, for real-time application, when dispatcher makes small changes in weighting factors to reduce emission or when system load changes insignificantly. In [35] the OPF problem is solved by applied voltage security constrains on the Fast Newton Raphson economic dispatch. By shifting generation to satisfy the economic dispatch solution, voltage at some busses in the system may fall below acceptable values. This method uses reactive power sensitivity factors of bus voltage magnitude to regulate load bus voltage to secure limits.
2.2.3 Power Quality Application

Sensitivity analysis was also applied in harmonic design. In [36], a method for distribution system harmonic design based on eigenvalue sensitivities to system parameters was presented. The system state equations are used to compute system poles and zeros and there sensitivity to parameters changes. The resulted sensitivities can help in capacitor bank placement to improve harmonic system design.

A method to compute voltage dips using sensitivity for large-scale power system is presented in [37]. The sensitivity analysis is derived relative to changes in operating conditions such as generation and load levels, voltage profile etc. The sensitivity technique can be used to maintain voltage dip within pre-specified limits.

Sensitivity analysis can be combined with optimization technique to optimize the location and size of PFC in radial distribution system. The sensitivity algorithm identifies candidate locations for PFC. This will reduce the search space of the optimization procedure and hence enhance the performance of the algorithm. In [38-39], sensitivities of system real power losses to nodal reactive power injections were formulated to identify candidate locations for the PFC that will have maximum impact on power losses.

2.3 Application of MNF in Power System

The MNF has generally been used in dealing with Power Electronic Circuits simulation. Switching modeling in power electronic circuits using MNF was presented in [40]. The MNF was also used in [41] to find the equilibrium points of closed loop
switching PWM converter under large signal disturbances. In [42], a computer-aided analysis of power electronic circuits based on Modified Nodal Formulation was proposed. By using the MNF the formulation of state-equations were avoided and hence the system equations can be solved much faster by solving a set of first order differential equations.

In this research, a sensitivity analysis approach based on the MNF is proposed as a power system analysis tool. The MNF is a well-known circuit analysis technique that can model an electric circuit as a set of linear equations and first order differential equations. The modified formulation was introduced to overcome modeling capabilities and better computational efficiency than the conventional Nodal Admittance Formulation (NAF). The enhanced capabilities of the MNF are summarized as follows:

- It can handle ideal voltage sources
- It can handle short-circuits
- It is possible to handle all the four types of controlled sources
- Impedance branch elements such as inductors can be modeled as first order differential equation system.
CHAPTER III

MATHEMATICAL FORMULATIONS

3.1 Introduction

Analyzing an electrical network involves three major steps; the first step is the formulation of system equations that describe the relationships between the variables. The second step is to solve these equations in the time and/or frequency domains. The third step is to evaluate the solution. Several methods have been used to formulate system equations such as loop matrices formulation and nodal-admittance/impedance formulations (NAF) [44]. All formulations are based on natural laws of balance Kirchhoff Voltage Law, KVL or Kirchhoff Current Law, KCL. Each method goal is to find a method, which is simple and general with respect to implementation in a computer program.

The loop formulation is useful in the hand calculations of simple circuits. It is based on the KVL, which states that the sum of the voltage drops around any loop visiting a set of nodes is zero. The method is difficult to apply for non-planar circuits. The Nodal Admittance Formulation (NAF) is based on the KCL, which states that the sum of the
currents leaving a closed subset of a circuit must be zero, e.g. the sum of the currents leaving an element is zero or the sum of the currents leaving a node is zero. A closed subset of a circuit is a super node or a multi terminal.

The variables chosen in the basic nodal formulation (NAF) are the node voltages, i.e. the node potentials with respect to an arbitrary reference. This implies that short circuits, ideal voltage sources and ideal current controlled sources cannot be handled. Furthermore inductive and capacitive elements will give rise to integro-differential equations or second order differential equations. Therefore drawbacks of the node admittance formulation can be summarized as follows [43]:

1. It is not possible to handle ideal voltage sources.
2. It is not possible to handle short-circuits.
3. It is possible to handle only the voltage controlled current source (VCCS) among the four types of controlled sources.
4. Impedance branch elements as e.g. the inductor may involve a second order differential equation system.

The following section demonstrates how it is possible to overcome these drawbacks by means of extending the node admittance formulation while preserving the basic properties of this formulation. This will be followed by rigorous derivation for the proposed Modified Nodal Formulation Sensitivity Analysis (MNFS).
3.2 Mathematical Formulation of the MNF [43]

One of the limitations with the NAF is the limitation when formulating equations for elements described by voltage constitutive equations.

![Figure 3.1 Ideal voltage source.](image)

An ideal voltage source, as shown in Figure 3.1, is described by

\[ v_k - v_j = E_{kj} \quad (3.1) \]

Since there is no current variable in this equation, it is not possible to formulate the nodal KCL equations, if the voltage source is the only branch in that circuit. This problem can be seen in the circuit of Figure 3.2, if \( E_1 \) became an ideal voltage source (i.e. \( G_2 = \infty \)). The nodal admittance matrix becomes singular as \( G_2 \) becomes much larger than the other conductances. One solution is to use a gyrator circuit to simulate an ideal voltage source. As shown in Figure 3.3 a current source of \( E \) amperes is connected to one port of the gyrator producing a voltage \( v_1 = E \) volts at the other port.
The implementation of this concept for the circuit of Figure 3.2 is shown in Figs. 3.3. It can be seen that a new node is introduced. The circuit is described by the following system of equations

\[
G = \begin{bmatrix}
G_1 & -G_1 & 1 \\
-G_1 & G_1 + G_2 & -1 \\
-1 & 1 & 0
\end{bmatrix}
\quad v = \begin{bmatrix}
v_1 \\
v_2 \\
v_3
\end{bmatrix}
\quad i = \begin{bmatrix}
J_1 \\
0 \\
-E_2
\end{bmatrix}
\]

(3.2)

Note that the third, new, equation is

\[
v_1 - v_2 = E_2
\]

(3.3)

which corresponds to (3.1). The voltage of the third node is numerically equal to the current flowing through the branch with the ideal voltage source.
Figure 3.3 (a) Use of gyrator for ideal voltage source simulation, (b) Implementation of the gyrator.
This leads to the idea of introducing a current as a new circuit variable when an ideal voltage source has to be described. This may also be useful for cases where the branch current is needed as an output variable irrespective of the type of branch element. Introduction of a branch current as a new variable means augmenting the number of unknowns; so one additional equation is needed.

As an example of this concept, consider the circuit of Figure 3.4. In this case $i_E$ has to be introduced as a circuit variable in order to describe the ideal voltage source, while $i_3$ is required as an output variable. For two new currents, two additional equations are needed. One represents the voltage source

\[ v_1 - v_2 = E \]  \hspace{1cm} (3.3)

And the other represents the conductance branch

\[ v_2 - i_3 Z_1 = 0 \]  \hspace{1cm} (3.4-a)
The nodal equations are

\[
\begin{align*}
-J + G_1v_1 + G_2(v_1 - v_2) + i_E &= 0 \\
G_2(v_2 - v_1) - i_E + i_3 &= 0.
\end{align*}
\] (3.4-b)

Augmenting equations (3.3), (3.4), the system of equations matrix describing the circuit is

\[
\begin{bmatrix}
G_1 + G_2 & -G_2 & 1 & 0 \\
-G_2 & G_2 & -1 & 1 \\
1 & -1 & 0 & 0 \\
0 & 1 & 0 & -Z_s
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2 \\
i_E \\
i_3
\end{bmatrix}
= \begin{bmatrix}
J \\
0 \\
J
\end{bmatrix}.
\] (3.5)

This system can be divided as shown. The first two equations, being nodal KCL equations, are distinguished from the second two, which are branch constitutive equations. The unknown variables are separated, too. The first part has the circuit node voltages while the second contains the new variables (currents). Finally, the RHS vector is also divided; the upper part has current excitations while the lower contains voltage excitations. Note that the gyrator method would not allow \(i_3\) to be introduced as a network variable so this is a new method.

The above formulations were described by Ho et al (1975) and named the Modified Nodal Formulation method or (MNF). It is now the most common method used in circuit analysis programs. It has also become a technique for explaining general ideas in circuit theory.

Generalization of (3.5) leads to the following equation:
\[ T \cdot x = W \]  \hspace{1cm} (3.6)

where

\[
T = \begin{bmatrix} G & B \\ C & D \end{bmatrix}, \quad x = \begin{bmatrix} v \\ i \end{bmatrix}, \quad W = \begin{bmatrix} J \\ E \end{bmatrix} \hspace{1cm} (3.7)
\]

In (3.7) the following notation is used:

- **G** is the reduced nodal conductance matrix whose size is \( n \times n \). Its elements are the derivatives of the nodal KCL equations with respect to the node voltages.
- **B** is an \( n \times m \) matrix where \( m \) is the number of new variables (currents). Its elements are the derivatives of the nodal KCL equations with respect to the new variables.
- **C** is an \( m \times n \) matrix whose elements are obtained as derivatives of the new equations with respect to the node voltages.
- **D** is an \( m \times m \) matrix with elements representing the derivatives of the new equations with respect to the new variables. **B** and **C** may have only 0, 1 and -1 as matrix elements.
- **v** is the vector of \( n \) node voltages.
- **i** is the vector of \( m \) new branch currents.
- **J** is the vector of node current-excitations. The \( k^{th} \) element of **i** is the sum of all independent current sources flowing into the node \( k \).
• $E$ is the vector of branch voltage sources. An element of $e$ will be non-zero if an ideal voltage source is described by the corresponding equation.

Unlike the matrix $G$, $T$ is not symmetric and is not diagonally dominant. This is seen in the second row of (3.5) with $G_2$ and the $1$. In addition, zero-valued diagonal elements may be introduced. For a grounded ideal voltage source (if connected between node $k$ and ground, such a source is described by $v_k = E$), a zero valued diagonal element and only one off-diagonal matrix element is introduced. This matrix element is frequently called a singleton.

From inspection of (3.5) it can be seen that the new equations are ordered at the end of the system taking numbers starting with $n+1$. This is also true for the numbering of the current variables. The first new variable is numbered $n+1$, where $n$ is the number of circuit nodes. Reordering the equations means renumbering the variables, which in turn means that the current variables will be rearranged in the vector $x$. A similar rearrangement will happen with $T$ and $W$.

### 3.3 Modeling Basic Power System Elements Using MNF

To model a power system using the MNF, the network is disseminated to its most basic components (resistance, conductance, inductance and capacitance, etc). Each of these basic elements is sequentially incorporated in the model using a MNF element stamp. Following are the MNF stamps for circuit elements related to the proposed
research work. Other circuit elements stamps, which are primarily related to circuit analysis theory, can be found in the literature [1-2, 43].

![Ideal Current Source](image)

Figure 3.5: Ideal Current Source

3.3.1 **Stamp for Ideal Current Source**

In power system applications, ideal current source is used in representing a harmonic source, performing impedance frequency scan studies and in representing the initial condition of a capacitor. The ideal current source, shown in Figure 3.5, is represented by two terminals (nodes) with a current injection (J) from one node \(j\) to the other one \(j'\). To model an ideal current source into the MNF equations, complex value of the current source is added to the \(j^{th}\) and the \(j'^{th}\) elements of output vector \(W\) or the Right Hand Side (RHS) of the system equations. The addition of an ideal current source does not make any change to the system matrix nor does it add a new state variable to the unknown vector \(x\). The stamp of an ideal current source is mathematically represented as follows:

<table>
<thead>
<tr>
<th>Equation (j)</th>
<th>(J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation (j')</td>
<td>(-J)</td>
</tr>
</tbody>
</table>

\[(3.8)\]
3.3.2 Stamp for Ideal Voltage Source

Voltage source are used to represent a utility system, generators or initial condition of an inductor. The ideal voltage source is represented by two terminals (nodes j and j') with a potential difference between the two nodes equals to the source voltage (E), as shown in Figure 3.6. To model an ideal voltage source, three steps are required: 1) the size of \( W \) vector is increased by one and the complex value correspond to the voltage source is added to the last element of the vector \( (k+1) \). 2) The size of \( T \) matrix has also to be increased by one row and one column. Values of the new row and columns elements are set to one at \{k+1,j\},\{k+1,j'\}, \{j,k+1\}, \{j',k+1\} and zero elsewhere; 3) The state variable vector \( (x) \) is increased by one \( (k+1) \) and a new state variable, representing the current passing throw the voltage source is introduced. After solving the system equations, current passing throw the ideal voltage source is automatically calculated. This is not possible in the case of nodal formulation method. The stamp for an ideal voltage source is as follows:

<table>
<thead>
<tr>
<th>Column j</th>
<th>Column j'</th>
<th>Column k+1</th>
<th>State Vector ((x))</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_j )</td>
<td>( V_j )</td>
<td>( I_{j-j'} )</td>
<td>( V_j )</td>
<td>( V_j' )</td>
</tr>
</tbody>
</table>

Were \( k \) is the size of the system matrix before adding this voltage source
The following stamp is added to the $x$ vector

$$E$$

![Figure 3.6 Ideal Voltage Source](image)

### 3.3.3 Stamp for Admittance Element

Using MNF to model admittance is simple and similar to the conventional nodal formulation. An admittance ($y$) connected between node $j$ and $j'$ is simply modeled by adding ($y$) to the $\{j,j\}$ and $\{j',j'\}$ (self) elements of the system matrix and ($-y$) to the $\{j,j'\}$ & $\{j',j\}$ (mutual) elements of $T$. The stamp for an admittance element is as follows:

<table>
<thead>
<tr>
<th>Column $j$</th>
<th>Column $j'$</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_j$</td>
<td>$V_j$</td>
<td>$V_j$</td>
</tr>
</tbody>
</table>

Equation $j$

Equation $j'$

$$\begin{align*}
\text{Equation } j & : y_{j'j} & y_{j'j'} & V_j \\
\text{Equation } j' & : -y_{j'j} & y_{j'j'} & V_{j'}
\end{align*}$$

![Figure 3.7 Admittance](image)

### 3.3.4 Stamp for Impedance Element

An impedance ($z$) connected between nodes $j$ and $j'$ is represented in the MNF by increasing the size of output vector $W$ and state vector $x$ by one to $(k+1)$ with new state
variable at \(k+1\) representing the current passing through the impedance. The system matrix size is increased by one row and one column \((k+1)\). The modification to the \(T\) matrix when adding impedance to a system is similar to that of voltage source except in this case element \(\{k+1,k+1\}\) is set to be equal to \(z\). The stamp of an impedance element is as follows:

<table>
<thead>
<tr>
<th>Column (j)</th>
<th>Column (j')</th>
<th>Column (k+1)</th>
<th>State ((x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V_j)</td>
<td>(V_j)</td>
<td>(I_{j-j'})</td>
<td>Vector ((x))</td>
</tr>
<tr>
<td>Equation (j)</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equation (j')</td>
<td>-1</td>
<td></td>
<td>(3.11)</td>
</tr>
<tr>
<td>Equation (k+1)</td>
<td>1</td>
<td>-1</td>
<td>(-z_{j-j'})</td>
</tr>
<tr>
<td>I_{j-j'}</td>
<td></td>
<td></td>
<td>(I_{j-j'})</td>
</tr>
</tbody>
</table>

Where \(k\) is the size of the system matrix before adding this inductance element.

\[ j \quad \begin{array}{c} I_{j-j'} \\ \downarrow \quad + V \quad - \quad Z_{j-j'} \quad \downarrow \quad j' \end{array} \]

Figure 3.8 Impedance

By being able to simultaneously model inductive elements as impedances and capacitive elements as admittances, the \(s\)-term introduced by these elements will appear in the system equations as first order differential equation. This will have significant computational advantage both in time domain and frequency domain analysis. As shown in equation 3.12, this formulation, also, allows the complex matrix \(T\) to be structured into two real matrices to improve the computational efficiency. This representation shows some physical characteristics of the system such as system damping and capacitive and inductive nodes in the system.
\[
\begin{pmatrix}
G_{11} & G_{12} & \ldots & G_{1n} \\
G_{21} & G_{22} & \ldots & G_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
G_{n1} & G_{n2} & \ldots & G_{nn}
\end{pmatrix}
\begin{pmatrix}
C_{11} & C_{12} & \ldots & C_{1n} \\
C_{21} & C_{22} & \ldots & C_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
C_{n1} & C_{n2} & \ldots & C_{nn}
\end{pmatrix}
\]
\[
T = G + sC
\]

Where

\( [G] \): is an \((m+n) \times (m+n)\) matrix constituting the real part of \( T \)

\( [C] \): is an \((m+n) \times (m+n)\) matrix constituting the imaginary part of \( T \)

\( n \): is the number of voltage variables (number of system nodes)

\( m \): is the number of current variables (number of impedance and voltage source elements)

\( S = 2\pi f \)

3.4 Derivation of Modified Nodal Sensitivities

Analysis (MNFSA)

The research work of this thesis is based on developing a parametric sensitivity utilizing the MNF to perform power system sensitivity analysis. The proposed formulation shall be able to determine the sensitivity of any power system state (node voltage or branch current), due to changes in any system parameters. As shown in the previous section, the MNF models both voltages and currents as system variables. Therefore, it is possible to obtain the sensitivity of node voltages and branch currents with respect to any circuit parameters (inductances, resistances, capacitance, etc). Following is
the mathematical derivation of the proposed Modified Nodal Formulation Sensitivity Analysis (MNFAS):

As shown in the previous section, the system equation of the MNF can be written as

\[ T(h_{i})x(h_{i}) = W(h_{i}) \]  \hspace{1cm} (3.13)

Where \( h_{i} \) is the parametric value of arbitrarily chosen system element \( i \), i.e. \( h_{i} \) can be resistor \( (R_{i}) \) inductor \( (L_{i}) \) capacitor \( (C_{i}) \) or conductance \( (G_{i}) \).

Taking the derivative of Equation (3.13) with respect to parametric value \( h_{i} \)

\[ \frac{\partial T}{\partial h_{i}} x + T \frac{\partial x}{\partial h_{i}} = \frac{\partial w}{\partial h_{i}} \] \hspace{1cm} (3.14)

Rearranging equation (3.14)

\[ T \frac{\partial x}{\partial h_{i}} = \frac{\partial w}{\partial h_{i}} - \frac{\partial T}{\partial h_{i}} x \] \hspace{1cm} (3.15)

\[ \frac{\partial x}{\partial h_{i}} = T^{-1} \left( \frac{\partial T}{\partial h_{i}} x - \frac{\partial w}{\partial h_{i}} \right) \] \hspace{1cm} (3.16)

Define a scalar variable \( \varphi \), where \( \varphi \) is the \( j^{th} \) unknown variable in the vector \( x \), this is to deal with one system variable at a time; then \( \varphi \) can be mathematically defined as

\[ \varphi = \varphi' x \] \hspace{1cm} (3.17)
Where $d$ is a constant vector with one at the $j^{th}$ element and zero in the rest.

Take the derivative of equation 3.17 with respect to $h_i$

$$\frac{\partial \varphi}{\partial h_i} = d' \frac{\partial x}{\partial h_i} \quad (3.18)$$

Substitute equation (3.16) into equation (3.18), we get

$$\frac{\partial \varphi}{\partial h_i} = d' \{T^{-1} (\frac{\partial T}{\partial h_i} x - \frac{\partial w}{\partial h_i})\} \quad (3.19)$$

Define an adjoint vector $x^a$ so that

$$(x^a)' = d' T^{-1} \quad (3.20)$$

Rewrite (3.20)

$$T' x^a = d \quad (3.21)$$

Substituting equation (3.21) into equation (3.19), we get

$$\frac{\partial \varphi}{\partial h_i} = (x^a)' \frac{\partial T}{\partial h_i} x - (x^a)' \frac{\partial w}{\partial h_i} \quad (3.22)$$

Equation (3.22) describes the sensitivity of any state variable $\varphi$ to any system element parametric value $h_i$. 
3.5 Applications and Advantages of the MNFSA

The details of sensitivity analysis formulations and derivations were discussed in the previous sections. It has been shown that it is possible to find the differential sensitivity of any arbitrarily chosen unknown state variable (voltage or current) from the vector \((\mathbf{x})\) with respect to any chosen system parameter \((h_i)\). These sensitivities will be used as indices to analyze power system issues related to power quality.

In order to focus the research efforts and demonstrate the capabilities of the MNFSA, key system parameters and phenomena of concern, namely harmonic resonance will be identified and evaluated for different case studies.

3.5.1 Application of MNFSA in Power Quality Issues

The research work will concentrate on dealing with harmonic and resonance issues to identify sources of harmonic amplification and resources of harmonic suppression in a typical power system.

The MNFSA provides a comprehensive tool that assist power system designers to identify the potential of harmonic amplification and to maintain harmonic levels to acceptable levels using various design approaches including filters to absorb harmonic or detune the system away from the harmonic source frequencies. This will improve the efficiency and quality of power system design.

The MNFSA can be applied more effectively to determine harmonic resonance and amplification of a large system than that of conventional tools such as frequency
scans. If a harmonic source exists in a system, under some operating conditions in which the system is tuned at or near to the harmonic source then, there is a potential for harmonic amplification. This can lead to undesirable consequences such as damage to equipment insulators due to over voltages. Such problems can be analyzed by the MNFSA without the need of conducting many scenarios.

Impedance frequency scan and voltage response are useful tools commonly used to identify resonance points in the system and the magnitude of harmonic amplification at different frequency range [45]. However, for relatively large system with many different operating scenarios, frequency scans may not be the best solution due to the huge number of scenarios. Furthermore, the frequency scan and voltage response cannot identify the network elements that are responsible of creating these resonance points. These analyses also cannot provide quantitative indices for measuring response improvement when designing filters, Power Factor Correction capacitors (PFC’s), shunt reactors etc.

3.5.2 Advantages of the MNFSA in Power Quality

Following are some of the MNFSA program advantages:

- A measure of contribution each system parameter has on the system response at any system frequency. This helps identifying the network parameters that causes phenomenon such as resonance.
- A parametric sensitivity evaluation for system response.
- A quick measure of how system response is sensitive to system frequency at different harmonics relative to fundamental frequency (for harmonic analysis).
- Compare alternative design solutions and optimize design parameters.
In the first type of studies, the MNFSA is used to identify those system parameters that have more influence on the system response. Therefore, almost all system parameters will be initially evaluated. Once identified, the dominant elements in the system can be further evaluated. This type of studies can be considered as screening for the parametric sensitivity evaluation. The parametric analysis examines the variation in the response of the sensitivity as some of the dominant parameters changes. In this case only those dominant parameters that are expected to vary will be evaluated.

For power quality studies, an alternative to performing complete frequency scan runs at all possible operating scenarios, performing sensitivity analysis to system response at frequencies where harmonics are expected (e.g. 5th, 7th etc) can be used as a screening tool to determine the cases of concern for further analysis.

The MNFSA can also provide qualitative comparison between designs and solutions to system problems. The sensitivities can be used as indices to compare these alternative designs and solutions. If, for example, the concern is the presence of a 5th harmonic source and the existence of a response near that frequency, then different solutions will result in reducing the sensitivity of system response to 5th harmonic. The lesser the sensitivity value the more effective is the solution. On the other hand, one should also avoid creating another resonance at a frequency that coincides with other existing harmonic source.
CHAPTER IV

MNFSA PROGRAMMING

4.1 Introduction

To verify the capabilities of the proposed MNFSA in analyzing a typical power system, the MNFSA is structured in a modular format suitable for computer program. The formulation is then developed into a Fortran based code, which was successfully integrated into a power system analysis package that was developed in [1] to create an integrated power system analysis tool.

Derivations of equations used in the development of the MNFSA program are detailed in this chapter. The structure of the algorithm used to develop the code is then described. The different types of output results the program can produce are described. Finally the program capabilities, applications and type of analysis that can be performed are also discussed.
4.2 Formulating Parametric Sensitivities for Basic Power System Elements

To develop the MNFSA into a computer program, equation 3.22, which calculates the parametric sensitivities, must be further simplified into a more programmable format. The parametric sensitivity equation is:

\[
\frac{\partial \varphi}{\partial h_i} = (x^a)' \frac{\partial T}{\partial h_i} x - (x^a)' \frac{\partial w}{\partial h_i} \tag{4.1}
\]

Careful analysis to equation 4.1 will help in developing a compact and efficient algorithm to solve this equation using a computer program. Clearly the equation has two distinct parts separated by a minus sign, the first part involves \( \frac{\partial T}{\partial h_i} \) while the second part involves \( \frac{\partial w}{\partial h_i} \). From chapter 3, it can be seen that the \( T \) and \( W \) are element-type dependent, and therefore, their derivatives with respect to \( h_i \) will depend on the type of element to which the sensitivity is being calculated. Once \( \frac{\partial T}{\partial h_i} \) and \( \frac{\partial w}{\partial h_i} \) are calculated, the remaining computations to obtain \( \frac{\partial \varphi}{\partial h_i} \) are the same for all types of system parameters.
Derivations and formulation of sensitivities for basic power system parameters are detailed in following sections.

4.2.1 Sensitivity With Respect To System Elements

As previously mentioned in chapter 3, the MNF can model any basic circuit element as either impedance or admittance. Impedance-type elements are incorporated differently into the MNF than admittance-type element when building the system equations.

4.2.1.1 Inductance Elements: Consider the element shown in Figure 4.1 to be an inductance \( L_i \) (\( Z_{ji} = L_i \)). In order to calculate the sensitivity of any system variable (\( x_k \), where \( x_k = \phi \)) with respect to any inductance (\( L_i \), where \( L_i = h_i \)), equation 4.1 is re-written as

\[
\frac{\partial x_k}{\partial L_i} = (x^a)^t \frac{\partial T}{\partial L_i} x - (x^a)^t \frac{\partial w}{\partial L_i}
\]

(4.2)

\[ \begin{align*}
\text{Figure 4.1: Impedance}
\end{align*} \]

To calculate \( \frac{\partial T}{\partial L_i} \), it is important to observe how \( Z_{ji} \) was incorporated into the T-matrix.

As discussed in Chapter 3, equation 3.11, shows how to incorporate \( Z_{ij} \) into \( T \)
<table>
<thead>
<tr>
<th>Column $j$</th>
<th>Column $j'$</th>
<th>Column $p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_j$</td>
<td>$V_j$</td>
<td>$I_{jj'}$</td>
</tr>
</tbody>
</table>

| Equation $j$ | 1 |
| Equation $j'$ | -1 |
| Equation $(p)$ | 1 | -1 | $-z_{jj'}$ |

(4.3)

Where $p$ is a pointer which correspond to order of the current variable introduced when $Z_{jj'}$ was incorporated in the MNF system equations. The first current variable is numbered $n+1$, (i.e. $x_{n+1} = I_1$, ..., $x_{n+p} = I_p$, ..., $x_{n+m} = I_m$) where $n$ is the number of circuit nodes and $m$ is the number of unknown currents.

Taking the derivative of $T$ with respect to $L_i$, results in:

\[
\frac{\partial T}{\partial L_i} = \begin{bmatrix}
0 & 0 & . & . & . & 0 \\
0 & 0 & . & . & . & . \\
. & . & . & . & . & . \\
. & . & . & . & . & . \\
. & . & . & . & . & . \\
0 & . & . & . & . & 0 \\
\end{bmatrix}
\]

(4.4)

For inductance element

\[
Z = sL_i
\]  

(4.5)
Where

\[ S = j2\pi f \]

\( f \): is the system frequency

Therefore, \[ \frac{\partial}{\partial L_i} (Z_{ii}) = s \] \hspace{1cm} (4.6)

There is only one non-zero element in \( \frac{\partial T}{\partial L_i} \) and it is equal to (s) which can be represented by an integer pointer instead of a matrix. The derivative of the RHS vector \( (W) \) with respect to \( L_i \) \( \left( \frac{\partial W}{\partial L_i} \right) \) is equal to zero, if the initial condition \( L_i(0) \) is zero.

\[ \frac{\partial W}{\partial L_i} = 0 \] \hspace{1cm} (4.7)

Substituting 4.6 into 4.4 and substitute 4.4 & 4.7 into 4.2

\[ \frac{\partial x_k}{\partial L_i} = [x_1 \ldots \ldots x_n \ldots x_{n+1} \ldots \ldots x_p \ldots x_{n+m}] S. \]

\[ \begin{bmatrix} 0 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & -1 & \ldots \\ 0 & \ldots & \ldots & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ x_{n+1} \\ x_p \\ \vdots \\ x_{n+m} \end{bmatrix} \] \hspace{1cm} (4.8)

Equation 4.8 produces a scalar, which is then reduced to

\[ \frac{\partial x_k}{\partial L_i} = -s.x_p.x_p \] \hspace{1cm} (4.9)
4.2.1.2 Resistance Element: If the element shown in Figure-4.1 is a resistance $R_i$ (where $Z_{jj} = R_i$) and the state variable of interest is $x_k$ (where $x_k = \phi$), then equation 4.1 is re-written as

$$\frac{\partial x_k}{\partial R_i} = (x^a)^{t} \frac{\partial T}{\partial R_i} x - (x^a)^{t} \frac{\partial w}{\partial R_i}$$

(4.10)

If the element is modeled as an impedance in the MNF then $\frac{\partial T}{\partial R_i}$ is calculated as

$$\frac{\partial T}{\partial R_i} = \begin{bmatrix}
0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 0 \\
\end{bmatrix}
\begin{bmatrix}
\frac{\partial}{\partial R_i} (Z_{ij}) \\
\vdots \\
\vdots \\
\frac{\partial}{\partial R_i} (Z_{ij}) \\
\end{bmatrix}$$

(4.11)

Since

$$Z_{jj} = R_i$$

(4.12)

Therefore,

$$\frac{\partial}{\partial R_i} (Z_{ji}) = I$$

(4.13)
There is only one non-zero element in $\frac{\partial T}{\partial R_i}$ and it is equal to (1) which can be represented by an integer pointer instead of a matrix. The derivative of $W$ vector with respect to $R_i$ ($\frac{\partial W}{\partial R_i}$) is equal to zero.

$$\frac{\partial W}{\partial R_i} = 0 \quad \text{(4.14)}$$

Substituting 4.13 into 4.11 and substitute 4.11 & 4.14 into 4.10,

$$\frac{\partial x_k}{\partial R_i} = \begin{bmatrix} x_1^a & \ldots & x_n^a & x_{n+1}^a & \ldots & x_p^a & \ldots & x_{n+m}^a \end{bmatrix} \cdot \begin{bmatrix} 0 & \ldots & 0 & \ldots & 0 & \ldots \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ x_{n+1} \\ x_p \\ \vdots \\ x_{n+m} \end{bmatrix} \quad \text{(4.14)}$$

Equation 4.14 produces a scalar value can be reduced to

$$\frac{\partial x_k}{\partial R_i} = -x_p^a x_p \quad \text{(4.15)}$$

4.2.1.3 Capacitance Element: Consider the element shown in Figure-4.2 to be a capacitor ($C_i$). In order to calculate sensitivity of any system variable $x_k$ (where $x_k = \varphi$) with respect to any capacitance in the system $C_i$ (where $h_i=C_i$) equation 3.22 can be re-written as
\[
\frac{\partial x_k}{\partial C_i} = (x^a)^t \frac{\partial T}{\partial C_i} x - (x^a)^t \frac{\partial w}{\partial C_i}
\] 

(4.16)

To calculate \( \frac{\partial T}{\partial C_i} \), it is important to observe how \( C_i \) was incorporated into the T-matrix.

Equation 3.10, shows how to incorporate \( Y_{jj} \) into \( T \):

<table>
<thead>
<tr>
<th></th>
<th>Column j</th>
<th>Column j'</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_j )</td>
<td>( y_{jj} )</td>
<td>(-y_{jj'})</td>
</tr>
<tr>
<td>( V_j )</td>
<td>(-y_{jj'})</td>
<td>( y_{jj'} )</td>
</tr>
</tbody>
</table>

(4.17)

Using equation 4.17 to calculate sensitivity of any system parameter with respect to \( C_i \) yields to;
Since $Y_i$ is a sussetance,

$$Y_i = sC_i \quad (4.19)$$

Therefore \( \frac{\partial}{\partial L_i} (SC_i) = s \) \quad (4.20)

The derivative of $W$ vector with respect to $C_i$ equals to zero, if the initial condition $C_i(0^*)$ is zero.

$$\frac{\partial W}{\partial C_i} = 0 \quad (4.21)$$

Substituting 4.20 into 4.18, and substitute 4.20 and 4.21 into 4.16 to get:
\[
\frac{\partial x_k}{\partial C_i} = \begin{bmatrix}
1 & x_1 & \cdots & x_n & x_{n+1} & \cdots & x_{n+m}
\end{bmatrix}_i S
\begin{bmatrix}
x_1 \\
x_i \\
x_{n+1} \\
x_n \\
x_{n+n-m}
\end{bmatrix}
\]

Equation 4.22 produces a scalar value that can be reduced to

\[
\frac{\partial x_k}{\partial C_i} = s(x_j - x_j')x_j - s(x_j - x_j')x_j'
\]  

(4.23)

### 4.2.1.4 Conductance Element: If the element shown in Figure-4.2 is a conductance \( G_i \) (where \( Y_{j,j'} = G_i \)) and the state variable of interest is \( x_k \) (where \( x_k = \phi \)), then equation 4.1 is re-written as

\[
\frac{\partial x_k}{\partial G_i} = (x^a)' \frac{\partial T}{\partial G_i} x - (x^a)' \frac{\partial W}{\partial G_i}
\]  

(4.24)

If the element is modeled as an admittance in the MNF then \( \frac{\partial T}{\partial G_i} \) is calculating as
\[
\begin{bmatrix}
0 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \ddots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \frac{\partial}{\partial G_i} (Y_{ij}) & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \ddots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \frac{\partial}{\partial G_i} (Y_{ij'}) & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \ddots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & \cdots & \ddots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\end{bmatrix}
\]

Row \(j\)

Row \(j'\)

\[
\frac{\partial \tau}{\partial G_i} =
\begin{bmatrix}
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\end{bmatrix}
\]

\(\frac{\partial \tau}{\partial G_i} = \begin{bmatrix} 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix} \quad (4.25)\)

Since

\[Y_{jj'} = G_i\]  \(\quad (4.26)\)

Therefore, \[\frac{\partial}{\partial G_i} (Y_{jj'}) = I\]  \(\quad (4.27)\)

There are four non-zero elements in \(\frac{\partial \tau}{\partial R_i}\), which can be represented by integer pointers instead of a matrix. The derivative of \(W\) vector with respect to \(G_i\) equals to zero.

\[\frac{\partial W}{\partial G_i} = 0\]  \(\quad (4.28)\)

Substituting 4.27 into 4.25 and substitute 4.24 & 4.28 into 4.24,
\[ \frac{\partial x_k}{\partial G_i} = \begin{bmatrix} x_i^a & x_j^a & \ldots & x_j^a & x_n^a & \ldots & x_n^a \end{bmatrix} \begin{bmatrix} 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & 0 & \ldots & 0 & 1 & 0 \\
\ldots & \ldots & \ldots & \ldots & 0 & 1 \\
\ldots & \ldots & \ldots & \ldots & 0 & 1 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\end{bmatrix} \begin{bmatrix} x_1 \\
\vdots \\
x_j \\
\vdots \\
x_n \\
\vdots \\
x_{n+m} \end{bmatrix} \]  

(4.29)

Equation 4.29 produces a scalar value and can be reduced to

\[ \frac{\partial x_k}{\partial G_i} = \{x_j^a - x_i^a\} x_j - \{x_j^a - x_i^a\} x_j. \]  

(4.30)

4.2.2 Sensitivity With Respect To System Frequency

One of the capabilities of the proposed MNFSA is to calculate the sensitivity of any system variable (voltage or current) to system frequency. Equation 3.12 shows that the complex system equations of the MNF can be structured into two real matrices as follows

\[ T x = W \]  

(4.31)

and

\[ T = G + sC \]  

(4.32)

Where G and C are real matrices correspond to the real and the imaginary parts of T respectively.
To calculate sensitivity of any system variable $x_k$ (where $x_k = \varphi$) with respect to system frequency equation 3.22 can be re-written as

$$\frac{\partial x_k}{\partial f} = (x^a)^t \frac{\partial T}{\partial f} x - (x^a)^t \frac{\partial w}{\partial f} \quad (4.33)$$

From equation 4.32

$$\frac{\partial T}{\partial f} = \frac{\partial}{\partial f} (G + sC) \quad (4.34)$$

$$\frac{\partial T}{\partial f} = \frac{\partial G}{\partial f} + \frac{\partial}{\partial f} (sC) \quad (4.35)$$

$$\frac{\partial T}{\partial f} = 2\pi C \quad (4.36)$$

$$\frac{\partial w}{\partial f} = 0 \quad (4.37)$$

Substitute equations 4.36 & 4.37 in 4.33

$$\frac{\partial x_k}{\partial f} = 2\pi (x^a)^t.C.x \quad (4.38)$$

Equation 4.38 produces a scalar value represents the sensitivity of any variable with respect to system frequency.
4.3 The MNFSA Algorithm

The parametric sensitivity equations derived in the previous section are summarized in Table 4.1 along with MNF stamps for different types of system parameters. These equations are used to develop the flowchart, shown in Figure 4.3, of the MNFSA algorithm. The proposed sensitivity analysis algorithm is developed into a Fortran program.
Table 4.1: Summary of Power System Elements Parametric Sensitivity Formulations

<table>
<thead>
<tr>
<th>No</th>
<th>System Parameter</th>
<th>MNF Stamp</th>
<th>MNFSA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>C</td>
<td>[ \begin{array}{c} \text{Col-}j' \ (V_{i}) \ \text{Row} j' \end{array} \begin{array}{c} sC_{ij} \ -sC_{ij} \ sC_{ij} \end{array} ]</td>
<td>[ \frac{\partial x_k}{\partial C_i} = s(x_j^a - x_j^a)x_j - s(x_j^a - x_j^a)x_j ]</td>
</tr>
<tr>
<td>2</td>
<td>L</td>
<td>[ \begin{array}{c} \text{Col-}j' \ (V_{i}) \ \text{Row} j' \end{array} \begin{array}{c} sL_{ij} \ -sL_{ij} \ sL_{ij} \end{array} ]</td>
<td>[ \frac{\partial x_k}{\partial L_i} = -sx_p^a x_p ]</td>
</tr>
<tr>
<td>3</td>
<td>G</td>
<td>[ \begin{array}{c} \text{Col-}j' \ (V_{i}) \ \text{Row} j' \end{array} \begin{array}{c} sG_{ij} \ -sG_{ij} \ sG_{ij} \end{array} ]</td>
<td>[ \frac{\partial x_k}{\partial G_i} = (x_j^a - x_j^a)x_j - (x_j^a - x_j^a)x_j ]</td>
</tr>
<tr>
<td>4</td>
<td>R</td>
<td>[ \begin{array}{c} \text{Col-}j' \ (V_{i}) \ \text{Row} j' \end{array} \begin{array}{c} sR_{ij} \ -sR_{ij} \ sR_{ij} \end{array} ]</td>
<td>[ \frac{\partial x_k}{\partial R_i} = -sx_p^a x_p ]</td>
</tr>
<tr>
<td>5</td>
<td>f</td>
<td>[ \begin{array}{c} \text{Col-}j' \ (V_{i}) \ \text{Row} j' \end{array} \begin{array}{c} sL_{ij} \ -sL_{ij} \ sL_{ij} \end{array} ]</td>
<td>[ \frac{\partial x_k}{\partial f} = 2 \pi \cdot \left( x^a \right)^t \cdot C \cdot x ]</td>
</tr>
</tbody>
</table>
Figure 4.3: Flowchart of the MNFSA Algorithm

Notes
1. \( k \) = Number of system parameters
2. \( f_{in}, f_{fin}, Df \): initial, final frequency and steps

- Start
  - Initialization
    - Read Input Data
    - Formulate \( T \) & \( W \)
      - \( f = f_{in} \cdot Df \)
    - Perform LU-Factorization to \( (T) \)
      - Fid \( x \): Forward & Backward Substitution \((Tx=W)\)
      - Find \( x^2 \): Forward & Backward Substitution \( (T'x^2=d) \)
  - Apply MNFSA Equations (What is the type of parameters?)
    - \( \frac{\partial \phi}{\partial C} \)
    - \( \frac{\partial \phi}{\partial L_i} = -s(x_i^u - x_i^v)x_i \)
    - \( \frac{\partial \phi}{\partial G} = \{x_i^u - x_i^v\}x_i \)
    - \( \frac{\partial \phi}{\partial R} = -\{x_i^u - x_i^v\}x_i \)
    - \( \frac{\partial \phi}{\partial f} = 2\pi(x^u)' \cdot C \cdot x \)
  - No \( i > k \)
    - Save output
  - Yes
    - No \( f > f_{fin} \)
    - Yes
      - Stop
4.4 MNFSA Program Input and output data format

The MNFSA program reads the input data from an ASCII-text file. The program output results are tabulated into an ASCII-text file that can be easily exported to other applications such as Excel. The sample circuit shown in Figure 4.4 is used to describe the structure of the input data and the output results. In the following sections the sequence and format of the input data and types and format of the output results for this example are detailed.

4.4.1 Input Data Structure

The input data consists of two main parts, general study data and system model details. For the system shown in Figure 4.4 the input data is shown in Table 4.2. The first part shows the general study data while the second part represents the system parameter data.
Figure 4.4: Sample System

Table 4.2 MNFSA Input Data For The Sample System

<table>
<thead>
<tr>
<th>NEL</th>
<th>NODES</th>
<th>INIT_FREQ</th>
<th>END_FREQ</th>
<th>STEP_FREQ</th>
<th>OUTNOD</th>
<th>RENODE</th>
<th>TYP</th>
<th>VBASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>4</td>
<td>0</td>
<td>400.0</td>
<td>50.0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>13800.0</td>
</tr>
</tbody>
</table>

SECTION II: SYSTEM PARAMETERS DATA

<table>
<thead>
<tr>
<th>ETYP</th>
<th>N1</th>
<th>N2</th>
<th>VAL</th>
<th>SC</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>1</td>
<td>2</td>
<td>0.0016</td>
<td>0</td>
</tr>
<tr>
<td>L2</td>
<td>2</td>
<td>4</td>
<td>0.03056</td>
<td>1</td>
</tr>
<tr>
<td>R1</td>
<td>3</td>
<td>0</td>
<td>5.36</td>
<td>0</td>
</tr>
<tr>
<td>R2</td>
<td>4</td>
<td>0</td>
<td>4.4</td>
<td>1</td>
</tr>
<tr>
<td>C1</td>
<td>2</td>
<td>3</td>
<td>0.207E-3</td>
<td>1</td>
</tr>
<tr>
<td>E1</td>
<td>1</td>
<td>0</td>
<td>13800.0</td>
<td>0</td>
</tr>
</tbody>
</table>

Where

NEL: Number of power system elements involved in the power system model.

NODES: Number of the power system nodes

INIT_FREQ: The initial frequency for the study
FINAL_FREQ: Final frequency

STEP_FREQ: Simulation frequency step

OUTNOD: Output variable; defines the output node voltage or branch current of interest

REFNOD: Defines the reference node (0 means ground)

TYP: Define the type of analysis (TYP=1 frequency scan, TYP=3 parametric sensitivity, TYP=4 frequency sensitivity)

BV: Base voltage. The program uses this value to normalize the parametric sensitivity.

ETYP: Type of element (L Inductance, R resistance, C capacitance, G Conductance, E Ideal voltage source, J Ideal current source)

N1: The “From” node or terminal

N2: The “To” node or terminal

VAL: Actual value of the parameter (L in Henry, C in Farad, R in Ohm, G in Siemens, Voltage source (E) in volts, Current Source (J) in Amperes)

SC: Sensitivity analysis control (If SA=1 the program will perform sensitivity with respect to the parameter, SA=0 the program will not carry sensitivity calculation with respect to this parameter)

As an example, in Table 4.2 the program is directed to perform sensitivity analysis for node-2 voltage ($V_1$) with respect to parameters $L_2$, $C_1$ & $R_2$ for frequency range from 0 to 350Hz in 50Hz steps.
4.4.2 Output Results

The MNFSA program calculates the sensitivity of any system variable (voltage or current), with respect to any circuit element parameter or the sensitivity of any system variable with respect to system frequency.

Table 4.3 shows the MNFSA output results that correspond to the input data file described in Table 4.2. In addition to calculating the sensitivity of any node voltage to any system parameter, the program can also calculate similar sensitivities to branch current variables. To calculate the sensitivities of load current \( I_1 \), output variable Output Variable in Table 4.2 is changed to \( \text{OUTNOD}=5 \), incorporating current variables in the MNF equations is described in detail in section 3.3. Table 4.4 shows the sensitivity of load current with respect to parameters \( L_2, C_1 \) and \( R_2 \).
### Table: 4.3 Sample Output For Parametric Sensitivity of Node-2 Voltage at Different Frequencies

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>SV2L2</th>
<th>SV2R2</th>
<th>SV2C1</th>
<th>SV2L2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00E+00</td>
<td>0.00E+00</td>
<td>0.00E+00</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>50</td>
<td>4.10E-02</td>
<td>1.88E-02</td>
<td>2.77E-02</td>
<td>4.10E-02</td>
</tr>
<tr>
<td>100</td>
<td>5.21E-02</td>
<td>1.19E-02</td>
<td>9.22E-02</td>
<td>5.21E-02</td>
</tr>
<tr>
<td>150</td>
<td>5.83E-02</td>
<td>8.90E-03</td>
<td>1.60E-01</td>
<td>5.83E-02</td>
</tr>
<tr>
<td>200</td>
<td>6.06E-02</td>
<td>6.95E-03</td>
<td>2.08E-01</td>
<td>6.06E-02</td>
</tr>
<tr>
<td>250</td>
<td>5.95E-02</td>
<td>5.46E-03</td>
<td>2.32E-01</td>
<td>5.95E-02</td>
</tr>
<tr>
<td>300</td>
<td>5.61E-02</td>
<td>4.29E-03</td>
<td>2.36E-01</td>
<td>5.61E-02</td>
</tr>
<tr>
<td>350</td>
<td>5.15E-02</td>
<td>3.37E-03</td>
<td>2.27E-01</td>
<td>5.15E-02</td>
</tr>
</tbody>
</table>

### Table: 4.4 Sample Output for Parametric Sensitivity of Source Current at Different Frequencies

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>SISL2</th>
<th>SISR2</th>
<th>SISC1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00E+00</td>
<td>2.85E-03</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>50</td>
<td>1.05E-03</td>
<td>4.81E-04</td>
<td>7.09E-04</td>
</tr>
<tr>
<td>100</td>
<td>6.67E-04</td>
<td>1.53E-04</td>
<td>1.18E-03</td>
</tr>
<tr>
<td>150</td>
<td>4.97E-04</td>
<td>7.60E-05</td>
<td>1.37E-03</td>
</tr>
<tr>
<td>200</td>
<td>3.88E-04</td>
<td>4.45E-05</td>
<td>1.33E-03</td>
</tr>
<tr>
<td>250</td>
<td>3.05E-04</td>
<td>2.80E-05</td>
<td>1.19E-03</td>
</tr>
<tr>
<td>300</td>
<td>2.39E-04</td>
<td>1.83E-05</td>
<td>1.01E-03</td>
</tr>
<tr>
<td>350</td>
<td>1.88E-04</td>
<td>1.23E-05</td>
<td>8.31E-04</td>
</tr>
</tbody>
</table>
The sensitivities shown in Tables 4.3 and 4.4 are normalized by multiplying the actual sensitivity with the parameter actual value divided by the base value. For example, the sensitivity of node 2 with respect to L₂ (SV2L2) is calculated based on the following equation:

\[ SV2L2 = \frac{\partial V_2}{\partial L_2} \cdot \frac{L_2}{BV} \]

Where SV2L2 is the sensitivity of node-2 voltage to system parameter L₂.

To calculate the sensitivity of node-2 voltage (V₂) with respect to system frequency, the only change to the input data is to change Type of analysis from (TYP=3) to (TYP=4). The results of the MNFSA are shown in Table 4.5. The results showed both the magnitude and the phase angle of the network sensitivity. Similar sensitivity for the source current (I₅), as shown in Table 4.6, can be obtained by choosing OUTNOD = 4.
Table: 4.5 Sample Output for Sensitivity of Node-2 Voltage with respect to Frequency

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>SV2F(Mag)</th>
<th>SV1F(Phase)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.07E+01</td>
<td>180.0001</td>
</tr>
<tr>
<td>50</td>
<td>1.05E+01</td>
<td>-115.95</td>
</tr>
<tr>
<td>100</td>
<td>2.53E+01</td>
<td>-148.211</td>
</tr>
<tr>
<td>150</td>
<td>3.24E+01</td>
<td>-173.659</td>
</tr>
<tr>
<td>200</td>
<td>3.46E+01</td>
<td>164.626</td>
</tr>
<tr>
<td>250</td>
<td>3.37E+01</td>
<td>146.2357</td>
</tr>
<tr>
<td>300</td>
<td>3.12E+01</td>
<td>130.7622</td>
</tr>
<tr>
<td>350</td>
<td>2.82E+01</td>
<td>117.7515</td>
</tr>
</tbody>
</table>

Table: 4.6 Sample Output for Sensitivity of Source Current with respect to Frequency

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>SISF Magnitude</th>
<th>SISF Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.26E+02</td>
<td>180.0001</td>
</tr>
<tr>
<td>50</td>
<td>2.81E+01</td>
<td>13.6923</td>
</tr>
<tr>
<td>100</td>
<td>1.44E+01</td>
<td>-51.4495</td>
</tr>
<tr>
<td>150</td>
<td>1.10E+01</td>
<td>-98.9134</td>
</tr>
<tr>
<td>200</td>
<td>9.31E+00</td>
<td>-135.511</td>
</tr>
<tr>
<td>250</td>
<td>8.03E+00</td>
<td>-164.238</td>
</tr>
<tr>
<td>300</td>
<td>6.93E+00</td>
<td>172.8298</td>
</tr>
<tr>
<td>350</td>
<td>5.98E+00</td>
<td>154.2364</td>
</tr>
</tbody>
</table>
4.5 Applications and Capabilities of the Algorithm

The MNFSA program is designed to perform a wide range of sensitivity analysis; more specifically the code is designed to compute the sensitivity of all system states variables (voltages and currents) with respect to any network parameters. Since this research is concerned with specific power system issues, only those network elements related to these issues are incorporated in the MNFSA program. These components are:

- Resistances
- Conductances
- Capacitors
- Inductors

4.5.1 Capabilities of the MNFSA Program

The MNFSA program can perform the following:

- Perform sensitivity analysis for any voltage with respect to any system parameter.
- Perform sensitivity analysis for any current with respect to any system parameter.
- Perform sensitivity analysis for any voltage with respect to system frequency
- Perform sensitivity analysis for any current with respect to system frequency
- Depending on the type of analysis, the program can perform the above sensitivities over a frequency or any parameter range
• The use of MNF provides extra flexibility of modeling ideal voltage sources and ideal switches.

4.5.2 Application and System Analysis

In the Chapter 5, the program will be utilized to perform the following analysis:

• Quantify the significance of each parameter on system response relative to other parameters

• Analyze the effect of changes in system parameter on sensitivities.

• Perform a quick assessment to system frequency response at different harmonic frequencies.

• The above analysis can be used in power system design optimization.

4.5.3 Computational Advantages

A close inspection to the above equations reveals the following:

1. Sensitivity of any system variable (branch current or node voltage) to any system parameter can be calculated.

2. The algorithm is generic for current and voltage variables, which means that the same equations are used for calculation of current and voltage sensitivities.

3. Inversion of $T$ is needed only once.

4. Although the size of system matrices has increased due to the use of MNF, the structure of system equations has been greatly simplified by modeling the system as a set of linear and first order differential equations.
CHAPTER V

APPLICATION OF THE MNFSA

5.1 Introduction

The mathematical basis and the programming aspect used to develop the proposed parametric MNF Sensitivity Analysis program (MNFSA) were presented in Chapters 3 and 4. In this chapter, the MNFSA program is applied in evaluating several power systems and to demonstrate the ability of the proposed method in analyzing power system issues related to harmonic, harmonic amplification, power system resonance and voltage stability. Application of the program to study these power systems issues involves calculating the parametric sensitivities and analyzing the results in order to identify the route causes of certain problems such as harmonic amplification as well as identifying the most effective mitigation methods to such problems.
5.2 Application of the MNFSA in Power System Analysis

Three power systems are evaluated using the proposed MNFSA. The first power system is a radial distribution system, the second power system is a 14-bus multi-machine industrial power system and the third system is the IEEE 30-bus test system. Two case studies are evaluated based on the first system, four case studies are based on the second system, and two based on the third system. Following are the case studies simulation and analysis using the MNFSA:

- Case study 1: A radial power system with a PFC at the load end
- Case study 2: A radial power system with a de-tuned capacitor at the load end
- Case studies 3-6: A 12-bus industrial power system supplied from local generation with different harmonic mitigation methods
- Case studies 7-8: The IEEE 30-Bus Test Power System

Results of the MNFSA are presented in graphical and tabular format. These results are evaluated to investigate the use of the MNFSA in analyzing certain power system problems and solution methods. In the following sections, a brief description of each system followed by presentation of the sensitivity analysis results then a discussion on the result from each case study.
5.3 Study System 1: Radial Systems

Radial systems are used to examine the ability of the proposed MNFSA in identifying circuit parameters that characterize power system phenomena such as parallel and series resonance and how these parameters influence system response at different operating conditions. In radial system, identifying these parameters can be done by inspection. In more complex systems however, identifying these parameters and establishing means to quantify the relative impact of varying these parameters on system response is usually a more difficult task. Furthermore, harmonic problems are usually localized to areas where the non-linear load is connected. Therefore, the analysis is usually performed on a reduced power system and, in many cases, this reduced system has a radial structure.

5.3.1 Application of the MNFSA to Study Problems Associated With PFC Installations in Industrial Systems

In the following cases, the MNFSA is applied to study the effect of installing a Power Factor Correction Capacitor (PFC) at the load bus. These capacitors are used in power systems to improve the efficiency at which electrical energy is utilized. A high power factor avoids penalties imposed by utilities and makes better use of the available
capacity of a power system. Introducing a capacitor in power system can lead to resonance phenomenon [47].

Resonance is a condition whereby the capacitive reactance of a system, offsets its inductive reactance leaving the resistive elements in the network as the only impedance. The frequency at which this offsetting effect takes place is called the resonant frequency of the system. Depending on how the reactive elements are arranged throughout the system, the resonance can be of a series or a parallel type. At the system's resonant frequency, the parallel combination of the capacitor bank and the source reactance appears as large impedance. If this frequency happens to coincide with one generated by the harmonic source, then dangerous voltages and currents will increase disproportionately, causing damage to capacitors and other electrical equipment.

Series resonance occurs when the capacitors are located toward the ends of feeder branches. The line impedance, in this case, appears in series with the capacitor, from the harmonic source perspective. At, or close to, the resonant frequency of this series combination, its impedance will be very low [48-49]. If the harmonic source generates currents near this resonant frequency, they will flow through this low impedance path causing interference in communication circuits along the resonant path, and excessive voltage distortion at the capacitor. Harmonic currents may load capacitors beyond their limit causing them to fail.

5.3.2 Case 1: Plain Power Factor Correction Capacitor

A radial system feeding a group of induction motors load (20MVA) operating at 60% lagging power factor is evaluated. The load is fed from a 13.8kV utility company via
a 15kV cable. Power Factor Correction compensation (PFC) of 15MVAR is connected at the load end. The single line diagram of the system is shown in Figure 5.1 and the Circuit diagram is shown in Figure 5.2. The system equipment data are as follows:

**Utility data**

Short Circuit Level (MVA$_{sc}$) = 350 MVA

$X/R$ ratio = 15

Voltage level = 13.8 kV

**Cable data**

Cable size 350 MCM, 3-Coconductors/Phase (3-1/350MCM)

Cable length = 3 km

Cable Impedance = $0.0427 + j0.0386$ ohm/km, $Y_c = 0.2$ $uF/km$

**Load data**

20 MVA Induction motors load

Power factor = 60% lagging

![Figure 5.1: Single Line Diagram for Case Study 1 (Plain PFC)](image-url)
5.3.2.1 Modified Nodal Formulation Modeling

*Utility Source Equivalent Impedance*

\[ Z_s = \frac{K V^2}{MVA_{sc}} \]
\[ Z_s = \frac{13.8^2}{500} = 0.38088 \, \text{Ohm} \]

\[ Z_s = \sqrt{R_s^2 + X_s^2} \]
\[ Z_s = X_s \sqrt{(R_s / X_s)^2 + 1} \]

\[ X_s = \frac{Z_s}{\sqrt{(R_s / X_s)^2 + 1}} = 0.38 \, \text{Ohm} \]

\[ L_s = L1 = \frac{X_s}{2\pi f} = 1.0078 \, \text{mH} \]

\[ R_s = R_l = X_s (R_s / X_s) = 0.0253 \, \text{Ohm} \]

*Cable impedance*

\[ R_c = 0.0427 \times 3 = 0.1281 \, \text{Ohm} \]

\[ L_c = L2 = \frac{X_c}{2\pi f} = \frac{0.0386 \times 3}{377} = 1.0078 \, \text{mH} \]

\[ C_c = C_2 = C_3 = 0.2 \times 3 = 0.6 \, \mu \text{F} \]

*Load equivalent impedance*

\[ Z_L = \frac{K V^2}{MVA_L} \]
\[ Z_L = \frac{13.8^2}{20} = 9.522 \, \text{Ohm} \]

\[ R_L = R_3 = p.f \times Z_L = 5.713 \, \text{Ohm} \]

\[ X_L = MVA_L \times \sqrt{1 - p.f^2} = 7.617 \, \text{Ohm} \]

\[ L_L = L_3 = \frac{X_L}{2\pi f} = 20.20 \, \text{mH} \]
Power Factor Correction Capacitor

\[ X_{C1} = \frac{K V^2}{MVAR_{PFC}} \]

\[ Z_L = \frac{13.8^2}{15} = 12.696 \text{ Ohm} \]

\[ C_{PFC} = C_1 = \frac{1}{2\pi f X_L} = 208.9 \text{ uF} \]

---

**Figure 5.2: Circuit Diagram for Case Study 1 (Plain PFC)**

5.3.2.2 Frequency Scan: Impedance frequency scan and voltage response of node 5 (load bus), shown in Figures 5.3 and 5.4 indicate that the system has a parallel resonance near the 5\textsuperscript{th} harmonic (315Hz). The circuit has an amplification factor of 14.02 p.u at the resonance frequency. Due to this amplification, the presence of a 5\textsuperscript{th} harmonic current source near this bus will lead to over voltages and possible damage to the PFC capacitor and other equipment. Nevertheless, this resonance frequency is very close to one of the commonly encountered characteristic harmonics that can be produced from nonlinear loads such as 6-pulse converter circuit.
5.3.2.3 Sensitivity Analysis Results: The MNFSA program is applied to calculate parametric sensitivities for node 5 at the 300 Hz and 60 Hz with and without the PFC. The parametric sensitivities are used to analyze the causes of the parallel resonance near the 300 Hz. Results shown in Table 5.1, shows a significant increase in the sensitivity of node-5 voltage to all system parameters at 300 Hz as compared to the 60 Hz case. The increase in the sensitivity is consistent with the frequency scan results and can be used as another means to locate the system resonance frequencies. In fact the sensitivity curves maintain the same profile as the frequency scan curve. This can be demonstrated by comparing node 5 the frequency scan curve shown in Figure 5.3 with node-5 parametric sensitivity versus frequency curves shown in 5.5. From these curves it is clear that the sensitivity curve can be used instead of normal frequency scan to calculate series and parallel resonance frequencies.

Moreover, results in Table 5.1 shows that \( V_5 \) sensitivities with respect to certain parameter are substantially higher than others. Fore example, in the case where the PFC is connected to the load bus, the sensitivities for the parameters \( C_1, L_1 \) and \( L_3 \) are 81.04, 58.33 and 5.232 respectively. Therefore parameters \( L_1, C_1 \) and \( L_2 \) are the main cause of the 300 Hz resonance. When the PFC removed from the system, the sensitivities shown in Table 5.1 are comparatively smaller at both the 60 Hz and the 300 Hz, which indicate that the 300 Hz is no longer a system resonance frequency.
Figure 5.3: Impedance Frequency Scan at Node 5

(Case 1: Radial System With a Plain PFC Capacitor)

Figure 5.4: Frequency Response For Node 5 Voltage

(Case 1: Radial System With a Plain PFC Capacitor)
Figure 5.5: Parametric Sensitivities of Node 5 Voltage versus Frequency

(Case 1: Radial System With a Plain PFC Capacitor)
Table 5.1: Sensitivity of load bus voltage (Node 5) to System Parameters at Different Frequencies with and without PFC

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<thead>
<tr>
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<th>Without PFC</th>
<th></th>
</tr>
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<td>60</td>
<td>300</td>
<td>60</td>
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<td>0.01273</td>
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<td>5.232</td>
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<td>0.05615</td>
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</table>

5.3.3 Confirmation of the MNFSA Program Results

To confirm the parametric sensitivities results obtained from the MNFSA program for the system shown in Figure 5.1, two scenarios in which the impact of small changes in C₁ and L₃ are evaluated. The parameters are chosen because the MNFSA identified them as the key system parameters causing harmonic amplifications at the 300 Hz and also because the value of these two parameters can be different under different operating conditions. It is not always practical to change cable reactance because cable impedance is related to the cable size, length and type.

5.3.3.1 Scenario 1: A 5 % Increase in C₁: The MNFSA results in Table 5.1 shows that V₅ is highly sensitive to C₁ at the 300 Hz frequency. This means for a very small change in this parameter there will be a large change in node 5 voltage. In this
scenario the parameter $C_1$ (PFC capacitance) is increased by 5% ($218.86\mu F$) and then $V_5$ is calculated with using the frequency scan subroutine.

Results shown in Table 5.2 indicates that an increase in the value of $C_1$ by 5%, which correspond to $10.4\mu F$, caused an increase to the magnitude of node-5 voltage by $76.2\text{kV}$ (60.6% increase in magnitude). This situation should be avoided since the system is very sensitive to the PFC capacitance changes at the $5^{th}$ harmonic frequency. It is advisable that the designer utilizes other measures to reduce the high sensitivity obtained, such as reducing the PFC capacitance or installs a series reactor, which will lead to shifting the resonance to other non-harmful frequency.

5.3.3.1 Scenario 2: A 5 % Increase in $L_3$: The MNFSA results in Table 5.1 shows that $V_5$ is much less sensitive to $L_3$ than $C_1$, therefore, the change will not result in significant change to node 5 voltage as compared to the first scenario. In this scenario the parameter $L_3$ (Load Reactance) is increased by 5% ($1.01.86\text{mH}$) and then $V_5$ is calculated with using the frequency scan subroutine.

In the first scenario, a 5% increase in the value of $C_1$, which correspond to $10.4\mu F$, resulted in increase to the magnitude of node-5 voltage by $76.2\text{kV}$ (60.6% increase in magnitude). While a similar percentage increase in $L_3$ (1.01mH) resulted in 2.44% decrease in $V_5$ magnitude. Figure 5.6 illustrates the effect of changing the PFC capacitance on the parametric sensitivities and on shifting the system resonance frequency. As the value of the PFC increases or decreases the resonance frequency is moved lower or higher. The pattern of the parametric sensitivity at the resonance frequency can also be obtained from the envelop of the sensitivities family of curves. As
the resonance frequency increases, the parametric sensitivity increases in an exponential manner.

Results from the MNFSA can also help in selecting the appropriate values of certain power system parameters. Figures 5.7 and 5.8 show the sensitivity of the load bus voltage and utility busses voltage to the PFC capacitance as a function of the PFC capacitance. As $C_1$ increases the parametric sensitivity increases in a non-linear fashion. The sensitivity is at its peak when the system resonance point coincides with the supply frequency. Further increase or decrease in $C_1$ will move the system resonance point away from the supply frequency and the sensitivity will start decreasing in a non-linear fashion. Due to the proximity of node-3 to the constant voltage source $E_1$, the sensitivity of the utility side bus voltage ($V_3$) has a lower value compared with the load side at all times.

Figure 5.9 shows the changes in the parametric sensitivities due to 50% ($+/-25\%$) variation in reactive load equivalent impedance. As the reactive load increases, due to increase or decrease in the load power factor, the sensitivity increases indicating that this load increase bring the system resonance frequency closer to the 300 Hz. This change in the sensitivity is more modest and closer to linearity than the change cased by $C_1$. From design perspective, the sensitivity curve shown in Figures 5.7, 5.8 and 5.9 can help in selecting adequate ranges for the PFC capacitance, to avoid harmonic amplification.
Table 5.2: Evaluation the Effect of Changing Parameters $C_1$ and $L_3$ on Actual Value of Node 5 Voltage at 300 Hz Frequency

| Scenario No.            | $V_5$ Magnitude Volt | $V_5$ Angle Degree | $|\Delta V_5|/|V_5|_{Base} \times 100$ |
|-------------------------|-----------------------|--------------------|---------------------------------|
| Normal Case             | 125832.6              | -38.52             | 0                               |
| Scenario 1: 5% Increase to $C_1$ | 171611.8              | -62.46             | 76243                           |
| Scenario 2: 5% Increase to $L_3$ | 129119.45             | -39.1              | 3072                            |

Figure 5.6: Parametric Sensitivities versus Frequency as the PFC Capacitance ($C_1$) Changes (Node 5 For Case 1)
Figure 5.7: Parametric Sensitivities versus PFC Capacitance
While the Frequency is Fixed at 300 Hz (Node 5 For Case 1)

Figure 5.8: Parametric Sensitivities versus PFC Capacitance
While the Frequency is Fixed at 300 Hz (Node 3 For Case 1)
Figure 5.9: Parametric Sensitivities versus Reactive Load Equivalent Impedance ($L_3$) (Node 5 For Case 1)
5.3.4 Case II: De-Tuned Capacitor

De-Tuned Capacitor (DTC) banks are normally considered for the purpose of power factor correction. The DTC will not have a significant contribution in removing exiting harmonic distortion but will allow the installation of a large capacitor bank without adverse system interactions. The DTC has an anti-resonance frequency that can be selected to prevent the system from having a resonance near a potential characteristic harmonic.

A DTC capacitor consists of a capacitor in series with an inductance. As shown in Figure 5.10, a DTC capacitor is connected at the load bus (node-5) of the system. In this case study the capacitive element is selected to provide 15 MVAR positive reactive injection while the reactive element tunes the circuit to 140 Hz. This frequency is selected since it is fare from the frequencies of harmonics commonly encountered in this type of industrial plants. The circuit diagram of the system is shown in Figure 5.11

![Figure 5.10: Single Line Diagram for Case Study 2 (De-tuned Capacitor)]
The specifications of the system elements are as follows:

**Utility data**

Short Circuit Level (MVA$_{SC}$) = 350 MVA

X/R ratio = 15

Voltage level = 13.8 kV

**Cable data**

Cable size 350 MCM, 3-Conductors/Phase (3-1/350MCM)

Cable length = 3 km

Cable Impedance = 0.0427 + j0.0386 ohm/km, $Y_c$ = 0.2 $uF/km$

**Load data**

20 MVA Induction motor load

Power factor = 60% lagging

5.3.4.1 Modified Nodal Formulation Calculations and Modeling

*De-Tuned Capacitor*

$$C_{cl} = \frac{1}{2\pi f X_{cl}} = 208.44 \text{ } uF$$

$$X_{cl} - X_{L4} = 0 \quad \text{(at 140 Hz)}$$

$$L_4 = \frac{1}{(2\pi f)^2 C_1} = \frac{1}{(2\pi f \times 140)^2 \times 208.44 \times 10^{-6}} = 6.2 \text{ } mH$$
5.3.4.2 Frequency Scan: Due to the DTC reactor the system resonance point can be shifted to a desired frequency. Impedance frequency scan and voltage response of node 5 (load bus), are shown in Figures 5.12 and 5.13 respectively. Both figures indicate that, the DTC has created a series resonance at 140 Hz. At this frequency, the system has a series resonance that was introduced by the DTC. The system has also a new parallel resonance near 130 Hz.

5.3.4.3 Sensitivity Analysis Results: Results of the sensitivity analysis performed on the system of Figure 5.11 are presented in Table 5.3. In this Table, two cases are evaluated, the de-tuned system and the un-tuned system (with $L_4$ short circuited). The un-tuned case evaluated at 300 Hz shows that the system has a harmonic resonance near 300 Hz, caused by $C_1$, $L_1$ and $L_2$. Parametric sensitivities of the de-tuned system show that at 60 and 300 Hz frequencies $C_1$, $L_1$ and $L_2$ are no longer key parameters to $V_5$. On the other hand at the 140 Hz, the MNFSA results has identified that, the de-tuned capacitor elements are the most significant system elements that influence $V_5$. These results also
illustrate that the DTC parameters are the key parameters for node 5 voltage ($V_5$). Furthermore, the sensitivity of $V_5$ with respect to $L_4$ equals to the sensitivity with respect to $C_1$. This can be explained by the fact that, the capacitor ($C_1$) and inductor ($L_4$) of the DTC have the same impedance magnitude at the resonance point, 140 Hz.

When the tuning reactor is removed from the DTC, the parametric sensitivity versus frequency curves shown in Figure 5.14, confirm that the system has a parallel resonance near the 300 Hz frequency. To remove this resonance, the circuit is de-tuned using the reactance of $L_4$. The sensitivities of node-5 voltage to DTC parameters verses frequency are shown in Figure 5.15.

In Figure 5.16 the system frequency is fixed at 130 Hz and value of the DTC capacitance gradually increased while $L_4$ is fixed. The sensitivity of node 5 voltage to $L_4$ and $C_1$ increased linearly until the circuit is tuned close to 130 Hz frequency. The sensitivity increases exponentially as the value of $C_1$ tune the system towards the resonance frequency.
Figure 5.12 Impedance Frequency Scan For Node 5
(Case 2: Radial System With A DTC Capacitor)

Figure 5.13 Frequency Response For Node 5 Voltage
(Case 2: Radial System With A DTC Capacitor)
Figure 5.14: Parametric Sensitivities versus Frequency
Un-Tuned System (Node 5 Voltage For Case 2)
Figure 5.15: Parametric Sensitivities versus Frequency

De-Tuned System (Node 5 Voltage For Case 2)
<table>
<thead>
<tr>
<th>Frequency</th>
<th>With De-tuned Capacitor</th>
<th>Un-tuned Capacitor</th>
</tr>
</thead>
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<td></td>
<td>130 (Hz)</td>
<td>140 (Hz)</td>
</tr>
<tr>
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<td>0.003872</td>
</tr>
<tr>
<td>SV5R1</td>
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<td>0.000111</td>
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<td>SV5L2</td>
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<td>SV5R2</td>
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<td>SV5R3</td>
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<td>SV5L3</td>
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<td>SV5L4</td>
<td>70.47</td>
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<tr>
<td>SV5C1</td>
<td>81.74</td>
<td>4.671</td>
</tr>
</tbody>
</table>
Fig 5.16: Parametric Sensitivities versus DTC Capacitance at 130 HZ
L₄ is Fixed (Node 5 For Case 2)
5.3.5 Discussion of the Radial System Cases

Frequency scan analyses were performed to study the effect of installing the PFC on the system frequency response. If there is a resonance frequency at or close to a nearby harmonic source frequency, then there is a potential of harmonic amplification and over voltages. The frequency scan however, does not provide information on what are the system elements that are causing this resonance? How changes in these parameters change the resonance? Addressing these questions, using the MNFSA will help in identifying the exact causes of the resonance problem and assist in devising appropriate solutions.

The MNFSA was applied to locate the system resonance frequency. Furthermore the sensitivity analysis identified system parameters that are causing this phenomenon and the effect of changing these parameters on system response. In the previous case studies, the MNFSA was used to study different approaches of PFC installations in industrial systems. The results of the MNFSA were confirmed using small perturbation and the frequency scan analyses.

5.4 Study System 2: A 14-Bus Power System

In the following three(3) case studies, the MNFSA is applied to analyze a 14-bus (25 Nodes), three (3)-generators industrial distribution system.
5.4.1 Description of the Power System

The MNFSA is used to analyze an industrial distribution system that consists of three main substations interconnected via a 34.5kV sub-transmission system and operates at three different voltage levels, 34.5kV, 13.8kV and 4.16kV. Two of the substations have local generation connected at the 13.8kV voltage level. Loads are connected at the 4.16kV and 13.8kV levels and consist of a combination of large and small induction motors in addition to a large UPS system located at Substation 2. The one-line diagram of the system is shown in Figure 5.17 and the specifications of the power system elements are as follows:

Load Centers

One 8000 Hp Induction Motor operating at 80% PF

Four small load centers as follows

<table>
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<th>MVA</th>
<th>PF</th>
</tr>
</thead>
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<tr>
<td>Load Center 1</td>
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</tr>
<tr>
<td>Load Center 2</td>
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<tr>
<td>Load Center 3</td>
<td>0.5</td>
</tr>
<tr>
<td>Load Center 4</td>
<td>1</td>
</tr>
</tbody>
</table>

Generation

Three generation units

\[ 2 \times 10 \text{ MW} \ 60 \text{ Hz}, \ 13.8 \text{ kV}, \ 80\% \text{ pf}, \ X_s = 8\% \]

\[ 1 \times 15\text{MW} \ 60\text{Hz}, \ 13.8 \text{kV}, \ 80\% \text{ pf}, \ X_s=7.5\% \]
The two-(2) 10 MW-Generators (Gen #1 and Gen #2) are located in Substation 2 and the 15 MW-Generator (Gen# 3) is located in Substation-1.

**Cables:** 3-35kV Submarine Cables, Size 500MCM 1-C/Pase

\[ Z_{c2} = 0.178 + j0.23, \quad Y_{c1} = j3.88 \text{ mS} \]
\[ Z_{c3} = 0.089 + j0.1152, \quad Y_{c1} = j2.228 \text{ mS} \]
\[ Z_{c4} = 0.089 + j0.1152, \quad Y_{c4} = j2.228 \text{ mS} \]

2-15kV Cables, size 350MCM 1-C/Pase

\[ Z_{c1} = 1.540 + j1.338, \quad Y_{c1} = j0.05 \text{ mS} \]
\[ Z_{c6} = 0.50 + j0.4, \quad Y_{c6} = j0.00 \]

1-5 kV Cable, size 500 MCM 1-C/Phase

\[ Z_{c5} = 1.69 + j1.244, \quad Y_{c5} = j3.77 \text{ uS} \]

**Transformers**

1-13.8/4.16kV 2.5/3.125 MVA, Z=5.8% Transformers

2-4.16/34.5kV 15/20 MVA, Z=8.1% Transformers

2-34.5/13.8kV 25/33 MVA, 6.5% Transformers

4-13.8/4.16 15/20 MVA, Z=10% Transformers

**5.4.2 The Modified Nodal Formulation Calculations and Modeling**

Parameters of each of the power system elements described in the previous section are calculated and summarized in Table 5.8. The circuit diagram for the MNFSA modeling is presented in Figure 5.18. The 14-bus power system is converted to a 25-nodes circuit with 43 circuit parameters. These parameters constitute the basic models for each of the power system elements. In this system, a cable is modeled using its π
equivalent model, a generator is modeled using its synchronous reactance in series behind a constant e.m.f, a transformer is modeled by its leakage reactance and loads are modeled as a constant impedance as shown in Figure 5.19.
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<td>H</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R8</td>
<td>19</td>
<td>16</td>
<td>1.780E-01</td>
<td>Ohm</td>
</tr>
<tr>
<td></td>
<td>CABLE3</td>
<td>C5</td>
<td>9</td>
<td>0</td>
<td>5.910E-06</td>
<td>F</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C6</td>
<td>16</td>
<td>0</td>
<td>5.910E-06</td>
<td>F</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>16</td>
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<td>H</td>
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<td></td>
<td></td>
<td>R5</td>
<td>9</td>
<td>15</td>
<td>8.900E-02</td>
<td>Ohm</td>
</tr>
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<td>9</td>
<td>0</td>
<td>5.910E-06</td>
<td>F</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C4</td>
<td>11</td>
<td>0</td>
<td>5.910E-06</td>
<td>F</td>
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<td></td>
<td></td>
<td>L8</td>
<td>10</td>
<td>11</td>
<td>3.056E-04</td>
<td>H</td>
</tr>
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<td></td>
<td></td>
<td>R4</td>
<td>9</td>
<td>10</td>
<td>8.900E-02</td>
<td>Ohm</td>
</tr>
<tr>
<td></td>
<td>CABLE5</td>
<td>C10</td>
<td>21</td>
<td>0</td>
<td>1.000E-08</td>
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<td></td>
<td></td>
<td>C9</td>
<td>8</td>
<td>0</td>
<td>1.000E-08</td>
<td>F</td>
</tr>
<tr>
<td></td>
<td></td>
<td>L16</td>
<td>8</td>
<td>20</td>
<td>3.300E-03</td>
<td>H</td>
</tr>
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<td></td>
<td></td>
<td>R10</td>
<td>20</td>
<td>21</td>
<td>1.697E00</td>
<td>Ohm</td>
</tr>
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<td></td>
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<td>24</td>
<td>25</td>
<td>1.330E-03</td>
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<td></td>
<td></td>
<td>R11</td>
<td>25</td>
<td>12</td>
<td>4.000E-01</td>
<td>Ohm</td>
</tr>
</tbody>
</table>
Figure 5.19: Typical Equivalent Power System Elements Models for MNFSA
5.4.3 Case Study 3: Base Case for the 14-Bus Industrial System

The first case to be evaluated using the MNFSA is the base case in which the power system is simulated using the circuit diagram shown in Figure 5.16 and the data presented in Table 5.8. In the following sections, frequency scan is performed to calculate the basic system frequency response, resonance frequencies and potential over voltages. The MNFSA then applied to analyze the base case for any power system problems related to harmonic amplification resonance, and harmonic over voltages to finds the causes of the problems. In cases 4-6 the MNFSA is applied to evaluate alternative solutions and mitigation methods to these problems.

5.4.3.1 Frequency Scan Analysis: Frequency scan analysis is performed at different busses in the power system to calculate the system frequency response at these locations. The frequency scan identifies system resonance frequencies and calculate the amount of harmonic amplification at these frequencies. On the other hand, harmonic surveys are usually performed to detect the presence of any harmonics in the system and identify their magnitude, phase and frequency. Availability of a harmonic source with a frequency near to a system parallel resonance frequency, will give rise to harmonic amplification and over-voltages problems. In this power system, a seventh (7th) and fifth (5th) harmonic sources are expected at Bus# bb1 where a large non-linear load is connected.

For the system of Figure 5.19, frequency scan analyses are performed at the location, of the non-linear load. The magnitude and phase angle of system impedance as seen from Bus# bb1 are shown in Figures 5.20 and 5.21. The frequency scan results show that the system has a parallel resonance at around 415 Hz, which very
close to the $7^{th}$ harmonic frequency in a 60 Hz power system. As shown in Figure 5.20, this parallel resonance will cause a significant amplification to the $7^{th}$ harmonic voltage.
Figure 5.20 Impedance Frequency Scan For Bus # bb1

(Case 3: 14 Bus Industrial System, Base Case)

Figure 5.21 Voltage Frequency Response at Bus # bb1

(Case 3: 14 Bus Industrial System, Base Case)
5.4.3.2 Parametric Sensitivity Analysis: The MNFSA program is applied to calculate the power system parametric sensitivities in order identify the parameters that are creating the 420 Hz parallel frequency. Analyzing parametric sensitivities will also facilitate the identification of the appropriate means for dealing with this phenomenon.

Parametric sensitivities were calculated for bus voltages at selected location in each of the system substations, bb1, bb4, bb8 and bb10 and the results are presented in Figures 5.22, 5.23, 5.24 and 5.25. Parametric sensitivities at the 60 Hz and 420 Hz are shown in Table 5.9. The 420 Hz was selected because it is the frequency of concern while the 60 Hz was selected to serve as a base for comparison purpose.

Results presented in Table 5.9, indicates a significant increase in all parametric sensitivities at the 420 Hz as compared with the 60 Hz case. This is a clear indication from the MNFSA results that the 420 Hz frequency is very close to a parallel resonance point. There are certain parameters that cause higher sensitivities than others. To focus the analysis and to identify the main causes of the resonance problem, ten parameters are selected for further analysis and summarized in Table 5.10. These parameters were selected because they are causing the highest sensitivities at the 7th harmonic level. The following can be observed by comparing the parametric sensitivities summarized in Table 5.10:

1. The results indicate that bb8 has the highest parametric sensitivities among the other busses. This bus is located in Substation# 3 which is not loaded and do not have local generation.
2. The synchronous reactance of Generator 3 ($L_{17}$), which is the largest generator, has the highest parametric sensitivities at the 420 Hz frequency. Therefore, in order to make a significant change in the frequency response of the system, the reactance between Generator#3 voltage source ($E_3$) and the system should be changed.

3. The charging capacitances of the 34.5kV cable 2 have the highest sensitivities among the capacitive parameters of the system. This cable is 10,000 ft long while the other two 34.5kV cables are 5,000 ft long each.

These observations help in understanding the system frequency response and hence clear alternative solutions to reduce harmonic amplification and shift the resonance frequency are identified by correlating these phenomena to relevant system parameters. In the following cases, these observations are utilized to devise practical solutions to the problems of this particular power system.
Table 5.9: Parametric Sensitivities for The 14 Bus Industrial System at 60 and 420 Hz With Respect to Different bus Voltages

<table>
<thead>
<tr>
<th>System Parameter</th>
<th>Bus # bb1</th>
<th>Bus # bb4</th>
<th>Bus # bb8</th>
<th>Bus # bb10</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>1.79E-05</td>
<td>6.81E-06</td>
<td>0.00234</td>
<td>3.84E-06</td>
</tr>
<tr>
<td>C10</td>
<td>1.01E-06</td>
<td>1.05E-05</td>
<td>1.27E-05</td>
<td>2.6E-06</td>
</tr>
<tr>
<td>C2</td>
<td>4.36E-05</td>
<td>1.66E-05</td>
<td>1.1E-9</td>
<td>9.34E-06</td>
</tr>
<tr>
<td>C3</td>
<td>0.000496</td>
<td>0.00227</td>
<td>0.00476</td>
<td>0.0215</td>
</tr>
<tr>
<td>C4</td>
<td>0.000469</td>
<td>0.00215</td>
<td>1.98E-05</td>
<td>0.0219</td>
</tr>
<tr>
<td>C5</td>
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<td>0.0215</td>
</tr>
<tr>
<td>C6</td>
<td>0.000487</td>
<td>0.00223</td>
<td>0.0202</td>
<td>0.0217</td>
</tr>
<tr>
<td>C7</td>
<td>0.000818</td>
<td>0.00375</td>
<td>0.0117</td>
<td>0.0383</td>
</tr>
<tr>
<td>C8</td>
<td>0.00085</td>
<td>0.00389</td>
<td>0.000115</td>
<td>0.0378</td>
</tr>
<tr>
<td>C9</td>
<td>1.17E-06</td>
<td>5.36E-06</td>
<td>5.11E-06</td>
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<tr>
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</tr>
<tr>
<td>L11</td>
<td>2.3E-10</td>
<td>1.05E-09</td>
<td>0.00002</td>
<td>1.02E-09</td>
</tr>
<tr>
<td>L12</td>
<td>2.15E-06</td>
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<td>L13</td>
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<td>0.000375</td>
<td>1.28E-09</td>
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<tr>
<td>L14</td>
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<td>3.05E-10</td>
<td>0.000426</td>
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</tr>
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<td>1.24E-05</td>
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<tr>
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<td>0.00907</td>
<td>0.000909</td>
<td>157.5</td>
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<tr>
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<td>0.0612</td>
<td>0.00768</td>
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<td>0.00603</td>
<td>0.000373</td>
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<td>0.000127</td>
<td>9.39E-05</td>
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<td>0.000431</td>
<td>2.19</td>
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<tr>
<td>L4</td>
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<td>0.000335</td>
<td>0.00256</td>
<td>0.0189</td>
</tr>
<tr>
<td>L5</td>
<td>0.00821</td>
<td>0.00312</td>
<td>0.00167</td>
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<tr>
<td>L6</td>
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<td>0.0127</td>
<td>0.00584</td>
<td>125</td>
</tr>
<tr>
<td>L7</td>
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<td>0.00301</td>
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<td>------------------------</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>BB1</td>
<td>BB4</td>
<td>BB8</td>
<td>BB10</td>
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<td>65.24</td>
<td>48.2</td>
</tr>
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<td>67.69</td>
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<td>82.3</td>
<td>110</td>
<td>81.4</td>
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<tr>
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<td>88.3</td>
<td>118.1</td>
<td>87.2</td>
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<td>60.7</td>
<td>81.1</td>
<td>58</td>
</tr>
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<td>L17</td>
<td>24.1</td>
<td>118</td>
<td>157.5</td>
<td>118</td>
</tr>
<tr>
<td>L19</td>
<td>16</td>
<td>78.4</td>
<td>104.7</td>
<td>78.3</td>
</tr>
<tr>
<td>L6</td>
<td>10.1</td>
<td>55.6</td>
<td>125</td>
<td>53.8</td>
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</table>
Figure 5.22 Parametric Sensitivities Versus Frequency For # bb1
(Case 3: 14 Bus Industrial System, Base Case)

Figure 5.23 Parametric Sensitivities Versus Frequency For # bb4
(Case 3: 14 Bus Industrial System, Base Case)
Figure 5.24 Parametric Sensitivities Versus Frequency For # bb8
(Case 3: 14 Bus Industrial System, Base Case)

Figure 5.25 Parametric Sensitivities Versus Frequency For # bb10
(Case 3: 14 Bus Industrial System, Base Case)
5.4.4 Case 4: Connecting A Small Load to Substation 3

As presented in the system input data, Table 5.8, the connected load at Substation 3 is practically zero. As a result of this loading condition, parametric sensitivities summarized in Table 5.10 shows that Bus #bb8 voltage has the highest sensitive among the other busses. This can be observed by comparing the sensitivities of different busses with respect to the same parameter. For example, Table 5.10 shows the sensitivity of Bus #bb1 to L_{17} is 24 pu while the sensitivity of Bus #bb8 to the same parameter equals to 157.5 pu.

Connecting a small resistive load of 0.1 MW to Bus# bb1 at Substation 3 will not shift the resonance point, but will reduce the parametric sensitivities of this node, and all other system nodes and reduce the amount of harmonic amplification throughout the system. As shown in Figure 5.27, the sensitivity of bb8 to parameter L_{17} is reduced from 157 pu to 135pu due to this load addition. The sensitivities of Busses bb1, bb4 and bb10 to parameter L_{17} with and with out the 0.1MW load at bb8 are presented in Figures 5.26, 5.28 and 5.29 respectively. The effect of adding this small load on the voltage amplification can be observed by comparing the frequency response results for Bus # bb1 voltage presented in Figures 5.30. The voltage amplification was reduced from 4.2pu in the no load case to 3.6 pu with 0.10MW at bb8.
Figure 5.26 Sensitivity of Bus#bb1 Voltage to Parameter L_{17} With and Without the 0.1 MW Load at Bus#bb8 (Case 4: 14 Bus Industrial System)

Figure 5.27 Sensitivity of Bus#bb4 Voltage to Parameter L_{17} With and Without the 0.1 MW Load at Bus#bb8 (Case 4: 14 Bus Industrial System)
Figure 5.28 Sensitivity of Bus#bb8 Voltage to Parameter L_{17} With and Without the 0.1 MW Load at Bus#bb8 (Case 4: 14 Bus Industrial System)

Figure 5.29 Sensitivity of Bus#bb10 Voltage to Parameter L_{17} With and Without the 0.1 MW Load at Bus#bb8 (Case 4: 14 Bus Industrial System)
Figure 5.30 Frequency Response For Bus # bb1 Voltage with and Without the 0.1-MW Load at Bus# bb8 (Case 4: 14 Bus Industrial System)
5.4.5 Case 5: Reducing System Capacitance by Taking Cable-2 Out of Service

The capacitance available in the power system will be reduced and hence, the resonance point will be shifted to a new higher resonance frequency as a result of removing any of the 34.5kV cables from the system. Since cable 2 capacitances were identified by the MNFSA as the one that causes the highest sensitivities, removing this cable will have the highest impact on the system frequency response. This action will reduce the system capacitance and hence the system resonance frequency will be shifted to a higher frequency.

As shown in Figures 5.31 to 5.34, the new resonance point is shifted to 580 Hz when cable-2 is taken out of service. This frequency is between the 9th and 10th harmonic. Results of the parametric sensitivities when cable#2 is taken out from the system are presented in Table 5.11. The results demonstrate that, small deviation in any critical system parameter, at the 420 Hz frequency, will have minimal effect on the system frequency response. Furthermore, these results indicate that the new resonance frequency is near the 9th harmonic, which is consistent with the frequency scan results. The possibility of harmonic amplification is minimized because triple order harmonic current, such as 9th harmonic, can be contained using proper transformer’s connections, and they are generally of no major concern.
Figure 5.31 Impedance Frequency Scan For Bus # bb1

(Case 5: 14 Bus Industrial System, Cable 2 out)

Figure 5.32 Impedance Frequency Scan For Bus # bb4

(Case 5: 14 Bus Industrial System, Cable 2 out)
Figure 5.33 Impedance Frequency Scan For Bus # bb8

(Case 5: 14 Bus Industrial System, Cable 2 out)

Figure 5.34 Impedance Frequency Scan For Bus # bb10

(Case 5: 14 Bus Industrial System, Cable 2 out)
<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>BB1</th>
<th>BB4</th>
<th>BB8</th>
<th>BB10</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>420</td>
<td>560</td>
<td>420</td>
<td>560</td>
</tr>
<tr>
<td>L5</td>
<td>0.1009</td>
<td>1.045</td>
<td>0.03546</td>
<td>1.151</td>
</tr>
<tr>
<td>L17</td>
<td>0.1133</td>
<td>4.656</td>
<td>0.3376</td>
<td>13.88</td>
</tr>
<tr>
<td>L6</td>
<td>0.05919</td>
<td>2.091</td>
<td>0.2267</td>
<td>7.232</td>
</tr>
<tr>
<td>L7</td>
<td>0.01362</td>
<td>0.6004</td>
<td>0.04058</td>
<td>1.789</td>
</tr>
<tr>
<td>L10</td>
<td>0.05901</td>
<td>2.399</td>
<td>0.1758</td>
<td>7.15</td>
</tr>
<tr>
<td>C5</td>
<td>0.09421</td>
<td>3.709</td>
<td>0.2807</td>
<td>11.06</td>
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<td>0.09651</td>
<td>3.875</td>
<td>0.2876</td>
<td>11.55</td>
</tr>
<tr>
<td>C3</td>
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<td>3.709</td>
<td>0.2807</td>
<td>11.06</td>
</tr>
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<td>L19</td>
<td>0.07535</td>
<td>3.097</td>
<td>0.2245</td>
<td>9.229</td>
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</tbody>
</table>
5.4.6 Case 6: Installing a De-Tuned Capacitor at Bus bb3

One of the common methods of shifting resonance frequency in power systems is to install a De-Tuned Capacitor (DTC). A 0.5MVAR, tuned to 402Hz (6.7*60Hz) is connected bus #bb3 to shift the system resonance points to non-characteristic frequencies. The parameters of the DTC are as follows:

De-Tuned Capacitor

\[ X_{c15} = \frac{KV^2}{MVAR_{DTC}} = 380.88 \text{ Ohm} \]

\[ C_{15} = \frac{1}{2\pi f X_{c15}} = 6.964 \text{ uF} \]

\[ X_{c1} - X_{L4} = 0 \quad \text{(at 402 Hz)} \]

\[ L_{20} = \frac{1}{(2\pi f)^2 C_{15}} = \frac{1}{(2\pi f \times 402)^2 \times 6.964 \times 10^{-6}} = 22.5 \text{ mH} \]

The MNFSA is applied to study the affect of installing the DTC on shifting the system resonance frequency. Table 5.12 shows the parametric sensitivity results at characteristic harmonics of concern, namely 5\text{th} and 7\text{th} harmonics. These results indicate that, with the DTC the parametric sensitivities have been reduced significantly, indicating that the 420 Hz is no longer the critical frequency of the system. This conclusion is confirmed from the bb1 frequency scan presented in Figure 5.35, which shows that the DTC has shifted the system frequency resonance to 340Hz and 475Hz. As a result of this shift, the system impedance as seen from Bus# bb1 at the 420 Hz is reduced from 8.3 ohm to 4.8 ohm. The MNFSA also confirms that the DTC did not introduce a new resonance point near the other characteristic harmonic frequency, in this case the main concern is the 5\text{th} harmonic.
Figure 5.35 Frequency Scan at Bus # bb 1

(Case 6: 14 Bus Industrial System: With and Without the DTC at bb3)
Table 5.12: Parametric Sensitivities for The 14 Bus Industrial System Parameters With the DTC at Bus # bb8

<table>
<thead>
<tr>
<th>Parameter</th>
<th>BB1</th>
<th></th>
<th>BB4</th>
<th></th>
<th>BB8</th>
<th></th>
<th>BB10</th>
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<td>300Hz</td>
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<td>300Hz</td>
<td>420Hz</td>
<td>300Hz</td>
<td>420Hz</td>
</tr>
<tr>
<td>C15 (DTC)</td>
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<td>2.391</td>
<td>8.413</td>
<td>1.769</td>
<td>6.223</td>
</tr>
<tr>
<td>L20 (DTC)</td>
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<td>1.411</td>
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<td>6.791</td>
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<td>0.1876</td>
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5.4.7 Discussion of the 14-bus Case study results:

The MNFSA is applied to investigate harmonic amplification problems in a 14-bus, multi-machine power system. The MNFSA was used as a tool to analyze a harmonic amplification and resonance problems, identify the root causes of the problem and help in evaluating different alternative solutions. Frequency scan revealed that under some operating conditions the system has a parallel resonance very close to the 420 Hz frequency. Since a 7th harmonic current producing source exist at bb1, the presence of this resonance can result in harmonic amplification and subsequent damage or mal operation of some electrical equipment. The indices obtained from the MNFSA program was used to:

1. Help in identifying design deficiencies or abnormal operating condition by analyzing relative difference between the parametric sensitivities of similar system states at different locations in power system. In case 5, high sensitivities of Bus# bb8 voltages, helped in identifying the light loading condition of Substation 3 as the main cause of the excessive harmonic amplification.

2. The MNFSA identified the parameters that caused the resonance problem and ranked them based on their effect on the system response. In this system the 34.5kV charging capacitances, mainly line-2, with generator#3 synchronous reactance and the leakage reactance of cable 6 were the main causes of the problem.
3. The MNFSA provided a quantitative approach to determine how the interaction between the system capacitive and reactive elements caused the 420 Hz parallel resonance and harmonic amplification problems.

4. MNFSA help in evaluating mitigation methods to resonance and harmonic amplification problems. Finding adequate solutions and mitigation method to these problems is possible, once the main parameters were identified and sufficient knowledge on how changes of these parameters affect the system is gained.

5.5 Study System 3: The IEEE 30-Bus Power System

The MNFSA program is applied on the IEEE 30-bus system. Two case studies are evaluated using the MNFSA program and frequency scan analysis program.

5.5.1 Description of the IEEE 30-Bus Power System

The IEEE 30-bus system is part of the American Electric Power Service Corporation Network that is used as a standard test system to study different power system problems and evaluate programs designs to analyze such problems [50]. The one line diagram of the system is shown in Figure 5.36 and the system parameters are presented in Tables 5.13 to 5.16. The MNFSA input data shows that the system consists of 91-nodes and 155-circuit elements.
Table 5.13: Branch Data for the IEEE 30-Bus Test System

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<th>Line ID</th>
<th>FROM BUS</th>
<th>TO BUS</th>
<th>Phantom Node</th>
<th>R (pu)</th>
<th>X(pu)</th>
<th>1/2 B(pu)</th>
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Table 5.14: Load Data for the IEEE-30 Bus Test System

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### Table 5.15: Machines Data for the IEEE 30-Bus Test System

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<th>P Gen MW</th>
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### Table 5.16: Shunt Capacitor Data for the IEEE 30-Bus Test System

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5.5.2 Case 7: IEEE 30-Bus System Base Case

5.5.2.1 Frequency Scan Analysis: Frequency scan analysis performed at bus 10, where the 19MVAR capacitor bank is connected and the result is presented in Figure 5.37. From the frequency scan results, it can be seen that this power system has a resonance frequency at 380Hz. There are two other resonance points with significantly lower impedances at 640Hz and 840Hz. As a result of the 380Hz resonance, the system has significantly high impedance at 420Hz (7th harmonic) and may cause some harmonic amplification if such harmonic exist in the power system.

5.5.2.2 Sensitivity Analysis: The MNFSA is applied to evaluate the power system at the characteristic harmonics of concern, namely the 5th, 7th, 11th and the 13th harmonics. The results are presented in Table 5.17. The MNFSA indicates C25 (the 19MVA Capacitor bank) has the highest parametric sensitivity and that bus-10 voltage has a relatively high sensitivity to C25 at the 420Hz frequency. This result is confirmed from the frequency scan analysis, which indicates that the system has high impedance at this frequency.
Figure 5.37 Frequency Scan at Bus # 10

(Case 7: IEEE 30-Bus Test System: Base Case)
Table 5.17: Parametric Sensitivities for The
IEEE 30-Bus Test System (Base Case)

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5.5.3 Case 8: Additional Reactive Power Injection at Bus 10

5.5.3.1 Frequency Scan Analysis: In this case study the capacitor bank at bus-10 is adjusted to create a system resonance at a characteristic harmonic frequency. A 30MVAR capacitor in lieu of the 19MVAR in the original system was connected to bus 10. As shown in Figure 5.38, this change resulted in shifting the system resonance frequency from 380Hz to 300Hz.

5.5.3.2 Sensitivity Analysis: The system was intentionally tuned to create a parallel resonance near the 5th harmonic. This was done to demonstrate MNFSA capability to detect such phenomena on the IEEE 30-Bus test system. Parametric sensitivities at the characteristic harmonics of concern are summarized in Table 5.18. At the 300Hz frequency, the sensitivity of bus-10 voltage to C25 has increased from 3.5 pu in case#7 to 64.8 pu in case#8, indicating that the system has a resonance point close or at the 5th harmonic.
Figure 5.38 Frequency Scan at Bus # 10

(Case 7: IEEE 30-Bus Test System: 30MVAR Shunt Capacitor at Bus 10)
<table>
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<th>System Parameter</th>
<th>60</th>
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<th>660</th>
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<td>0.02959</td>
</tr>
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CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 CONCLUSIONS

1. A Parametric Sensitivity Analysis approach based on the Modified Nodal Formulation for power system analysis (MNFSA) has been proposed. The MNFSA approach provides the following:

- Help in understanding how parametric variation of power system equipment such as motor, generators, transformer, cable and PFC capacitors influences the power system response.

- Provide a tool to help system designers and operation engineers in comparing the quality of alternative power system design having the same purpose. The tool can also provide an assessment mechanism to changes in the power system due to addition of new component such as power factor correction and harmonic filters.
• Provide insight into power quality issues specifically in systems with non-linear loads to which PFC capacitors are applied.

2. The proposed MNFSA formulation has a modular and general mathematical algorithm that can incorporate the parameters of all power system elements.

3. The MNFSA algorithm was translated into a Fortran code computer program that was applied to study power system resonance and harmonic over-voltages and amplification problems. The program can perform the following parametric sensitivity calculations:
   • Sensitivity of any power system voltage with respect to any system parameter.
   • Sensitivity of any power system current with respect to any system parameter.
   • Sensitivity of any power system voltage with respect to system frequency.
   • Sensitivity of any current with respect to system frequency.

4. The MNFSA program was applied as a power system analyses tool to perform the following engineering studies on different power systems:
   • Study the causes and suggest mitigations for harmonic amplification and over voltage problems that can arise when installing a Power Factor Correction Capacitor in a power system. The MNFSA was applied to study a typical case of a PFC installation in an industrial power system that exhibited such
problems. The sensitivity approach was used identify the system parameters that were causing the resonance and harmonic amplification problems. The sensitivity indices obtained from the program helped in selecting the adequate, type and design parameters for the PFC installation.

- Analyze harmonic amplification and over voltage problems in an industrial power system. The system was found to have a parallel resonance at the 7th harmonic frequency, which coincided with a harmonic generated from a nonlinear load.

- The MNFSA was applied on the IEEE-30 bus test system. Results from the MNFSA program were found to be consistent with the frequency scan analysis.

- The MNFSA was applied to identify the root causes of the harmonic amplification and over-voltage problems and help in suggesting and evaluating alternative solutions.

5. The MNFSA can provide a quantitative approach to determine how the interaction between the system capacitive and reactive parameters caused resonance and harmonic amplification problems.

6. The MNFSA combines the MNF superior modeling capabilities and the computational efficiency of the sensitivity analyses. The develop program has the following computation advantages:
• The algorithm is generic for current and voltage variables, which means the same equations are used for calculation of current and voltage sensitivities.

• Inversion of system matrix is needed only once.

• The structure of system equations has been greatly simplified by modeling the system as a set of linear equations.

7. The MNFSA limitation is that it requires some additional memory and computational speed due to the increased size of system matrices. The size of system matrices has increased due to the increase in the number of system nodes and the additional current variables introduced by the impedance and voltage source elements.

6.2 Recommendations for Future Work and Potential Applications

1. Most of the published PFC optimization techniques for radial distribution feeders are based on losses and installation cost minimization with no or little attention made to the potential adverse effects of the PFC interaction with power system. The sensitivity indices obtained from the MNFSA can be integrated with some of these tools to offer a comprehensive PFC placement and sizing tool.
2. The MNFSA can be formulated in the time domain to serve as a switching transient, dynamic analysis and motor acceleration assessment tool. The advantage of using the MNF is that, power system equations can be represented as a set of linear and first order differential equations. Due this simplification, this tool could be used as an on-line simulation tool for power system and plants operators.

3. Incorporate constant power models and generators reactive power limits into the MNFSA program. This will allow more adequate power system representation for voltage stability sensitivity analysis.

4. Apply the MNFSA to investigate other power system issues such as voltage stability analysis.
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