Some Quality and Reliability Aspects of Aluminum Extrusion Process

by

Muhammad Kamran Raza

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES
KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the Requirements for the Degree of

MASTER OF SCIENCE

In

MECHANICAL ENGINEERING

December, 1998
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MUHAMMAD KAMRAN RAZA

under the direction of his Thesis Advisor and approved by his Thesis Committee,

has been presented to and accepted by the Dean of Graduate Studies, in partial

fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

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Date 12/6/99
Dedicated

to

My Parents

Brothers & Sisters

Ashi & Saad
Acknowledgements

In the name of Allah, Most Gracious, Most Merciful

First and foremost, all praise to Allah, Subhanahu-wa-ta’ala, the Almighty, Who gave me an opportunity, courage and patience to carry out this work. I feel privileged to glorify His name in the sincerest way through this small accomplishment. I seek His mercy, favor, and forgiveness. May He Subhanahu-wa-ta’ala, guide us and whole humanity to the right path (Ameen)

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Abstract

Name: Muhammad Kamran Raza
Title: Some Quality and Reliability Aspects of Aluminum Extrusion Process
Major Field: Mechanical Engineering
Date of Degree: December, 1998

In today's world, aluminum alloys with improved mechanical properties are in great demand. The properties such as formability, machinability, light weight, and high strength-to-weight ratio enable these alloys to undergo almost all types of principal production processes like casting, forging, extrusion etc. However for most applications extrusion is the most common.

The stages of extrusion process are, forcing preheated billets through a die orifice, performing heat treatments to improve its mechanical properties, and then painting or anodizing for good surface finish.

Defects at different stages deteriorate the quality of the final output of extrusion. A comprehensive study of defects found in the products of a large extrusion plant in Gulf area is conducted using various quality assurance techniques and tools. A cost model is proposed in a generalized framework, which incorporates probabilities of producing a defective part at each stage. The plant experience indicates that in many types of defects the dies are at the root of the problem. Various modes of die failure are investigated. Die life is characterized by an appropriate reliability model. For prediction of die reliability of any complexity two approaches, Multiple Regression and Neural Network are used. In both approaches die reliability model parameters are expressed as a function of various die features characterizing its complexity.

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Department of Mechanical Engineering
King Fahd University of Petroleum and Minerals
Dhahran, Saudi Arabia
December, 1998
الخلاصة

urrection

|

| القسم | المقدمة
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| التخصص | د. د. | ديسمبر 1998 م |

تعدد حالات العالم باضطراب إلى سياسات الألوميوم ذات الأسوأ الخصائص الميكانيكية، فمن قابلية التشبيك وقابلية المشكل بالمكبات وحتى الزيادة في نسبة القوة إلى الوزن لسياسات الألوميوم هي عوامل مفيدة لتصنيعها باستخدام طرق الإنتاج المختلفة مثل القص والطرق والرقاب، وعلى العكس يعتبر البثق أهم طرق تصنيع هذه السياق.

تتكون عملية التشبيك بالبثق من ثلاثة مراحل هي: دفع كتلة معدن مصغرة الخلاصة عبر فتحة قيابة لتصنيع المشكل ثم معالجته حرارة ثم معالجة السطح بالطلاء أو بالمعالجة الألودية.

إن المعرفة أن هناك عوامل عديدة تؤدي إلى جودة الأجزاء المصنعة بالبثق مما يضمن على المصممين حصر أنواعها وأسبابها.

قام هذا البحث بإعداد دراسة شاملة للعوامات الموجودة في متنجة سياسات الألوميوم مصنعة بالبثق في أحد أكبر مصانع تشبيك سياسات الألوميوم بالبثق في منطقة الخليج العربي وذلك باستخدام أنواع معددة من عوامل مشكلة وعدد من مراحل التصنيع الثلاث.

وأما أنه من المعرفة كذلك أن أنواع عديدة من العوامل تكون متميزة عن تدفق حالة قيابة للتشبيك أثناء التصنيع فقد تم بحث آخر الأنواع المختلفة لإضافة القوالب وقد تم التعبير عن عوامل البثق بموجزه اعتمادية مناسب، وللتميز باعتمادية القوالب معدة المشكل ثم استخدام طرق الإخبار المعدة وطرق المكبات المخصصة لإستنبط نماذج للاعتمادية وقد تم التعبير عن معامل تجاوز الاعتمادية بواسطة دوال للمستويات المميزة لشكل القوالب.

درجات الاجامر

قسم الهندسة الميكانيكية

جامعة الملك فيد للبروت وورشاد

الظهرازن المملكة العربية السعودية

ديسمبر 1998
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<tr>
<td>COV</td>
<td>Coefficient of Variation ($\sigma/\mu$)</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>PCR</td>
<td>Process Capability Ratio</td>
</tr>
<tr>
<td>USL</td>
<td>Upper Specification Limit</td>
</tr>
<tr>
<td>LSL</td>
<td>Lower Specification Limit</td>
</tr>
<tr>
<td>MSD</td>
<td>Mean Square Deviation</td>
</tr>
<tr>
<td>$R^2$</td>
<td>Coefficient of Determination</td>
</tr>
<tr>
<td>$t_{\text{min}}$</td>
<td>Minimum Thickness of Extrusion</td>
</tr>
<tr>
<td>N</td>
<td>Number of Ports in a Die</td>
</tr>
<tr>
<td>W</td>
<td>Weight of Extrusion Profile</td>
</tr>
<tr>
<td>L</td>
<td>Length of Extrusion Profile</td>
</tr>
<tr>
<td>P</td>
<td>Perimeter of Extrusion Profile</td>
</tr>
<tr>
<td>$W_p$</td>
<td>weighted Perimeter</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>$\bar{T}$</td>
<td>Average die Life</td>
</tr>
<tr>
<td>$\lambda(t)$</td>
<td>Failure Rate</td>
</tr>
<tr>
<td>$R(t)$</td>
<td>Reliability Function</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Shape parameter of Weibull Distribution</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Scale parameter of Weibull Distribution</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Mean</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Standard Deviation</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

In today’s world, aluminum and its alloys with improved mechanical properties are in great demand. The desirable properties, which include light weight, high strength-to-weight ratio, good corrosion resistance, high thermal and electrical conductivity, good availability, recycling ability, a wide range of surface finishes, formability, machinability, and weldability, make aluminum alloys a popular choice for engineering applications. These properties enable aluminum alloys to undergo almost all types of principal production processes like casting, forging, extrusion etc. However for uniform sections, continuous products required for structural frames, and for certain other applications extrusion is the most adopted production process. Even a variety of parts for structural requirements of aircraft and automotive industry are produced by extrusion. Most commercially available aluminum alloys can be extruded into practically any intricate shape desired. For example, Al-Cu-Mg alloys system (7xxx series) and
Al-Zn-Mg-Ag-Cu alloys systems (2xxx series) are strongest among wrought aluminum alloys and are used in aircraft industry. Similarly Al-Mg-Si alloys systems (6xxx series) represent the highest volume (90%) of the western world extruded products. In 1990, 23 percent of world’s aluminum consumption was the extruded products of 6xxx aluminum alloys [1-3].

A brief description of production processes for Aluminum extrusion is given below:

1. Production Processes for Aluminum Extrusion

Extrusion is the plastic deformation in which material is forced under pressure to flow through one or more die orifices to produce products of desired shapes. Different sequential steps involved in the production of aluminum extrusion are listed below.

- Billet Preparation
- Extrusion
- Aging treatment
- Anodizing or Painting
- Packing

1.1 Billet Preparation

Aluminum alloys that are to be extruded are generally cast in the form of cylindrical logs of diameter compatible with cylindrical container of extrusion press. These logs are sawed or sheared into billets of varying lengths depending on the cross-sectional area and the length of the product to be extruded. Specimens are taken to check the composition of the billets. There are limits for different impurities for each aluminum alloy. Table 1.1
shows the minimum and maximum limits of different impurities for commonly used 6063 alloy [4].

Table 1.1: The standard limits of alloying elements in 6063 alloy. The rest is Aluminum [4]

<table>
<thead>
<tr>
<th>Element</th>
<th>Si</th>
<th>Cu</th>
<th>Fe</th>
<th>Mn</th>
<th>Cr</th>
<th>Zn</th>
<th>Ti</th>
<th>Mg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum(%)</td>
<td>0.42</td>
<td>0.0001</td>
<td>0.16</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.005</td>
<td>0.45</td>
</tr>
<tr>
<td>Maximum(%)</td>
<td>0.48</td>
<td>0.0200</td>
<td>0.22</td>
<td>0.0500</td>
<td>0.0200</td>
<td>0.0200</td>
<td>0.020</td>
<td>0.51</td>
</tr>
</tbody>
</table>

Before they are extruded aluminum billets are frequently homogenized by heat treatment. This treatment improves the extrudability of the material and surface finish produced. Billet temperature is important for aluminum extrusion. A billet temperature that is too high can cause blisters or other surface defects including cracking. A temperature that is too low increases the pressure requirements for the extrusion and shortens tool life.

1.2 Extrusion Process

After preheating, billets go to the extrusion press. During extrusion process the following factors play an important role:

1.2.1 Pressure Requirement

The pressure required for hot extrusion depends on the following factors:

1. Metal to be extruded and its condition
2. Length and temperature of billet
3. Complexity of the cross section to be extruded
4. Speed of extrusion (For more details section 1.2.2)
5. Extrusion or reduction ratio

6. Lubricant used and resultant friction

Analytical results are available in literature in terms of various above mentioned factors to calculate the pressure requirements.

1.2.2 Stem Speed

Optimal stem speeds are essential for hot extrusion. Excessive stem speed can cause overheating as well as tears and other defects. A speed that is too slow reduces the productivity and increases the required extrusion pressure because of the billet cooling. Slow speeds can also decrease tool life because of prolonged contact time between the tool and hot billet.

1.2.3 Dies

After preheating billets are placed in the container and forced through die to give the desired shape of extrusion profile. A die for aluminum extrusion must be able to provide the following:

- Accurate dimensions and product shape, to avoid the need for any corrective work.
- Maximum possible working life
- Maximum length of extruded section
- A good-quality surface finish maintained over many extrusions i.e. infrequent die cleaning
- Good performance at high extrusion speeds
- Low manufacturing costs
The die plays a central role in determining the overall economics of the extrusion process. The quality and reliability of the dies also have a major impact on the quality of extruded product. Die life estimation and its implication on the quality of the product are major areas of interest in extrusion industry. Root cause of many extrusion defects is die.

1.3 Aging Treatment

Heat treatments are done to enhance or modify the properties of metals and alloys. This process changes the physio-chemical and mechanical properties of the alloys by changing their microstructure. For most aluminum alloys age hardening treatment is used to increase strength, hardness, and other mechanical properties. It is a sequential process, which involves three major steps; solution heat treatment, quenching, and finally aging at some predetermined temperature. The properties achieved depend on control parameters i.e., solution treatment temperature, quenching rate, aging time and temperature.

1.4 Anodizing

Anodizing is an electrolytic treatment that produces a stable oxide layer on a metallic surface. Anodized coating are used primarily for decorative purposes, they also provide wear and corrosion protection. Anodizing can be described as an artificially induced, self-terminating corrosion process. As aluminum is highly reactive metal, a thin hard transparent oxide film is formed on its surface. Anodizing is a relatively massive thickening of this film induced by electrochemical means.

The anodizing mechanism starts at the original surface of the profile and progresses inward. It is the reverse of electroplating process in which the deposited coating grows outward.
1.5 Painting

Applying a good organic finish to aluminum extrusions provides protection against oxidation and corrosion to which mill finish aluminum is susceptible. This also provides a stable, attractive appearance over many years of exposure with minimum maintenance.

The most widely used system for painting extrusions today is the electrostatic spray process, in which the extrusions are either hung vertically or racked in a horizontal plane on a conveyORIZED line, either fully or semiautomatic. [5]

The advantages of electrostatic spray application in such a system are:

1. High volume production
2. Good wrap and coverage on multi-channel configuration
3. Better film thickness control
4. Good appearance in terms of smoothness and flow
5. Better economy and efficiency with a high paint utilization factor
6. Ease of changing color

The finishing system in the electrostatic spray process consists of following steps:

1. Cleaning and pretreatment of the extrusions
2. Dry-off of water after pretreatment and cooling
3. Spray application of the coating
4. Flash-off of the solvents from the coating
5. Baking or curing of the coating
1.6 Packing

After painting and anodizing aluminum extrusions are tied into bundles and wrapped with plastic sheets. Proper packing is necessary to avoid finished products from damages during loading, unloading and transportation.

1.7 Objectives of This Thesis

Various defects occur at different production stages of Aluminum extrusions as discussed above. These defects deteriorate the quality of final output of the plant as well as cause greater loss due to increase in the process scrap. Therefore it is necessary to systematically study these defects, their causes and their effect on overall cost of the product. This will help to run the plant economically and will result into a better competition in the market.

For this purpose a formal study of quality and reliability aspects of aluminum extrusion process of ALUPCO (Aluminum Products Company) was done. ALUPCO is the largest aluminum extrusion plant in Gulf area, and a systematic study of defects in their process and its proper analysis will be quite beneficial for overall cost reduction and production improvement.

The following are the main objectives of this study:

1: To carry out a detailed literature survey of defects found in aluminum extrusion and their causes.

2: Define the quality and identify various quality assurance techniques and control tools used in Statistical Process Control (SPC). To study the defects found at different stages in the products of ALUPCO using above quality assurance techniques and tools
find the relative contribution of various types of defects and possible reasons of these defects including tool quality.

3: To propose a cost model in generalized framework, which incorporates probability of producing a defective part at each stage.

4: To study different modes of failure of dies and characterize the die life by an appropriate reliability die life model, and express its shape and scale parameters as a function of various die features using different modeling techniques e.g. Multiple Regression, and Artificial Neural Network.

5: To provide useful recommendations for further improvement in quality and cost reduction.

1.8 Report Layout

Chapter 1 deals with production processes at different stages for aluminum extrusion and brief description of defects that can be produces at these stages. Chapter 2 describes a detailed literature survey of defects found in extrusion production and investigations carried out to find the root cause of different defects. Some quality evaluation and control tools and techniques are discussed in chapter 3, while chapter 4 describes the use of above tools and techniques at various stages of manufacturing in evaluating the quality of aluminum extruded products at ALUPCO. Chapter 5 deals reliability of dies, different reliability models used for die life modeling and methods used to find the parameters of these models by taking data from ALUPCO die shop as a case study.
1.9 Data Collection

Data used in this study is collected for most commonly used 6063 alloys. The following three resources were used to collect or generate the data.

1. Personal collection of data by taking measurements for paint and anodizing film thickness, and hardness before and after aging. (This data was used in Chapter 4. The data about chemical analysis was retrieved from data files of ALUPCO.)

2. Data on dies already collected by previous researchers from ALUPCO.

3. Additional data collected from die shop of ALUPCO using die life cards. (This data combined with the previous data on die life was used in Chapter 5.)
Chapter 2

Literature Review

Quality improvement is a never ending process. Although aluminum extrusion is an old process and a lot of research has been done to characterize the defects and their causes in aluminum extrusion products, a considerable part of an aluminum extrusion plant production is still recycled due to various defects. Initially extrusion was used to produce cylindrical shapes only. The defects found in cylindrical shape extrusions are centerbrust, piping, and surface cracking[6]. Centerbrust is an internal crack that develops as a result of tensile stress along the centerline of billet during extrusion. The greater movement of the material in the outer regions stretches the material along the center of the billet. If these stresses are high enough, centerbrust occurs. Conditions that
promote centerbrust are high die angles, low extrusion ratios, and impurities in the billet. 

*Piping* is a defect associated with direct extrusion. It is the formation of a sinkhole at the end of the billet. Surface cracking results from high temperatures at the surface of billets. Laue [7] showed that defects close to extrusion surface resulting from the formation of scales or *blisters* develop due to improper lubrication in the container and friction at the die surface. Braynnt [8] did metallurgical investigation of defects found most frequently in hot extruded products. He facilitated the investigation of these defects by suitable classification of defects. Such a classification is carried out most conveniently by considering the outward appearance and the symptoms associated with the technical metallurgical processes occurring in extrusion.

Sheppard [9] found that *pick up* defects occur by two mechanisms. Firstly due to transient sticking between extrudate and die land results in accumulation of extrudate material. Secondly due to removal of debris from the aluminum coating on the die land area. The later mechanism is the most common in AA6063 extrusions. He also found that homogenization treatment is essential to reduce pick up.

Clode [10] investigated the die line phenomena during the extrusion of AA 6063, and the effect of temperature and strain rate on it. He found that the surface formation mechanism in extrusion is one of generations and modifications; the surface is generated at the choked portion of the die land and is modified subsequently by interacting with the remainder of the die surface. Die lines are mainly the result of interaction between the extrudate and choked portion of the die land. Optimum process conditions can be established, together with a minimum die land length. The optimum die land length is that which gives sufficient stability together with a land length that is totally choked.
Zajac [11] studied the effect of addition of Mn in extrusion billets. He found that extrudability of aluminum alloys can be significantly improved and homogenization time is reduced producing less pick up defects, if 0.03-0.05 % of Mn is added. Wilcox [12] studied the effect of lubrication on cold extrusion of aluminum and copper at slow speeds and discussed various surface defects arising from it. Rogers [13] and Kulkarni [14] investigated factors influencing lubrication behavior in hot extrusion. Tottle [15] showed that sometimes extrusions have zones in which crystals are extremely coarse, in comparison with the rest of the structure. Such coarse grained regions have relatively poor mechanical properties and any attempt to work the material further frequently leads to cracking or splitting of stock.

Northcott [16] showed that surface irregularities might arise from central billet segregate. Longitudinal streaks on the surface of extruded aluminum alloy bars, found to result in low circumferential strength and laminated fractures, were due to string of particles of intermetallic compounds. Joseph Pepe [17] revealed that central bursts develop from regions of structural damage and grow to completion in a two-stage fracture process at a location near the die orifice. The first stage consists of the tensile growth of micro cracks to a critical size defect, followed by a period of unstable metal flow, which results in growth of central burst to completion, by intense shear fracture. Suh [18,19] investigated the defect in extruded cups and transmission gear blank. Lugosi [20] studied surface defect development during hydrostatic extrusion of Aluminium wire. Miki [21] performed experiments to study the factors causing internal defects in multistage extrusion and found that the extrusion limit of one material has a very close relation with the tensile fracture ductility or the reduction in area of material center if the
various forming conditions are kept constant. Berezhnoy [22] presented friction-assisted extrusion as an alternate to indirect and direct extrusion in the production of hard aluminum alloys for higher quality and variety. Colbert [23] reviewed the extrusion of medical tube products and the improvement in extrusion technology designed to meet the demand for quality and efficiency when producing disposal medical devices. Fielding [24] discussed the processes of grain refining, degassing and filtration of liquid aluminum to improve the quality of extrusion billets. Dion [25] discussed the die contribution to surface quality and the role of die design on final quality of extruded product. He showed that bearing surface, bearing length transitions, bearing clearance, pocket and weld chambers in the die face, feeder plates and weld chambers, pads and armpits in hollow dies contribute to final quality of extrusion product.

Sam [26] studied different problem faced during anodizing. He found that many problems experienced in anodizing such as burning, unpredictable build of anodic film thickness, and power recovery are integrally related to the rate of formation of the oxide as influenced by current density and time. He also found that temperature effects influence the chemical dissolution part of the equilibrium. For this reason, the sulfuric acid electrolyte should be thoroughly agitated by a fine stream of air bubbles and/or mechanical circulation of the solution through the tank to prevent localized heating.

Franz [27] studied the effects of metallurgical factors on response of AA 6063 alloy extrusion to caustic etched and clear anodized finishes. He found that the iron contents have the most significant compositional effects. Higher Fe concentration (> 0.2%) produces better billet homogenization and surface finish.
Anon [28] described that by implementing a Total Quality Management (TQM) across all operations in one of the world's largest aluminum extrusion company in 1992 has enabled the company to achieve an on-time delivery rate 20% above the industry standard. Also ongoing scrutiny of processes revealed that the on-time delivery could be further improved by increasing finishing line productivity.

Philips [29] studied the defect detection on extrusion profiles by means of digital image processing. He showed that high speed image processing algorithms in combination with fuzzy-logic led to a successful implementation in extrusion lines.
Quality Evaluation and Control:

Some Tools and Techniques

3.1 Quality

Quality can mean different things to different people and can be interpreted in a variety of ways by an individual. From manufacturing point of view quality is simply conformance to specifications. The ultimate customer could describe quality as fitness for use. When trying to edge out the competition, quality can be interpreted as producing the very best product. Therefore quality is:[30]

1. Fitness for use

2. Conformance to specification
3. Producing the very best products

4. Total customer satisfaction

5. Exceeding customer expectation

### 3.2 General Aspects of Quality

There are two general aspects of quality, quality of design and quality of conformance.

#### 3.2.1 Quality of Design

By quality of design we mean the different grades or levels of performance, reliability, serviceability, and function that are result of deliberate engineering and management decisions [31]. For example aluminium extrusion products are generally divided into two classes, architectural applications and non-architectural applications. These design differences include the types of materials used in construction, tolerances in manufacturing, and use of equipment. Product development roughly falls into three categories:

1. Product design
2. Process design
3. Manufacturing

Product design encompasses both conceptual and detailed stages. In conceptual design the customer wants are translated into performance specifications and both the functional principles and physical configuration of the product are synthesized. In detailed design the detailed configuration of the components and parts is set forth and parts parameters and tolerances are specified.
Process design also includes conceptual and detailed phases in which the manufacturing processes to be employed are first chosen and then the detailed tooling specifications are made.

After product and process design, manufacturing begins and monitored. To obtain high quality products it is necessary to effectively incorporate the customer’s demands into the design process, and to consider concurrently the manufacturing process that are to be employed as the product is designed. The desirable performance characteristics with a minimum of variability and cost can only be achieved with strong efforts to integrate the product design with the selection of manufacturing process.

3.2.2 Quality of Conformance

The Quality of conformance measures how well the product conforms to the specifications and tolerances required by the design. Smaller the spread of dimensions or characteristics about their targeted values, better is the quality. Quality of conformance is influenced by a number of factors. These include the choice of manufacturing process, the training and supervision of work force, the type of quality assurance system (process controls, tests, inspection activities etc.) used, the extent to which these quality-assurance procedures are followed, and the motivation of the work force to achieve the targeted quality.

3.3 Quality Evaluation and Control Tools

Quality improvement effects are primarily focused on attempts to reduce the variability in processes and product characteristics, since variation can only be described in statistical terms statistical methods are of considerable use in quality improvement efforts.
Dedication to constant improvement in quality and productivity is needed to prosper in today's economic climate. Yesterday's standards are not good enough. A company's product has competition from companies throughout the world because modern communication and transportation modes have created a world marketplace. In order to compete the quality of a product has to be world class, as good as the best in the world. Consumers are looking for the best combination of price and quality before they decide to purchase the desired goods.

Today each company employee must be committed to the use of effective methods to achieve optimum efficiency, productivity, and quality to produce competitive goods. Statistical process control (SPC), in its broad sense, is a collection of production methods, management concepts and practices that can be used throughout the organization. SPC involves the use of statistical signals to identify sources of variation, to improve performance, and to maintain control of production at higher quality levels. It can be applied to any area where work is done.

Statistical process control can effectively be done by seven major SPC tools:

[30,31]

1. Histograms
2. Check sheets
3. Pareto charts
4. Cause and effect diagrams
5. Defect-concentration diagram
6. Scatter diagrams
7. Control charts
3.3.1 Histograms

A histogram is a bar graph with a measurement scale on one axis and a frequency or percentage scale on the other. The adjacent bars share a common side and all bars are of equal width. The histogram is a picture of a frequency distribution and is generally used to show tile distribution pattern of a large sample of data.

The following steps are used to make a histogram

**Step 1:** Set the scale for the measurement axis using the group endpoints from the frequency distribution or the individual measurements. If the measurements have specifications, include the specification limits on the measurement scale.

**Step 2:** Set an appropriate scale for the frequency or percentage axis.

**Step 3:** Make a bar of the proper height for each group or measurement. The adjacent bars should share a common side. When the bars represent a single measurement, the sides of the bars would be at the “half” measurement.

3.3.2 Checksheets

A checksheet is a data gathering device that consists of a list of the different types of data to be gathered and a row or column in which to put tally marks or brief descriptive remarks. The heading on the checksheet should contain information such as the name of the individual gathering the data, the time frame in which the data are gathered, and any specific information about the source and type of data. The information from a checksheet of this type could be put in a Pareto chart for prioritizing problems.

Another type of checksheet is used in the inspection process. The checksheet contains a list of specific items that have to be checked or it could contain a picture of the
item being inspected with inspection areas highlighted or numbered. Some also contain codes that are used to specify the problem area and the type of problem.

3.3.3 Pareto Diagrams

Pareto diagrams are important tools in the quality improvement process. Named after the Italian economist Alfredo Pareto, Pareto diagrams were first applied to the area of quality control by Joseph Juran[32]. Like Pareto, who found that the distribution of wealth was concentrated in a few people, Juran realized that similar principle holds in other fields. For instance in manufacturing or service, most problems are created by a few causes, even though there are many causes that may influence the occurrence of these problems. These categories of problems were identified as the vital few and the trivial many, respectively.

The Pareto principle lends support to the 80/20 rule, which states that 80 percent of the problem (nonconformities or defects) are created by 20 per cent of the causes. Pareto diagrams help management quickly identify the critical areas (those causing most of the problems) which deserve immediate attention. They identify the important problems, the resolution of which will lead to substantial improvements in quality. They provide guidance in the allocation of limited resources to problem-solving activities. Through the use of Pareto diagrams, problems may be arranged in order of importance. ‘Importance’ may refer to the financial impact of a problem (which is usually most appropriate) or the relative number of occurrences of the problem.

The steps for constructing a Pareto diagram are as follows:
Step 1: Decide on the means of classifying the data—say, by problem causes, type of non-conformity (critical, major, minor), or whatever else seems appropriate.

Step 2: Determine how relative importance is to be judged, that is, whether it should be based on associated dollar values or the frequency of occurrence.

Step 3: Rank the categories from most important to least important.

Step 4: Compute the cumulative frequency of the data categories in their chosen order.

Step 5: Plot a bar graph, showing the relative importance of each problem area in descending order. Identify the vital few that deserve immediate attention.

3.3.4 The Cause-And-Effect Diagram

It is sometimes called a fishbone diagram because of its shape or an Ishikawa diagram after Professor Ishikawa of Japan, who first used the technique in the 1960s [33]. The cause-and-effect diagram is a useful tool because it organizes the ideas presented. Basically they are used to identify and systematically list the different causes that can be attributed to a problem. By first listing the various causes, such diagrams can help in determining which of several causes have the greatest effect. A cause-and-effect diagram can aid in identifying reasons, which cause a process to go out of control. Alternatively, if a process is stable, such a diagram can provide guidance on causes to be investigated for process improvement.

3.3.5 Defect Concentration Diagram

The defect concentration diagram is just a sketch or diagram of the product, with the most frequently occurring defects shown on the part.
3.3.6 Scatterplots

The scatterplot is a graph of measurement pairs that shows whether there is correlation between the measurements. When correlation exists, changes in one measurement will be accompanied by proportionate changes in the other. If there is positive correlation, the changes will be in the same direction. When the first measurement increases, the second increases, and when the first measurement decreases, the second decreases. When the correlation is negative, the two measurements move in opposite directions. If the first measurement increases, the second decreases, and vice versa.

Two relationships may exist between measurements that are correlated. If a common-cause relationship exists, then the measurements are similarly, affected by the common-cause variation in the process. So, if one measurement is in control, the other one will be too. Likewise, trouble with one can signal trouble with the other. In a cause-and-effect relationship a change in one measurement will cause the other measurement to change as well.

Scatterplots can be useful in both quality control and problem solving. If measurements have a common-cause correlation, just one of them has to be tracked on a control chart. If measurements have cause-and-effect relationship, the important measure can be optimized and controlled with the other. Also, if improvements on the process of the first measurement result in decreased variability, the second measurement will often undergo a similar improvement. When process changes are made, however, the correlation factor should be verified.
3.3.7 Control Charts

A major objective of statistical process is to quickly detect the occurrence of assignable causes or process shifts so that investigation of the process and corrective action may be undertaken before many nonconforming units are manufactured. The control chart is an online process-monitoring technique widely for this purpose. Control charts may also be used to estimate the parameters of a production process and through this information, to determine process capability. The control charts can also provide information that is useful in improving the process.

There are two basic types of control charts:

1. Variables Control chart
2. Attributes Control chart

*Variables charts* use actual measurements for charting. The types of variables control charts are the following:

- Average and range charts
- Average and standard deviation charts
- Median and range charts
- Individual and moving range chart
- Run chart

*Attributes control charts* use pass-fail information for charting. An item passes inspection when it conforms to the standards; a nonconforming item fails inspection. The types of attributes control charts include the following:
• p chart
• np chart
• c chart
• u chart

Control charts for variables are more useful than charts for attributes, but attributes charts work best in some situations, such as when tracking paint flaws or surface smoothness. Ongoing process analysis is also better with variables charts because the location (the middle value) is charted along with the piece-to-piece variability (the range); the shape of the distribution of measurements can be determined as well.

Control charts are the basic tools of SPC, and variables control charts provide most information. They can quickly indicate if a process is in statistical control at specific process points, and their analysis can suggest causes of any out-of-control occurrence. They also provide the basis for a process capability study by indicating when a study can be made and provides the data needed for the study.

3.4 Process Capability Ratio

It is usually necessary to obtain some information about the capability of process i.e. the performance of the process when it is operating in control. Two graphical tools, the tolerance chart and the histogram are helpful in assessing process capability.

Another way to express process capability is in terms of an index that is called Process Capability Ratio (PCR). It is defined as follow [32]
The 6σ width, (3σ on either side of mean) is sometimes called the basic capability of the process. The limits 3σ on either side of the process are sometimes called natural tolerance limit, because these represent limits that an in-control process should meet with most of units produces. The PCR has a natural interpretation: \((1/\text{PCR})100\) is just the percentage of specification width used by the process.

The definition of the PCR in equation 3.1 implicitly assumes that the process is centered at the nominal dimension. If the process is running off-center, its actual capability will be less than the indicated by the PCR. It is convenient to think of PCR as a measure of capability with a centered process. If the process is not centered, then a measure of actual capability is often used. This ratio, called \(\text{PCR}_k\) is defined below.

\[
\text{PCR}_k = \min \left[ \frac{\text{USL} - \mu}{3\sigma}, \frac{\mu - \text{LSL}}{3\sigma} \right]
\]  

(3.2)

\(\text{PCR}_k\) is a one-sided process capability ratio that is calculated relative to the specification limit nearest to the process mean.

### 3.5 Taguchi Philosophy on Quality

Some of the fundamental ideas that are embraced by the Taguchi’s philosophy are as follows.[34,35]
According to Taguchi, "Quality is the loss imparted to society from the time a product is shipped." Taguchi attaches a monetary value to quality because he feels that this will make quality improvement understood by all-technical personnel as well as management. Typical examples of loss to society include failure to meet the customer's requirements and unsatisfactory performance that leads to loss of goodwill and reduced market share. The purpose of quality improvement, then, is to discover innovative ways of designing products and processes that will save society more than they cost in the long run. All products ultimately cause loss because they break and then need to be repaired or replaced or because they wear out after an adequate performance in their functional life. The degree to which the product or service meets consumers' expectations will influence the magnitude of loss. Taguchi contends that the loss due to a product's variation in performance is proportional to the square of the performance characteristic's deviation from its target value. [36]

The quality and cost of a manufactured product is influenced by the engineering design of the product as well as the process. By concentrating on product and process designs that are robust, that is, less sensitive to uncontrollable factors such as temperature, humidity, and manufacturing variation (the noise factors), the Taguchi concept attempts to reduce the impact of noise rather than eliminate it. Frequently, the elimination of noise factors is neither practical (because it is often very costly) nor feasible. Manufacturing variability cannot be totally eliminated since incoming components and raw material often exhibits considerable variation. By dampening the impact of noise factors and by selecting the controllable factor levels which force the
desirable quality characteristics to stay close to target value, robust design of the product and thereby the process is achieved.

3.6 Loss Functions

In the Taguchi philosophy, quality is the loss imparted to society from the time that a product is shipped. It includes the loss incurred while the product is being manufactured. The components of loss include the expense, waste, and lost opportunity that result when a product fails to meet the target value exactly. During production, costs such as inspection, scrap, and rework contribute to loss. Although those costs are easy to account for, there are others that are much more difficult to measure, such as the loss associated with customer dissatisfaction that arises from the variation of products and services. Even though profit-increasing measures such as productivity improvement and waste reduction are important, they are bounded by such factors as labor and material costs and technology. On the other hand, real growth in terms of market share is very much influenced by society, which again is dependent on customer satisfaction. Price and quality, are critical factors in this context. Any cost incurred by the customer or any loss resulting from poor quality will have a significant negative impact. On other hand, a satisfied customer who realizes savings from using the product or service will turn it over many times to greatly improve market share. All of these ideas lead to the conclusion that variability is the key, concept that relates product quality to dollars by means of a loss function.

Since the quality loss function is stated in financial terms, it provides a common language for various entities within an organization, such as management, engineering,
and production. It can also be related to performance measures such as the signal-to-noise ratio, which is used in the parameter design phase. The loss function is assumed to be proportional to the square of the deviation of the quality characteristic from the target value.

3.6.1 Traditional Loss Function

The traditional notion of the loss function is that as long as the product's quality characteristic is within certain specification limits or tolerances, no loss is incurred. Outside these specifications, the loss takes the form of a step function with a constant value $A$. Figure 3.1 shows this traditional loss function. There are drawbacks inherent to this loss function that can possibly be limited to the cost of scrap and rework.

![Traditional Loss Function Diagram](image)

Fig 3.1: Traditional Loss Function [32]
Obviously, the function makes no distinction between a product whose characteristic is exactly on target versus one whose characteristic is just below the USL or just above the LSL. There will be performance differences in these products, but the loss function does not reflect these differences. Furthermore, it is difficult to justify the sudden step increase in loss as the quality characteristic just exceeds the specification limits, because there is no great functional difference between a product with a quality characteristic value of USL-δ and that with a value of USL+δ, where δ is very small. Additionally, it is unreasonable to say that the loss remains constant, at a value A, for all values of the characteristic beyond the specification limits. There will obviously be a significant functional difference between products that are barely above the USL and those that are far above the USL. These differences would cause a performance variation, so the loss to society would be different. The Taguchi loss function overcomes these deficiencies, because the loss increases quadratically with increasing deviation from the target value. The following subsections give expressions for the loss functions based on three situations that may arise in practice: nominal is best, smaller is better, and larger is better.

### 3.6.2 Nominal is Best

Consider characteristics for which a target, or nominal, value is appropriate (that is, the nominal value is best). As the quality characteristic deviates from the target value, the loss increases in a quadratic manner. Figure 3.2 is an example of such a loss function. Examples of quality characteristics in this category are product dimensions such as
length, thickness, and diameter; a product characteristic such as the viscosity of an oil; and a service characteristic such as the degree of management involvement.

![Diagram showing loss function](image)

**Fig 3.2: Nominal is the Best Loss Function [32]**

The loss function is given by

$$L(y) = k(y - m)^2$$  \hspace{1cm} (3.3)

where \(k\) is a proportionality constant, \(m\) is the target value, and \(y\) is the value of the quality characteristic. Note that when \(y = m\), that is, when the value of the quality characteristic is on target, the loss is zero. The constant \(k\) can be evaluated if the loss \(L(y)\) is known for any particular value of the quality characteristic; it is influenced by the financial importance of the quality characteristic. For instance, if a critical dimension of the braking mechanism in an automobile deviates from a target value, the loss may increase drastically.

To determine the value of the constant \(k\), suppose the consumer's average loss is \(A\) when the quality characteristic is at limit of functional tolerance \(\Delta\) in Figure 3.2. Then
\[ k = \frac{A}{\Delta^2} \]  
(3.4)

and above equation becomes

\[ L(y) = (A/\Delta^2)(y - m)^2 \]  
(3.5)

The expected loss is the mean loss over many instances of the product. The expectation is taken with respect to the distribution of the quality characteristic \( Y \). We have

\[
\mathbb{E}[L(y)] = \mathbb{E}
\left[k(y - m)^2\right]
\]
\[
\mathbb{E}[L(y)] = k\left[\text{Var}(y) + (\mu - m)^2\right]
\]  
(3.6)
\[
\mathbb{E}[L(y)] = k(\text{MSD})
\]

Here MSD represents the mean square deviation of \( y \) and is estimated as

\[
\text{MSD} = \frac{\left(\sum_{i=1}^{n}(y_i - m)^2\right)}{n}
\]  
(3.7)

over a sample of \( n \) items.

### 3.6.3 Smaller Is Better

Some quality characteristics are nonnegative and for which the ideal value is zero (that is, smaller values are better) e.g. fuel consumption in automobiles, impurities in aluminum billets. Fig. 3.3 shows the loss function for this case which is given by

\[ L(y) = ky^2 \]  
(3.8)
If the loss caused by exceeding the customer's tolerance level $\Delta$ is $A$, the value of proportionality constant $k$ is given in equation 3.4.

$$E[L(y)] = kE(y^2)$$
$$= \frac{(A / \Delta^2)E(y^2)}{\Delta}$$
$$= \left(\frac{A}{\Delta^2}\right)[\text{Var}(y) + \mu^2]$$
$$= \left(\frac{A}{\Delta^2}\right)\text{MSD} \quad (3.9)$$

Where MSD represents mean square deviation and is estimated by

$$\text{MSD} = \frac{\sum_{i=1}^{n} y_i^2}{n} \quad (3.10)$$

for a sample of $n$ items.
3.6.4 Larger Is better

Some quality characteristics are nonnegative and have an ideal target value that is infinite (that is, larger values are better) e.g. hardness of aluminum extrusions. Here no target value is predetermined. Figure 3.4 shows the loss function for this, which is given by

\[ L(y) = k \left( \frac{1}{y^2} \right) \]  \hspace{1cm} (3.11)

If the loss is \( A \) when the quality characteristic falls below the customer’s tolerance level \( \Delta \), the value of the proportionality constant \( k \) is given by

\[ k = A\Delta^2 \]  \hspace{1cm} (3.12)

![Graph showing Larger is Better Loss Function](image).

Fig. 3.4: Larger is Better Loss Function [32]

The expected or average loss is given by the expectation with respect to probability distribution of the quality characteristic. The expected loss is
\[ E[L(y)] = kE(1/y^2) = (A\Delta^2)E(1/y^2) \quad (3.13) \]

The quantity of \( E(1/y^2) \) can be estimated from a sample of \( n \) items as

\[ E \left( \frac{1}{y^2} \right) = \frac{\sum_{i=1}^{n} \frac{1}{y_i^2}}{n} \quad (3.14) \]

In all cases enhancing the signal, but reducing the noise (variation of the values of characteristic) will reduce the loss, this will increase the quality of the product. But before using these tools, a broad picture of variability of various output characteristics must be comprehended.

### 3.7 Fitting a Distribution Model to a Quality Characteristic

If the quality characteristics (such as dimension, strength, hardness, composition of an element) can be measured, and represent a continuos variable. We can often fit well known probability models to this data. These models are tabulated as \( f(t; \alpha_1, \alpha_2, \ldots) \) in Table 3.1 with the relationship of their parameters \( \alpha_1, \alpha_2 \), with mean and variance of the distribution (approximately reflecting the mean and variance of the data). The CDF \( F(x) \) characterize \( P(X \leq x) \), where \( X \) represents the quality characteristic being modeled. If data of quality characteristic is arranged in an ascending order \( x_1 < x_2 < x_3 \ldots < x_n \), then \( F(x_i) = i/N+1 \) is determined for \( i = 1, 2, 3, \ldots, n \). This value can be inserted in Table 3.2 and the function can be transferred such that it represents a straight line \( Y = mZ + c \). The equations for \( Y, Z, m \) and \( c \) are given in Table 3.2. Using Excel or any other spread sheet
program, the model can be fitted to the data and \( R^2 \) (Coefficient of Determination) can be used to identify the best fitted model.

In any appropriate model selected for characterizing the data, one parameter represents the average or mean value of the signal (quality characteristics) and other parameter is related to its spread or variation around the average values (i.e. the noise of quality characteristic). The objective of quality improvement is to enhance the signal and reduce the noise.
Table 3.1: PDF, Mean and Variance of Different distributions

<table>
<thead>
<tr>
<th>Distributions</th>
<th>( f(x; \alpha_1, \alpha_2) )</th>
<th>( \alpha_1 )</th>
<th>( \alpha_2 )</th>
<th>( \mu )</th>
<th>( \sigma^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>( \frac{1}{\sqrt{2\pi} \sigma} \exp \left{ -\frac{1}{2} \left( \frac{x - \mu}{\sigma} \right)^2 \right} )</td>
<td>( \mu )</td>
<td>( \sigma )</td>
<td>( \mu )</td>
<td>( \sigma^2 )</td>
</tr>
<tr>
<td>Lognormal</td>
<td>( \frac{1}{\sqrt{2\pi} \omega y} \exp \left{ -\frac{1}{2\omega^2} \left[ \ln \left( \frac{y}{y_0} \right) \right]^2 \right} )</td>
<td>( \ln y_0 )</td>
<td>( \omega )</td>
<td>( y_0 \exp(\omega^2/2) )</td>
<td>( y_0^2 \exp(\omega^2) \left( \exp(\omega^2) - 1 \right) )</td>
</tr>
<tr>
<td>Weibull</td>
<td>( \frac{\beta}{\theta} \left( \frac{x}{\theta} \right)^{\beta-1} e^{-\left( \frac{x}{\theta} \right)^\beta} )</td>
<td>0</td>
<td>( \beta )</td>
<td>( \theta \left( 1 + \gamma_n \right) )</td>
<td>( \theta^2 \left{ \Gamma \left( 1 + \frac{1}{\gamma_n} \right) \right} )</td>
</tr>
<tr>
<td>Exponential</td>
<td>( \frac{1}{\theta} e^{-\gamma_0} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( \theta^2 )</td>
</tr>
<tr>
<td>Extremes value</td>
<td>( \frac{1}{\lambda} e^{-\left( x-\lambda \right)/\lambda} )</td>
<td>0</td>
<td>( u )</td>
<td>( y_m - 0 \Gamma \left( 1 + \gamma_m \right) )</td>
<td>( \theta^2 \left{ \Gamma \left( 1 + \gamma_m \right) - \left[ \Gamma \left( 1 + \gamma_m \right) \right]^2 \right} )</td>
</tr>
</tbody>
</table>
Table 3.2: CDF, Slope and Intercept of Different Distributions

<table>
<thead>
<tr>
<th>Distribution</th>
<th>$F(x)$</th>
<th>$Y$</th>
<th>$X$</th>
<th>Slope</th>
<th>Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>$\phi\left[\frac{\alpha - \mu}{\sigma}\right]$</td>
<td>$\phi^{-1}(F)$</td>
<td>$x$</td>
<td>$\frac{1}{\sigma}$</td>
<td>$-\frac{\mu}{\sigma}$</td>
</tr>
<tr>
<td>Log normal</td>
<td>$\phi\left[\frac{1}{\omega}\ln\left(\frac{y}{y_0}\right)\right]$</td>
<td>$\phi^{-1}(F)$</td>
<td>$\ln y$</td>
<td>$\frac{1}{\omega}$</td>
<td>$-\frac{1}{\omega}\ln y$</td>
</tr>
<tr>
<td>Weibull</td>
<td>$1 - \exp\left[-\left(\frac{x}{y}\right)^\theta\right]$</td>
<td>$\ln\ln\left[\frac{1}{1 - F(x)}\right]$</td>
<td>$\ln x$</td>
<td>$\beta$</td>
<td>$-\beta\ln \theta$</td>
</tr>
<tr>
<td>Exponential</td>
<td>$1 - e^{-\frac{x}{\theta}}$</td>
<td>$\ln\ln\left[\frac{1}{1 - F(x)}\right]$</td>
<td>$x$</td>
<td>$\frac{1}{\theta}$</td>
<td>$\frac{u}{\theta}$</td>
</tr>
<tr>
<td>Min Extreme Value</td>
<td>$1 - \exp\left[-e^{\frac{x-u}{\theta}}\right]$</td>
<td>$\ln\ln\left[\frac{1}{1 - F(x)}\right]$</td>
<td>$x$</td>
<td>$\frac{1}{\theta}$</td>
<td>$\frac{u}{\theta}$</td>
</tr>
</tbody>
</table>
Chapter 4

Using Quality Control Tools In Evaluating

The Quality Of Aluminum Extruded

Products In ALUPCO

ALUPCO is one of the largest Aluminum Extrusion Plant in the Gulf region with an average yearly production capacity of 20,000 Tons of Aluminum products. About 600 different cross section of various level of complexity are extruded.

The plant is ISO 9000 certified and have up-to-date documentation. However merely being ISO 9000 certified does not mean that all quality problem are solved. It is necessary to judiciously interpret the available data or create new relevant data to qualitatively evaluate the quality while simultaneously reducing the cost due to rejections. This chapter is essentially a case study of illustrating how some of the previously described SPC tools and Taguchi concepts can be used to evaluate the quality. The various stages of Aluminum Extraction are illustrated in Fig. 4.1
Well established SPC tools mentioned in section 3.3 were used to analyze the data collected at different stages during aluminum extrusion from ALUPCO.

4.1 Billet preparation (First stage)

As shown in Figure 4.1, first stage of extrusion process is billet preparation. ALUPCO has its own melting furnace. Raw material in the form of ingots comes from Bahrain. Process scrap from different stages is also mixed with these ingots. The molten alloy from the furnace is cast into continuous cylindrical logs which are cut into billets. From the melting furnace samples are taken after every 4-5 hrs to check the composition of billets.

4.1.1 Histograms and Model fitting for Compositions of various elements in billets

Figure 4.2 and 4.3 show histograms of various elements in the composition of raw material. These histograms are based on a sample size of 100 in each case. Different probability distribution models were fitted to these histograms using raw data. Some typical results are plotted in Fig. 4.4-4.7 for Fe only. Such model fitting was done for all
Fig 4.2: Histograms of % of Mg, Mn, Fe and Si in Billets
Fig 4.4: Normal distribution of %Fe in Al Billets (Xi : Fe %)

$y = 41.37x - 7.5358$

$R^2 = 0.9804$
Fig 4.5: Inverted Normal Distribution of %Fe in Al Billets (X): Fe %

\[ y = 1.2841x + 7.1315 \]

\[ R^2 = 0.9369 \]
Fig 4.6: Lognormal distribution of %Fe in Al Billets (Xi : Fe %)

\[ y = 7.1689x + 12.257 \]

\[ R^2 = 0.9636 \]
Fig 4.7: Weibull distribution of %Fe in Al Billets (X1: Fe %)
other element's composition and the results are tabulated in table 4.1. Generally either normal or lognormal distribution was found better to fit the data.

Table 4.1: $R^2$ Values Different Distributions Fitted to Composition Data

<table>
<thead>
<tr>
<th>Element</th>
<th>Normal</th>
<th>Lognormal</th>
<th>Inverted normal</th>
<th>Weibul</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zn</td>
<td>0.907</td>
<td>0.960</td>
<td>0.751</td>
<td>0.917</td>
</tr>
<tr>
<td>Ti</td>
<td>0.965</td>
<td>0.989</td>
<td>0.970</td>
<td>0.942</td>
</tr>
<tr>
<td>Si</td>
<td>0.767</td>
<td>0.693</td>
<td>0.608</td>
<td>0.852</td>
</tr>
<tr>
<td>Mn</td>
<td>0.832</td>
<td>0.975</td>
<td>0.884</td>
<td>0.934</td>
</tr>
<tr>
<td>Mg</td>
<td>0.762</td>
<td>0.686</td>
<td>0.579</td>
<td>0.854</td>
</tr>
<tr>
<td>Fe</td>
<td>0.980</td>
<td>0.968</td>
<td>0.935</td>
<td>0.968</td>
</tr>
<tr>
<td>Cu</td>
<td>0.436</td>
<td>0.951</td>
<td>0.924</td>
<td>0.843</td>
</tr>
<tr>
<td>Cr</td>
<td>0.701</td>
<td>0.905</td>
<td>0.987</td>
<td>0.779</td>
</tr>
</tbody>
</table>

4.1.2 Tolerance Charts for Composition

Tolerance chart for respective element composition using the technique highlighted in Montgomery [31] are plotted in Fig.4.8-4.15. These tolerance charts show that % Cr, Cu, Zn, and Ti, is within limits. % Fe is mostly either between center line and LSL or below LSL. It is of greater concern because higher Fe concentration (> 0.2%) produces better billet homogenization and surface finish[28]. Amounts of Si and Mg are mostly out of control (less than LSL) and this needs continuous monitoring and adjustment in the furnace.

The extrudability of aluminum alloys depends to a large extent on chemical composition of billet. An ideal billet should have a uniform structure with a low flow stress, and homogeneity through out its cross-section. An out of tolerance limit composition can cause many surface defects during extrusion e.g. Mn is added to increase
Fig 4.8: Tolerance Chart for Mn percentage in Al Billets
Fig 4.9: Tolerance Chart for Cr percentage in Al Billets
Fig 4.12: Tolerance Chart for Mg percentage in Al Billets
Fig 4.13: Tolerance Chart for Si percentage in Al Billets
Fig 4.14 : Tolerance Chart for Zn percentage in Al Billets
Fig 4.15: Tolerance Chart for Ti percentage in Al Billets
the ductility of billet, but if its composition is less, then it can cause pick up defects leading to poor surface finish.

4.1.3 Process Capability Analysis

Using 'Normal Distribution Model' for each composition and upper and lower specification limits provided by ALUPCO standards, Equation 3.2 is used to calculate process capability ratios (PCR, PCR_{min}, PCR_{max}, PCR_{k}) and is tabulated in Table 4.2.

<table>
<thead>
<tr>
<th>Element</th>
<th>USL</th>
<th>LSL</th>
<th>Mean</th>
<th>SD</th>
<th>PCR</th>
<th>PCR_{min}</th>
<th>PCR_{max}</th>
<th>PCR_{k}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mn</td>
<td>0.05</td>
<td>0.0001</td>
<td>0.0413</td>
<td>0.0192</td>
<td>0.4330</td>
<td>0.1515</td>
<td>0.7146</td>
<td>0.1515</td>
</tr>
<tr>
<td>Cr</td>
<td>0.02</td>
<td>0.0001</td>
<td>0.0017</td>
<td>0.0005</td>
<td>6.1080</td>
<td>11.2548</td>
<td>0.9613</td>
<td>0.9613</td>
</tr>
<tr>
<td>Cu</td>
<td>0.02</td>
<td>0.0001</td>
<td>0.0030</td>
<td>0.0022</td>
<td>1.4873</td>
<td>2.5380</td>
<td>0.4366</td>
<td>0.4366</td>
</tr>
<tr>
<td>Fe</td>
<td>0.22</td>
<td>0.16</td>
<td>0.1820</td>
<td>0.0242</td>
<td>0.4137</td>
<td>0.5240</td>
<td>0.3034</td>
<td>0.3034</td>
</tr>
<tr>
<td>Mg</td>
<td>0.51</td>
<td>0.45</td>
<td>0.4295</td>
<td>0.1019</td>
<td>0.0981</td>
<td>0.2634</td>
<td>0.0672</td>
<td>0.0672</td>
</tr>
<tr>
<td>Si</td>
<td>0.48</td>
<td>0.42</td>
<td>0.4300</td>
<td>0.0532</td>
<td>0.1880</td>
<td>0.3133</td>
<td>0.0627</td>
<td>0.0627</td>
</tr>
<tr>
<td>Ti</td>
<td>0.02</td>
<td>0.005</td>
<td>0.0129</td>
<td>0.0027</td>
<td>0.9353</td>
<td>0.8812</td>
<td>0.9894</td>
<td>0.8812</td>
</tr>
<tr>
<td>Zn</td>
<td>0.02</td>
<td>0.0001</td>
<td>0.0051</td>
<td>0.0021</td>
<td>1.5862</td>
<td>2.3741</td>
<td>0.7982</td>
<td>0.7982</td>
</tr>
</tbody>
</table>

In some of the cases process capability ratio is less than 1, which means that continuous manipulation of the composition of these elements in the furnace is needed to adjust the values with in the desirable limits. This increases the cost due to two reasons:

- Cost of material addition to maintain the composition.
- Cost of increase in operation time of furnace

4.1.4 Taguchi Loss Function

Taguchi Loss Function for alloying elements is calculated and tabulated in Table 4.3.
Table: 4.3: Expected loss using Taguchi Loss functions for different composition

<table>
<thead>
<tr>
<th>Process</th>
<th>Mean</th>
<th>SD</th>
<th>Loss Function</th>
<th>Expected Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mn ( % in Billets)</td>
<td>0.041271</td>
<td>0.019205</td>
<td>Nominal is Best</td>
<td>1.395A₁</td>
</tr>
<tr>
<td>Cr ( % in Billets)</td>
<td>0.001666</td>
<td>0.000543</td>
<td>Nominal is Best</td>
<td>0.98A₂</td>
</tr>
<tr>
<td>Cu ( % in Billets)</td>
<td>0.003021</td>
<td>0.00223</td>
<td>Nominal is Best</td>
<td>0.4385A₃</td>
</tr>
<tr>
<td>Fe ( % in Billets)</td>
<td>0.182</td>
<td>0.024172</td>
<td>Nominal is Best</td>
<td>0.664A₄</td>
</tr>
<tr>
<td>Mg ( % in Billets)</td>
<td>0.429453</td>
<td>0.101948</td>
<td>Nominal is Best</td>
<td>9.65A₅</td>
</tr>
<tr>
<td>Si ( % in Billets)</td>
<td>0.43</td>
<td>0.0532</td>
<td>Nominal is Best</td>
<td>1.458A₆</td>
</tr>
<tr>
<td>Ti ( % in Billets)</td>
<td>0.012934</td>
<td>0.002673</td>
<td>Nominal is Best</td>
<td>0.1299A₇</td>
</tr>
<tr>
<td>Zn ( % in Billets)</td>
<td>0.005107</td>
<td>0.002091</td>
<td>Nominal is Best</td>
<td>0.285A₈</td>
</tr>
</tbody>
</table>

Where A₁, A₂, ..., A₈ are the costs in $/ton of each element being out of limits. 'Nominal is the best' is used because both upper and lower specification limits are given. Average of both limits was taken as target value.

This function shows the loss imparted to the manufacturer by producing billets which have the composition different from the target value (nominal in the case), even if it is in the specification limits.

4.2 Extrusion Press Output (Second Stage)

Aluminum billets are heated up to suitable temperature depending upon the alloy being extruded. This treatment also known as homogenization, improves the extrudability of the material and surface finish produced, and reduces the appearance of a streaked texture that often results from anodizing.

After preheating, the billets are placed in the press container. These billets are then forced through the die orifice by applying high pressure. Desired shape comes out of the die, it is cooled and cut according to required length. Since some part of billet remain
in the die and container, when new billet is put in the press and forced, being at a high
temperature it makes a weld with the butt, and product is extruded in a continuous
fashion. However near the joint on both sides the product is defective and can be cut,
removed and recycled.

These extrusion profiles are then transferred to aging ovens.

4.2.1 Press Defects

Different press defects found in the ALUPCO products are described below:

• **Black / White Lines**
  
  These are lines found on the surface of extrusion profiles when they come out of the press.

• **Silicon marks**
  
  These marks are formed when silicon in the billets separates at surface due to excessive heating.

• **Graphite / Run out marks**
  
  These marks are formed when the extrusion profiles move on the conveyer rollers as they come out of the die.

• **Blisters / Pick up**
  
  Following are reasons for these defects

  1. Impurities and improper composition

  2. Transient sticking between extrudate and dieland results in accumulation of extrudate material

  3. Debris removal from the aluminum coating on the die land area
• Die stop marks
  These marks are formed due to jerks by trapped air or fusion of one billet with other.

• Stain/Oil patches
  These marks are due to improper lubrication.

• Scratches Damages
  These defects are due to mishandling

• Twist / Bends
  These defects are due to faulty die.

• Out of angle, Concave/Convex
  These are also due to faulty die.

Figure 4.16 shows some of the above mentioned defects found in press shop.

4.2.2 Pareto Analysis

Pareto Charts are plotted for defects in press shop by using monthly and annually rejection data. Also charts are plotted for total number of defects during last 6 years. The plots for annual defects and total defects for last 6 years are shown in Figure 4.17 - 4.23.

These plot show that in press shop black/white lines are the most frequently occurring defects. The reason for black lines was found to be the extrusion of the defective material. Therefore it is very important to keep the percentage of different impurities with in recommended level. The white lines, which are also called die lines are mainly the result of interaction between the extrudate and choked portion of the die land.[10]

The second important cause of defects was scratches and damages. The reason of some of the scratches is found to be defective die. The main reason of scratches and damages is improper cutter and damages during shifting of extrusions from press to
Surface Tearing

Pick Up

Figure 4.16: Some Press Defects
Figure 4.17: Pareto Chart & Analysis for Press Defects 1992
Figure 4.18: Pareto Chart & Analysis for Press Defects 1993

- Black/White Lines: 100686
- Scratches/Damages: 51951
- Other Defects: 25323
- Stain/Oil Patches: 24586
- Silicon Marks: 19920
- Low Hardness: 14130
- Graphite/Runout Mark: 4183
- Concave/Convex: 2958
- Twist/Bends: 2590
- Out of Angle: 2027
- Blisters/Die Stop: 1094
- Rough Surf./B. Hole: 54
- Off Dimension: 7

Number of Defects vs. Percent of Defects Chart.
Figure 4.19: Pareto Chart & Analysis for Press Defects 1994

- Off Dimension
- Out of Angle
- Concave/Convex
- Twist/Bends
- Blisters/Die Stop
- Rough Surf./B. Hole
- Graphite/Rout. Mark
- Low Hardness
- Silicon Marks
- Stain/Oil Patches
- Scratches/Damages
- Other Defects
- Black/White Lines
Figure 4.20: Pareto Chart & Analysis for Press Defects 1995

- Off Dimension
- Out of Angle
- Concave/Convex
- Silicon Marks
- Stain/Oil Patch
- Graphite/Rumout Mark
- Low Hardness
- Twist/Bends
- Rough Surf./B. Hole
- Blister/Die Stop
- Other Defects
- Scratches/Damages
- Black/White Lines
- Number of Defects
- Percent of Defects
Figure 4.22: Pareto Chart & Analysis for Press Defects 1997

Number of Defects vs Percent of Defects

- Black/White Lines: 27785
- Graphite/Rnout Mark: 14077
- Low Hardness: 11993
- Other Defects: 11878
- Blisters/Die Stop: 7106
- Silicon Marks: 3569
- Scratches/Damages: 3148
- Twist/Bends: 2058
- Stain/Oil Patches: 880
- Rough Surf./B. Hole: 845
- Concave/Convex: 371
- Out of Angle: 13
- Off Dimension: 0
stretcher and from stretcher to aging ovens. No record of cutter life is maintained at ALUPCO. Therefore it is recommended that record of cutter life should be maintained. From this record reliable life of a new cutter could be predicted, which will facilitate the replacement of cutter in time utilizing preventive replacement strategy. Suitable training of operators can reduce the damages due to mishandling. Some minor defects which were categorized as other defects were the third major cause. This shows that these minor defects need to be further analyzed in detail and should be categorized separately in accordance with their relative importance.

4.3 Age Hardening

After press the aluminum extrusions are taken to the third stage i.e. Aging ovens that are used to increase the hardness of extrusions. For a typical extrusion product 100 measurements of hardness measurements were taken before and after aging process and the following tools were used for analysis of data.

4.3.1 Histograms for Hardness

Histograms for hardness before and after aging were drawn and shown in Fig 4.24. Other details are given below in Table 4.4.

<table>
<thead>
<tr>
<th></th>
<th>μ</th>
<th>σ</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardness (Before Aging)</td>
<td>4.4625</td>
<td>1.1356</td>
<td>.2545</td>
</tr>
<tr>
<td>Hardness (After Aging)</td>
<td>13.43</td>
<td>0.7818</td>
<td>0.0582</td>
</tr>
</tbody>
</table>
Fig 4.24: Histogram for Hardness without and after Aging

Hardness without Aging

No of obs

Hardness (weber)

Hardness after Aging

No of obs

Hardness (weber)
Coefficient of variation decreases after age hardening which shows that there is a less variation in the output and that also results in a decrease in Taguchi Loss function.

4.3.2 Model Verification

Different probability distribution models were fitted and results are shown in Table 4.5.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Hardness (Before Aging)</th>
<th>Hardness (After Aging)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>0.873</td>
<td>0.858</td>
</tr>
<tr>
<td>Lognormal</td>
<td>0.868</td>
<td>0.855</td>
</tr>
<tr>
<td>Inverted normal</td>
<td>0.846</td>
<td>0.851</td>
</tr>
<tr>
<td>Weibul</td>
<td>0.829</td>
<td>0.842</td>
</tr>
</tbody>
</table>

Normal probability distribution was the best fit and it is plotted in Figure 4.24 with histograms. The $\mu$ and $\sigma$ for this distribution are given below in Table 4.6. The probability plots for hardness before aging are shown in Figure 4.25-4.28 and for hardness after aging are shown in Figure 4.29-4.32. In these plots each data point represents the mid point of the corresponding cell in the histogram where all cells consist of 80 data points for hardness before aging and 100 data points for hardness after aging.

<table>
<thead>
<tr>
<th></th>
<th>$\mu$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardness (Before Aging)</td>
<td>4.4625</td>
<td>1.28</td>
</tr>
<tr>
<td>Hardness (After Aging)</td>
<td>13.44</td>
<td>0.89565</td>
</tr>
</tbody>
</table>
Fig 4.25: Normal distribution for Hardness Before Aging

\[
y = 0.7812x - 3.4762
\]

\[
R^2 = 0.8883
\]
Fig 4.26: Lognormal distribution for Hardness Before Aging

\[ y = 3.7215x - 4.8876 \]

\[ R^2 = 0.9539 \]

\( \Phi^{-1}(F) \)

\( \ln (X_i) \)
Figure 4.27: Weibull distribution for Hardness Before Aging

\[ y = 5.0021x - 7.407 \]

\[ R^2 = 0.9567 \]

\( X_i \): Hardness (Before Aging)
Figure 4.28: Inverted normal distribution for Hardness Before Aging

\[ y = -11.803x + 3.432 \]

\[ R^2 = 0.9038 \]

Xi : Hardness (Before Aging)
Fig 4.29: Normal distribution for Hardness after Aging

\[ y = 0.7812x - 3.4762 \]

\[ R^2 = 0.8883 \]

\( X_i \): Hardness (After Aging)
Figure 4.30: Lognormal Distribution for Hardness after Aging

\[ y = 16.157x - 41.334 \]

\[ R^2 = 0.9934 \]
Figure 4.31: Inverted normal Distribution for Hardness after Aging

$Y = -216.18X + 16.785$

$R^2 = 0.994$

$X_i$: Hardness (After Aging)
4.3.3 Process Capability Study

Using normal model for both cases and lower specification limits provided by the ALUPCO standard Process Capability Ratio (PCRₖ) was shown in Table 4.7.

<table>
<thead>
<tr>
<th></th>
<th>LSL</th>
<th>μ</th>
<th>σ</th>
<th>PCRₖ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardness (Before Aging)</td>
<td>10</td>
<td>4.4625</td>
<td>1.28</td>
<td>0.8984</td>
</tr>
<tr>
<td>Hardness (After Aging)</td>
<td>1</td>
<td>13.44</td>
<td>0.89565</td>
<td>1.2806</td>
</tr>
</tbody>
</table>

The PCRₖ for hardness after aging is 1.28. A value of PCRₖ >1 shows that process natural tolerance limits lie inside the specifications and very few defective or non conforming units will be produced. For hardness before aging PCRₖ < 1 which shows that the process is very yield sensitive and number of defectives will be greater if used in such state.

4.3.4 Taguchi Loss Function

Normally for hardness Taguchi Loss Function is calculated using ‘Larger is the Best’ loss function. But after a certain time, rate of increase in hardness is greatly reduced as the operating time of aging oven increases. Therefore an optimal hardness which will be with in a cost effective range was taken as an upper limit for hardness after aging and expected loss was calculated using ‘Nominal is the Best’ loss function. While for hardness without aging ‘larger is the best’ loss function was used and results are shown in Table 4.8.
Table 4.8: Taguchi Loss Functions for Hardness

<table>
<thead>
<tr>
<th>Process</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>Loss Function</th>
<th>Expected Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardness (Before Aging)</td>
<td>4.4625</td>
<td>1.1356</td>
<td>Larger is Better</td>
<td>.063253 $A_1$</td>
</tr>
<tr>
<td>Hardness (After Aging)</td>
<td>13.43</td>
<td>0.7818</td>
<td>Nominal is the Best</td>
<td>0.2339$A_2$</td>
</tr>
</tbody>
</table>

Where $A_1$ and $A_2$ are costs for being out of specification

4.4 Anodizing

Aluminum extrusions after age hardening are taken to anodizing or painting shop depending upon the surface finish required for final product. The cost of anodizing is about half of painting cost. It is relatively a low cost process that gives good surface finish.

4.4.1 Anodizing Defects

Different anodizing defects observed in the ALUPCO products are following

- **Black Pits**
  
  These are black spots found on the surface after anodizing.

- **Dull Finish**
  
  ALUPCO has different grades of surface finish for final anodizing output that are made according to customer's requirements. If the final finish does not fulfill the required quality it is called dull finish.

- **Corrosion**
  
  Corrosion takes place during storage after aging and before anodizing
• **Caustic Patches / Non uniform etching**

  These patches are found on some extrusions due to non uniform anodizing

• **Wall thickness reduced**

  It is due to less anodizing thickness on the final anodizing output

• **Damaged / scratches**

  It is due to mishandling in transferring extrusions from one tank to other.

Figure 4.33 shows some of the above defects.

### 4.4.2 Pareto Analysis

Pareto Charts are plotted for defects in anodizing shop by using monthly and annually rejection data. Also charts are plotted for total number of defects during last 6 years. The plots for annual defects and total defects for last 6 years are shown in Figure 4.34-4.37. Process scrap was the most occurring defect in anodizing shop. Number of extrusions are made greater than that are ordered by customer due to expected loss in press and anodizing shop. If the final anodized output has number of extrusions greater than that of ordered, they are scraped and are called process scrap.

*This requires a proper feedback from anodizing shop to planning department.*

*Anodizing process needs further improvement to enhance the productivity of the plant.*

Second important cause of defects is corrosion. Corrosion takes place when extrusions are stored due to delay between press and anodizing shop.

*Therefore, there should be some proper arrangements to prevent corrosion during storage.*
Figure 4.33: Some Anodizing Defects [26]
Fig 4.35(a) : Pareto Chart & Analysis for Anodizing Defects 1994

Fig 4.35(b) : Pareto Chart & Analysis for Anodizing Defects 1995
Fig 4.36(a): Pareto Chart & Analysis for Anodizing Defects 1996

Fig 4.36(b): Pareto Chart & Analysis for Anodizing Defects 1997
Fig 4.37: Pareto Chart for Total Anodizing Defects 1992-1997
4.4.3 Histogram for Anodizing Thickness

Thickness of anodizing film is very important characteristic to decide the quality of surface finish of anodized extrusion. According to ALUPCO standard, upper specification limit for anodizing film is 30\(\mu\)m and lower limit is 10\(\mu\)m. Histogram was drawn taking 100 samples of measurements of anodized thickness of finished products, which is shown in Fig. 4.38 and other details are given in Table 4.9

<table>
<thead>
<tr>
<th></th>
<th>(\mu)</th>
<th>(\sigma)</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anodizing</td>
<td>22.64</td>
<td>3.422</td>
<td>0.15114</td>
</tr>
</tbody>
</table>

4.4.4 Model Verification

Different probability distributions models were fitted and results are shown in Table 4.10.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>(R^2) Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>0.986</td>
</tr>
<tr>
<td>Lognormal</td>
<td>0.982</td>
</tr>
<tr>
<td>Inverted normal</td>
<td>0.959</td>
</tr>
<tr>
<td>Weibull</td>
<td>0.964</td>
</tr>
<tr>
<td>Exponential</td>
<td>0.848</td>
</tr>
<tr>
<td>Extreme value</td>
<td>0.930</td>
</tr>
</tbody>
</table>
Fig 4.38: Histogram for Anodizing film Thickness
Normal probability distributions fitted well to data, and plotted with histogram in Fig. 4.38, other details of normal distribution are given in Table 4.11

<table>
<thead>
<tr>
<th>Anodizing</th>
<th>(\mu)</th>
<th>(\sigma)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>22.63</td>
<td>3.5463</td>
</tr>
</tbody>
</table>

Probability plots of all distributions are shown in Fig. 4.39-4.43.

4.4.5 Tolerance Charts and \(\text{PCR}_K\) for Anodizing

Tolerance chart was drawn using above data of 100 samples. The results are shown in Fig. 4.44. Tolerance chart shows that most of the points lie between centerline (CL) and upper specification limit, that mean the process is non central.

The \(\text{PCR}_K\) was calculated using Equation 3.2, which is shown below in Table 4.12

<table>
<thead>
<tr>
<th>Anodizing</th>
<th>USL</th>
<th>LSL</th>
<th>(\mu)</th>
<th>(\sigma)</th>
<th>(\text{PCR}_K)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30</td>
<td>10</td>
<td>22.64</td>
<td>3.422</td>
<td>.693</td>
</tr>
</tbody>
</table>

\(\text{PCR}_K < 1\) shows that there is large probability of producing out of specification limit extrusions at the end of anodizing process.
Fig. 4.39: Normal distribution for Anodizing Thickness

\[ y = 0.2802x - 6.3431 \]

\[ R^2 = 0.986 \]
Fig. 4.41: Inverted Normal distribution for Anodizing Thickness

$y = -131.35x + 5.9134$

$R^2 = 0.9627$

$\Phi^{-1}(F)$

$1/XI$

$\Xi$: Anodizing Thickness
Fig. 4.42: Exponential distribution for Anodizing Thickness

$y = 0.2168x - 3.9829$

$R^2 = 0.8687$
Fig. 4.43: Weibull Distribution for Anodizing Thickness

\[ y = 7.9768x - 25.335 \]

\[ R^2 = 0.9661 \]
Fig 4.44: Tolerance Chart for Anodizing Thickness
4.4.6 Taguchi Loss Function

Taguchi loss function was calculated using ‘Nominal is the Best’ loss function and taking mean of ALUPCO specifications as target value and was found to be 0.1886 A. where A is the cost/ton for being out of specification, to manufacturer.

Table 4.13: Taguchi Loss Functions for Anodizing

<table>
<thead>
<tr>
<th>Process</th>
<th>μ</th>
<th>σ</th>
<th>Loss Function</th>
<th>Expected Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anodizing</td>
<td>22.64</td>
<td>3.422</td>
<td>Nominal is the Best</td>
<td>0.1886 A</td>
</tr>
</tbody>
</table>

Where A is the cost of being out of specifications.

4.4.7 Cause and Effect Diagram

Figure 4.45 shows a cause and effect diagram. It shows how defects at different stages contribute to the final quality of an anodized extrusion product.

4.5 Painting.

For high quality surface finish aluminum extrusions are painted after age hardening. Details of this process are given in section 1.5. Painting shop has a variety of color schemes. Therefore painted aluminum extrusions can be used for decorative purposes.

4.5.1 Painting Defects

- Less Thickness
  
  It is due to insufficient paint thickness.

- Orange Peel
Fig 4.45: Cause and Effect Diagram for Anodized Products
It is due to greater thickness of paint

- **Blisters / overlapping**

  These are due to touching of two extrusions before drying.

- **Damaged / scratches**

  Due to mishandling during different steps.

### 4.5.2 Pareto Analysis

Pareto Charts are plotted for defects in painting shop by using monthly and annually rejection data. Also charts are plotted for total number of defects during last 6 years. The plots for annual defects and total defects for past 6 years are shown in Figure 4.46-4.49.

In painting shop, minor defects which were categorized as other defects were found to be most frequently occurring defects. Therefore it needed to study them individually and categorize them separately. Other important causes are orange peel, less micron and sticking of dust particles. The root cause of less micron and orange peel is faulty spray gun. If it is choked then it sprays less paint causing less micron. While if it is worn out it sprays more causing orange peel.

*Therefore it is necessary to clean and replace the spray gun at proper time.*

### 4.5.3 Histogram for Painting Thickness

Similar to the anodizing the thickness of paint film is also very important characteristic to decide the quality of surface finish of painted extrusion. According to ALUPCO standard, upper specification limit for painting film is 150µm and lower limit is 60µm. Histogram was drawn taking 100 samples of painting thickness on finished products, which is shown in Fig. 4.50. and other details are given in Table 4.14.
Fig 4.46(a): Pareto Chart & Analysis for Painting Defects 1992

Fig 4.46(b): Pareto Chart & Analysis for Painting Defects 1993
Fig 4.47(a): Pareto Chart & Analysis for Painting Defects 1994

Fig 4.47(b): Pareto Chart & Analysis for Painting Defects 1994
Fig 4.49: Pareto Chart for Total Painting Defects 1992-1997

- Others: 306967
- D. Particil/Orange Peel: 169124
- Less Microns: 152399
- Damages / Scratches: 87167
- Blisters/Overlapping: 82840

Legend:
- Value
- Cumul. Percent
Table 4.14: Table for Mean and SD of Paint Thickness

<table>
<thead>
<tr>
<th></th>
<th>( \mu )</th>
<th>( \sigma )</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paint Thickness</td>
<td>22.64</td>
<td>3.422</td>
<td>0.15114</td>
</tr>
</tbody>
</table>

4.5.4 Model Verification

Different probability distributions models were fitted and results are shown in Table 4.15.

Table 4.15: Model Verification for Paint Thickness

<table>
<thead>
<tr>
<th>Distribution</th>
<th>( R^2 ) Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>0.949</td>
</tr>
<tr>
<td>Lognormal</td>
<td>0.946</td>
</tr>
<tr>
<td>Inverted normal</td>
<td>0.860</td>
</tr>
<tr>
<td>Weibull</td>
<td>0.978</td>
</tr>
<tr>
<td>Exponential</td>
<td>0.913</td>
</tr>
<tr>
<td>Extreme value</td>
<td>0.859</td>
</tr>
</tbody>
</table>

Normal and Weibull probability distributions fitted well to data, but due to ease in use normal distribution was used and plotted with histogram in Fig.4.50. Probability plots for all distributions are shown in Fig. 4.51-4.55.

4.5.5 Tolerance Charts and PCR\(_K\) for Painting

Tolerance chart was drawn using above data of 100 samples. The results is shown in Fig. 4.56. Chart shows that most of the points lie between center line (CL) and lower specification limit and 17 points or out of specification limits
Fig. 4.50: Histogram for paint thickness

Expected Normal
Fig. 4.51: Normal distribution for Paint Thickness

(y = 0.0257x - 2.4314
R^2 = 0.949)

(XI: Paint Thickness)
Fig. 4.53: Inverted Normal distribution for Paint Thickness

($\Phi^{-1} (F)$ vs. $1/X_i$)

$y = -169.23x + 2.0341$

$R^2 = 0.8635$

($X_i$: Paint Thickness)
Fig. 4.54: Exponential distribution for Paint Thickness

(Y = 0.0243x - 1.3153)

$R^2 = 0.9134$
Fig. 4.55: Weibull distribution for Paint Thickness

\( y = 2.8378x - 13.239 \)
\( R^2 = 0.9784 \)

(XI : Paint Thickness)
Fig 4.56 : Tolerance Chart for Paint Thickness
The PCR$_K$ was calculated using Equation 3.2 which is shown below in Table 4.16.

<table>
<thead>
<tr>
<th>Table 4.16: PCR$_K$ Values for Painting</th>
</tr>
</thead>
<tbody>
<tr>
<td>USL</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>Painting</td>
</tr>
</tbody>
</table>

Such a small value of PCR$_K$ show that there is large probability of producing out of specification limit extrusions, thus further reduction of variability at painting stage is highly desirable.

### 4.5.6 Taguchi Loss Function

Taguchi loss function was calculated using ‘Nominal is the Best’ loss function and taking mean of ALUPCO specifications as target value and was found to be 0.1886 A. where A is the cost/ton for being out of specification, to manufacturer.

<table>
<thead>
<tr>
<th>Table 4.17: Taguchi Loss Functions for Painting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>Painting</td>
</tr>
</tbody>
</table>

Where A is the cost of being out of specifications.

### 4.5.7 Cause and effect diagram

Cause and effect diagram for a painted extrusion is shown in Figure 4.57. It shows that how defects at different stages contribute to final quality of a painted extrusion product.
Fig 4.57: Cause and Effect Diagram for Painted Products
4.6 Cost Model and Plant Efficiency

As shown in above sections there is a loss of production at different stages due to defects. These defective parts are taken back to melting shop and are recycled. It is necessary to calculate the loss due to these defectives and its effect on overall cost of product and plant efficiency.

A cost model is proposed in a generalized framework, which incorporates the probabilities of producing a defective part at each stage.

Figure 4.59: Plant lay out showing the block diagram for cost analysis and fraction of defectives at each stage

Let

\[ P_i = \text{Fraction (probability) of defectives at different stages, } i = 1, 2, 3, 4a \text{ or } 4b \]

\[ C_i = \text{Individual cost of different stages} \]
\[ T_{cl} = \sum_i C_i \]

\[ T_{CA} = C_1 + P_1 C_1 + C_2(1-P_1) + P_2(1-P_1) (C_1+C_2) \]
\[ + C_3(1-P_1)(1-P_2) + P_3(1-P_1)(1-P_2) (C_1+C_2+C_3) \]
\[ + C_4(1-P_1)(1-P_2)(1-P_3) + XP_4(1-P_1)(1-P_2)(1-P_3) (C_1+C_2+C_3+C_4) \]
\[ + (1-X)P_5(1-P_1)(1-P_2)(1-P_3) (C_1+C_2+C_3+C_4+C_5) \]

\[ T_{Cl} = C_1 + P_i + \beta \sum_{i=2}^n \left[ C_i \left[ \prod_{j=1}^{i-1} (1-P_j) \right] + P_i \left[ \prod_{j=1}^{i-1} (1-P_j) \sum C_i \right] \right] \]

Where \( \beta \) is \( X \) for 4a stage and \( (1-X) \) for 4b stage. For all other stages \( \beta \) is 1.

\( T_{Cl} \) = Ideal Total Cost/Ton of final product if there were no defectives at any stage

\( T_{Ca} \) = Actual Total Cost/Ton of final product considering fraction of defectives at each stage

\[ \text{Plant Efficiency} = \frac{T_{CA}}{T_{Cl}} \]

Table 4.18 shows number of defectives at different stage during 1992-1997 and their total.

Table 4.18: % of Defectives in different shops (Pi x 100)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Press</td>
<td>1.89</td>
<td>1.29</td>
<td>0.97</td>
<td>1.69</td>
<td>0.89</td>
<td>0.62</td>
<td>1.26</td>
</tr>
<tr>
<td>Anodizing</td>
<td>1.49</td>
<td>1.31</td>
<td>1.10</td>
<td>1.50</td>
<td>0.99</td>
<td>0.62</td>
<td>1.25</td>
</tr>
<tr>
<td>Painting</td>
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<td>2.00</td>
<td>1.86</td>
<td>2.10</td>
<td>2.23</td>
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</tbody>
</table>
As it can be seen that during 1992 the probability of defect at each stage was much higher, it was gradually reduced to 1995. Still it was substantial. Further reduction is observed in pressing and anodizing stage (particularly due to a new extrusion press installation with a highly accurate computer control system, ensuring more consistent output). However the process of painting is still producing about two tons of scrap out of every 100 ton of extrusion. Thus processes in painting shop need to be further studied to eliminate or minimize this scrap.
Chapter 5

Reliability Aspects of Extrusion Process

5.1 Reliability

Most of the defects in extruded product at stage 2 are attributed to initial or gradually progressing defects in extrusion die. Thus the quality of product is directly influenced by the quality of die. The quality of die deteriorates with time. This deterioration of quality of dies with respect to time is reflected by its reliability. Reliability R(t) is defined as the probability that a tool will perform properly for a specified period of time under a given set of operating conditions. Mathematically it is defined as R(t)=P(T>t).

The reliability R(t) is the complementary function of probability of failure at time ‘t’ F(t) = P(T\leq t) which is also known as commulative distribution function (F(t) = 1-R(t)).
The rate of change of the cumulative distribution function is known as probability density function. i.e., \( f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt} \)

Another important function defining the reliability is the failure rate function or hazard function \( \lambda(t) = \frac{f(t)}{R(t)} = \frac{-dR(t)/dt}{R(t)} \). Thus reliability function \( R(t) \) and hazard function \( \lambda(t) \) can be expressed as \( R(t) = e^{-\int_0^t \lambda(t) dt} \). The hazard function or failure rate function of a die in this case will indicate whether it is constant or increasing with respect to time. Since we often experience aging type of die failure (time dependent quality deterioration of die) we can expect that the failure rate should be an increasing function of time. Before we further investigate the nature of reliability function which characterize the die, it is important to see what type of failures lead to termination of die life and what is the relative importance of each mode of failure.

### 5.2 Failure of Dies

Tools and dies, although made from the correct grade, well designed, and properly machined and heat treated, can fail after limited service due to improper operation or mechanical problems. Mechanical factors that may cause premature failures include overloading, overstressing or alignment and clearance problems. Excessive temperature may be a factor in hot working die failures, perhaps due to inadequate cooling between operations. Failures can occur during assembly e.g. during shrink fitting of one part onto other. Stamp marks, in addition to causing heat treatment failures, can cause service failure due to stress concentration. Alignment problems are a common cause of failure of tools used in shearing operations. While most tools and dies fail either in a brittle manner, or due to fatigue failure. In most case the fatigue failure is located at a change in
section size at a sharp corner. Tools used in hot working applications generally fail due to development of heat cracks. These result due to thermal stresses from alternate heating and cooling.

5.3 Failure Modes of Extrusion Dies

A number of causes are considered to be responsible for initiating the damage of an extrusion die before its designated life is reached. An extrusion die is exposed to high temperatures derived not only from the heated billet but also from the heat generated by deformation and friction. In addition, die is subjected to high pressure, and in the area of die land, considerable frictional forces.

The extrusion dies work in the temperature range of 400 to 470°C, while the stress or pressure is in the range of 2000 to 2700psi. The stresses experienced by an extrusion die in service are both mechanical and thermal in origin. The thermal stresses arising from temperature difference are generally quite moderate in extrusion, while wear fracture are very pronounced in extrusion. The nature of stress is cyclic with the maximum reaching upto 2700psi and a minimum of zero. Action of cyclic stress in presence of initial cracks ( which are often generated during machining and heat treatment ), can lead to crack growth and ultimate fatigue failure.

The commonly observed modes of die failure are

- Crack initiation or formation of inherent micro quench cracks during heat treatment, and their propagation with respect to time to a threshold value.
- Progression of Wear to a limiting value
- Plastic deformation of dies or deflection of mandrels
5.3.1 Fatigue due to Crack Growth

Most tools and dies fail due to fatigue failure as their primary cause. In most cases, the fatigue failure is located at a change in section size, at a sharp corner, or at stamp marks. The nature of various operational aspects of the operating nature of these dies suggest a potential for failure resulting from the growth of cracks on the bearing surface. First, during normal operations, the dies, are subjected to large, cyclic stresses. Second the cavities in both dies create regions of high stress concentration specially at race ways and at intricate braces, where cracks can initiate and grow resulting a catastrophic failure. Third, in order to produce profiles of well controlled shape and dimensions, the dies are made of high strength, hardened material e.g. H-13. As such they must be prone to brittle fatigue fracture.

5.3.2 Wear Failures

Wear also constitutes a sizable cause of die failure. During the preheating of the aluminum billet, aluminum oxide(Al_2O_3) is formed on the surface-. This oxide of aluminum is very hard. It’s hardness is comparable to the hardness of some compounds of diamond. When the die is heated prior to extrusion, a film of iron oxide is also formed on the die bearing surface. The combine effect of these two phenomena results in the wear of the bearing surface. It is observed that the wear is more pronounced at locations of high stress concentration, resulting in wearing the die cavity unevenly.

Die material should have a wear resistant surface, hence a proper surface treatment is essential to create such a surface and tool material should be adaptable to this treatment without a greater sacrifice of toughness.
5.3.3 Deflection of Mandrel and other Plastic Deformation Related Failures

One of the main causes that promote the extrusion die failure is the die deflection which causes part of the die surface to get deflected due to high extrusion pressures acting at the center of opening and decreasing radially outwards. This is observed particularly when the die opening is wide making the die weak at the corners of the opening with higher stress intensity at that point. The movement of the die surface along the corners of the opening is considered to be responsible for the die deflection.

Some dies also fail due to the mandrel deflection or due to the plastic deformation. The primary reason for this type of failure is the unbalance stress acting on the die face, either due to misalignment or due to poor die design. This type of failure is often observed in dies which extrude un-symmetric profiles. The extent of the deflection can be discovered by observing cracks on the mandrel. Figure 5.1 shows some typical failures of aluminum extrusion dies.

5.3.4 Die Failure Modes Observed in ALUPCO

Analyzing around 2400 die failure histories obtained from ALUPCO, the dies were categorized according to the cause of failure and results are given below:

- Wear 24%
- Fatigue Crack 40%
- Deflection and Plastic Deformation 36%

The results are also shown as a pie chart in figure 5.2.
Cracked Extrusion Die

Plastic Deformation on Extrusion Die

Extrusion Die Failed due to underhardening

Figure 5.1: Die Failure Modes
Figure 5.2: Pie Chart showing the major causes of extrusion die failure (Data from ALUPCO)
5.4 Probabilistic Modeling of die Life

The die material and its specific properties are inherently probabilistic in nature. This, in turn is responsible for the variation of life of an individual extrusion die used in industry. The reliable prediction of the tool life is important for an economical production because of the associated loss due to an unscheduled failures of the dies, particularly in an automated system when the cost of machine stoppage is quite high due to productivity losses and poor quality product. In addition, the expenses involved in unscheduled change of the tool are much higher, and thus the unscheduled replacement may contribute to a significant part of the total expenditure involved in production. The fact that tool life in general is typically stochastic in nature and hence governed by the laws of probability has been well appreciated in literature. The probability theory can be used to define the random behavior of die life by appropriate statistical distributions.

5.5 Statistical Distributions

In statistical literature variety of distributions are used to model reliability of systems and devices. These distributions \((F(t)=1-R(t))\) are [36]

- Normal
- Lognormal
- Extreme value of smallest extreme
- Weibull
- Exponential

The cumulative distribution function \(F(t)\) of these distributions are summarized in Table 3.1 with their mean \((\mu)\) and variance \((\sigma^2)\) given by
\[ \mu = \int_0^\infty t \cdot f(t) \, dt = \int_0^\infty R(t) \, dt \]

and \[ \sigma^2 = \int_0^\infty t \cdot f(t) \, dt - \mu^2 \]

respectively. The mean is the measure of central tendency of data and standard deviation is measure of variability of data around its mean.

Each individual distribution can be justified either by

- Physical reasoning i.e., by characterizing the damaged process leading to failure
- Empirical reasoning; first, fitting the data to a non-parametric \( F(t) \) and then by suitable transformation technique to make a best fit, the most appropriate distribution is selected

In the case of die life study, Asif [37] provided physical justification to use a Weibull model. In this study we will use empirical fit to a feasible group of distributions which can fit the die life data. Then by comparing the degree of fit we will determine the best fitting model.

### 5.7 Reliability Analysis of Dies at ALUPCO

In this section reliability of extrusion dies will be studied using real life data from ALUPCO. An analysis of 1340 die data was earlier reported by Asif [37]. That data was essentially fitted to Weibull model, by assuming this distribution a priori, using the physical reasoning. In this study we have included an additional 1200 die life data. It is important to emphasize that the real life data such as obtained from industry is very valuable, since the cost of setting up such prototype identical laboratory test facility would cost million of Riyals. So this comprehensive data was used to fit a spectrum of
distributions as shown in Table 3.1 and 3.2, to check the assumption of using Weibull distribution as best choice to fit die failure data. The life of an extrusion die is the duration up till which, it performs satisfactorily in terms of quality of the product being extruded. Hence if lines appear on the extruded product surface or die cavity wears out of tolerance rendering unacceptable weight/length of the product, the die is considered failed. The life of a die can be expressed in a number of ways.

- Number of pushes or number of billets extruded, after which the die fails
- The total weight of metal extruded

In this analysis we will use total weight of metal extruded as a measure of die life. Also the data was later analyzed to correlate the parameters of selected reliability model with various geometric features of the dies (i.e. a measure of its complexity).

A summarized version of data, and a degree of fit to data based on coefficient of determination ($R^2$) value are shown in Table 5.1. These results also show that Weibull distribution is the best distribution to describe the failure of dies in each categories. Parameters of fitted Weibull distribution are shown in Table 5.2. Figures 5.3 - 5.11 show some of the Weibull probability plots of die failure data for various dies illustrated before each graph. Some reasons due to which Weibull distribution is the best distribution to describe failure rate data are given by Asif [38] as follows.

- The Weibull model is versatile and has necessary flexibility and the ability to be expressed in a closed form to accommodate a number of shapes of distributions by varying its parameters.
- When a system is composed of a number of components and failure is due to the weakest element of the system, or when several modes of failure or damage
Table 5.1: $R^2$ Values for Different Statistical Distribution fitted to Die Failure Data

<table>
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<th>Sr. No.</th>
<th>Die No.</th>
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<th>Log Normal</th>
<th>Inverted Normal</th>
<th>Weibull</th>
<th>Exponential</th>
<th>Extreme Value</th>
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Table 5.2: Details of Weibull Distribution

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<th>$\eta$</th>
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<td>0.946</td>
</tr>
<tr>
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<td>0.965</td>
</tr>
<tr>
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<td>H5178</td>
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<td>0.992</td>
</tr>
<tr>
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<td>H5179</td>
<td>20</td>
<td>3.010</td>
<td>40221.7</td>
<td>0.980</td>
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<tr>
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<td>H7233</td>
<td>39</td>
<td>1.520</td>
<td>21799.9</td>
<td>0.990</td>
</tr>
<tr>
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<td>H9008</td>
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<td>2.049</td>
<td>46603.2</td>
<td>0.973</td>
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<td>0.993</td>
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<td>2.144</td>
<td>37193.9</td>
<td>0.982</td>
</tr>
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<td>15032.6</td>
<td>0.965</td>
</tr>
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</tr>
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<td>41407.5</td>
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<td>39740.9</td>
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<td>22</td>
<td>1.290</td>
<td>26281.0</td>
<td>0.967</td>
</tr>
</tbody>
</table>
Figure 5.3: Shapes of some extrusion profiles produced by different dies. [Number below each profile represents ALUPCO die shop number]
Fig 5.5: Probability Plot for Weibull distribution of Die H1984
(Time to failure $X_i$=Kg of metal extruded by die before failure)
Fig 5.6: Probability Plot for Weibull distribution of Die H2392
(Time to failure $X_i=Kg$ of metal extruded by die before failure)

$y = 1.8286x - 18.57$

$R^2 = 0.9953$
Fig 5.7: Probability Plot for Weibull distribution of Die H5178
(Time to failure $X_i$ in Kg of metal extruded by die before failure)

$y = 1.8819x - 19.837$

$R^2 = 0.992$
Fig. 5.8: Probability Plot for Weibull distribution of Die H5179 (Time to failure Xi=Kg of metal extruded by die before failure)

\[ y = 3.0059x - 31.869 \]

\[ R^2 = 0.9801 \]
Fig 5.9: Probability Plot for Weibull distribution of Die H7233
(Time to failure $X_i$=Kg of metal extruded by die before failure)
Fig 5.10: Probability Plot for Weibull distribution of Die H9019
(Time to failure $X_i$=Kg of metal extruded by die before failure)
Fig 5.11: Probability Plot for Weibull distribution of Die H9025
(Time to failure $X_i$=Kg of metal extruded by die before failure)
• mechanism are operating simultaneously, the most severe damage manifestation (or extreme value) will cause failure, then time to failure distribution will be Weibull.

• The Weibull model as compared to other models is the most conservative reliable life prediction model.

• It is the life model obtained as an outcome of extreme value phenomenon of tool damage which is quite relevant to the die failure phenomenon in metal forming (i.e. extreme damage due to crack propagation or/and wear reaching a critical level).

• Monotonically increasing failure rate of the tools or dies is best characterized by the power law type Weibull failure rate model.

5.8 Die Reliability Parameters as a Function of Die Geometry

As shown in Table 5.1 the CDF for Weibull model is

\[ F(t) = 1 - \exp\left\{-(t/\eta)^\beta\right\} \]  \hspace{1cm} (5.1)

Therefore for the Weibull distribution reliability \( R(t) = 1 - F(t) \) is given by

\[ R(t; \eta, \beta) = \exp\left\{-(t/\eta)^\beta\right\} \]  \hspace{1cm} (5.2)

Also the Weibull model reliability can be expressed as a function of average life \( \bar{T} \) and coefficient of variation (COV) i.e. \( \sigma/\mu \), using the following relationship.

\[ \bar{T} = \eta \Gamma\left(1 + \frac{1}{\beta}\right) \]  \hspace{1cm} (5.3)

\[ \frac{\sigma}{\mu} = \frac{\sqrt{\Gamma\left(1 + 2/\beta\right) - \Gamma^2\left(1 + 1/\beta\right)}}{\Gamma\left(1 + 1/\beta\right)} \]  \hspace{1cm} (5.4)
Our objective in this study was to express these parameters in terms of various die features, i.e.

\[ \eta = \eta(X_1, X_2, X_3, \ldots, X_n) \]
\[ \beta = \beta(X_1, X_2, X_3, \ldots, X_n) \]

or

\[ \tilde{T} = T(X_1, X_2, X_3, \ldots, X_n) \]
\[ \sigma/\mu = \sigma/\mu(X_1, X_2, X_3, \ldots, X_n) \]

Where \( X_1, X_2, X_3, \ldots, X_n \) are important die features.

The above data was distributed in different categories depending upon following seven important die features which are normally provided with the detail drawings of the dies:

- The weight per unit length (W/L)
- The circumscribing circle diameter (CCD)
- The profile perimeter (P)
- The minimum wall thickness of the profile (\( t_{\text{min}} \))
- The profile area (A)
- The number of profile cavities (N)
- The press container diameter (C.D.)

Two techniques are used to calculate the Weibull model parameters as a function of above die features.

1. Regression Analysis
2. Artificial Neural Networks
5.8.1 Regression Analysis

In many problems two or more variables are inherently related, and it is important to model and explore the nature of relationship. Regression analysis is a statistical technique for modeling and investigating the relationship between two or more variables. Regression methods are frequently used to analyze data from unplanned experiments and this analysis is also very useful in designed experiments. The overall objectives of regression analysis can be summarized as follows:

- To determine whether or not a relationship exists among the variables
- To describe the nature of relationship, should one exist, in the form of a mathematical equation
- To assess the degree of accuracy of prediction achieved by the regression equation
- To assess the relative importance of different predictor variables in their contribution to variation in the dependent variable

5.8.1.1 Types of Regression Analysis

If the independent variable or regressor is only one then the model is called simple regression. But many regression problems involve more than one regression variable. The general form for fitting multiple linear regression is given as

\[ Y = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \cdots + \alpha_k X_k + \varepsilon \]

Where \( X_1, X_2, \ldots, X_k \) are the designed operational factors and various interaction terms are ignored. The unknown parameters "\( \alpha \)'s" are called regression coefficients and "\( \varepsilon \)" is an error. The methodology of the least square is used to estimate the regression coefficients in the above equation [31].
5.8.2 Artificial Neural Networks

Interest in artificial neural network has grown rapidly over the past few years. Researchers from different branches of engineering are interested to explore the potential offered by this emerging modeling and analysis tool to their respective disciplines. A neural network has the ability to learn the underlying relationship among a number of variables through exposure to the examples of relationship. A neural network can capture relationships that are difficult to relate analytically. It offers a simple algorithmic way of capturing the relationship without paying any attention to it's complexity.

5.8.2.1 History

Modern era of neural networks begun with the pioneering work of McCulloch and Pitts [38] in 1943, in which they have described a logical calculus of neural networks. In 1956 Uttley [39] demonstrated that neural networks with modified synapses may learn to classify binary patterns. Rosenblatt [40] proposed the perceptron convergence theorem in 1958 and proved his theorem in 1960. In 1969 Minsky and Papert [41] demonstrated that there is a fundamental limit to one-layer perceptrons computation. They further stated that any of the virtues of the one-layer perceptrons carry over to the many-layered version. Werbos [42] in 1974 developed the back propagation algorithm to propagate the error at output back to the hidden layer.

The most influential publications are, the 1982 paper by Hopfield [43] and the two-volume book by Rumelhart and McLelland [44] which are responsible for the resurgence of interest in neural networks in the 1980s. Since then there is an exponential rise in the research contributions to this field.
5.8.2.2 Architecture

The artificial neural networks are biologically inspired information processing units. They consist of simple processing elements called neurons. These neurons have the ability to work in parallel and are interconnected. The strength of interconnection is denoted by the parameter called synaptic weights. The model of a neuron is shown in Fig.(5.12), where $x_1, x_2, \ldots, x_p$ are the input signals (Different die features in our case), $w_{k1}, w_{k2}, \ldots, w_{kp}$ are the synaptic weights of neuron $k$, $w_{k0}$ is the bias, $v_k$ is the linear combiner output, $\varphi(\cdot)$ is the activation function, and $y_k$ is the output signal (Die life and coefficient of variation of die life, $\sigma/\mu$ in our case) of the neuron. In mathematical terms the neuron $k$ is described as [45]

$$v_k = \sum_{j=1}^{p} w_{kj} x_j + w_{k0}$$

$$y_k = \varphi(v_k)$$

(5.1)

Fig.(5.12): Model of a neuron [45]
5.8.2.3 Activation Function

The activation function, denoted by $\varphi(\cdot)$, defines the output of a neuron in terms of the activity level at its input. Various types of non-linear activation functions can be used. Some of them are shown in the Fig. (5.13). For the problem under study sigmoid or hyperbolic tangent sigmoid function can be used in the hidden layers and any other function can be used for the output layer. The hyperbolic tangent sigmoid function is defined as:

$$
\varphi(v_k) = \frac{1 - \exp(-av_k)}{1 + \exp(-av_k)}
$$

(5.2)

![Graphs of activation functions]

Fig.(5.13): Types of activation function [45]
5.8.2.4 Multilayer Feed-forward Networks

Multilayer feed-forward networks consists of an input layer, a number of hidden layers and an output layer as shown in Fig.(5.14). It is the most popular neural network structure. Input layer is essentially a direct link to the inputs of the first hidden layer. The hidden layer consists of neurons with non-linear activation function in our application. The output neuron in our case is a linear activation function. The output of each neuron is connected to the inputs of all the neurons in the next layer. Signal is unidirectional i.e., flows from input to output.

![Diagram](https://via.placeholder.com/150)

Fig. 5.14: Multilayer Feed Forward Network [46]

5.8.2.5 Training of Feed-forward Neural Networks

Typical training of a multilayer feed-forward network using the back-propagation algorithm can be described as follows: Starting with a suitable network configuration, set all the synaptic weights and biases of the network to small uniformly distributed random
numbers. The set of the input data and the desired outputs are collected. The input set of data is applied to the neural networks and the output of the network is compared with desired outputs. The error so generated is used to train the feed-forward network through back-propagation of this error. The synaptic weights are adjusted while minimizing an objective function, typically sum of the squared error. The synaptic weights are adjusted iteratively until a stopping criterion is met.

_Nomenclature:_

\[ w_{ji}(.) \quad \text{weight connecting the output of neuron } i \text{ to the input of neuron } j \]

\[ \Delta w_{ji}(.) \quad \text{correction factor.} \]

\[ e_j(.) \quad \text{error signal at the output of neuron } j \]

\[ \varepsilon(.) \quad \text{instantaneous sum of the square error} \]

\[ \varepsilon_{av} \quad \text{average squared error} \]

\[ d_j(.) \quad \text{desired response of neuron } j \]

\[ y_j(.) \quad \text{output of neuron } j \]

\[ \eta \quad \text{learning rate parameter} \]

\[ \varphi'_j(.) \quad \text{activity function} \]

### 5.8.2.6 Derivation of Back-propagation Algorithm

Back-propagation is a first order gradient scheme to minimize the error between the actual and the desired output. The error signal at the output of neuron \( j \) at iteration \( n \) as shown in Fig. (5.15) is defined by

\[ e_j(n) = d_j(n) - y_j(n) \quad (5.3) \]
Let the instantaneous squared error for neuron \( j \) be \( \frac{1}{2} e_j^2(n) \). Therefore instantaneous sum of squared error of the network is given by

\[
\varepsilon(n) = \frac{1}{2} \sum_{j \in C} e_j^2(n)
\]  

(5.4)

where \( C \) includes all the neurons in the output layer of the network.

![Signal flow graph showing the details of output neuron \( j \) [46]](image)

If \( N \) denotes the total number of patterns in the input, then the average squared error is given as

\[
\varepsilon_{av} = \frac{1}{N} \sum_{n=1}^{N} \varepsilon(n)
\]  

(5.5)
The average squared error $\varepsilon_{av}$ is a function of all the free parameters (synaptic weights and biases). The objective of the learning process is to adjust the free parameters of the network so as to minimize $\varepsilon_{av}$.

The net internal activity level $v_j(n)$ produced at the input of the nonlinearity associated with $j$ is

$$v_j(n) = \sum_{i=0}^{p} w_{ji}(n)y_i(n) \tag{5.6}$$

where $p$ is the total number of inputs (Total number of die features in our case).

$$y_j(n) = \varphi_j(v_j(n)) \tag{5.7}$$

The corrections $\Delta w_{ji}(n)$ to the synaptic weights $w_{ji}$, is proportional to the instantaneous gradient $\partial\varepsilon(n)/\partial w_{ji}(n)$. According to the chain rule

$$\frac{\partial \varepsilon(n)}{\partial w_{ji}(n)} = \frac{\partial \varepsilon(n)}{\partial e_j(n)} \frac{\partial e_j(n)}{\partial y_j(n)} \frac{\partial y_j(n)}{\partial v_j(n)} \frac{\partial v_j(n)}{\partial w_{ji}(n)} \tag{5.8}$$

Differentiating Eq.(5.3), Eq.(5.4), Eq.(5.6) and Eq.(5.7) with respect to the appropriate variable following relation is obtained

$$\frac{\partial\varepsilon(n)}{\partial w_{ji}(n)} = -e_j(n)\varphi_j'(v_j(n))y_i(n) \tag{5.9}$$

The corrections $\Delta w_{ji}(n)$ to the synaptic weights $w_{ji}(n)$ is defined by the delta rule:

$$\Delta w_{ji}(n) = -\eta \frac{\partial\varepsilon(n)}{\partial w_{ji}(n)} \tag{5.10}$$

$$\Delta w_{ji}(n) = \eta \delta_j(n)y_i(n) \tag{5.11}$$

where $\delta_j(n)$ is called local gradient defined as
\[ \delta_j(n) = -\frac{\partial \varepsilon(n)}{\partial e_j(n)} \cdot \frac{\partial e_j(n)}{\partial y_j(n)} \cdot \frac{\partial y_j(n)}{\partial v_j(n)} \]

(5.12)

\[ = e_j(n) \varphi_j'(v_j(n)) \]

(5.13)

Above equation shows that the key factor involved in the weight adjustment \( \Delta w_{ji}(n) \) is the error signal at the output of the neuron. In this context, there are two distinct cases depending upon where neuron \( j \) is located:

Case I: Neuron \( j \) is an output node

Case II: Neuron \( j \) is a hidden node

**Case I: Neuron \( j \) Is An Output Node**

When neuron \( j \) is in the output layer, we have the desired response of that neuron. So we can determine the error using Eq.(5.3) and can find the local gradient using Eq.(5.12).

**Case II: Neuron \( j \) Is A Hidden Node**

When neuron \( j \) is located in the hidden layer, there is no specific desired response of the neuron. But here again it shares the responsibility for any error made at the output of the network. Consider the situation shown in Fig.(5.16) depicting \( j \) as a hidden neuron [46].

Let the local gradient is redefined as

\[ \delta_j(n) = -\frac{\partial \varepsilon(n)}{\partial y_j(n)} \cdot \frac{\partial y_j(n)}{\partial v_j(n)} \]

(5.14)

\[ = \frac{\partial \varepsilon(n)}{\partial y_j(n)} \varphi_j'(v_j(n)) \]

(5.15)

The instantaneous sum of squared error of the network is given as

\[ \varepsilon(n) = \frac{1}{2} \sum_{k \in C} \varepsilon_k^2(n) \quad \text{neuron k is an output node} \]

(5.16)
The partial derivative of the above equation with respect to \( y_j(n) \) is

\[
\frac{\partial \varepsilon(n)}{\partial y_j(n)} = \sum_k e_k \frac{\partial e_k(n)}{\partial y_j(n)}
\]

(5.17)

Using chain rule above equation can be written as

\[
\frac{\partial \varepsilon(n)}{\partial y_j(n)} = \sum_k e_k \frac{\partial e_k(n)}{\partial v_k(n)} \frac{\partial v_k(n)}{\partial y_j(n)}
\]

(5.18)

As neuron \( k \) is in the output layer so we have access to its error.
\[
\frac{\partial e_k(n)}{\partial v_k(n)} = -\varphi'_k(v_k(n)) \tag{5.19}
\]

From Fig.(6.5) it is also clear that activity at neuron k is

\[
v_k(n) = \sum_{j=0}^{q} w_{kj} y_j(n) \tag{5.20}
\]

\(q\) is the total number of inputs to neuron k which will give

\[
\frac{\partial v_k(n)}{\partial y_j(n)} = w_{kj} \tag{5.21}
\]

Putting Eq.(5.19) and Eq.(5.21) in Eq.(5.18) we have

\[
\frac{\partial e(n)}{\partial y_j(n)} = \sum_k \delta_k(n) w_{kj}(n) \tag{5.22}
\]

Now putting Eq.(5.22) in Eq.(5.15) local gradient for neuron j results as

\[
\delta_j(n) = \varphi'_j(v_j(n)) \sum_k \delta_k(n) w_{kj}(n) \tag{5.23}
\]

i.e., the local gradient is the product of the associated derivative \(\varphi'_j(v_j(n))\) and the weighted sum of \(\delta's\) for the neuron in the next hidden or output layer.

In words back-propagation algorithm can be expressed as:

\[
\begin{bmatrix}
\text{Weights correction} \\
\Delta w_{ji}(n)
\end{bmatrix} = \begin{bmatrix}
\text{Learning rate parameter} \\
\eta
\end{bmatrix} \begin{bmatrix}
\text{Local gradient} \\
\delta_j(n)
\end{bmatrix} \begin{bmatrix}
\text{input signal} \\
y_i(n)
\end{bmatrix}
\]

5.8.2.6 Stopping Criteria

The back-propagation algorithm is considered to have converged when the cost function or sum square error (SSE) measure of the network has reached a pre-determined value, acceptable value or the change in the SSE is almost constant.
5.9 Results and Discussion

In this study \( \bar{T} \) and coefficient of variation \( \sigma/\mu \) (method of moments) was used to calculate reliability.

Previously Asif [37] defined \( \bar{T} \) and \( \sigma/\mu \) as a function of die complexity, and through predictive equation of \( \bar{T} \) and \( \sigma/\mu \), parameters of Weibull model determined for any complexity of die.

In this study we used an attractive alternative which is more precise, that uses Artificial Neural Network analysis. By appropriately training the neural network with all available information it is possible to predict quite accurately the reliability life parameters of any extrusion die of any complexity. This is quite promising approach as will be demonstrated.

Asif developed a model by using weighted multiple regression to predict \( \bar{T} \) and \( \sigma/\mu \). His data is given in Table 5.5. The equation of his predicted model is given below:

\[
\text{The Predicted } \bar{T} = \frac{A \times (C.D)^x(N)^y(A)^u(P)^v}{(t_{min})^w(CCD)^x(W_p)^y(W/L)^z}
\]

The values of his regression coefficients are given below:

<table>
<thead>
<tr>
<th>( A_0 )</th>
<th>s</th>
<th>t</th>
<th>u</th>
<th>v</th>
<th>w</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10)^28.3</td>
<td>2.39</td>
<td>1.03</td>
<td>68.19</td>
<td>1.17</td>
<td>1.86</td>
<td>0.82</td>
<td>2.61</td>
<td>66.18</td>
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</table>

The regression equation for coefficient of variation for Asif's data is as follows:

\[
COV = \frac{A_0(Perim)^x(t)^y(W/L)^z}{(A)^x(C.D)^y(CCD)^z(N)^z}
\]
Table 5.5: Table of factors influencing the die complexity, Predicted average life and Proposed complexities by Asif [37]

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Die</th>
<th>Scale Parameter</th>
<th>Shape Parameter</th>
<th>Average Life</th>
<th>C.D.</th>
<th>t</th>
<th>N</th>
<th>A</th>
<th>CCD</th>
<th>Perim</th>
<th>W.P.</th>
<th>W/L</th>
<th>Predic. Life</th>
</tr>
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<td>1</td>
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<td>9659.2</td>
<td>190</td>
<td>1.3</td>
<td>1</td>
<td>357</td>
<td>109</td>
<td>550</td>
<td>820</td>
<td>0.964</td>
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</tr>
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<td>1.3</td>
<td>2.53</td>
<td>182</td>
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<td>2</td>
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<td>1</td>
<td>1</td>
<td>236</td>
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</table>
The actual and regression model predicted average die life and coefficient of variation are plotted in figures 5.17 and 5.18. Each input point in these figures represents a row of CD, t, N, A, CCD, Perimeter, W.P, and W/L. Statgraphics output showing ANOVA and other details of regression model are given in Table 5.6 and 5.7.

We initially used this data for training the neural network by using Neural Network Toolbox in MATLAB and prediction by neural network is shown in Figure 5.19-5.20. The comparison of both regression model and neural network output are shown in Figure 5.21-5.22.

While developing regression equation for average die life $\bar{T}$ Asif added a new parameter called weighted parameter, in addition to seven parameters discussed above. This weighted parameter was developed by the multiple regression of each die category by considering all of its original data. Also he used weighted number of ports and container diameter by using weighted average of number of ports, when an extrusion is sometime made on a single port die and the other time it is made on a multiport die. This average has caused a non-integer number of ports in several cases, which is not true because the actual number of ports are only integers. Due to these complexities this model provide some limitations when using it for predicting the life of new dies since weighted parameter will be difficult to correct for this new die.

Therefore this data was modified as follow:
Fig 5.17: Output of Regression Analysis for Average Die Life (Kg of metal extruded before failure) of Asif's data
Fig 5.18: Output of Regression Analysis for COV (σ/μ) of Asif’s data
Table 5.4: Multiple Regression Analysis for average life of Asif's data

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<tr>
<th>Parameter</th>
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<th>Standard Error</th>
<th>T statistic</th>
<th>P-value</th>
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</thead>
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<td>W/L</td>
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Analysis of Variance

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<th>F-Ratio</th>
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$R^2 = 90.40\%$

Table 5.5: Multiple Regression Analysis for COV of Asif's data

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Analysis of Variance

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$R^2 = 50.038$
Fig 5.19: Neural network Prediction for Average Die Life (Kg of metal extruded before failure) of Asif's Data
Fig 5.20: Neural Network Prediction for $\sigma/\mu$ of Asif's Data
Fig 5.22: Comparison of Neural Network and Regression Models for COV (σ/µ) of Asif's Data
The weighted parameter was removed

Weighted number of ports and container diameter was replaced with actual number of ports and container diameter as they appear in die drawing

New data of about 1200 additional die obtained from ALUPCO was added

This modified data is shown in Table 5.8. Various regression models were applied and the equation of the best fit for $\bar{T}$ and $\sigma/\mu$ are given below:

$$
\text{The Predicted } \bar{T} = \frac{A_s(C.D)^t(A)^r(P)^u(t_{min})^w}{(CCD)^x(N)^y(W/L)^z}
$$

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<th>s</th>
<th>t</th>
<th>$U$</th>
<th>$v$</th>
<th>$w$</th>
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$$
\sigma/\mu = \text{COV} = \frac{(CCD)^x(N)^y(t)^z(W/L)^d}{A_s(A)^r(CD)^f(Perim)^g}
$$

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<th>b</th>
<th>C</th>
<th>d</th>
<th>e</th>
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The actual and regression model predicted average die life and COV are plotted in figures 5.23-5.24. Statgraphics output showing ANOVA and other details of regression model are given in Table 5.8-5.9.

We also used this data (Table 5.8) for training of neural network and corresponding prediction by neural network are shown in Figure 5.25-5.26. The comparison of both
regression model and neural network output are shown in Figure 5.27-5.28. The figures show that in this case neural network prediction is far better than the regression model.

After predicting coefficient of variation (σ/µ), ‘β’ can be calculated using Figure 5.29 which is a plot of equation 5.3. After calculating ‘β’ we can calculate ‘η’ by using equation 5.4.

β and η were calculated by using both regression analysis and Neural Network for two sample dies and respective probability distribution functions are plotted in figure 5.31-5.33 and the shapes of corresponding extrusion profiles are shown in figures 5.30, 5.30-5.32. Also the probability distribution function was calculated using above procedure for a die whose data was not used for regression analysis and neural Network training and result are shown in figure 5.34-5.35.
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Fig 5.23: Prediction of Regression Analysis for Average Die life (Kg of metal extruded before failure) of Modified Data
Table 5.7: Multiple Regression Analysis for average die life of Modified data

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Analysis of Variance

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$R^2 = 54.25\%$

Table 5.8: Multiple Regression Analysis for COV of Modified data

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Analysis of Variance

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$R^2 = 38.29\%$
Fig 5.25: Output of Neural Network for Average Die Life (Kg of metal extruded before failure) of Modified Data
Fig 5.26: Neural Network Prediction for $\sigma/\mu$ of Modified Data
Fig 5.27: Comparison of Neural Network and Regression Analysis for Average Die Life (Kg of metal extruded before failure) of Modified Data
Fig 5.28: Comparison of Neural Network and Regression Models for COV ($\sigma/\mu$) of Modified Data
Fig. 5.29: Coefficient of Variation Vs. "β" Values of Weibull Distribution
Figure 5.30: Extrusion Profile for Die H1464
Fig 5.31: Comparison of Neural Network and Regression Prediction for Die H1464 [See Fig 5.30]
Figure 5.30: Extrusion Profile for Die H9019
Figure 5.30: Extrusion Profile for Die H4524
Fig 5.35: Comparison of Neural Network and Regression Prediction for Die H4524

[See Fig 5.34]
5.10 Effect of Tool Life Scatter on the Quality Loss

Function

Reliability of a die is a manifestation of a set of performance characteristics embedded in its design, material selection, manufacturing and heat treatment. Reliability R(t) of a die deteriorate with respect to time which could be viewed as a degradation of its embedded quality. The quality is related to the loss to the society caused by a product during its life cycle. The loss to the society caused is composed of the costs incurred in the production process as well as the cost encountered during use by the customer in the case of tool and dies the loss is which occurred in the production process, and the corresponding cost transmitted to the customer of the product in terms of the high price. A truly high quality tool or die will have a minimal loss of manufacturer or consumer. This fact is quantitatively expressed by Taguchi’s loss function recognizes the customer’s (manufacturer in this case) desire to have dies that are more consistent in their performance and help to produce a low cost product. The Taguchi’s strategy is to enhance quality by determining conditions which provides a robust design to encourage uniform tools, and reduces cost of product at the point of consumption. When the life of a tool is a measure of its quality, then we would like to have a higher the better situation.

The loss function for higher-a-better characteristics is shown in Fig. 3.4 and the expected loss \(E[L(T)]\) is given by \(k/t^2\) and in terms of average \(\bar{T}\) and standard deviation \(\sigma\) of life is given by:

\[
E[L(t)] = k\left(1/\bar{T}\right)\left[1 + 3\sigma^2/\bar{T}^2\right]
\]
The ratio $\sigma / \bar{T}$ is related to the Weibull shape parameter $\beta$ as illustrated in Fig. 5.29. As $\sigma / \bar{T}$ decreases, $\beta$ increases and an approximate relationship between these quantities is $\sigma / \bar{T} = 1/\beta$. Using this approximation above equation can be written as

$$E[L(t)] = k\left(1 / \bar{T}\right)^{1 + 3(1 / \beta)^2}$$

Thus expected loss $E[L(t)]$ is minimum when both $\bar{T}$ and $\beta$ are highest ($\sigma / \mu$ is smallest). The reliable life of a die to meet a target reliability $R(t_p) = 1-F$ is given by

$$t_p = \left(\frac{\bar{T}}{\Gamma(1 + 1 / \beta)}\right)\ln\left(- R(t_p)\right)^{1/\beta}$$

Thus for a specified reliability level $R(t_p) = R^*(target\ reliability)$, tool A is better than tool B if

$$\frac{T_A}{T_B} > \left\{\frac{\Gamma(1 + 1 / \beta_A)}{\Gamma(1 + 1 / \beta_B)}\right\}\left\{\ln(- R^*)\right\}^{1/\beta_A - 1/\beta_B}$$

Two special cases are as follows: (i) If scatter in both tools is the same ($\beta_A = \beta_B$), then the tool A is better than tool B if $T_A / T_B > 1$ and (ii) If the average life of both tools is the same ($T_A = T_B$), but the scatter is not the same ($\beta_A \neq \beta_B$), than the tool A is better than tool B if $1 > \left\{\frac{\Gamma(1 + 1 / \beta_A)}{\Gamma(1 + 1 / \beta_B)}\right\}\left\{\ln(- R^*)\right\}^{1/\beta_A - 1/\beta_B}$

These equation provide a rationale of comparative evaluation of tools supplied by two or more suppliers. The role of scatter parameter $\beta$ is significant in determining the tool quality, and every effort is needed to maximize it, in addition to enhancing the average tool life $T$.

The above equation can also be used to evaluate the effectiveness of two heat treatment processes A and B.
Some methods found in literature to improve the tool life are given below:

**5.11 Methods of Die Life Improvement**

The important parameters which influence the die life are:

- Die material
- Heat treatment of die
- Design and manufacturing of die
- Working conditions e.g. alloy being extruded, extrusion temperature and speed

Die and mandrel material should be such that it has high strength, at high temperature and the geometry should be such which provide high stiffness. Some other desirable aspects of extrusion dies to minimize the failure are [

- *Mechanical design* must be compatible with the steel grade selected, the procedures required to manufacture the tool or die, and the use of tool or die.

- *Grade Selection* must be compatible with the design chosen, the manufacturing process used to produce the tool or die, and the intended service conditions and desired life.

- *Steel Quality* must be microstructurally sound, free of harmful inclusions to the degree required for the applications, and free of harmful surface defects.

- *Machining Process* must not alter the surface microstructure or surface finish and must not produce excessive residual stresses that will promote heat treatment problems or service failures.

- *Heat Treatment Operations* must produce the desired microstructure, hardness, toughness and hardenability at the surface and interior.
• *Grinding and Finishing Operations* must not impair surface integrity of the component.

• *Tool and Die setup* alignment must be precise to avoid irregular, excessive stresses that will accelerate wear or cause cracking

• *Tool and Die operation* overloading must be avoided to ensure achievement of desired tool life.
Chapter 6

Concluding Remarks and Recommendations

1. The types of defects and problems observed at various stages of aluminum extrusion process (i.e. billet preparation, extrusion press, heat treatment, anodizing and painting) were categorized and analyzed. The following observations were made:

   - It was found that at *billet preparation (stage 1)*, there is a constant need of adjusting the composition in the furnace to get billets of consistent composition. This frequent adjustment of alloying elements increases the operating time of furnace and cost of the melting process. To minimize this problem, a better control should be maintained on acceptance of ingots from the supplier. In addition to that the melting of process scrap should be done separately.

   - At *extrusion press (stage 2)* Pareto diagrams showed that about 75% of the defects are due to black and white lines, scratches and damages and other defects (defects for which exact reasons have not been identified). The causes for these defects include improper composition, faulty die and cutter, and mishandling. For the dies, since the data was available, a detailed study of die failure modes and die reliability analysis was done. It was also desirable to perform a similar analysis.
for the cutters, but due to lack of data such analysis was not performed. However it is recommended that a proper record of cutter failure times should be maintained. Which can be used to predict the reliable life of cutter and to determine when to replace them. Mishandling losses can be decreased by properly training the operators.

- After *age hardening (stage 3)* it was found that the coefficient of variation reduces after completion of aging process. It means that aging process increases the average hardness as well as provides consistency in output. Process capability ratio after aging is 1.28, which is quite good. The aging process characteristics (hardness) have normal distribution. For a process following normal distribution, if the process capability is ‘1’ then there is probability of producing 0.27% defective components. When process capability ratio is greater than ‘1’, then the number of defective components is are even smaller. But for *hardness before aging* coefficient of variation is greater as well as process capability is low, this need to investigate the impact of operating parameters and billet composition on the hardness of the product to suggest appropriate improvements particularly for the cases where some customers need extrusions with out aging.

- At *anodizing shop (stage 4a)* three major causes of defects were the process scrap, damages and scratches, and corrosion. Process scrap, which is the extrusions produced greater than ordered, need a proper feed back from anodizing shop to planning department. Damages and scratches are due to mishandling of extrusions while shifting them from one tank to another and during jigging. At present shifting of extrusions from one tank to other is done by operators using
hydraulic overhead cranes. Damages and scratches can be reduced by using some computer controlled overhead crane. This will also help in forming uniform anodizing film by keeping the extrusions in a tank exactly up to the time required for producing a certain thickness. Properly covering the extrusions during storage and avoiding unnecessary delays between aging and anodizing can decrease corrosion defects. Tolerance chart for anodizing film thickness showed that most of the data points lie between centerline and upper specification limit that mean the process is highly non-central. The process capability is less than ‘1’. By making suitable adjustment in voltage and current process mean can be shifted close to centerline. This will increase the process capability ratio and decrease the probability of producing defectives.

- **Painting shop** produces a major part of plant defectives. As it is a final stage therefore rejection at this stage causes maximum loss. The root cause of many problems in this shop is faulty spray gun. If it is choked then it sprays less paint causing less thickness, while if it is worn out it sprays more causing orange peel. Therefore it is necessary to clean and replace the spray gun after optimal intervals. Process capability ratio is very low. Tolerance chart showed that in a sample of 100 components 17 were out of specification limits, again the process is non-central. Also most of the points were between the centerline and the lower specification limit. By suitable adjustment in spray gun pressure, the process mean can be shifted close to the centerline. This will increase process capability ratio and decrease the probability of number of defectives.
2. A cost model was proposed in a generalized framework, which incorporates the probability of producing defectives at each stage. By knowing the exact cost of each stage we can calculate the actual cost of overall process and can determine the plant efficiency.

3. The quality of metal deforming tool deteriorates with respect to time and has significant effects on product quality. Different modes of die failure were studied using data of 2400 dies. It was found that 40% of dies failed due to fatigue, 36% by deflection or plastic deformation of die or mandrel and rest 24% failed by wear. Dies of same features operating under same conditions showed different times to failures. This variation can only be characterized by statistical models. Different statistical methods were used to fit the die life data. Two-parameter Weibull distribution was found to be the best distribution to fit this data. Two approaches, Regression Analysis, and Artificial Neural Network, were used to estimate the parameters \( \eta \) and \( \beta \) of the Weibull distribution from various die features. Neural network showed very good results. Its predictions were better both for the average die life and the coefficient of variation.

4. This knowledge of reliability model can be used for:
   - Predicting number of dies needed for a given production run
   - Comparing reliability of two dies of different material
   - Comparing reliability of two dies subjected to different treatments
   - Developing Taguchi loss function of a given die to assess the expected loss due to its scatter.
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