Time-Weighted State and Output Feedback Control of Power Systems with Mode Assignment

by

Abdul Quddus

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

ELECTRICAL ENGINEERING

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This thesis, written by **Abdul Quddus** under the direction of his thesis adviser, and approved by his Thesis Committee has been presented to and accepted by the Dean of College of Graduate Studies in partial fulfilment of the requirements of the degree of

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بسم الله الرحمن الرحيم

Dedicated to

loving parents

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ABSTRACT

NAME : ABDUL QUDDUS, ABDUL QADIR RANDHAWA

TITLE : TIME-WEIGHTED STATE AND OUTPUT FEEDBACK CONTROL OF POWER SYSTEMS WITH MODE-ASSIGNMENT

MAJOR: ELECTRICAL ENGINEERING

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The application of state and output feedback Time-Weighted Linear Quadratic Regulator (TWLQR) control for the dynamic performance improvement of power systems is investigated in this thesis.

For state and output feedback control, Linear Quadratic Regulator (LQR) and TWLQR strategies are tested on the linearised models of a single machine infinite bus power system (SMIB) with coordinated exciter-governor control as well as multi-machine power system (MMPS) with excitation control. The mode-assignment using TWLQR control design has been applied to SMIB model The simulations have been performed while the system is subjected to small pulse disturbances either in system torque or at system reference voltage.

The simulation results show that by using TWLQR control strategies, superior electromechanical mode dynamic responses of a power system can be obtained.

MASTER OF SCIENCE DEGREE

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

Dhahran, Saudia Arabia

خلاصة الرسالة

اسم الطالب : عبدالقدوس عبدالقادر رندهاوا

عنوان البحث : التحكم في أنظمة القوة باستخدام طريقة المردودات الحالية والمدخلية الموزونه زمنياً مع تحديد الحالة .

التخصص : هندسة كهربائية

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في هذه الرسالة قمنا بتطبيق التحكم عن طريق الحالات والمخارج في أنظمة الطاقة وذلك باستخدام المنظم الخطي التربيعي الموزون بالوقت .كما أنه تم إختبار المنظم الخطي التربيعي والمنظم الخطي التربيعي الموزون بالوقت على أنظمة خطيه لمولد أحادي موصل إلى نظام غير محدود وكذلك لأنظمة المولادات العدة .ايضاً ، لقد استخدمت نظرية قيم آيقن مع المنظم الخطي التربيعي للمولد الأحادي الموصل إلى نظام غير محدود .أما للحصول على نتائج المحاكاة باستخدام الكمبيوتر فقد استخدم اضطراب في عزم النظام أو في الفولتيه المرجعيه للنظام .واخيراً ، لقد أضهرت نتائج المحاكاة بأن استخدام المنظم الخطي التربيعي الموزون بالوقت يؤدي إلى نتائج افضل في التحكم بأنظمة الطاقة .

درجة الماجستير في العلوم
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Chapter 1

Introduction and Literature Review

The extensive growth of electric power systems and the development of high-voltage long-distance transmission systems, separating generation from load, have accentuated the importance of increasing the dynamic and transient stability limits of synchronous machine and is one of the motivation to search out and apply more efficient control strategies for power system stability enhancement.

The application of optimal control is very well established for power system stabilization. The review of optimal control theory applications, starting from early seventies to-date is given here in detail ranging from earlier applications to the latest optimal output feedback control applications. The evolution of time-weighted linear quadratic regulator (TWLQR) is given next, which includes its first appearance in 1969, till its latest applications.

1.1 Linear State and Output feedback control

A control system is a dynamic system which as time evolves behaves in a certain prescribed way ; and control theory deals with the analysis and synthesis of control systems. Since in a control system, the actual operation is continuously compared to the designed operation, feedback control systems are able to operate satisfactorily despite adverse conditions. Under the complete controllability conditions, a time-invariant linear system can always be stabilized by a linear state feedback law. The closed-loop poles can be located anywhere in the complex plane, the system can be stabilized; but moreover, by choosing the closed-loop poles far to the left in the complex plane, the convergence to the zero state can be made arbitrarily fast .To make the system faster , however, large input amplitudes are required. In any practical problem the input amplitudes must be bounded; this imposes a limit on the distance over which the closed-loop poles can be moved to the left. These considerations lead quite naturally to the formulation of the optimization problem, which gives rise to the popular Linear Quadratic Regulator (LQR).

An LQR is based on the concept of minimization of the state variable variations and the control effort at the same time. To find the control, a performance index of the quadratic form, which shall be a function of both the state deviations and the control effort, is chosen. Finally the state equations are appended to the performance index by a co-state variable vector to find the LQR. The solution of the co-state variable vector is derived by solving the *Matrix Riccatti Equation*, and in turn the required feedback gain matrix is found. In deriving a state feedback LQR it is assumed that the complete state vector of the plant can be accurately measured at all

times and is available for feedback, which is an unrealistic assumption for many practical control systems. In many practical situations only observed variables are available for measurements and feedback, so the control system where the observed variables serve as input to the controller is called output LQR feedback control.

1.2 Modern Control of Power Systems

The application of modern control theory for the dynamic performance improvement is very well established in literature. Optimal feedback control based on state and output feedback control gains has found a large number of applications in single machine as well as multi-machine power systems. Research is still going on to find more efficient optimized controls. Such needs stem from the fact that the modern power systems are becoming highly inter-connected and complex. Disturbances occurring in such a system can propagate through it resulting in unacceptable system operation. Systems which have long transmission distances between the load centers and generating stations may exhibit poorly damped or even negatively damped oscillations on certain disturbance conditions.

1.2.1 Early Application of Optimal Control

In mid 60's, an enormous amount of work was done in the area of system optimization by control engineers; many theoretical results were published but application examples were mainly on low-order systems. The application of optimal control theory to the power system was first started in early 70's. The problem of

optimization of synchronous machine performance was considered, by minimizing a scalar performance index in both system state variables and system inputs.

The first LQR design was made for a hydroelectric power plant [1]. The plant was modeled as an eighth-order system, a third-order for the synchronous machine, a second-order for the excitation, and a third-order for the governor and hydraulic power. The main motivation behind LQR design was to obtain an optimal controller comparable or even better than conventional excitation controller, existing at that time, to improve the stability and dynamic response of the system. Such a comparison was made in [2] between LQR and conventional PSS and the signals derived from lower order linear model were tested on a high order non-linear single-machine infinite-bus as well as multi-machine systems.

Whereas the objectives of the conventional Power System Stabilizer (PSS) design is to improve the system mechanical mode damping alone, that of the LQR is to minimize the system state variation in conjunction with the control effort. The other difference is in the realization of control schemes. Whereas the conventional PSS has been designed with a signal input using phase compensation and for a narrow band of oscillating frequencies, the LQR synthesizes the control input from many state variable signals that themselves have different phases, has no need of compensation blocks, and is good over a wide band of frequencies.

By a proper selection of the weighting matrix for the state variables in the performance index, or even modifying the feedback matrix in a prescribed way, the dominant eigenvalues can be shifted to a pre-specified location , to improve the

system stability. The main advantage gained by eigenvalue assignment is to apply the LQR over a wide range of operating conditions [3], while keeping the power system effectively stable.

One of the limitations of earlier design of LQR, the use all the state variables, was overcome by using only those states in system modeling which could be available easily for measurement as output [4]. The LQR design was proved superior where a situation might arise in large interconnected systems, where more comprehensive controllers in the form of LQR were required, not just for one machine but designed and coordinated for multi-machine [5], not just for one set of operating conditions but for wide range of operating conditions [3 , 5].

In [7], A. K. De Sarkar et. Al, presented the stabilization of a synchronous machine through output feedback control. The sub-optimal controls corresponding to the feedback of the machine slip velocity and its derivatives produced substantial improvement of the machine response.

1.2.2 Recent Control Applications

The paper by A. B. R. Kumar and E. F. Richards [8], discussed the mode assignment for the improvement of dynamic stability of power system. A fourth order system with coordinated exciter-governor control was considered and the mode assignment was

done with respect to a performance index minimizing the sum of the squares of each control gain element.

In [9], eigenvalue analysis was performed to characterize the plant modes and the effect of stabilization on these modes, and the minimum number of stabilizers. Reference [10] describes an iterative algorithm based on decentralized pole assignment to determine the parameters of the power system stabilizers.

Chow and J. J. Sanchez-Gasca [11] described four pole-placement techniques for the design of power system stabilizers, with the emphasis on frequency response characteristics of the controllers. While D. Ostojic, and B. Kovacevic [12] presented the eigenvalue control strategy which utilizes an adaptive power system stabilizer for decentralized control of damping and oscillation frequency.

Multivariable frequency domain techniques for the systematic design of large-scale power systems are discussed in [13]. In [14], the authors gave an outline of the pole shifting control algorithm which was physically implemented on a 400 MW thermal generating unit.

1.2.3 Output Feedback Applications

The implementation of optimal linear regulator control requires the design of state estimators, which results in increased cost and reduced reliability of the control system. These are the reasons that such controls schemes which use only some

desirable state variables such as torque angle and speed, are preferred for practical implementations. So in recent years many such control strategies have been developed [15 - 20] for power systems stabilization, which utilized output feedback as a main ingredient of the control law.

In 1982 J. H. Anderson et. al. [15] formulated a control law which used absolute output variables as actually measured in practice, rather than their deviations, and the effectiveness of this controller as a wide range stabilizer for a micro-alternator was tested experimentally.

An algorithm for the design of decentralized output feedback stabilizers for large-scale electric power systems was proposed in [16]. It was based on decentralized pole assignment for the determination of the stabilizers parameters, which used local generator output as their feedback signals.

The idea of output feedback with prespecified eigenvalues was applied to power system stabilizer design by Huang et. al [17]. The proportional-integral stabilizers were derived by reduced order model. It was shown that the result of eigenstructure assignment by output feedback is more stable than the assignment based on the whole system.

The design of two-level power system stabilizers using an optimal reduced order model was addressed in [18]. In this scheme local controllers determined

depend only on local output information pertaining to the subsystem, while the global controller use the output state variables of the overall system via optimal reduced order model. Actually it is an extension of optimal reduced order method proposed in [19 , 20] and the two-level optimal stabilization method proposed in [21].

A 1995 paper [22], presents a modified optimal controller for an interconnected power system. The design method is independent of the specification of weighting matrices and places the electromechanical and exciter modes in a prespecified vertical strip. Here the output feedback gains are obtained from a transformation of the state feedback matrix.

A recent paper by Y. L. Abdel-Magid , Maamar Bettayeb & Shokri Z. Selim [23] demonstrates the use of the *Genetic Algorithm* to design output feedback power system stabilizers. Various performance indices are developed permitting the selection of the output feedback gains to optimally place the closed-loop eigenvalues either in the left-hand side of a vertical line in the complex s-plane, or within an open sector in s-plane or within a vertical strip in the complex s-plane.

1.3 Time-Weighted Linear Quadratic Control

In Time-Weighted Linear Quadratic Regulator (TWLQR), a performance index having a time factor with quadratic state variables part is minimized, the direct consequence of which is a heavy penalty imposition on the sustained transients,

resulting in a high accuracy in the steady state response. So a TWLQR control can stabilize the system even before the LQR to steady state conditions and suppress the oscillations faster as time elapses. The mode-assignment using TWLQR control was first introduced in [34], and was mainly restricted to the assignment of real valued modes for low order systems. The utilization of such desirable properties of TWLQR control in the area of power systems stabilization is , to the author's knowledge, the first attempt made, as to date no such applications have been reported in the literature.

1.4 Evolution in Time-Weighted Control

The first paper in the literature which suggested the use of TWLQR control was by F. T. Man and H. W. Smith [24]. Their method was, however, found to be flawed as explained by B. Ramaswami and K. Ramar [25]. P. Fortins and G. Parkins [26], explained that the TWLQR control law must contain time-varying gains as opposed to the time-invariant gains as suggested by [24], which was also overlooked by [25].

In 1974, N. Ramani and D.P. Atherton [27], put forth a *statistical index*, so that the performance index was no longer dependent on the initial states, because it was taken as a random vector with unit variance. By using *Kleinman's lemma* [28] as exploited in [29], they also derived the necessary conditions to be satisfied by the constant state feedback gain matrix of TWLQR control law. Subbayyan et. al. [30] presented a simpler form of the derivation in [27] by means of the *Hamiltonian* approach. They pointed out by the use of a third order example that the transient

response based upon TWLQR criterion is not always less oscillatory for all the state variables, as was the case for second order example in [27]. This contradiction was due to the fact that the closed-loop eigen-values of the system based on LQR and TWLQR are in general completely different. The use of TWLQR was made by Saturu Fukata et al [31, 32, 33], for the determination of feed-back gains of gains linear continuous and discrete time system.

The mode-assignment using TWLQR was first introduced by T.M Abdul Moneim and N.N. Sorial [34]. They worked out an example to reveal the contradiction of the conclusions made in [24, 27, 30] and ended up with the conclusion that in general the transient responses decay faster and exhibit small overshoots than those based on the usual quadratic performance index with the same closed loop eigen-values. In 1986, B.H. Kwon [35], reconsidered the problem of TWLQR mode-assignment. He derived the necessary conditions which were simple in form and his algorithm gave a considerable amount of computational time savings compared to the previous algorithm [34]. The mode-assignment for TWLQR observer design was also given by B.H. Kwon in [36]. In 1988, B.H. Kwon [37] presented the discrete time version of [35].

In 1989, P. Agathoklis et al [38], discussed the lower bounds for the stability margin of linear time invariant continuous systems. These bounds were obtained as a function of the TWLQR criterion computed from the measurement of the system impulse response.

The optimal regulators with steady state trajectory insensitivity were designed in [39]. This design was based on a Quadratic performance index which includes a time-multiplied quadratic function of the trajectory sensitivity vector. In [40], the design of optimal discrete-time linear time invariant system with TWLQR criterion was studied by Tsu- Tian Lee et al.

In 1991, B.H. Kwon and E. H. Song [41], designed a current source using 12-pulse phase controlled rectifier incorporating TWLQR control. As another application the authors of [41], applied TWLQR control to obtain an enhanced dynamic response for direct digital control of a phase control rectifier [42]. In 1992, B. H. Kwon again applied the TWLQR control for the design of a highly stable electromagnetic power supply [43]. An optimal proportional and measurable variable feedback controller was designed in [44], using TWLQR, to have an improved dynamic and static response than that of LQR based design.

With the aid of *Lagrangian* approach, Cheok [45] investigated the sensitivity analysis of cost indices for time-discrete optimal feedback systems with a class of Nth order time-weighted performance indices.

M.A Zohdi and J.D Aplevich [46] derived the necessary conditions for optimality for TWLQR output feed back control of linear time invariant continuous systems. The discrete time version of [46] was then considered in [47, 48]. Of

lately in 1994, an efficient numerical algorithm with a claim of global convergence was presented in [49] for the same problem as discussed in [46]. In [50] state and output feedback control using LQR and TWLQR was considered by P. Wang et al, the control design was derived using sensitivity derivatives of the performance index with respect to the feedback gain.

The reason why the time-weighted performance index is so promising is that time-varying factor imposes a heavy penalty on the sustained transients, which results in high accuracy in the steady-state system response while allowing relaxed control effort in the initial stage.

1.5 Scope of the Thesis

It is obvious from the review of TWLQR control that :

- TWLQR has been implemented only to simple, real eigenvalued low-order state-space models.
- There is no power system example, in author's knowledge, used to implement TWLQR control.
- TWLQR control is confined mainly to state feedback cases.

The above factors motivate the incorporation of TWLQR state and output feedback control of power system for dynamic stability analysis. The linearised

models of both single machine infinite bus system and multi-machine power system are considered for this purpose.

The following TWLQR control strategies are evaluated and tested on power system models.

- TWLQR state feedback control
- TWLQR output feedback control
- TWLQR state feedback control with mode assignment

The simulations are performed for a 10 percent pulse disturbances of three cycles either in system reference voltage or in system torque for the aforementioned power system models and the results are compared with their counterparts in LQR control.

1.6 Outline

In the *second* chapter theoretical background for state and output feedback control using LQR and TWLQR is given for its application to power system, furthermore mode assignment using TWLQR is also discussed for state feedback case. Algorithmic implementation of above methods is illustrated with the help of flow charts in the same chapter. In chapters *three* the control strategies described in chapter 2 have been applied to compute both state and output feedback control gains for

SMIB linearised model. In chapter *four* the design applications have been made for MMPS linearised model. The closed loop system response simulation for the said models have been carried out in respective chapters for different dynamic stability cases. In chapter *five* the TWLQR mode assignment design has been applied to SMIB system. The conclusions of the work and recommendations for future work are summarized in chapter *six*.

Chapter 2

Control Designs and Implementations

The mathematical formulation for LQR and TWLQR output feedback control strategies are given in detail. The implementation of each control design is explained by presenting simple step-by-step algorithms and programming flow charts. The mode-assignment using LQR and TWLQR criterion is also included along-with comprehensive implementation algorithms and corresponding flow charts.

2.1 LQR Output Feedback

The LQR output feedback problem is one of the most important open question in control engineering. A general output feedback problem can be described as; given a linear time-invariant system, find a static output feedback so that the closed loop system has some desirable characteristics, or determine that such a feedback does not exist.

The stabilization by the state feedback is very well established in the literature, and the LQR with state feedback has well known analytical solutions. Optimal state feedback for LQR will be obtained as a special case of LQR output feedback.

2.1.1 Problem Development

Consider the time-invariant system described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &\equiv Cx(t) \end{aligned} \tag{2.1 a}$$

under the influence of static output feedback of the form :

$$u(t) = Ky(t) = KCx(t) \tag{2.1 b}$$

The closed-loop system is

$$\begin{aligned} \dot{x}(t) &= (A + BKC)x(t) \\ \dot{x}(t) &\equiv A_C x(t) \end{aligned} \tag{2.2}$$

here $A_C = A + BKC$, the state $x(t) \in \Re^n$, the control input $u(t) \in \Re^m$, and the output $y(t) \in \Re^p$.It is desired here to select K to minimize, subject to the constraint (2.2), the performance index

$$J = \int_0^\infty (x^T Q x + u^T R u)\, dt \tag{2.3}$$

where Q is positive semi-definite and R is positive definite

2.1.2 Necessary Conditions for Optimality

In [51 - 54], necessary conditions for optimality were given as:

$$\partial J / \partial K = RKCPC^T + B^T SPC^T = 0 \tag{2.4}$$

where S and P are given by the following *Lyapunov* equations

$$A_C^T S + SA_C + Q + C^T K^T RKC = 0 \tag{2.5}$$

$$A_C P + PA_C^T + X_o = 0 \tag{2.6}$$

with $X_o = x(0)\,x(0)^T$ and $A_C = A+BKC$.

To eliminate the dependence of (2.4 - 2.6) on the specific initial conditions, the expected values of the performance index (2.3) may be taken, so that $X = E\{x(0)x(0)^T\}$ in (2.6). It is generally assumed that $x(0)$ is uniformly distributed on the unit sphere so that $X = I$ [52].

Assumptions for Optimal K

I : There exist a K for which the closed loop matrix Ac, is asymptotically stable.

II : The weighting matrix R and the initial state covariance matrix X_o are strictly positive definite.

II : The system outputs are linearly independent, i.e., *rank* $C = p$.

The assumptions (*I* - *III*) imply that $J(K)<\infty$ for control $u = K\,y\,(t)$, if and only if the matrix A_C is asymptotically stable.

2.1.3 Results for State Feedback

The state feedback can be considered as a special case of output feedback problem with the output vector equal to the states of the system i.e., $C = I_n$, the identity matrix of order equal to the number of states in the system. So under the controllability

conditions, (2.4 - 2.6) become necessary and sufficient conditions leading to the optimal state feedback solution

$$u(t) = -R^{-1}B^{T}Sx(t) \tag{2.7}$$

Where S is the unique positive definite solution of the *Algebraic Ricatti Equation* (2.5) with $C = I_n$, as given below

$$A_C^T S + SA_C + Q - SBR^{-1}B^T S = 0 \tag{2.8}$$

2.1.4 Design approaches and limitations

All the algorithms discussed here are iterative in nature, and two of the efficient ones are presented in [54, 55]. The algorithm in [54] requires repetitive solution of (2.5) and (2.6) for fixed values of K so that they are considered as two *Lyapunov* equations, and the form

$$K = -R^{-1}B^{T}SPC^{T}(CPC^{T})^{-1} \tag{2.9}$$

as a candidate for the next choice for K. Note however that it guarantees only a local minimum. Unfortunately, iterative algorithms such as these require the selection of an initial stabilizing gain. A direct procedure for finding such a K is still an open problem.

Conditions for the existence and global uniqueness of solutions to (2.4 - 2.6) such that P and S are positive definite and (2.2) is stable are not known.

2.1.5 LQR Output Feedback Algorithm

The detailed iterative algorithm for the computation of the optimal gain matrix K for LQR output feedback case is illustrated in *Fig 2.1* with the help of a flow chart and is given below in words:

Step 1: Set iteration no. $i=0$, and choose an initial guess for K_0, stabilizing the given system.

Step 2:

i) Compute the closed loop system matrix A_C.

ii) Compute the positive definite solution S from (2.5).

iii) Compute P from (2.6).

Step 3: Compute the gradient matrix $\partial J / \partial K_{(i)}$, from (2.4).

Step 4: Evaluate the stopping criteria $\|\partial J / \partial K_{(i)}\| \leq \varepsilon$,if it is satisfied then go to *step 7*, otherwise go to *step 5*, where ε is a pre-specified small positive real number.

Step 5: Increment the iteration counter i and update the gain matrix as follows

$$K_{(i+1)} = K_{(i)} - \alpha(\partial J / \partial K_{(i)}) \qquad (2.10)$$

where step length α can be chosen optimally by using some gradient based technique [56].

Step 6: Repeat *step 2* to *step 5*.

Step 7: Stop and accept $K_{(i+1)}$ as an optimal feedback gain matrix.

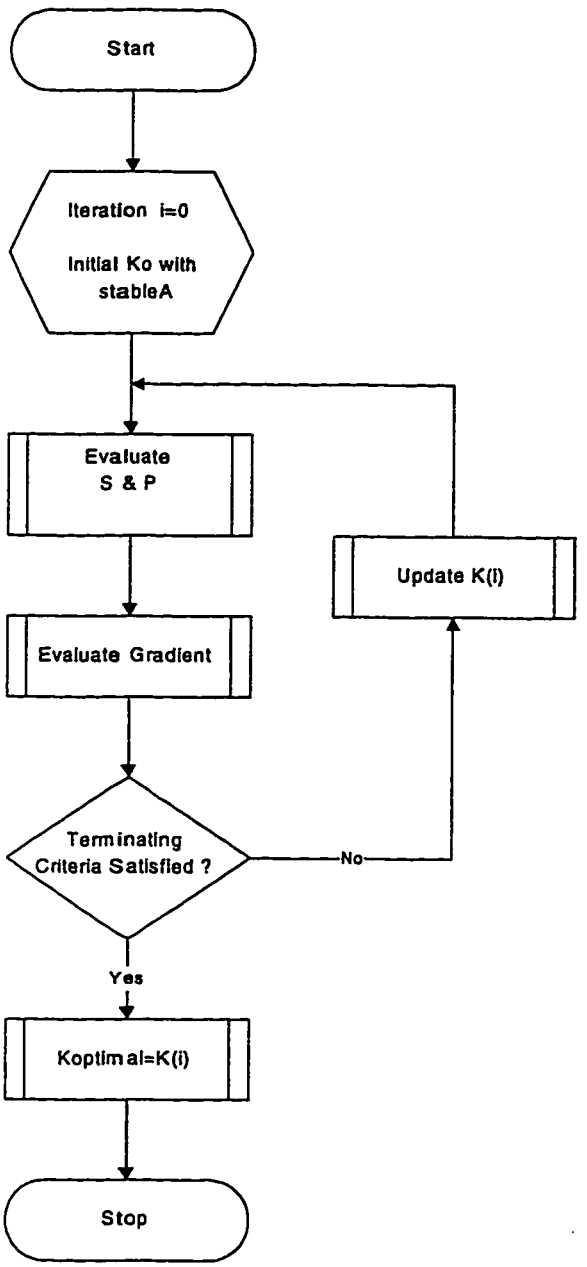


Figure 2. 1 : Flow Chart for LQR Output Feedback Algorithm

2.2 TWLQR Output Feedback

A general TWLQR output feedback problem can be described as; given a linear , time-invariant system, find a static output feedback so that the closed loop system has some desirable characteristics, or determine that such a feedback does not exist.

The state feedback can be considered as a special case of output feedback problem with the output vector equal to the states of the system i.e., $C = I_n$, the identity matrix of order equal to the number of states in the system.

2.2.1 Problem Development

Consider the time-invariant system described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &\equiv Cx(t) \end{aligned} \tag{2.11 a}$$

under the influence of static output feedback of the form :

$$u(t) = Ky(t) = KCx(t) \tag{2.11 b}$$

The closed-loop system is

$$\begin{aligned} \dot{x}(t) &= (A + BKC)x(t) \\ \dot{x}(t) &\equiv A_C x(t) \end{aligned} \tag{2.12}$$

where $A_C = A + BKC$, the state $x(t) \in \Re^n$, the control input $u(t) \in \Re^m$, and the output $y(t) \in \Re^p$.Now it is desired to select K such that the following statistical time-weighted quadratic performance index [30] is minimized:

$$J = E\left\{\int_0^\infty (tx^T Qx + u^T Ru)dt\right\} \tag{2.13}$$

Using (2.12), the performance index is rewritten as [46]

$$J = E\left\{\int_0^\infty (tx^T Qx + x^T C^T K^T RKCx)dt\right\} \tag{2.14}$$

where Q is positive semi-definite and R is positive definite, constant matrices.

2.2.2 Evaluation of Performance Index

From (2.12), it follows that

$$x = e^{A_c t}\, x(0) \tag{2.15}$$

substituting (2.11) and (2.12) into (2.13), it follows that

$$J = E\left\{\int_0^\infty x^T(0)e^{A_c^T t}(tQ + C^T K^T RKC)e^{A_c t}x(0)dt\right\} \tag{2.16}$$

Under the assumption that the initial state $x(0)$ is a random vector, uniformly distributed over the unit sphere, with $X_o = E\{x(0)x(0)^T\}$, a modified performance index , independent of the initial state, may be introduced [27 - 29] :

$$J_1 = tr\{Q_2 X_o\} \tag{2.17}$$

where, $X_o = E\{x(0)x(0)^T\}$ and

$$Q_2 = \int_0^\infty e^{A_c^T t}(tQ + C^T K^T RKC)e^{A_c t}dt = P_1 + P_2 \tag{2.18}$$

where

$$P_1 = \int_0^\infty te^{A_c^T t} Q e^{A_c t} dt \tag{2.19}$$

and,

$$P_2 = \int_0^\infty e^{A_c^T t}(C^T K^T RKC)e^{A_c t}dt \tag{2.20}$$

Suppose that the control law (2.11) makes the system (2.12) asymptotically stable, then :

$$\lim_{t\to\infty} e^{A_c t} = o, \lim_{t\to\infty} t e^{A_c t} = o \tag{2.21}$$

Let

$$Q_1 = \int_0^\infty e^{A_c^T t} Q\, e^{A_c t} dt \tag{2.22}$$

One obtains [46]

$$A_C^T Q_1 + Q_1 A_C = -Q \tag{2.23}$$

$$A_C^T P_2 + P_2 A_C = -C^T K^T RKC \tag{2.24}$$

$$A_C^T P_1 + P_1 A_C = -Q_1 \tag{2.25}$$

Now from (2.23 - 2.25), Q_2 satisfies :

$$A_C^T Q_2 + Q_2 A_C + Q_1 + C^T K^T RKC = 0 \tag{2.26}$$

$$A_C^T Q_1 + Q_1 A_C + Q = 0 \tag{2.27}$$

So, Q_2 is available with the *Lyapunov* equations (2.26) and (2.27). Therefore it is straightforward to find out the required performance index J_1 from (2.17).

2.2.3 Necessary Conditions for Optimality

The gradient of J_1 in (2.17) with respect to any system parameter is given by [46]

$$\partial J_1 / \partial\beta = tr\{\partial Q_2 / \partial\beta\} = 2tr\{U_2 Q_2 \partial A_C / \partial\beta\} + tr\{Q_2 \tfrac{\partial}{\partial\beta}(Q_1 + C^T K^T RKC)\} \tag{2.28}$$

where

$$A_C U_2 + U_2 A_C^T + X_o = 0 \tag{2.29}$$

Now (2.28) may be proved by differentiation of (2.26), the use of (2.29) and the properties of the *trace* operator. Similarly, it can be shown that

$$tr\{U_2(\partial Q_1 / \partial\beta)\} = tr\{(\partial Q_1 / \partial\beta)U_2\} = 2tr\{U_1 Q_1(\partial A_C / \partial\beta)\} + tr\{U_1(\partial A_C / \partial\beta)\} \tag{2.30}$$

where

$$A_C U_1 + U_1 A_C^T + U_2 = 0 \tag{2.31}$$

The necessary conditions for optimality require that $\partial J / \partial K = 0$.

Therefore viewing β, as an element of K and employing (2.28) & (2.30), the following conditions must be satisfied :

$$\partial J / \partial K = B^T(Q_2 U_2 + Q_1 U_1)C^T + RKCU_2C^T = 0 \tag{2.32}$$

Assumptions for Optimal K [49]

I : There exist a K for which the closed loop matrix Ac, is asymptotically stable.

II : The weighting matrix R and the initial state covariance matrix X_o are strictly positive definite.

II : The system outputs are linearly independent, i.e., *rank* $C = p$.

The assumptions (*I* - *III*) imply that $J(K)<\infty$ for control $u = K\,y\,(t)$, if and only if the matrix A_C is asymptotically stable.

2.2.4 TWLQR Output Feedback Algorithm

The details of the iterative algorithm for the computation of the TWLQR output feedback control gain matrix K are given below whereas the flow chart for the same algorithm is presented in *Fig 2.2.*

Step 1: Set iteration no. $i=0$, and choose an initial guess for K_0, stabilizing the given system.

Step 2:

i) Compute the positive definite solution $Q_{1(i)}$ from (2.27).

ii) Compute the positive definite solution $Q_{2(i)}$ from (2.18) by solving (2.24) and (2.25).

iii) Compute $U_{2(i)}$ and $U_{1(i)}$ from (2.29) and (2.31).

Step 3: Compute the gradient matrix $\partial J / \partial K_{(i)}$, from (2.32).

Step 4: Evaluate the stopping criteria $\|\partial J / \partial K_{(i)}\| \leq \varepsilon$,if it is satisfied then go to *step 7*, otherwise go to *step 5*, where ε is a prespecified small positive real number.

Step 5: Increment the iteration counter i and update the gain matrix as follows

$$K_{(i+1)} = K_{(i)} - \alpha(\partial J / \partial K_{(i)}) \qquad (2.33)$$

where step length α can be chosen optimally by using some gradient based technique [56].

Step 6: Repeat *step 2* to *step 5.*

Step 7: Stop and accept $K_{(i+1)}$ as an optimal feedback gain matrix.

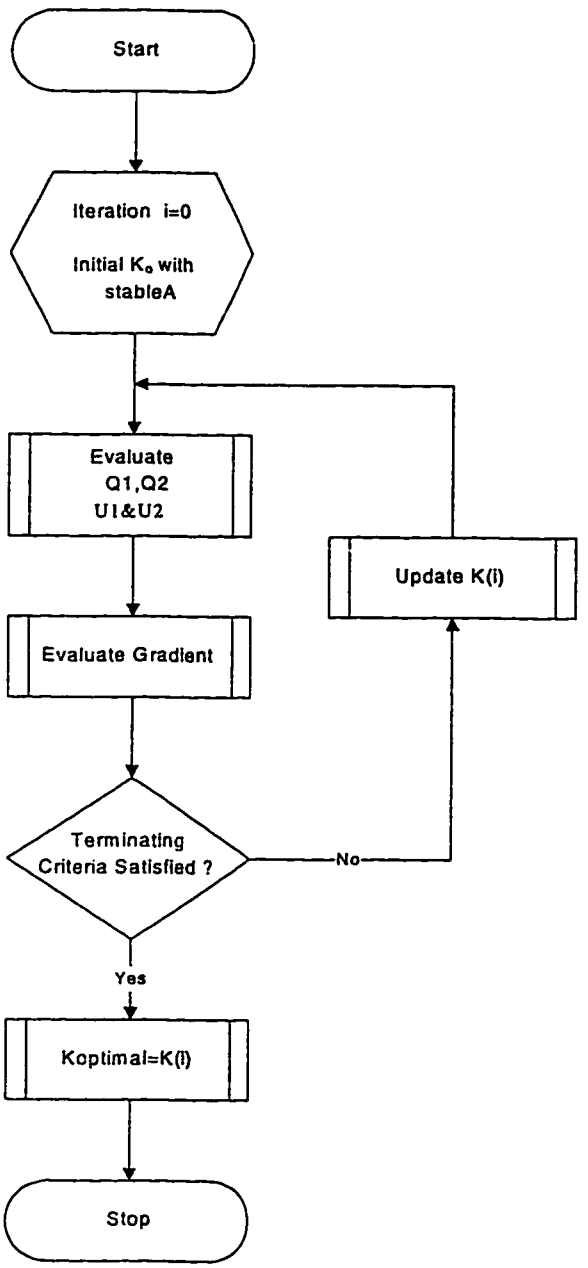


Figure 2. 2 : Flow Chart for TWLQR Output Feedback Algorithm

2.3 Mode Assignment using LQR State Feedback

While the mode-assignment problem for multivariable systems is fairly complicated, Moore [57] showed that the problem of assigning both eigen-values and eigen-vectors has a straightforward solution, and the necessary conditions for optimality are simpler in form. The essentials are summarized as follows :

2.3.1 Problem Development

It is well known that the constant feedback gains giving desired closed-loop eigenvalues in the multi-input systems are not unique. Now using pole placement and optimization technique [34], this freedom can be used to minimize a linear quadratic performance index.

Consider the linear time-invariant system described by:

$$\begin{aligned} \dot{x}(t) &= A\,x(t) + B\,u(t) \\ x(0) &= x_0 \end{aligned} \tag{2.34}$$

where the state $x(t) \in \Re^n$, the control input $u(t) \in \Re^m$, A and B are constant matrices of appropriate dimensions. It is assumed that the above system is completely controllable and the matrix B is of full rank.

The design problem is to find the constant feedback control

$$u(t) = F x(t) \tag{2.35}$$

which assigns prescribed closed-loop eigen-values and at the same time minimizes the expected value of the linear quadratic performance index :

$$J = E\left\{\int_0^{\infty} (x^T Q x + u^T R u)\, dt\right\} \tag{2.36}$$

Here Q and R are symmetric positive semi-definite and positive definite matrices, respectively, and initial state $x(0)$ is assumed to be a random vector, with is given as [29] $X_o = E\{x(0)x(0)^T\} = I$.The closed-loop system is

$$\begin{aligned} &\dot{x}(t) = (A + BF)x(t) \\ &or \\ &\dot{x}(t) \equiv A_C x(t) \end{aligned} \tag{2.37}$$

where $A_C = A + BF$

The closed-loop eigen-values are given as any desired set of distinct eigen-values $\Lambda = \{\lambda_1, \ldots. \lambda_n\}$.

2.3.2 Mode Assignment

All the classes of controllers satisfying pole constraint can be characterized through a set of parameter vectors p_i ($i = 1, \ldots.., n$) as follows [58, 59] :

1) Compute the maximal rank matrix, N_i , satisfying $(\lambda_i I - A , B) N_i = 0$ as :

$$N_i = \begin{bmatrix} N_{1i} \\ N_{2i} \end{bmatrix}, i = 1, \ldots, n \tag{2.38}$$

where $N_i \in \Re^{(n+m)xm}$ and $N_{1i} \in \Re^{nxm}$.

2) Define a parameter vector p_i ($i = 1, \ldots.., n$) as :

$$p_i = (p_{i1}, \ldots.., p_{im})^T \tag{2.39}$$

and form the following matrices

$$V = V_o P, \quad W = W_o P \tag{2.40}$$

where

$V_o = (N_{11} , ..., N_{1n})$, $W_o = (N_{21} , ..., N_{2n})$, and $P = diag (p_i)$, $i = 1, ..., n$.

3) For each value of the parameter vector p_i where $det\ V \neq 0$, compute the feedback gain matrix.

$$F = -W V^{-1} \tag{2.41}$$

A simple algorithm for the computation of the maximal rank matrix (2.38) is given in *Appendix D*. Thus , the degree of freedom remaining after the eigen-values being assigned is expressed as any parameter vector p_i which makes the matrix V nonsingular.

2.3.3 Optimization

A lemma given in [61] is useful in computing the gradient matrix and it is easily derived by using the *Lagrangian* approach. Now, using (2.35) and (2.36), the performance index J can be expressed as [62]

$$J = E\{x(t)^T Sx(t)\} = tr\{SX_o\} \tag{2.42}$$

where S is the solution of the following *Lyapunov* equation :

$$A_c{}^T S + SA_c + Q + F^T RF = 0 \tag{2.43}$$

To determine the optimal parameter vector p_i which minimizes the cost (2.36) subject to the constraint in (2.43) , the *Hamiltonian* for this problem can be expressed as :

$$H = tr(S) + tr\{L^T[A_C^T S + SA_C + Q + F^T RF]\} \tag{2.44}$$

where L is the symmetric *Lagrange* Multiplier matrix, given by the following *Lyapunov* equation.

$$A_C L + L A_C^T + X_o = 0 \qquad (2.45)$$

The necessary conditions for the solution are derived by taking the partial derivatives of (2.44) with respect to S and L and p_i and equating them to zero. The gradient vector, $\partial H/\partial p_i$, can be simply determined through the matrix form of $\partial H/\partial P$ which can be derived using [61].

2.3.4 Necessary Conditions for Optimality

In order that the constant feedback gain matrix F be optimal with respect to the performance index (2.36) under the stable eigen-value constraint, it is necessary that [35] :

$$\partial H / \partial p_i = 0 , \qquad i = 1 , \ldots\ldots, n \qquad (2.46)$$

The vector in (2.46) is a block diagonal part of the following equation :

$$\partial H / \partial P = 2(W_o - F V_o)^{-1} [R F L + B^T S L](V^T)^{-1} \qquad (2.47)$$

where S and L satisfy the *Lyapunov* equations as given in (2.42) and (2.45).

2.3.5 Algorithm for LQR State Feedback Mode Assignment

The step by step details of the LQR mode assignment algorithm using state feedback for the optimal parameter vector computation of are presented here. Moreover the flow chart representation is also provided in *Fig2.3*.

Step 1: For each eigenvalue λ_i ,calculate Ni .

Step 2: Set iteration no. $i=0$, and select any initial parameter vector p_i such that the matrix V is nonsingular.

Step 3:

i) Compute the the feedback gain $F_{(il)}$.and closed loop system matrix A_C from (2.41) and (2.37) respectively.

ii) Compute the positive definite solution $S_{(i)}$ from (2.43).

iii) Compute $L_{(i)}$ from (2.45).

Step 4: Compute the performance Index J from (2.42) and the gradient matrix $\partial J \ / \ \partial P_{(i)}$, from (2.47).

Step 5: Evaluate the stopping criteria, $\|\partial J \ / \partial P_{(i)}\| \leq \ \varepsilon$, if it is satisfied then go to *step 8*, otherwise go to *step 6*, where ε is a prespecified small positive real number.

Step 6: Increment the iteration counter i and find a new value of parameter vector p_i using

$$P_{(i+1)} = P_{(i)} - \alpha(\partial J \ / \ \partial P_{(i)}) \qquad (2.48)$$

where step length α can be chosen optimally by using some gradient based technique [56].

Step 7: Repeat *step 2* to *step 5*.

Step 8: Stop and accept $P_{(i+1)}$ as an optimal parameter vector.

Step 9: After the optimal parameter vector P is determined using the above algorithm, the feedback gain matrix F is given from (2.40) and (2.41).

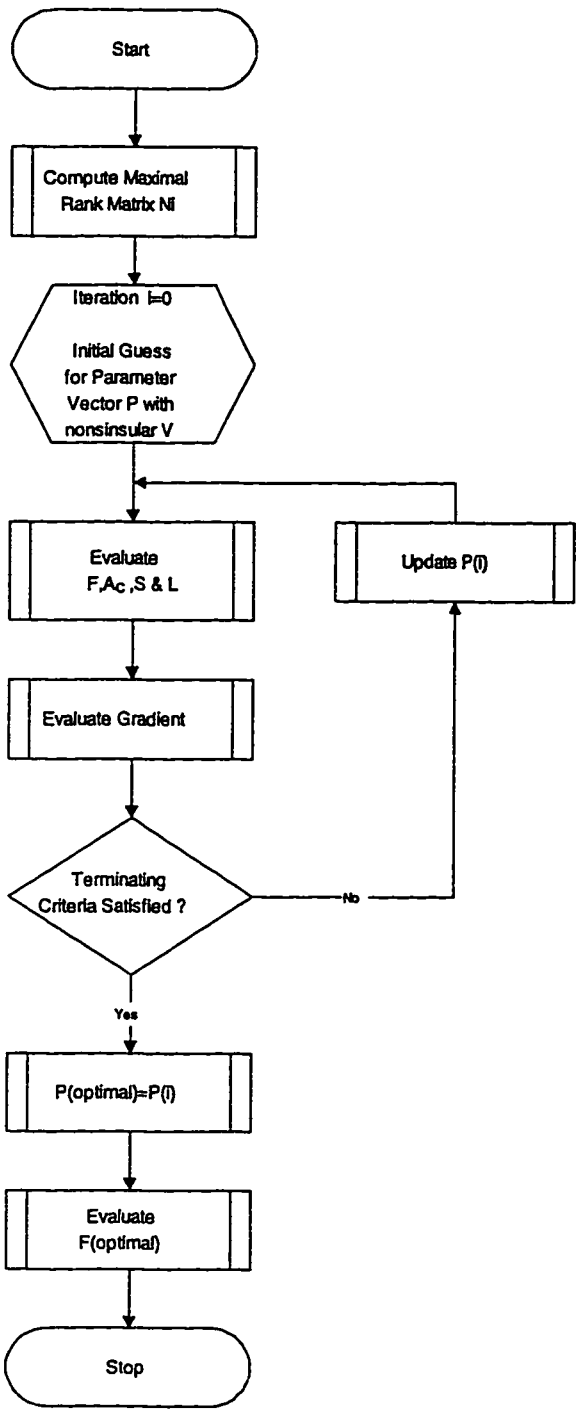


Figure 2. 3 : Flow Chart for LQR State Feedback Mode-assignment Algorithm

2.4 Mode Assignment using TWLQR State Feedback

The TWLQR State Feedback Control with mode-assignment was first proposed by T. M. Abdel-Moneim et. al [34], and later improved by Bong-Hwan Kwon et al [35], who gave the necessary conditions simpler in form and the algorithm presented, saved considerable amount of computational time. The essentials of the control design are summarized as follows :

2.4.1 Problem Development

As described earlier the constant feedback gains giving desired closed-loop eigenvalues in the multi-input systems are not unique. This freedom can be utilized to minimize a time-weighted quadratic performance index.

Consider the linear time-invariant system described by:

$$\begin{aligned} \dot{x}(t) &= A\,x(t) + B\,u(t) \\ x(0) &= x_0 \end{aligned} \tag{2.49}$$

where the state $x(t) \in \Re^n$, the control input $u(t) \in \Re^m$, A and B are constant matrices of appropriate dimensions. It is assumed that the above system is completely controllable and the matrix B is of full rank.

The design problem is to find the constant feedback control

$$u(t) = F x(t) \tag{2.50}$$

which assigns prescribed closed-loop eigenvalues and at the same time minimizes the expected value of the time-weighted quadratic performance index :

$$J = E\left\{\int_0^{\infty} (tx^T Q x + u^T R u)\, dt\right\} \tag{2.51}$$

Here Q and R are symmetric positive semi-definite and positive definite matrices, respectively, and initial state $x(0)$ is assumed to be a random vector, which is given [29] as $X_o = E\{x(0)x(0)^T\} = I$.The closed-loop system is

$$\begin{aligned} &\dot{x}(t) = (A + BF)x(t) \\ &or \\ &\dot{x}(t) \equiv A_C x(t) \end{aligned} \tag{2.52}$$

where $A_C = A + BF$

The closed loop eigen values are given as any desired set of distinct eigen values

$\Lambda = \{\lambda_1, \ldots \lambda_n\}$

2.4.2 Mode Assignment

As described in preceding section, all classes of controllers satisfying pole constraint can be characterized through a set of parameter vectors p_i ($i = 1, \ldots, n$), the general procedure for mode assignment may be given as :

1) Compute the maximal rank matrix N_i satisfying $(\lambda_i I - A, B)N_i = 0$ as :

$$N_i = \begin{bmatrix} N_{1i} \\ N_{2i} \end{bmatrix}, i = 1, \ldots, n \tag{2.53}$$

where $N_i \in \Re^{(n+m)xm}$ and $N_{1i} \in \Re^{nxm}$.

2) Define a parameter vector p_i ($i = 1, \ldots, n$) as :

$$p_i = (p_{i1}, \ldots, p_{im})^T \tag{2.54}$$

and form the following matrices

$$V = V_o P, \quad W = W_o P \tag{2.55}$$

where

$V_o = (N_{11} , ..., N_{1n})$, $W_o = (N_{21} , ..., N_{2n})$, and $P = diag (p_i)$, $i = 1, ..., n$.

3) For each value of the parameter vector p_i where $det\ V \neq 0$, compute the feedback gain matrix.

$$F = -W V^{-1} \tag{2.56}$$

Thus the degree of freedom remaining after the eigen-values being assigned is expressed as any parameter vector p_i which makes the matrix V nonsingular and is used in optimizing the control gain with respect to the given performance index.

2.4.3 Optimization

Now, using (2.50) and (2.51) , the performance index J can be expressed as [62]

$$J = E\{x(t)^T Q_2 x(t)\} = tr\{Q_2 X_o\} \tag{2.57}$$

where Q_2 can be determined by the solution of following *Lyapunov* equations :

$$A_C{}^T Q_1 + Q_1 A_C + Q = 0 \tag{2.58}$$

$$A_C{}^T Q_2 + Q_2 A_C + Q_1 + F^T R F = 0 \tag{2.59}$$

To determine the optimal parameter vector p_i which minimizes the performance index (2.51) subject to the constraints in (2.58) and (2.59) , the *Hamiltonian* for this problem can be formulated as below :

$$H = tr(Q_2) + tr\{L_1^T [A_C^T Q_1 + Q_1 A_C + Q] + L_2^T [A_C^T Q_2 + Q_2 A_C + Q_1 + F^T R F]\} \tag{2.60}$$

where L_1 and L_2 are the symmetric *Lagrange* Multiplier matrix, and are given by the following *Lyapunov* equations.

$$A_C L_2 + L_2 A_C^T + X_o = 0 \qquad (2.61)$$

$$A_C L_1 + L_1 A_C^T + L_2 = 0 \qquad (2.62)$$

The necessary conditions for the solution are derived by taking the partial derivatives of (2.60) with respect to Q_1 ,Q_2 ,L_1, L_2, and p_i and equating them to zero.

2.4.4 Necessary Conditions for Optimality

The necessary conditions for the optimality of the constant feedback gain matrix F with respect to the performance index (2.51) under the stable eigen-value constraint, are given as follows [35]:

$$\partial H / \partial p_i = 0 , \qquad i = 1 , \ldots\ldots, n \qquad (2.63)$$

The vector in (2.63) is a block diagonal part of the following equation :

$$\partial H / \partial P = 2(W_o - F V_o)^{-1} [R F L_2 + B^T Q_1 L_1 + B^T Q_2 L_2](V^T)^{-1} \qquad (2.64)$$

where Q_1 ,Q_2 ,L_1, and L_2, satisfy the *Lyapunov* equations as given in (2.58, 2.59, 2.61 & 2.62)

2.4.5 Algorithm for TWLQR State Feedback Mode Assignment

Here the details of the iterative algorithm for the computation of the optimal parameter vector required for the computation of TWLQR mode assignment control using State Feedback are summarized along-with the equivalent flow chart presentation of *Fig2.4*.

Step 1: For each eigenvalue λ_i ,calculate Ni .

Step 2: Set iteration no. $i=0$, and select any initial parameter vector p_i such that the matrix V is nonsingular.

Step 3:

i) Compute the the feedback gain $F_{(i)}$ and closed loop system matrix A_C from (2.58) and (2.52) respectively.

ii) Compute the positive definite solution $Q_{1(i)}$ and $Q_{2(i)}$ from (2.58) and (2.59).

iii) Compute $L_{2(i)}$ and $L_{1(i)}$from (2.61) and (2.62).

*Step 4:*Compute the performance Index J from (2.57) and the gradient matrix $\partial J \ / \ \partial P_{(i)}$, from (2.64).

*Step 5:*Evaluate the stopping criteria $\|\partial J \ / \partial P_{(i)}\| \leq \varepsilon$. if it is satisfied then go to *step 8*, otherwise go to *step 6*, where ε is a prespecified small positive real number.

*Step 6:*Increment the iteration counter i and find a new value of parameter vector p_i using

$$P_{(i+1)} = P_{(i)} - \alpha(\partial J \ / \ \partial P_{(i)}) \qquad (2.65)$$

where step length α can be chosen optimally by using some gradient based technique.

*Step 7:*Repeat *step 2* to *step 5*.

*Step 8:*Stop and accept $P_{(i+1)}$ as an optimal parameter vector.

*Step 9:*After the optimal parameter vector P is determined using the above algorithm, the feedback gain matrix F is given from (2.55) and (2.56).

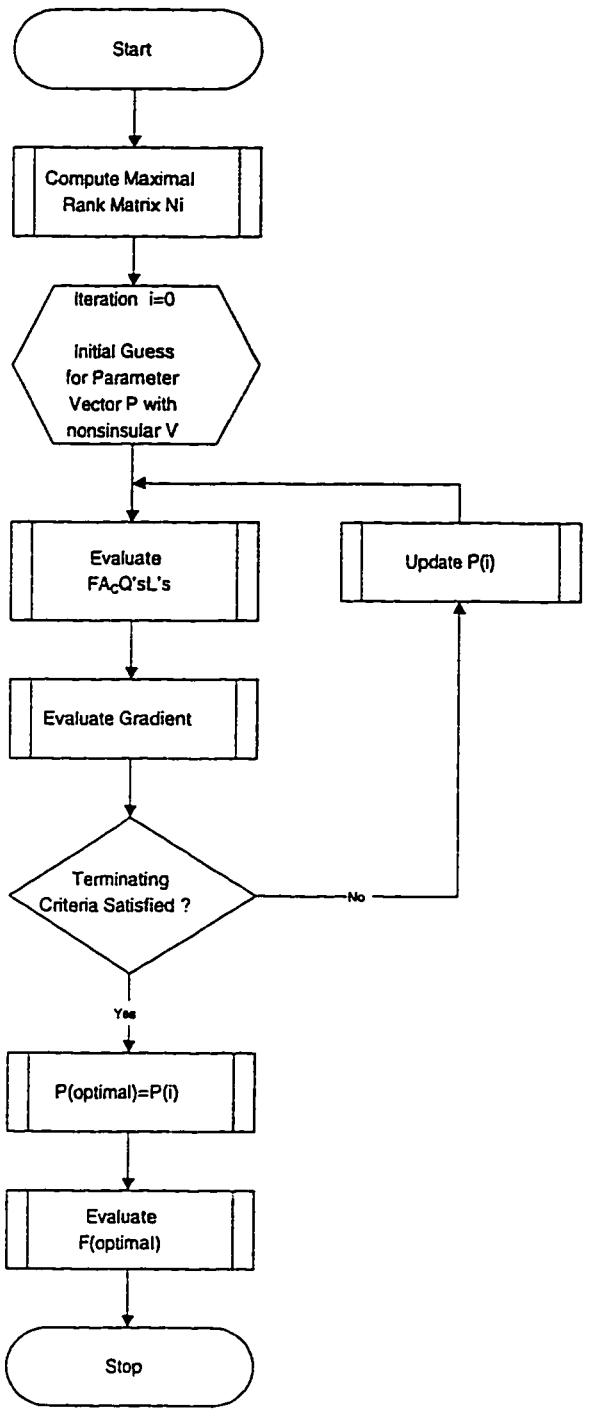


Figure 2. 4 : Flow Chart for TWLQR State Feedback Mode-assignment Algorithm

2.5 Software Implementation using *MATLAB*

To implement the algorithms discussed in the preceding sections, *MATLAB Optimization Toolbox* [63] is used. Since the problem presented here is to minimize a given performance index while satisfying the constraints imposed by the necessary conditions for optimality therfore the constrained minimization function *constr* has been customized for the particular problem requirements of each minimization case.

2.5.1 Constrained Minimization Function

2.5.1.1 General Description

Constr Finds constrained minimum of scalar function of several variables, starting at an initial estimate. This is generally referred to as constrained nonlinear optimization, and is mathematically stated as

$$\textbf{minimize } f(\mathbf{X}) \textbf{ subject to } G(\mathbf{X}) \leq 0$$

Where bold upper case characters indicate that X and $G(X)$ are matrices and plain lower case characters indicate that $f(X)$ is a scalar function. An expression can be used with X representing the independent variable and with f and g representing the function and constraints. For example

$$\mathrm{X} = constr\ (\ \text{'f} = fun(\mathrm{X});\quad \mathrm{g} = cstr(\mathrm{X});\text{'}, \mathrm{X}_0)$$

The objective function, f, is minimized such that $g < zeros(g)$.

$$\mathrm{X} = constr\ (\text{'fun'},\ \mathrm{X}_0,\ \textbf{options})$$

defines a vector of optional parameters. There are a total of eighteen options associated with *constr* minimization function. The more important of these options are described as follows :

- **options(1)** controls display. Setting this to a value of 1, produces a tabular display of intermediate results.
- **options(2)** controls the accuracy of x at the solutions.
- **options(3)** controls the accuracy of f at the solution.
- **options(4)** sets the maximum constraint g violation that is acceptable.
- **options(13)** sets the number of equality constraints. These constraints are placed in the first elements of the variable g.
- **options(14)** sets the maximum number of iterations used in optimization.
- **options(16)** sets the minimum change in variables for finite difference gradient calculation.
- **options(17)** sets the maximum change in variable for finite difference gradient calculation.
- **option (18)** sets the step size parameter.

The proper choice of these options may leads to more accurate and efficient results, and it depends entirely on the nature of particular problem application.

2.5.1.2 Algorithm

Constr uses a Sequential Quadratic Programming (SQP) method. In this method, a Quadratic Programming (QP) sub-problem is solved at each iteration. An estimate of the Hessian of the Lagrangian is updated at each iteration using the formula of

Broyden [64], Flethcer [65], Goldfarb [66], and Shanno [67],(BFGS) which considered to be the most effective for the use in constrained minimization.

A line search is performed using a merit function similar to that proposed by Han [68],and Powell [69 , 70]. The QP sub-problem is solved using an active set strategy similar to that described in Gill, Murray and Wright [71].

2.5.2 Application of *constr* for Optimization

For the application of *MATLAB* optimization function *constr* , the initial guess is taken as a starting step. Here this function evaluates the given performance index and the associated optimality constraints. The updating of the parameter vector within a particular optimization procedure is performed by the *constr* function itself is done, by first determining *search direction* and finding *optimal step size.* This procedure will be repeated by the optimization function itself until there is no further minimization both in the value of performance index and maximum value of active constraints and results obtained will be considered as one of the optimal solution with respect to that particular starting guess. Several starting guesses may be taken to assure the global convergence.

2.5.2.1 Initial Guesses for LQR and TWLQR Output Feedback

For state feedback case, the gain matrix obtained for LQR is used as a initial starting guess for TWLQR control gain computations, and in most of the cases algorithm leads to convergence. However, it is not the case for output feedback, as several different initial guesses have to be given for both LQR and TWLQR control gain

computations, and the gain matrix with which the objective function is minimum and the constraints are also satisfied is chosen as the required solution.

2.5.2.2 Initial Guesses for TWLQR State Feedback Mode Assignment

For TWLQR mode assignment initial guess for parameter vector P_o is given such that the matrix V is non-singular. A sufficiently large number of initial guesses are given to ensure the global optimal solution of the parameter vector P needed for the evaluation of TWLQR mode assignment control gain matrix F.

Chapter 3

TWLQR Control of Single Machine Infinite Bus System

A single machine infinite bus model having coordinated exciter-governor control is selected here to make comparison between LQR and TWLQR control responses for dynamic stability analysis. Both state and output feedback control strategies have been considered. The corresponding feedback gains are computed using the algorithms presented in *chapter two* and the closed loop responses are simulated for small pulse disturbances made in load torque or reference voltage for three cycles of the system frequency.

3.1 Model Description

The power system under investigation consists of a synchronous machine unit connected to an infinite bus through a transmission line [3, 4, 72] as shown in *fig 3.1*. It has both exciter-voltage regulator and speed governor *figs 3.2 (a) & 3 2 (b)*.

3.1.1 System State Variables

The overall system is of fifth order, with the synchronous machine represented by third order model, and both exciter-voltage regulator and the governor system represented by first order dynamics. The linearised system model can be written as

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t)$$

where the state variables are described as below

$$x = \begin{bmatrix} \Delta\omega & \Delta\delta & \Delta e_q' & \Delta V_F & \Delta P_m \end{bmatrix}$$

The system coefficient matrix A and input coefficient matrix B along-with initial operating conditions and machine constants are given in *Appendix A*.

3.1.2 Exciter-Voltage Regulator System

A control signal u_E is fed into the summing junction of the exciter-voltage regulator system through a transfer function as shown in *Fig 3.2(a)*. The transfer function of the exciter itself is also shown by the block diagram in the forward branch of the same figure.

3.1.3 Governor System

A control signal u_g is fed into the summing junction of the governor system as shown in the *Fig 3.2(b)*. The governor system transfer function is shown by the block diagram in the forward branch of the figure.

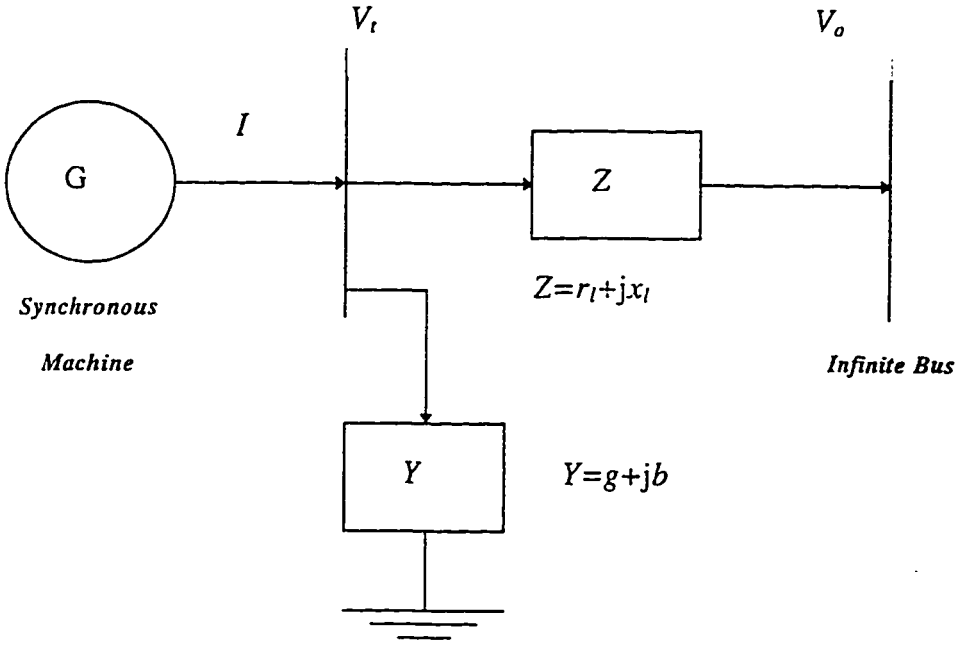
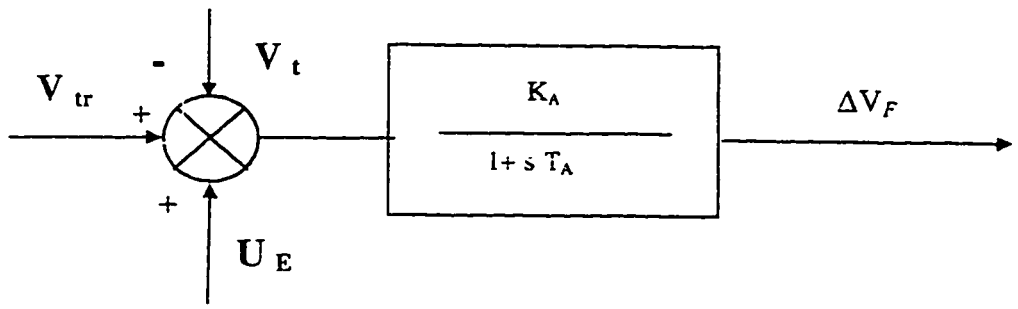
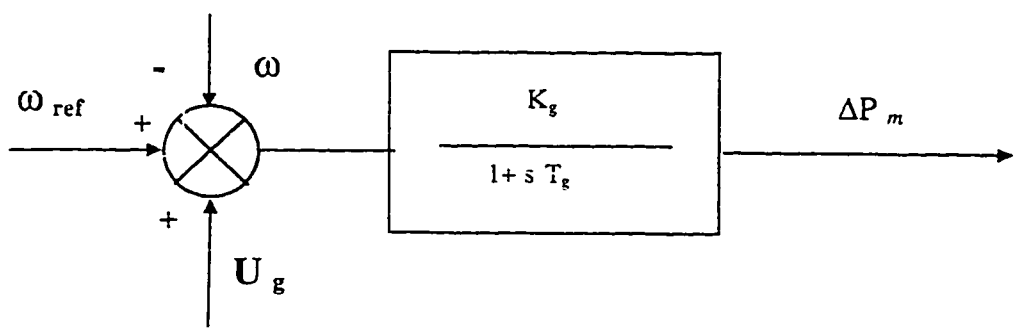


Figure 3. 1 : Synchronous machine infinite bus system



(a)



(b)

Figure 3. 2 : (a) Exciter-voltage regulator system, (b) Governor system

3.1.4 Open Loop System Behavior

The eigenvalues of the unstabilized open loop system are given in *Table 3.1.* The first two rows of eigenvalues correspond to the machine dynamics, δ and ω, also called the electro-mechanical mode of the power system.

Table 3. 1 : Open loop eigen-values

	Machine
Electro-mech. Eigenvalues	0.2928 + j 4.9692
	0.2928 - j 4.9692
	-0.9963
	-10.3925 + j 3.2834
	-10.3925 - j 3.2834

The eigenvalue analysis of the open loop unstabilised system shows that the electro-mechanical modes are unstable and needs feedback stabilization.

3.2 System Dynamic Responses

The dynamic performance evaluation of the power system under consideration can be studied when the system is subjected to a small pulse disturbance for a duration.of three cycles of the system frequency [20].In this situation the system is described as

$$\dot{x}(t) = A_d x(t) + B_d d$$

where

A_d is open loop matrix for unstabilised system simulation and closed loop matrix for respective LQR or TWLQR gain matrices

d is the disturbance input taken as 10 percent of nominal values for dynamic reponse analysis.

B_d is the disturbance input matrix , defined particularly for each case.

The following two cases have been considered

Case 1 : *Change in Load Torque*

As the mechanical power is taken as one of the state variable for the single machine infinite bus system. so the electrical torque is changed for simulation purposes, hence the B_d and d are defined as

$$B_d = [\ -1/M \quad 0 \quad 0 \quad 0 \quad 0\ ]$$

d is 10 percent of nominal load torque.

Case 2 : *Change in Reference Voltage*

As clear in the exciter block diagram that the terminal reference voltage V_{tr} and the machine terminal voltages V_t are fed at the summing junction where the exciter control input signal U_E is applied, see *fig 3.2(a)*, so in this case B_d and d are defined as

$$B_d = [\ 0 \quad 0 \quad 0 \quad K_A/T_A \quad 0\ ]$$

d is 10 percent of nominal reference voltage V_{tr}

The disturbance d is a pulse of duration taken as three cycles of the system frequency.

3.3 State Feedback Design

3.3.1 Selection of States and Control weighting matrices

As in the case of state feedback all the states are assumed to be available as output, so the respective Q and R matrices are given as follows:

$$Q = \text{diag}\,[\,20 \quad 2 \quad 1 \quad 0.1 \quad 1\,] \qquad\qquad R = 5$$

where each diagonal entry of the Q matrix shows the weightage given to the respective state of the system and the single element of R shows that only *excitation control* is used for state feedback stabilization. Here the weightage given to speed deviation is 10 times as that of torque angle deviation, which is the usual practice for power systems.

3.3.2 Computation of State Feedback Gains

The computations of the state feedback gains are carried out for the initial operating conditions as listed in the *appendix A*. The LQR and TWLQR state feedback gains for exciter control by using the algorithm given in *chapter* 2 are listed in the following table.

Table 3. 2 : LQR & TWLQR State Feedback Control Gains

Conrtol Design	$\Delta\omega$	$\Delta\delta$	$\Delta e'_q$	ΔV_F	ΔP_m
LQR	26.3601	-0.5455	-1.9813	-0.1246	1.5617
TWLQR	61.6513	-0.9587	-3.9263	-0.1229	2.7168

The corresponding closed loop modes of the system are given in *Table 3.3*

Table 3. 3 : Closed Loop eigen-values

	LQR	**TWLQR**
Electro-mech.	-0.5958 + j 4.8320	-1.1057 + j 4.8586
Eigenvalues	-0.5958 - j 4.8320	-1.1057 - j 4.8586
	-0.9999	-0.9982
	-1.5218	-2.3482
	-142.0876	-138.5134

The closed loop modes shows that the system is sufficiently stable and the system damping requirements are fulfilled, specially for TWLQR case.

3.3.3 State Feedback Simulation Results

The following two cases, as described in previous section, are considered here for dynamic response evaluations.

Case 1 : **10 percent load torque pulse disturbance**

The open loop system dynamic response is shown in *fig 3.3*, while the closed loop responses for both LQR and TWLQR are compared in *figs. 3.4 - 3.5* for system electromechanical modes and control inputs respectively.

Case 2 : **10 percent pulse disturbance at the reference voltage of the machine**

For this case the open loop system response is shown in *fig 3.6*, and the closed loop responses for speed and angle deviation are shown in *fig. 3.7* whereas the exciter control input is shown in *fig. 3.8*.

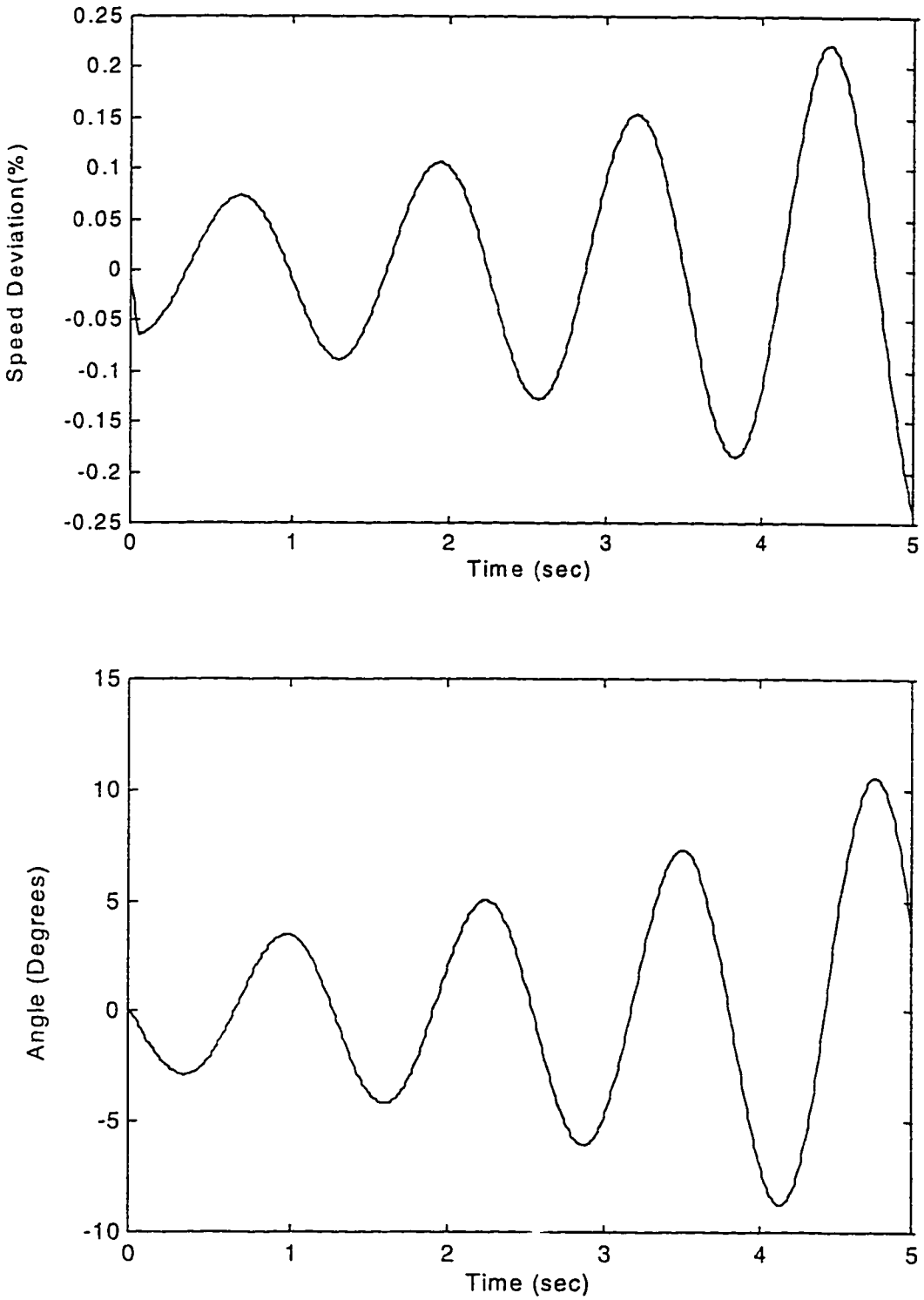
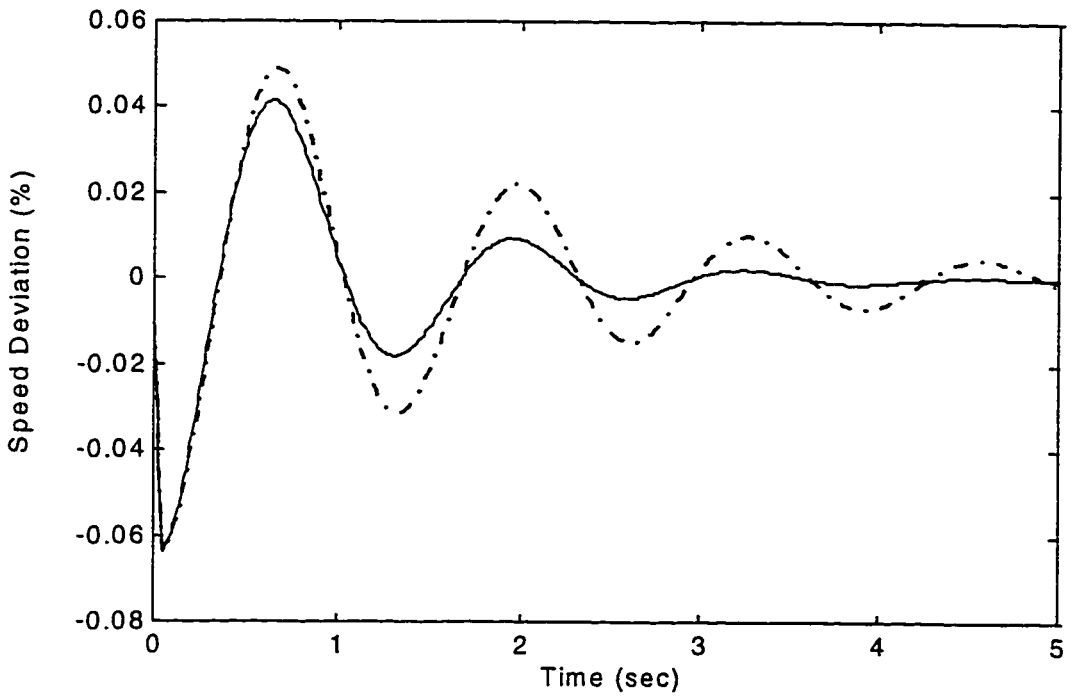


Figure 3. 3 : Open Loop Response For Case 1



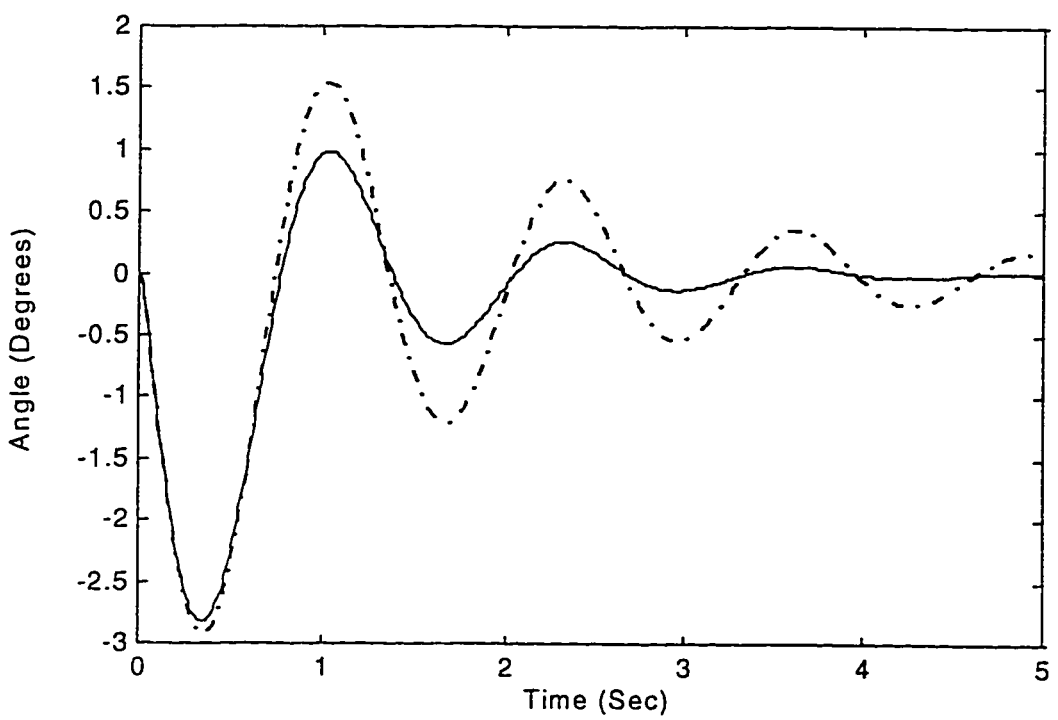


Figure 3. 4 : System State Feedback Response for Case 1

LQR -.-.-.-.-.-.-.- TWLQR ————

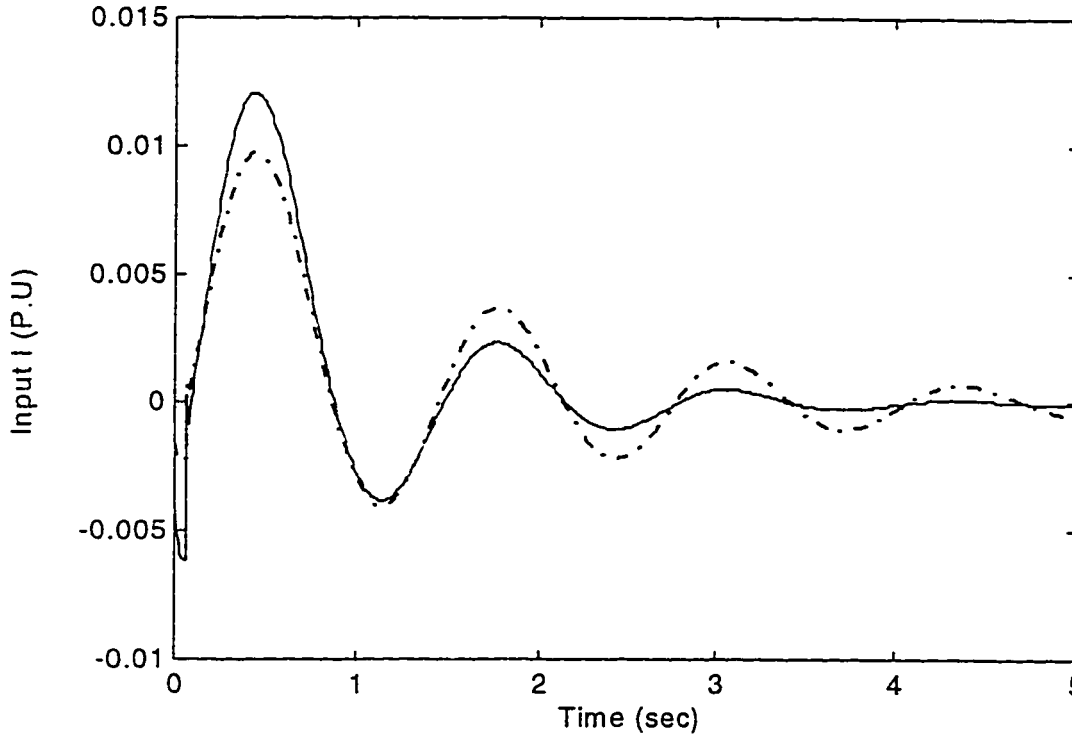
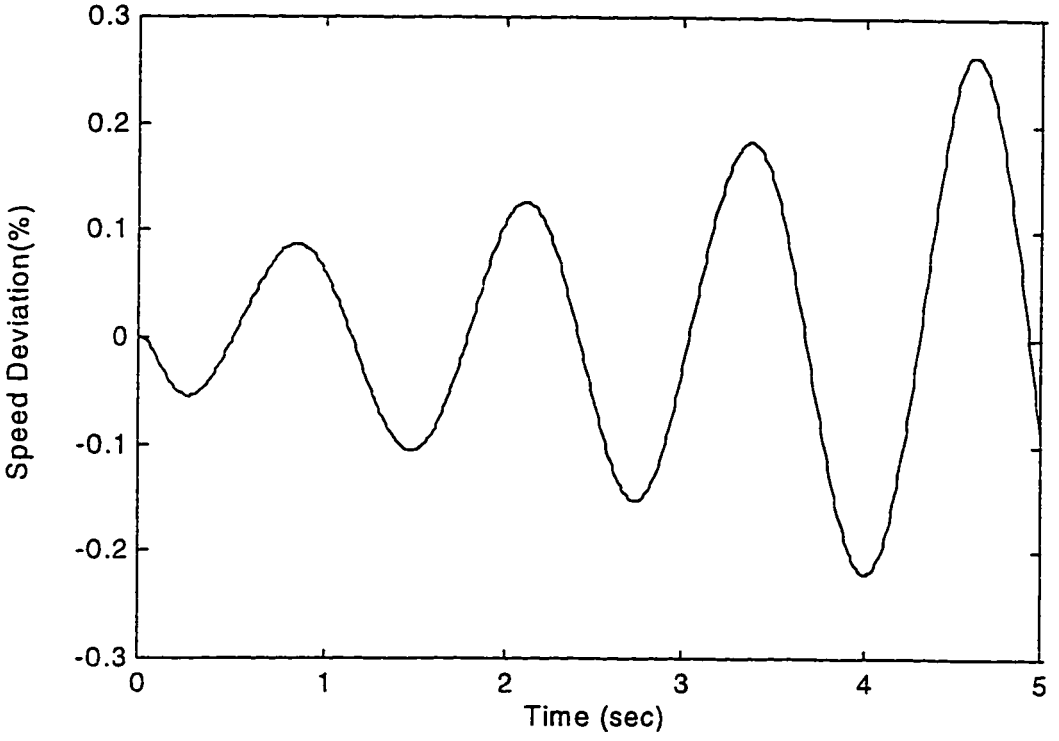


Figure 3. 5 : Exciter Control Input for Case I

LQR _ _ _ _ _ _ _ _ _ _ _ _ TWLQR _____



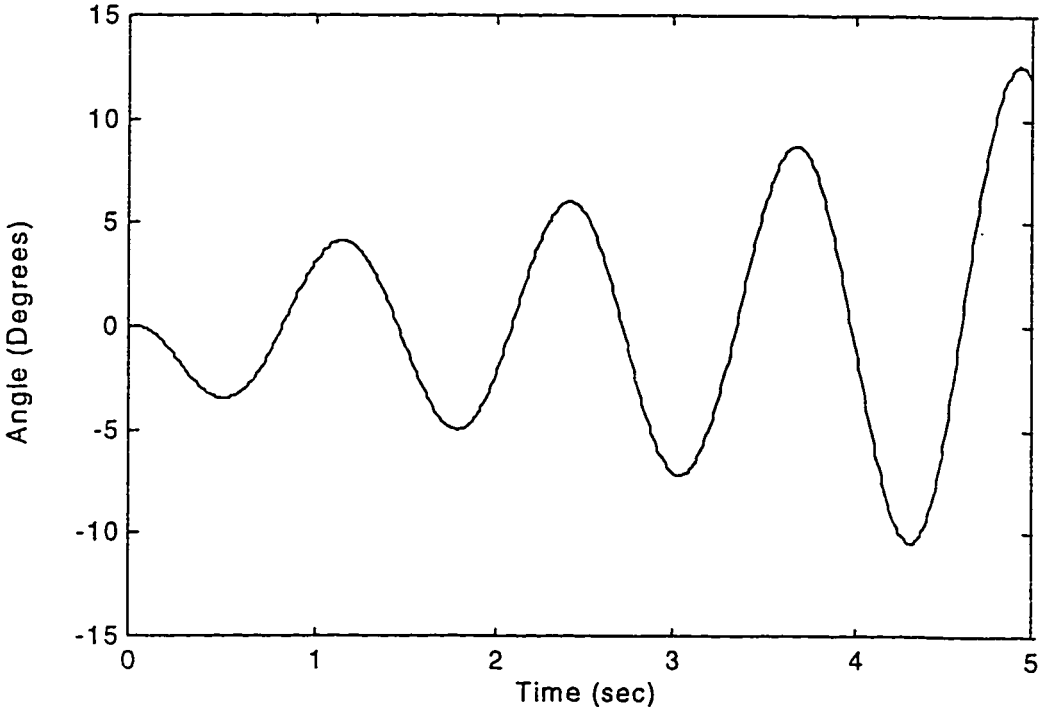
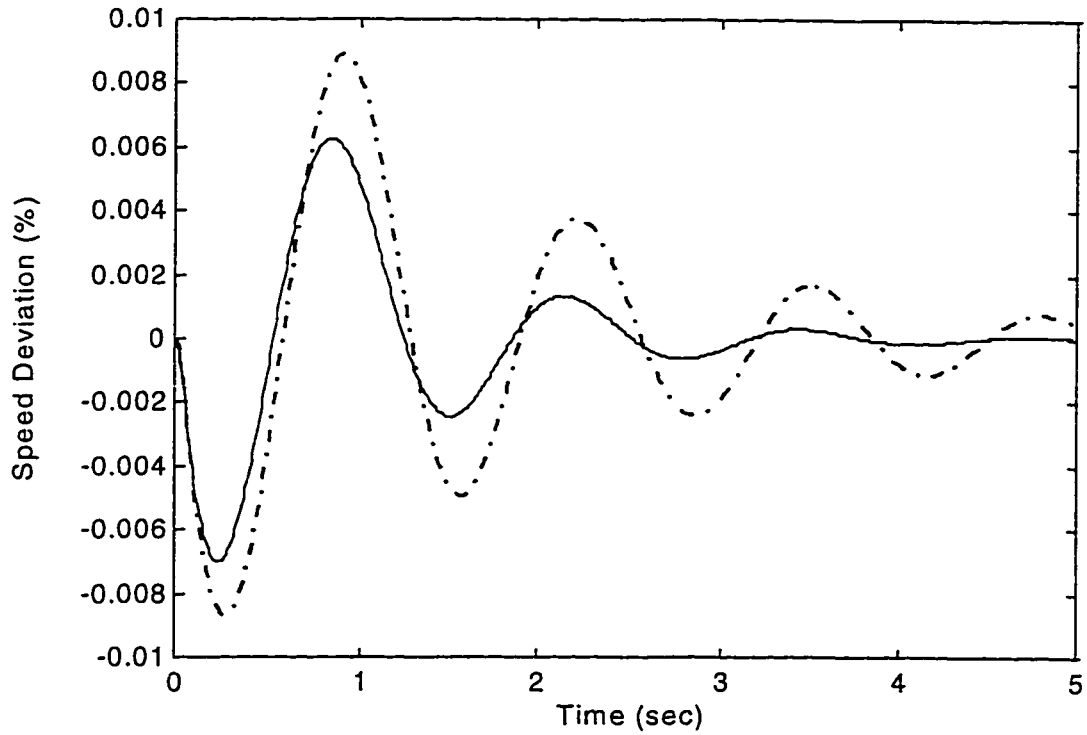


Figure 3. 6 : Open Loop Response for Case 2



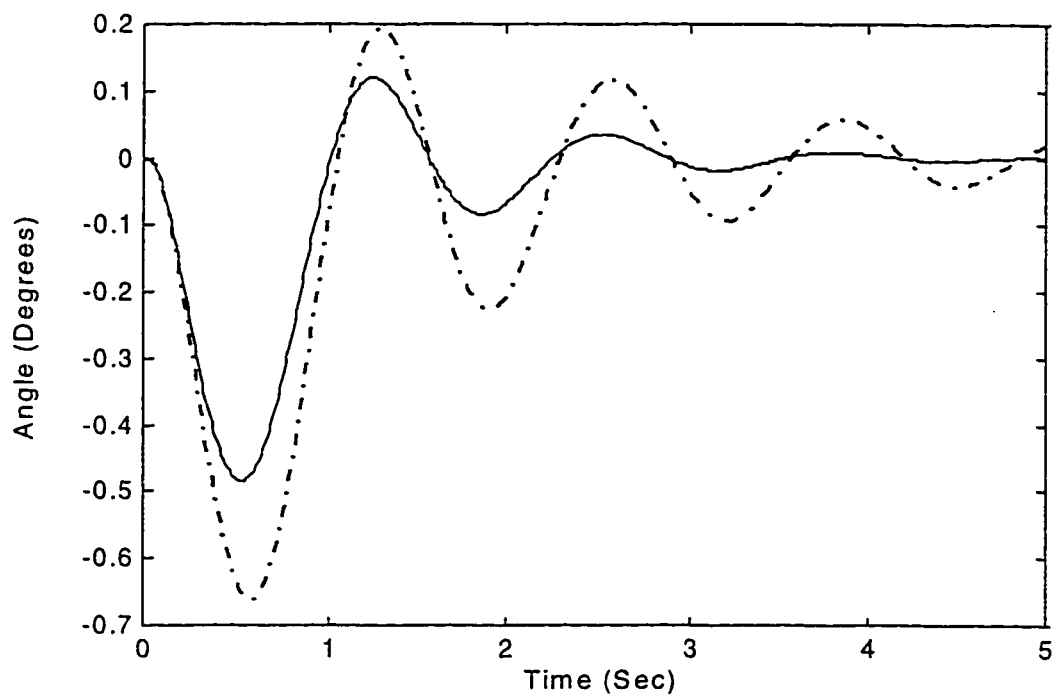
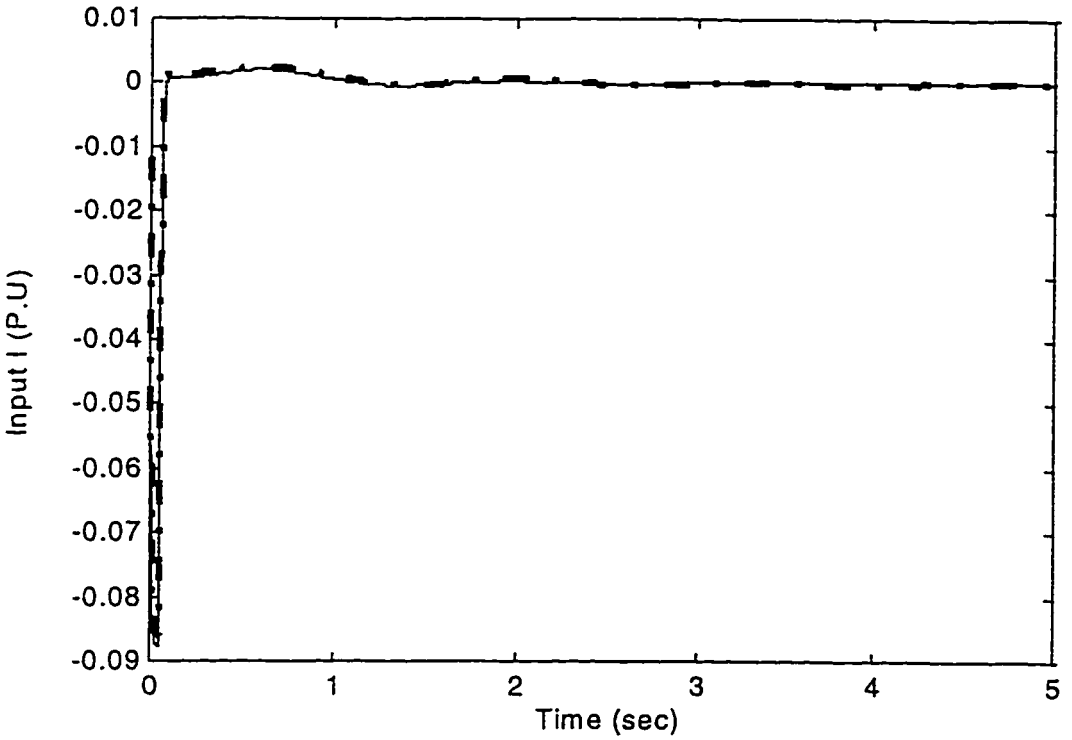


Figure 3. 7 : Closed Loop Response for Case 2

LQR -.-.-.-.-.-.-.- TWLQR ————————



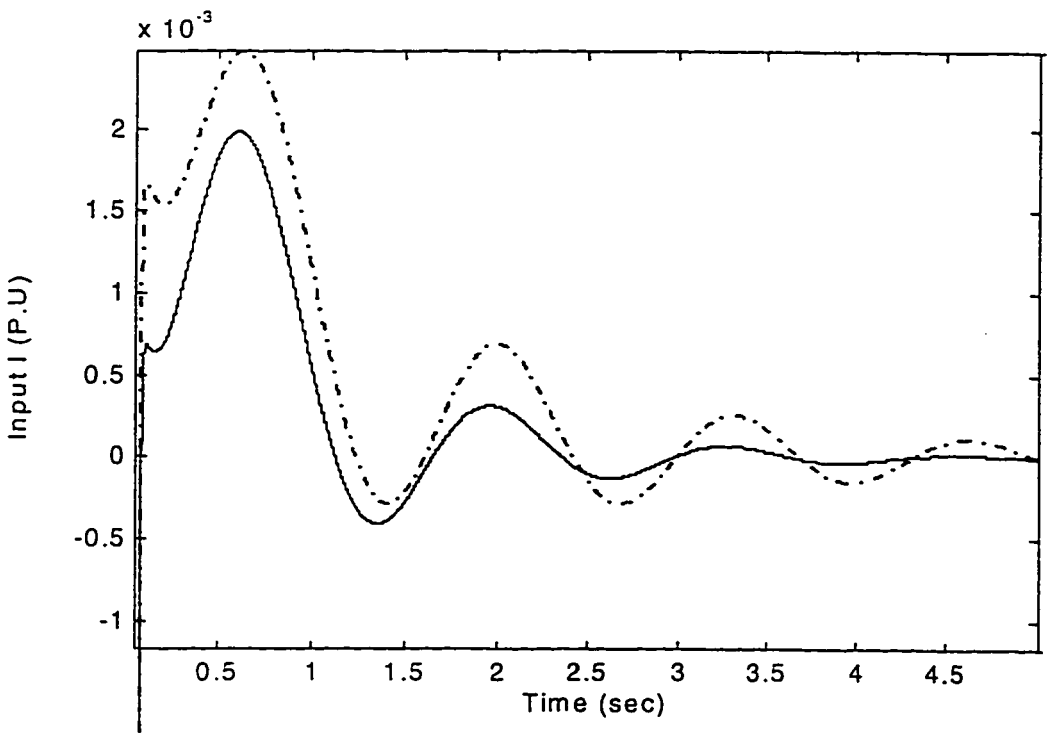


Figure 3. 8 : (a) Exciter Control Input for Case 2 (b) Magnified view

LQR -.-.-.-.-.-.-.- TWLQR ————

3.4 Output Feedback Design

3.4.1 Selection of States and Control weighting matrices

Since the angular speed is the most favorite candidate for output feedback due to its easy accessibility for output measurements, so $\Delta\omega$ is taken as the output variable. The respective Q and R matrices are then given below :

$$Q = 1000 \qquad\qquad R = [\,0.001 \quad 0.01\,]$$

Here both exciter and governor *coordinated* control is used to obtain an effective output feedback response. The high weightage is given to the selected output variable whereas the exciter and governor control inputs are given very small weightages to ensure enough damping of the system.

3.4.2 Computation of Output Feedback Gains

The computations of the output feedback gains are carried out for same operating conditions as for state feedback but with coordinated exciter governor control.The algorithms described in *chapter 2* are used here to evaluate both LQR and TWLQR output feedback gains , the corresponding numarical values are listed below.

Table 3. 4 : LQR Output Feedback Control Gains

Control Inputs	$\Delta\omega$	$\Delta\delta$	$\Delta e'_q$	ΔV_F	ΔP_m
U_1	85.3102	0	0	0	0
U_2	265.2435	0	0	0	0

Table 3. 5 : TWLQR Output Feedback Control Gains

Control Inputs	$\Delta\omega$	$\Delta\delta$	$\Delta e'_q$	ΔV_F	ΔP_m
U_1	76.1312	0	0	0	0
U_2	295.8156	0	0	0	0

The corresponding closed loop modes of the system are given in *Table 3.6*

Table 3. 6 : Closed Loop eigen-values for Output Feedback

	LQR	**TWLQR**
Electro-mech.	-0.7729 + j 8.2582	-1.1208 + j 7.5970
Eigenvalues	-0.7729 - j 8.2582	-1.1208 - j 7.5970
	-0.5787 + j 1.4036	-0.4240 + j 1.6021
	-0.5787 - j 1.4036	-0.4240 - j 1.6021
	-18.4925	-18.1062

The closed loop modes shows that the system electro-mechanical modes are sufficiently damped specially the electro-mechanical modes for TWLQR design.

3.4.3 Output Feedback Simulation Results

The simulation are performed for same disturbance cases as described for state feedback design and are again summarized below.

Case 1 : **10 percent load torque pulse disturbance**

For this case the open loop response is the same as shown in *fig 3.3,* while the closed loop responses for both LQR and TWLQR are compared in *figs. 3.9 & 3.10* for system electromechanical modes and control inputs respectively.

Case 2 : **10 percent pulse disturbance at the reference voltage of the machine**

Again the open loop system response is already shown in *fig 3.6,* and the closed loop responses for speed and angle deviation are shown in *fig. 3.11* whereas the exciter and governor control inputs are shown in *fig. 3.12* for both TWLQR and LQR control designs.

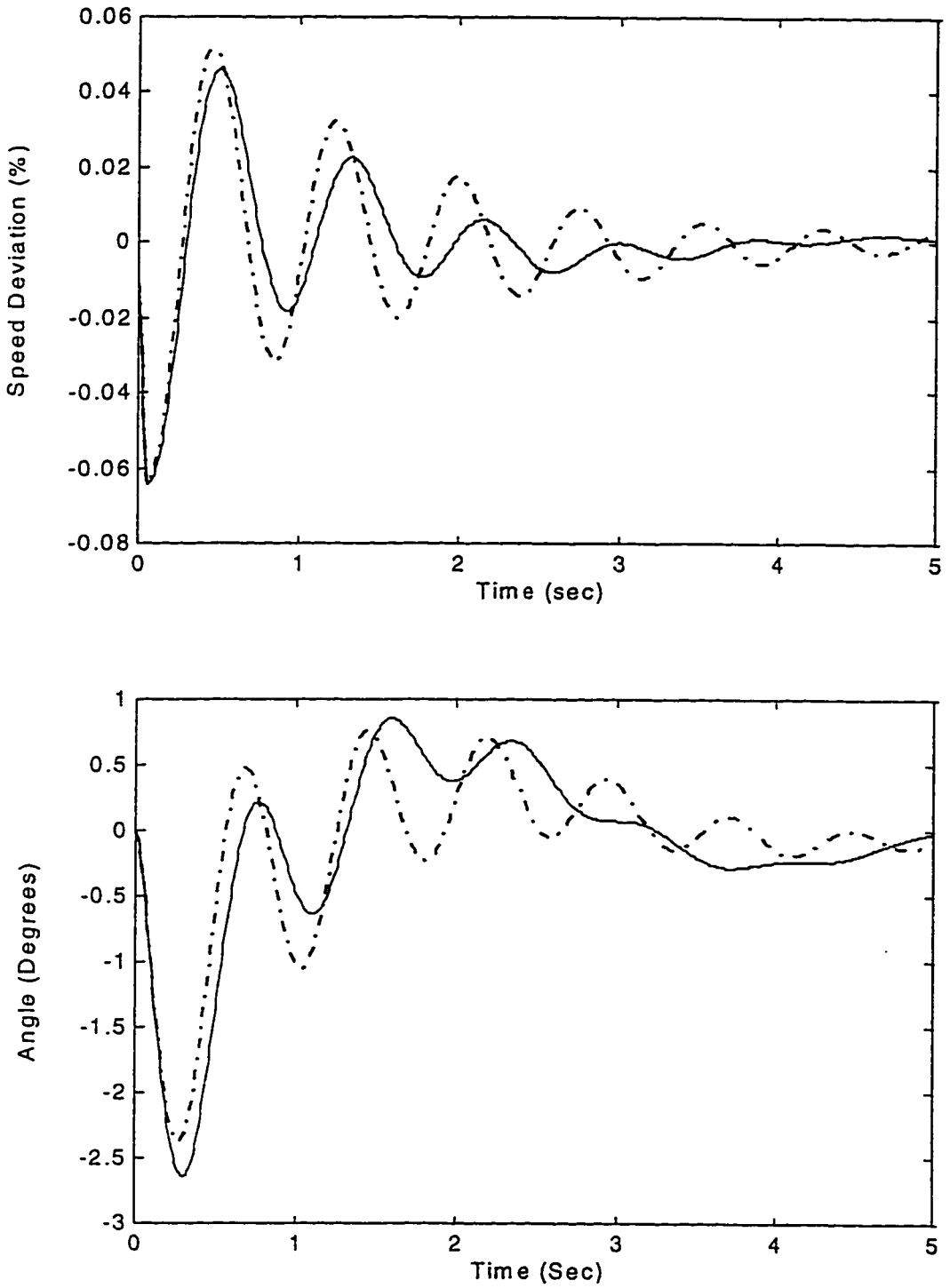
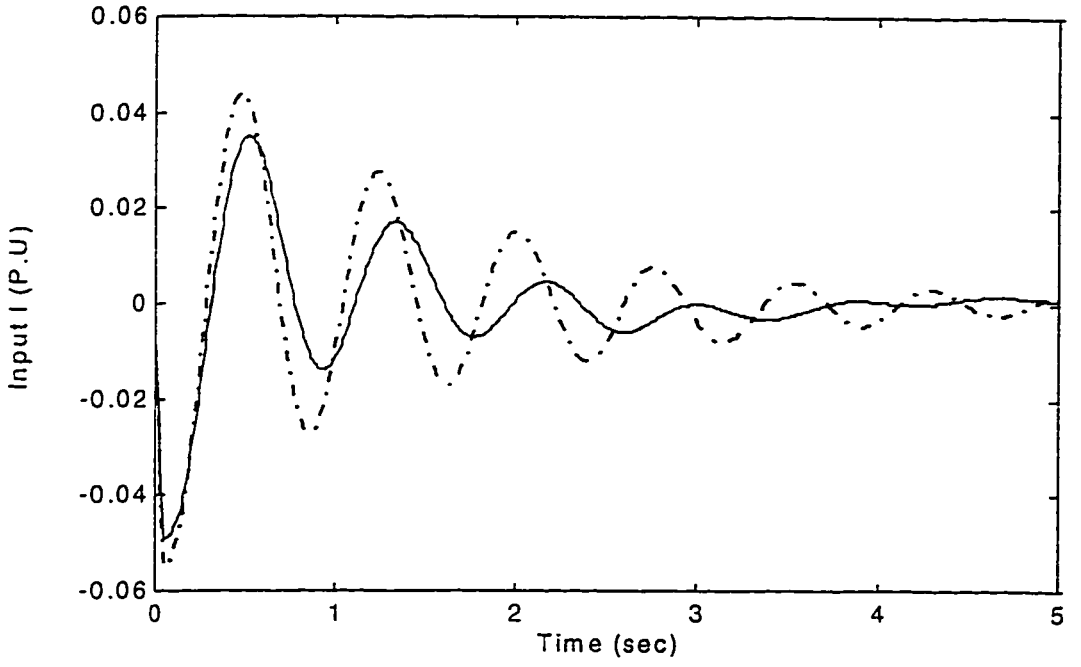


Figure 3. 9 : System Output Feedback Response Case 1

LQR ..... TWLQR _____



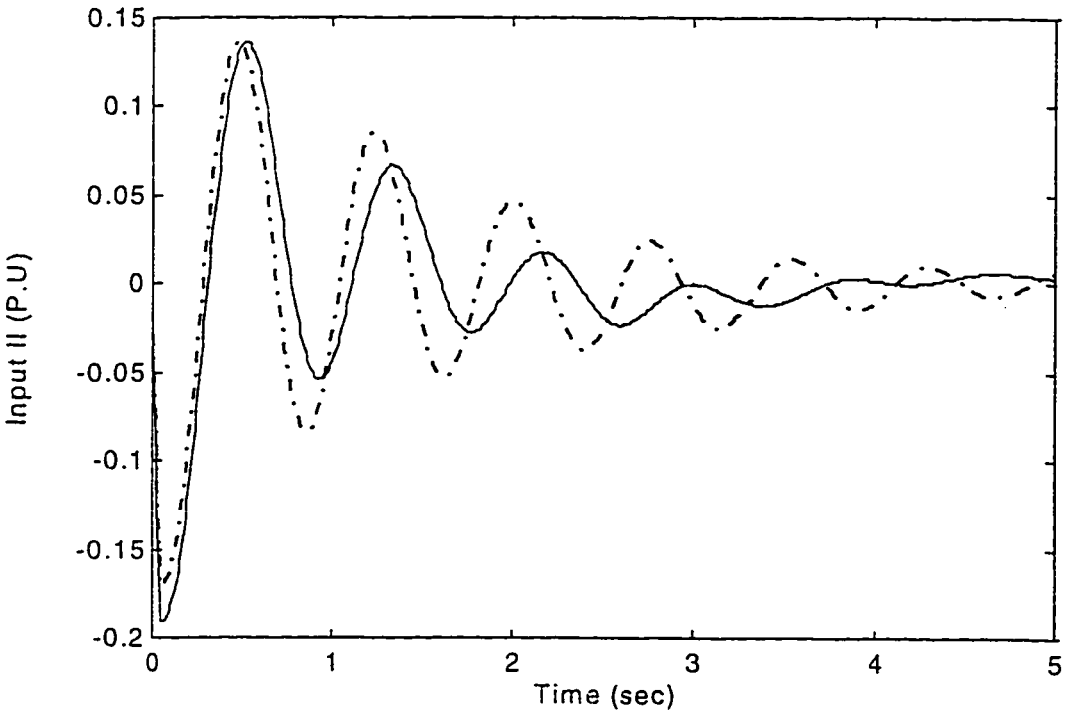
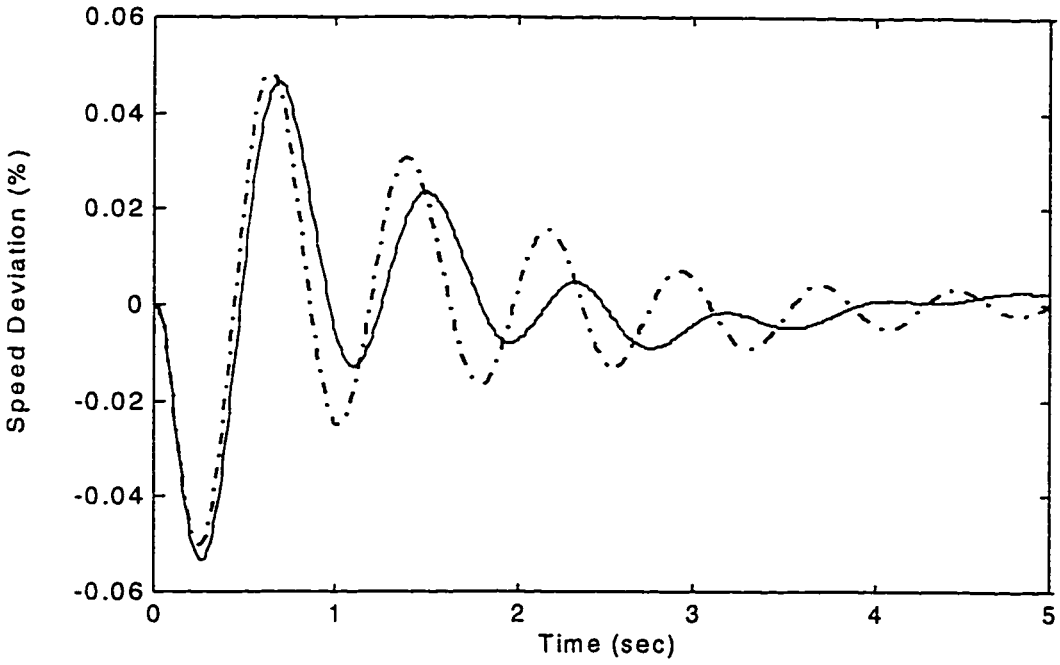


Figure 3. 10 : Output Feeback Control Inputs for Case I

LQR -.-.-.-.-.-.-.- TWLQR ————————



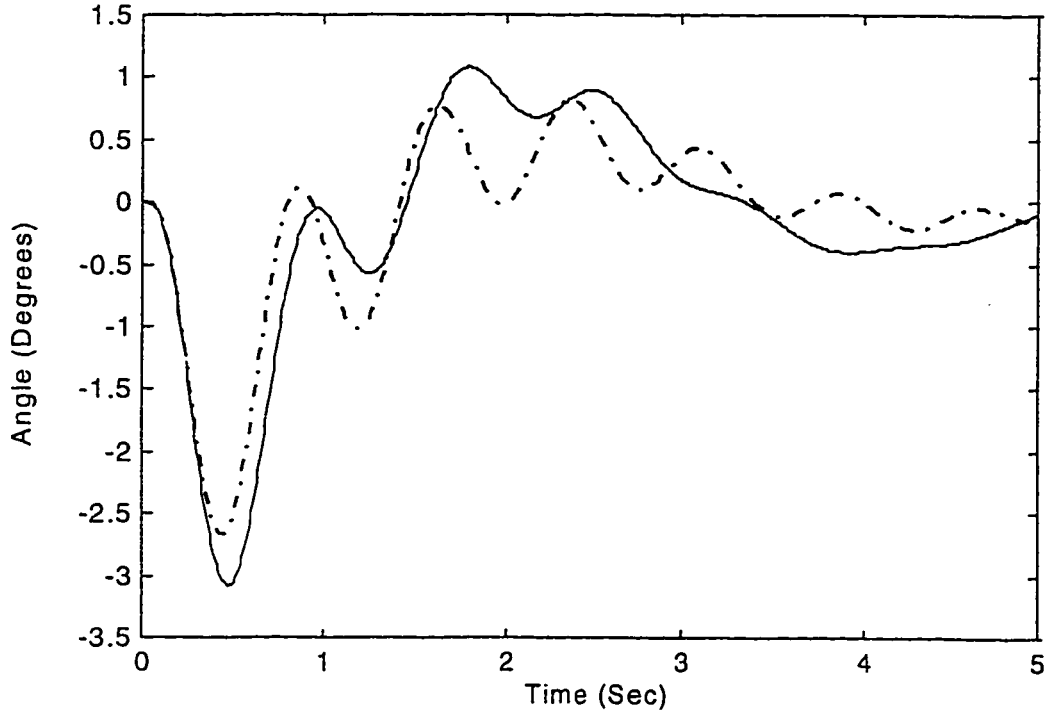
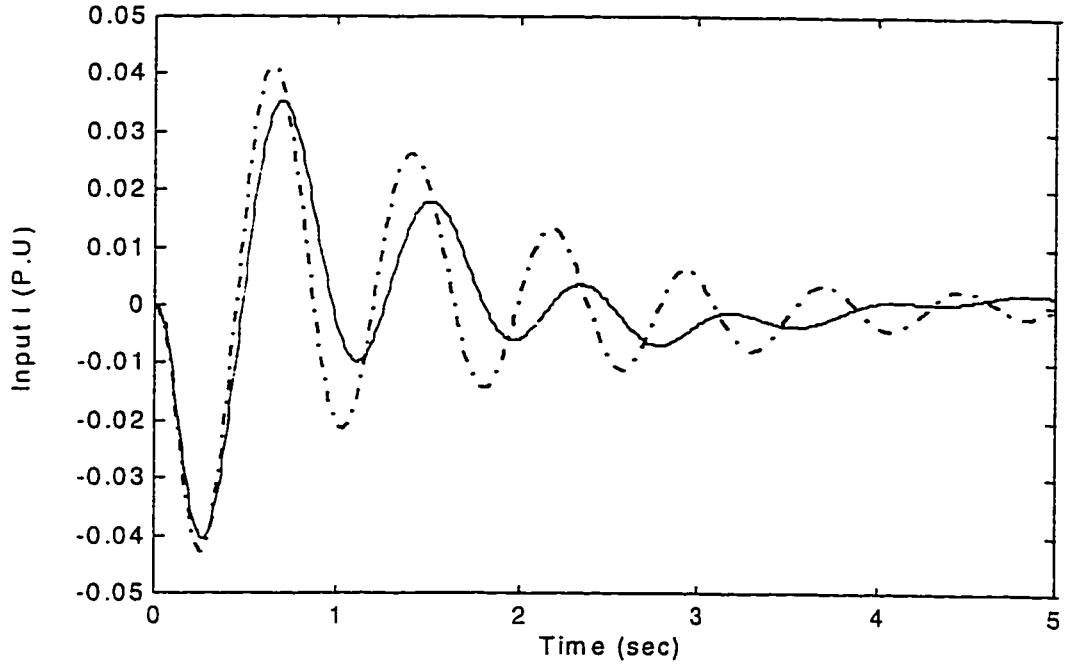


Figure 3. 11 : System Output Feedback Response for Case 2

LQR -.-.-.-.-.-.- TWLQR ————



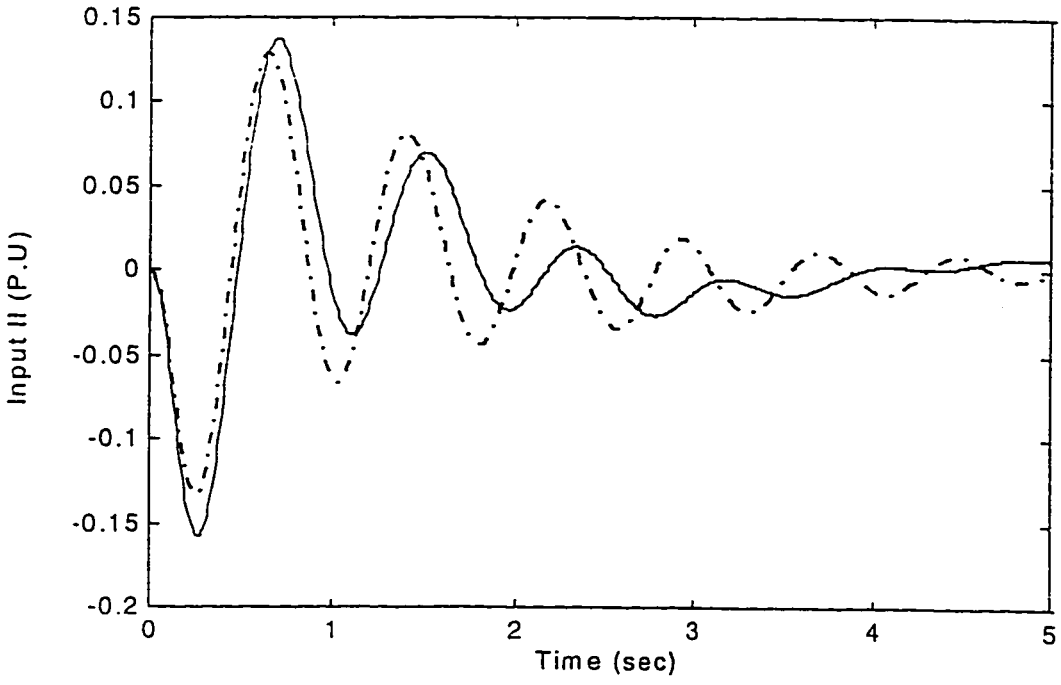


Figure 3. 12 : Output Feedback Control Inputs for Case 2

LQR -.-.-.-.-.-.-.- TWLQR ————————

3.5 Discussions

It can be inferred from the simulation results of both state feedback and output feedback control designs that the TWLQR response is better in all of the above listed cases, irrespective of the disturbance case considered. However in most of the cases ,the control inputs required to achieve these improved system responses are more in magnitude in the start but then decay to zero faster than their LQR counterparts.

Chapter 4

TWLQR Applications to Multi-Machine System

The LQR and TWLQR control strategies have been applied to a multi-machine linearized power system model consisting of three machines, having exciter voltage regulator control on each machine. The LQR and TWLQR control gains are computed for both state and output feedback cases, and are used for closed loop response simulations while the power system is subjected to either small pulse disturbance in mechanical torque or in reference voltage for three cycles of system frequency.

4.1 Multi-Machine Power System Model

The three machine infinite-bus power system given by Yu et al.[6], is chosen for study. This system consists of one thermal and two hydro machines which are mutually coupled and radially connected to an infinite bus, as shown in *Fig 4.1(a)*. The system data and terminal conditions are given in *appendix B*.

4.1.1 State Space Representation

The overall system is of 12th order, with each plant modeled as a fourth order system, represented by a third-order synchronous machine plus a first order exciter-regulator system. The linearised system equations are written as :

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + B \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

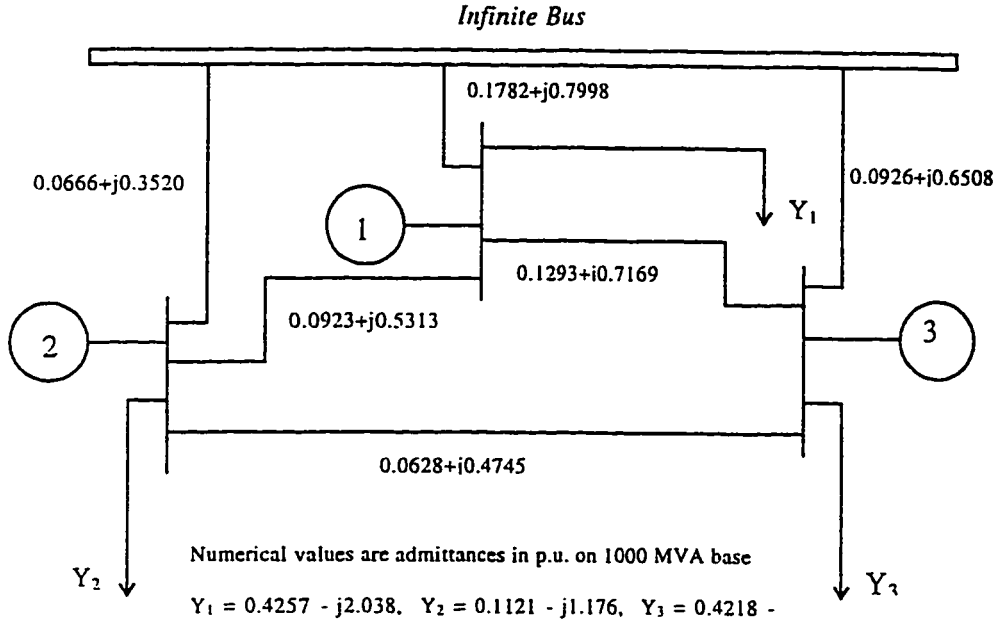
where $x_i = \left[\Delta\psi_{Fi} \quad \Delta V_{Fi} \quad \Delta\delta_i \quad \Delta\omega_i\right]^T for, i = 1,2,3$

4.1.2 System Control Inputs

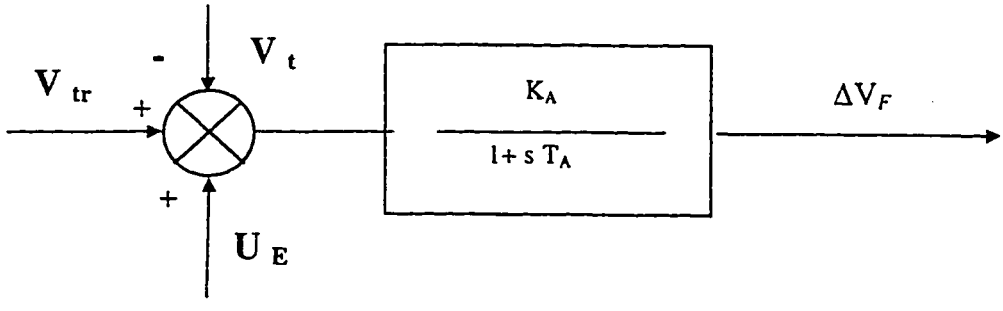
The individual control input to each machine is generated through the exciter-voltage regulator system. A control signal u_E is fed into the summing junction of the exciter-voltage regulator system as shown in *Fig 4.1 (b)*. The transfer function of the exciter itself is shown by the block diagram in the forward branch of the same figure. Therefore system control input matrix may be given as follows

$$B = \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix}$$ where each B_i i=1...3, shows the individual machine control input.

The numerical values of system coefficient matrix A and control input matrix B are given in *appendix B*.



(a)



(b)

Figure 4. 1 : (a) Multi-Machine Power System, (b) Excitation System

4.1.3 Open Loop System Behavior

The eigenvalues of the unstabilized open loop multi-machine system are given in *Table 4.1.* Although there is dynamic coupling among all of the three plants, roughly, the three column eigenvalues correspond to three plants respectively. Also the first two rows of eigenvalues of each column correspond to the machines dynamics, δ and ω (also called the electromechanical mode of each machine).

Table 4. 1 : Open loop eigen-values

	Machine # 1	Machine # 2	Machine # 3
Electro-mech.	-0.0128+ j 7.8157	- 0.0181 + j 7.4196	0.1775 + j 3.9778
Eigenvalues	- 0.0128 - j 7.8157	- 0.0181 - j 7.4196	0.1775 - j 3.9778
	- 1.8468 + j 1.3506	-1.0350 + j 1.3064	-3.8469
	- 1.8468 - j 1.3506	-1.0350 - j 1.3064	-16.6157

The eigenvalue analysis of the open loop unstabilised multi-machine system shows that the electro-mechanical modes of *machine 3* are unstable as the real parts of these eigen-values are positive.

4.2 State Feedback Case

4.2.1 Optimal Selection of States and Control Weighting Matrices

To make the comparison more realistic, the Q and R matrices used here are the same, as given in [6]. The Q matrix is given by the listed values times 10^{-3} :

$$Q = diag\begin{bmatrix} Q_{11} & Q_{22} & Q_{33} \end{bmatrix}$$

where

$Q_{11} = \begin{bmatrix} 0.146 & 0.001 & 2.64 & 97.2 \end{bmatrix}, Q_{22} = \begin{bmatrix} 4.65 & 3.36 & 3.11 & 93.6 \end{bmatrix}$ and

$Q_{33} = \begin{bmatrix} 0.1 & 0.0007 & 4.02 & 88.2 \end{bmatrix}$

The weighting matrix elements of the three plant control is given below

$R = \text{diag}\ [\ 1 \quad 2 \quad 10\ ]$

4.2.2 Computation of Feedback Gains

The algorithm presented in *chapter two*, is applied to compute TWLQR feedback gains. The gain matrices for LQR and TWLQR are given in *Tables 4.2 - 4.4* and *Tables 4.5 - 4.7* respectively. The corresponding closed loop eigen-values for LQR and TWLQR are given in *Table 4.8* and *Table 4.9*.

Table 4. 2: LQR Gains for Control Input I

	$\Delta\psi_F$	ΔV_F	$\Delta\delta$	$\Delta\omega$
Machine # 1	-0.1067	-0.0291	-0.6387	-0.1802
Machine # 2	-0.0510	-0.0039	-0.5875	0.0317
Machine # 3	-0.0733	-0.0026	0.1271	0.1377

Table 4. 3: LQR Gains for Control Input II

	$\Delta\psi_F$	ΔV_F	$\Delta\delta$	$\Delta\omega$
Machine # 1	0.0085	-0.0042	0.5383	0.0220
Machine # 2	-0.0690	-0.0394	-0.8233	-0.1318
Machine # 3	-0.0405	-0.0015	0.0106	0.1004

Table 4. 4: LQR Gains for Control Input III

	$\Delta\psi_F$	ΔV_F	$\Delta\delta$	$\Delta\omega$
Machine # 1	-0.0057	-0.072	0.3803	-0.0224
Machine # 2	-0.0083	-0.0037	0.0709	-0.0518
Machine # 3	-0.1124	-0.0050	-0.7184	0.1159

Table 4. 5: TWLQR Gains for Control Input I

	$\Delta\psi_F$	ΔV_F	$\Delta\delta$	$\Delta\omega$
Machine # 1	-0.1881	-0.0853	-0.5003	-0.2545
Machine # 2	-0.0674	-0.0394	-0.6415	0.0271
Machine # 3	-0.1266	-0.0197	-0.1607	0.1992

Table 4. 6: TWLQR Gains for Control Input II

	$\Delta\psi_F$	ΔV_F	$\Delta\delta$	$\Delta\omega$
Machine # 1	0.0742	0.0316	0.9710	0.0930
Machine # 2	-0.0762	-0.0332	-0.8865	-0.1776
Machine # 3	0.0332	0.0015	0.0280	0.0266

Table 4. 7: TWLQR Gains for Control Input III

	$\Delta\psi_F$	ΔV_F	$\Delta\delta$	$\Delta\omega$
Machine # 1	-0.0058	-0.0110	0.3302	-0.0156
Machine # 2	-0.0054	-0.0061	0.0744	-0.0442
Machine # 3	-0.0840	-0.0005	-0.6181	0.0928

Table 4. 8: Closed Loop Eigen-values for LQR control

	Machine # 1	Machine # 2	Machine # 3
Electro-mech.	- 1.0160 + j 7.6387	- 0.4443 + j 7.8862	- 2.0554 + j 4.0375
Eigenvalues	- 1.0160 - j 7.6387	- 0.4443 - j 7.8862	- 2.0554 - j 4.0375
	- 1.9429 + j 1.0993	-1.7057	-3.0299
	- 1.9429 - j 1.0993	-2.7616	-16.6550

Table 4. 9: Closed Loop Eigen-values for TWLQR control

	Machine # 1	Machine # 2	Machine # 3
Electro-mech.	- 1.1707 + j 7.6407	- 0.6436 + j 7.9106	- 1.9206 + j 3.9483
Eigenvalues	- 1.1707 - j 7.6407	- 0.6436 - j 7.9106	- 1.9206 - j 3.9483
	- 2.9535 + j 2.8699	-1.5467 + j 0.9420	-3.2976
	- 2.9535 - j 2.8699	-1.5467 - j 0.9420	-12.3639

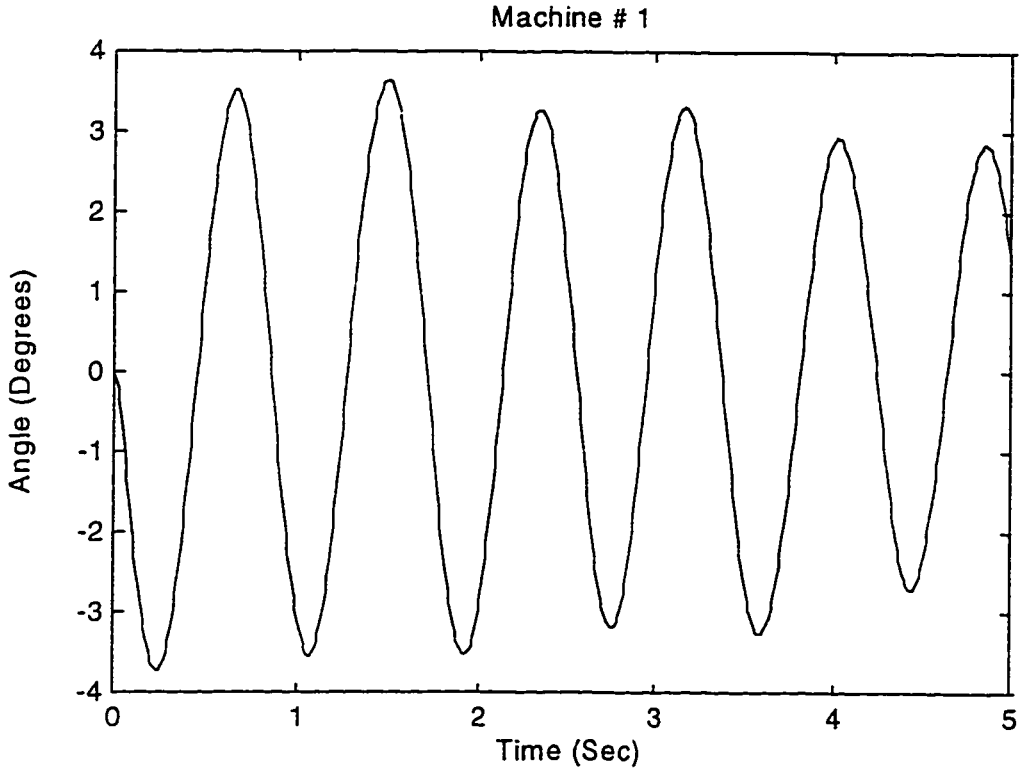
4.3 Simulation Results for State Feedback

For the sake of small perturbation analysis, simulations have been carried out while one of the machine is subjected to a 10 percent pulse disturbance either in its mechanical torque or in reference voltage for a duration of three cycles of system frequency. The simulations for different combinations of disturbances have been performed extensively , however due to the space limitations only those cases showing the variety in response are selected for presentation here.

Case 1* : 10 percent mechanical torque pulse disturbance on *machine 1

A 10-percent torque pulse is applied at the shaft of *machine 1* for a duration of three cycles. The open-loop torque and speed deviation responses for *machine 1* are shown in *fig. 4.2,* showing the oscillatory and poorly damped nature of the system. The LQR and TWLQR closed loop responses for *machines 1, 2 and 3* are shown in *figs. 4.3 - 5* respectively, while the respective control inputs exerted by the excitation control of each machine are shown in *fig. 4.6.*

From *figs. 4.3 - 5,* it can be observed that TWLQR performs better than LQR control design, in damping electromechanical modes, and the oscillations for TWLQR response decay faster than LQR design, however the former exerts more control efforts in the start but even then decays to zero before LQR counterpart as shown in *fig. 4.6.*



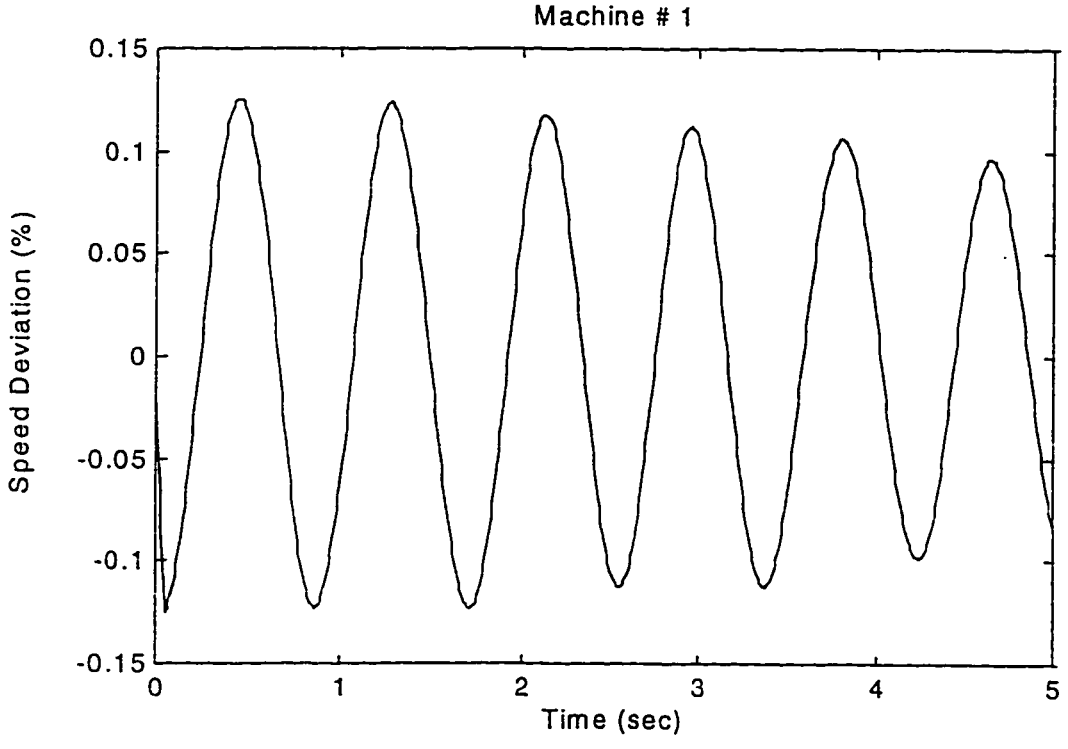


Figure 4. 2 : Open Loop Response of Machine # 1 for Case 1

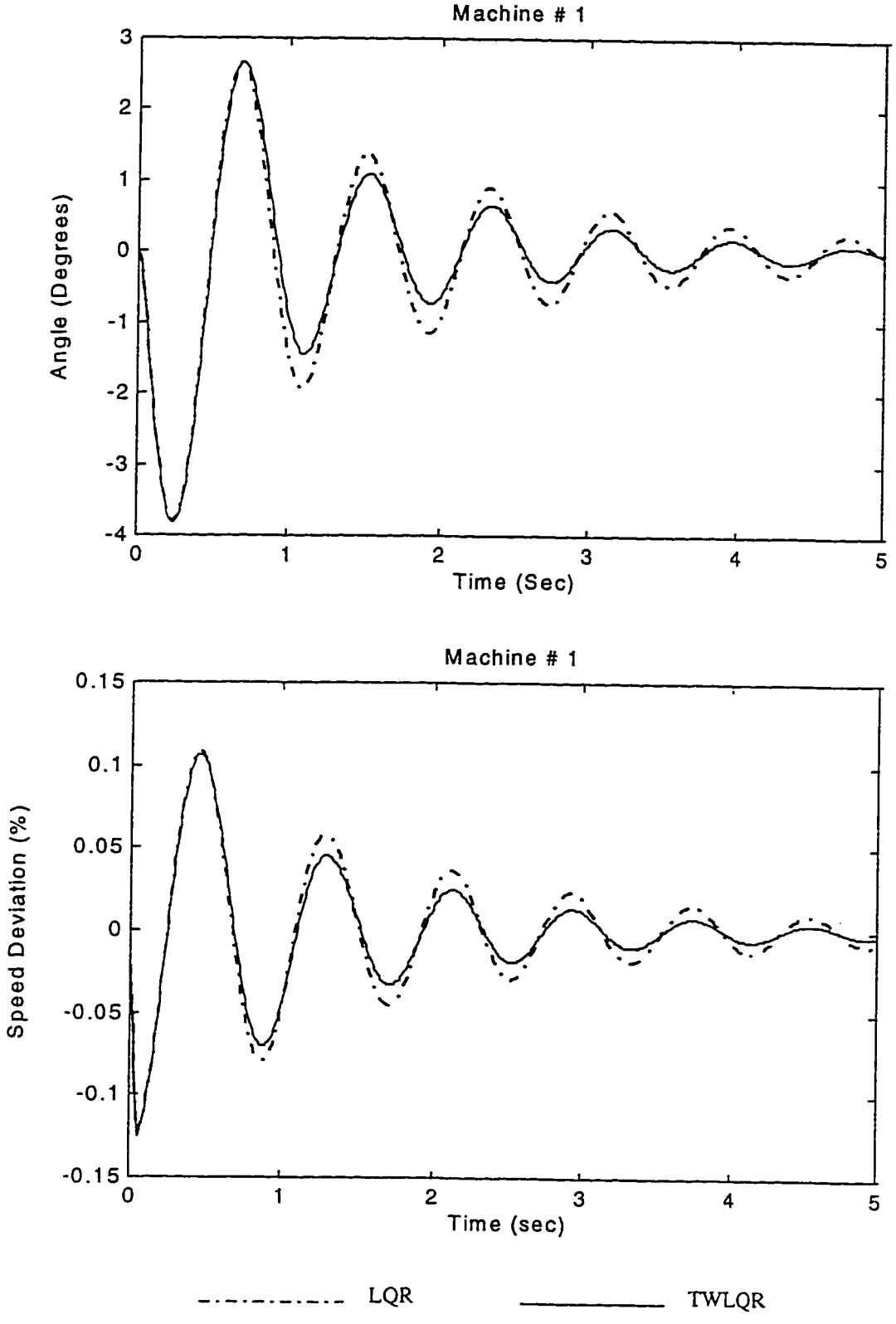


Figure 4. 3 : State Feedback Response of Machine # 1 for Case 1

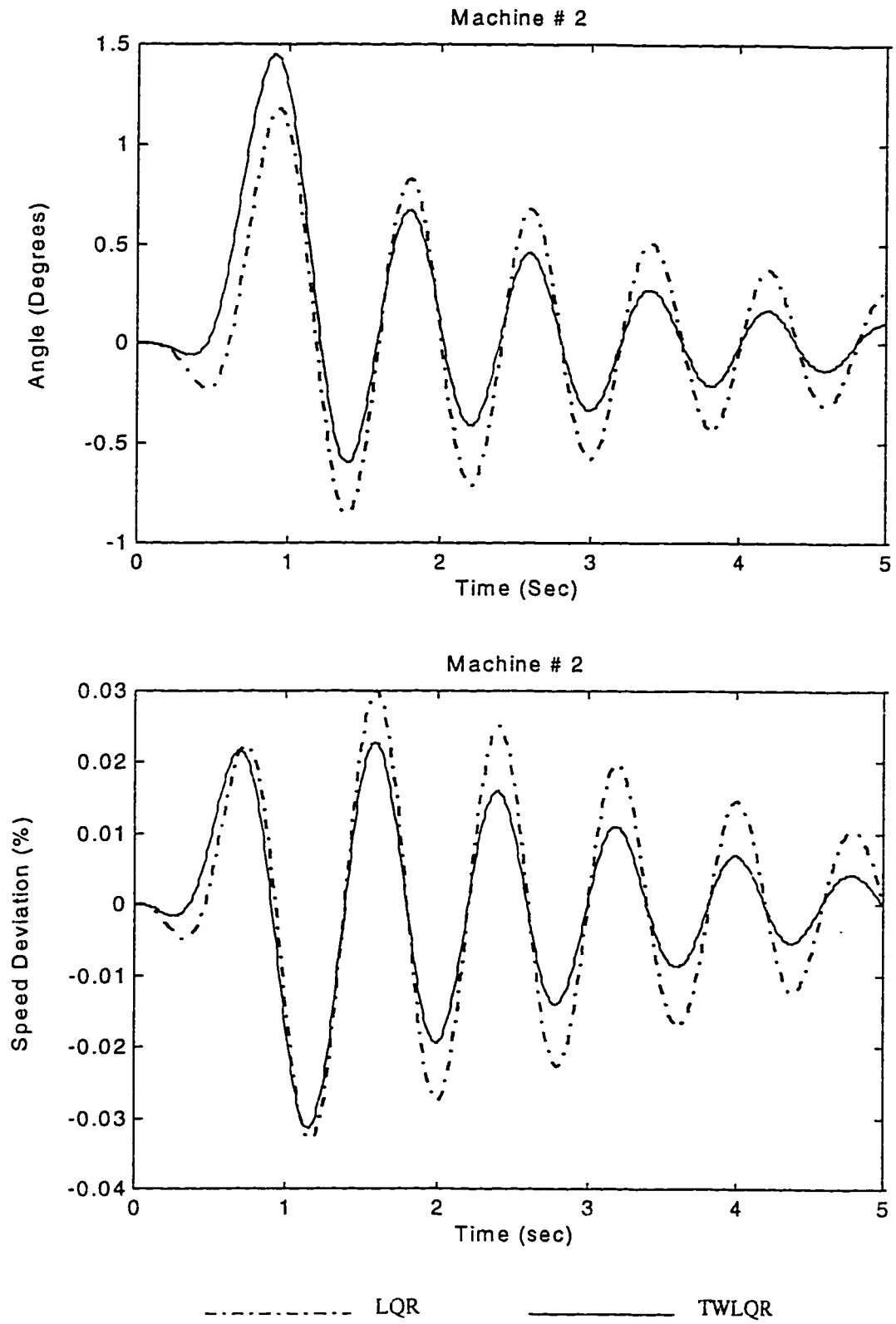


Figure 4. 4 : State Feedback Response of Machine # 2 for Case 1

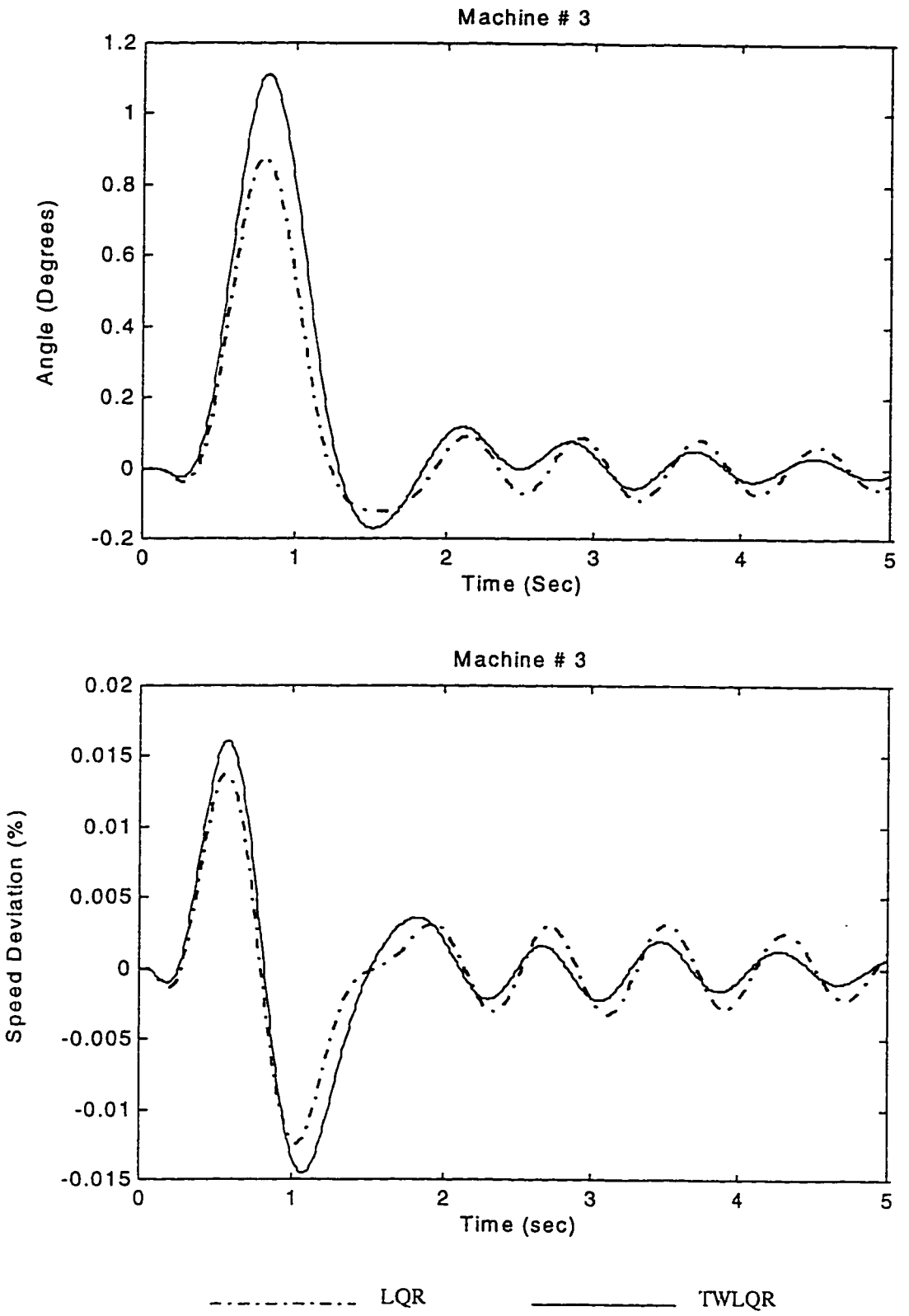
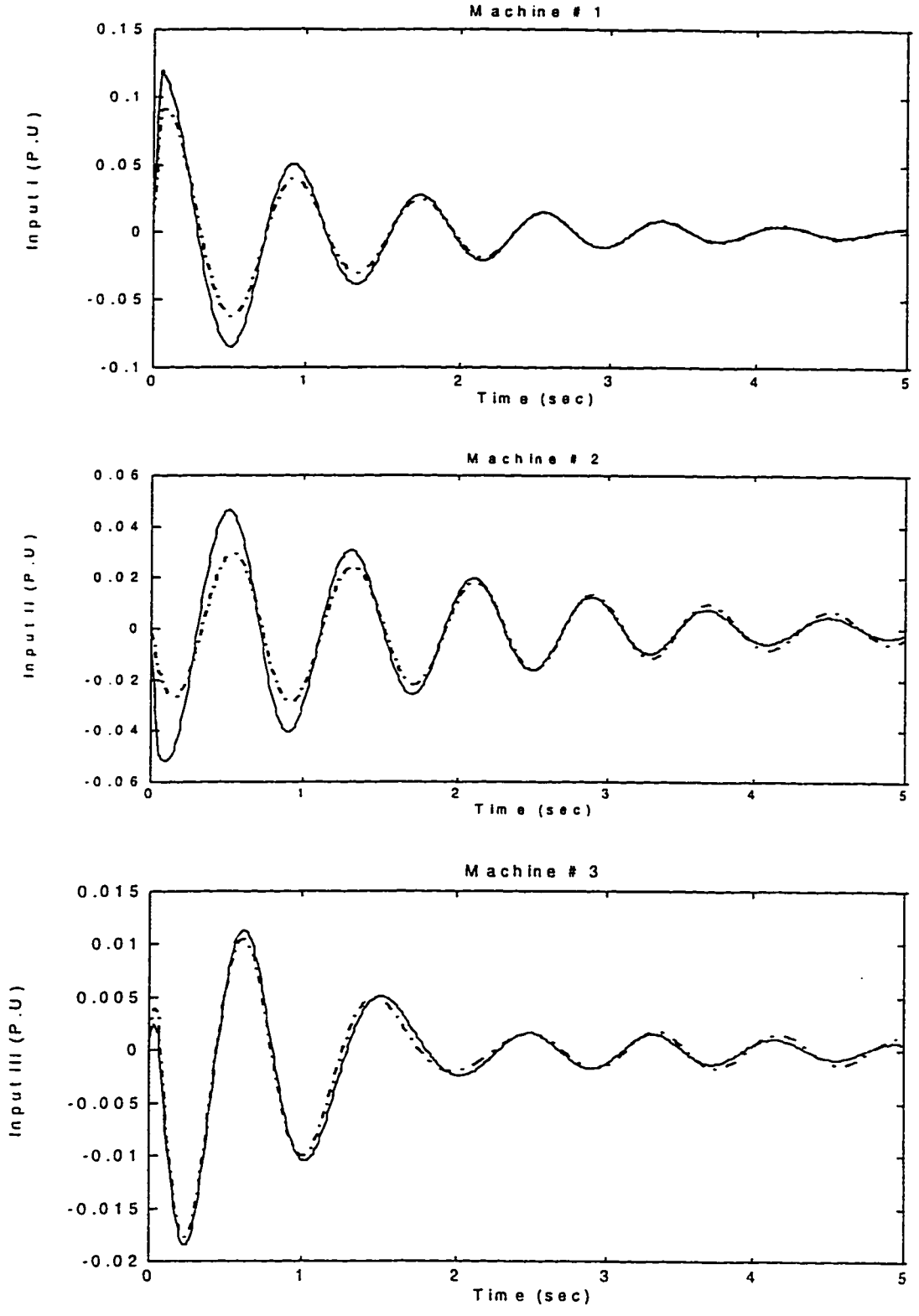


Figure 4. 5 : State Feedback Response of Machine # 3 for Case 1

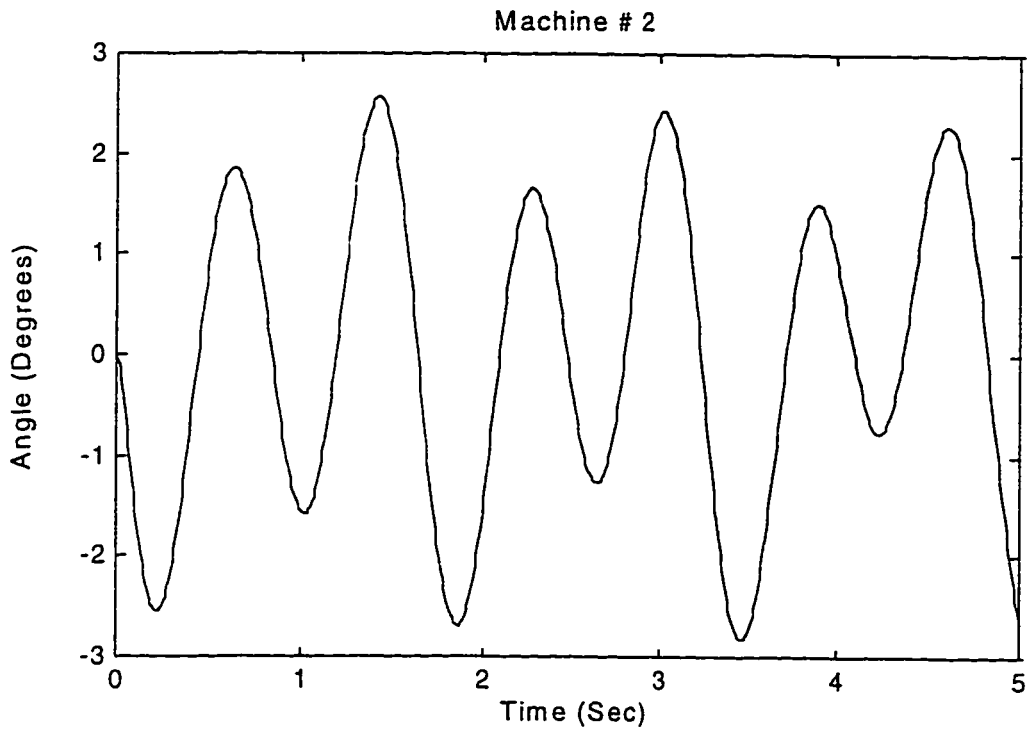


LQR TWLQR

Figure 4. 6 : State Feedback Control Inputs for Case 1

Case 2 : **10 percent mechanical torque pulse disturbance on *machine 2***

The open-loop torque and speed deviation responses for *machine 2* are shown in *fig. 4.7,* following a 10 percent torque pulse disturbance injected at the shaft of this machine for a duration of three cycles. The corresponding closed loop responses for LQR and TWLQR control for *machines 1, 2 and 3* are shown in *figs. 4.8 - 10* respectively, with the respective control inputs shown in *fig. 4.11.*The system dynamic responses of *figs. 4.8 - 11* show the same behavior as in *case 1.*



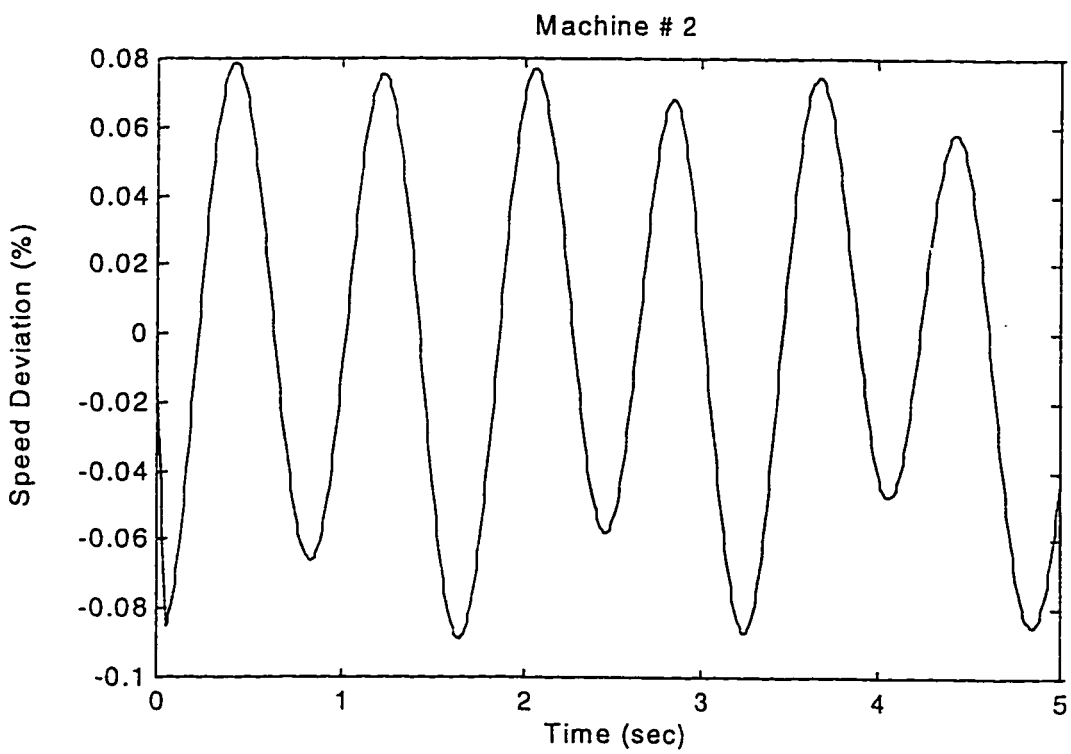


Figure 4. 7 : Open Loop Response of Machine # 2 for Case 2

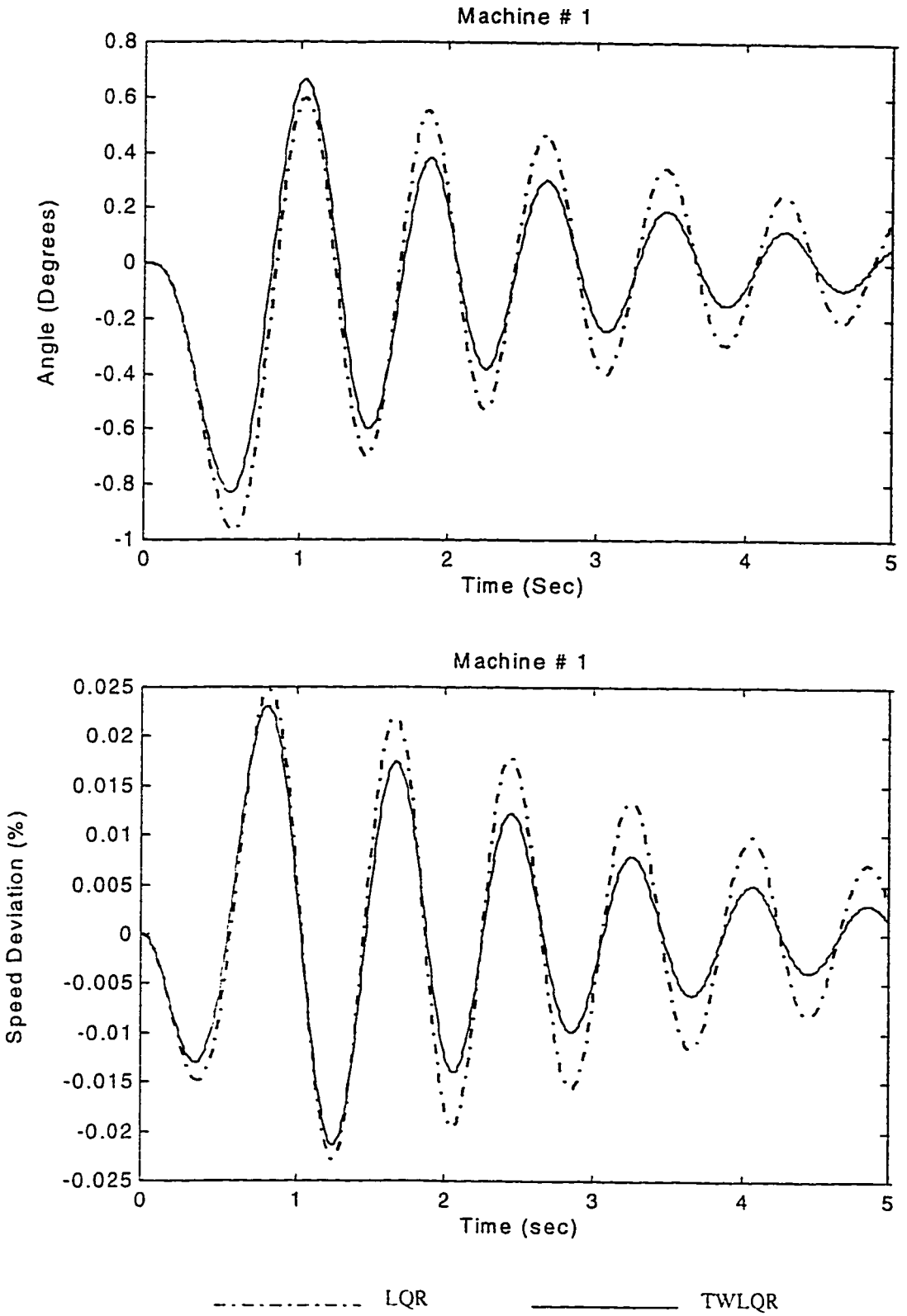


Figure 4. 8 : State Feedback Response of Machine # 1 for Case 2

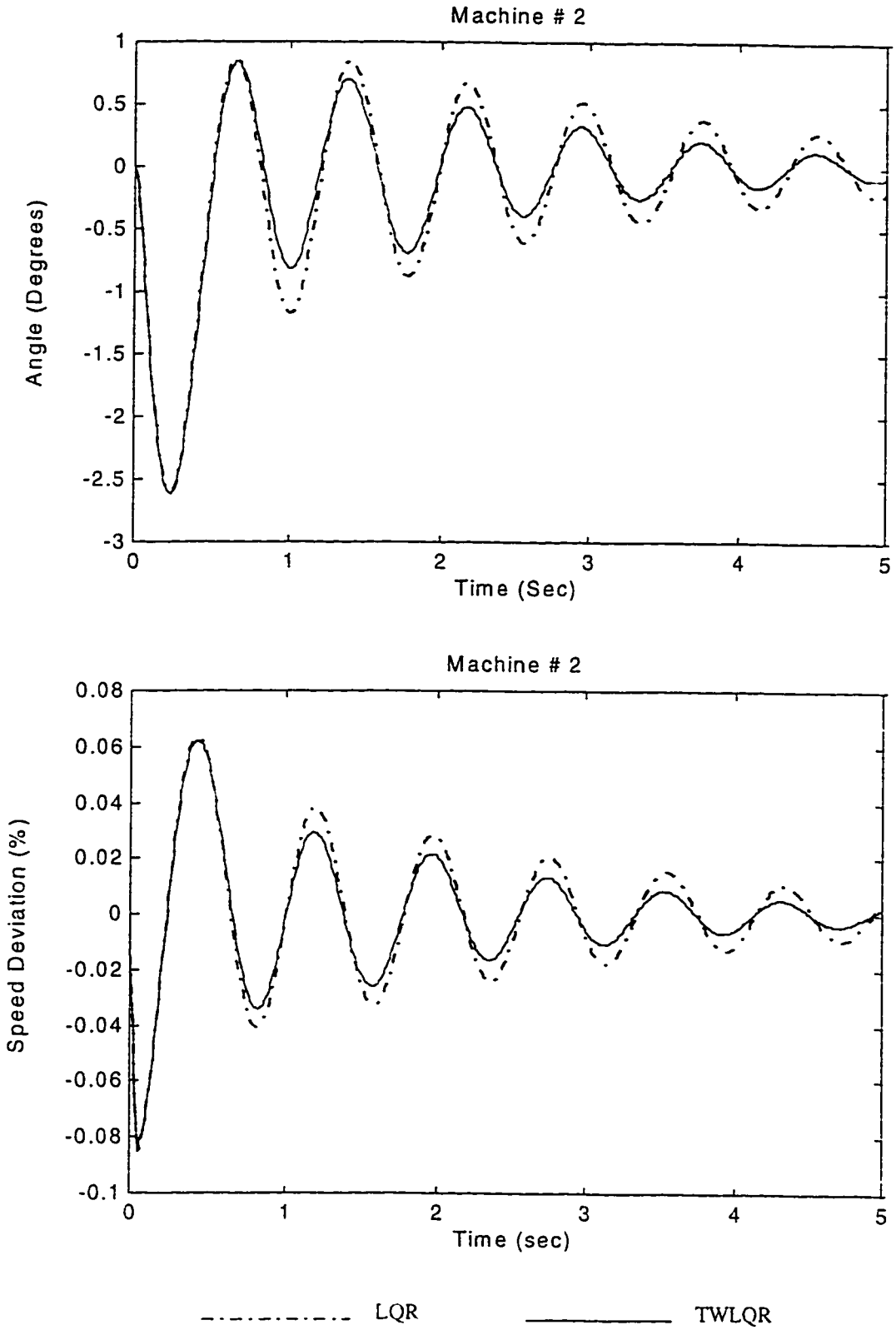


Figure 4. 9 : State Feedback Response of Machine # 2 for Case 2

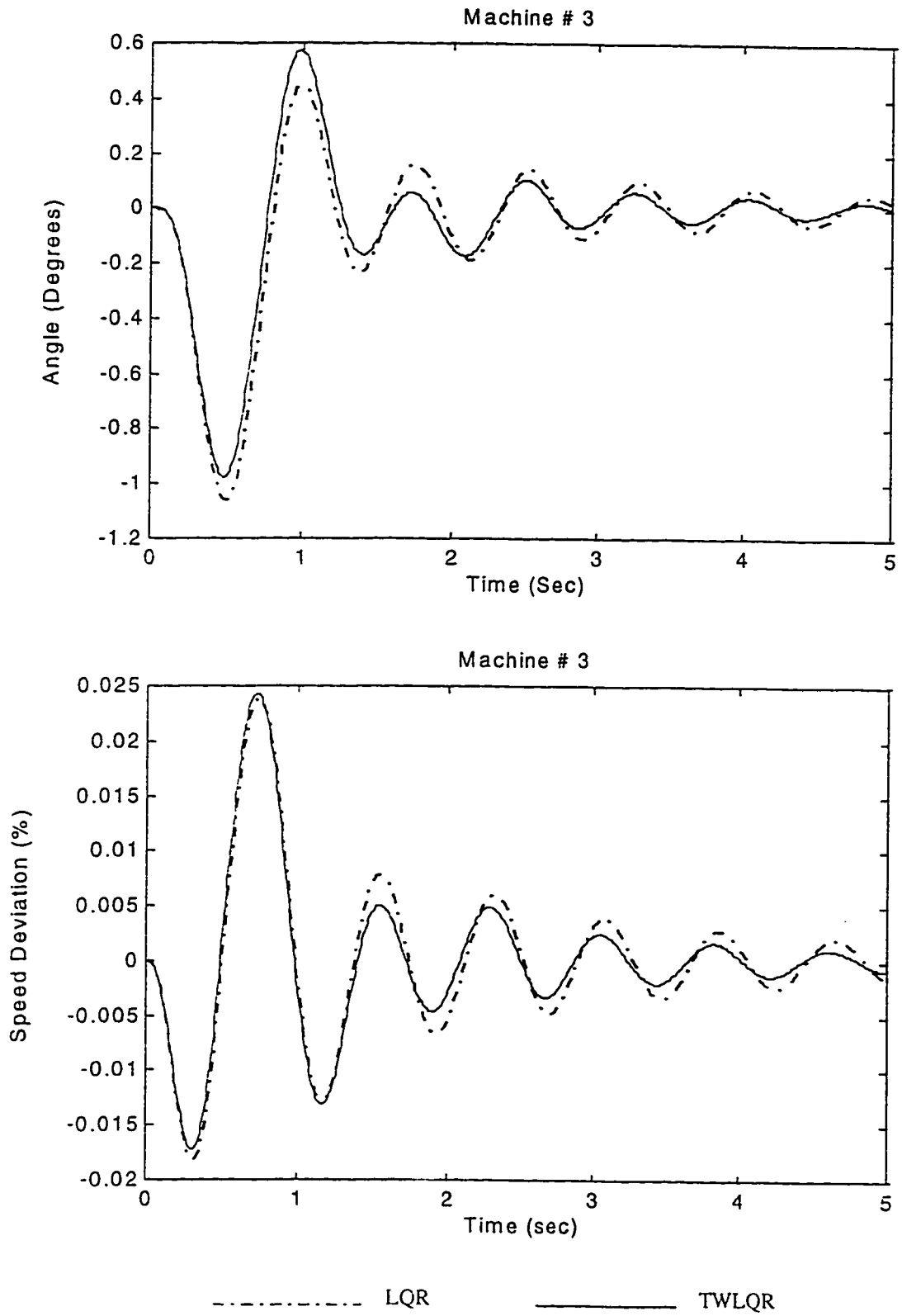


Figure 4. 10 : State Feedback Response of Machine # 3 for Case 2

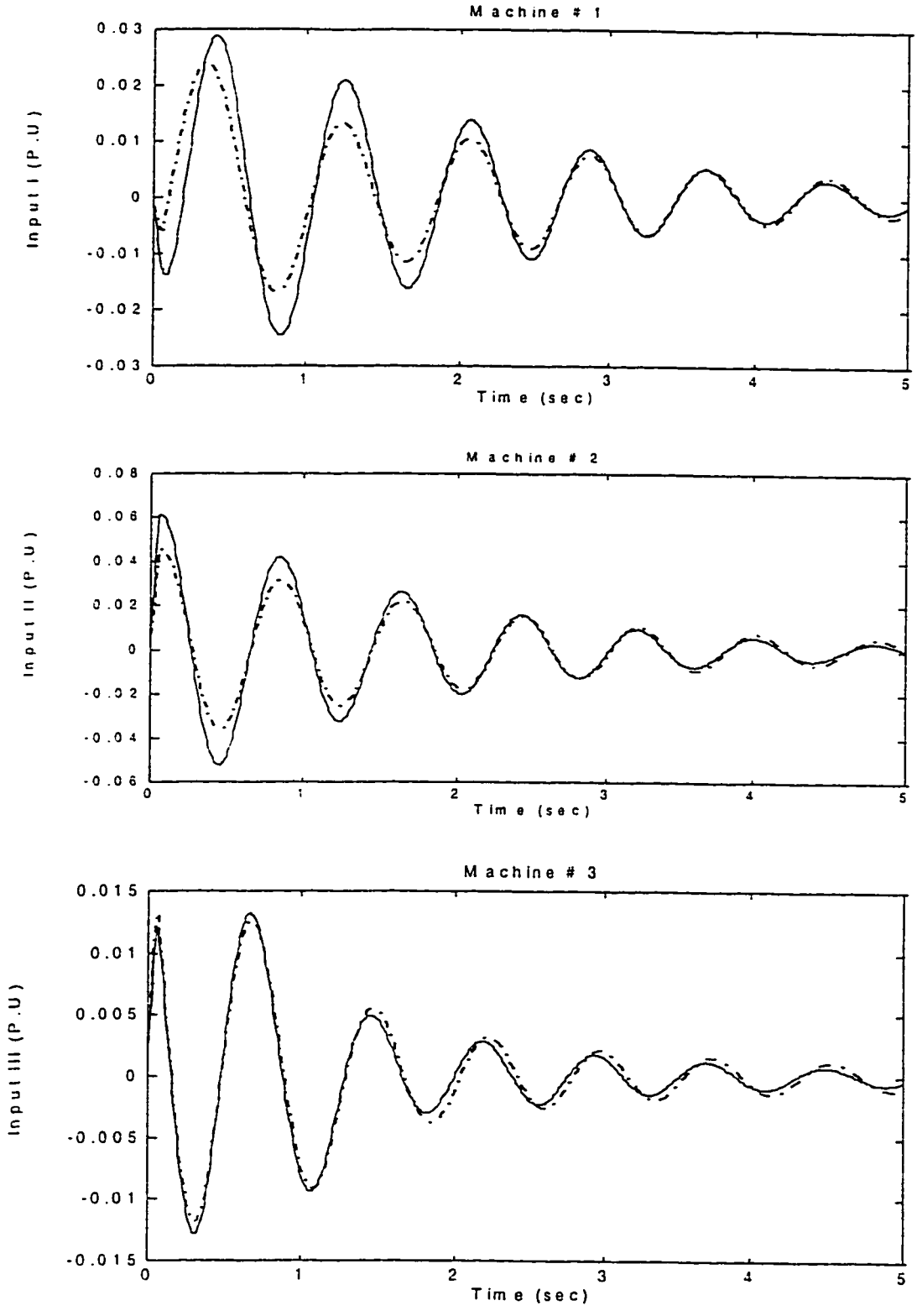


Figure 4. 11 : State Feedback Control Inputs for Case 2

Case 3 : 10 percent pulse disturbance at reference voltage of *machine 1*

Here the 10 percent pulse disturbance is injected at the reference voltage of *machine 1* for three cycles. The open-loop responses of *fig. 4.12,* for *machine 1* show the undamped oscillatory nature. The corresponding system responses for *machines 1, 2 and 3* for LQR and TWLQR feedback controls are shown in *figs. 4.13 - 15* respectively, with *fig. 4.16* showing the respective control inputs of each machine. The results of *figs. 4.13 - 16* show the consistent superior behavior of TWLQR over LQR design as for as system torque angle and speed responses are concerned, however the former offer extra control inputs in the start to damp out the system oscillations following a pulse disturbance.

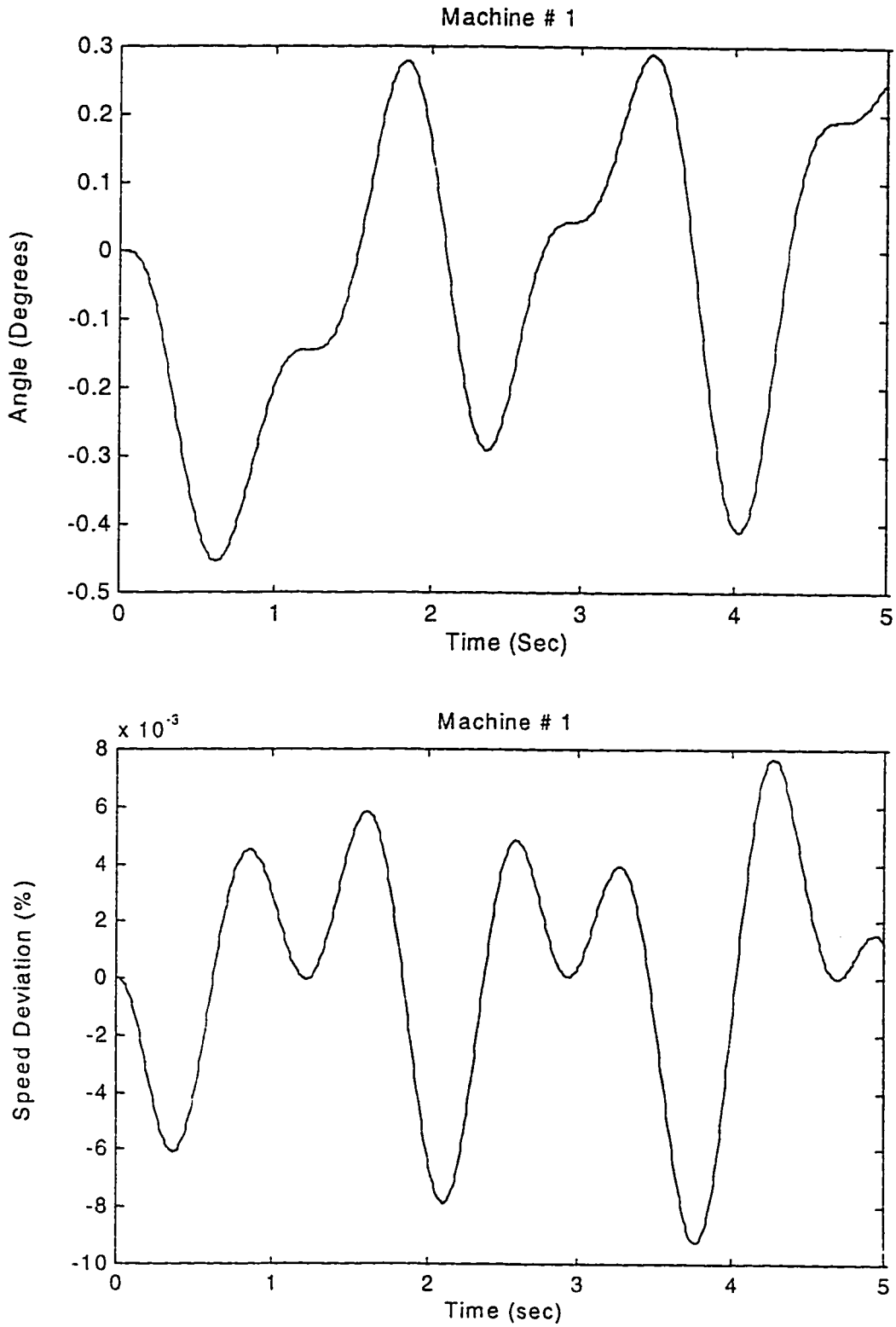


Figure 4. 12 : Open Loop Response of Machine # 1 for Case 3

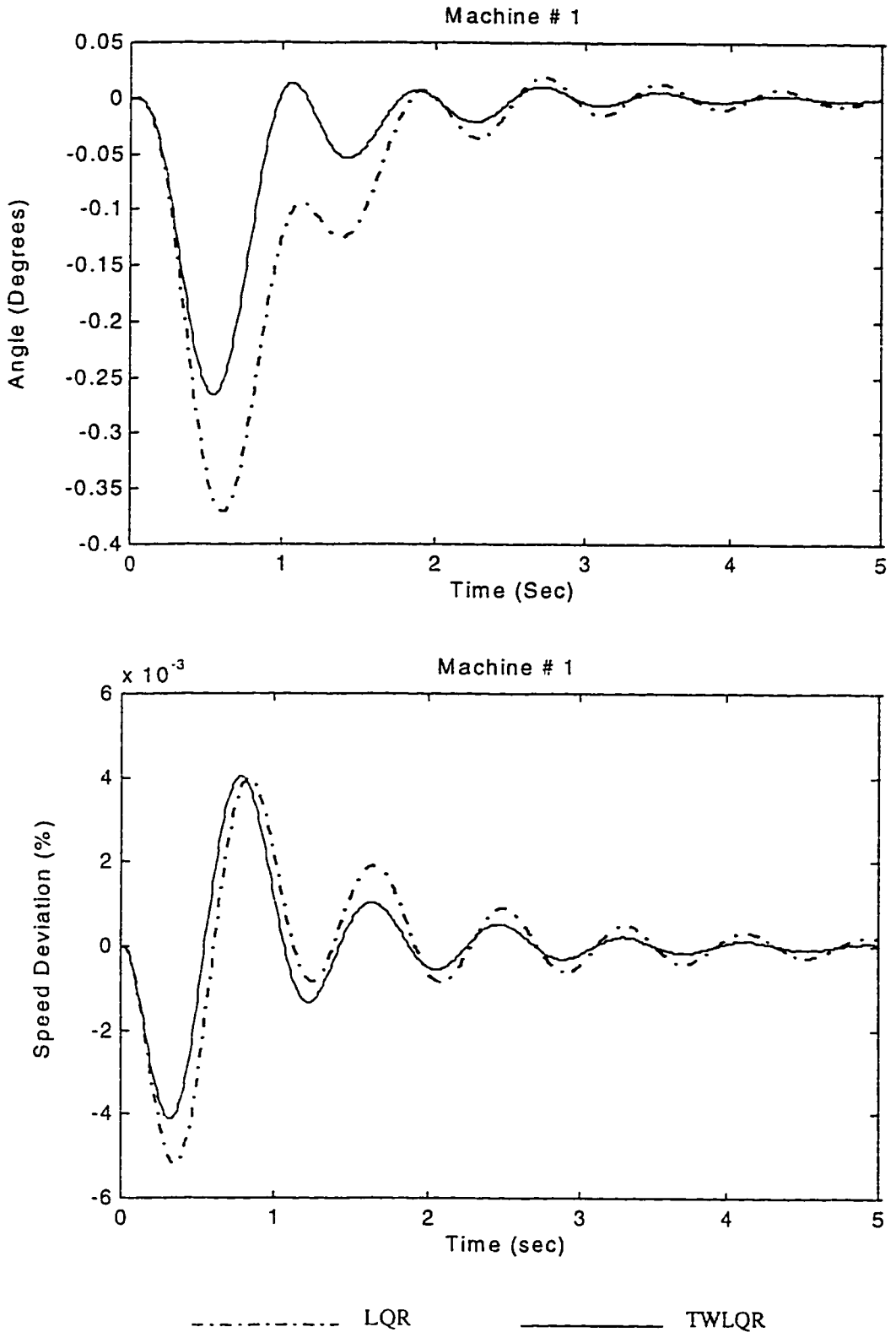


Figure 4. 13 : State Feedback Response of Machine # 2 for Case 3

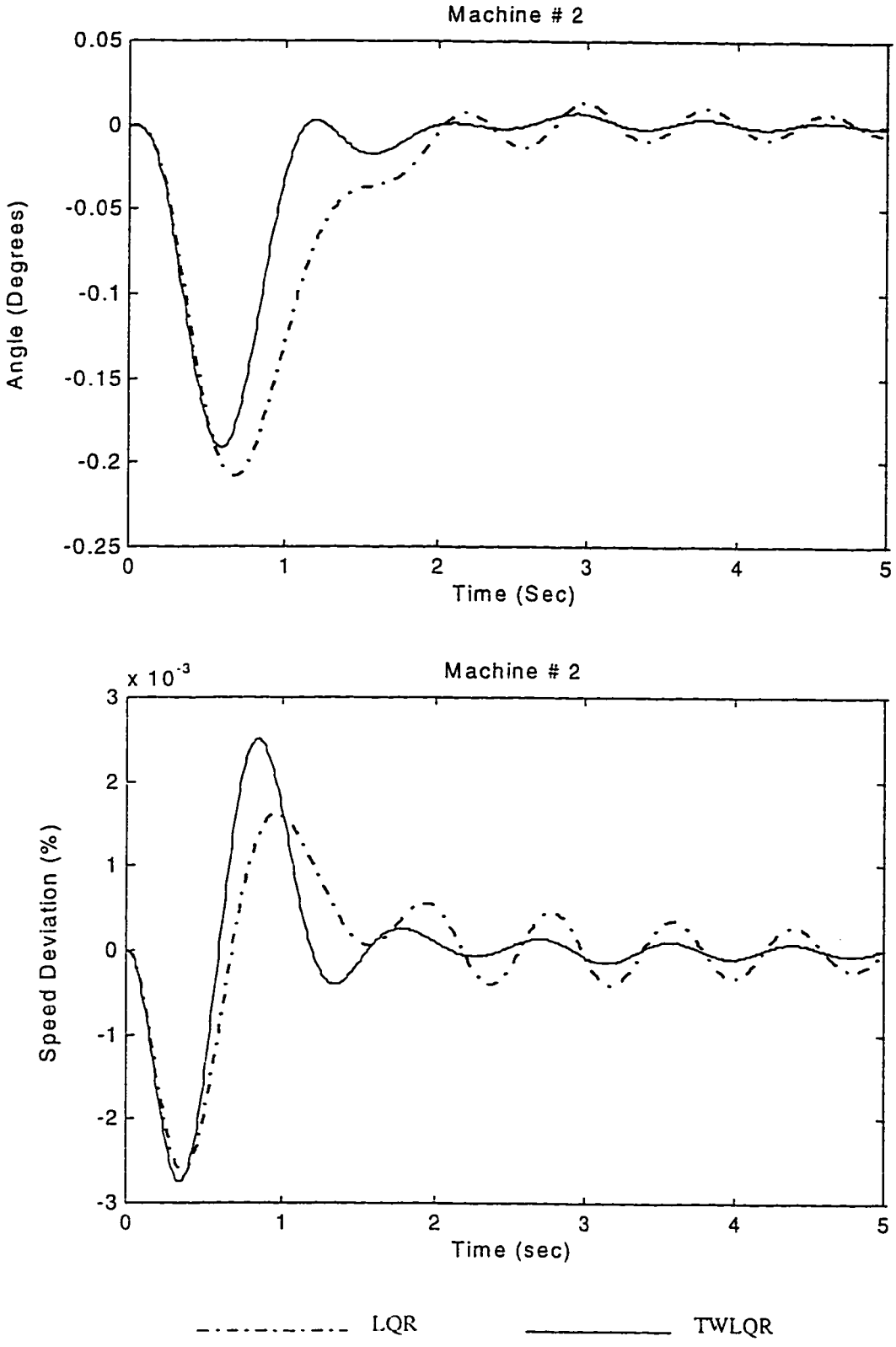
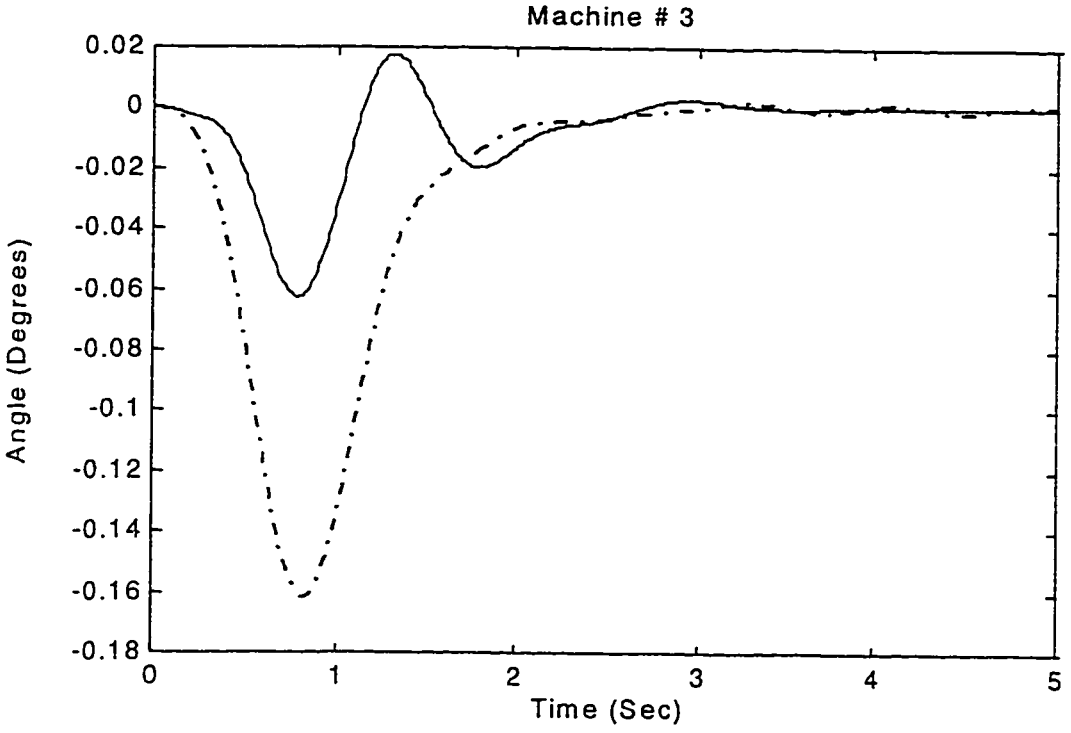
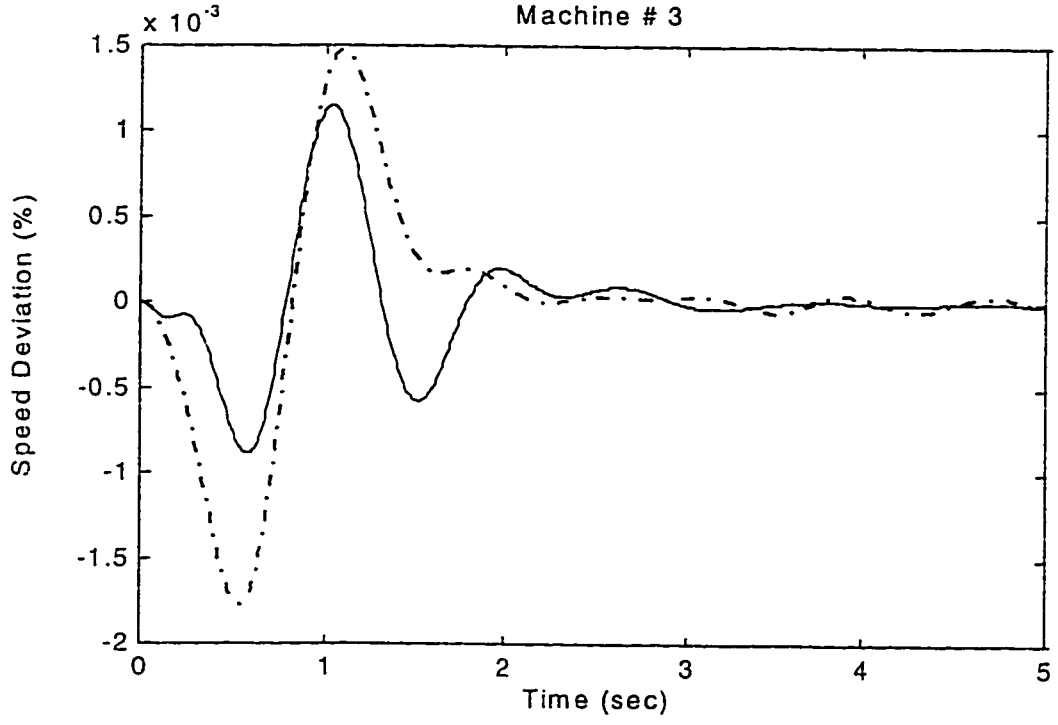


Figure 4. 14 : State Feedback Response of Machine # 2 for Case 3





-·-·-·-·-·-·- LQR ———— TWLQR

Figure 4. 15 : State Feedback Response of Machine # 3 for Case 3

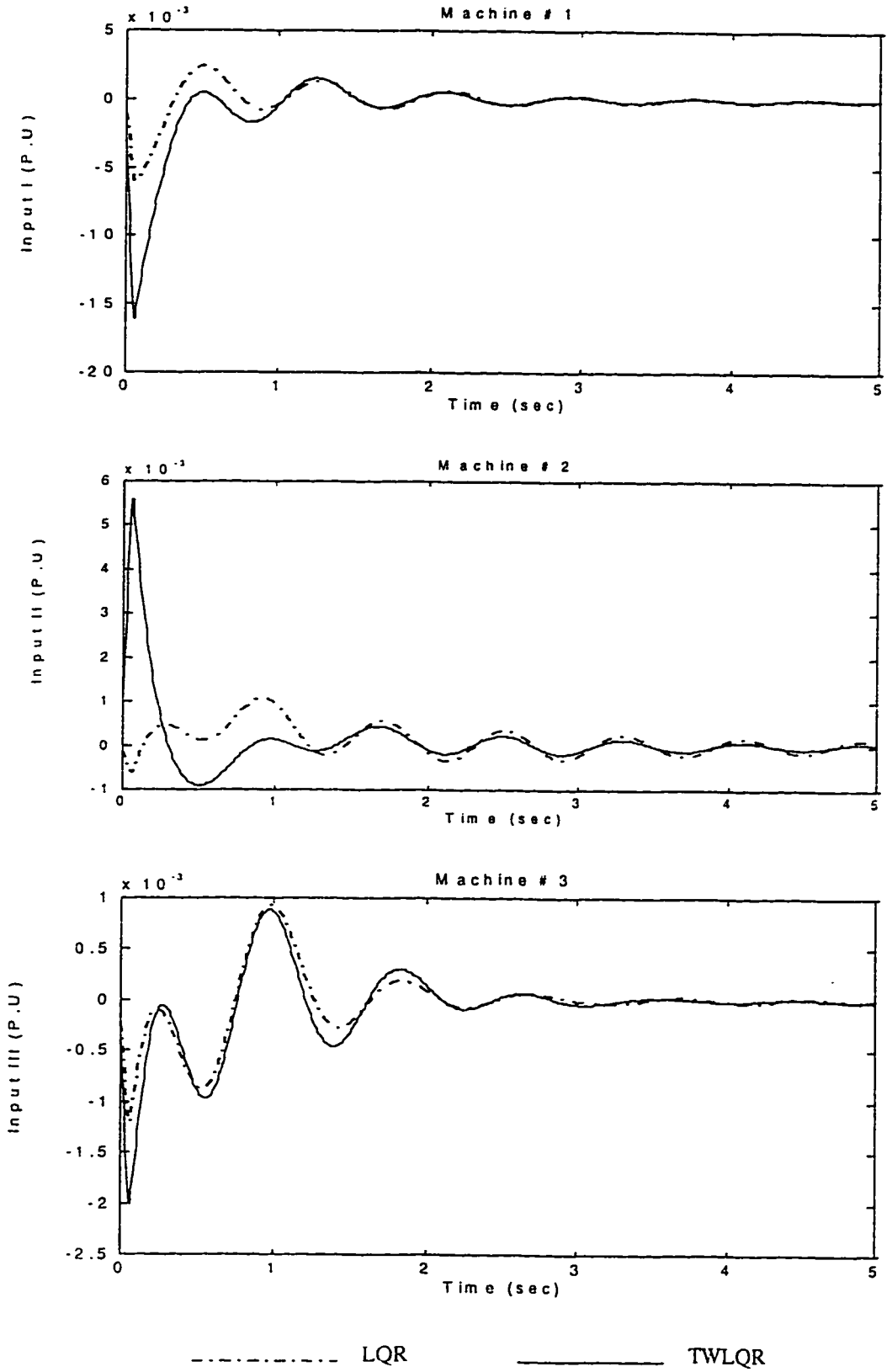
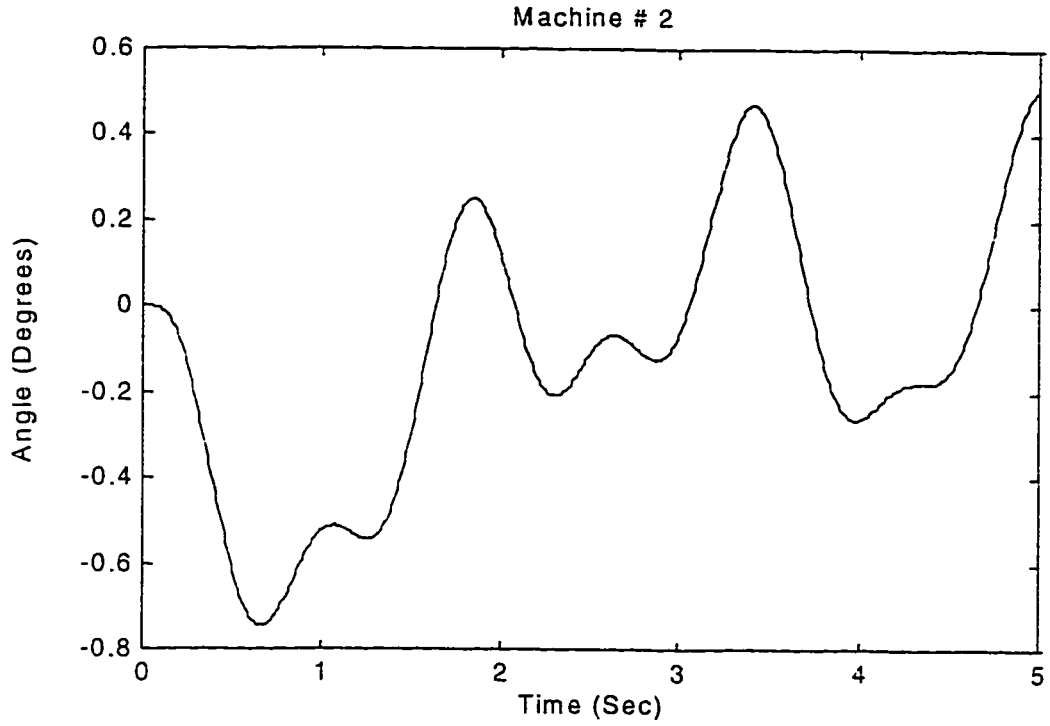


Figure 4. 16 : State Feedback Control Inputs for Case 3

Case 4 :* 10 percent pulse disturbance at reference voltage of *machine 2

In *case 4*, the *machine 2* is subjected to a 10 percent pulse disturbance at its reference voltage for three cycles with corresponding open-loop responses of *fig. 4.17*, for *machine 2* show the undamped oscillatory nature. The corresponding system responses for *machines 1, 2 and 3* for LQR and TWLQR feedback controls are shown in *figs. 4.18 - 20* respectively, wheras the control inputs of each machines are shown in *fig. 4.21*. The results of *figs. 4.18 - 20* show the consistency in superior behavior of TWLQR control to LQR.



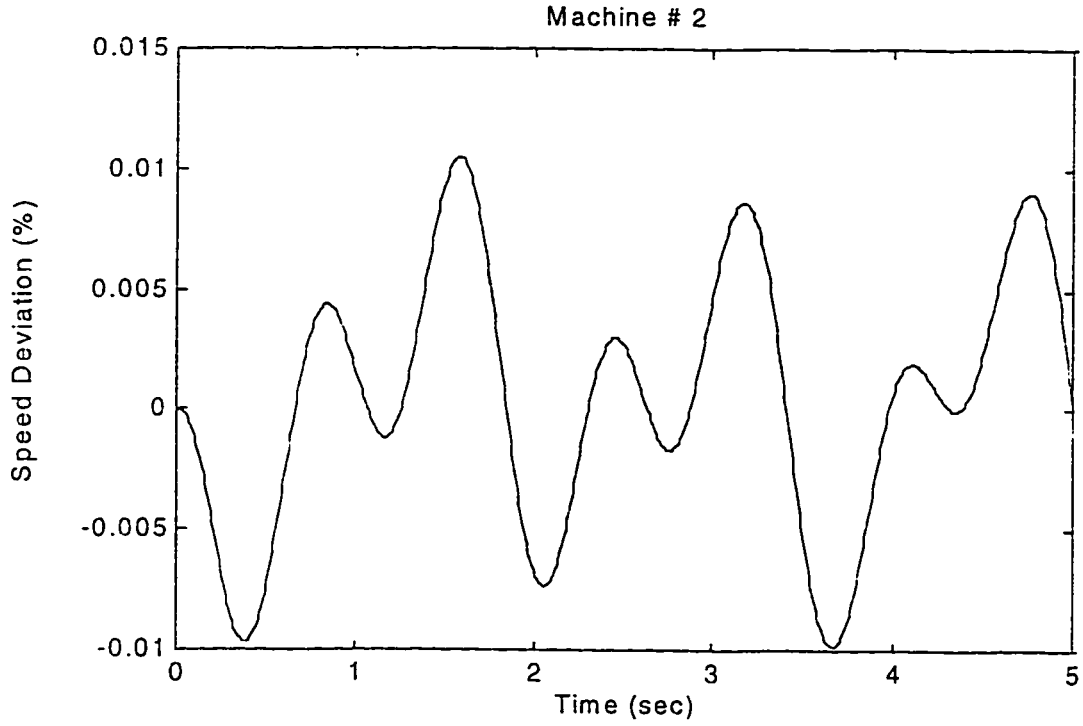


Figure 4. 17 : Open Loop Response of Machine # 2 for Case 4

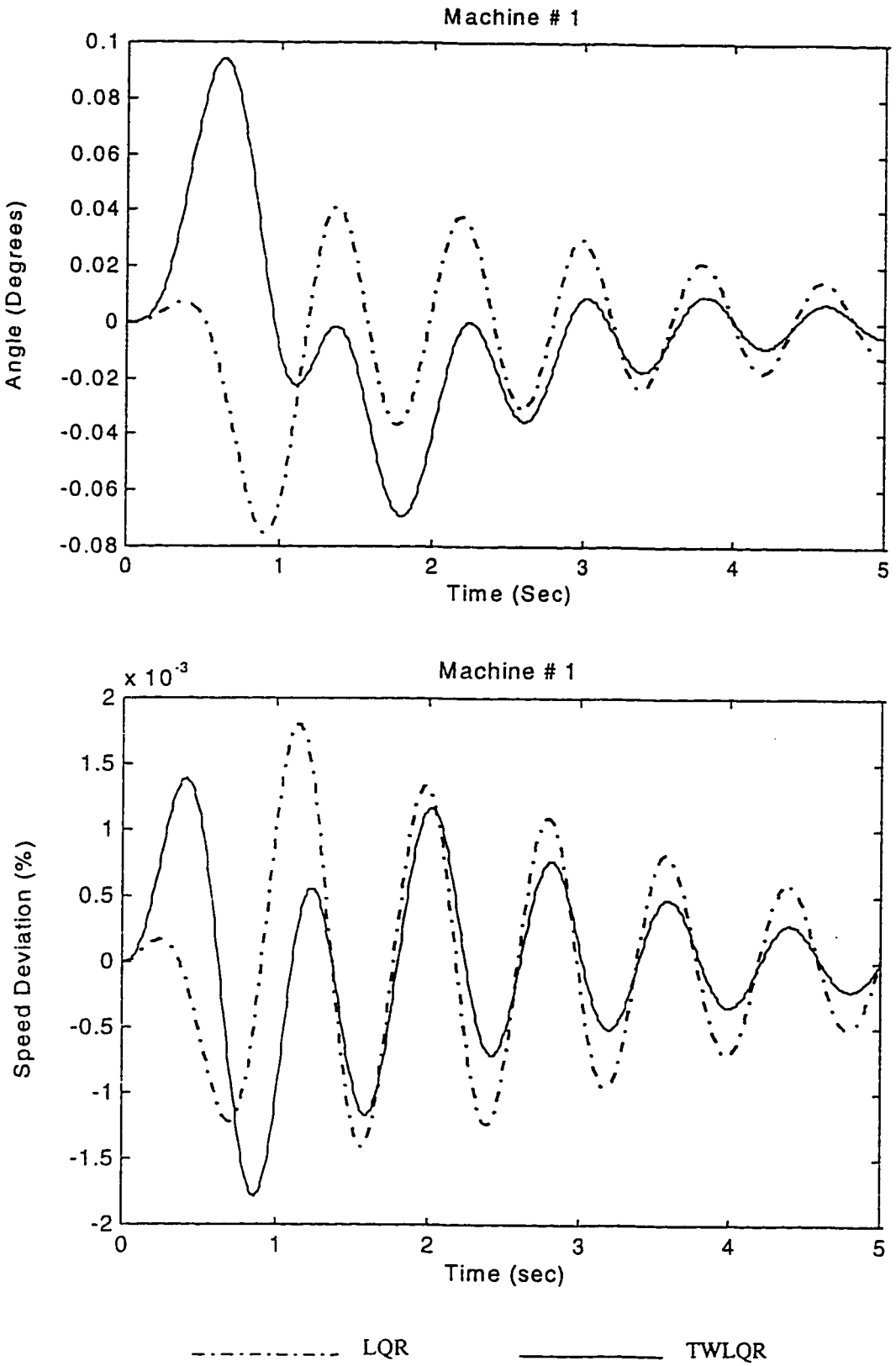


Figure 4. 18 : State Feedback Response of Machine # 1 for Case 4

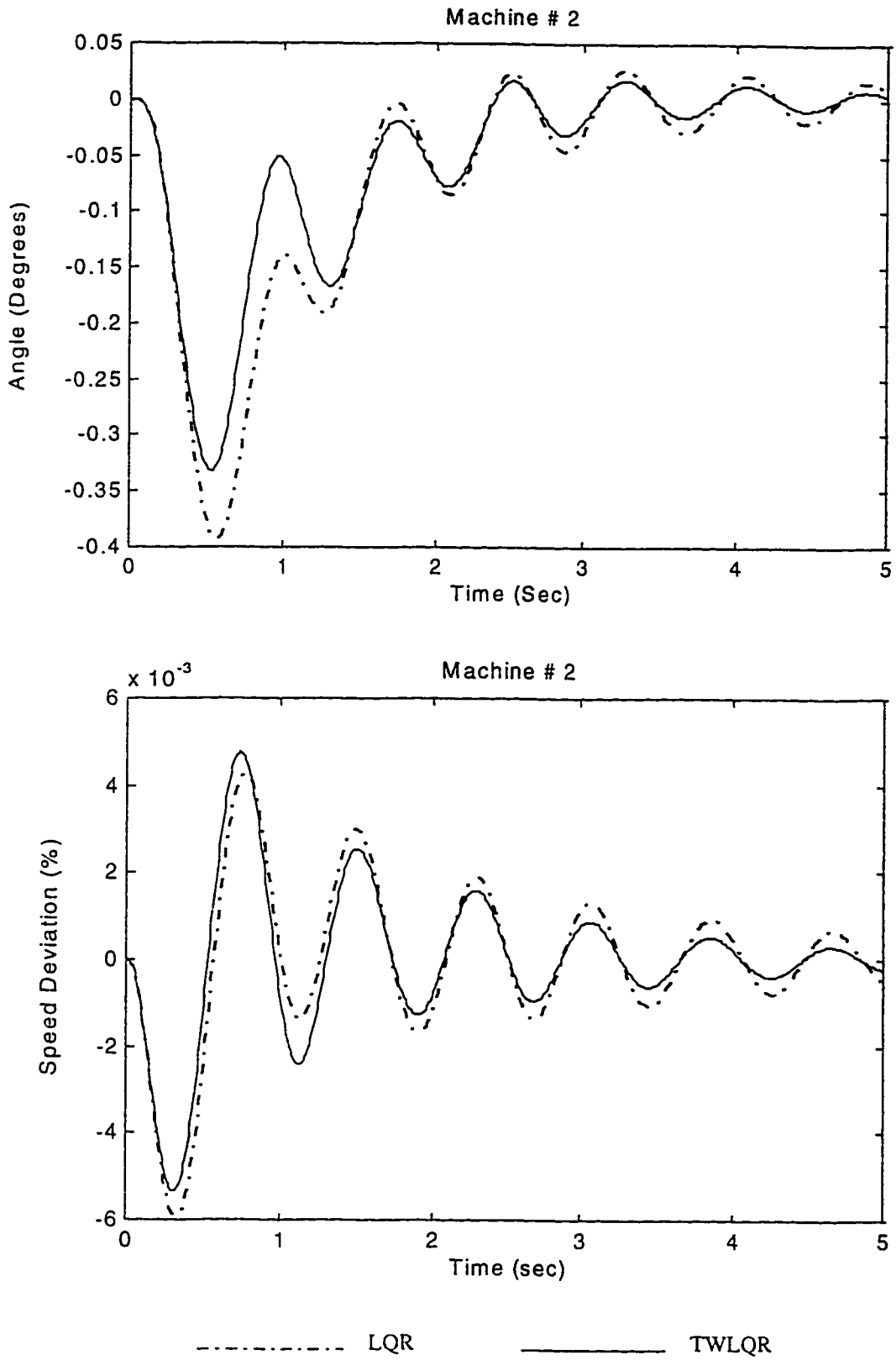


Figure 4. 19 : State Feedback Response of Machine # 2 for Case 4

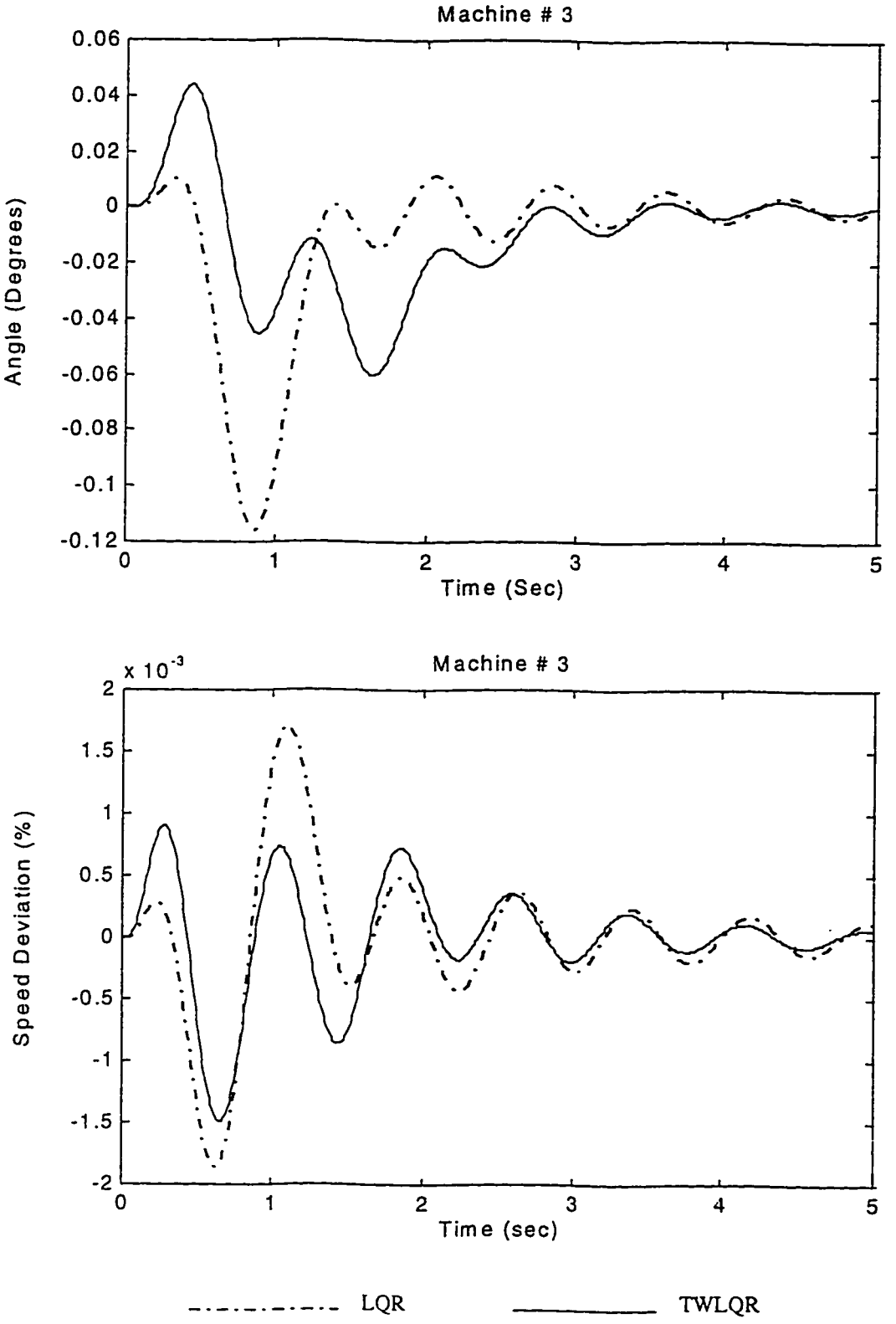
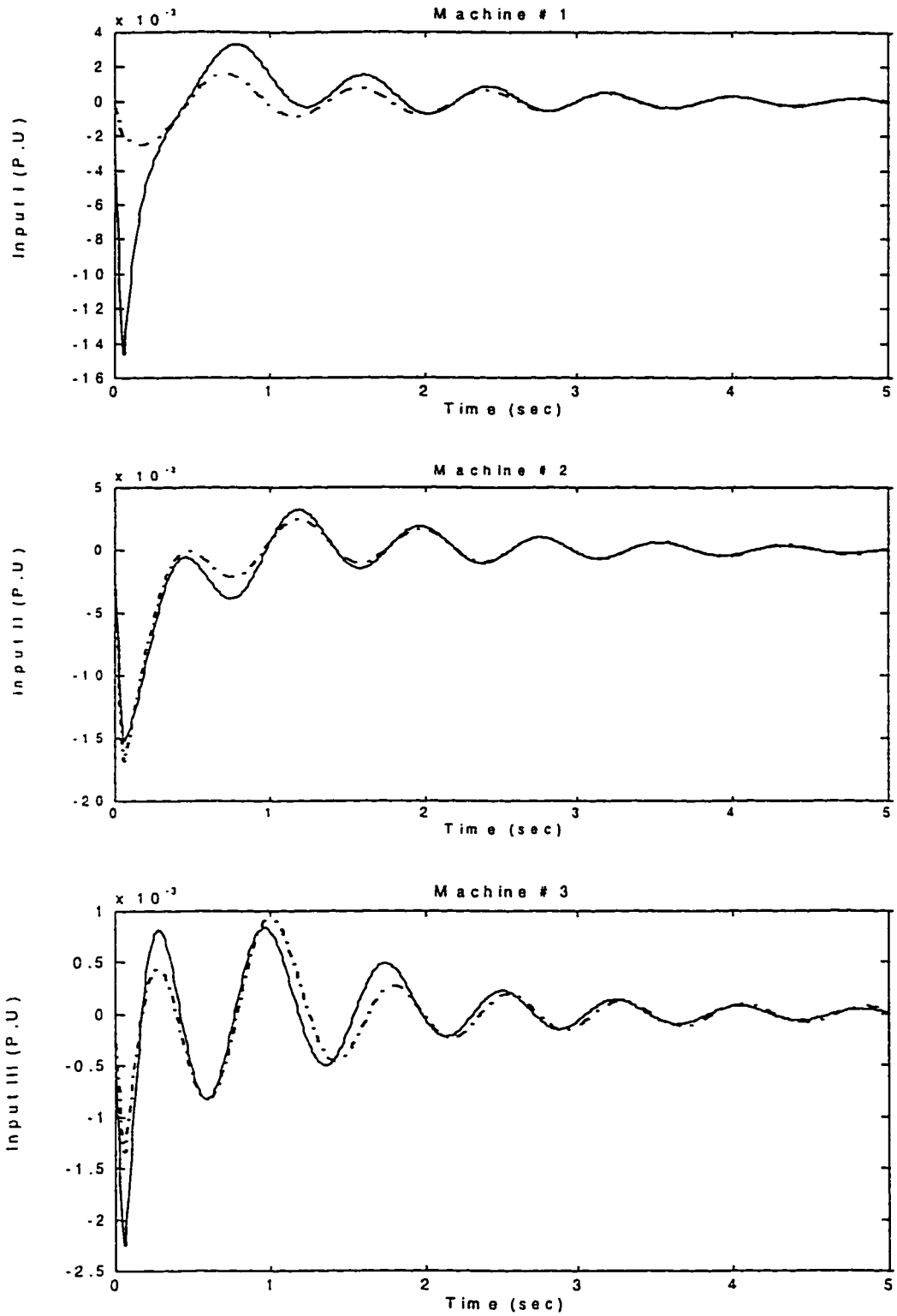


Figure 4. 20 : State Feedback Response of Machine # 3 for Case 4



-.-.-.-.-.- LQR ———— TWLQR

Figure 4. 21 : State Feedback Control Inputs for Case 4

4.3.1 Discussions of Results for State Feedback

While comparing the closed loop responses of *case 1* for LQR and TWLQR, both torque angle deviation and speed deviation decay faster for the later as compared to the former control, moreover in most of the cases the initial overshoot is also lesser in case of TWLQR control and it acquires the zero steady state before LQR control.

While studying the results of other three cases, almost same trends as found in *case 1*, have been observed regarding the improvement in dynamic responses of the electromechanical modes of each of the machines by using TWLQR control. The control inputs response for both LQR and TWLQR show that these improved dynamic behavior is on the expense of exerting extra control effort by the TWLQR control, but it seems quite natural, and moreover in most of the cases, the TWLQR control is not much deviated from the LQR control, and as time proceeds its decaying rate increases.

4.4 Output Feedback Case

As already mentioned, for Single Machine Infinite Bus Case, that it is not always possible to feedback all the state of the system, so as a practical point of view, here the angular speed, commonly used as output variable, is taken for output feedback design.

4.4.1 Optimal Selection of States and Control Weighting Matrices

To make the comparison more realistic, the Q and R matrices used here are the same, as given in [6]. The Q matrix is given by the listed values times 10^{-3} :

$$Q = diag\left[Q_{11} \quad Q_{22} \quad Q_{33}\right]$$

where

$$Q_{11} = 97.2, \qquad Q_{22} = 93.6\,, \quad \text{and} \quad Q_{33} = 88.2$$

The weighting matrix elements of the three plant control is given below

$$R = \text{diag}\ [\ 1 \quad 2 \quad 10\ ]$$

4.4.2 Computation of Output Feedback Gains

Now applying the Algorithm of chapter *two* for the computation of both LQR and TWLQR Output Feedback Gains. The respective gain matrices are given in *Tables 4.10.& 4.11*. The corresponding closed loop eigen-values for LQR and TWLQR controls are given in *Table 4.12*.

Table 4. 10 : LQR Output Feedback Gains

Control Input	$\Delta\omega_1$	$\Delta\omega_2$	$\Delta\omega_3$
Machine # 1	-0.2027	-0.1136	0.1434
Machine # 2	-0.0686	-0.1570	-0.0050
Machine # 3	-0.0086	-0.0103	0.0273

Table 4. 11 : TWLQR Output Feedback Gains

Control Input	$\Delta\omega_1$	$\Delta\omega_2$	$\Delta\omega_3$
Machine # 1	-0.3650	-0.0300	0.1406
Machine # 2	0.0872	-0.2465	0.0343
Machine # 3	0.0029	-0.0222	0.0472

Table 4. 12 : LQR and TWLQR Output Feedback Closed loop Eigenvalues

LQR	TWLQR
- 0.1626 ± j 7.8062	- 0.4242 ± j 7.9381
- 0.3821 ± j 7.4533	- 0.4709 ± j 7.2440
- 0.4181 ± j 3.9519	- 0.4776 ± j 1.7561
- 0.4776 ± j 1.9705	- 0.5308 ± j 4.1688
- 1.3725 ± j 1.4579	- 1.0111 ± j 2.0296
- 3.2962, -17.0110	- 2.8715, -17.2325

4.5 Simulation Results for Output Feedback

Similar to the State Feedback case, the dynamic stability analysis of the multi-machine power system model case for LQR and TWLQR output feedback gains, the simulations have been performed for every practical combinations, while the system is subjected to a 10 percent pulse disturbance either in mechanical torque or in reference voltage for a duration of three cycles of system frequency, to study every aspect of proposed design. Again due to the space limitations, the following three cases have been chosen to show the variety in responses :

Case 1 : **10 percent mechanical torque pulse disturbance on *machine 1 & 2***

A 10-percent torque pulse is applied at the shafts of *machine 1 and* 2 simultaneously for a duration of three cycles. The LQR and TWLQR closed loop responses for *machines 1, 2 and 3* are shown in *figs. 4.22 - 24* respectively, while the respective control inputs exerted by the excitation control of each machine are shown in *fig. 4.25*.

From *figs. 4.22 -25*, it can be observed that although the TWLQR control response is identical to LQR control response in the start but as the time proceeds the oscillations in TWLQR response decays faster than LQR counterpart.

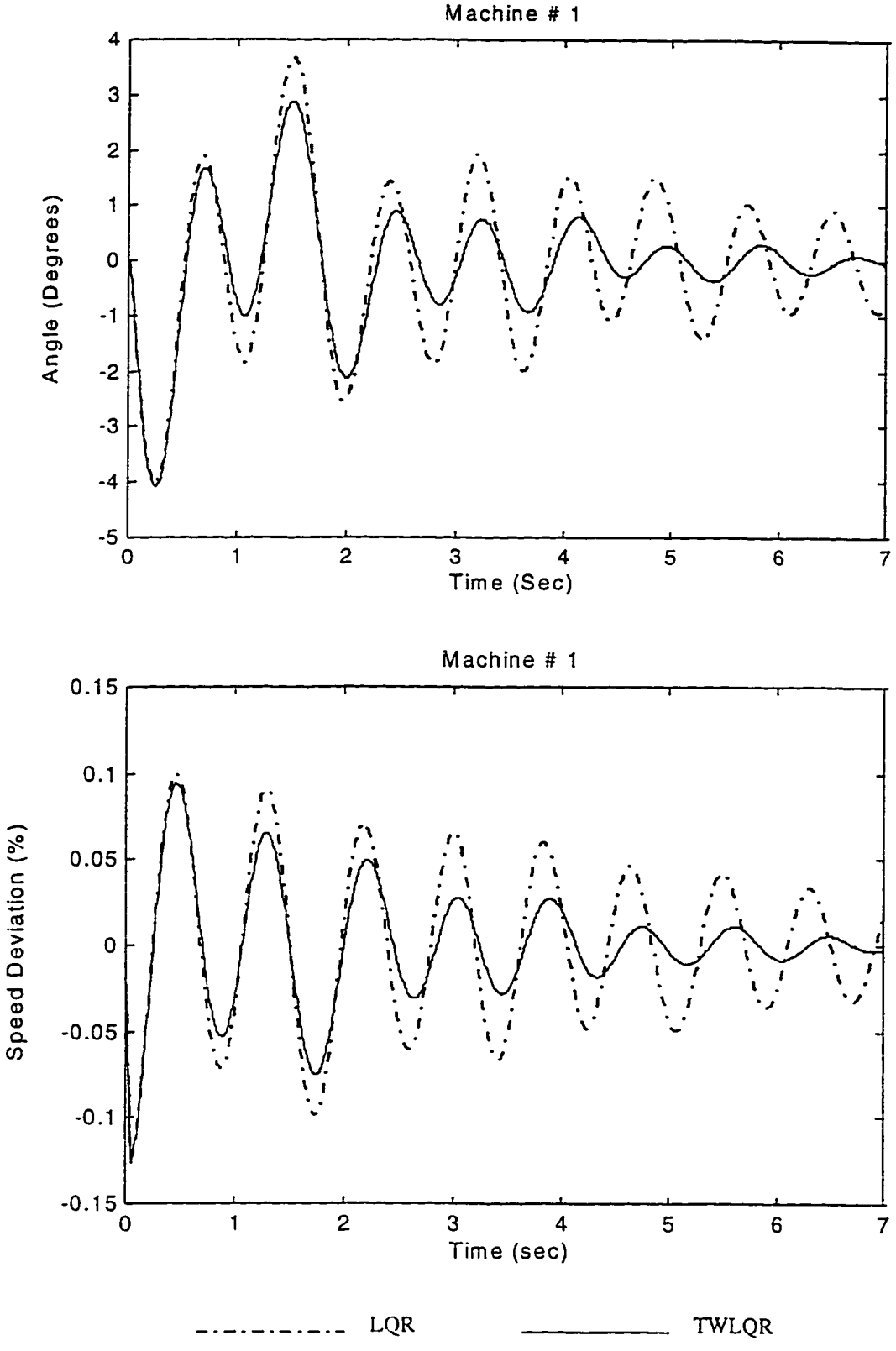


Figure 4. 22 : Output Feedback Response of Machine # 1 for Case 1

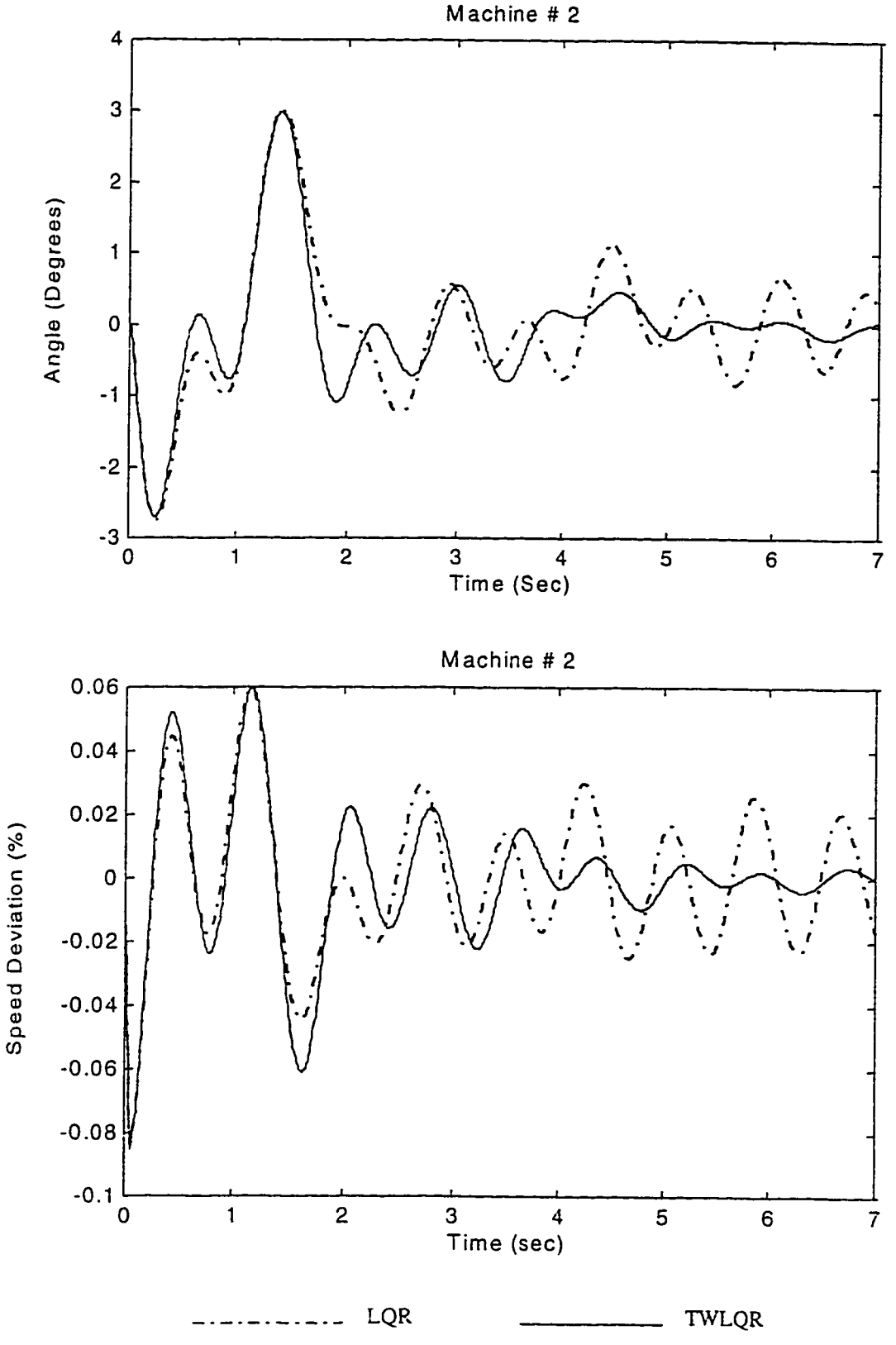


Figure 4. 23 : Output Feedback Response of Machine # 2 for Case 1

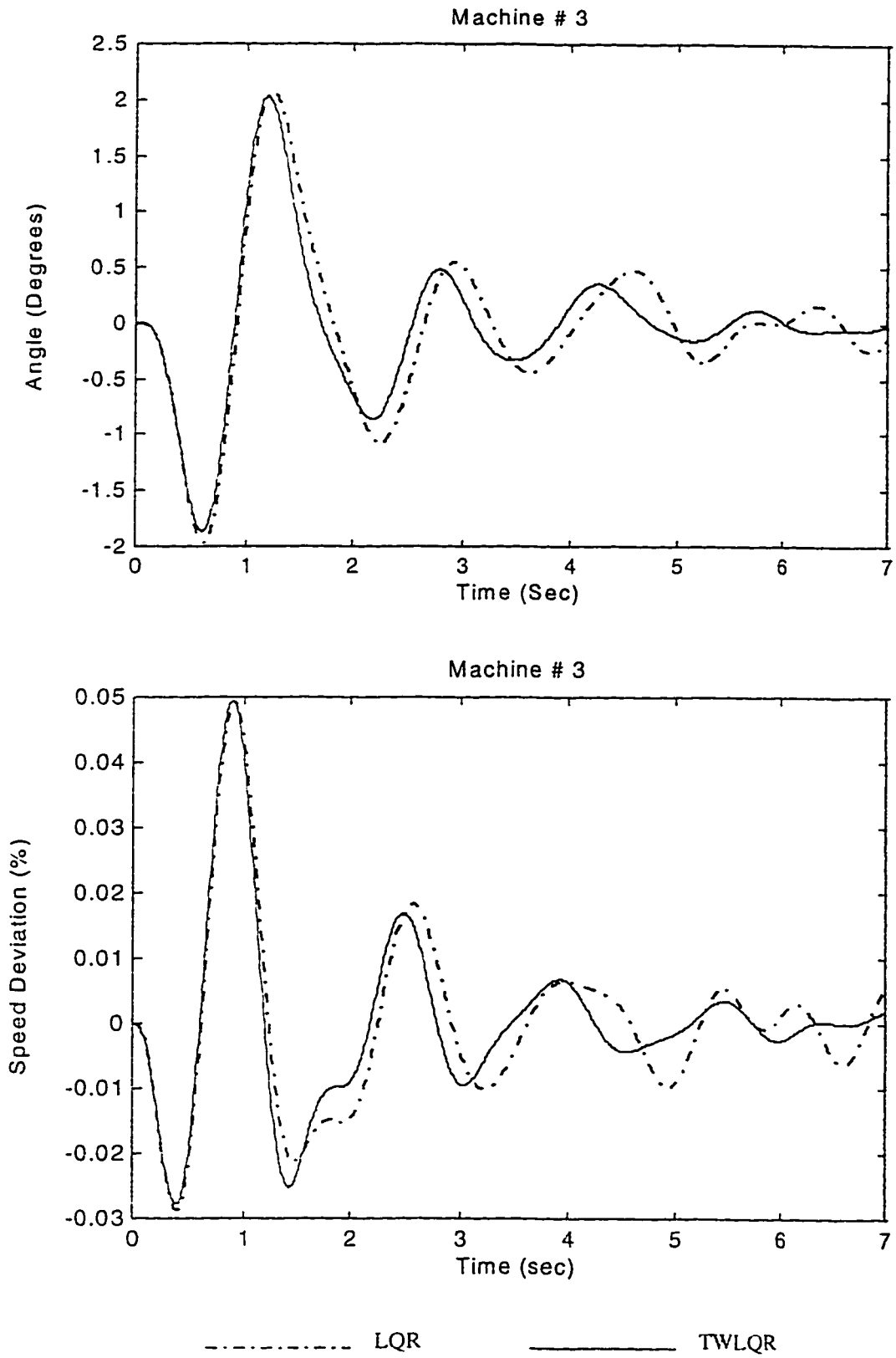


Figure 4. 24 : Output Feedback Response of Machine # 3 for Case 1

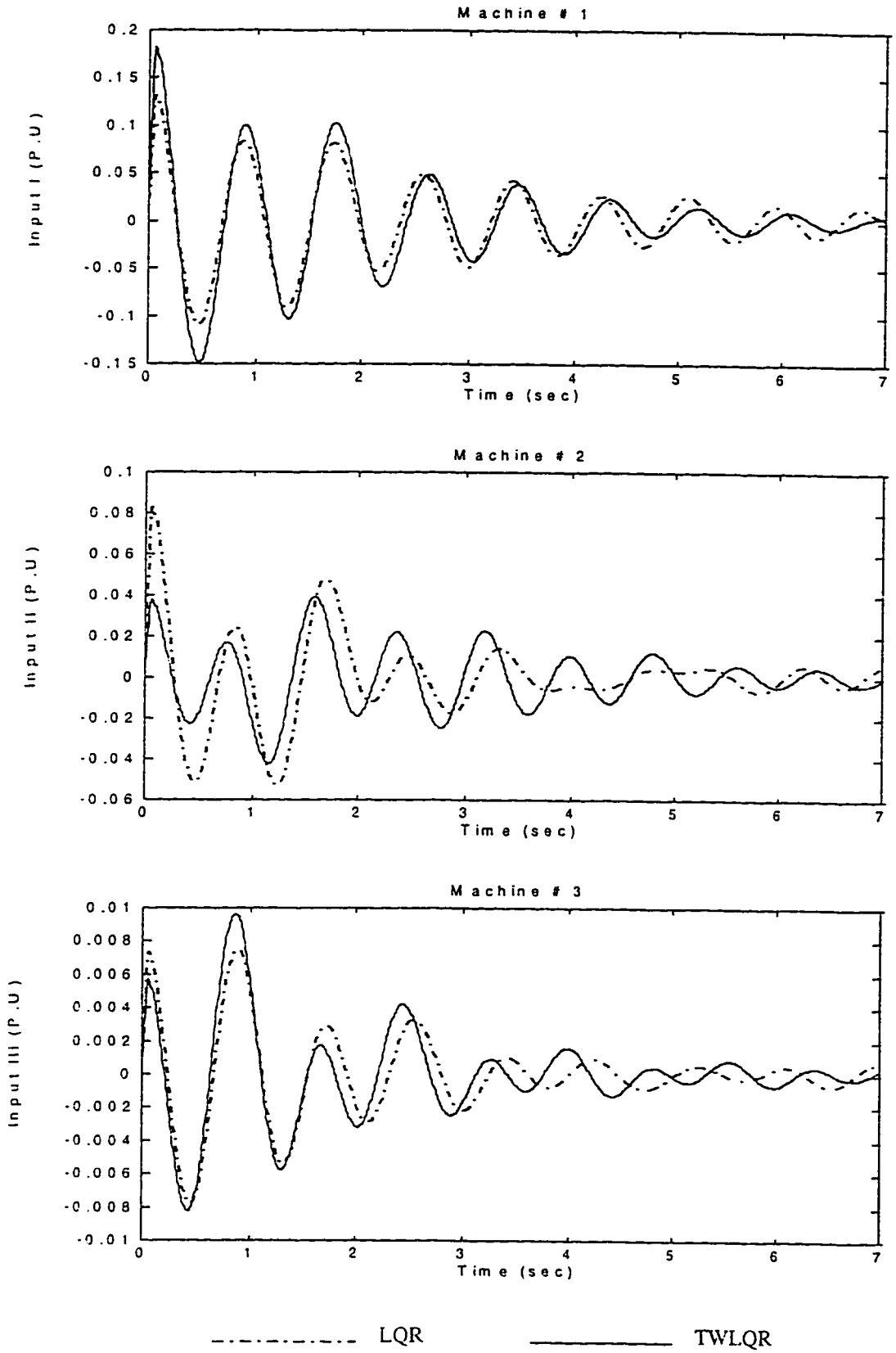


Figure 4. 25 : Output Feedback Control Inputs for Case 1

Case 2 : 10 percent mechanical torque pulse disturbance on *machine 3*

The open-loop torque and speed deviation responses for *machine 3* are shown in *fig. 4.26,* following a 10 percent torque pulse disturbance injected at the shaft of this machine for a duration of three cycles, as clear from the figure both angle and speed deviations are increasing with time due to the unstable openloop nature of this machine. The corresponding closed loop responses for LQR and TWLQR control for *machines 1, 2 and 3* are shown in *figs. 4.27 - 29* respectively, with the respective control inputs shown in *fig. 4.30.* These system dynamic responses of *figs. 4.27 - 30* show the same behavior as in *case 1.*

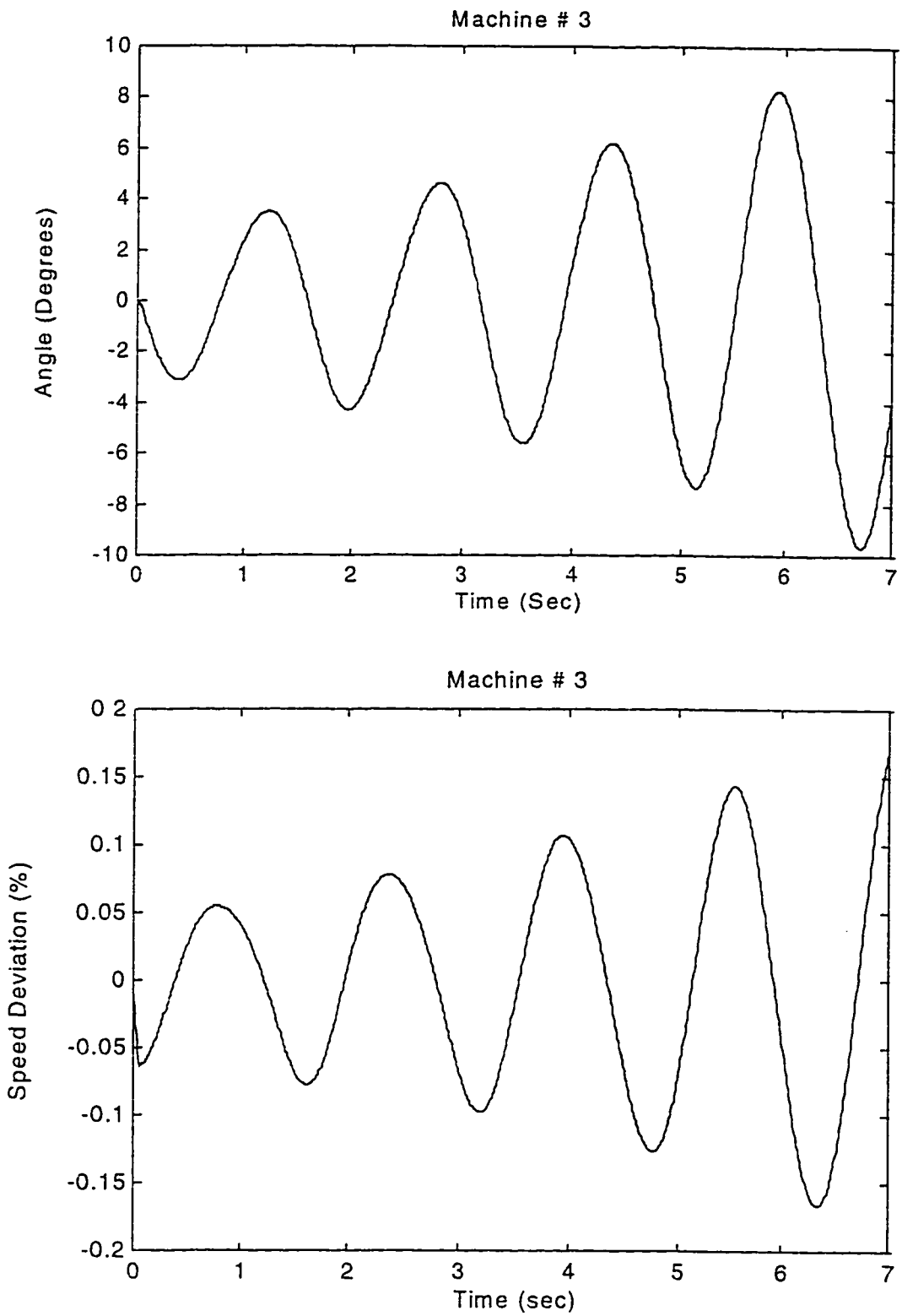


Figure 4. 26 : Open Loop Response of Machine # 3 for Case 2

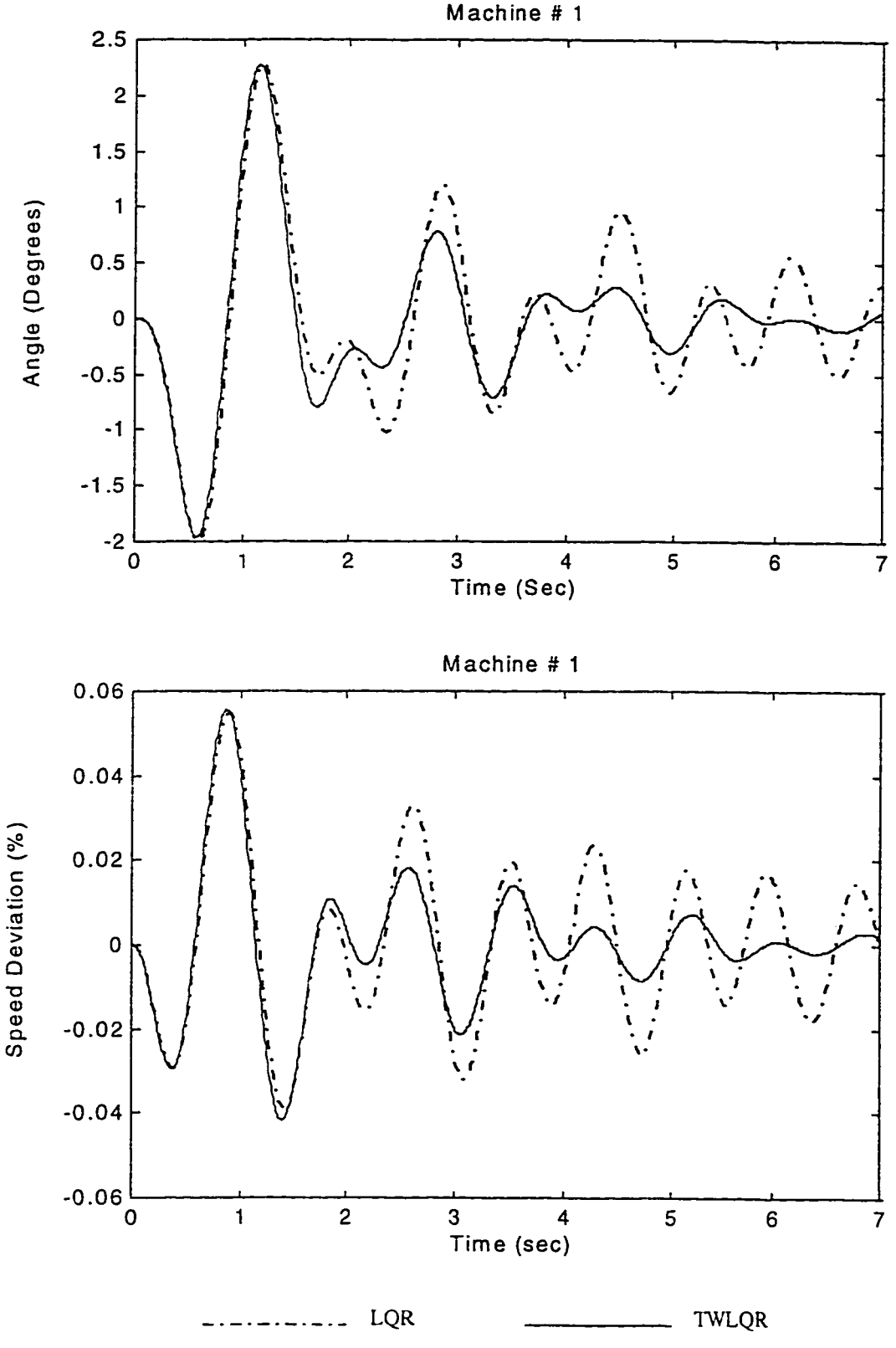


Figure 4. 27 : Output Feedback Response of Machine # 1 for Case 2

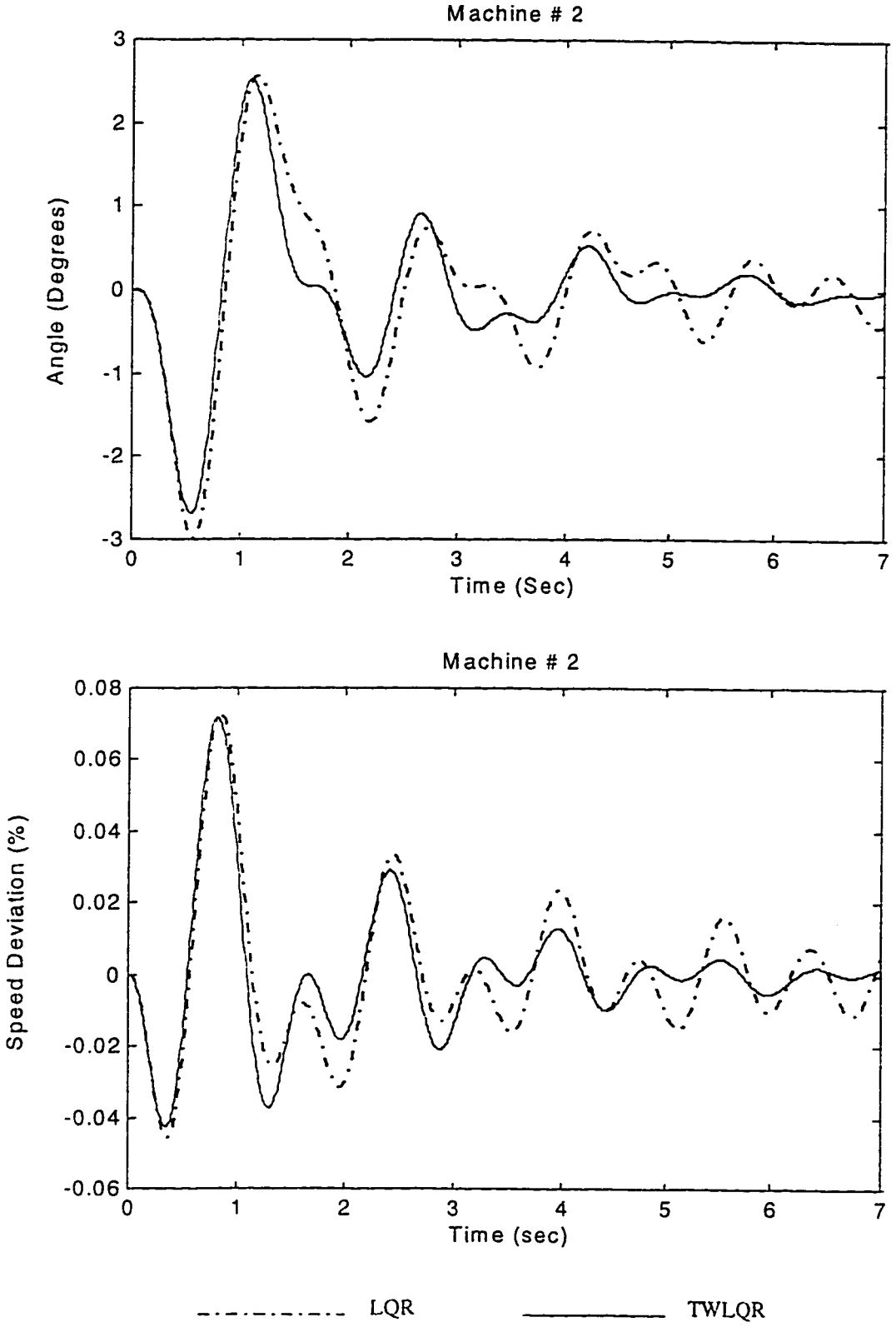


Figure 4. 28 : Output Feedback Response of Machine # 2 for Case 2

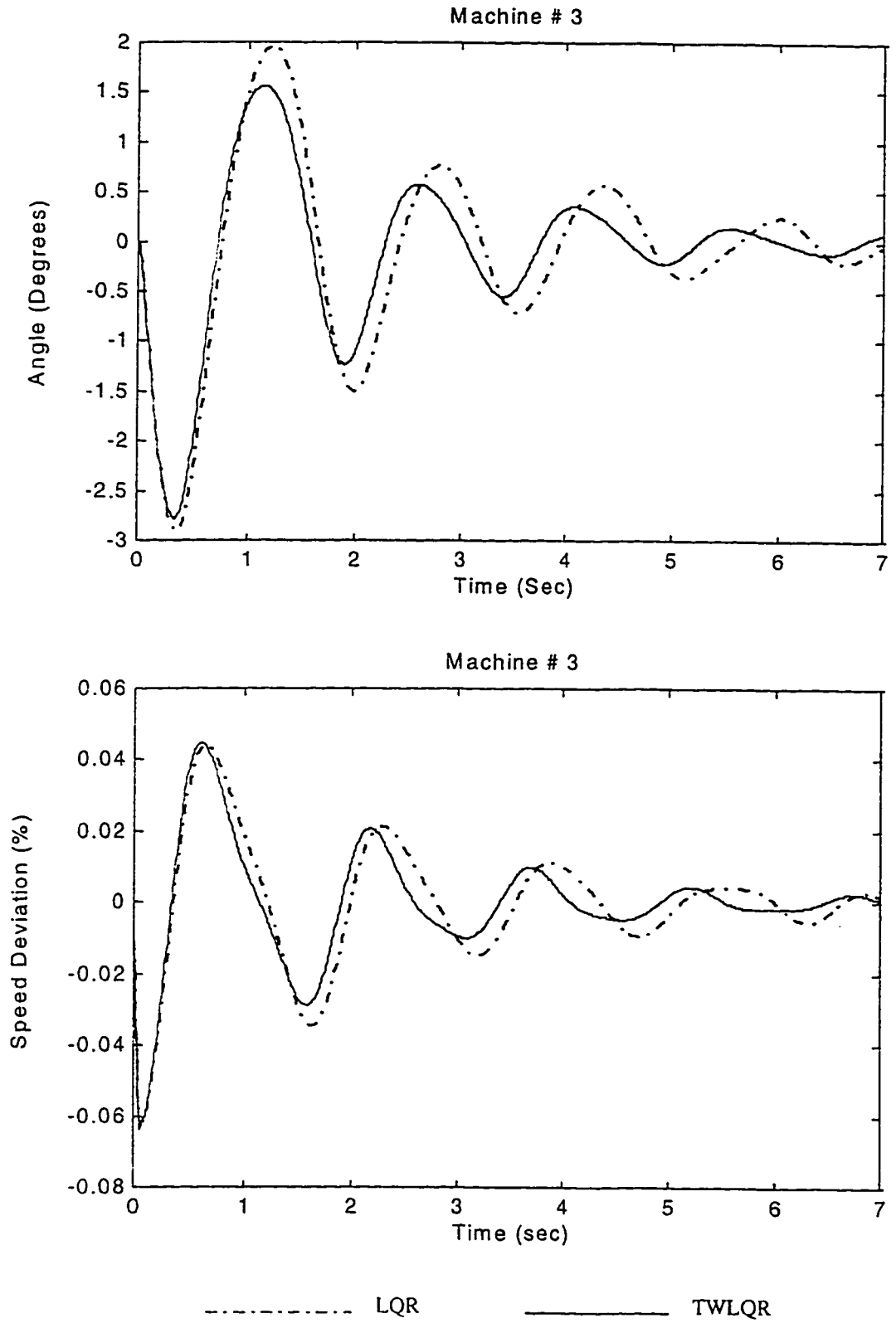


Figure 4. 29 : Output Feedback Response of Machine # 3 for Case 2

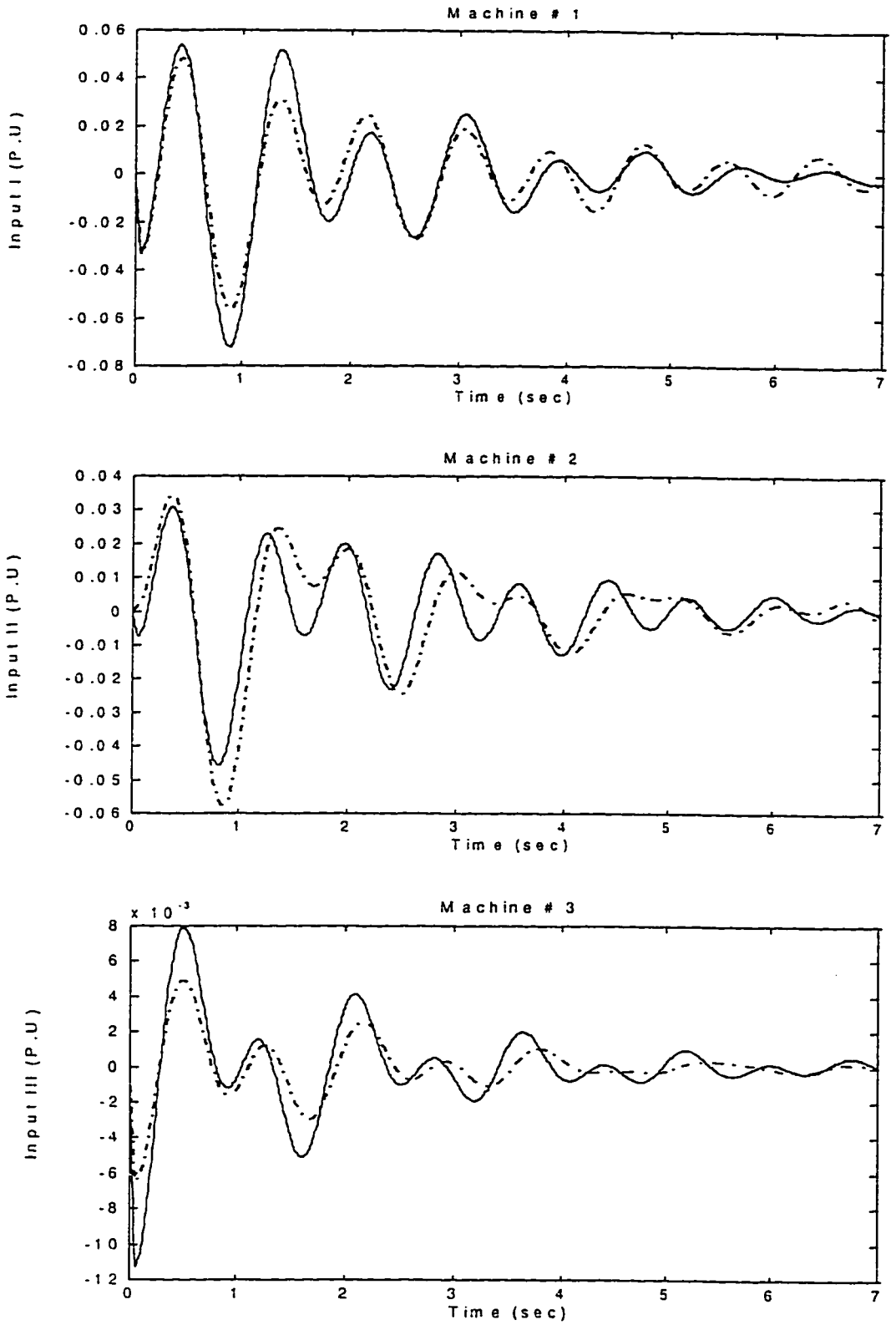


Figure 4. 30 : Output Feedback Control Inputs for Case 2

Case 3 :* 10 percent pulse disturbance at reference voltage of *machine 2 & 3

Here the 10 percent pulse disturbance is injected at the reference voltage of *machine 2 and 3* simultaneously for three cycles. The system LQR and TWLQR control dynamic responses of *machines 1, 2 and 3* are shown in *figs. 4.31 - 33* respectively, with *fig. 4.34* showing the corresponding control inputs of each machine. The results of *figs. 4.31 - 34* show the consistent superior behavior of TWLQR over LQR design as for as system torque angle and speed responses are concerned, however the former offer extra control inputs in the start to damp out the system oscillations following a pulse disturbance.

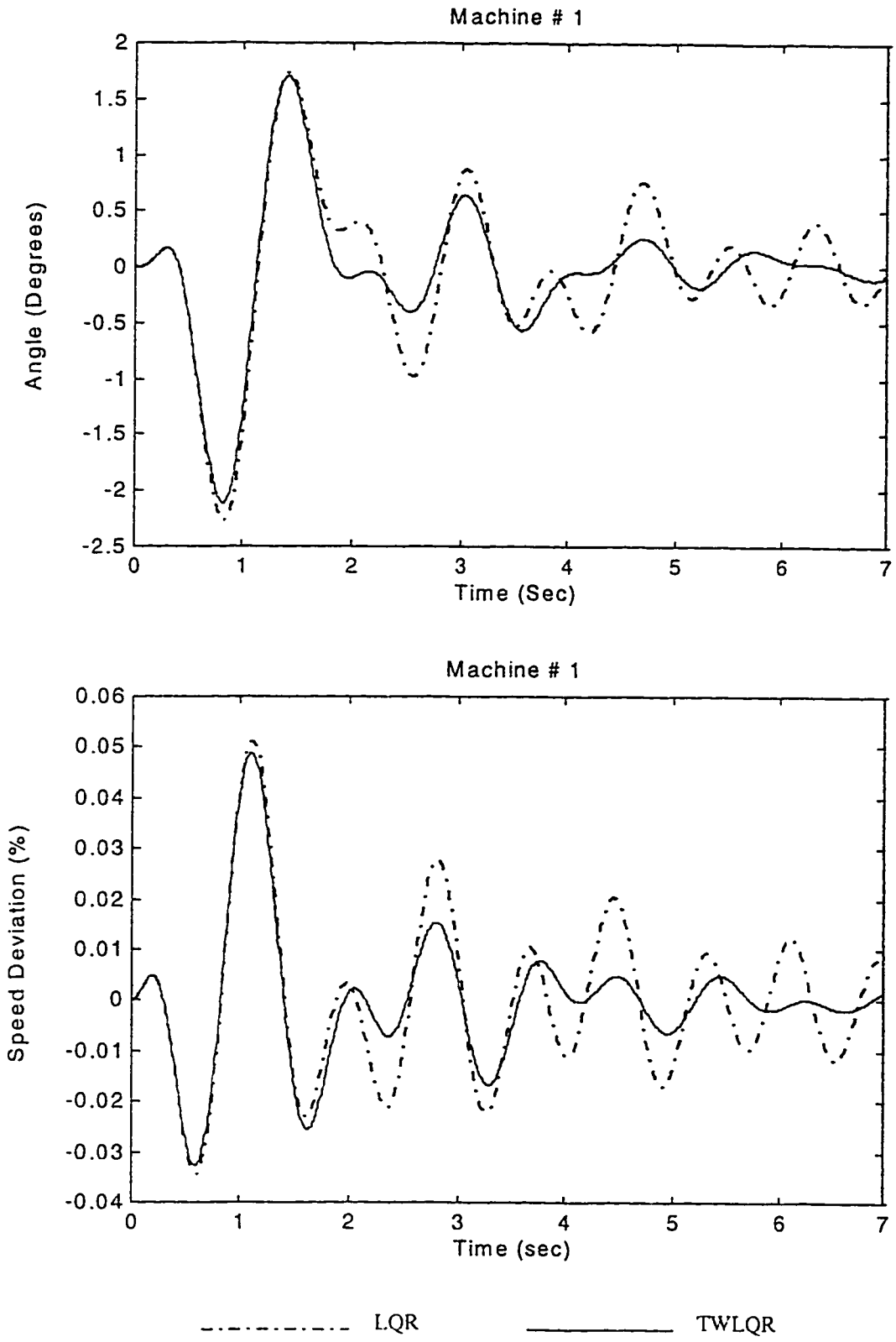


Figure 4. 31 : Output Feedback Response of Machine # 2 for Case 3

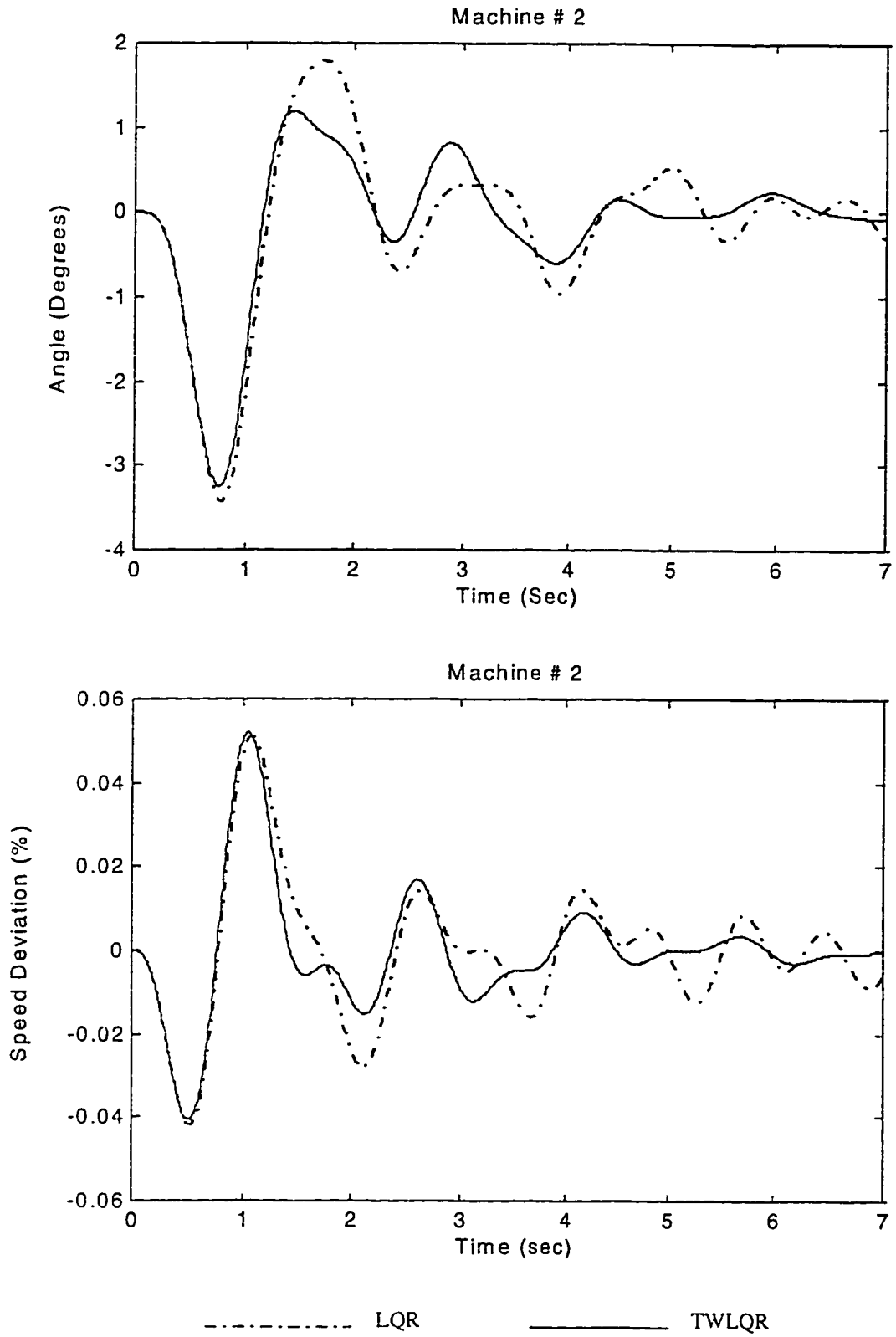


Figure 4. 32 : Output Feedback Response of Machine # 2 for Case 3

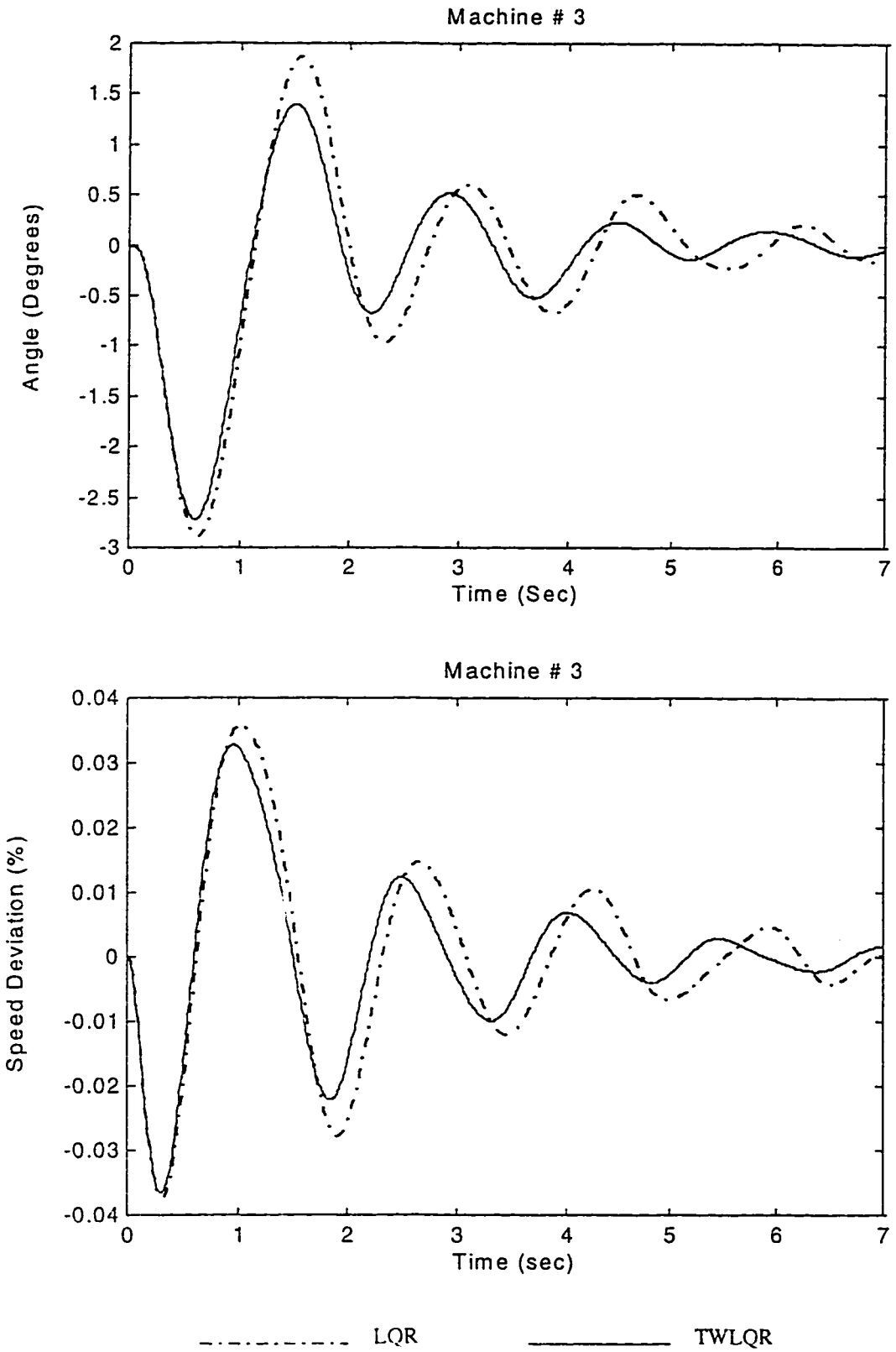


Figure 4. 33 : Output Feedback Response of Machine # 3 for Case 3

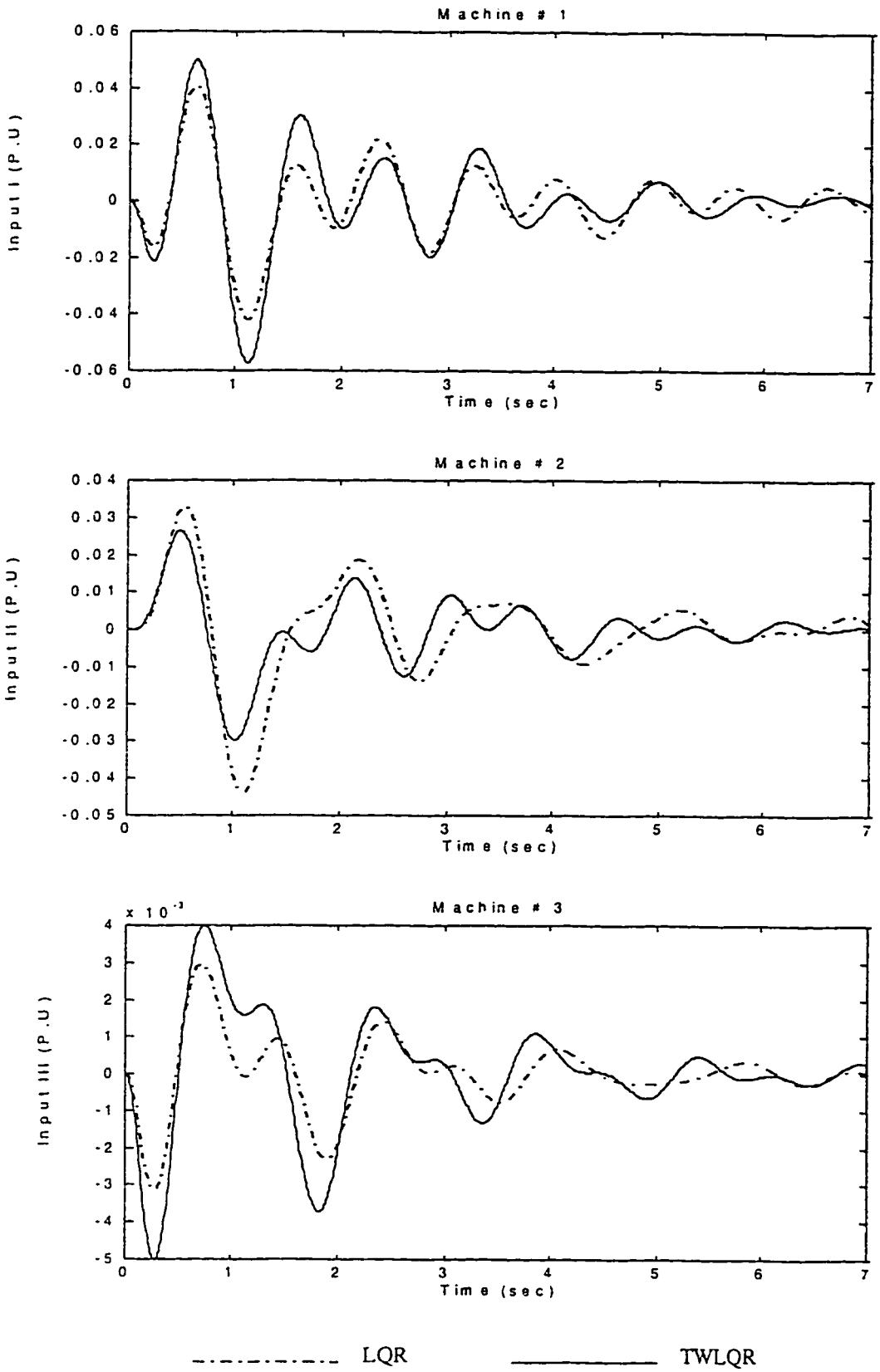


Figure 4. 34 : Output Feedback Control Inputs for Case 3

4.6 Disscussions of Results

In this chapter the simulation results for both state feedback and output feedback control of a MMPS have been considered for LQR and TWLQR control designs. Two different types of pulse disturbances were used for dynamic response studies. The overall results shows that the TWLQR is not only superior to LQR for the state feedback but it can also be applied efficiently for output feedback control.

Therefore the comparison made between the two methods shows the clear superiority of TWLQR control design over LQR control, as for as the states are concerned. However the former exert more control efforts to stabilize the system and tries to bring the system to its steady state values as the time proceeds, due to the time factor involved in its performance index.

Chapter 5

TWLQR Mode Assignment Control of Power System

The application of TWLQR control for the mode-assignment of single machine infinite bus power system with coordinated exciter governor control is considered in this chapter. The LQR control gains evaluated for a particular Q and R are used to find closed-loop modes of the system and the modes of system are placed on the same values by computing TWLQR mode-assignment control gains as explained in the algorithm given in *chapter two*. The control gain computations have been carried out for two Q and R combinations to have variety of system closed loop modes. The dynamic response simulations are performed while the system is subjected to either small load torque pulse disturbance or reference voltage pulse disturbance for a duration of three cycles of system frequency.

5.1 Model Description

The same single machine infinite bus power system model of *chapter 3* is considered here for mode assignment design.

5.2 Different Eigenvalues Configurations for Mode Assignment

Two mode assignment designs have been taken to illustrate the effectiveness of the proposed TWLQR mode assignment control over LQR counterpart.

5.2.1 Mode-Assignment Design I

In first case, the Q and R values are chosen such that the closed loop system should have enough damping for electromechanical modes of the system.

$$Q = \text{diag}\,[\,20 \quad 2 \quad 2 \quad 0.2 \quad 2] \qquad\qquad R = \text{diag}\,[1 \quad 10]$$

where each diagonal entry of the Q matrix shows the weightage given to the respective state of the system. The corresponding eigenvalues of the system for LQR and the eigen-values achieved by TWLQR assignment are given in *Table 5.1* while the corresponding feedback gains are given in *Tables 5.2 & 5. 3.*

Table 5. 1 : TWLQR Mode-Assignment Design I

	LQR Modes	TWLQR Mode-Assignment
Electro-mech.	-0.5772 + 4.8175 j	-0.5771 + 4.8170 j
Eigenvalues	-0.5772 - 4.8175 j	-0.5771 - 4.8170 j
	-0.7931	-0.7931
	-1.4386	-1.4386
	-447.4254	-447.4254

Table 5. 2 : LQR Feedback Gains For Mode Assignment Design I

Control Inputs	$\Delta\omega$	$\Delta\delta$	$\Delta e'_q$	ΔV_F	ΔP_m
U_1	40.4696	-0.8558	-4.1806	-0.4289	2.6316
U_2	-11.8967	0.2108	0.9168	0.0003	-0.7541

Table 5. 3 : TWLQR Feedback Gains For Mode Assignment Design I

Control Inputs	$\Delta\omega$	$\Delta\delta$	$\Delta e'_q$	ΔV_F	ΔP_m
U_1	-4.7059	-0.1815	-0.3713	-0.4279	-0.6197
U_2	-28.2312	0.4599	2.8896	0.1086	-1.6977

5.2.2 Mode-Assignment Design II

In this case, the control weighting matrix R is modified to get different LQR gains, and consequently different set of closed loop modes of the system. Now for these new modes, the TWLQR mode-assignment is performed. The modified Q and R are as follows

$$Q = \text{diag}\,[\,20 \quad 2 \quad 2 \quad 0.2 \quad 2] \qquad\qquad R = \text{diag}\,[0.5 \quad 5]$$

The corresponding eigenvalues for LQR control and achieved by TWLQR control are given in *Table 5.4* with their respective feedback gains listed in *Tables 5.5 & 5.6*. Note that the electromechanical modes are more damped and other eigenvalues are more negative due to the relaxation offered by halfing the penalty imposed on control inputs for modified R.

Table 5. 4 : TWLQR Mode-Assignment Design II

	LQR	TWLQR
Electro-mech.	-0.6757 + 4.8540 j	-0.6755 + 4.8530 j
Eigenvalues	-0.6757 - 4.8540 j	-0.6755 - 4.8530 j
	-0.7358	-0.7358
	-1.6665	-1.6665
	-632.61	-632.61

Table 5. 5 : LQR Feedback Gains For Mode Assignment Design II

Control Inputs	$\Delta\omega$	$\Delta\delta$	$\Delta e'_q$	ΔV_F	ΔP_m
U_1	54.3008	-0.9744	-5.4290	-0.6139	3.1607
U_2	-22.3777	0.3398	1.5558	0.0003	-1.2879

Table 5. 6 : TWLQR Feedback Gains For Mode Assignment Design II

Control Inputs	$\Delta\omega$	$\Delta\delta$	$\Delta e'_q$	ΔV_F	ΔP_m
U_1	-12.8001	-0.2419	-0.1681	-0.6130	-1.2064
U_2	-38.8951	0.5349	3.4631	0.0977	-2.1709

5.3 Simulation Results

For the two mode assignment designs with different sets of control weighting matrices R, the simulation results have been obtained for the same disturbances cases as considered in *chapter 3*.

Case 1 : **10 Percent Pulse Disturbance at Reference Voltage for three cycles**

Case 2 : **10 Percent Load Torque Pulse Disturbance for three cycles**

The open loop system responses are the same as shown in *chapter 3* for respective disturbance cases. The simulation results for LQR and TWLQR control, for the electromechanical modes and for the corresponding control inputs are shown in *figs 5.1 - 5.5* for mode-assignment design I and in *figs 5.6 - 5.10* for mode-assignment design II.

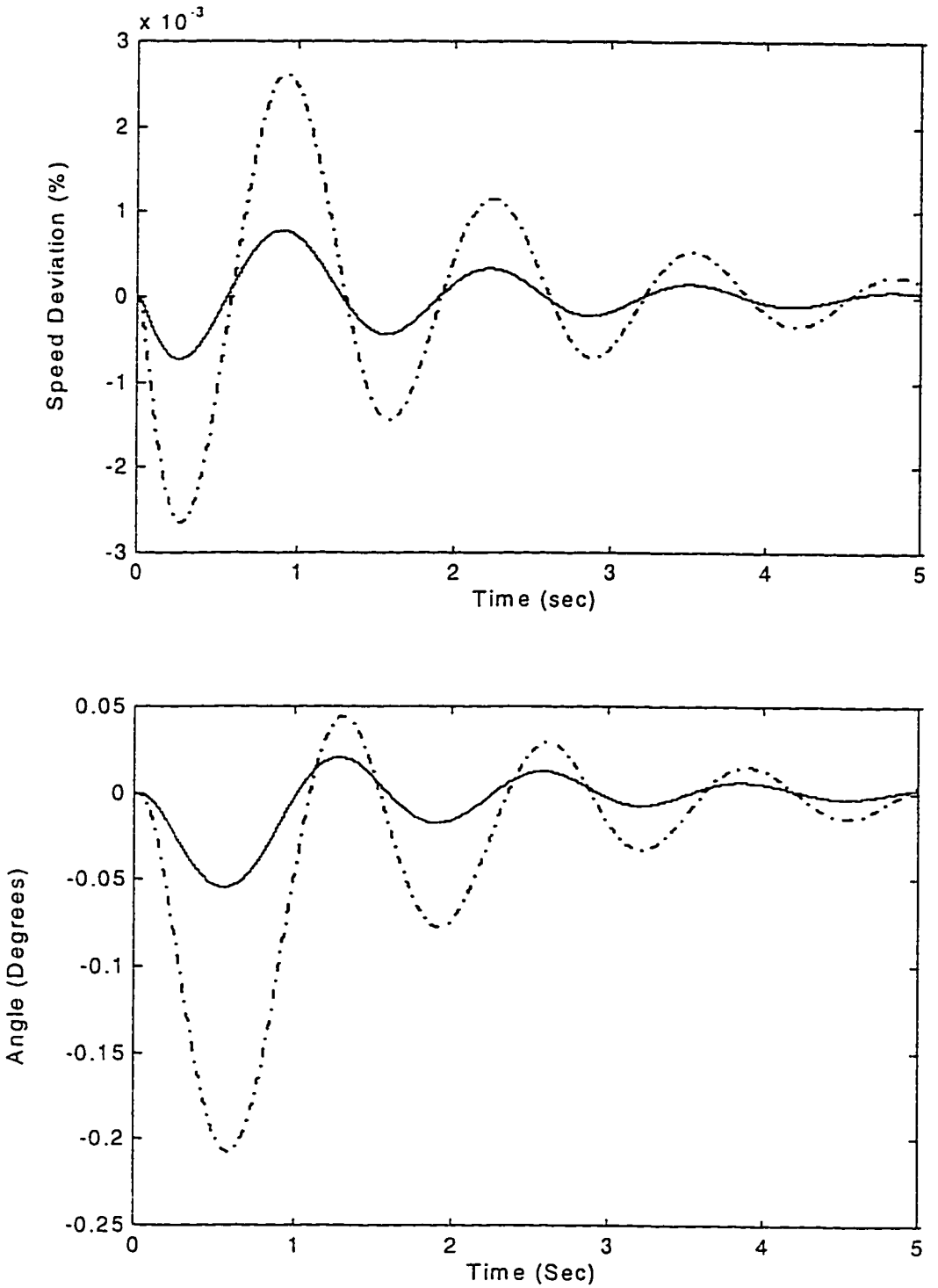
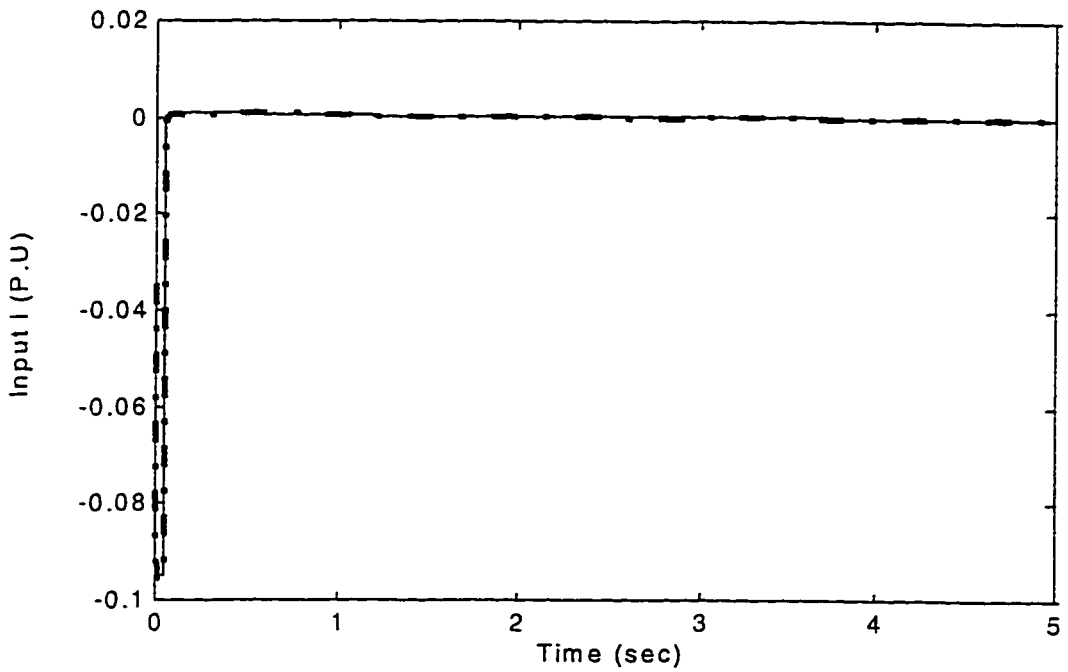


Figure 5. 1 : Design I System Response for Case 1

LQR _._._._._._._ TWLQR ________



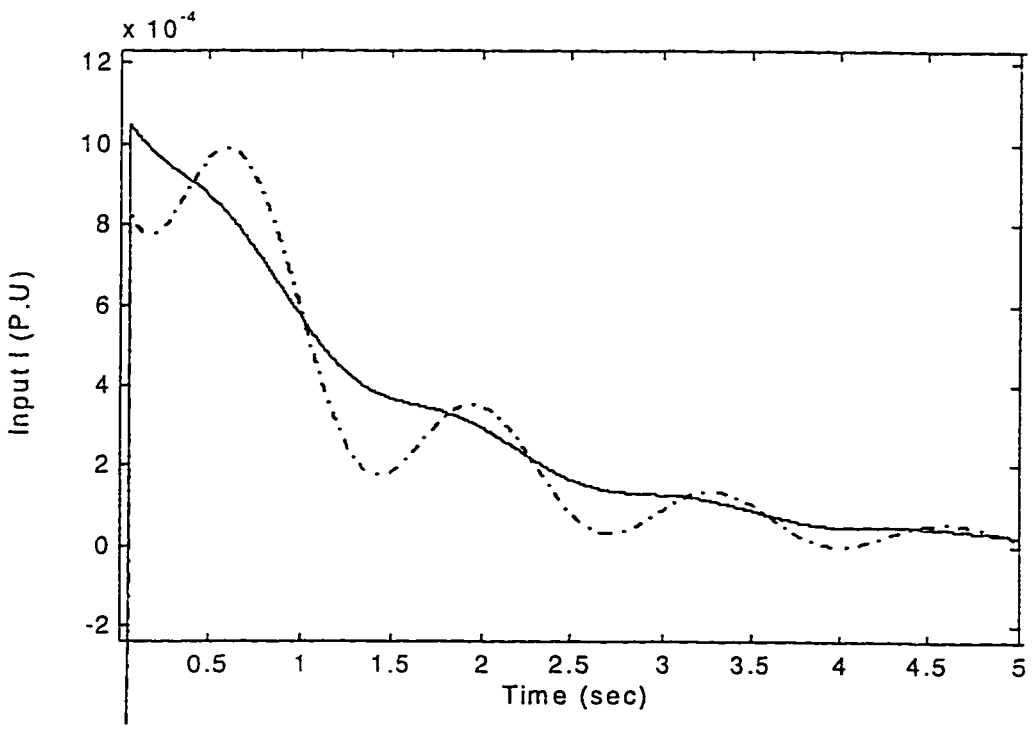


Figure 5. 2 : (a) Design I Control Input I for Case 1 (b) Magnified View

LQR TWLQR

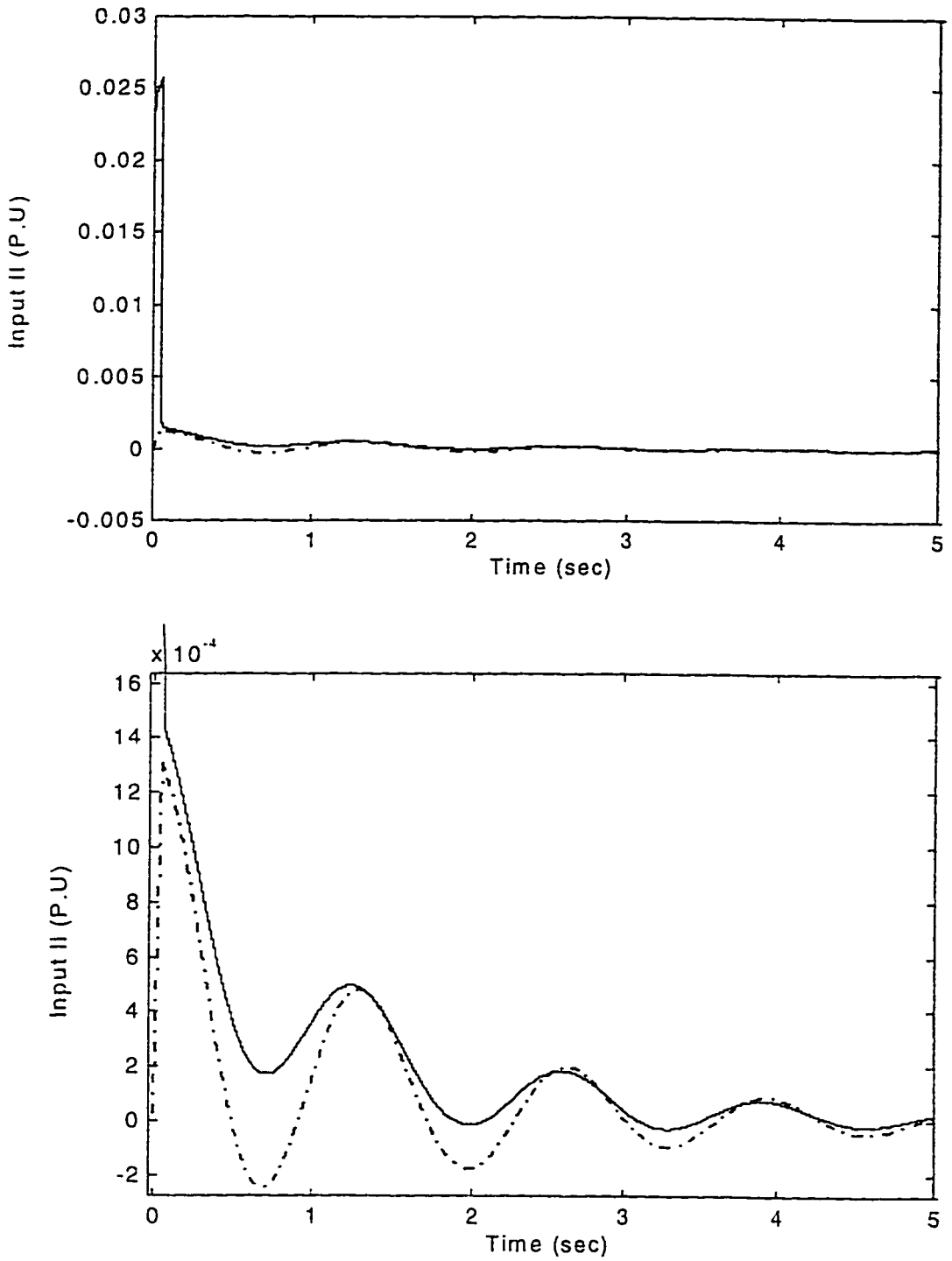


Figure 5. 3 : (a) Design I Control Input II for Case 1 (b) Magnified View

LQR ..... TWLQR _____

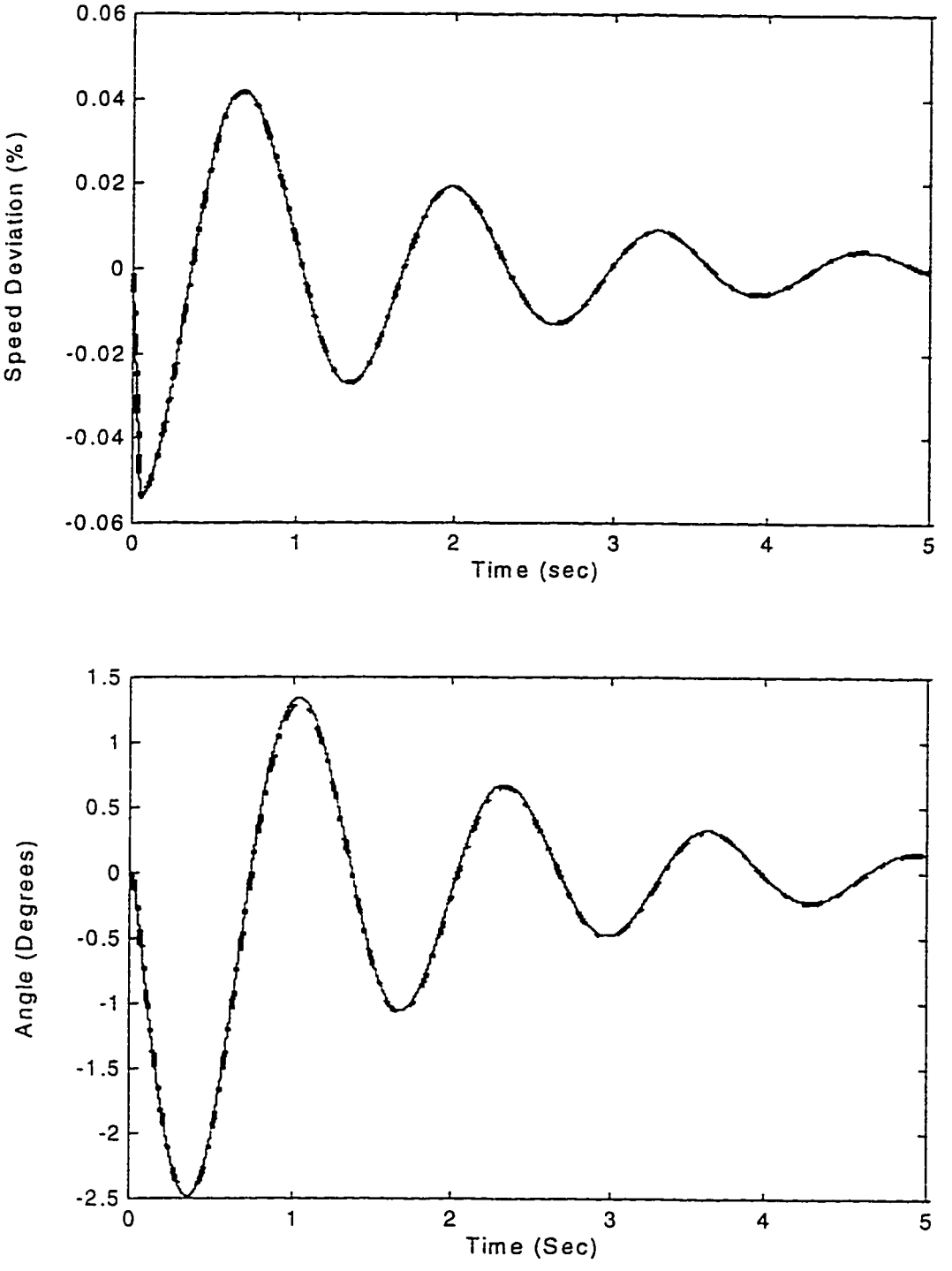


Figure 5. 4 : Design I System Response for Case 2

LQR ..... TWLQR _____

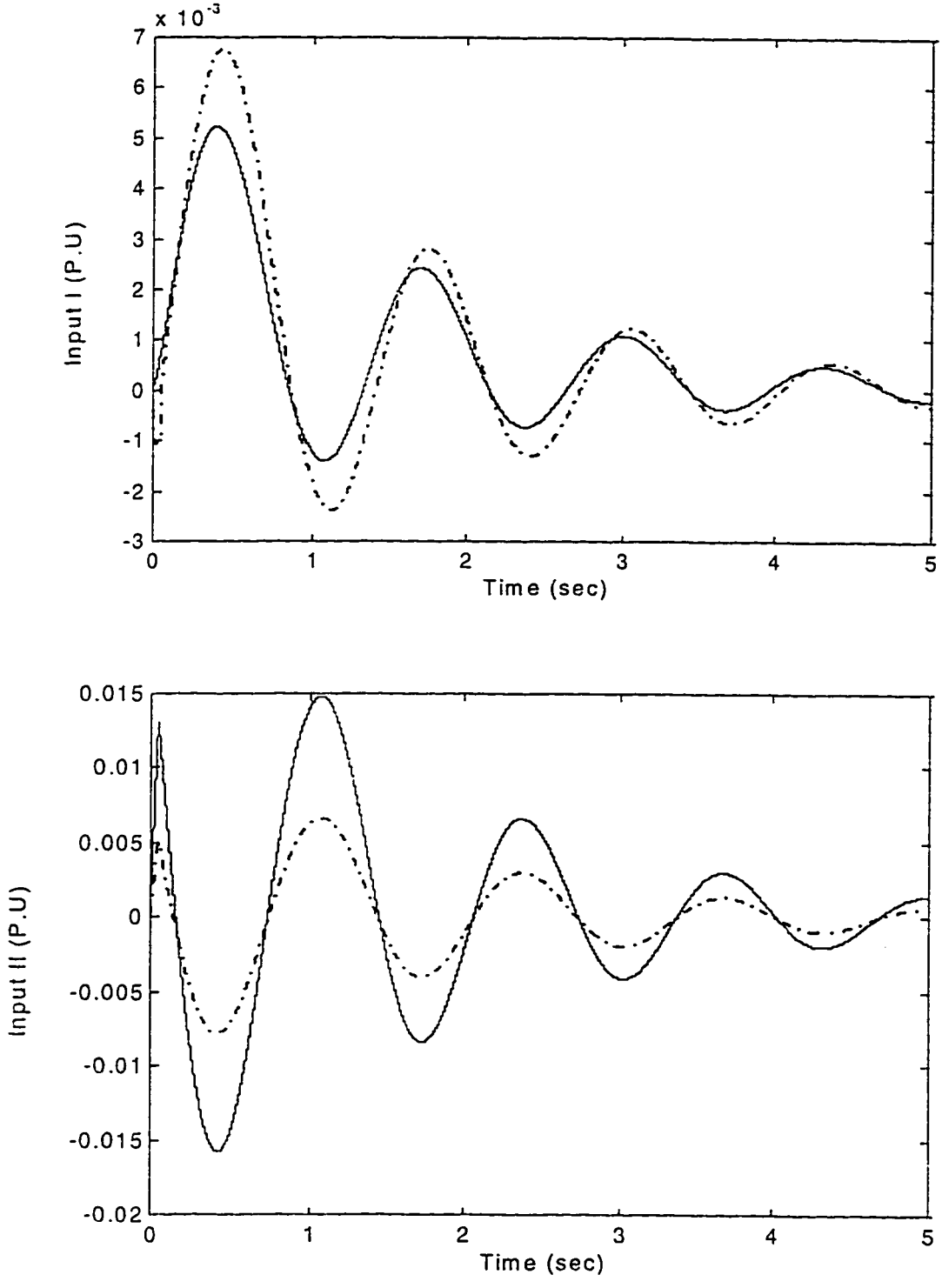


Figure 5. 5 : Design I Control Inputs for Case 2

LQR - - - - - - - - - - - - TWLQR ————————

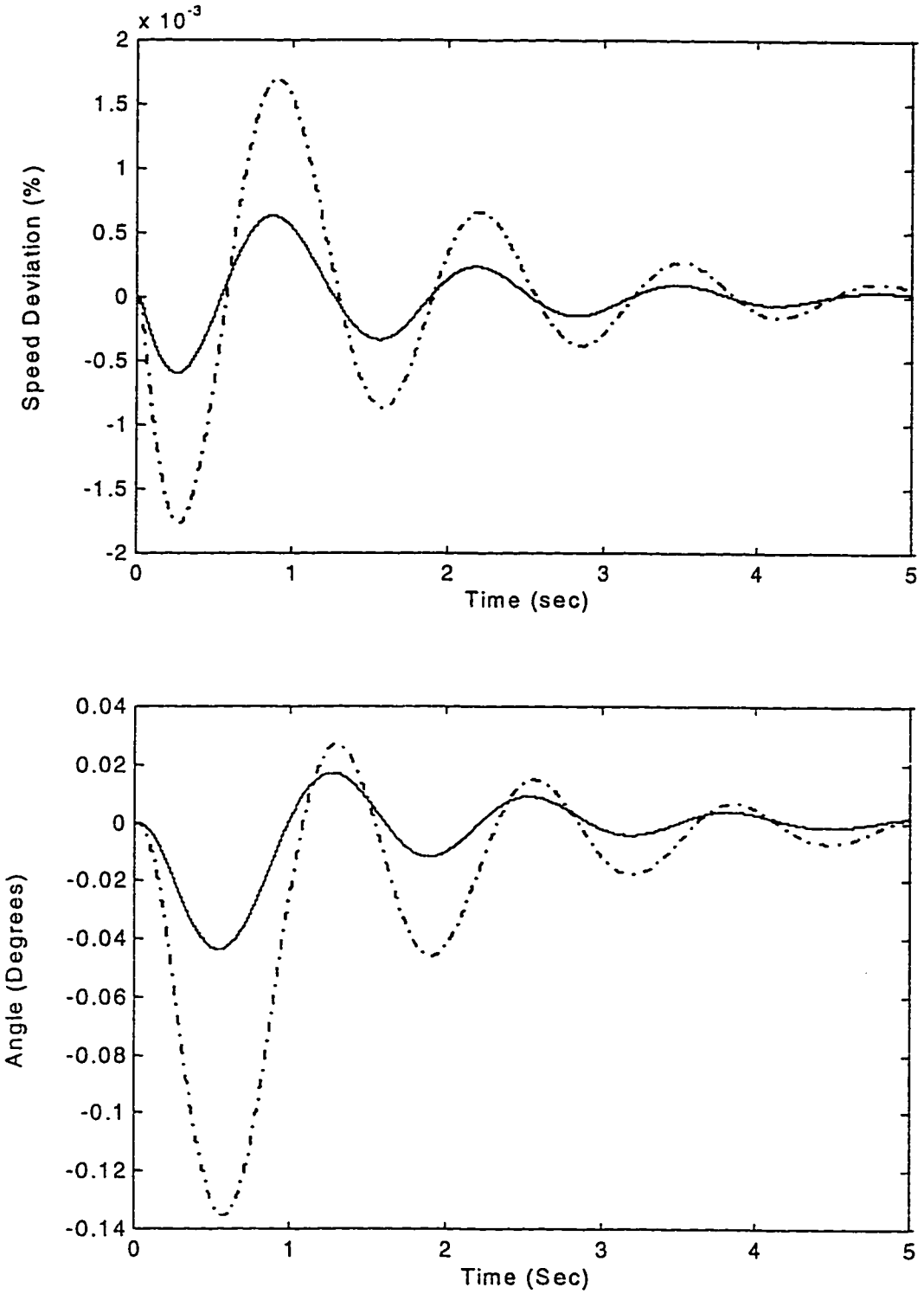


Figure 5. 6 : Design II System Response for Case 1

LQR ..... TWLQR _____

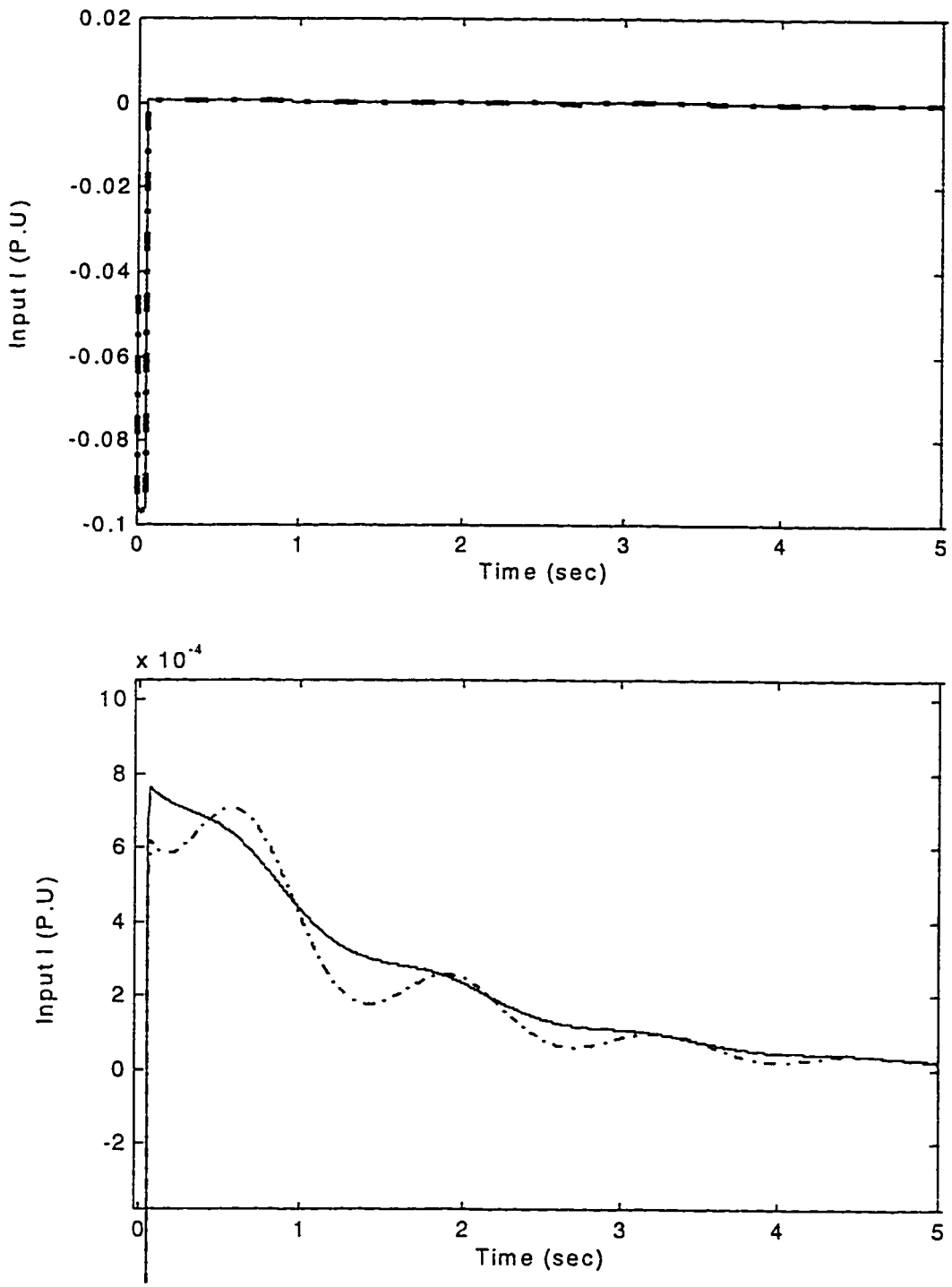


Figure 5. 7 : (a) Design II Control Input I for Case 1 (b) Magnified View

LQR TWLQR

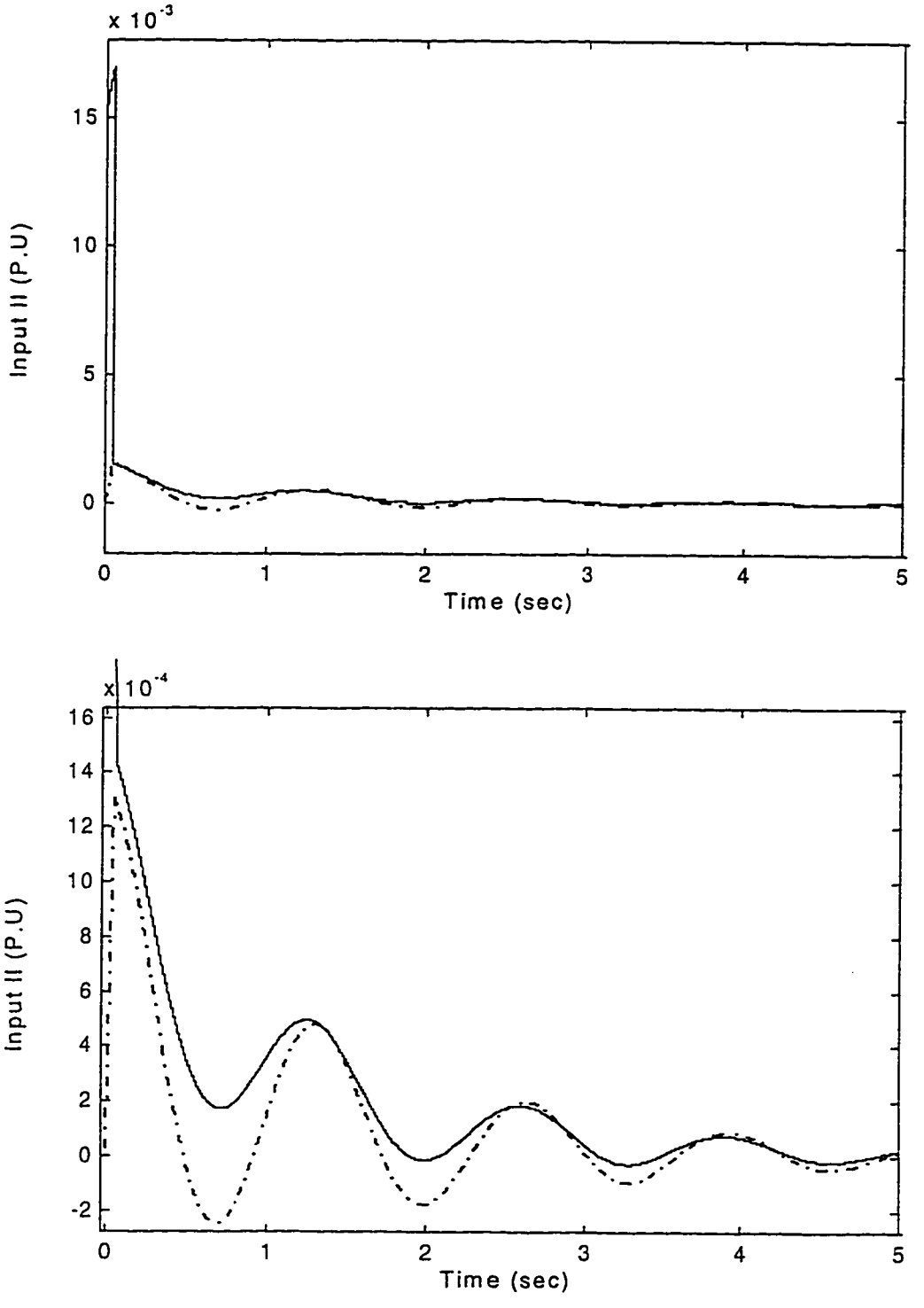


Figure 5. 8 : (a) Design II Control Input II for Case 1 (b) Magnified View

LQR -.-.-.-.-.-.-.- TWLQR ————

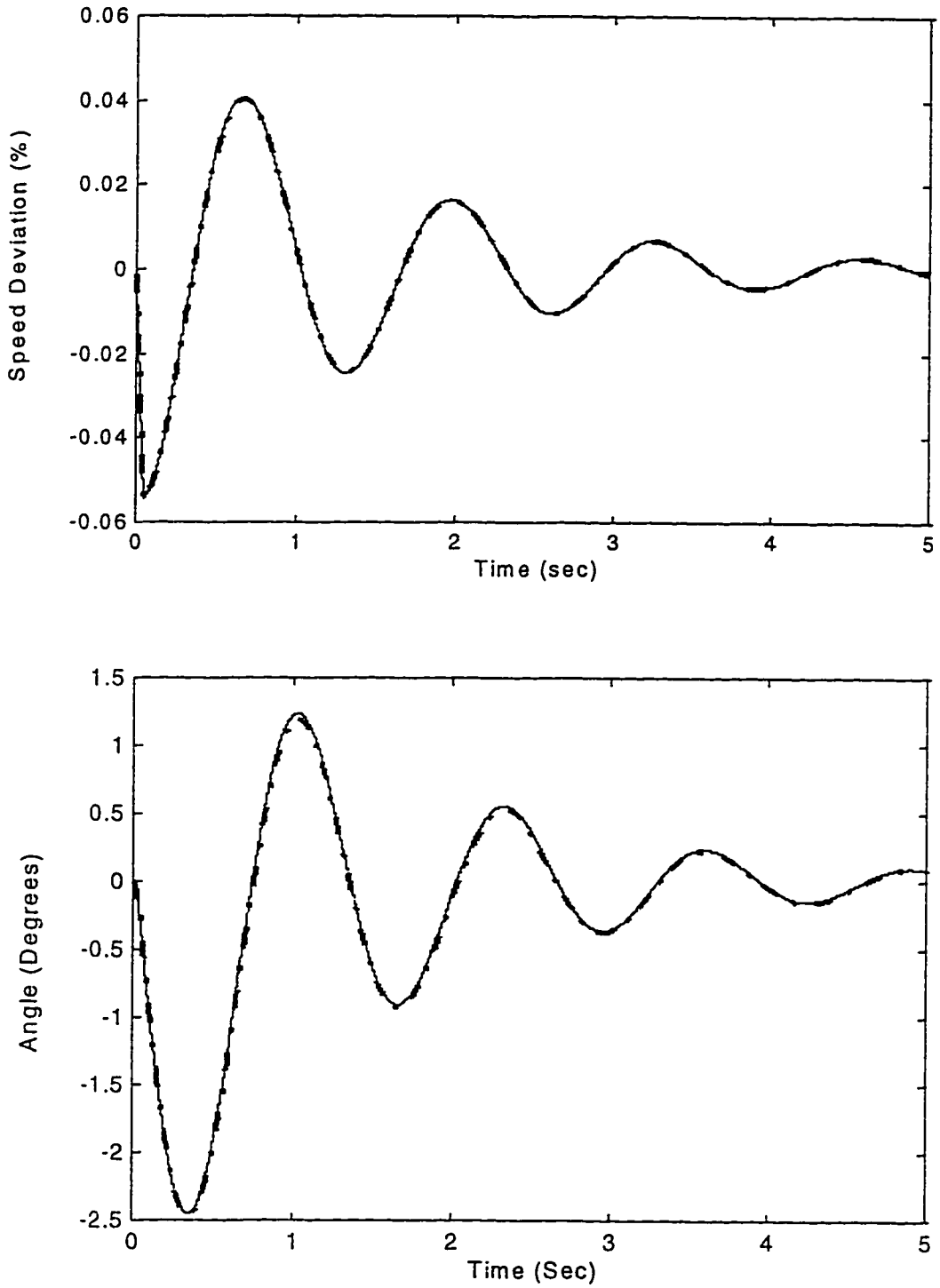


Figure 5. 9 : Design II System Response for Case 2

LQR ..... TWLQR _____

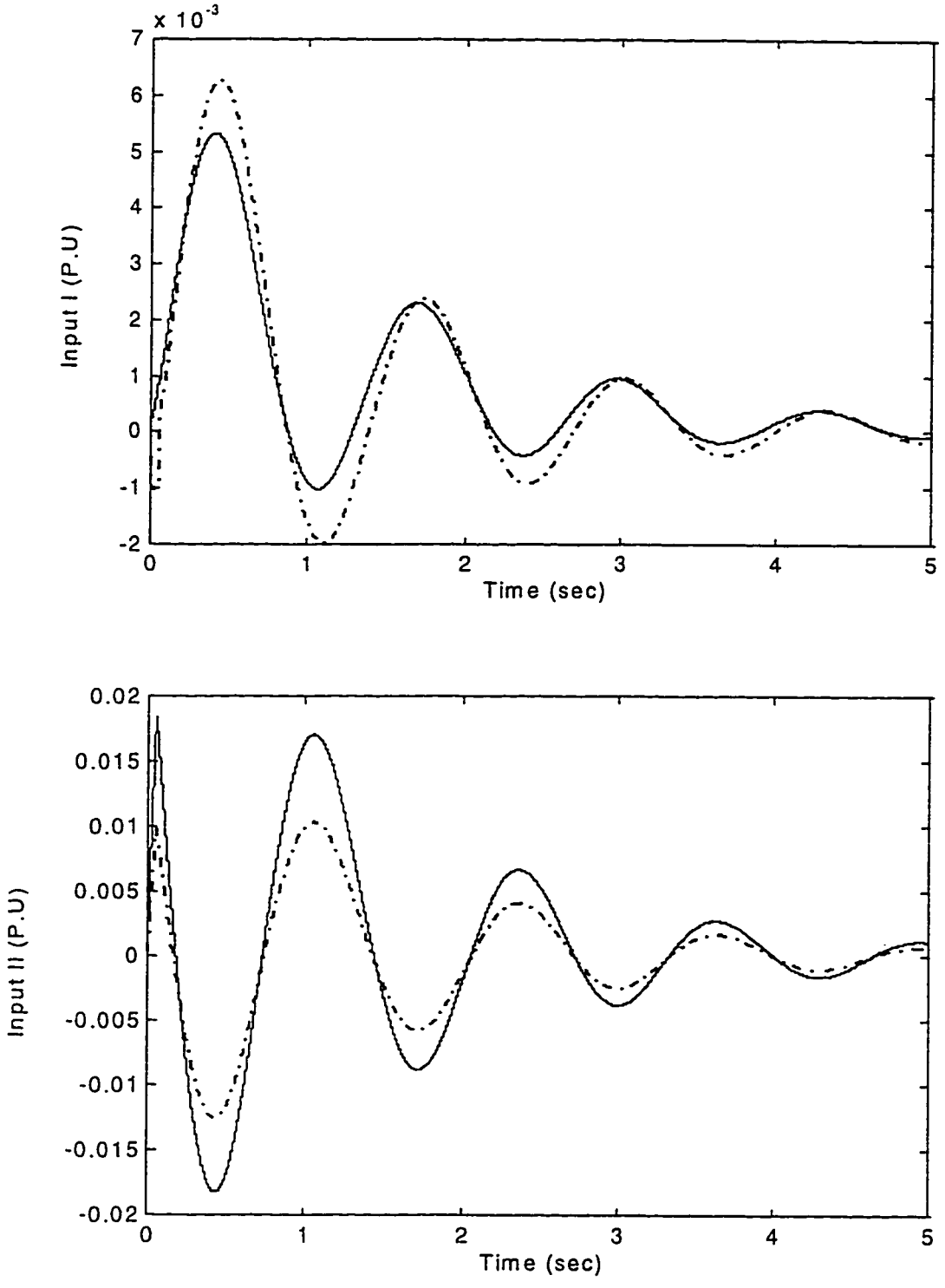


Figure 5. 10 : Design II Control Inputs for Case 2

LQR -.-.-.-.-.-.-.- TWLQR ————

5.4 Discussions

It is obvious from the simulation results that both designs are consistent in their dynamic performance behavior. While analysing the reference voltage pulse disturbance response of both designs of *fig 5.1* and *fig. 5.6* , the TWLQR control performed much better than LQR counterpart for electromechanical modes. While considering the control inputs as shown in *figs.5.2* , *3* and *figs. 5.7* , *8* for design I and II respectively , it is observed that the exciter control input behavior is nearly identical for both TWLQR and LQR designs whereas for governor control input, the TWLQR control exert more effort in the start. Now coming to torque pulse response of both designs in *figs 5.4 - 5.9* , the responses for both LQR and TWLQR designs are identical. This can be due to the fact that for TWLQR, the solution obtained is not necessarily a global minimum.

Chapter 6

Conclusions and Recommendations

6.1 Conclusions

The dynamic performance of various linearized power systems has been analyzed for LQR and TWLQR state and output feedback controls when subjected to various small pulse disturbances for three cycles. The TWLQR mode assignment control design has also been carried out for small perturbation analysis of the SMIB power system.

For both SMIB and MMPS models presented in chapter *three* and *four*, the control strategies presented in chapter *two* are employed for the computations of LQR and TWLQR feedback controls for both state and output feedback cases. While keeping the practical implementation in view, only angular speed, the most practical state taken as output variable, is used for output feedback case. The third and final

application of TWLQR control is the mode-assignment of SMIB power system with coordinated exciter-governor control. The LQR control gains evaluated for a particular Q and R are used to find closed loop modes and the system modes are placed on the same values by computing TWLQR mode-assignment control gains as explained in the algorithm given in chapter *two*.

Once the controls have been derived, the closed loop dynamic responses have been simulated to study the dynamic behavior while the power system subjected to different small pulse disturbances either in load or mechanical torque or at reference voltage for a duration of three cycles of system frequency. The simulation results for deviation in speed and torque angle have been plotted along with corresponding control input signals. The results show the distinct effectiveness of TWLQR control over LQR for both the states and control inputs, as in most of the cases TWLQR responses decay faster than LQR as time elapses

6.2 Recommendations For Future work

The following recommendations are made based on the analysis of the present work:

- The work may be extended to include non-linear simulations to study the robustness of the TWLQR over LQR control for the transient stability analysis of the power systems.
- More efficient optimization techniques may be searched out and applied to ensure the evaluation of global minimum of the objective function.
- The *Genetic Algorithm* may be used, to find the optimal control gains, as it has already been successfully applied to the stabilization and control of power systems in [23 , 73].
- The mode assignment using TWLQR may be extended to multi-machine power systems and mathematical formulation should be worked out to implement it for output feedback control for both single machine infinite bus system and multi-machine cases.

APPENDICES

A. Single Machine Infinite Bus Model

A.1 System Parametric Values

The system under investigation has the following parametric values [3, 4, 72]:

A.1.1 Machine Data (p.u)

$x_d = 0.973,$ $x_d' = 0.190,$ $x_q = 0.550,$

$T_{do}' = 7.76,$ $M = 9.26\ sec,$ $r_a = 0.00435$

A.1.2 Exciter and Governor Data

$K_A = 50,$ $T_A = 0.05(sec)$ $K_g = 1,$ $T_g = 1.0(sec)$

A.1.3 Load and Line parameters

$x_l = 0.997,$ $r_l = -0.034$

$g = 0.249,$ $b = 0.262$

A.1.4 Initial loading Conditions (p.u)

$P_e = 1.00,$ $Q_e = 0.015,$ $V_{to} = 1.05$

$I_{do} = 0.4354,$ $I_{qo} = 0.8471,$

$V_{do} = 0.4659,$ $V_{qo} = 0.9410,$

$V_o = 1.0509, \qquad \delta_o = 68.0148\ (degrees)$

A.2 State Space Model Representation

The fifth order Concordia-de-Mello Model with Coordinated Exciter-Governor Control [3, 4, 72]

$$\begin{bmatrix} \Delta\dot{\omega} \\ \Delta\dot{\delta} \\ \Delta\dot{e}'_q \\ \Delta\dot{V}_F \\ \Delta\dot{P}_m \end{bmatrix} = \begin{bmatrix} \frac{D}{M} & \frac{-K_1}{M} & \frac{-K_2}{M} & 0 & \frac{1}{M} \\ \omega_o & 0 & 0 & 0 & 0 \\ 0 & \frac{-K_4}{T'_{do}} & \frac{-1}{K_3 T'_{do}} & \frac{1}{T'_{do}} & 0 \\ 0 & \frac{-K_A K_5}{T_A} & \frac{-K_A K_6}{T_A} & \frac{-1}{T_A} & 0 \\ \frac{-K_g}{T_g} & 0 & 0 & 0 & \frac{-1}{T_g} \end{bmatrix} \begin{bmatrix} \Delta\omega \\ \Delta\delta \\ \Delta e'_q \\ \Delta V_F \\ \Delta P_m \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{K_A}{T_A} & 0 \\ 0 & \frac{K_g}{T_g} \end{bmatrix} \begin{bmatrix} u_E \\ u_g \end{bmatrix}$$

A.3 Numerical Values

The numerical values of the constants $K_1 \ldots K_6$ are given below for above mentioned operating conditions

$K_1 = 0.5441 \qquad K_2 = 1.2067 \qquad K_3 = 0.6584$

$K_4 = 0.6981 \qquad K_5 = -0.0955 \qquad K_6 = 0.8159$

Substituting the values of various parameters yields numerical values of the matrices A and B

$$A = \begin{bmatrix} 0.000 & -0.0588 & -0.1303 & 0 & 0.1080 \\ 376.9911 & 0 & 0 & 0 & 0 \\ 0 & -0.0900 & -0.1957 & 0.1289 & 0 \\ 0 & 955320 & -815.9298 & -20.00 & 0 \\ -1.00 & 0 & 0 & 0 & -1.00 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$$

B. Multi-Machine Power System Model

B.1 State Space Numerical Data

The numerical values of the coefficient matrix A of the system is given below [6]:

$$A_{11} = \begin{bmatrix} -0.922 & 1 & -0.266 & -0.009 \\ -2.75 & -2.78 & -1.36 & -0.037 \\ 0 & 0 & 0 & 1 \\ -4.95 & 0 & -55.5 & -0.039 \end{bmatrix} \quad A_{22} = \begin{bmatrix} -0.21 & 1 & -1.6 & -0.005 \\ -1.9 & -1.8 & 9.3 & -0.12 \\ 0 & 0 & 0 & 1 \\ -3.1 & 0 & -56 & 0.032 \end{bmatrix}$$

$$A_{33} = \begin{bmatrix} -0.197 & 1 & -1.2 & -0.003 \\ -54.4 & -20 & 70.1 & -2.37 \\ 0 & 0 & 0 & 1 \\ -3.4 & 0 & -21 & -0.017 \end{bmatrix} \quad A_{12} = \begin{bmatrix} 0.024 & 0 & -0.087 & 0.002 \\ -0.158 & 0 & 1.11 & -0.011 \\ 0 & 0 & 0 & 0 \\ 0.222 & 0 & 8.17 & 0.004 \end{bmatrix}$$

$$A_{13} = \begin{bmatrix} 0.072 & 0 & -0.25 & 0.003 \\ -0.46 & 0 & 2.8 & -0.02 \\ 0 & 0 & 0 & 0 \\ 0.924 & 0 & 17.5 & 0.02 \end{bmatrix} \quad A_{21} = \begin{bmatrix} 0.021 & 0 & 0.121 & 0.003 \\ -1.1 & 0 & -1.62 & -0.015 \\ 0 & 0 & 0 & 0 \\ -2.43 & 0 & 1.37 & -0.034 \end{bmatrix}$$

$$A_{23} = \begin{bmatrix} 0.06 & 0 & 0.46 & 0.002 \\ -1 & 0 & 1.49 & -0.04 \\ 0 & 0 & 0 & 0 \\ -0.12 & 0 & 29.8 & -0.028 \end{bmatrix} \quad A_{31} = \begin{bmatrix} -0.002 & 0 & 0.083 & 0 \\ -6.78 & 0 & -10.1 & -0.09 \\ 0 & 0 & 0 & 0 \\ -1.24 & 0 & 0.498 & -0.017 \end{bmatrix}$$

$$A_{32} = \begin{bmatrix} 0.011 & 0 & 0.22 & 0 \\ -2.1 & 0 & 1.7 & -0.123 \\ 0 & 0 & 0 & 0 \\ -0.07 & 0 & 6.37 & -0.011 \end{bmatrix}$$

The input matrix B may be written as

$$B = \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix}$$

where $B_1 = [0 \quad 36.1 \quad 0 \quad 0]^T$,

$B_2 = [0 \quad 78.9 \quad 0 \quad 0]^T$ &

$B_3 = [0 \quad 1000 \quad 0 \quad 0]^T$

B.2 Machine Data

The machine data based on the base MVA and kV of the multi-machine system are summarised in the following Table

Table B. 1 : Machine Data

Machine	x_d	x_q	x_d'	T_{do}'	H	K_A	T_A	Base Quantities MVA	kV
# 1	1.68	1.66	0.32	4.0	2.31	13	0.21	360	13.8
# 2	0.88	0.53	0.33	8.0	3.40	45	0.07	503	13.8
# 3	1.02	0.57	0.20	7.76	4.63	50	0.02	1673	13.8

B.3 Initial Terminal Conditions

The initial active and reactive power flow at each bus with its terminal voltages and initial torque angels are given below :

Table B. 2 : Initial Terminal Conditions

Bus #	*Power*	*Flow*	*Ter. voltage*	*Torque Angel*
	P_{o}, MW	Q_{o},MVAR	V_{to} ,p .u.	δ_{o} ,degrees
1	26.0	37.0	1.030	10.00
2	518.0	-31.5	1.025	32.52
3	1582.0	-69.9	1.030	45.82
4	410.0	49.1	1.060	20.69

C. Maximal Rank Matrix

The algorithm for the computation of the maximal rank matrix N_i satisfying $(\lambda_i I_n - A , B) N_i = 0$ is given in the following theorem, where the synthesis of state feedback regulators by eigenstructure assignment is greatly facilitated [60] :

C.1 Theorem :

If the pair { A , B } is controllable and the matrix B is of full rank, the maximal rank matrix satisfying $(\lambda_i I_n - A , B) N_i = 0$ can be determined through column operations as follows :

$$\approx \begin{bmatrix} \lambda_i I_n - A, B \\ I_{n+m} \end{bmatrix} = \begin{bmatrix} I_n & 0 \\ E_1 & E_2 \end{bmatrix}$$

where $\approx$ [ ] represents the column operation of the matrix [ ]. Then $N_i = E_1$

C.2 Proof of Theorem

The rank of the matrix $(\lambda_i I_n - A , B)$ is n if and only if the pair {A , B} is controllable [moore]. Therefore, we can always obtain the following matrix form through column operations

$$\approx \begin{bmatrix} \lambda_i I_n - A, B \\ I_{n+m} \end{bmatrix} = \begin{bmatrix} I_n & 0 \\ E_1 & E_2 \end{bmatrix}$$

The column operations imply

$$\approx \begin{bmatrix} \lambda_i I_n - A, B \\ I_{n+m} \end{bmatrix} = \begin{bmatrix} \lambda_i I_n - A, B \\ I_{n+m} \end{bmatrix} \begin{bmatrix} E_1 & E_2 \end{bmatrix}$$

$$\equiv \begin{bmatrix} (\lambda_i I_n - A, B)E_1 & (\lambda_i I_n - A, B)E_2 \\ E_1 & E_2 \end{bmatrix}$$

D. Listing of Programs

The listing of some of the essential programs used in the software implementations of the algorithms discussed in *chapter two*, is given as follows. These programs are written in *MATLAB*

D.1 LQR and TWLQR state and output feedback minimization

D.1.1 Function for LQR state feedback minimization

```
function [ fo , K ] = lqr_sfd( A , B , Q , R , X o ,Ko)
[ n , m ] = size(B);
options = foptions ;
options ( 1 ) =1;
options ( 13 ) = m * n ;
options (14) = 200*m*n;
%calling the minimization function Constr
[fo , K ] = constr ( ' lqrk ' , Ko , options , [ ] , vlb , vub , A , B , Q , R ,X o );
% vlb = lower bound on independent variable
% vub = upper bound on independent variable
end
```

D.1.2 Function for TWLQR state feedback minimization

```
function [ f1 , K ]=twlqr_sfd( A , B , Q , R , Xo, Ko)
 [ n , m ] = size( B ) ;
options = foptions ;
options ( 1 ) =1;
options ( 13 ) = m * n ;
options (14) = 200*m*n;
[f1,K] = constr ( ' twlqrk ' , Ko , options , [ ] , vlb , vub , A , B , Q , R , X o );
end
```

D.1.3 Function for LQR output feedback minimization

```
function [ p0 , K ] = lqrop ( A , B , C , Q , R , X o , Ko )
[ n , m ] = size ( B );
[ l , n ] = size ( C ) ;
options = foptions ;
options ( 1 ) = 1 ;
options ( 13 ) = m * l ;
options ( 14 ) = 200 * m * l ;
[ p0 , K ] = constr ( ' lqrpok ' , Ko , options , [ ] , [ ] , [ ] , A , B , C , Q , R , X o );
end
```

D.1.4 Function for TWLQR output feedback minimization

```
function [ p1 , K ] = twlqrop ( A , B , C , Q , R , X o , K o )
```

```
[ n , m ] = size ( B );
[ l , n ] = size ( C ) ;
options = foptions ;
options ( 1 ) = 1 ;
options ( 13 ) = m * l ;
options ( 14 ) = 200 * m * l ;
[ p1 , K ] = constr ( ' twlqropk' , Ko, options ,[ ],[ ] , [ ] , A , B , C , Q , R , X o );
end
```

D.1.5 Sub-function for LQR state feedback minimization

```
function [ f , g ] = lqrk ( F , A , B , Q , R , X o )
[ n , m ] = size ( B ) ;
Ac = A + B * F ;
U2 = Xo ;
U1 = lyap( Ac , U2 );
Q0 = Q ;
Q1 = lyap ( Ac ' , Q0 + F' * R * F );
f = trace ( Q1 * U2 ) + trace ( U1' * ( Ac ' * Q1 + Q1 * Ac + Q0 + F' * R * F ) ) ;
dHQ1 = Ac * U1 + U1 * Ac ' + U2 ;
dHU1 = Ac ' * Q1 + Q1 * Ac + Q0 + F ' * R * F ;
gg = F + inv ( R ) * B ' * Q1 ;
g = [gg ; dHQ1 ; dHU1 ];
end
```

D.1.6 Sub-function for TWLQR state feedback minimization

```
function [ f , g ] = twlqrk ( F , A , B , Q , R , X o )

Ac = A + B * F ;

U3 = Xo ;

U2 = lyap ( Ac , U3 ) ;

U1 = lyap ( Ac , U2 ) ;

Q0 = Q ;

Q1 = lyap (Ac ' , Q0 ) ;

Q2 = lyap ( Ac ' , Q1 + F ' * R * F ) ;

f = trace ( Q2 * U3 ) + trace ( U1 ' * ( Ac ' * Q1 + Q1 * Ac + Q0 ) ) + trace ( U2 '* (

Ac ' * Q2 + Q2 * Ac + Q1 + F' * R * F ) ) ;

dHQ2=Ac*U2+U2*Ac'+U3;

dHQ1=Ac*U1+U1*Ac'+U2;

dHU1=Ac'*Q1+Q1*Ac+Q0;

dHU2=Ac'*Q2+Q2*Ac+Q1+F'*R*F;

gg = F + inv (R)*B'*(Q2+Q1*U1*inv(U2));

g = [ gg ; dHQ2 ; dHQ1 ; dHU1 ; dHU2 ];

end
```

D.1.7 Sub-function for LQR output feedback minimization

```
function [ f , g ] = lqrok ( F , A , B , C , Q , R , X o )

Ac = A + B * F * C ;

U2=Xo;

U1=lyap(Ac,U2);

Q0=Q;

Q1=lyap(Ac',Q0+C'*F'*R*F*C);

f = trace (Q1*U2) + trace (U1'*(Ac'*Q1+Q1*Ac+Q0+C'*F'*R*F*C));

dHQ1 = Ac * U1 + U1 * Ac ' + U2 ;

dHU1 = Ac' * Q1 + Q1 * Ac + Q0 + C' * F' * R * F * C;

gg = F + inv ( R ) * B '* ( Q1 * U1 ) * C '* inv (C*U1*C');

g = [gg ; dHQ1 ; dHU1 ];

end
```

D.1.8 Sub-function for TWLQR output feedback minimization

```
function [ f , g ] = twlqropk ( F , A , B , C , Q , R , Xo )

Ac = A + B * F * C ;

U3=Xo;

U2=lyap(Ac,U3);

U1=lyap(Ac,U2);

Q0=Q;

Q1=lyap(Ac',Q0);
```

```
Q2=lyap(Ac',Q1+C'*F'*R*F*C);

f = trace(Q2*U3)+trace(U1'*(Ac'*Q1+Q1*Ac+Q0))...

+ trace(U2'*(Ac'*Q2+Q2*Ac+Q1+C'*F'*R*F*C));

dHQ2=Ac*U2+U2*Ac'+U3;

dHQ1=Ac*U1+U1*Ac'+U2;

dHU1=Ac'*Q1+Q1*Ac+Q0;

dHU2=Ac'*Q2+Q2*Ac+Q1+ C'*F'*R*F*C;

gg = B'*(Q2*U2+Q1*U1)*C'+R*F*C*U2*C';

g = [ gg ; dHQ2 ; dHQ1 ; dHU1 ; dHU2 ];

end
```

D.2 Mode-assignment Minimization

D.2.1 Main Function for LQR and TWLQR mode-assignment minimization

```
clear all

%Load system A,B,C,D ;

%Define no of iterations, N

%Initilize different parameters as below

iter=0;diso=0;Xopt=[ ];

%Define the mode of the system to be assigned , s

%Evaluate the maximal rank matrices Vo and Wo

[ nn , mm ] = size( B ) ;
```

```
%Open the file to write the data
fd1=fopen ('file . m '.' wt +')
%Outer loop starts here
for it=1:N
%Give the initial guess for parameter vactor,p
        iter=iter+1;operat=1;
        for ii=1:n
                k=m*ii; j=m-1;
                P(k-j:k.ii)=p(:,ii);
        end
        V = Vo * P; W = Wo * P; GG0 =-W* inv (V); Ac = A + B * GG0 ;
        if  abs ( det (V)) < = eps
                fa=0;
                fprintf (fd1. '%7.0f  %22.12f    %1.0f;\n',iter,fa);
                operat=0;
        end
        if operat==1
                diso=diso+1;
                if diso==1
                        disp('closed loop poles after Opt');
                        eig (Ac);
                end
```

```
            if operat==1
                options = foptions ;
                options(1)=1; options(13)=n * m; options(14)=250 * m * n :
                [fa,x]=constr('fun',p,options,[ ],vlb,vub,A,B,Q,R,Wo,Vo, Xo );
                %fun=twlqrma                for TWLQR mode assignment
                %fun=lqrma                  for LQR mode assignment
                fprintf(fd1, '%7.0f %   22.12f  .0f; \n' , iter ,fa );
            end
            if  iter > = N;  break; end
        end
        fclose(fd1);
end
Xopt;fmin;pp=Xopt';
for ii=1:n
        k=m*ii; j=m-1;
        P(k-j:k,ii)=pp(:,ii);
end
pp1=P;V=Vo*pp1;W=Wo*pp1;GG0=-W*inv(V)Ac=A+B*GG0;
disp('closed loop poles after Opt');
eig(Ac)
```

D.2.2 Sub-function for LQR mode-assignment minimization

```
function [f , g] = lqrma ( p . A , B , Q , R , Wo , Vo , Xo )
```

```
[ n ,m]=size(B);
pp = zeros ( m * n , n);
p1=p';
for ii=1:n
        k=m*ii; j=m-1;
        pp(k-j:k,ii)=p1(:,ii);
end
V = Vo * pp ;
W = Wo * pp ;
F = - W * inv ( V ) ;
Ac = A + B * F ;
U2 = Xo ;
U1 = lyap (Ac,U2);
Q0= Q ;
Q1=lyap2(Ac',Q0+F'*R*F);
f = trace(Q1*U2)+trace(U1'*(Ac'*Q1+Q1*Ac+Q0+F'*R*F));
dHP = (Wo - F* Vo )'*(2*B'*(Q1*U1)+2*R*F*U1)*inv(V');
for ii=1:n
        k = m *ii; j=m-1;
        dHPI (:,ii ) = dHP (k-j : k ,ii);
end
dHPII (:) = dHPI;
```

```
g = dHPII;
end
```

D.2.3 Sub-function for TWLQR mode-assignment minimization

```
function [ f , g ] = twlqrma (p , A , B , Q , R , Wo , Vo , Xo )
[ n , m ]=size(B);
pp = zeros ( m * n , n );
p1=p' ;
for ii=1:n
        k = m * ii; j=m-1;
        pp (k-j: k , ii)=p1(:,ii);
end
V = Vo * pp ;
W = Wo * pp ;
F = - W * inv (V) ;
Ac = A + B * F ;
U3 =Xo;
U2 =lyap(Ac,U3) ;
U1 =lyap(Ac,U2) ;
Q0 = Q ;
Q1 = lyap ( Ac' , Q0 ) ;
Q2 = lyap (Ac' , Q1 + F ' * R * F ) ;
f = trace ( Q2 * U3 ) + trace ( U1 '* ( Ac ' * Q1 + Q1 * Ac + Q0 ) ) ...
```

```
+ trace ( U2 ' * ( Ac'*Q2+Q2*Ac+Q1+F'*R*F));
dHP = (Wo - F * Vo ) ' * ( 2 * B '*(Q1*U1+Q2*U2)+2*R*F*U2) * inv ( V' ) ;
for ii=1:n
        k=m*ii;j=m-1;
        dHPI (:,ii)= dHP ( k-j : k , ii);
end
dHPII (:)=dHPI;
g = dHPII;
end
```

D.2.4 Sub-function for Maximal Rank Computations

```
function [Vo,Wo] = maxrank (AA,BB,s)
%KERBASE computatizon of the Kernel bases Vo & Wo.
%       KERBASE(A) produces the Kernel bases of a given matrix A.
[nn,mm]=size(BB);
%Calculation of Ni, Solve [siI - A  B].Ni = 0
%Defining The matrix dimensions
Vo=zeros(nn,nn*mm);
Wo=zeros(mm,nn*mm);
M=zeros(2*nn+mm,nn+mm);
for ii=1:nn
        M(1:nn,1:nn)=s(ii,:).*eye(nn) - AA;
        M(1:nn,nn+1:nn+mm)=BB;
```

```
M(nn+1:2*nn+mm,:)=eye(nn+mm);

%M is Original matrix from where ker bases to be found

M1 = rot90(M);

A=M1;

[m,n] = size(A);

% Compute the default tolerance if none was provided.

if (nargin < 4), tol = max([m,n])*eps*norm(A,'inf'); end

% Loop over the entire matrix.

i = 1;  j = 1; k = 0;

[q,r]=size(BB');

while (i <= r) & (j <= r)

% Find value and index of largest element in the remainder of column j.

[p,k] = max(abs(A(i:m,j))); k = k+i-1;

if (p <= tol)

        % The column is negligible, zero it out.

        A(i:m,j) = zeros(m-i+1,1);j = j + 1;

elseif i ~= k

        % Swap i-th and k-th rows.

        A([i k],:) = A([k i],:);

end

% Divide the pivot row by the pivot element.

A(i,j:n) = A(i,j:n)/A(i,j);
```

```
        % Subtract multiples of the pivot row from all the other rows.
        for k = 1:m
                if k ~= i
                        A(k,j:n) = A(k,j:n) - A(k,j)*A(i,j:n);
                end
        end
        i = i + 1;    j = j + 1;
end
end
% Swaping :
for ll=1:q
        A([ll r+ll],:)=A([r+ll ll],:);
end
M2 = rot90(A);  M3 = rot90(M2);  M4 = rot90(M3);
kk = mm*ii; jj=mm-1;
Vo(:,kk-jj:kk)=M4(nn+1:2*nn,nn+1:nn+mm);
Wo(:,kk-jj:kk)=M4(2*nn+1:2*nn+mm,nn+1:nn+mm);
end
Wo=rot90(Wo);Wo=rot90(Wo);
```

D.3 Linear State Space Simulation

D.3.1 function for Linear State Space Simulation

```
clear all

%Load System A,B,C,D

% Load the control Gain Matrix K1 and K2 for LQR and TWLQR control design

Tm1=input('the diturbance at reference input 1');

Tm2= input('the diturbance at reference input 2');

%Defining simulation times:

offset=input('Enter the sampling time : ');

tf=input('Enter the final time : ');

%Defining Initial conditions :

XIC=zeros(1,n);

[n,m]=size(B);

D=zeros(1,m);ukk=[];Xc=[];

% Open loop Response :

Dst=[Tm1 Tm2];

Kop=zeros(m,n);

[Xc,uc,T]=simulat(A,B,C,D,Kop,Dst,XIC,offset);

Xop=[Xc];Uop=[uc];

%For LQR:

Dst=[Tm1 Tm2];
```

```
[Xc1,uk1,T]=simulat(A,B,C,D,K1,Dst,XIC,dft,offset);
[nn,mm]=size(Xc1);
X0 = Xc1(nn,:);
X1=[Xc1];U1=[uk1];
%For TWLQR:
Dst=[Tm1 Tm2];
[Xc1,uk2,T]=simulat(A,B,C,D,K2,Dst,XIC,dft,offset);
[nn,mm]=size(Xc1);
X2=[Xc1];U2=[uk2];
```

D.3.2 Sub-function for Simulation

```
function [X,U,T]=simulat(A,B,C,D,K,Dst,XIC,ft,offset)
X1=[];Y1=[];T=[];
U=[];Xc=[];X0=XIC;[n,m]=size(B);
for t=0:offset:dft
UK=K*X0';  tt=[t t+offset];
UKK=[(UK(1,1)+Dst(1,1)).*ones(length(tt),1)UK(2,1)+Dst(1,2)).*ones(length(tt),1)];
UKa=[UK(1,1).*ones(length(tt),1)   UK(2,1).*ones(length(tt),1)];
[Y x]=lsim(A,B,C,D,UKK,tt,X0);
X0=x(length(tt),:);  Xc=[Xc;X0]; T=[T t];
UKK1=UKa(length(tt),:);
U=[U ; UKK1];
end
```

D.4 Initial Conditions for SMIB model

```
clear all

%Machine data

M=9.26; Tdop=7.76; D=eps: xd=0.973; xdp=0.190; xq=0.550;

f=60; Wo=2*pi*f;


%excitation Ke=50; Te=0.05:

%GovernorTg=1;Kg=1;


 %Line & load R=-0.034; X=0.997; G=0.249; B=0.262;

 %Initial state Peo=1; Qeo=0.015; Vto=1.05;


%Calculation of different constants

C1 =1+R*G-X*B ;C2 = X*G +R*B ;

R1 = R-C2*xdp;R2 = R-C2*xq;

X1 = X+C1*xq;X2 = X+C1*xdp;

Ze2 = R1 * R2 + X1 * X2 ;

Yd = ( C1 * X1 - C2 * R2 ) / Ze2 ;

Yq = ( C1 * R1 + C2 * X2 ) / Ze2 ;


Vdo=Peo*Vto*i/(sqrt(Peo^2+((Qeo+Vto^2/xq)^2)))

Vqo=sqrt(Vto^2-Vdo^2)
```

```
Iqo=Vdo/xq

Ido=(Peo-Iqo*Vqo)/Vdo

Ido=(Qeo+Iqo*Vdo)/Vqo

eqpo=Vqo+xdp*Ido

Vod=C1*Vdo-C2*Vqo-R*Ido+X*Iqo

Voq=C2*Vdo+C1*Vqo-X*Ido-R*Iqo

del0=atan(Vod/Voq)

delli=del0*180/pi

Vo=sqrt(Vod^2+Voq^2)


sd = sin(del0) ;cd = cos(del0) :

Fd = (Vo/Ze2)*(-R2*cd+X1*sd) ;

Fq = (Vo/Ze2)*(X2*cd+R1*sd) ;


%Calculation of constants K1 ... K6

K1 = Fd*(xq-xdp)*Iqo + Fq*(eqpo+(xq-xdp)*Ido)

K2 = Iqo + Yd*(xq-xdp)*Iqo + Yq*(eqpo+(xq-xdp)*Ido)

K3 = inv(1+(xd-xdp)*Yd)

K4 = (xd-xdp)*Fd

K5 = [Fd Fq]*[-xdp*Vqo/Vto : xq*Vdo/Vto]

K6 = Vqo/Vto + [Yd Yq]*[-xdp*Vqo/Vto ; xq*Vdo/Vto]
```

```
%System coefficient matrix A

A=[ D/M   -K1/M      -K2/M          0       1/M;
   Wo    0          0            0       0;
   0    -K4/Tdop   -1/(K3*Tdop)   1/Tdop  0;
   0    -Ke*K5/Te  -Ke*K6/Te      -1/Te    0;
  -Kg/Tg 0          0            0       -1/Tg];

%for  excitation control
 B=[0 0 0 Ke/Te 0]';
%for  coordinated control
B=[0 0 0 Ke/Te 0;
   0 0 0 0    Kg/Tg]';
%Output variable
C=[1 0 0 0 0];
%Torque disturbance input matrix
Btd=[-1/M  0  0   0     0 ]'
%Reference voltage disturbance input matrix
Bvd= [0   0  0   Ke/Te  0 ]';
```

Nomenclature

State Space Notations

A	Matrix coefficient of state variables of system without control
A_C	Matrix coefficient of state variables of system with optimal control
B	Matrix coefficient
Q	State variable weighting matrix
R	Control signal weighting matrix
J	Performance Index
x	State Variable Vector
u	Control Input Vector
s	Laplace operator
o	Subscript denoting an initial condition
T	Superscript denoting matrix transpose
Δ	Prefix denoting a linearized variable

System Parameters (in per unit)

ω_o	Synchronous speed (377 rad / sec)
δ	torque angle
ω	speed (rad / sec)
D	Damping coefficient

M	Inertia constant, $M=2\ H$
x_d	Direct axis synchronous reactance
x_d'	Direct axis transient reactance
x_q	Quadrature axis synchronous reactance
T_{do}'	Open-circuit time constant of the field
V_F	Exciter field voltage
K_A	Exciter gain
T_A	Exciter time constant
T_g	Governor time constant
K_g	Governor gain
P_m	Mechanical power input
P_e	Generator active power output
Q_e	Generator reactive power output
$[\ ]_o$	Steady state operating value of [ ].
e_q'	Voltage behind transient reactance of the machine at q- axis
V_o	Infinite bus voltage
V_t	Machine terminal voltage
V_{tr}	Terminal reference voltage
V_d	Direct-axis component of V_t
V_q	Quadrature-axis component of V_t
I	Generator stator current
I_d	Direct-axis component of I

I_q Quadrature-axis component of I

ψ_F field flux linkage

$r_l + j\,x_l$ Equivalent series impedence of transmission system

$g + j\,b$ Terminal load admittance

u_E Excitation control input

u_g Governor control input

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