Second-Law-Based Thermoeconomic Analysis and Optimization of Thermal-Energy-Storage Systems

by

Muammar Abdullah Al-Naglah

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

MECHANICAL ENGINEERING

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This thesis, written by

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Date : 21 - 7 - 97

This Thesis is Dedicated to

My Father for keep pushing me to the best

and to

My Mother for the love she gave to me

ACKNOWLEDGMENT

I must thank ALLAH almighty. Without the will and help of ALLAH I would not have finished this work.

I would like to offer my indebtedness and appreciation to my thesis committee chairman and advisor Dr. Syed M. Zubair who has been the main source of help and encouragement to complete this thesis. He was always available when I needed. I also appreciate the cooperation and support extended by Dr. B. S. Yilbas and Dr. A. K. Sheikh who served as committee members.

I am also indebted to the department chairman, Dr. Mohammed O. Budair and other faculty members for their support and encouragement during my undergraduate and graduate studies.

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THESIS ABSTRACT

NAME OF STUDENT	:	*MUAMMAR ABDALLAH AL-NAGLAH*
TITLE OF STUDY	:	*Second-law-based thermoeconomic analysis and optimization of thermal-energy-storage systems*
MAJOR FIELD	:	*Mechanical Engineering*
DATE OF DEGREE	:	*June, 1997.*

A closed-form model for the second-law-based thermoeconomic analysis and optimization of thermal-energy-storage systems is derived and discussed. The analytical procedure is generalized since it is applicable to all types of thermal-energy-storage systems that do not have chemical reactions. The derived procedure is applied on three types of energy-storage systems, namely: Sensibly Joulean-heated, sensible-heat, and latent-heat, thermal-energy-storage systems. In the analysis, a complete second-law derivation is given for each one of the three systems to obtain the entropy generated (or irreversible losses) in each of the storage and removal processes. Monetary values are attached to the irreversible losses caused by the finite temperature difference heat transfer and pressure drop in the storage system. The systems analyzed are optimized using a new performance criterion described as the Cost Rate Number. This number relates the cost rate of all the irreversible losses to the cost rate that is supplied to the system. The cost rate number is minimized with respect to number of transfer units, in addition to other important variables of the systems. The effect of unit cost parameters on the performance of all three types of storage systems are discussed in detail.

MASTER OF SCIENCE DEGREE

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

Dhahran, Saudi Arabia

June, 1997

خلاصة الرسالة

الاسم : معمر عبدالله النقلة

عنوان الرسالة : التحليل الاقتصادي الحراري القائم على القانون الثاني للخزانات الحرارية

التخصص : هندسة ميكانيكية

تاريخ الشهادة : يونيو ١٩٩٧

في هذا البحث يتم اشتقاق ومناقشة صيغة لتحليل وايجاد الحل الأنسب للاقتصادية الحرارية القائمة على القانون الثاني للخزانات الحرارية. هذه الصيغة تعمم بشكل يجعلها قابلة للتطبيق على جميع أنواع الخزانات الحرارية التي لا تحتوي على تفاعل كيميائي. يتم تطبيق المراحل الاشتقاقية على ثلاثة أنواع من الخزانات الحرارية وهي الخزانات التي تسخن بالكهرباء، الخزانات متغيرة درجة الحرارة والخزانات التي تخزن بها الطاقة بشكل طاقة كامنة. في المناقشة يتم اشتقاق كامل للقانون الثاني للخزانات الحرارية الثلاث وذلك لايجاد قيمة الإنتروبية المتولدة. يتم ارفاق قيم مالية الى الإنتروبية المتولدة وبعد ذلك يتم ايجاد القيمة المثلى باستخدام رقم جديد يربط بين القيمة المفقودة والقيمة الممولة للنظام. يتم ايجاد القيمة الصغرى للرقم بالنسبة للمتغيرات المهمة. كما يتم دراسة تأثير القيم المالية على أداء النظام.

درجة الماجستير في العلوم

جامعة الملك فهد للبترول والمعادن

الظهران - المملكة العربية السعودية

يونيو ١٩٩٧

CHAPTER 1

INTRODUCTION

Developing techniques for designing efficient and cost-effective energy systems is one of the foremost challenges that energy engineers face today. In a world with finite natural resources and increasing energy demand by developing countries, it becomes increasingly important to understand the mechanisms which degrade energy and resources and to develop systematic approaches for improving the design of energy systems which can help reduce the impact on the environment.

Thermal-energy storage has a great importance in applications where thermal energy generation and consumption occur at different times. These storage systems may operate by changing the temperature of the heat-storage medium (sensible-heat storage), or by heat transfer to a phase change material (latent-heat storage). Typically, the design and rating of thermal-energy-storage systems is solely based on the first law of thermodynamics. From the first-law perspective, the efficiency of a thermal-energy-storage system can be assessed in terms of how much thermal energy the system can store. This approach produces generally workable designs, but not necessarily those with the highest-thermodynamic efficiencies. This led Bejan (1982a) to define an optimum

heat-transfer system as the least irreversible system that the designer can afford and to develop thermal-design techniques based on minimization of entropy generation.

In a pioneering study, Bejan (1978b) applied his second-law techniques to the analysis of a sensible heat, thermal-energy-storage system. The power of his analysis resides in Bejan's insight that the primary purpose of a 'thermal energy storage system' is not, as the name implies, to store energy, but rather, to store useful work, that is, thermodynamic availability. Thus, his approach is based on minimizing the destruction of thermodynamic availability (or exergy) as opposed to maximizing the total amount of thermal-energy stored. Bejan (1978b) considered the same scheme in almost all thermodynamic analysis of both sensible and latent-heat, thermal-energy-storage systems.

Although the analyses of Bejan and others have firmly established the need to employ second-law techniques to design thermodynamically efficient thermal-energy-storage systems, these studies do not attach monetary values to the component irreversibilities generated in these systems. The second law of thermodynamics combined with economics represents a very powerful tool for the systematic design and optimization of energy systems. This combination forms the basis of the relatively new field of second-law-based thermoeconomic analysis, which is further explored in this thesis with respect to optimization of thermal-energy-storage systems, i.e., Joulean, sensible-heat, and latent-heat storage systems. In this regard, a cost rate ratio number is introduced to study the optimal performance of these systems.

A background and an extensive literature review on thermal-energy-storage systems is given in chapter 2. In this chapter, a general discussion and literature review on thermal-energy storage is described first and then each storage system is described in detail. It is then followed by background and discussion on previous work related to thermoeconomic analysis of thermal systems.

Chapter 3 describes a general scheme of thermoeconomic analysis that can be used to study all types of thermal-energy-storage systems. The validity and applicability of the procedure discussed in this chapter is explained by applying it to three examples of thermal-energy-storage systems in the following three chapters.

Chapters 4, 5 and 6 present thermoeconomic analysis and optimization of sensibly Joulean heated, sensible-heat, and latent-heat, thermal-energy-storage systems, respectively. The general structure of these three chapters is the same, that is; a general description of the system to be analyzed is described first and it is then followed by a second-law-based thermoeconomic analysis and optimization of the storage system. To optimize the storage systems, numerical solution for each variable to be optimized is given with respect to the important economic parameters on the system performance.

Finally, chapter 7 gives conclusions and recommendations about the use of the derived thermoeconomic scheme systems and its application to the three examples of thermal-energy storage.

CHAPTER 2

BACKGROUND AND LITERATURE REVIEW

2.1 Thermal-Energy-Storage Systems - A General Background

The recent books on thermal-energy storage by Schmidt and Willmott (1981) and Beckmann and Gilli (1984) are indicative of the continuing interest in this field of thermal engineering. These books summarize much of the literature on thermal-energy-storage systems for a wide variety of residential-, commercial-, and industrial-scale applications. On the other hand, the thermodynamic analysis and optimization of thermal-energy-storage systems were addressed in many papers and books where sensible heat, sensible heat with Joulean heating and latent-heat, thermal-energy-storage systems are fully described and analyzed based on the second law of thermodynamics (e.g., Bejan, 1978b, 1982a, 1988; Schmidt and Willmott, 1981; Krane, 1985a, 1985b; De Lucia and Bejan, 1990, 1991; Adebiyi, 1991; Aceves-Saborio et al., 1992; Charach and Zemel, 1992; Rosen, 1992).

The sensible heat storage, the latent heat storage and even combined sensible/latent heat storage systems are all, in essence, regenerative heat exchangers (or periodic flow regenerators). There is a reasonable success already in respect of design and rating of the periodic flow regenerators. A notable benchmark in setting out a design theory for

regenerators is due to Coppage and London (1953). More recently, Schmidt and Willmott (1981) have provided comprehensive thermal design theory for regenerators and thermal-energy-storage systems, although from the first law perspective, only. An update on thermal design theory for regenerators is also given by Shah (1981).

The impetus for a design theory and modeling of thermal energy storage systems was provided in recent times mostly by efforts to develop cost-effective solar energy systems. Duffie and Beckmann (1980) give an excellent overview of the modeling approaches that have been quite successfully applied, especially for the relatively low temperature thermal-energy-storage systems. A number of other researchers (Hughes et al., 1976; Schmidt et al., 1977; and Morrison and Abdel-Khalik, 1978) have presented design procedures which may be employed for either sensible or latent heat storage systems.

Adebiyi and Russell (1987) stated that the set of equations governing the behavior of periodic flow regenerators typically comprises the equations for material balances, momentum conservation (for the fluid media) and thermal energy balances for both the fluid media and the storage medium. By making certain reasonable assumptions, these equations may be reduced to a simpler set of partial differential equations which often include those of the hyperbolic type. Even with the simpler set of equations, a computer based numerical scheme is frequently the only viable route to the solution of the regenerator problems.

It should be noted that a second law analysis is of immense practical value in modeling of a thermal energy storage system especially as regards optimization of the storage medium configuration relative to fluid flow through the system. An excellent demonstration of how this can be accomplished is given by Bejan (1977) for counter flow heat exchangers with the heat exchanger geometric parameters optimized on the basis of minimum irreversibility. The analysis does not eliminate the need for the traditional analysis based on the first law. Rather, it makes the first law analysis more purposeful by helping to resolve the first major design consideration which has to do with the configural aspects of the heat exchanger design. One may mention also in passing that a second law analysis could serve as additionally as thermoeconomic analysis (Reistad and Gagioli, 1980; Wepfer, 1980; and El-Sayed and Tribus, 1983).

2.2 Thermal-Energy-Storage Systems with Joulean Heating of the Storage Element

2.2.1 Background

The operation of electrically-heated thermal energy storage systems differs from that of other types of thermal energy storage systems in an important way, namely, their storage and removal cycles are rigidly constraints since they must coincide with the off-peak and peak periods of electrical power usage. Typically, storage is accomplished during an eight hour off-peak period at night and the removal takes place during the remaining 16 hours of the day, when the demand for electrical power increases.

In the sensible-heat, thermal-energy-storage systems with Joulean heating of the storage element, the storage element is a solid body that is heated up and cooled down without phase change. The sensible part of enthalpy determines its storage capacity.

Figure (2.1) illustrates the temperature history of the storage element during a typical storage cycle (Krane, 1985b). The symbols are defined in the nomenclature section. Storage is accomplished by dissipating electrical energy in the Joulean heater depositing it in the storage element as sensible thermal energy. The temperature of the storage element rises continuously until it reaches its maximum value, T_{FS}, at time, t_S, ending the storage process. Thermal energy is removed by a heat exchanger passing through the storage element. A cold ideal gas or any other working fluid passes through the heat exchanger. This makes the temperature of the storage element to decrease continuously until it reaches its minimum value, T_{FR}, at the completion of a single cycle at time, t_R.

2.2.2 Literature Review

The desire to design thermal systems with the highest possible thermodynamic efficiencies has resulted in an emerging consensus in the heat transfer community to adopt new design practices that are based on the second law of thermodynamics. This trend is reflected in the expanding literature on second law analysis, much of which is summarized in the recent books by Bejan (1982b), Moran (1981) and Ahern (1980). The new design techniques are based on Bejan's definition (1982a) of an optimal thermal system as the least irreversible system that the designer can afford.

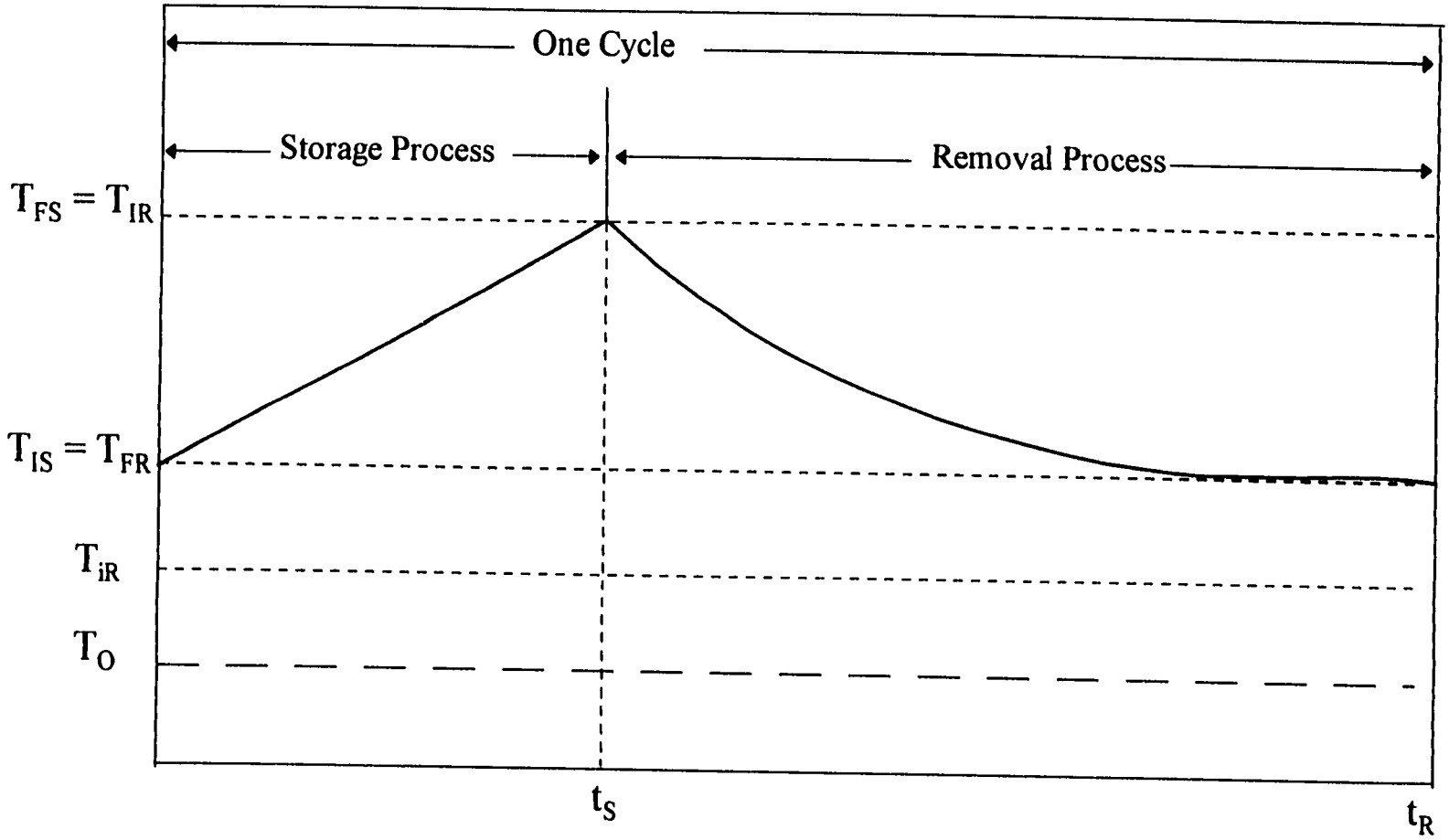

Figure 2.1 Temperature history of the storage element of sensible heat thermal energy storage systems with Joulean Heaters

A second-law analysis of sensible-heat, thermal-energy-storage system with Joulean heating of the storage element was demonstrated by Krane (1985b). He extended the second-law methodology developed for the sensible heat storage and applied it to the Joulean heating system. His work was focused on a system that uses low-cost, off-peak electrical energy to heat a well-insulated mass of material with a large capacity of storing sensible thermal energy. The stored thermal energy is extracted at a later time during a period of peak electrical power usage.

In Krane's analysis (1985b), optimization criterion was based on the ratio of availabilities, or what is called the entropy generation number (N_S), given by,

$$N_s = \frac{\textit{Total availability destroyed during an entire cycle}}{\textit{Total availability supplied during an entire cycle}} \tag{2.1}$$

This definition was also used by Bejan (1978b) to determine the optimization criterion of sensible heat thermal energy storage systems. The minimum the entropy generation number, the better the system performance is.

Krane (1985b) showed that from the stand point of classical thermodynamics, Joulean heaters are extremely inefficient devices. Although their first law efficiency is unity, they typically destroy a large percentage of the thermodynamic availability of their electrical power input. Thus, the thermodynamic performance of thermal energy storage systems which employ Joulean heaters is potentially very poor and such systems should receive

the closest possible scrutiny from the viewpoint of second law analysis before this technology is implemented on a large scale.

2.3 Sensible-Heat, Thermal-Energy-Storage Systems

2.3.1 Background

By maintaining a pressure above saturation pressure "pressurization" or a temperature below saturation temperature "subcooling", boiling of the liquid storage medium is prevented. Normally, the system is kept at constant pressure. Pressurization may be carried out by the atmosphere (for hot water storage below 100° C or for high boiling liquids), by pressurized inert gas, or by a two phase pressurizer maintaining the liquid pressure in a steam cushion by evaporating some liquid or condensing some steam (Beckmann and Gilli, 1984). The storage capacity is provided by changing only the sensible part of the enthalpy; this type of storage is therefore based on liquid phase only.

There is a large number of storage media suited for thermal storage of subcooled liquids than for liquids at saturation. Liquids with a boiling temperature (at atmospheric pressure) above 100°C are usually called "high boiling" (Beckmann and Gilli, 1984).

The working temperature is, in any case, between melting temperature and boiling temperature. For liquids with high viscosity at low temperatures (such as high

temperature oils), the minimum working temperature may be considerable above their melting temperature, and it may be given by the limits of pumping power. Figure (2.2) shows a storage element temperature history for a complete storage cycle. Thermal energy is stored by passing hot fluid through a heat exchanger impeded in the storage element as a sensible heat until the time reaches t_S. In the same way heat is removed but by passing cold fluid through the heat exchanger until the storage element temperature reaches the same temperature at the beginning of the storage process ending a one complete cycle.

Possible applications of this type of storage include processes with heat transmission by means of subcooled liquid or process cycles of which one step consists of heating up the working medium, such as feed water in steam power stations. So far, hot water storage in district heating systems, storage for feed water heating in steam power stations, and storage of hot oil or molten salts in solar power stations- where heat transfer medium and storage medium are identical- have been realized in practice (Krane, 1985a).

2.3.2 Literature Review

In a series of work, Bejan (1978b, 1982a, 1987, 1988) considered the minimization of entropy generation in sensible-heat-storage systems that are designed for the subsequent production of power. However, when Bejan analyzed the sensible heat storage systems (1978b), he did not consider the complete storage-removal cycle, but rather, he

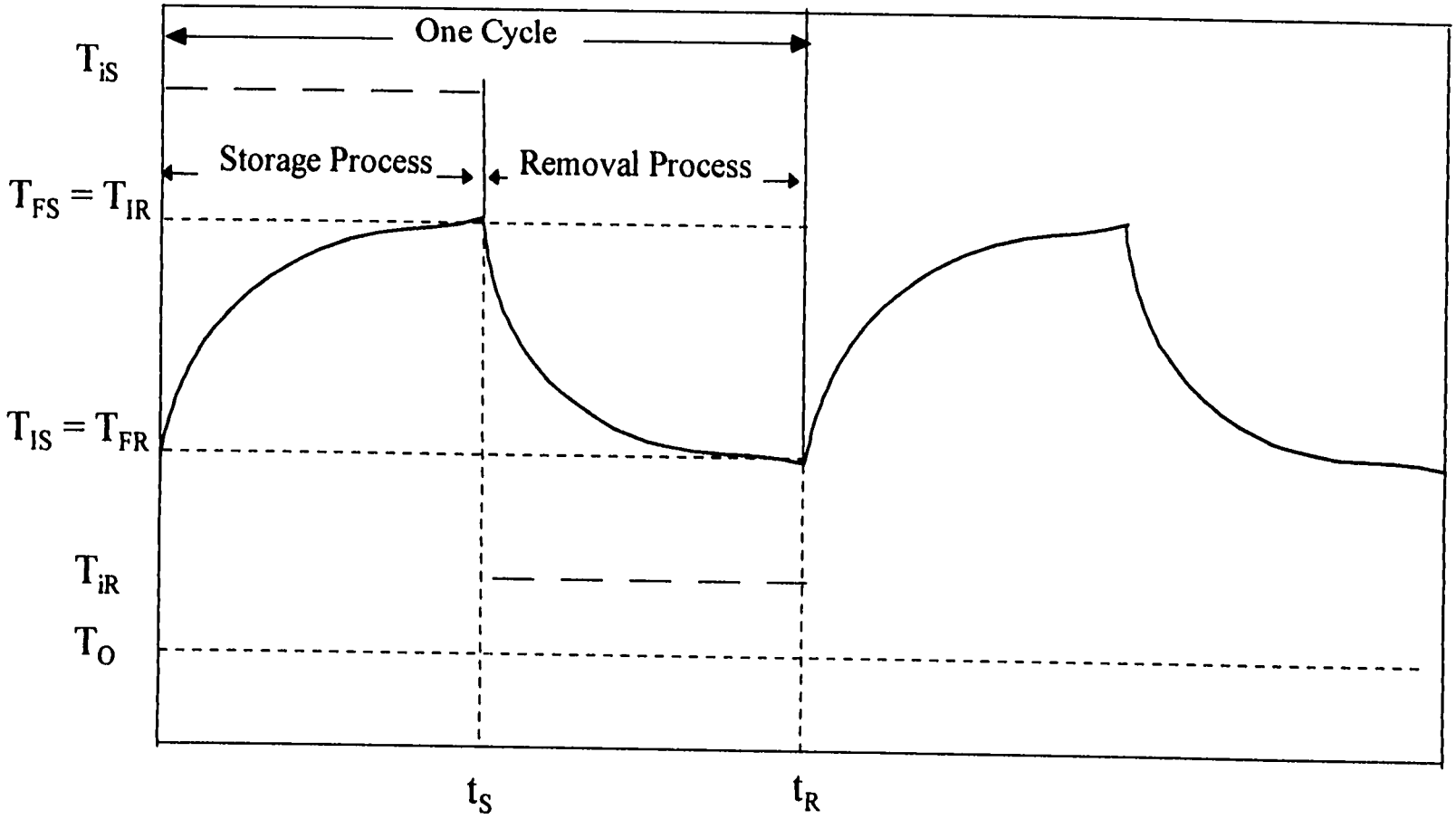

Figure 2.2 Temperature history of the storage element of sensible heat thermal energy storage systems

considered the storage process alone. This would give different sizes of the storage system than the complete storage cycle.

Krane (1985a) extended Bejan's analysis of a sensible-heat-storage system by considering an entire storage-removal cycle. In Bejan (1978b, 1982a, 1987, 1988) and Krane (1985a), sensible heat storage systems are addressed, by modeling the storage unit as "lumped" elements interacting with a heat-transfer fluid. These models account only for the external irreversibilities due to the heat transfer and fluid flow and its interaction with the storage element.

In Krane's analysis (1985a) for the sensible heat thermal energy storage system as well as Bejan's analysis (1978b), they used the entropy generation number [equation (2.1)] as the optimization criterion.

2.4 Latent-Heat, Thermal-Energy-Storage Systems

2.4.1 Background

Latent heat storage systems have several advantages, compared with sensible heat storage systems such as rock beds or water tanks, of greater storage capacity per unit weight or volume, less control complexity, and greater system efficiency.

Latent heat systems also utilize some sensible-heat storage above and below the phase change material melting point. The storage capacity of such system may be expressed as (Adebiyi and Russell, 1987)

$$Q = M\left[xL_{ph} + c_P\left(T_{ph} - T_{IS}\right) + \left(T_{FS} - T_{ph}\right)\right] \tag{2.2}$$

In the above equation, the first term represents the latent heat; the second term, the sensible heat below the melting point; and the third term, the sensible heat above the melting point. In practical phase change material systems the latent heat term predominates.

The storage capacity of latent heat is based not on temperature change but on phase change of the storage medium at constant temperature.

The phase change from the liquid state to the gaseous phase involves the latent heat. However, the volumetric storage capacity of the vapor phase is rather low. Therefore, this type of latent heat storage has not been used or suggested. For the most part, latent heat storage is understood to mean the storage of the melting heat which involves only small volume changes. Sometimes, the solid-liquid phase change is combined with a solid-solid phase change at a temperature somewhat below the melting point. Figure (2.3) shows a typical temperature history of the storage element of a latent-heat storage system excluding the sensible part of the cycle. It works the same way as sensible heat systems except the temperature of the storage element remains constant because thermal energy is stored as latent heat.

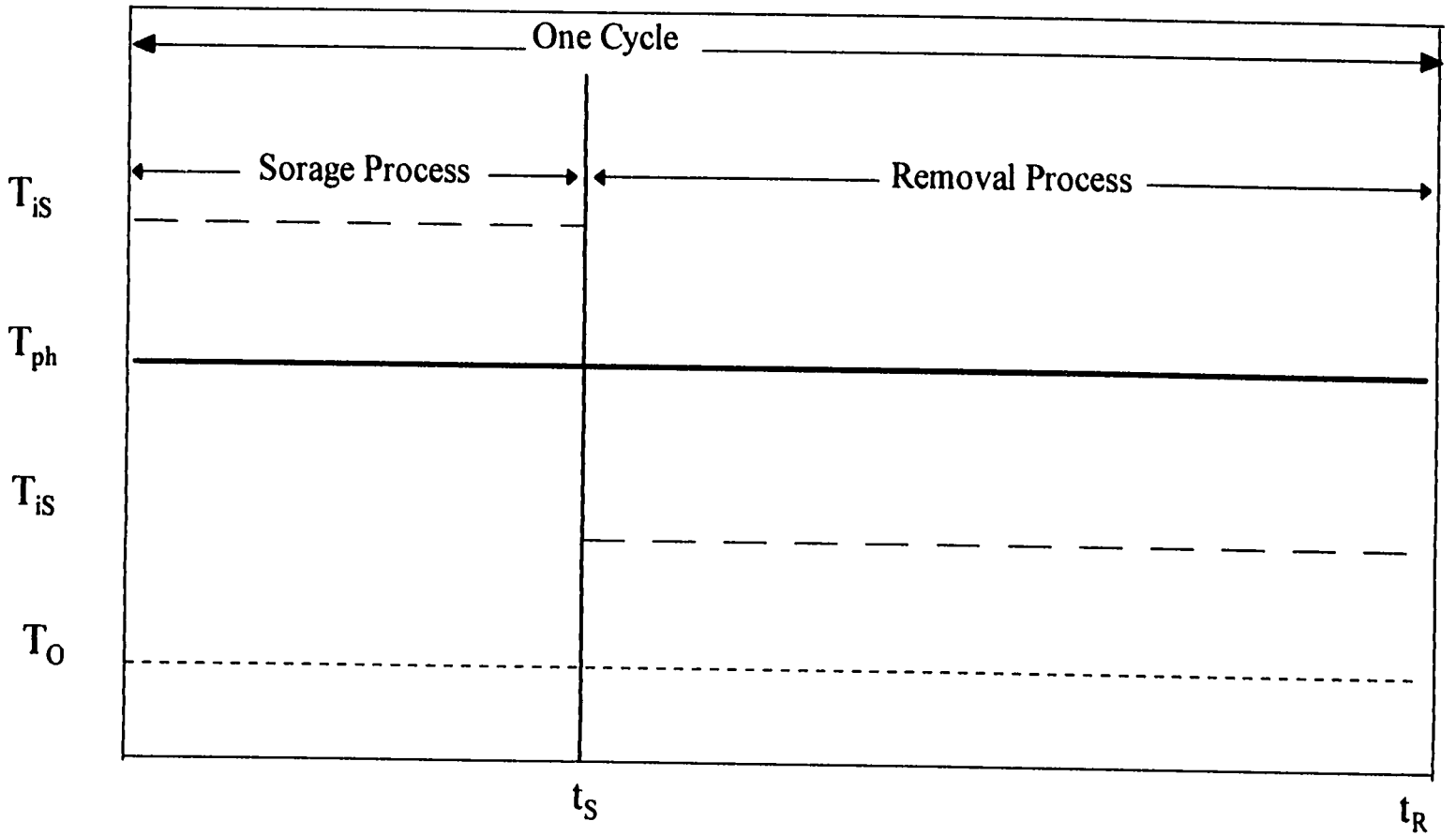

Figure 2.3 Temperature history of the storage element of latent-heat thermal-energy-storage systems

It is often suggested that use be made of the sensible heat of the liquid and/or solid phase in addition to the latent heat. This will indeed increase the heat capacity but it will also remove the advantage of heat supply at constant temperature.

Contrary to the previously discussed storage systems which are more or less state of the art, latent heat storage systems are still in the development stage. The incentives are: high heat capacity, constant temperature and low pressures. The problems are: low heat transfer rate between the storage medium and the working medium, high cost of heat exchangers, difficulty in finding suitable temperature levels and controlled atmosphere.

In choosing a latent heat storage material, Lane (1983), suggested the following properties:

1. Thermal properties
    a) Suitable phase-transition temperature
    b) High latent heat of transition
    c) Good heat transfer
2. Physical properties
    a) Favorable phase equilibria
    b) Low vapor pressure
    c) Small volume change
    d) High density
3. Kinetic properties

a) No supercooling

b) Sufficient crystallization rate

4. Chemical properties

a) Long-term chemical stability

b) Compatibility with materials of construction

c) No toxicity

d) No nuisance factor

e) No fire hazard

5. Economics

a) Abundance

b) Availability

c) Cost-effectiveness

2.4.2 Literature Review

Second-law analyses of the phase-change energy storage were first performed by Bjurstrom and Carlsson (1985) and by Adebiyi and Russell (1987). Both studies were based on the "lumped" system model, in which the details of the conduction phase-change process are overlooked. The most fundamental thermodynamic result of the lumped-system analysis is that the storage of exergy is maximized when the melting point of the phase change material is the geometric average of the energy source temperature (T_{iS}) and the environment temperature (T_O).

Farid and Kanzawa (1989) performed mathematical modeling to show the performance of a heat storage model using phase-change materials with different melting temperatures.

De Lucia and Bejan (1990) have recently attempted to consider the thermodynamic analysis of latent heat storage from the lumped element level to the continuum level. They considered one dimensional planar melting in a slab induced by a thermal contact with a stream of hot gas. Both conduction-controlled and convection-controlled phase change processes have been considered. The gas has been assumed to be very well mixed. Consequently, the instantaneous gas temperature in the heat transfer volume, adjacent to the phase change material, has been considered as spatially uniform, equal to the temperature at which the gas leaves this volume. The thermodynamic imperfections, accounted for in De Lucia and Bejan (1990), include the finite temperature gradients along the heat transfer fluid stream and the heat transfer from the gas through the phase change material to the moving interface. The analysis has been restricted to the quasi-steady regime corresponding to small Stefan numbers, and irreversibilities due to the pressure drop have been neglected. As a figure of merit for the system, the authors have defined the number of entropy generation units which is the ratio of the exergy destroyed during a given time interval to the total exergy contained in the hot fluid used during that period of time. Minimization of this quantity with respect to the melting point of the phase change material yields the geometric mean of the inlet and the dead state temperatures for the optimum melting temperature which is in line with the conclusions

of Bjurstrom and Carlsson (1985). The dependence of the entropy generation number on charging time and on the number of heat transfer units has also been investigated.

In a subsequent paper De Lucia and Bejan (1991) have extended the analysis of De Lucia and Bejan (1990) to the complete charge-discharge cycle of operation. For the melting (charging) stage, a numerical solution that fully accounts for the sensible heat, has been developed. Thus, the analysis was no longer restricted to the quasi-steady limit. It has been found that the sensible heat contributions entail deviations of the optimal melting temperature from the simple geometric mean law of Bjurstrom and Carlsson (1985) and De Lucia and Bejan (1990). Furthermore, a quasi steady solution has been developed for the complete cycle. The results demonstrate that the contribution of the irreversible losses of the removal process introduce deviations from the optimum phase change temperature calculated by the geometric mean of the inlet and the dead state temperatures, even for systems operating within the quasi-steady regime.

Aceves-Saborio et al. (1992) analyzed the lumped system of the latent-heat, thermal-energy-storage system using optimum efficiencies to obtain phase-change temperatures. From their analysis they concluded that the optimum phase change temperature is equal to the geometric mean of the inlet and ambient temperatures.

2.5 Thermoeconomic Perspectives

2.5.1 Background

A full assessment of an energy storage plant includes not only the economic comparison of several possible storage and peaking systems but also an optimization of the size of one or more storage systems in an energy supply system. Sometimes, reasons other than economic ones (flexibility, availability, operational advantages) will be decisive for the installation of storage systems.

It should be noted that the optimization of a storage plant will need even more care and knowledge of the energy demand and its time-dependence than the optimization of a pure conversion plant without storage. With an energy storage plant, not only the discharge power but also the storage capacity and the charge power are parameters that must be selected. Although typical cases may be treated in a general way, the number of parameters is such that each investment decision has to be considered on its own merits.

Installation costs of a storage plant consist of a part which depends on capacity and a part which depends on (charge and discharge) power (Beckmann and Gilli, 1984):

1. Capacity dependent cost
    - a) storage vessel with liner, thermal insulation, support, etc.
    - b) Storage medium

2. Discharge-power-dependent cost
   a) discharge equipments
   b) discharge piping
   c) heat disposal system
3. Charge-power-dependent cost
   a) charge equipment
   b) charge piping
   c) compression engine and motor

The remaining cost items such as buildings, foundations, control, installation, commissioning, etc. can not easily be allocated to the above categories.

2.5.2 Literature Review

During the sixties, Evans and Tribus (1966) conducted pioneering work in the field of thermoeconomy using sea-water-desalination systems as an illustration. A similar approach was used by Loper (1979) to determine the quantities of insulation required for any thermal system having multiple paths for heat exchange between the system and the environment. However, the comprehensive effort to apply thermoeconomics systematically to the analysis, optimization and design of energy systems did not start until the eighties. New methodologies have flourished, giving rise to new concepts with their own definitions and applications. London (1982) and London and Shah (1983) discussed in detail the thermodynamic irreversibilities that exist in any real engineering system. They also demonstrated a method to attach monetary values to the component

irreversibilities generated in the condenser of a powerplant with the purpose of developing various tradeoff factors.

In related contemporary work, Frangopuolos (1984) and Frangopuolos and Evans (1984) have placed Utility Functional Analysis on a rigorous mathematical formulation for thermal systems under the name "Thermoeconomic Functional Analysis". Von Spakovsky and Evans (1989, 1990) used a second law costing method for the optimum detailed design of thermal system components in a complex thermal system in which the energy-dissipation cost was balanced against capital cost. Detailed sizing rules for two-phase heat exchangers with respect to thermodynamic and economic optimization were developed by Zubair et al. (1985b). Recently, Badar et al. (1993) performed second-law-based thermoeconomic optimization on a sensible-heat, thermal-energy-storage system by considering only the storage process. Badar et al. (1993) showed in his analysis that the problem can be converted to thermodynamic optimization problem by neglecting the effect of the cost parameters. In a subsequent paper, Badar and Zubair (1995) described the same system of Badar et al. (1993) but with a numerical example to show how the procedure of thermoeconomic analysis can be applied. They also showed the sensitivity of various unit cost parameters in their example problem.

CHAPTER 3

THERMOECONOMICS OF THERMAL ENERGY STORAGE SYSTEMS

Most of the analytical studies of thermal energy storage systems are based on the second law of thermodynamics. Badar et al. (1993) demonstrated the application of thermoeconomic analysis on sensible heat thermal energy storage systems by considering the storage process alone. The analysis is based on Bejan's second law analysis (1978b) and they have considered capital cost of the system, in addition to unit cost parameters associated with irreversible losses. One can observe from Badar et al. analysis that Bejan's results can be obtained when the unit costs of lost work associated with heat transfer and pressure drop are equal and other monetary parameters are set to zero.

3.1 A Scheme for the Analysis

The energy dissipations are formulated from balances of the second law quantities "exergy", where the loss of these quantities are defined as "Lost Work", $T_O\dot{S}$. Here $\dot{S}$ is the amount of entropy generation and T_O is the absolute temperature of the environment. The cost rate of energy dissipation may usually be found as a function of $(T_O\dot{S})$ as shown in Figure (3.1). Here the cost rate is represented as $\dot{\Gamma}$. The realistic design range is

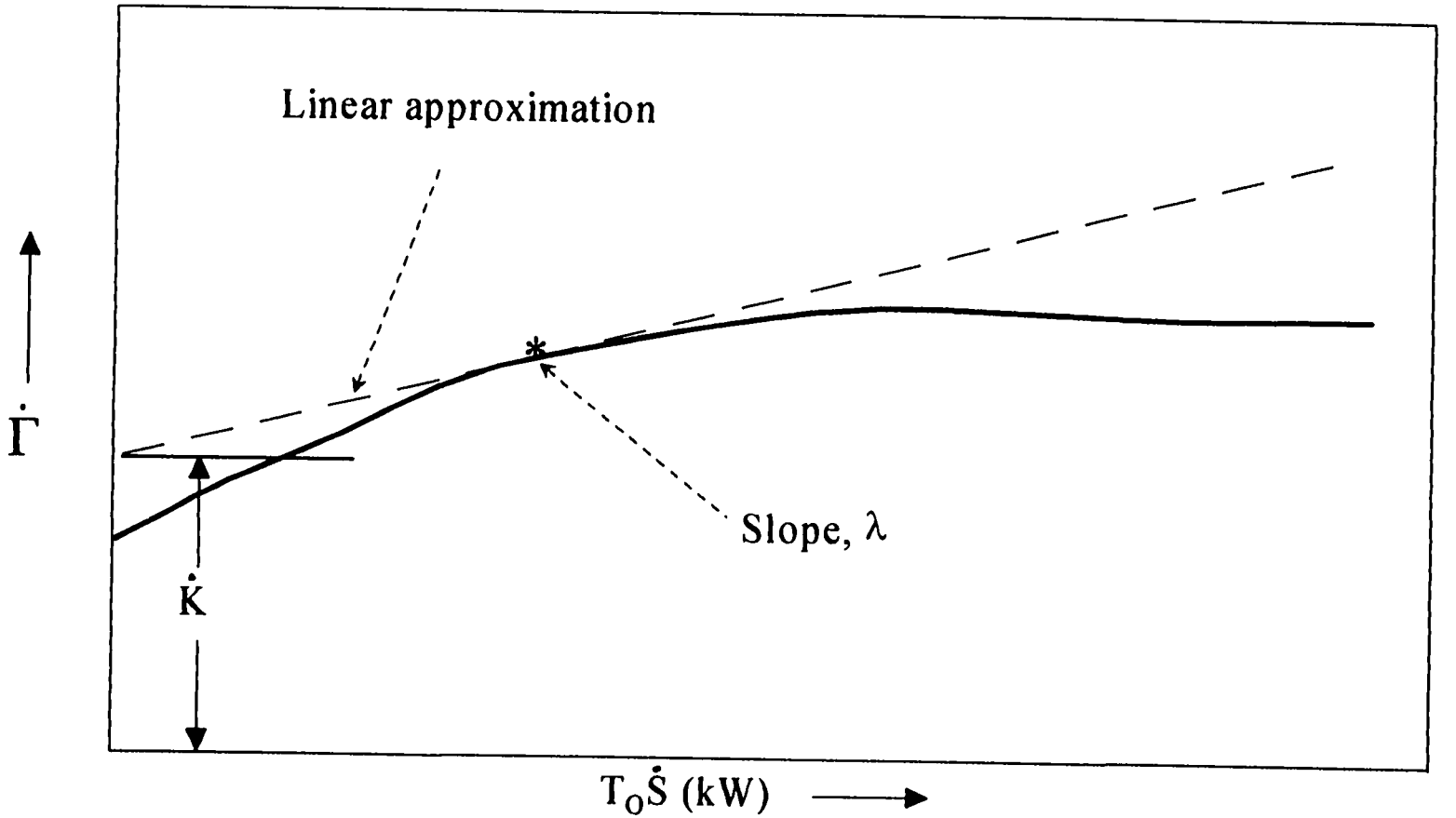

Figure 3.1 Annualized cost rate ($\dot{\Gamma}$) of the energy dissipation ($T_0\dot{S}$)

represented by the solid portion of the curve, which is approximated by a linear equation of the form (von Spakovsky and Evans, 1984)

$$\dot{\Gamma} = \lambda T_O \dot{S} + \dot{K}, \tag{3.1}$$

which is shown as a dashed line in Figure (3.1). Here $\dot{K}$ is the intercept while λ is the slope at the point of tangency,

$$\lambda \equiv \frac{\partial \dot{\Gamma}}{\partial (T_O \dot{S})}. \tag{3.2}$$

From the second law analysis of a thermal-energy-storage system it is generally observed that there are two main causes of entropy generation. The first is due to the frictional effect of the working fluid or what is called the pressure drop. The second irreversible loss is due to the finite temperature difference between the working fluid and the storage material and between the working fluid and the ambient if the working fluid is exhausted to the environment. For example, in thermal energy storage systems with Joulean heating the heat transfer occurs between the Joulean heater and the storage material.

Following equation (3.1), the costs of irreversibilities due to the frictional effects and due to the finite temperature difference can be formulated, respectively, as

$$\dot{\Gamma}_P = \lambda_P T_O \dot{S}_P + \dot{K}_P \tag{3.3}$$

and

$$\dot{\Gamma}_T = \lambda_T T_O \dot{S}_T + \dot{K}_T \tag{3.4}$$

Thus, the total annual cost rate ($\dot{\Gamma}$) for owning and operating the thermal energy storage system can be described as (von Spakovsky and Evans, 1984)

$$\dot{\Gamma} = \dot{\Gamma}_P + \dot{\Gamma}_T \tag{3.5}$$

Substituting equations (3.3) and (3.4) into equation (3.5) we get

$$\dot{\Gamma} = \lambda_P T_O \dot{S}_P + \lambda_T T_O \dot{S}_T + \dot{Z} \tag{3.6}$$

where

$$\dot{Z} \equiv (\dot{K}_P + \dot{K}_T) = \dot{z}A + \dot{K}_O \tag{3.7}$$

In equation (3.7), $\dot{z}$ represents the annualized capital cost per unit area of owning and maintaining the energy storage system and $\dot{K}_O$ is the sum of the fixed maintenance cost and any other extraneous annual costs that apply to the storage system as a whole.

It is more useful to create a parameter that relates some of the economic terms to the design and operational parameters of the heat exchanger. So we define the cost per unit heat conductance as (Zubair et al., 1987)

$$\gamma_{UA} = \frac{\dot{z}A + \lambda_P T_O \dot{S}_P}{UA} \tag{3.8}$$

We note that the number of transfer units (NTU), which is a thermal design parameter, is defined as

$$NTU = \frac{UA}{\dot{m}c_P} \tag{3.9}$$

Substituting equations (3.8) and (3.9) into equation (3.6), gives

$$\dot{\Gamma} = \dot{m}c_P \gamma_{UA} NTU + \lambda_T T_O \dot{S}_T + \dot{K}_O \tag{3.10}$$

Because $\dot{K}_O$ is fixed and predetermined cost, depending on the type of storage system, we can write equation (3.10) in terms of the modified cost rate equation as

$$\dot{\Gamma} - \dot{K}_O = \dot{m}c_P NTU \gamma_{UA} + \lambda_T T_O \dot{S}_T \tag{3.11}$$

In thermodynamic analysis, optimization is not carried out by minimizing the entropy generation or irreversibilities but rather by minimizing the ratio between the irreversibilities and the availability supplied to the system. In other words, we minimize what is called the entropy generation number (Bejan, 1978b). It is more convenient to follow the same analogy when using thermoeconomic analysis by defining the ratio between the cost rate of irreversibilities to the cost rate of the availability supplied to the system as

$$\Gamma^* = \frac{\dot{\Gamma} - \dot{K}_O}{\dot{\Gamma}_{as} - \dot{K}_O} \tag{3.12}$$

where ($\dot{\Gamma}$ - $\dot{K}_O$) is from equation (3.11), and $\dot{\Gamma}_{as}$ is defined as the cost rate of the availability supplied to the system. It can be generalized as

$$\dot{\Gamma}_{as} = \dot{z}A + \dot{K}_O + \sum \lambda_i \dot{W}_i \quad , \; i = P, T, E, ... \tag{3.13}$$

where P stands for pressure drop, T is the temperature difference, E is electrical energy and i is any other type of energy supplied. The idea behind defining equation (3.12) is that we need a base for comparison. It means that the system which looses less of the investment is better. To demonstrate the idea, consider two systems each one of them looses 1\$/s. For the first system we invested 2 \$/s and for the second one we invested 3 \$/s. According to equation (3.12) the former system destroys 50 % of the investment and the latter one destroys only 33.3 %. We conclude that the latter one is more economically feasible and gives better performance.

To check that thermodynamic results can be recovered from equation (3.12) we consider the same assumptions as that stated by Badar et al. (1993). That is,

$$\lambda_i = \lambda, \quad and \quad \dot{z} = \dot{K}_O = 0, \tag{3.14}$$

then the cost per unit overall conductance [equation (3.8)] reduces to

$$\gamma_{UA} = \frac{\lambda T_O \dot{S}_P}{UA} \tag{3.15}$$

Substituting the above equation into the cost rate ratio equation we get

$$\Gamma^{*} = \frac{\lambda\left(T_O \dot{S}_P + T_O \dot{S}_T\right)}{\lambda\left(\sum \dot{W}_i\right)} \tag{3.16}$$

By eliminating the cost parameter we get the entropy generation number of the cycle which converts the problem to a purely thermodynamic problem.

So, our aim is to minimize equation (3.12) with respect to different operational and design variables and to test the effect of cost parameters on the optimum values thus obtained.

In some cases we need to know the effect of the reduced cost parameters on the performance of the energy storage system. So we define the ratio of the unit cost of entropy generation associated with irreversible heat transfer to the cost per unit overall heat conductance of the heat exchanger as the dimensionless cost ratio, given by (Badar et al., 1993)

$$\beta = \frac{\lambda_T T_O}{\gamma_{UA}} \tag{3.17}$$

Then equation (3.11) can be rewritten as

$$\dot{\Gamma} - \dot{K}_O = \lambda_T T_O \left(\frac{\dot{m} c_P NTU}{\beta} + \dot{S}_T \right), \tag{3.18}$$

and the general form of the cost rate ratio will thus be expressed as

$$\Gamma^{*} = \frac{\lambda_T T_O \left(\frac{\dot{m} c_P NTU}{\beta} + \dot{S}_T \right)}{\dot{z}A + \sum \lambda_i \dot{W}_i}. \tag{3.19}$$

An optimum design could be developed from equation (3.12) or equation (3.19) by either the laborious procedure of direct trial and error, taking all possible variables into consideration, or by using the method of partial derivatives. Another simple but accurate optimization procedure is followed by using a computer-based solution of the above equations.

In this thesis, MathCad is used to obtain optimization of the systems described in the following chapters. This is because of the fact that the expressions derived are lengthy and most of the variables are related together that makes it very difficult to get accurate expressions using the method of partial derivatives.

CHAPTER 4

THERMOECONOMIC ANALYSIS AND OPTIMIZATION OF THERMAL-ENERGY-STORAGE SYSTEMS WITH JOULEAN-HEATING OF THE STORAGE ELEMENT

A system of this type uses low-cost, off-peak electrical energy to heat a well insulated mass of material with a large capacity for storing sensible thermal energy. The stored thermal energy is extracted at a latter time during a period of peak electrical power usage. Krane (1985b) performed a second law analysis to this system and concluded that it destroys approximately 60 to 80% of the entering availability, that is, the thermodynamic efficiency of a thermal energy storage system with Joulean heating of the storage element is very low. In this chapter, the method of thermoeconomic analysis described in chapter 3 will be applied to a thermal-energy-storage system with Joulean-heating of the storage element, where the working fluid is assumed to be an ideal gas.

4.1 Second Law Analysis

4.1.1 Description of the System to be Analyzed

The system consists of a well-insulated solid storage element of mass M and specific heat C. A Joulean, or electrical resistance heater, and a heat exchanger duct are embedded in the storage element. Energy is removed from the storage element by heat transfer to a gas

that is passed through the duct. The storage element remains in the solid phase. Since no phase change occur in the storage element, only sensible thermal energy is stored in the system.

The only source of entropy generation during the storage process is Joulean heating. There are two sources of entropy generation during the removal process: heat transfer through finite temperature differences between the gas in the heat exchanger duct and the storage element, and frictional losses in the heat exchanger duct.

4.1.2 Analysis of the Storage Process

Figure (4.1) shows a schematic diagram of the sensible heat thermal energy storage system with Joulean heating of the storage element. On applying the first and second law of thermodynamics to the control volume, results in

$$\dot{S}_{Si} = \frac{dS}{dt} = \frac{\dot{Q}_{SE}}{T} = \frac{MC}{T}\frac{dT}{dt} \tag{4.1}$$

Integrating equation (4.1) over the storage period we get

$$(S_{FS} - S_{IS}) = MC \ln\left(\frac{T_{FS}}{T_{IS}}\right) \tag{4.2}$$

Having energy balance on the storage element we get

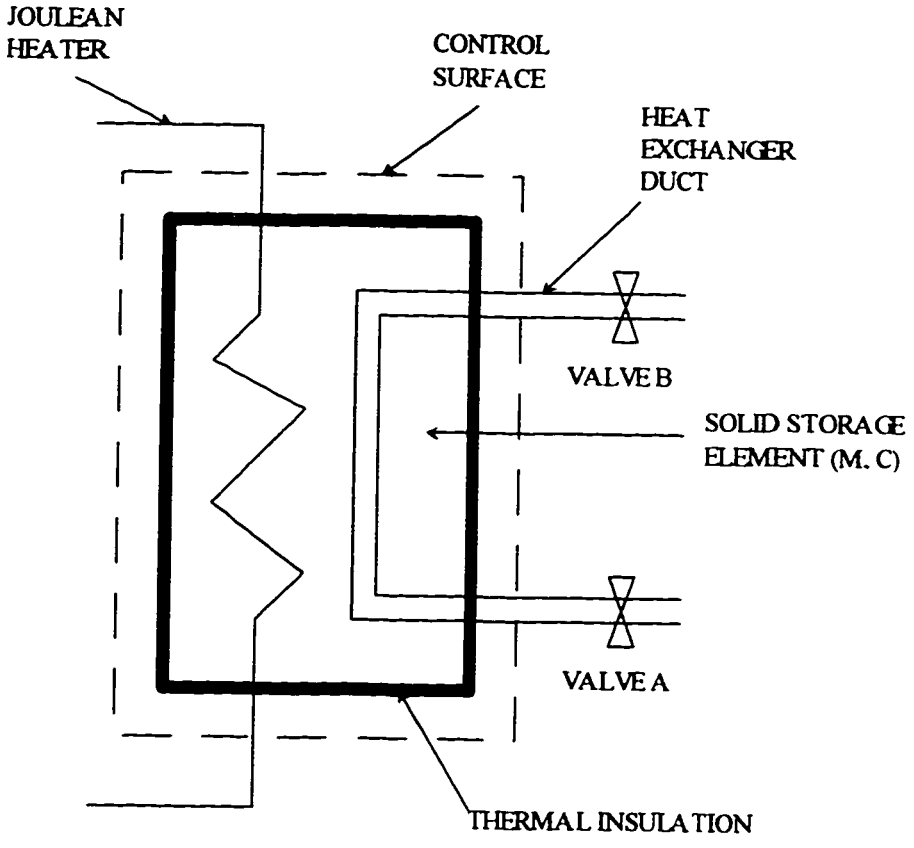

Figure 4.1 . Schematic representation of a generic thermal energy storage system with a Joulean heater

$$MC\frac{dT}{dt} = P \tag{4.3}$$

where P represents the power supplied by the Joulean heater. Integrating equation (4.3) over 0 to t and T_{IS} to T we get

$$T = T_{IS} + \frac{Pt}{MC} \tag{4.4}$$

Introducing

$$\tau_R = \frac{T_{iR} - T_O}{T_O}, \qquad \overline{P} = \frac{P}{\dot{m}_R c_P T_O}, \quad \text{and} \quad \theta = \frac{\dot{m}_R c_P t}{MC} \tag{4.5}$$

and assume that the temperature of the storage element at the beginning of the storage process exceeds the inlet temperature of the heated gas stream by an arbitrary amount, that is (Krane, 1985b)

$$T_{IS} = (1+\varepsilon)\, T_{iR} \qquad \varepsilon > 0 \tag{4.6}$$

Rewriting equation (4.4) in a non-dimensional form, we get

$$\frac{T}{T_O} = (1+\varepsilon)(1+\tau_R) + \overline{P}\theta \tag{4.7}$$

Evaluating equation (4.7) at the end of the storage process, gives

$$\overline{T}_{FS} \equiv \frac{T_{FS}}{T_O} = (1+\varepsilon)(1+\tau_R) + \overline{P}\theta_S. \tag{4.8}$$

Therefore, equation (4.2) can now be expressed in terms of dimensionless variables as

$$S_S = MC \ln\left[\frac{\overline{T}_{FS}}{(1+\varepsilon)(1+\tau_R)}\right] \tag{4.9}$$

4.1.3 Analysis of the Removal Process

The instantaneous rate of entropy generation in the system surrounded by the control volume of Figure (4.1), for the removal process, is given by

$$\dot{S}_{Ri} = \dot{m}_R c_P \ln\frac{P_{iR}}{P_O} + \dot{m}_R c_P \ln\frac{T_{eR}}{T_{iR}} + \frac{d}{d(t-t_S)}\left(MC \ln\frac{T}{T_{IR}}\right) \tag{4.10}$$

Integrating equation (4.10) with respect to time from t_S to t_R we get

$$S_R = \dot{m}_R c_P \left[\frac{R}{c_P}(t_R - t_S)\ln\left(\frac{P_{iR}}{P_O}\right) + \int_{t_S}^{t_R} \ln\left(\frac{T_{eR}}{T_{iR}}\right) dt\right] + MC \ln\left(\frac{T_{FR}}{T_{IR}}\right) \tag{4.11}$$

In order to solve the above equation we need to get expressions for (T_{eR} / T_{iR}) and (T_{FR} / T_{IR}). Here we assume that the initial storage element temperature of the storage process, T_{IS}, is equal to the final storage element temperature of the removal process, T_{FR}, and the final storage element temperature of the storage process, T_{FS}, is equal to the initial storage element temperature of the removal process, T_{IR}, or

$$T_{FR} = T_{IS} \qquad \text{and} \qquad T_{IR} = T_{FS} \tag{4.12}$$

which implies

$$\ln\left(\frac{T_{FR}}{T_{IR}}\right) = -\ln\left[\frac{\overline{T}_{FS}}{(1+\varepsilon)(1+\tau_R)}\right] \tag{4.13}$$

Applying the energy balance on the storage element, results in

$$MC\frac{dT}{dt} = \dot{m}_R c_P \left(T_{iR} - T_{eR}\right) \tag{4.14}$$

Integration, we get

$$\int_{T_{iR}}^{T} \frac{dT}{T_{iR} - T_{eR}} = \frac{\dot{m}_R c_P \left(t - t_S\right)}{MC} = \theta - \theta_S \tag{4.15}$$

From the analysis of the heat exchanger, we can express the temperature change in terms of the heat exchanger effectiveness ($y_R = 1 - e^{-NTU}$) as

$$T_{iR} - T_{eR} = y_R \left(T_{iR} - T\right) \tag{4.16}$$

or substituting in equation (4.15) and performing the integration, results in

$$T = T_{iR} + \left(T_{IR} - T_{iR}\right)e^{-y_R(\theta - \theta_S)} \tag{4.17}$$

Substituting equation (4.17) into equation (4.16), and after manipulation we get

$$\frac{T_{eR}}{T_{iR}} = 1 + y_R \left(\frac{T_{IR}}{T_{iR}} - 1 \right) e^{-y_R(\theta - \theta_S)} \tag{4.18}$$

Since

$$\frac{T_{IR}}{T_O} = \frac{T_{FS}}{T_O} = \overline{T}_{FS} \quad \text{and} \quad \frac{T_{iR}}{T_O} = 1 + \tau_R \tag{4.19}$$

then

$$\frac{T_{eR}}{T_{iR}} = 1 + y_R \left(\frac{\overline{T}_{FS}}{1 + \tau_R} - 1 \right) e^{-y_R(\theta - \theta_S)} \tag{4.20}$$

Substituting equations (4.13)and (4.20) into equation (4.11) and writing the equation in non-dimensional form, we get

$$S_R = MC \left\{ \frac{R}{c_P} (\theta_R - \theta_S) \ln \frac{P_{iR}}{P_O} - \ln \left[\frac{\overline{T}_{FS}}{(1+\varepsilon)(1+\tau_R)} \right] + \int_0^{\theta_R - \theta_S} \ln \left[1 + y_R \left(\frac{\overline{T}_{FS}}{1+\tau_R} - 1 \right) e^{-y_R \theta} \right] d\theta \right\} \tag{4.21}$$

The total entropy generated for the complete cycle is obtained by combining equations (4.9) and (4.21). This gives

$$S_{GEN} = MC\left\{\frac{R}{c_P}(\theta_R - \theta_S)\ln\left(\frac{P_{iR}}{P_O}\right) + \int_0^{\theta_R-\theta_S} \ln\left[1 + y_R\left(\frac{\overline{T}_{FS}}{1+\tau_R} - 1\right)e^{-y_R\theta}\right]d\theta\right\} \tag{4.22}$$

To obtain the average rate of entropy production for the complete cycle, we divide the above equation by the complete cycle time, t_R. It gives

$$\dot{S}_{GEN} = \frac{S_{GEN}}{t_R} = \dot{S}_P + \dot{S}_T, \tag{4.23}$$

where the rate of entropy generation due to pressure drop and finite-temperature difference can be expressed, respectively, as

$$\dot{S}_P = \frac{\dot{m}_R c_P}{\theta_R}\frac{R}{c_P}(\theta_R - \theta_S)\ln\left(\frac{P_{iR}}{P_O}\right), \tag{4.24}$$

and

$$\dot{S}_T = \frac{\dot{m}_R c_P}{\theta_R}\int_0^{\theta_R-\theta_S} \ln\left[1 + y_R\left(\frac{\overline{T}_{FS}}{1+\tau_R} - 1\right)e^{-y_R\theta}\right]d\theta \tag{4.25}$$

where θ_R is the dimensionless cycle time.

4.1.4 Relation Between the Storage Time and the Removal Time

Since the system is operating in a cycle, the dimensionless cycle time, θ_R, and the dimensionless storage time, θ_S, are not independent of each other. Thus it is possible to obtain a closed form analytical expression relating these two variables.

Noting that $T_{IR} = T_{FS}$ and that $T = T_{FR} = T_{IS} = (1 + \varepsilon)T_{iR}$ when $\theta = \theta_R$, reduces equation (4.17) to the form:

$$\frac{\varepsilon\, T_{iR}}{T_{FS} - T_{iR}} = e^{-y_R(\theta_R - \theta_S)} \tag{4.26}$$

However, the storage process in an electrically-heated thermal energy storage system typically takes place during eight hour, off-peak period at night, while the removal process occurs during the remaining sixteen hours of the day, when the demand for electrical power increases. Thus the storage time, t_S, and the total cycle time, t_R, are related by the simple expression $t_R = 3t_S$, which, in turn, yields a second relationship between the dimensionless storage and cycle times, given by

$$\theta_R = 3\theta_S \tag{4.27}$$

4.1.5 Calculating the Availability Supplied to the System

During the storage process, we assume that the electrical energy is completely converted into useful work. This useful work can be expressed in terms of electrical power (P), as

$$W_E = t_S P \tag{4.28}$$

From equation (4.4) we can write

$$W_E = MCT_O \left(\overline{T}_{FS} - \frac{T_{IS}}{T_O} \right) \tag{4.29}$$

Substituting equation (4.6) and rearranging gives

$$W_E = MCT_O \left[\overline{T}_{FS} - (1+\varepsilon)(1+\tau_R) \right] \tag{4.30}$$

During the removal process, the availability supplied is

$$W_R = \dot{m}_R (t_R - t_S)\left[(h_{iR} - h_O) - T_O (s_{iR} - s_O) \right] \tag{4.31}$$

For an ideal gas this expression is written in a nondimensional form as

$$W_R = MCT_O (\theta_R - \theta_S)\left[\tau_R - \ln(1+\tau_R) + \frac{R}{c_P} \ln\left(\frac{P_{iR}}{P_O} \right) \right]. \tag{4.32}$$

Therefore, the total availability supplied to the system is given by

$$W = MCT_O \left\{ (\theta_R - \theta_S)\left[\tau_R - \ln(1+\tau_R) + \frac{R}{c_P} \ln\left(\frac{P_{iR}}{P_O} \right) \right] + \overline{T}_{FS} - (1+\varepsilon)(1+\tau_R) \right\} \tag{4.33}$$

The rate of availability supplied to the complete cycle is obtained by dividing the above expression by the complete cycle time, t_R. It gives

$$\dot{W} = \frac{W}{t_R} = \dot{W}_E + \dot{W}_P + \dot{W}_T \tag{4.34}$$

where the rate of availability supplied by the Joulean heating, the pressure difference, and the temperature difference can be expressed, respectively, as

$$\dot{W}_E = \frac{\dot{m}_R c_P T_O}{\theta_R}\left[\overline{T}_{FS} - (1+\varepsilon)(1+\tau_R)\right], \tag{4.35}$$

$$\dot{W}_P = \frac{\dot{m}_R c_P T_O}{\theta_R}(\theta_R - \theta_S)\frac{R}{c_P}\ln\left(\frac{P_{iR}}{P_O}\right), \tag{4.36}$$

and

$$\dot{W}_T = \frac{\dot{m}_R c_P T_O}{\theta_R}(\theta_R - \theta_S)\left[\tau_R - \ln(1+\tau_R)\right] \tag{4.37}$$

4.2 Thermoeconomic Analysis

Substituting equations (4.24) and (4.25) into the cost rate equation [equation (3.6)], we get

$$\begin{aligned}\dot{\Gamma} - \dot{K}_O = \dot{z}A &+ \lambda_P T_O \left\{\frac{\dot{m}_R c_P}{\theta_R}\frac{R}{c_P}(\theta_R - \theta_S)\ln\left(\frac{P_{iR}}{P_O}\right)\right\} \\ &+ \lambda_T T_O\left\{\frac{\dot{m}_R c_P}{\theta_R}\int_0^{\theta_R-\theta_S}\ln\left[1 + y_R\left(\frac{\overline{T}_{FS}}{1+\tau_R} - 1\right)e^{-y_R\theta}\right]d\theta\right\}\end{aligned} \tag{4.38}$$

Introducing the cost per unit overall heat conductance based on the overall heat transfer coefficient of the removal process, we get

$$\gamma_{UA} = \frac{\dot{z}A + \lambda_P T_O \dot{S}_P}{U_R A} \tag{4.39}$$

and note that

$$NTU_R = \frac{U_R A}{\dot{m}_R c_P} \tag{4.40}$$

Substituting equations (4.39) and (4.40) into equation (4.38), the cost rate equation can be written as

$$\dot{\Gamma} - \dot{K}_O = \dot{m}_R c_P \left\{ \gamma_{UA} NTU_R + \frac{\lambda_T T_O}{\theta_R} \int_0^{\theta_R - \theta_S} \ln\left[1 + y_R \left(\frac{\overline{T}_{FS}}{1+\tau_R} - 1 \right) e^{-y_R \theta} \right] d\theta \right\} \tag{4.41}$$

In terms of the dimensionless cost ratio, β [equation (3.17)], equation (4.41) can be written as

$$\dot{\Gamma} - \dot{K}_O = \dot{m}_R c_P \lambda_T T_O \left\{ \frac{NTU_R}{\beta} + \frac{1}{\theta_R} \int_0^{\theta_R - \theta_S} \ln\left[1 + y_R \left(\frac{\overline{T}_{FS}}{1+\tau_R} - 1 \right) e^{-y_R \theta} \right] d\theta \right\} \tag{4.42}$$

The cost rate that is supplied to the storage system is composed of four parts namely: the annualized capital cost of owning and maintaining the energy storage system, the cost of electrical energy that is supplied during the storage process (Joulean heating), the cost of

availability carried by the working stream due to pressure difference, and the cost due to temperature difference. It can be expressed as

$$\dot{\Gamma}_{as} - \dot{K}_O = \dot{z}A + \lambda_E \dot{W}_E + \lambda_P \dot{W}_P + \lambda_T \dot{W}_T \tag{4.43}$$

Substituting equations (4.35), (4.36), and (4.37) into equation (4.43) we get

$$\begin{aligned}\dot{\Gamma}_{as} - \dot{K}_O = \dot{z}A + \frac{\dot{m}_R c_P T_O}{\theta_R}\Bigg\{ & \lambda_P (\theta_R - \theta_S)\frac{R}{c_P}\ln\left(\frac{P_{iR}}{P_O}\right) \\ & + \lambda_T (\theta_R - \theta_S)(\tau_R - \ln(1+\tau_R)) + \lambda_E \left[\overline{T}_{FS} - (1+\varepsilon)(1+\tau_R)\right]\Bigg\}\end{aligned} \tag{4.44}$$

It should be noted that the Joulean heater converts the electrical energy into heat. So, λ_T will have a higher value than λ_E. The value of λ_T equals the value of λ_E divided by the efficiency of conversion from electrical energy to heat. For an ideal case (i.e. 100% efficiency), both values will be equal. In this study we will consider that all the electrical energy is converted into heat by the Joulean heater without any loss. In other words, both λ_E and λ_T will have the same value.

By comparing equation (4.44) with equations (3.38) and (4.41), we can write

$$\begin{aligned}\dot{\Gamma}_{as} - \dot{K}_O = \dot{m}_R c_P \Bigg\{ & \gamma_{UA} NTU_R + \frac{T_O}{\theta_R}\{\lambda_T (\theta_R - \theta_S)(\tau_R - \ln(1+\tau_R)) \\ & + \lambda_E \left[\overline{T}_{FS} - (1+\varepsilon)(1+\tau_R)\right]\}\Bigg\}\end{aligned} \tag{4.45}$$

The dimensionless cost ratio (β) can be included in equation (4.45). This gives

$$\dot{\Gamma}_{as} - \dot{K}_O = \dot{m}_R c_P \lambda_T T_O \left\{ \frac{NTU_R}{\beta} + \frac{1}{\theta_R}\left[(\theta_R - \theta_S)(\tau_R - \ln(1+\tau_R)) + \frac{\lambda_E}{\lambda_T}\left[\overline{T}_{FS} - (1+\varepsilon)(1+\tau_R)\right]\right]\right\} \tag{4.46}$$

By substituting equations (4.41) and (4.45) into the cost rate number [equation (3.12)], we get

$$\Gamma^* = \frac{\gamma_{UA} NTU_R + \frac{\lambda_T T_O}{\theta_R}\int_0^{\theta_R-\theta_S} \ln\left[1 + y_R\left(\frac{\overline{T}_{FS}}{1+\tau_R} - 1\right)e^{-y_R\theta}\right]d\theta}{\gamma_{UA} NTU_R + \frac{T_O}{\theta_R}\left[\lambda_T(\theta_R - \theta_S)(\tau_R - \ln(1+\tau_R)) + \lambda_E\left(\overline{T}_{FS} - (1+\varepsilon)(1+\tau_R)\right)\right]} \tag{4.47}$$

or by introducing the dimensionless cost ratio [equations (4.42) and (4.46)], the cost rate number can be expressed as

$$\Gamma^* = \frac{NTU_R + \frac{\beta}{\theta_R}\int_0^{\theta_R-\theta_S} \ln\left[1 + y_R\left(\frac{\overline{T}_{FS}}{1+\tau_R} - 1\right)e^{-y_R\theta}\right]d\theta}{NTU_R + \frac{\beta}{\theta_R}\left[(\theta_R - \theta_S)(\tau_R - \ln(1+\tau_R)) + \frac{\lambda_E}{\lambda_T}\left(\overline{T}_{FS} - (1+\varepsilon)(1+\tau_R)\right)\right]} \tag{4.48}$$

4.3 Presentation and Interpretation of Results

4.3.1 Definition of the optimization criteria

The fundamental criterion for designing an optimum electrically heated thermal energy storage system is that the values of the independent variables be selected such that the cost rate number is minimized. This criterion is subject to some modifications, however,

by engineering judgment. For instance, it is generally considered to be desirable to make the heat exchanger size (NTU_R) as small as possible. It is also often considered to be advantageous to operate a thermal energy storage system such that the maximum temperature of the storage element in a storage-removal cycle (T_{FS}) is as large as possible, consistent with the subsequent use of the stored energy and the maximum allowable temperature of the storage element material, and that the system be operated over the maximum possible temperature range (in order to minimize the size of the storage element).

4.3.2 Description of the Optimization Study

The minimum cost rate number (Γ^*) and the optimum number of transfer units (NTU_R) were determined for each of twenty-four different systems. These systems are described and analyzed by Krane (1985b) based only on thermodynamic optimization. The procedure for calculating cost rate number is shown in Figure (4.2), where the dimensionless mass flux G is discussed in the appendix. The results of these calculations, which are presented in Tables (4.1)-(4.3), represent range of the independent variables which should include most systems of practical interest.

The removal gas inlet temperature ratio, τ_R, was assigned a value of 0.0 for systems 1-20 and a value of 0.1 for systems 21-24. These values represent the commonly encountered situations in which the gas inlet temperature is at, or slightly above, ambient temperature

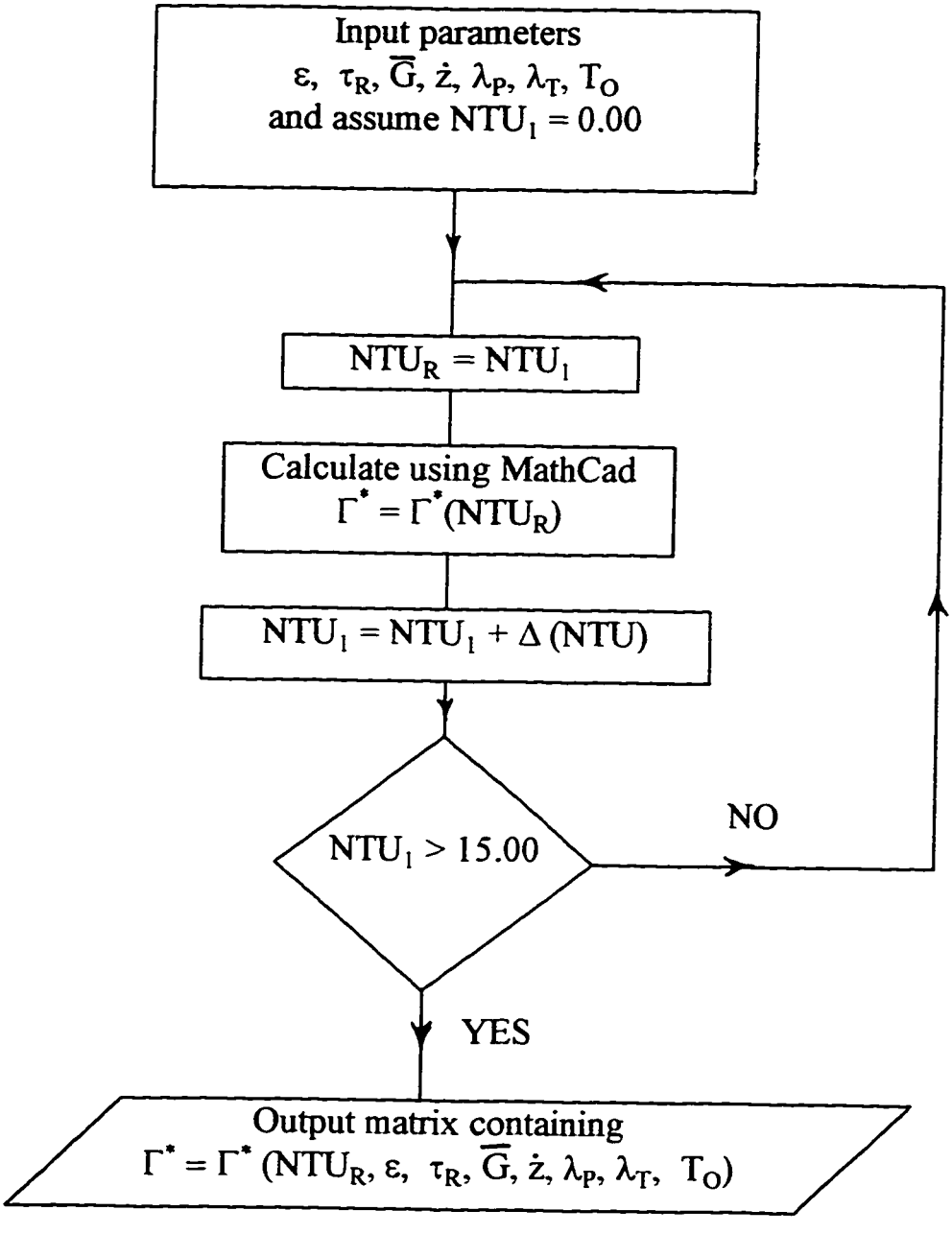

Figure 4.2 Flow diagram for calculating the cost rate number

Table 4.1 Results for medium temperature systems

System→ Variable↓	1	2	3	4	5	6	7	8	9	10
τ_R	0.0									
$\dot{z}$ ($\$ / m^2$ - s)	0.00377									
$\lambda_T = \lambda_E$ ($\$ / kW$-s)	.1									
λ_P ($\$ / kW$-s)	.04									
T_{FS}	3.00									
ε	.01	.01	.01	.01	.01	.1	.1	.1	.1	.1
V_R (m / s)	5	10	50	100	200	5	10	50	100	200
θ_S	2.651	2.651	2.654	2.661	2.679	1.499	1.499	1.499	1.502	1.508
θ_R	7.954	7.954	7.962	7.982	8.038	4.498	4.498	4.498	4.505	4.524
P_{iR}/P_0	1.000144	1.000571	1.012649	1.042386	1.128968	1.000154	1.000613	1.013676	1.046259	1.142233
$NTU_{R,opt}$	7.09	7.05	6.32	5.45	5.49	7.06	7.57	6.84	5.97	5.01
Γ^*_{opt}	.718128	.718165	.719244	.721992	.729595	.705573	.705598	.706318	.708166	.713338

Table 4.2 Results for high temperature systems

System→ Variable↓	11	12	13	14	15	16	17	18	19	20
τ_R	0.0									
$\dot{z}$ ($\$ / m^2$ - s)	0.00377									
$\lambda_T = \lambda_E$ ($\$ / kW$-s)	.1									
λ_P ($\$ / kW$-s)	.04									
T_{FS}	3.667									
ε	.01	.01	.01	.01	.01	.1	.1	.1	.1	.1
V_R (m / s)	5	10	50	100	200	5	10	50	100	200
θ_S	2.795	2.795	2.797	2.804	2.821	1.643	1.643	1.643	1.646	1.652
θ_R	8.385	8.385	8.392	8.411	8.462	4.928	4.928	4.928	4.937	4.955
P_{iR}/P_0	1.000146	1.000583	1.012926	1.043432	1.132567	1.000156	1.000622	1.013893	1.047074	1.145004
$NTU_{R,opt}$	7.22	7.19	6.46	5.59	4.63	7.71	7.68	6.95	6.08	5.12
Γ^*_{opt}	.667799	.667834	.66886	.671481	.678778	.656671	.656695	.657387	.659167	.664166

Table 4.3 Effects of elevated removal gas inlet temperature on system performance

System→ Variable↓	21	22	23	24
τ_R	0.1			
$\dot{z}$ ($\$ / m^2$ - s)	0.00377			
$\lambda_T = \lambda_E$ ($\$ / kW$-s)	.1			
λ_P ($\$ / kW$-s)	.04			
T_{FS}	3.00		3.667	
ε	.01	.1	.01	.1
V_R (m / s)	50	50	50	50
θ_S	2.583	1.427	2.732	1.577
θ_R	7.749	4.28	8.197	4.731
P_{iR}/P_0	1.012963	1.014222	1.013441	1.014612
$NTU_{R,opt}$	5.89	6.47	6.11	6.65
Γ^*_{opt}	.66796	.658936	.623927	.615248

during the removal process. The dimensionless maximum temperature of the storage element, T_{FS}, was set equal to either 3.00 or 3.667. These values correspond to typical storage temperatures for a medium temperature system (900 K) and a high temperature system (1100 K), respectively, for an ambient temperature of 300 K (Krane, 1985b). Values of 0.01 and 0.1 were assumed for the parameter ε. The value of $\varepsilon = 0.1$, which corresponds to a minimum temperature difference of approximately 30 °C between the storage element and the gas inlet during the removal process when $\tau_R = 0.0$, is taken to be the most realistic because it will improve the heat transfer between the working fluid and the storage element. In addition, the values of unit cost parameters are selected based on the work of Zubair et al. (1987) and Badar and Zubair (1995).

Figure (4.3) confirms the effects of $\overline{T}_{FS}$ on system performance by showing that system cost rate number decreases as the maximum storage temperature, $\overline{T}_{FS}$, increases. The effects of the parameter ε on the system performance are also shown in Figure (4.3). Increasing ε by a factor of 10 (from 0.01 to 0.1) is seen to result in relatively small improvements in system cost rate numbers. An increased value of ε, however, results in an increase in the size of the storage element required to store a fixed amount of thermal energy. Therefore, the choice of the value of ε must be left as a tradeoff to be made by the system designer depending on the requirements of the system to be designed.

Figures (4.3) and (4.4) also show the effects of the stream velocity on system performance (Γ^*_{opt}) and the number of transfer units ($NTU_{R,opt}$) of the heat exchanger, respectively. These figures reveal that values of (V_R) that are too small result in

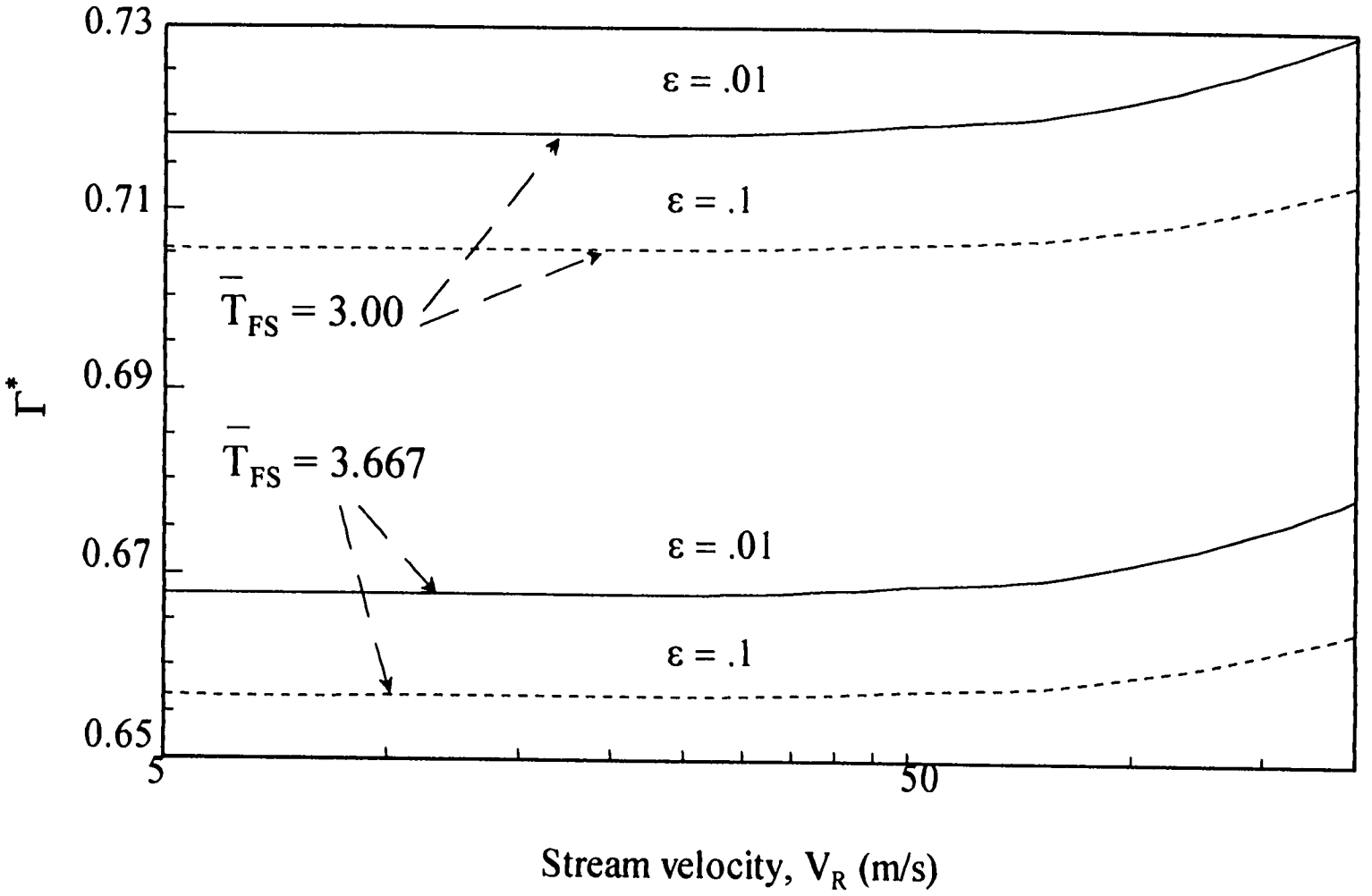

Figure 4.3 Effect of stream velocity on system performance

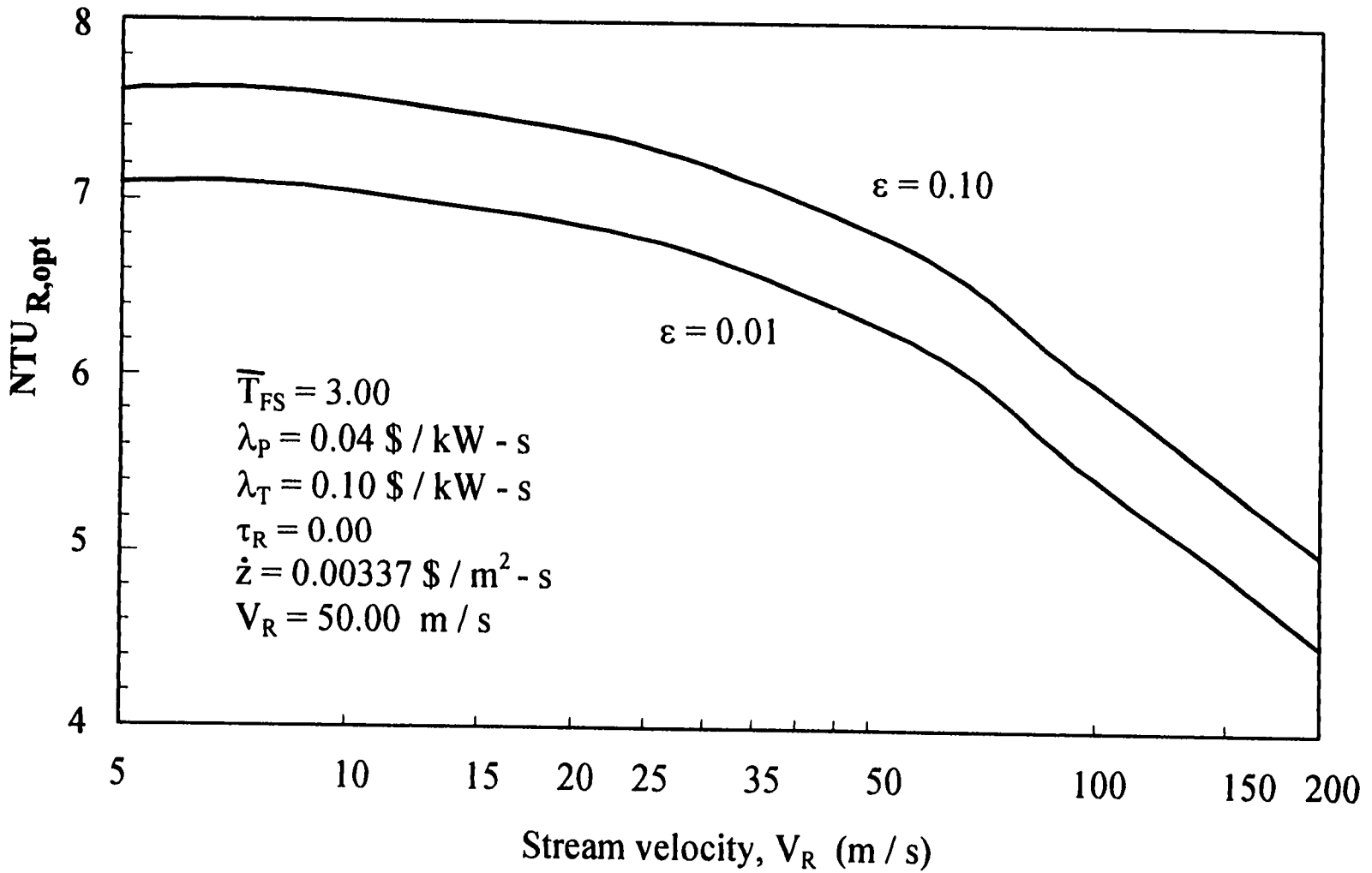

Figure 4.4 Effect of the stream velocity on heat exchanger size

undesirably large heat exchangers ($NTU_{R,opt}$'s), while values that are too large result in reduction in system performance (increases in Γ^*_{opt}). Such reductions in system performance are caused by greatly increased viscous losses in the heat exchanger duct as represented by the increase in the pressure ratio with the stream velocity (V_R) as shown in Tables (4.1) and (4.2). Inspection of Figures (4.3) and (4.4) suggests that a compromise between these two effects is obtained when (V_R) is selected to be around 50 m/s.

Systems 21 to 24 were included in the study in order to determine the effect of the dimensionless removal gas inlet temperature ratio, τ_R, on system performance. For systems 1-20, it was assumed that $\tau_R = 0.0$, that is, the inlet temperature of the removal gas was atmospheric. For systems 21-24, however, this temperature was assumed to be 10% greater than atmospheric pressure and that $\tau_R = 0.1$. Comparing system 21 with system 3, 22 with 8, 23 with 13, and 24 with 18 shows that increasing τ_R increases system performance (decreases Γ^*_{opt}) and decreases the heat exchanger size ($NTU_{R,opt}$). These trends are expected since increasing τ_R reduces the required amount of heat transfer to the removal gas and, therefore, the required amount of Joulean heating.

For system 3 we found that $NTU_{R,opt} = 6.33$. If we take $\dot{Z} = 0$ and $\lambda_P = \lambda_T = \lambda$, then $NTU_{R,opt} = 6.1737$ which is Krane's result (1985). This implies that cost parameters do influence the performance of the system and the influence of the unit cost parameters (λ_P and λ_T) on optimum number of heat-transfer units and the cost rate number is shown in Table (4.4). The same results are presented in Figures (4.5) and (4.6).

Table 4.4 Effect of the cost parameters on the system performance

for $V_R = 50$ m/s, $\dot{z} = .00377$ \$ / m^2-s, $\varepsilon = .01$, $\tau_R = 0.0$

λ_P (\$ / kW-s)	λ_T (\$ / kW-s)	$NTU_{R,opt}$	Γ^*_{opt}	β
.04	.04	5.43	.72192	1441
	.1	6.33	.719244	3608
	.2	7.01	.718217	7223
	.4	7.7	.717648	14460
	.6	8.11	.717443	21710
.1	.04	4.85	.725095	790
	.1	5.74	.720761	1980
	.2	6.42	.719067	3965
	.4	7.11	.718118	7942
	.6	7.51	.717774	11920

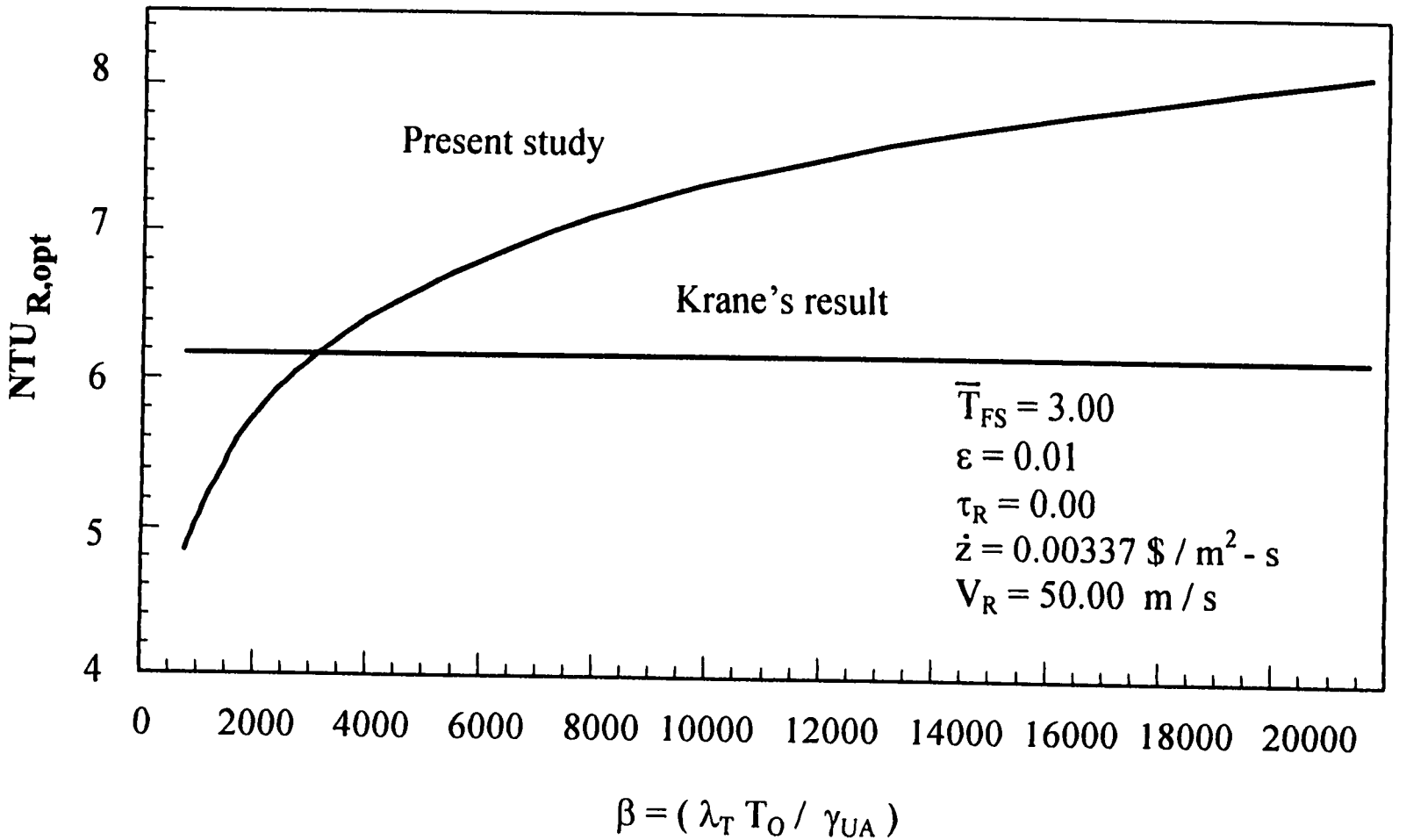

Figure 4.5 Optimum number of transfer units vs. the dimensionless cost ratio

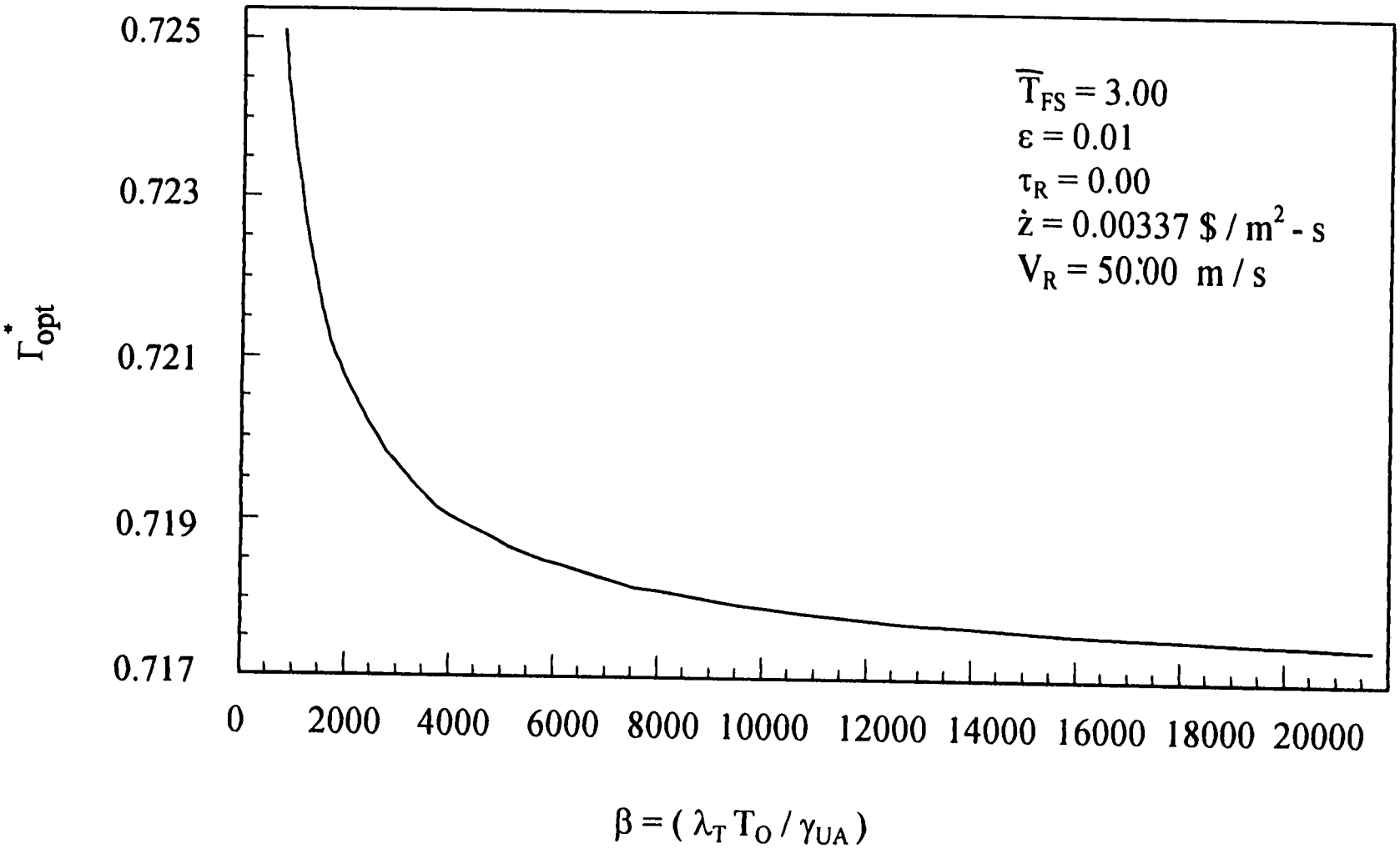

Figure 4.6 Optimum cost rate number vs the dimensionless cost

Figure (4.7) represents the influence of capital cost parameter $\dot{z}$ on optimum number of heat transfer units. From the figure we see that there is an influence of $\dot{z}$ on the optimum NTU_R values, for example as $\dot{z}$ increases we obtain a smaller size heat exchanger. However, Figure (4.8) demonstrate the effect of $\dot{z}$ on the system performance. We see that as $\dot{z}$ increases, Γ^{*}_{opt} increases which means that we have a lower system performance for a small-sized heat exchanger. The above analysis indicate that the effect of $\dot{z}$ should be considered for both the size of the heat exchanger and the system performance.

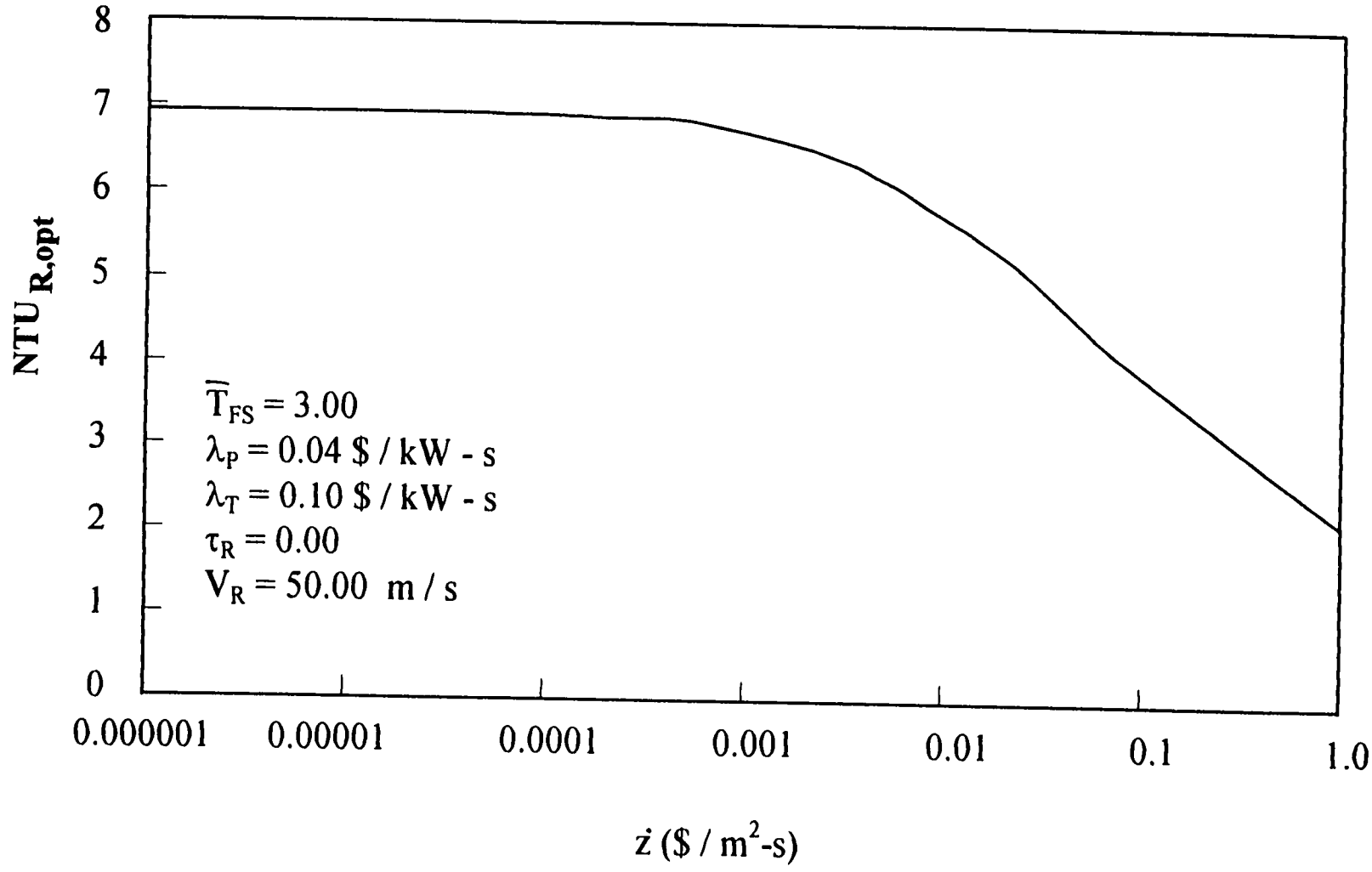

Figure 4.7 Influence of cost parameter z on optimum number of transfer units

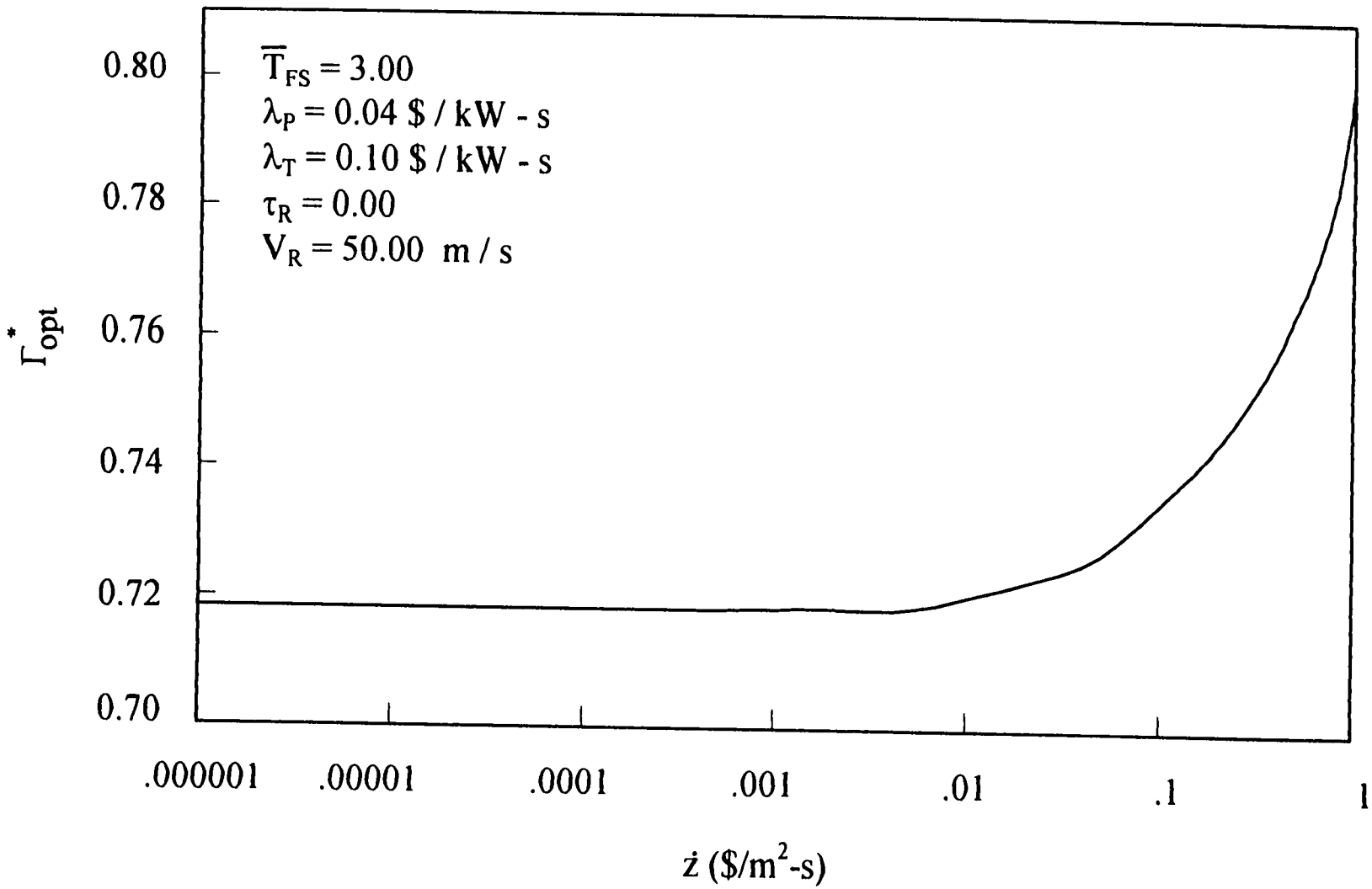

Figure 4.8 Influence of cost parameter z on cost rate number

CHAPTER 5

THERMOECONOMIC ANALYSIS AND OPTIMIZATION OF THE SENSIBLE-HEAT, THERMAL-ENERGY-STORAGE SYSTEM

Because of their simplicity of operation and maintenance and their relatively low cost, sensible-heat systems have emerged as the most important class of heat storage systems. They are commonly used in power generation, refrigeration and air-conditioning systems, and many solar assisted thermal systems.

In this chapter, thermoeconomic analysis and optimization techniques described previously in chapter 3 are applied to the systems described by Krane (1985a). Here, the working fluid is also considered to be an ideal gas.

5.1 Second Law Analysis of the System

5.1.1 System Description

The system consists of a large liquid bath of mass M and specific heat C placed in an insulated vessel. Hot gas enters the system through a heat exchanger immersed in the bath and eventually discharged into the atmosphere. Generally, the bath temperature T, as well as the gas outlet temperature T_{out}, approach the hot gas inlet charging temperature T_{iS}. The bath is filled with an incompressible liquid such as water or oil. It is required that the

pressure of the hot-gas supply P_{iS} must be raised slightly above the atmospheric pressure to overcome the pressure drop caused by friction in the heat exchanger. The specific heats c_P and C are assumed to be constant. Furthermore, there is no phase change during the process, the liquid bath is well agitated and of uniform temperature, the overall heat-transfer coefficient of the storage cycle is constant, and the heat capacity of the hot gas is much smaller than the bath heat capacity (MC).

5.1.2 Analysis of the Storage Process

The energy storage process has three irreversible losses. First, the heat transfer between the hot gas and the liquid bath always takes place across a finite ΔT. Second, the gas stream discharged into the atmosphere is ultimately cooled down to T_O, again by heat transfer across a finite ΔT. Third, the gas flow requires a net ΔP_S across the heat exchanger to overcome friction. Therefore, only a fraction of the total useful work carried by the supplied stream is ultimately stored. The remaining friction is dissipated through the thermodynamic irreversibilities present in the system. Quantitatively, the system irreversibility during the storage process is given by

$$\dot{I}_s = \dot{S}_s T_O \tag{5.1}$$

and the instantaneous rate of entropy generation in the system surrounded by the control volumes 1 and 2 in Figure (5.1) is

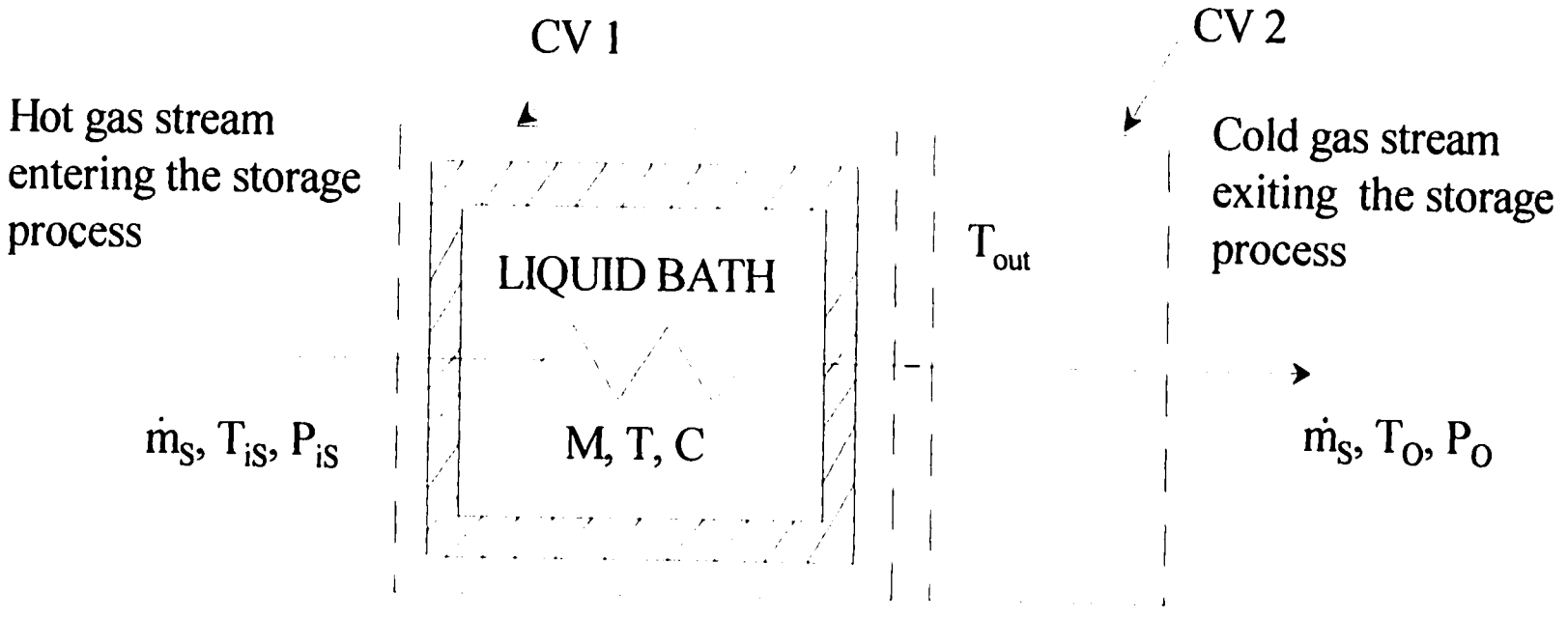

Figure 5.1. Schematic of storage process in sensible heat storage systems

$$\dot{S}_{Si} = \dot{m}_S c_P \ln\frac{T_O}{T_{iS}} + \dot{m}_S R \ln\frac{P_{iS}}{P_O} + \frac{d}{dt}\left(MC \ln\frac{T}{T_{IS}} \right) + \dot{m}_S c_P \left(\frac{T_{out} - T_O}{T_O} \right) \tag{5.2}$$

where the first two terms represent the entropy change experienced by the ideal gas stream from inlet to outlet, the third term stands for the time rate of change of the liquid bath entropy, and the last term accounts for the entropy generation by heat transfer to the environment at temperature T_O.

The total entropy generated during the storage process is obtained by integrating the above equation over time from 0 to t_S. Introducing for the storage process

$$\theta = \frac{\dot{m}_S c_P t}{MC}, \quad NTU_S = \frac{U_S A}{\dot{m}_S c_P}, \quad \text{and} \quad y_S = 1 - e^{-NTU_S} \tag{5.3}$$

and applying energy balance on the storage element and the heat exchanger duct, we get

$$\frac{T_{out}}{T_{iS}} = 1 + y_S \left(\frac{T}{T_{iS}} - 1 \right) \tag{5.4}$$

where

$$T = T_{iS} + (T_{IS} - T_{iS}) e^{-y_S \theta} \tag{5.5}$$

Following Krane's analysis (1985a), we assume

$$T_{IS} \equiv (1+\varepsilon) T_{iR} = T_{FR}, \quad \varepsilon > 0 \tag{5.6}$$

and introducing non-dimensional temperature differences as

$$\tau_S = \frac{T_{iS} - T_O}{T_O}, \quad \text{and} \quad \tau_R = \frac{T_{iR} - T_O}{T_O} \tag{5.7}$$

we get

$$\frac{T_{out}}{T_O} = \frac{T_{out}}{T_{iS}} \cdot \frac{T_{iS}}{T_O} = (1+\tau_S)\left\{1 + y_S\left[\frac{(1+\varepsilon)(1+\tau_R)}{(1+\tau_S)} - 1\right]e^{-y_S\theta}\right\}. \tag{5.8}$$

Notice that at the end of the charging process $t = t_S$, $\theta = \theta_S$ and $T = T_{FS}$, so from equation (5.5) we get

$$T_{FS} = T_{iS} + (T_{IS} - T_{iS})e^{-y_S\theta_S} \tag{5.9}$$

or after manipulation

$$\frac{T_{FS}}{T_{IS}} = \frac{1+\eta}{1+\varepsilon} \tag{5.10}$$

where

$$\eta = \left(1+\varepsilon - \frac{1+\tau_S}{1+\tau_R}\right)e^{-y_S\theta_S} + \frac{1+\tau_S}{1+\tau_R} - 1 \tag{5.11}$$

Since $T_{FS} = T_{IR}$ and $T_{IS} = T_{FR}$, we get

$$\frac{T_{FR}}{T_{IR}} = \frac{1+\varepsilon}{1+\eta} \tag{5.12}$$

On substituting equations (5.8)-(5.10) into equation (5.2), and after manipulation and integrating from 0 to θ_S, the total entropy generation of the storage process can be expressed as,

$$S_S = MC\left\{\theta_S\left[\frac{R}{c_P}\ln\frac{P_{iS}}{P_O} + \tau_S - \ln(1+\tau_S)\right] + \ln\left(\frac{1+\eta}{1+\varepsilon}\right) + (1+\tau_R)(\varepsilon-\eta)\right\} \tag{5.13}$$

One can combine equations (5.1) and (5.13) to estimate the useful work dissipated during the time interval from 0 to t_S, i.e., $I_S = T_O S_S$.

5.1.3 Analysis of the Removal Process

Consider the energy removal process of Figure (5.2); for time $t > t_S$. The instantaneous rate of entropy generation in the system surrounded by the control volume is

$$\dot{S}_{Ri} = \dot{m}_R c_P \ln\left(\frac{T_{eR}}{T_{iR}}\right) + \dot{m}_R R \ln\left(\frac{P_{iR}}{P_O}\right) + \frac{d}{d(t-t_S)}\left(MC\ln\frac{T}{T_{IR}}\right) \tag{5.14}$$

The total entropy generated can be obtained by integrating equation (5.14) over time ($t - t_S$) from 0 to ($t_R - t_S$). We define for the removal process,

$$\theta = \frac{\dot{m}_R c_P t}{MC}, \quad NTU_R = \frac{U_R A}{\dot{m}_R c_P}, \quad and \quad y_R = 1 - e^{-NTU_R}; \tag{5.15}$$

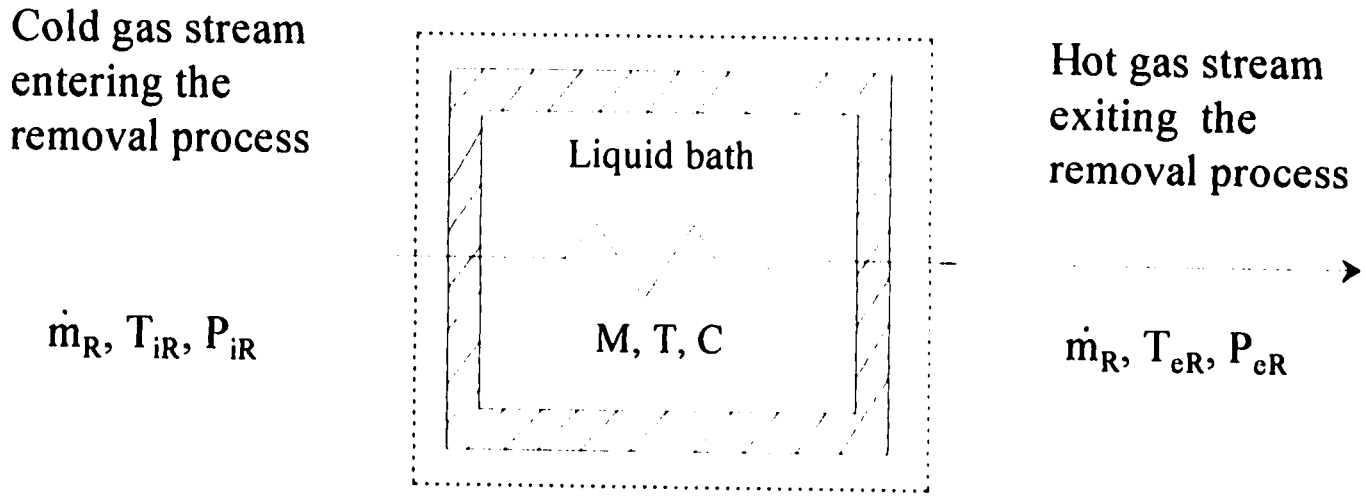

Figure 5.2 Schematic of removal process in sensible heat storage systems

On using equations (5.3) and (5.15) we can write

$$\frac{\dot{m}_R c_P (t - t_S)}{MC} = \theta - \frac{\dot{m}_R}{\dot{m}_S}\theta_S \tag{5.16}$$

Similar to equations (5.4) and (5.5), we have

$$\frac{T_{eR}}{T_{iR}} = 1 + y_R\left(\frac{T}{T_{iR}} - 1\right) \tag{5.17}$$

and

$$T = T_{iR} + \left(T_{IR} - T_{iR}\right)e^{-y_R\left(\theta - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right)} \tag{5.18}$$

Using equations (5.11), (5.17), and (5.18), we get

$$\frac{T_{eR}}{T_{iR}} = 1 + y_R\eta e^{-y_R\left(\theta - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right)} \tag{5.19}$$

Substituting into equation (5.14) and integrating the resulting equation over non-dimensional time from 0 to $[\theta_R - (\dot{m}_R / \dot{m}_S)\theta_S]$, we obtain

$$S_R = MC\left\{\frac{R}{c_P}\left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right)\ln\left(\frac{P_{iR}}{P_O}\right) - \ln\left(\frac{1+\eta}{1+\varepsilon}\right) + \int_0^{\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S} \ln\left(1 + y_R\eta e^{-y_R\theta}\right)d\theta\right\} \tag{5.20}$$

The total entropy generated for the complete storage-removal cycle is obtained by summing equations (5.13) and (5.20), results in

$$S_{GEN} = MC\left\{\frac{R}{c_P}\left[\theta_S \ln\left(\frac{P_{iS}}{P_O}\right)+\left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right)\ln\left(\frac{P_{iR}}{P_O}\right)\right] + (1+\tau_R)(\varepsilon-\eta)+\theta_S\left[\tau_S - \ln(1+\tau_S)\right] + \int_0^{\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S} \ln\left(1+y_R\eta e^{-y_R\theta}\right)d\theta\right\} \tag{5.21}$$

To obtain average rate of entropy production for the complete cycle, we divide the above equation by the total storage-removal time (t_R). It gives

$$\dot{S}_{GEN} = \frac{S_{GEN}}{t_R} = \dot{S}_P + \dot{S}_T \tag{5.22}$$

where the rate of entropy generation due to pressure drop and finite-temperature difference can be expressed, respectively, as

$$\dot{S}_P = \frac{\dot{m}_R c_P}{\theta_R}\left\{\frac{R}{c_P}\left[\theta_S \ln\left(\frac{P_{iS}}{P_O}\right)+\left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right)\ln\left(\frac{P_{iR}}{P_O}\right)\right]\right\}, \tag{5.23}$$

and

$$\dot{S}_T = \frac{\dot{m}_R c_P}{\theta_R}\left\{\theta_S\left[\tau_S - \ln(1+\tau_S)\right]+(1+\tau_R)(\varepsilon-\eta) + \int_0^{\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S} \ln\left(1+y_R\eta e^{-y_R\theta}\right)d\theta\right\} \tag{5.24}$$

5.1.4 Relation Between the Storage Time and the Removal Time

At the end of the removal process, $\theta = \theta_R$ and $T = T_{FR} = T_{IS} = T_O\ (1 + \varepsilon)\ (1 + \tau_R)$. Substituting into equation (5.18) and simplifying gives

$$\theta_R = \frac{\dot{m}_R}{\dot{m}_S}\theta_S + \ln\left(\frac{\eta}{\varepsilon}\right)^{1/\gamma_R} \tag{5.25}$$

5.1.5 Calculating the Availability Supplied to the System

During the storage process, the total availability supplied can be expressed as

$$W_S = \dot{m}_S t_S \left[h_{iS} - h_O - T_O\left(s_{iS} - s_O\right)\right] \tag{5.26}$$

By using the ideal gas assumption and writing the above equation in terms of non-dimensional variables, we get

$$W_S = MCT_O\theta_S\left[\frac{R}{c_P}\ln\left(\frac{P_{iS}}{P_O}\right) + \tau_S - \ln\left(1+\tau_S\right)\right] \tag{5.27}$$

Similarly for the removal process we can easily show that the availability supplied is given by

$$W_R = MCT_O\left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right)\left[\frac{R}{c_P}\ln\left(\frac{P_{iR}}{P_O}\right) + \tau_R - \ln\left(1+\tau_R\right)\right], \tag{5.28}$$

thus the total availability supplied to the complete storage removal cycle is given by

$$W_{total} = W_S + W_R \tag{5.29}$$

The rate of availability supplied is obtained by dividing equation (5.29) by the complete cycle time, t_R. This gives

$$\dot{W} = T_O \frac{\dot{m}_R c_P}{\theta_R} \left\{ \frac{R}{c_P} \left[\theta_S \ln\left(\frac{P_{iS}}{P_O}\right) + \left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right) \ln\left(\frac{P_{iR}}{P_O}\right) \right] \right.$$
$$\left. + \theta_S \left[\tau_S - \ln(1+\tau_S)\right] + \left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right)\left[\tau_R - \ln(1+\tau_R)\right] \right\}, \tag{5.30}$$

where the rate of pressure and temperature availabilities supplied are, respectively, expressed as

$$\dot{W}_P = T_O \frac{\dot{m}_R c_P}{\theta_R} \frac{R}{c_P} \left[\theta_S \ln\left(\frac{P_{iS}}{P_O}\right) + \left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right) \ln\left(\frac{P_{iR}}{P_O}\right) \right] \tag{5.31}$$

and

$$\dot{W}_T = T_O \frac{\dot{m}_R c_P}{\theta_R} \frac{R}{c_P} \left\{ \theta_S \left[\tau_S - \ln(1+\tau_S)\right] + \left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right)\left[\tau_R - \ln(1+\tau_R)\right] \right\} \tag{5.32}$$

5.2 Thermoeconomic Analysis

Substituting equations (5.23) and (5.24) into the cost rate equation [equation (3.6)] we get

$$\dot{\Gamma}-\dot{K}_O=\dot{z}A+\frac{\dot{m}_R c_P}{\theta_R}T_O\left\{\lambda_P\frac{R}{c_P}\left[\theta_S\ln\left(\frac{P_{iS}}{P_O}\right)+\left(\theta_R-\frac{\dot{m}_R}{\dot{m}_S}\theta_S\right)\ln\left(\frac{P_{iR}}{P_O}\right)\right]\right.$$
$$+\lambda_T\left[\theta_S\left[\tau_S-\ln(1+\tau_S)\right]+(1+\tau_R)(\varepsilon-\eta)\right.$$
$$\left.\left.+\int_0^{\theta_R-\frac{\dot{m}_R}{\dot{m}_S}\theta_S}\ln\left(1+y_R\eta e^{-y_R\theta}\right)d\theta\right]\right\} \tag{5.33}$$

Introducing the cost per unit heat conductance based on the overall heat transfer coefficient of the storage process as

$$\gamma_{UA}=\frac{\dot{z}A+\frac{\dot{m}_R c_P}{\theta_R}\lambda_P T_O\frac{R}{c_P}\left[\theta_S\ln\left(\frac{P_{iS}}{P_O}\right)+\left(\theta_R-\frac{\dot{m}_R}{\dot{m}_S}\theta_S\right)\ln\left(\frac{P_{iR}}{P_O}\right)\right]}{U_S A} \tag{5.34}$$

where the relation between U_S and U_R values is discussed in Appendix A. Therefore, the general form for the cost rate equation can be expressed as

$$\dot{\Gamma}-\dot{K}_O=\dot{m}_S c_P\gamma_{UA}NTU_S+\lambda_T T_O\frac{\dot{m}_R c_P}{\theta_R}\left[\theta_S\left[\tau_S-\ln(1+\tau_S)\right]\right.$$
$$\left.+(1+\tau_R)(\varepsilon-\eta)+\int_0^{\theta_R-\frac{\dot{m}_R}{\dot{m}_S}\theta_S}\ln(1+y_R\eta e^{-y_R\theta})d\theta\right] \tag{5.35}$$

Substituting the dimensionless cost ratio

$$\beta=\frac{\lambda_T T_O}{\gamma_{UA}} \tag{5.36}$$

into equation (5.35), we get

$$\dot{\Gamma} - \dot{K}_O = \lambda_T T_O \left\{ \frac{\dot{m}_S c_P NTU_S}{\beta} + \frac{\dot{m}_R c_P}{\theta_R} \left[\theta_S \left[\tau_S - \ln(1+\tau_S) \right] + (1+\tau_R)(\varepsilon - \eta) + \int_0^{\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S} \ln(1 + y_R \eta e^{-y_R \theta}) d\theta \right] \right\} \tag{5.37}$$

It should be noted that the cost rate which is supplied to the system is composed of three parts. These are the annualized cost of operating the storage system, the cost of availability supplied due to the pressure difference, and cost of availability supplied as a result of heat transfer. This can be expressed as

$$\dot{\Gamma}_{as} - \dot{K}_O = \dot{z}A + \lambda_P \dot{W}_P + \lambda_T \dot{W}_T \tag{5.38}$$

By substituting the values of W_P and W_T from equations (5.31) and (5.32), we get

$$\dot{\Gamma}_{as} - \dot{K}_O = \dot{z}A + \frac{\dot{m}_R c_P}{\theta_R} T_O \left\{ \lambda_P \frac{R}{c_P} \left[\theta_S \ln\left(\frac{P_{iS}}{P_O}\right) + \left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right) \ln\left(\frac{P_{iR}}{P_O}\right) \right] + \lambda_T \left[\theta_S \left(\tau_S - \ln(\tau_S + 1)\right) + \left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right)\left(\tau_R - \ln(\tau_R + 1)\right) \right] \right\} \tag{5.39}$$

The above equation can be written in terms of the cost per unit overall heat conductance as

$$\dot{\Gamma}_{as} - \dot{K}_O = \dot{m}_S c_P \gamma_{UA} NTU_S + \frac{\dot{m}_R c_P}{\theta_R} T_O \lambda_T \left[\theta_S \left(\tau_S - \ln(\tau_S + 1)\right) \right.$$

$$+\left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right)\left(\tau_R - \ln(\tau_R + 1)\right)\Bigg] \tag{5.40}$$

or expressing it in terms of the dimensionless cost ratio (β) it can be expressed as

$$\dot{\Gamma}_{as} - \dot{K}_O = \lambda_T T_O \left\{ \frac{\dot{m}_S c_P NTU_S}{\beta} + \frac{\dot{m}_R c_P}{\theta_R}\Big[\theta_S\left(\tau_S - \ln(\tau_S + 1)\right) + \left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right)\left(\tau_R - \ln(\tau_R + 1)\right)\Big]\right\} \tag{5.41}$$

5.3 Presentation and Interpretation of Results

5.3.1 Definition of the Optimization Criteria

Numerous design problems can be posed by selecting different sets of the independent variables. In the present study all the independent variables will be specified except for optimum charging time (an operational variable), and the number of transfer unit (a design variable), which were optimized using a MathCad software. The procedure for calculating the cost rate number is shown in Figure (5.3).

5.3.2 Description of the Optimization Study

The results for all 16 systems examined in this study are presented in Table (5.1). An efficient method of assimilating this information is to begin by focusing attention on the results for a 'typical' optimum system and then to proceed to a more general discussion of the overall results. System No. 8 was selected for this purpose because it is representative

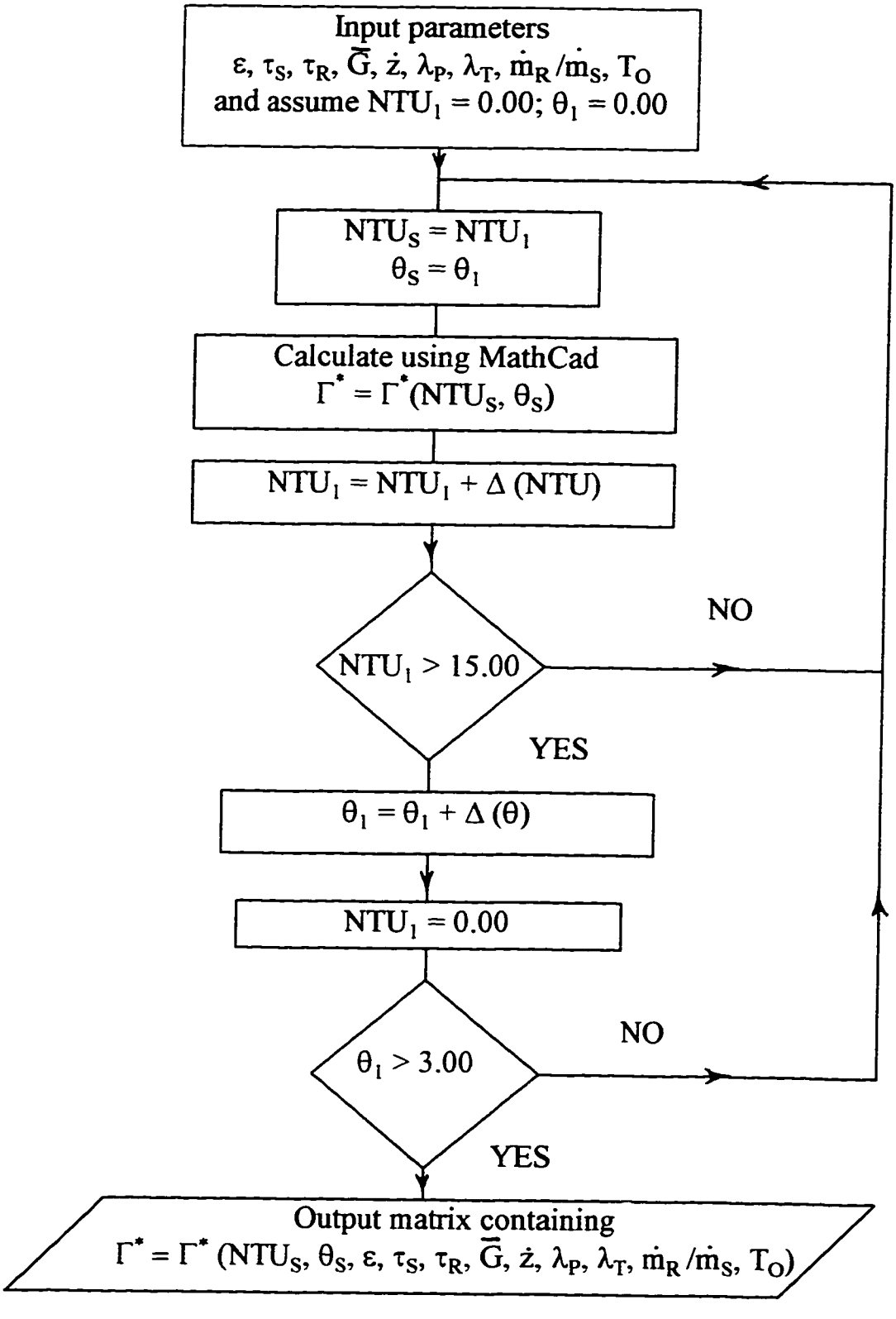

Figure 5.3 Flow diagram for calculating the cost rate number

Table 5.1 Optimization results for the systems used in the study

System →	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$\dot{z}$ ($\$/m^2$-s)	.00377															
λ_P (\$/kW-s)	.04															
λ_T (\$/ kW-s)	.1															
τ_S	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2
τ_R	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.1	.1
$\dot{m}_R/\dot{m}_S$	.5	.5	.5	.5	1	1	1	1	2	2	2	2	1	1	1	1
$\bar{G}$	.005	.005	.05	.05	.005	.005	.05	.05	.005	.005	.05	.05	.05	.05	.05	.05
ε	.01	.1	.01	.1	.01	.1	.01	.1	.01	.1	.01	.1	.01	.1	.01	.1
$\theta_{S,opt}$	1.15	.89	1.12	.87	1.11	.87	1.11	.86	1.09	.85	1.14	.88	.98	.86	.65	.56
$NTU_{S,opt}$	4.53	4.98	4.86	5.26	5.27	5.7	5.29	5.68	6.08	6.46	5.25	5.73	6.04	6.40	5.56	6.05
Γ^*_{opt}	.7624	.7371	.7568	.7334	.7549	.7320	.7548	.7323	.7506	.7293	.7611	.7366	.7130	.6984	.6317	.6267

of many medium-temperature, thermal-energy-storage systems of practical interest. This is the same system which was studied in somewhat more detail by Krane (1985a), while investigating only irreversible losses of a complete storage cycle of a sensible-heat, thermal-energy-storage system.

The cost rate number surface is shown in Figure (5.4) for system No. 8. This surface is representative of the surfaces for all the systems examined in this chapter. Visual inspection shows that there is no local minimum point that could be mistakenly identified as a global minimum by the optimization procedure.

Table (5.2) shows a part of the matrix that is used to generate the surface of Figure (5.4). From this matrix we can get two conclusions. The first conclusion is that for each number of transfer units, NTU_S, or for each dimensionless charging time, θ_S, there is an optimum cost rate number, Γ^*_{opt}. The second conclusion is that we need to optimize the number of transfer units and the dimensionless charging time at the same time to get the optimum cost rate number.

The curve formed by the intersection of the cost rate number surface with the plane for which $NTU_S = NTU_{S,opt}$ is shown in Figure (5.5). As expected, the curve indicates that Γ^* always increases when the storage time deviates from its optimum value of .86, while the curve formed by the intersection of the cost rate number surface with the plane for which $\theta_S = \theta_{S,opt}$ is given in Figure (5.6). This curve clearly indicates that Γ^* is a very weak function of NTU_S over a wide range of values near the optimum point, which is of some

system # 8

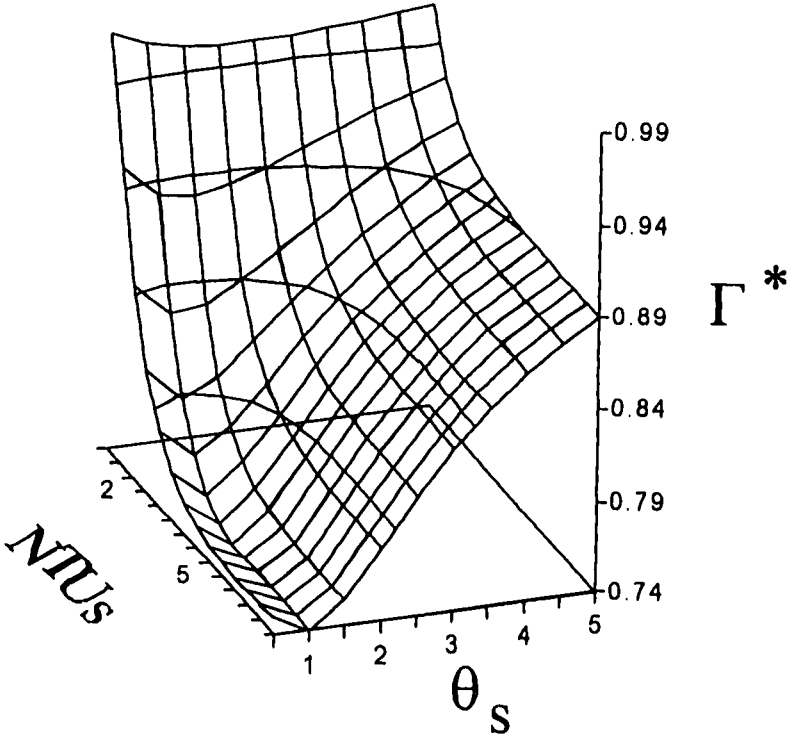

Figure 5.4 Typical cost rate number surface

Table 5.2 Part of the optimization matrix

$NTU_S \rightarrow$ $\theta_S \downarrow$	3	3.5	4	4.5	5	5.5	6	6.5	7
.5	.7659	.7569	.7518	.7491	.7478	.7474	.7476	.7480	.7487
.6	.7583	.7492	.7441	.7413	.7400	.7396	.7397	.7401	.7407
.7	.7535	.7445	.7394	.7366	.7353	.7349	.7350	.7353	.7359
.8	.7509	.7421	.7371	.7344	.7331	.7327	.7327	.7331	.7336
.9	.7502	.7416	.7368	.7341	.7329	**.7324**	.7325	.7328	.7333
1	.7508	.7426	.7379	.7354	.7341	.7337	.7337	.7340	.7345
1.1	.7526	.7447	.7402	.7377	.7366	.7361	.7362	.7365	.7369
1.2	.7552	.7477	.7433	.7410	.7399	.7395	.7395	.7398	.7402
1.3	.7585	.7513	.7472	.7450	.7439	.7435	.7435	.7438	.7442
1.4	.7624	.7555	.7516	.7494	.7484	.7480	.7481	.7483	.7487
1.5	.7666	.7600	.7563	.7543	.7533	.7530	.7530	.7532	.7536

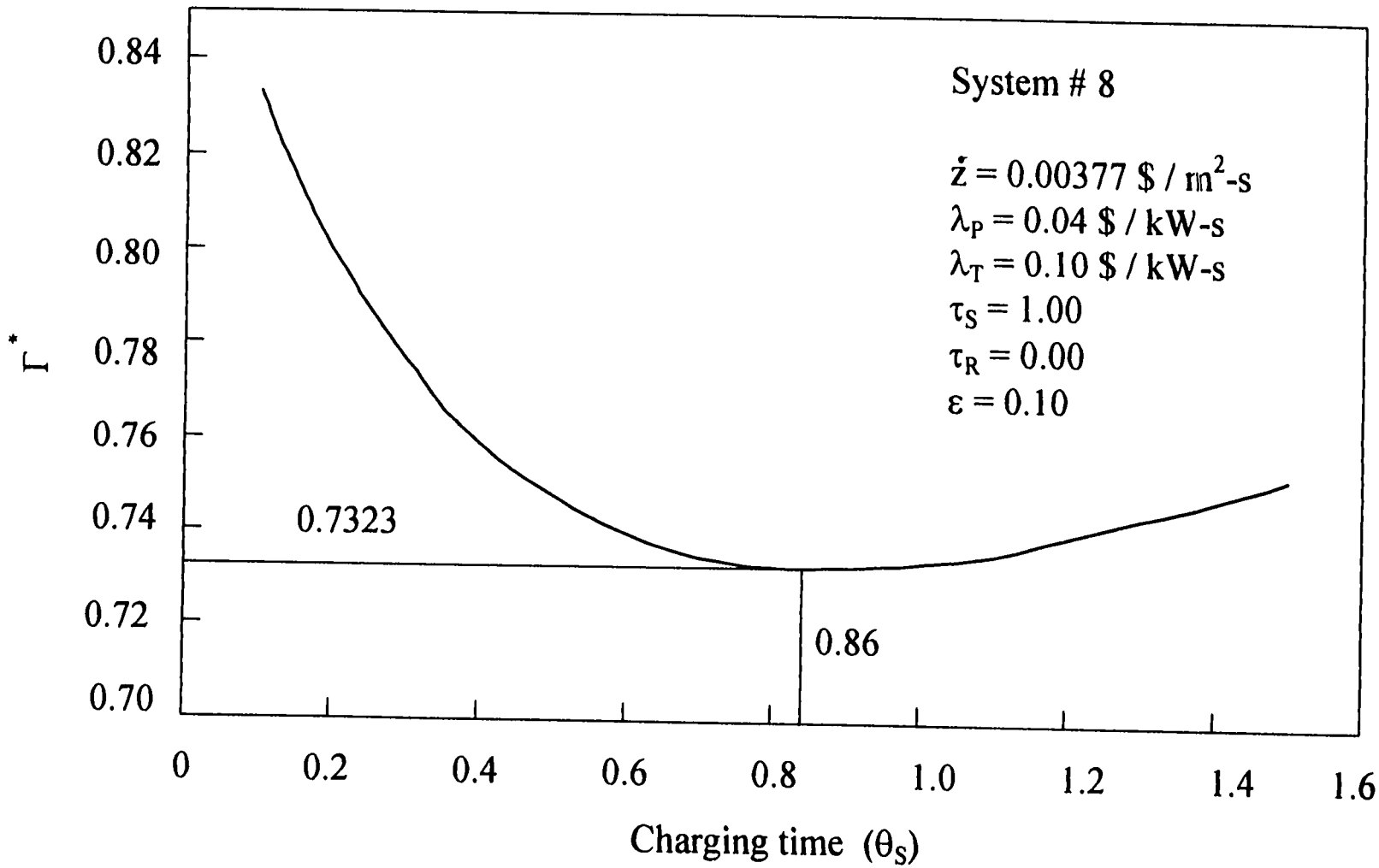

Figure 5.5 Effect of storage time on cost rate number

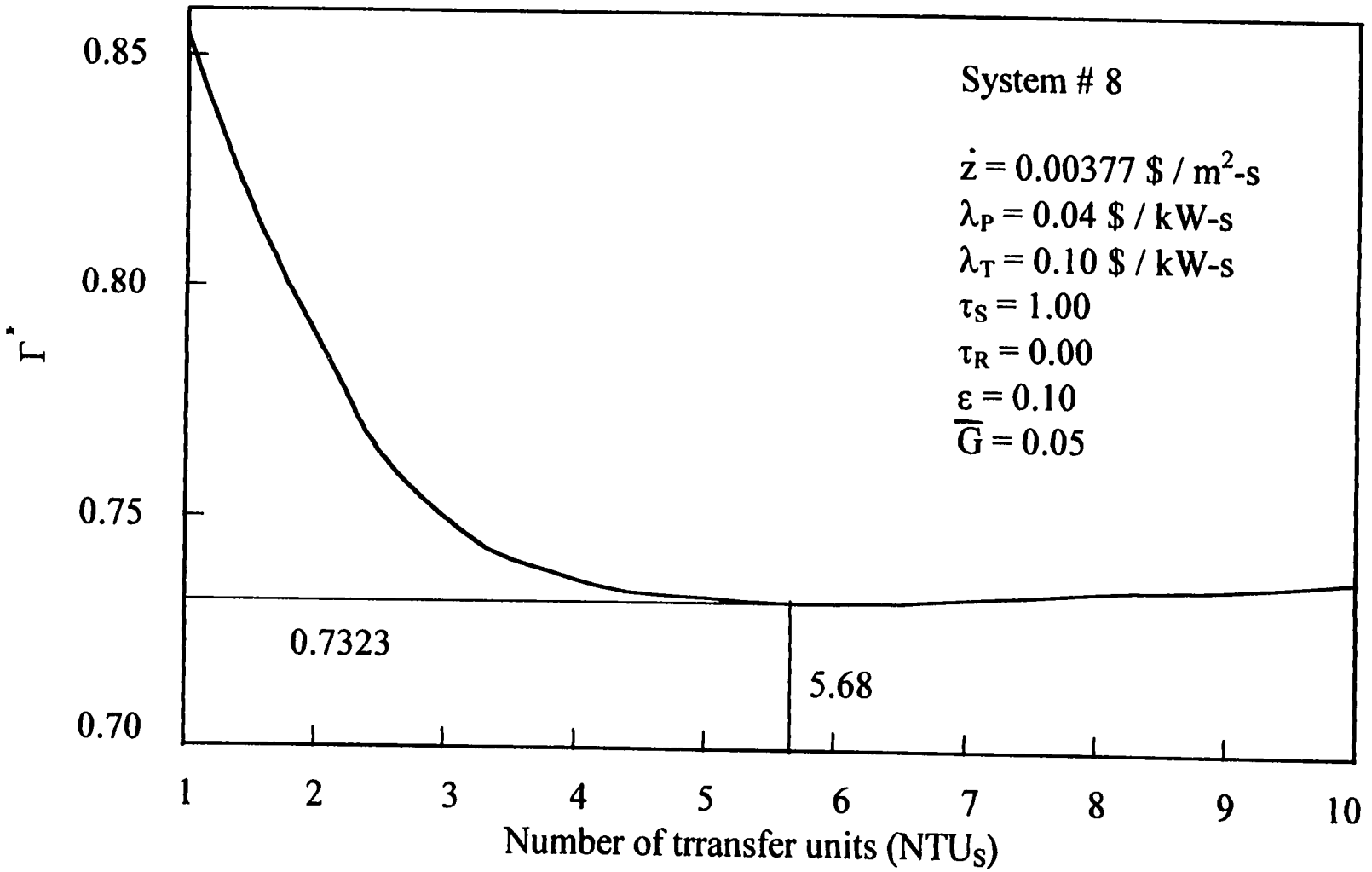

Figure 5.6 Effect of number of tranfer units on cost rate number

practical importance, since this shows that the number of transfer units could be reduced from the optimum value of 5.68 to values as low as 3.5 without seriously reducing the performance of the system.

For system 8 we found $NTU_{S,opt} = 5.68$. If we take $\dot{Z} = 0$ and $\lambda_P = \lambda_T = \lambda$, then $NTU_{S,opt} = 5.533$ which is Krane's result (1987). This implies that cost parameters do influence the performance of the system and the impact of the unit cost parameters (λ_P and λ_T) on optimum number of heat-transfer units and the cost rate number is shown in Table (5.3). The same results are presented in Figures (5.7) and (5.8), respectively. Krane's result (1985a) for the system No. 8, when neglecting the effect of the economic terms is also shown on Figure (5.7).

Figure (5.9) represents the influence of cost parameter $\dot{z}$ on optimum number of heat-transfer units and dimensionless charging time. The optimum values of NTU_S and θ_S for system 8 are also shown on this figure. We note that $NTU_{S,opt}$ is strongly influenced by $\dot{z}$, while $\theta_{S,opt}$ remains more-or-less constant with $\dot{z}$. For example when $\dot{z}$ increases from 0.0001 to 0.01, $NTU_{S,opt}$ decreases by 26.2% while $\theta_{S,opt}$ increases by only 3.7%. In addition, the effect of $\dot{z}$ on the system performance is shown in Figure (5.10), where Γ^*_{opt} is shown as a function of z for a fixed value of other parameters that are indicated in the figure. As $\dot{z}$ increases, Γ^*_{opt} increases which means that we have a lower system performance. So, the effect of $\dot{z}$ should be considered for both the size of the heat exchanger and the system performance.

Table 5.3 Influence of the dimensionless cost ratio on the optimum number of the transfer units and the system performance

λ_P ($ / kW-s)	λ_T ($ / kW-s)	$NTU_{S,opt}$	Γ^*_{opt}	β
.04	.04	4.81	.7446	623
	.1	5.68	.7323	1560
	.2	6.34	.7272	3128
	.3	6.74	.7253	4695
	.4	7.03	.7243	6264
	.5	7.25	.7236	7833
	.6	7.43	.7232	9403
.1	.04	4.33	.7561	365
	.1	5.17	.7384	914
	.2	5.84	.7308	1840
	.3	6.23	.7279	2763
	.4	6.51	.7263	3690
	.5	6.73	.7253	4616
	.6	6.91	.7247	5542

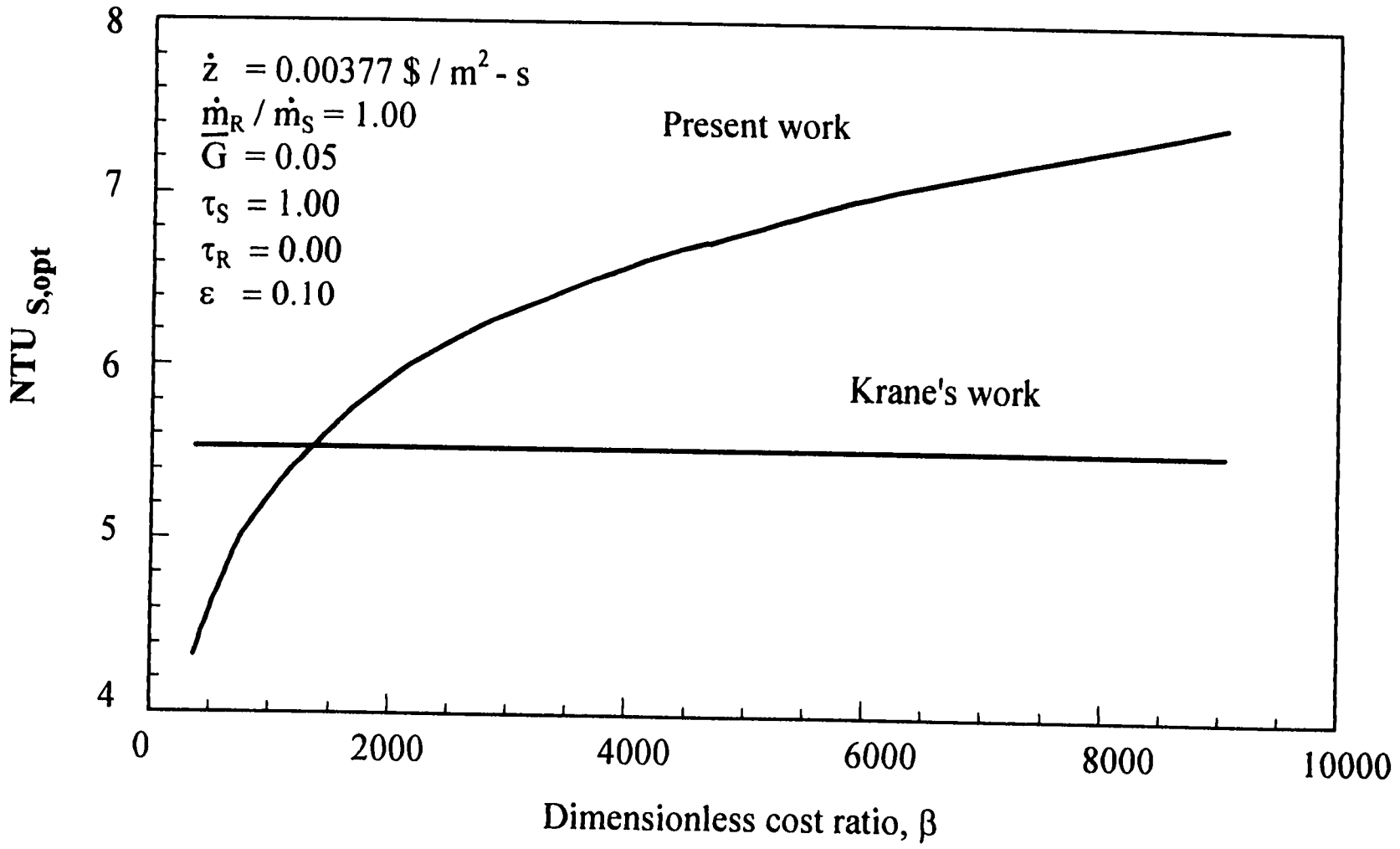

Figure 5.7 Optimum number of transfer units vs the dimensionless cost ratio

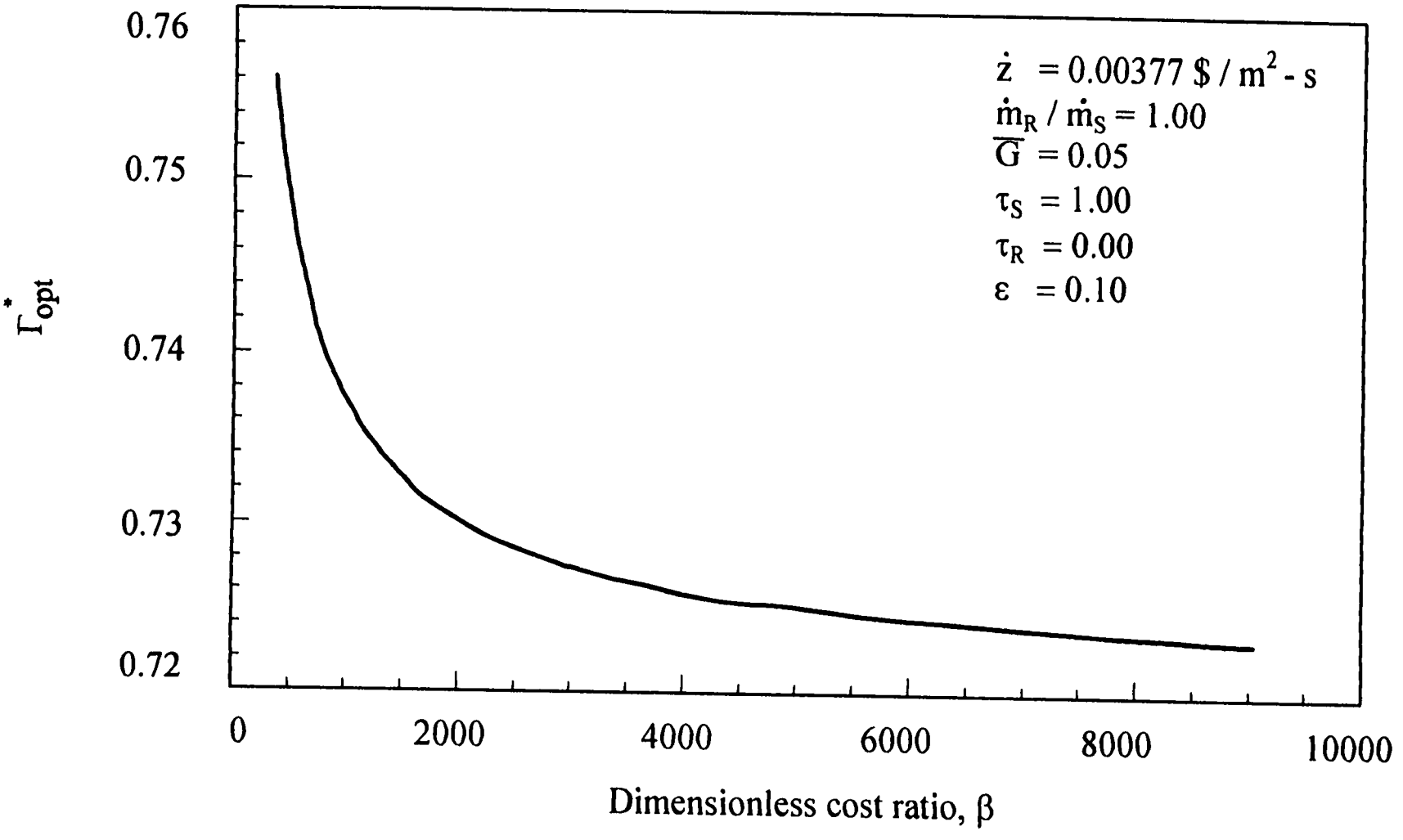

Figure 5.8 Optimum cost rate number vs the dimensionless cost ratio

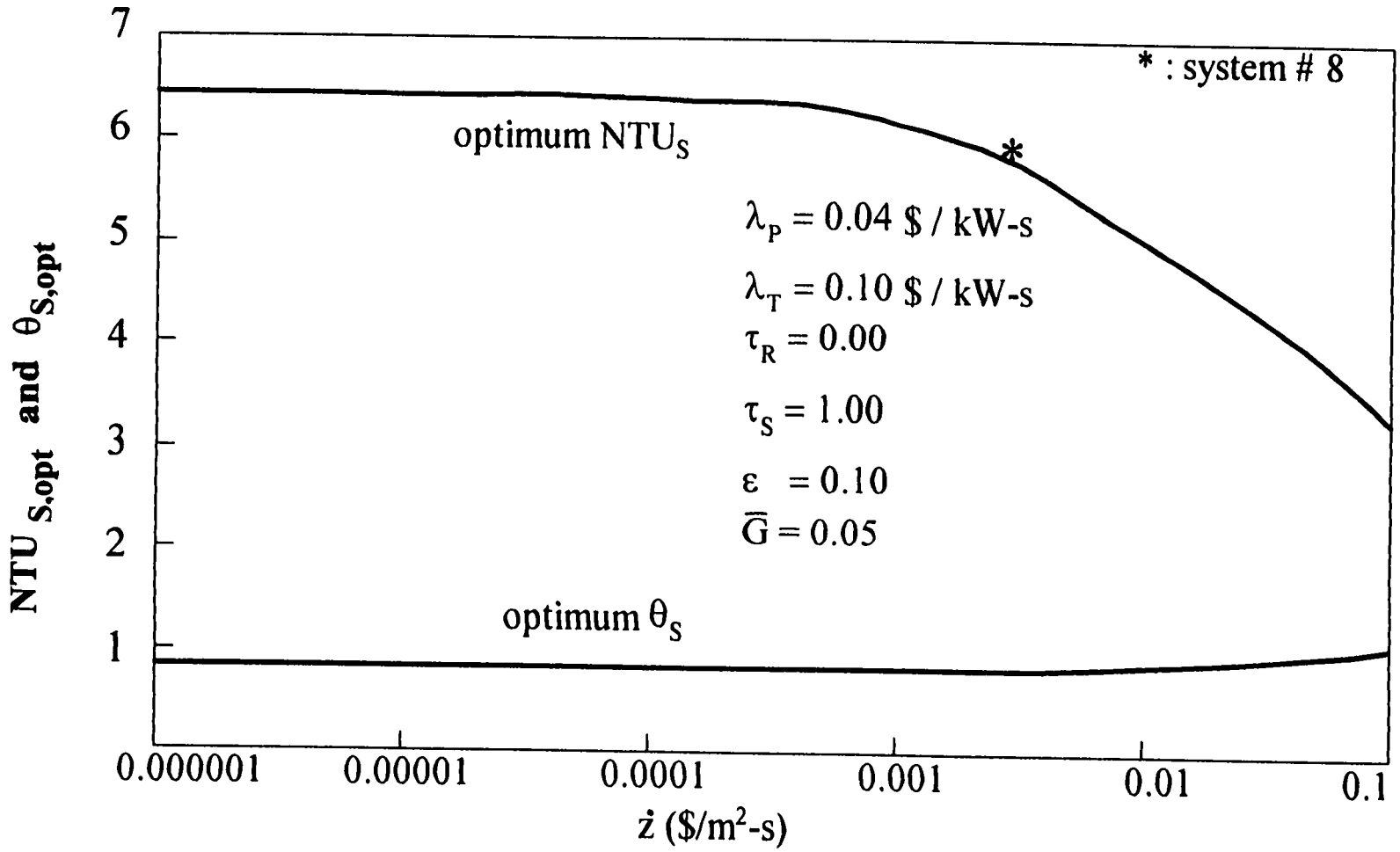

Figure 5.9 Influence of cost parameter $\dot{z}$ on optimum number of transfer units and the dimensionless charging time

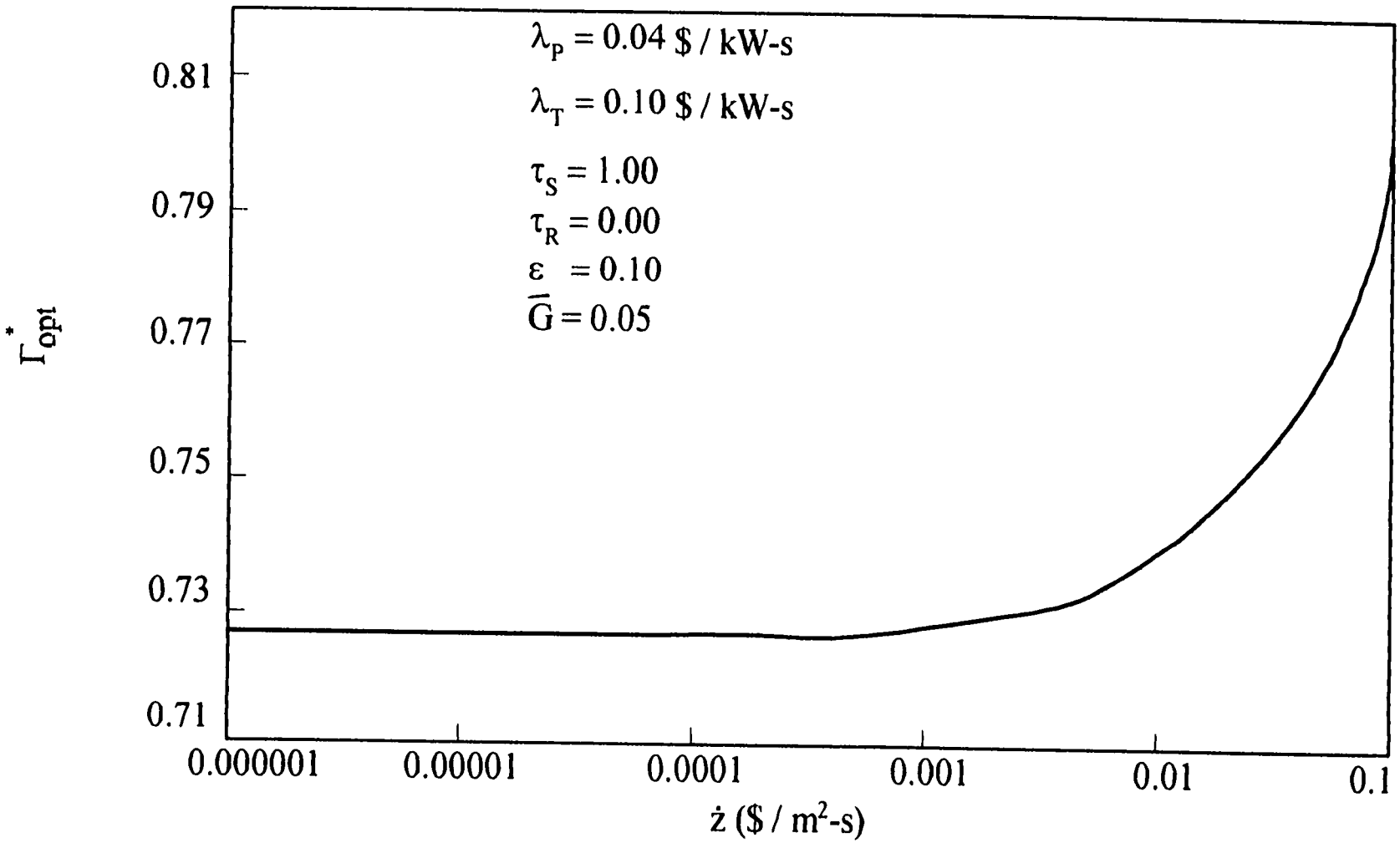

Figure 5.10 Influence of cost parameter $\dot{z}$ on optimum cost rate number

The constants A1 and A2, which are used to calculate the overall heat conductance, in equation (A12) of Appendix A, influence the overall heat transfer coefficient U_S while velocity V_S affects the loss of work due to pressure drop as well as the U_S (refer to equations (A10) and (A12)). Thus, γ_{UA} and β, and consequently the performance of the storage system are influenced by varying these parameters. Figure (5.11) shows the plots of $NTU_{S,opt}$ versus velocity for different values of A1 and A2. It can be seen from this figure that $NTU_{S,opt}$ peaks at some given values of velocity for different values of constants A1 and A2. The figure shows that for a fixed value A1 and increasing A2, the optimum value of NTU_S shifts towards the higher velocities. It should also be noticed that by increasing A2, the optimum value of NTU_S decreases. Figure (5.11) also shows that for a fixed value of A2, and increasing A1, the optimum value of NTU_S decreases and shifts towards the lower velocities.

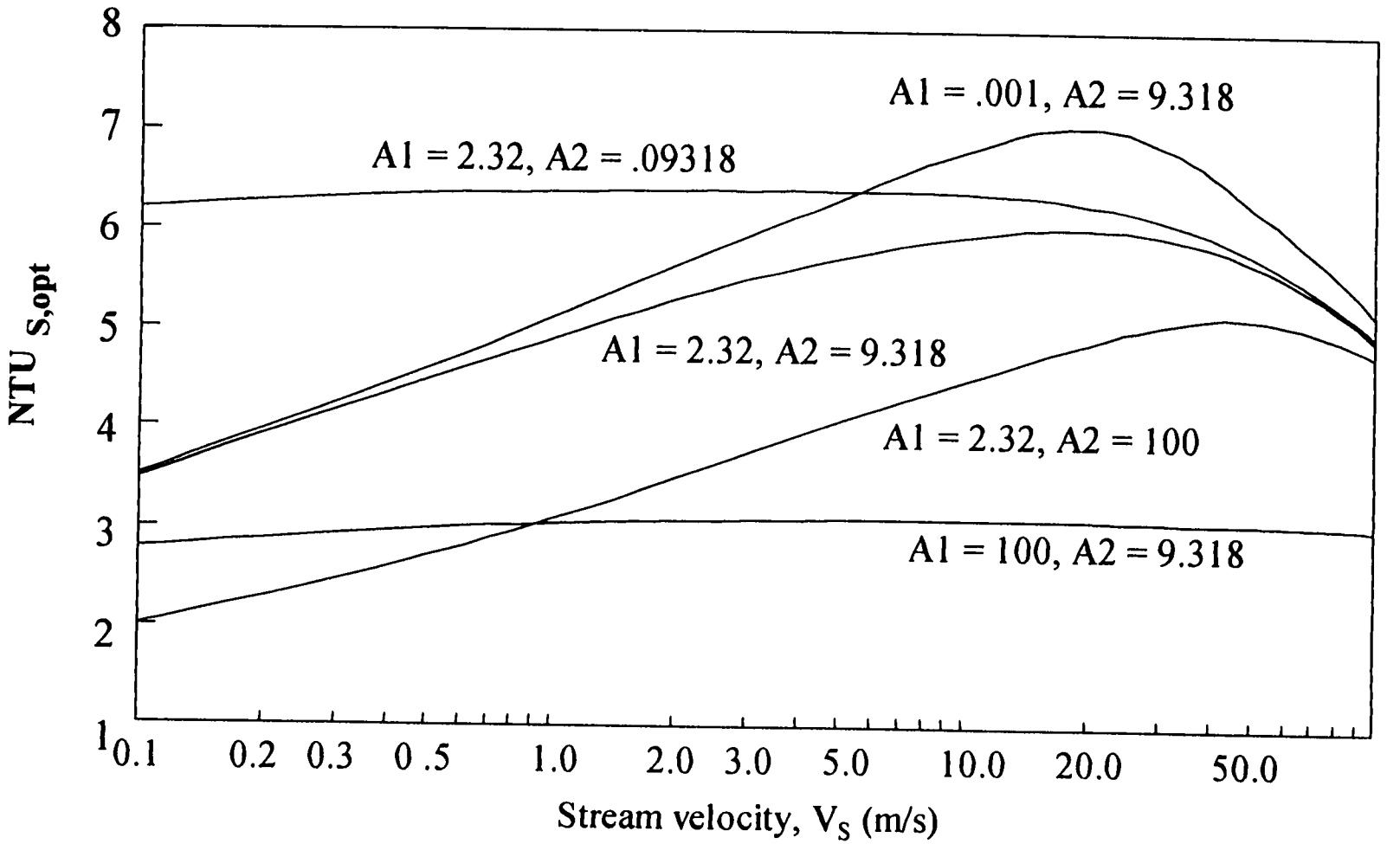

Figure 5.11 Influence of constants A1 and A2 of the equation $1/U = A1 + A2\ V^{-n}$ and velocity on the optimum number of transfer units.

CHAPTER 6

THERMOECONOMIC ANALYSIS AND OPTIMIZATION OF LATENT-HEAT, THERMAL-ENERGY-STORAGE SYSTEMS

Latent heat methods of heat storage have attracted many investigators in different fields such as solar energy and nuclear energy, since the available heat can be stored at the temperatures required in these applications. Most of the phase change materials have high storage capacity with a small temperature swing that makes them attractive for thermal storage. The major drawback in the phase change materials is their low thermal conductivity which limits the heat transfer rates in the storage units that use the phase change material. Similar to the Joulean and sensible heat storage systems, the working fluid is also here assumed to be an ideal gas.

6.1 Second Law Analysis

6.1.1 System Description

The system consists of a phase change material of mass M and specific heat C contained in a well insulated vessel. A heat exchanger passing through the phase change material allows heat transfer to occur between the gas passing through the system and the storage material. Other features and assumptions are similar to the sensible-heat, thermal-energy-storage system, described earlier in the previous chapter.

6.1.2 Analysis of the Storage Process

The instantaneous rate of entropy generation in the system surrounded by the control volume 1 in Figure (6.1) is given by (Bejan, 1988)

$$\dot{S}_{CV1} = \frac{dS}{dt} + \dot{m}_S \left(s_e - s_i \right) \tag{6.1}$$

Applying the relation ($Tds = dh - vdP$), and the ideal gas relations for the working fluid, we get

$$ds = c_P \frac{dT}{T} - R\frac{dP}{P} \tag{6.2}$$

Integrating equation (6.2) gives

$$\left(s_e - s_i \right) = c_P \ln\left(\frac{T_{out}}{T_{iS}} \right) + R\ln\left(\frac{P_{iS}}{P_O} \right) \tag{6.3}$$

Substituting equation (6.3) into equation (6.1) and integrating over time t_S gives

$$S_{CV1} = S_{FS} - S_{IS} + \dot{m}_S c_P t_S \left[\ln\left(\frac{T_{out}}{T_{iS}} \right) + \frac{R}{c_P} \ln\left(\frac{P_{iS}}{P_O} \right) \right] \tag{6.4}$$

In general, for a phase-change material during the melting process we can write (Adebiyi and Russell, 1987)

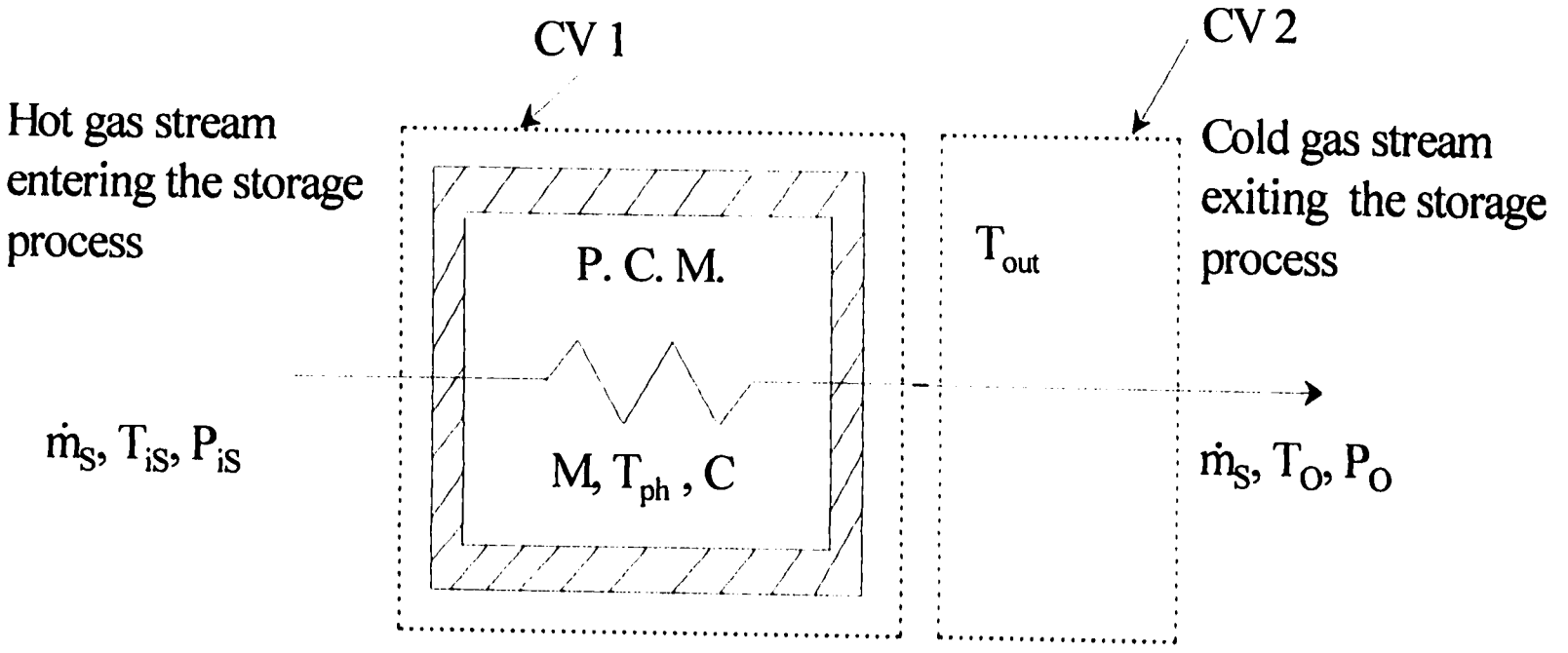

Figure 6.1 Schematic of storage process in latent heat storage systems

$$(S_{FS} - S_{IS}) = \frac{(x_{FS} - x_{IS})ML_{ph}}{T_{ph}}, \tag{6.5}$$

where L_{ph} is the latent heat of fusion and x is the liquid to total mass ratio. Therefore equation (6.4) can be written as

$$S_{CV1} = \frac{(x_{FS} - x_{IS})ML_{ph}}{T_{ph}} + \dot{m}_S c_P t_S \left[\ln\left(\frac{T_{out}}{T_{iS}}\right) + \frac{R}{c_P} \ln\left(\frac{P_{iS}}{P_O}\right) \right] \tag{6.6}$$

For the region marked by CV2, the second law of thermodynamics can be written as

$$\dot{S}_{CV2} = \dot{m}_S (s_O - s_e) - \frac{\dot{Q}_{CV2}}{T_O} \tag{6.7}$$

Using the relation ($Tds = dh - vdP$) and the ideal gas relation for the case when there is no change in pressure, we get

$$(s_O - s_e) = c_P \ln\left(\frac{T_O}{T_{out}}\right) \tag{6.8}$$

The heat transfer between the out going gas and the atmosphere is

$$\dot{Q}_{CV2} = -\dot{m}_S c_P (T_{out} - T_O) \tag{6.9}$$

Substituting equations (6.8) and (6.9) into equation (6.7) we get

$$\dot{S}_{CV2} = \dot{m}_S c_P \left[\ln\left(\frac{T_O}{T_{out}}\right) + \left(\frac{T_{out}}{T_O} - 1\right) \right] \tag{6.10}$$

Integrating equation (6.10) over time t_S gives

$$S_{CV2} = \dot{m}_S c_P t_S \left[\ln\left(\frac{T_O}{T_{out}}\right) + \left(\frac{T_{out}}{T_O} - 1\right) \right] \tag{6.11}$$

The entropy generation during the storage process is obtained by combining equations (6.7) and (6.11) to give

$$S_S = \dot{m}_S c_P t_S \left[\ln\left(\frac{T_O}{T_{iS}}\right) + \left(\frac{T_{out}}{T_O} - 1\right) + \frac{R}{c_P} \ln\left(\frac{P_{iS}}{P_O}\right) \right] + \frac{(x_{FS} - x_{IS}) ML_{ph}}{T_{ph}} \tag{6.12}$$

6.1.3 Analysis of the Removal Process

Consider the energy removal process of Figure (6.2), for time $t > t_S$. The instantaneous rate of entropy generation in the system surrounded by the control volume can be expressed as (Bejan, 1988)

$$\dot{S}_{Ri} = \frac{dS}{d(t - t_S)} + \dot{m}_R (s_e - s_i) \tag{6.13}$$

The total entropy generated can be obtained by integrating equation (6.13) over time ($t - t_S$) from 0 to ($t_R - t_S$). This gives entropy change for the removal process as

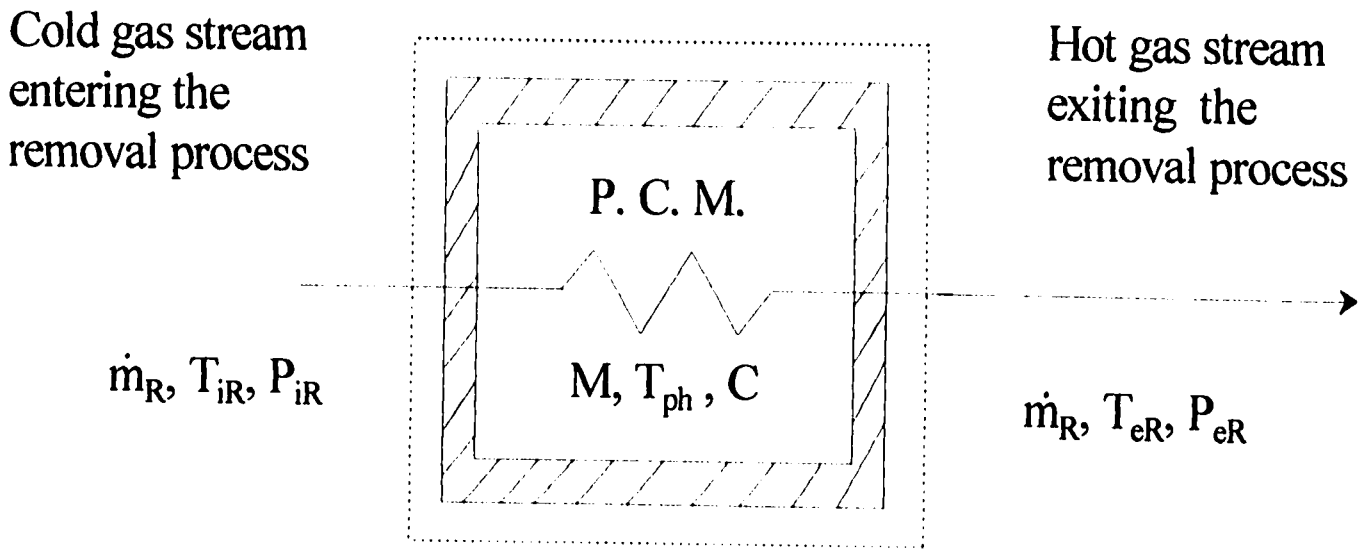

Figure 6.2 Schematic of storage process in latent heat storage systems

$$S_R = (S_{FR} - S_{IR}) + \dot{m}_R (t_R - t_S)(s_e - s_i) \tag{6.14}$$

Using the relation ($Tds = dh - vdP$) we have

$$(s_e - s_i) = c_P \ln\left(\frac{T_{eR}}{T_{iR}}\right) + R\ln\left(\frac{P_{iR}}{P_O}\right) \tag{6.15}$$

Also, the entropy change of the storage element material can be expressed as

$$(S_{FR} - S_{IR}) = -(S_{FS} - S_{IS}) = -\frac{(x_{FS} - x_{IS})ML_{ph}}{T_{ph}} \tag{6.16}$$

Substituting equations (6.15) and (6.16) into equation (6.14) gives

$$S_R = -\frac{(x_{FS} - x_{IS})ML_{ph}}{T_{ph}} + \dot{m}_R (t_R - t_S)\left[c_P \ln\left(\frac{T_{eR}}{T_{iR}}\right) + R\ln\left(\frac{P_{iR}}{P_O}\right)\right] \tag{6.17}$$

The total entropy generation for the complete storage-removal cycle is obtained by combining equations (6.12) and (6.17) to give

$$\begin{aligned} S_{GEN} &= \dot{m}_S c_P t_S \left[\ln\left(\frac{T_O}{T_{iS}}\right) + \left(\frac{T_{out}}{T_O} - 1\right) + \frac{R}{c_P}\ln\left(\frac{P_{iS}}{P_O}\right)\right] \\ &+ m_R (t_R - t_S)\left[c_P \ln\left(\frac{T_{eR}}{T_{iR}}\right) + R\ln\left(\frac{P_{iR}}{P_O}\right)\right] \end{aligned} \tag{6.18}$$

To write the above equations in nondimensional form, we define the following nondimensional parameters for the storage process,

$$\theta = \frac{\dot{m}_S c_P t}{MC}, \quad NTU_S = \frac{U_S A}{\dot{m}_S c_P}, \quad and \quad y_S = 1 - e^{-NTU_S} \tag{6.19}$$

and for the removal process

$$\theta = \frac{\dot{m}_R c_P t}{MC}, \quad NTU_R = \frac{U_R A}{\dot{m}_R c_P}, \quad and \quad y_R = 1 - e^{-NTU_R} \tag{6.20}$$

Also, we define the dimensionless inlet temperature ratios as

$$\tau_S = \frac{T_{iS} - T_O}{T_O}, \quad \tau_R = \frac{T_{iR} - T_O}{T_O} \quad and \quad \overline{T}_{ph} = \frac{T_{ph}}{T_O} \tag{6.21}$$

Also, for the heat exchanger duct we can write similar to Krane's approach (1985a)

$$\frac{T_{out}}{T_{iS}} = 1 + y_S \left(\frac{T_{ph}}{T_{iS}} - 1 \right). \tag{6.22}$$

From equation (6.20) we can write

$$\frac{T_O}{T_{iS}} = \frac{1}{1 + \tau_S} \tag{6.23}$$

Therefore, equation (6.22) becomes

$$\frac{T_{out}}{T_O} = \frac{T_{out}}{T_{iS}} \frac{T_{iS}}{T_O} = (1 + \tau_S) \left[1 + y_S \left(\frac{\overline{T}_{ph}}{1 + \tau_S} - 1 \right) \right] \tag{6.24}$$

In the same way it can be easily shown that

$$\frac{T_{eR}}{T_{iR}} = 1 + y_R \left(\frac{\overline{T}_{ph}}{1+\tau_R} - 1 \right). \tag{6.25}$$

Substituting equations (6.22)-(6.25) into equation (6.18) and writing the resulting expression in non-dimensional form, we get

$$S = MC \left\{ \theta_S \left[\frac{R}{c_P} \ln\left(\frac{P_{iS}}{P_O} \right) + (1+\tau_S) \left[1 + y_S \left(\frac{\overline{T}_{ph}}{1+\tau_S} - 1 \right) \right] - \ln(1+\tau_S) - 1 \right] \right.$$
$$\left. + \left(\theta_R - \frac{m_R}{m_S} \theta_S \right) \left[\frac{R}{c_P} \ln\left(\frac{P_{iR}}{P_O} \right) + \ln\left[1 + y_R \left(\frac{\overline{T}_{ph}}{1+\tau_R} - 1 \right) \right] \right] \right\}. \tag{6.26}$$

To obtain an average rate of entropy generation for the complete cycle, we divide equation (6.26) by the cycle time, t_R. It gives

$$\dot{S} = \frac{S}{t_R} = \dot{S}_P + \dot{S}_T \tag{6.27}$$

where the rate of entropy generation due to pressure drop and finite-temperature difference can be expressed, respectively, as

$$\dot{S}_P = \frac{\dot{m}_R c_P}{\theta_R} \cdot \frac{R}{c_P} \left[\theta_S \ln\left(\frac{P_{iS}}{P_O} \right) + \left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S} \theta_S \right) \ln\left(\frac{P_{iR}}{P_O} \right) \right], \tag{6.28}$$

and

$$\dot{S}_T = \frac{\dot{m}_R c_P}{\theta_R}\left\{\theta_S\left[(1+\tau_S)\left[1+y_S\left(\frac{\overline{T}_{ph}}{1+\tau_S}-1\right)\right]-\ln(1+\tau_S)-1\right]\right.$$
$$\left.+\left(\theta_R-\frac{\dot{m}_R}{\dot{m}_S}\theta_S\right)\ln\left[1+y_R\left(\frac{\overline{T}_{ph}}{1+\tau_R}-1\right)\right]\right\} \tag{6.29}$$

6.1.4 Duration of the Charge and Discharge Processes

The duration of the charge and discharge processes are related by the requirement that all the energy stored during the charge process be released during the discharge (neglecting energy losses). This results in the following expression

$$\dot{m}_R c_P (t_R - t_S)(T_{ph} - T_{iR})y_R = \dot{m}_S c_P t_S (T_{iS} - T_{ph})y_S \tag{6.30}$$

Or writing in terms of the dimensionless removal time, we get

$$\theta_R = \left[\frac{\dot{m}_R}{\dot{m}_S} + \frac{y_S}{y_R}\frac{(\tau_S+1)-\overline{T}_{ph}}{\overline{T}_{ph}-(\tau_R-1)}\right]\theta_S \tag{6.31}$$

6.1.5 Calculating the Availability Supplied to the Cycle

The availability supplied to the system can be calculated in the same way as described earlier for the sensible heat thermal energy storage systems, refer to section 5.1.4. Therefore, we can write the rate of availability supplied to the storage cycle as

$$\dot{W} = T_O \frac{\dot{m}_R c_P}{\theta_R} \left\{ \frac{R}{c_P} \left[\theta_S \ln\left(\frac{P_{iS}}{P_O}\right) + \left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right) \ln\left(\frac{P_{iR}}{P_O}\right) \right] \right.$$
$$\left. + \theta_S \left[\tau_S - \ln(1+\tau_S)\right] + \left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right)\left[\tau_R - \ln(1+\tau_R)\right] \right\}, \tag{6.32}$$

where the rate of pressure and temperature availabilities supplied are, respectively, expressed as

$$\dot{W}_P = T_O \frac{\dot{m}_R c_P}{\theta_R} \frac{R}{c_P} \left[\theta_S \ln\left(\frac{P_{iS}}{P_O}\right) + \left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right) \ln\left(\frac{P_{iR}}{P_O}\right) \right] \tag{6.33}$$

and

$$\dot{W}_T = T_O \frac{\dot{m}_R c_P}{\theta_R} \frac{R}{c_P} \left\{ \theta_S \left[\tau_S - \ln(1+\tau_S)\right] + \left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right)\left[\tau_R - \ln(1+\tau_R)\right] \right\} \tag{6.34}$$

6.2 Thermoeconomic Analysis of the System

Substituting equations (6.28) and (6.29) into the cost rate equation [equation (3.6)] we get

$$\dot{\Gamma} - \dot{K}_O = \dot{z}A + \frac{\dot{m}_R c_P}{\theta_R} T_O \left\{ \lambda_P \frac{R}{c_P} \left[\theta_S \ln\left(\frac{P_{iS}}{P_O}\right) + \left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right) \ln\left(\frac{P_{iR}}{P_O}\right) \right] \right.$$
$$+ \lambda_T \left[\theta_S \left[(1+\tau_S)\left[1 + y_S \left(\frac{\bar{T}_{ph}}{1+\tau_S} - 1 \right) \right] - \ln(1+\tau_S) - 1 \right] \right.$$
$$\left. \left. + \left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right) \ln\left[1 + y_R \left(\frac{\bar{T}_{ph}}{1+\tau_R} - 1 \right) \right] \right] \right\} \tag{6.35}$$

Introducing the cost per unit heat conductance based on the overall heat transfer coefficient of the storage process as [refer to equation (3.8)]

$$\gamma_{UA} = \frac{zA + \frac{\dot{m}_R c_P}{\theta_R}\lambda_P T_O \frac{R}{c_P}\left[\theta_S \ln\left(\frac{P_{iS}}{P_O}\right) + \left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right)\ln\left(\frac{P_{iR}}{P_O}\right)\right]}{U_S A} \tag{6.36}$$

where the relation between U_S and U_R values are discussed in appendix A. Therefore, the general form of the cost rate equation can be expressed as

$$\dot{\Gamma} - \dot{K}_O = \dot{m}_S c_P \gamma_{UA} NTU_S + \lambda_T T_O \frac{\dot{m}_R c_P}{\theta_R}\left\{\theta_S\left[(1+\tau_S)\left[1 + y_S\left(\frac{\overline{T}_{ph}}{1+\tau_S} - 1\right)\right] - \ln(1+\tau_S) - 1\right]\right.$$
$$\left. + \left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right)\ln\left[1 + y_R\left(\frac{\overline{T}_{ph}}{1+\tau_R} - 1\right)\right]\right\} \tag{6.37}$$

Introducing the dimensionless cost ratio as

$$\beta = \frac{\lambda_T T_O}{\gamma_{UA}} \tag{6.38}$$

and substitute into equation (6.37), we get

$$\dot{\Gamma} - \dot{K}_O = \lambda_T T_O \left\{\frac{\dot{m}_S c_P NTU_S}{\beta} + \frac{\dot{m}_R c_P}{\theta_R}\left[\theta_S\left[(1+\tau_S)\left[1 + y_S\left(\frac{\overline{T}_{ph}}{1+\tau_S} - 1\right)\right] - \ln(1+\tau_S) - 1\right]\right.\right.$$
$$\left.\left. + \left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right)\ln\left[1 + y_R\left(\frac{\overline{T}_{ph}}{1+\tau_R} - 1\right)\right]\right]\right\} \tag{6.39}$$

For the cost rate that is supplied to the system, the equations that are derived earlier in chapter 5 for the sensible heat thermal energy storage system are also applicable for the latent-heat storage system because the availability supplied is related to the working fluid regardless of the type of the system. These equations are

$$\dot{\Gamma}_{as} - \dot{K}_O = \dot{z}A + \frac{\dot{m}_R c_P}{\theta_R} T_O \left\{ \lambda_P \frac{R}{c_P} \left[\theta_S \ln\left(\frac{P_{iS}}{P_O}\right) + \left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right) \ln\left(\frac{P_{iR}}{P_O}\right) \right] \right.$$
$$\left. + \lambda_T \left[\theta_S \left(\tau_S - \ln(\tau_S + 1)\right) + \left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right)\left(\tau_R - \ln(\tau_R + 1)\right) \right] \right\} \tag{6.40}$$

In terms of the cost per unit heat conductance, equation (6.40) can be written in the form

$$\dot{\Gamma}_{as} - \dot{K}_O = \dot{m}_S c_P \gamma_{UA} NTU_S + \frac{\dot{m}_R c_P}{\theta_R} T_O \lambda_T \left[\theta_S \left(\tau_S - \ln(\tau_S + 1)\right) \right.$$
$$\left. + \left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right)\left(\tau_R - \ln(\tau_R + 1)\right) \right], \tag{6.41}$$

and in terms of the dimensionless cost ratio, the cost of availability supplied can be expressed as

$$\dot{\Gamma}_{as} - \dot{K}_O = \lambda_T T_O \left\{ \frac{\dot{m}_S c_P NTU_S}{\beta} + \frac{\dot{m}_R c_P}{\theta_R} \left[\theta_S \left(\tau_S - \ln(\tau_S + 1)\right) \right.\right.$$
$$\left.\left. + \left(\theta_R - \frac{\dot{m}_R}{\dot{m}_S}\theta_S\right)\left(\tau_R - \ln(\tau_R + 1)\right) \right] \right\} \tag{6.42}$$

The cost rate number (Γ^*) is obtained by substituting equations (6.35) and (6.40) into equation (3.12), i.e.,

$$\Gamma^{*} = \frac{\dot{\Gamma} - \dot{K}_O}{\dot{\Gamma}_{as} - \dot{K}_O} \tag{6.43}$$

6.3 Presentation and Interpretation of Results

6.3.1 Definition of the Optimization Criteria

The main objective of thermoeconomic study is to minimize the cost rate number. This is achieved by selecting the independent variables in a way that will give us a minimum Γ^{*}. In the design and analysis of latent heat storage systems, the phase change temperature of the storage medium and the number of transfer units (size of the heat exchanger) are the most important parameters that have to be considered. The procedure for calculating the cost rate number is shown in Figure (6.3).

6.3.2 Description of the Optimization Study

The effect of the storage gas inlet temperature ratio, τ_S, on the system performance is demonstrated in Table (6.1) and Figure (6.4) by showing that the system cost rate number, Γ^{*}_{opt}, decreases as the storage gas inlet temperature ratio, τ_S, increases. The curve asymptotically approaches zero as τ_S increases, but the improvement in system performance is slow, and very large values of τ_S are necessary before low cost rate numbers can be reached. For example, an increase in τ_S from a value of 1 to a value of 4 will give an improvement in the system performance of about 15%.

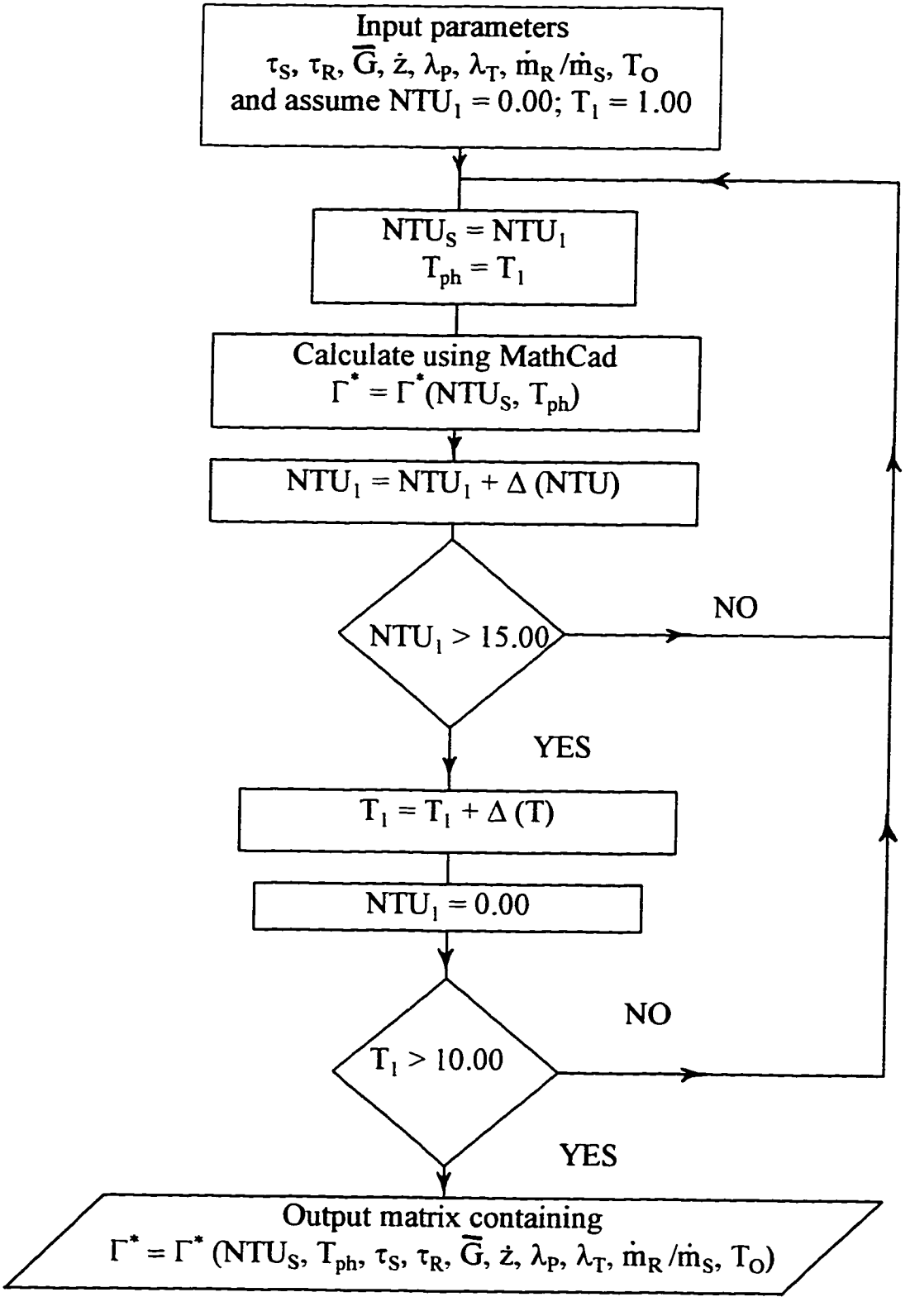

Figure 6.3 Flow diagram for calculating the cost rate number

Table 6.1 Effects of elevated storage gas inlet temperature on system performance

System→ Variable↓	1	2	3	4
τ_R	0.00			
τ_S	1.00	2.00	3.00	4.00
$\dot{z}$ ($\$ / m^2$-s)	0.00377			
λ_T ($\$ / kW$-s)	.1			
λ_P ($\$ / kW$-s)	.04			
V (m / s)	20	20	20	20
NTU_{opt}	6.4	7.2	7.6	7.8
$\mathbf{T}_{ph,opt}$	1.4	1.8	2.1	2.4
Γ^{*}_{opt}	.695	.650	.620	.595

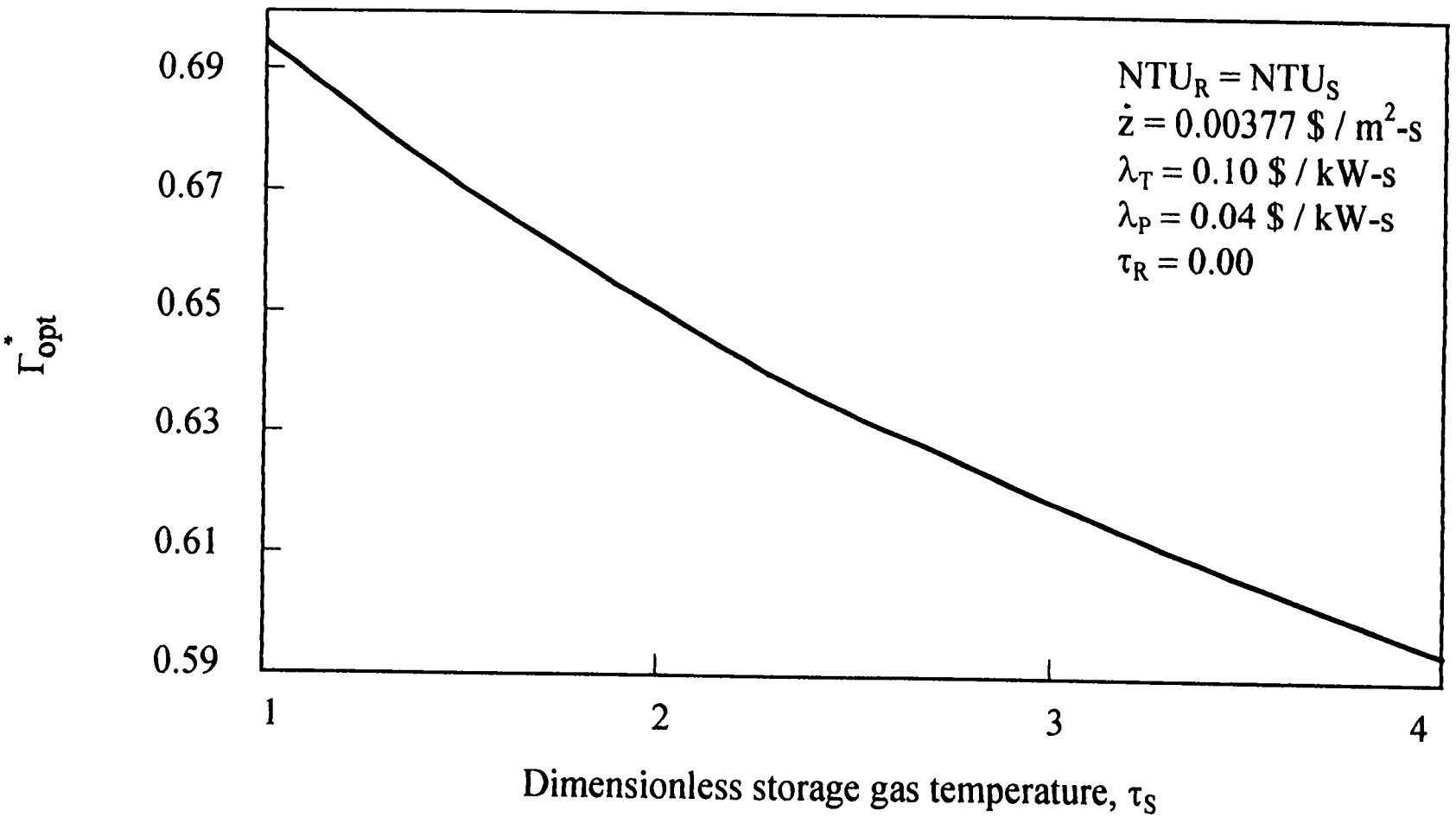

Figure 6.4 Optimum cost rate number as a function of dimensionless storage gas temperature

Figure (6.5) shows the influence of the storage gas inlet temperature ratio, τ_S, on the number of transfer units, NTU_{opt}. We see from the Figure that as τ_S increases we obtain a larger size heat exchanger for minimum cost rate number. Applying the results of Figures (6.4) and (6.5) on a design problem of latent-heat, thermal-energy-storage systems, the designer should compromise in selecting the value of τ_S in away that it gives acceptable values of relatively low cost rate number and small size heat exchanger. Also from Figure (6.5) we see that for $\lambda_T = 0.1$, $\lambda_P = 0.04$ and $\dot{z} = 0.00377$, we get $NTU_{opt} = 6.2$, and with varying these values we will get different values of NTU's. In thermodynamic optimization (i.e. , $\lambda_P = \lambda_T$ and $\dot{z} = \dot{K}_O = 0$) we will get a fixed value of $NTU_{opt} = 7.3$ which is the same as that obtained from the analysis of Adebiyi and Russell (1987).

The optimum phase change temperature as a function of dimensionless charging fluid inlet temperature, τ_S, is shown in Figure (6.6). The figure uses $NTU_R = NTU_S$ and $\tau_R = 0.0$ (ambient removal temperature), for simplicity. It can be seen from the figure that as the charging fluid inlet temperature increases we need to use a higher temperature phase change material in order to minimize the cost rate number (optimum performance). The effect of the heat exchanger size on the phase change temperature is also expressed in Figure (6.6). The phase-change temperature decreases as NTU increases until the optimum value of NTU is reached, for the same charging fluid inlet temperature.

The influence of the unit cost parameters, λ_P, and λ_T, which is represented in the dimensionless cost ratio, β, is shown in Figures (6.7) and (6.8) to study the optimum number of heat-transfer units and the cost rate number, respectively. These two figures

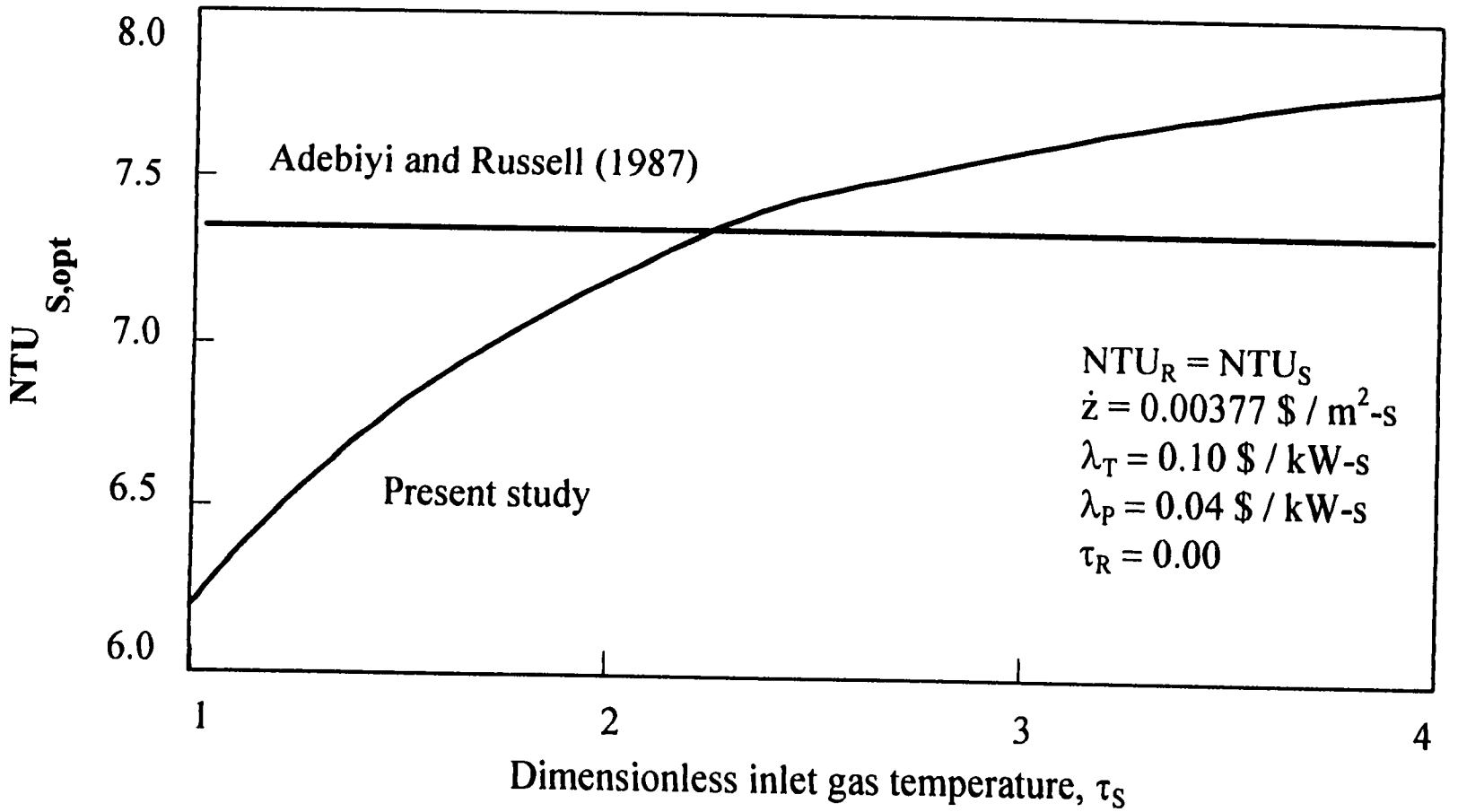

Figure 6.5 Optimum number of transfer units as a function of dimensionless inlet gas temperature

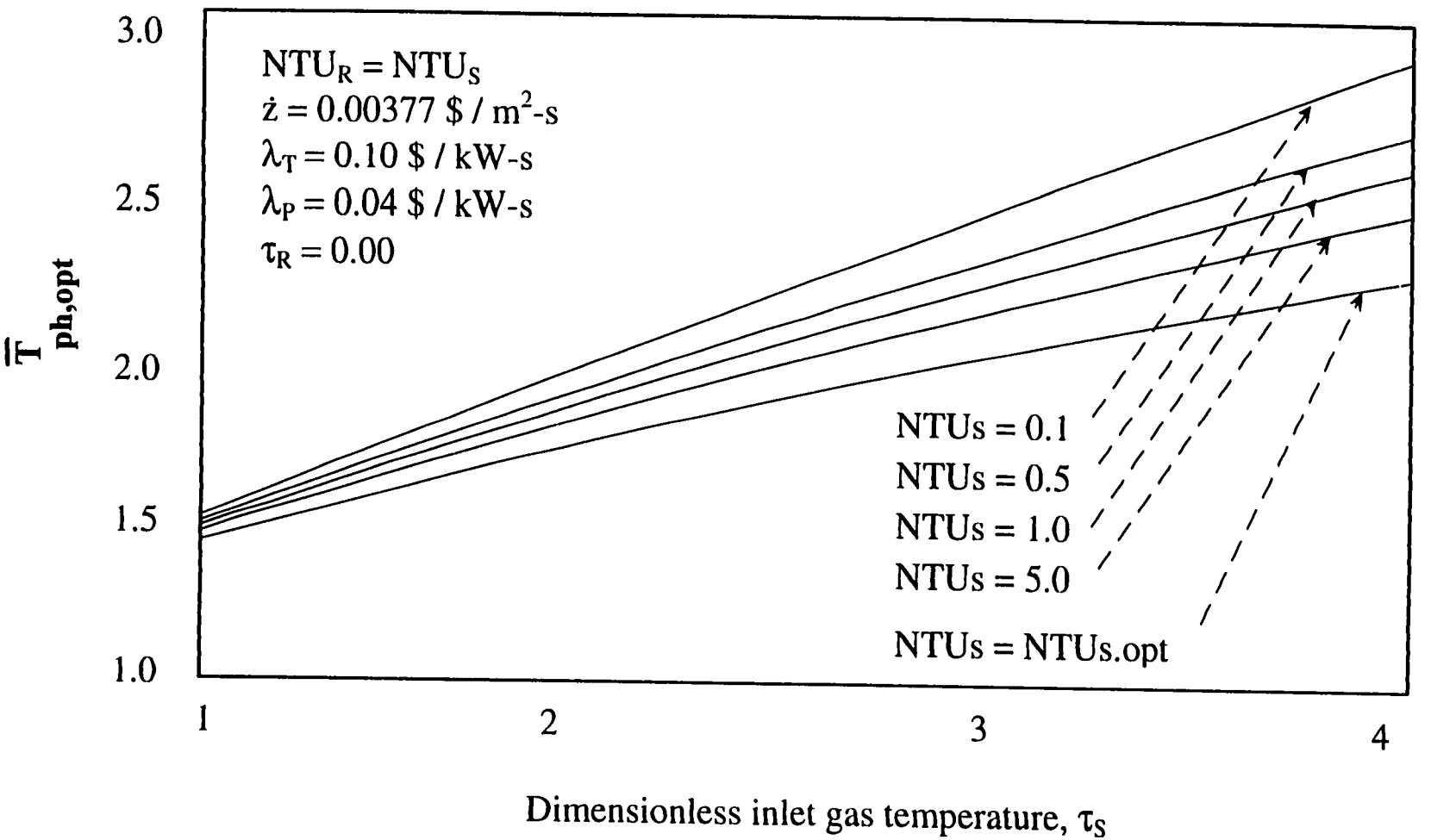

Figure 6.6 Optimum phase change temperature as a function of dimensionless charging fluid inlet temperature

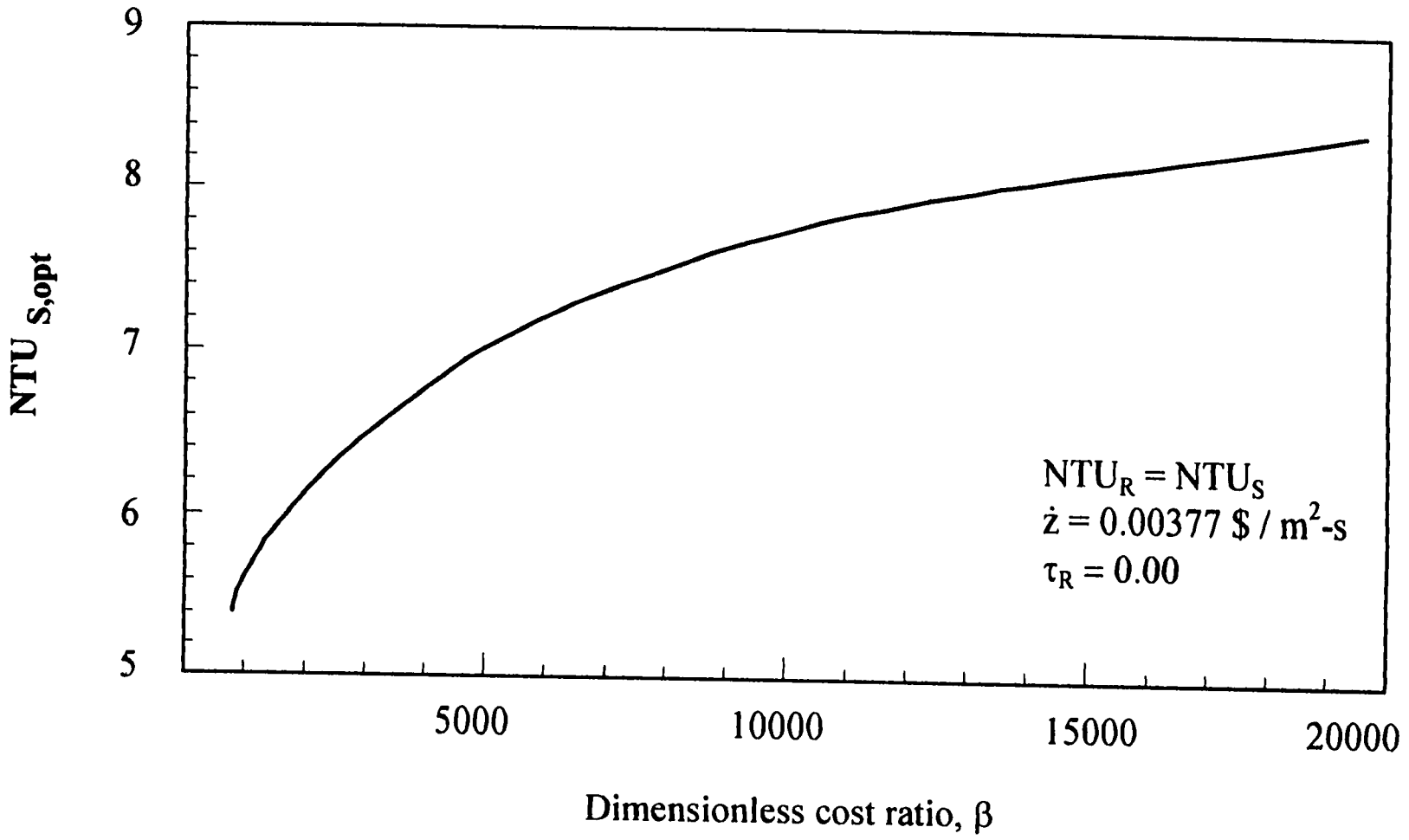

Figure 6.7 Optimum number of transfer units vs the dimensionless cost ratio

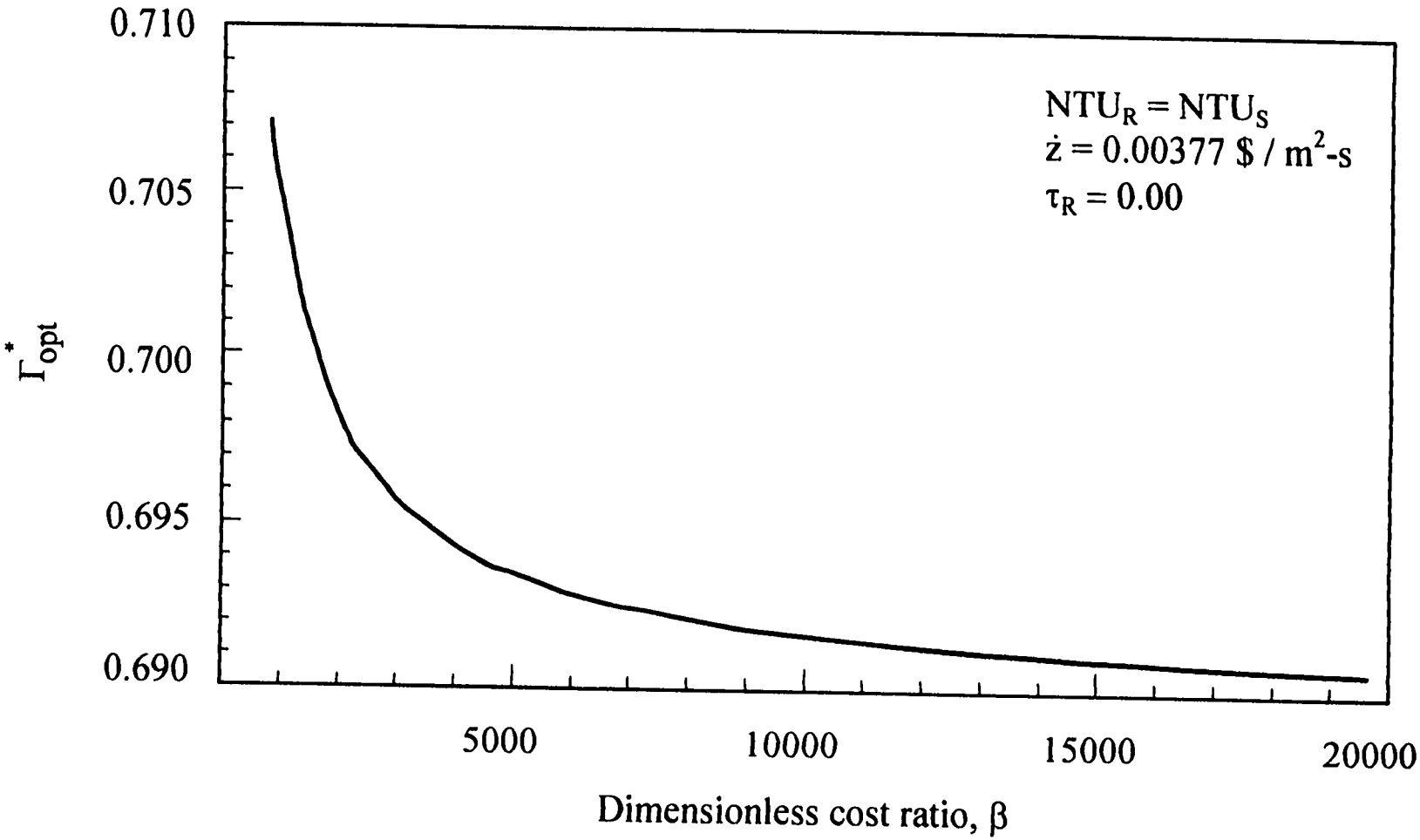

Figure 6.8 Optimum cost rate number vs the dimensionless cost ratio

show that for larger values of β, it results in the undesired large size heat exchanger, while for larger values of β they give better system performance (decreasing Γ^*). The effect of β on the size of the heat exchanger is obvious because β has a reverse proportionality with the cost per unit overall heat conductance which is directly proportional to the size of the heat exchanger.

Figure (6.9) represents the influence of cost parameter $\dot{z}$ on optimum number of heat-transfer. From the figure we see that the great influence of $\dot{z}$ on the optimum NTU values in a way that as $\dot{z}$ increases we obtain a smaller size heat exchanger. However, Figure (6.10) demonstrates the effect of $\dot{z}$ on the system performance. We see that as $\dot{z}$ increases, Γ^*_{opt} increases which means that we have a lower system performance. So, the effect of $\dot{z}$ should be considered for both the size of the heat exchanger and the system performance.

The effect of the stream velocity on the size of the heat exchanger and the system performance is shown in Figures (6.11) and (6.12), respectively. In general, varying the stream velocity would affect the overall heat transfer coefficient. Thus, γ_{UA} and β, and therefore the performance of the storage system are influenced by varying these parameters. It can be seen from Figure (6.11) that NTU_{opt} peaks at some given values of velocity. Combining the results of Figures (6.11) and (6.12) implies that the optimum value of NTU shifts towards the higher velocities.

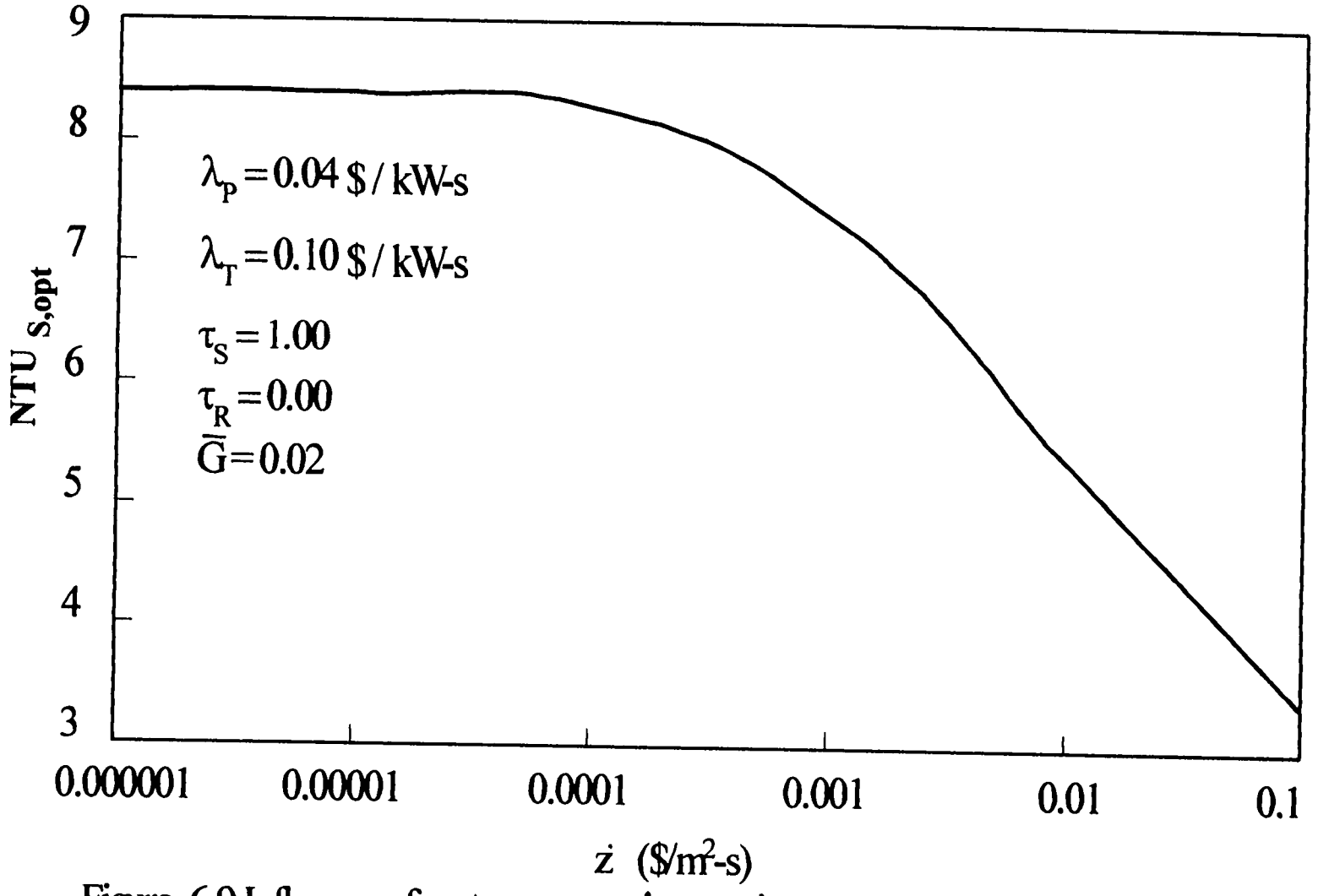

Figure 6.9 Influence of cost parameter $\dot{z}$ on optimum number of transfer units

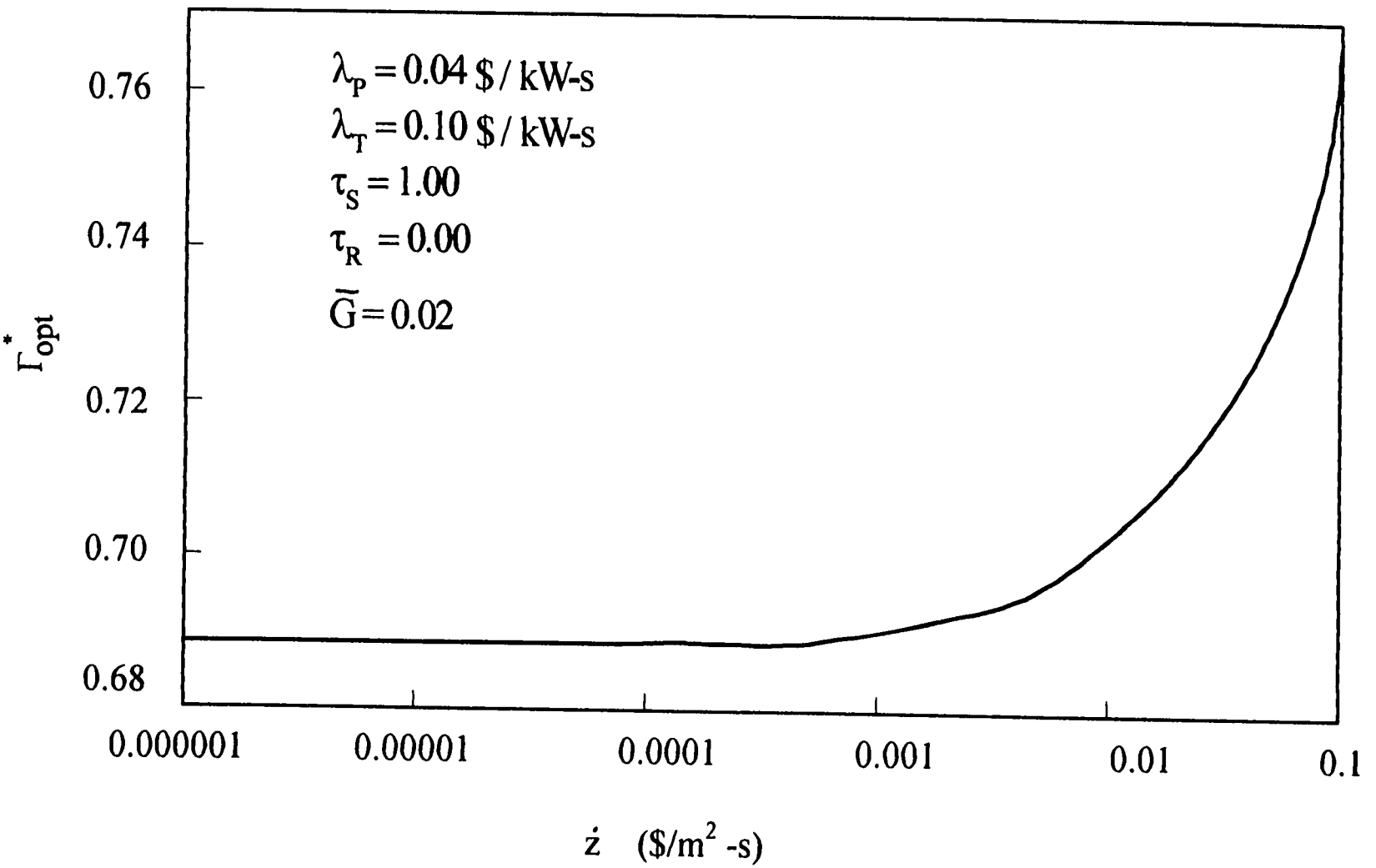

Figure 6.10 Influence of cost parameter $\dot{z}$ on optimum cost rate number

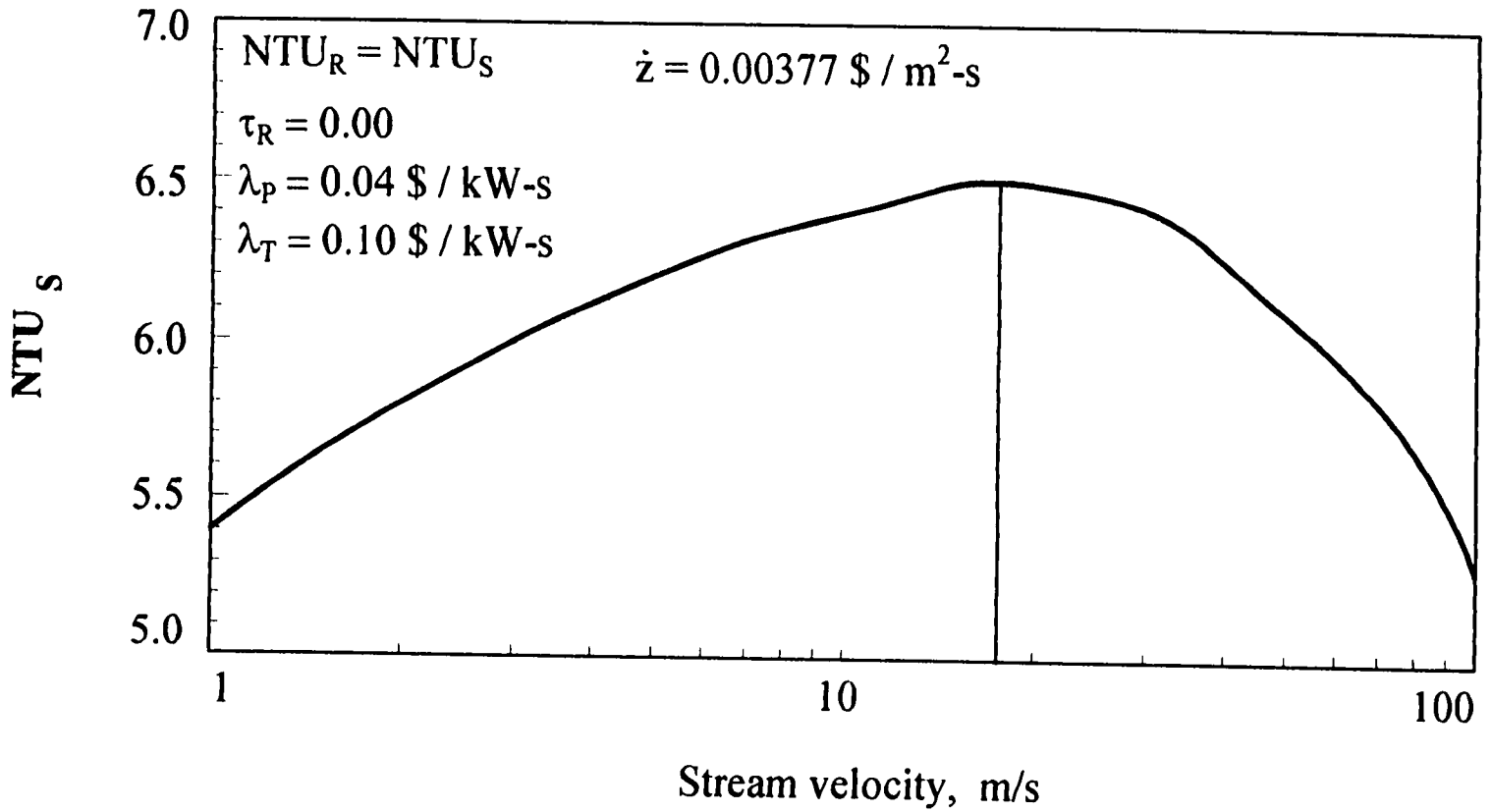

Figure 6.11 Effect of inlet stream velocity on size of heat exchanger

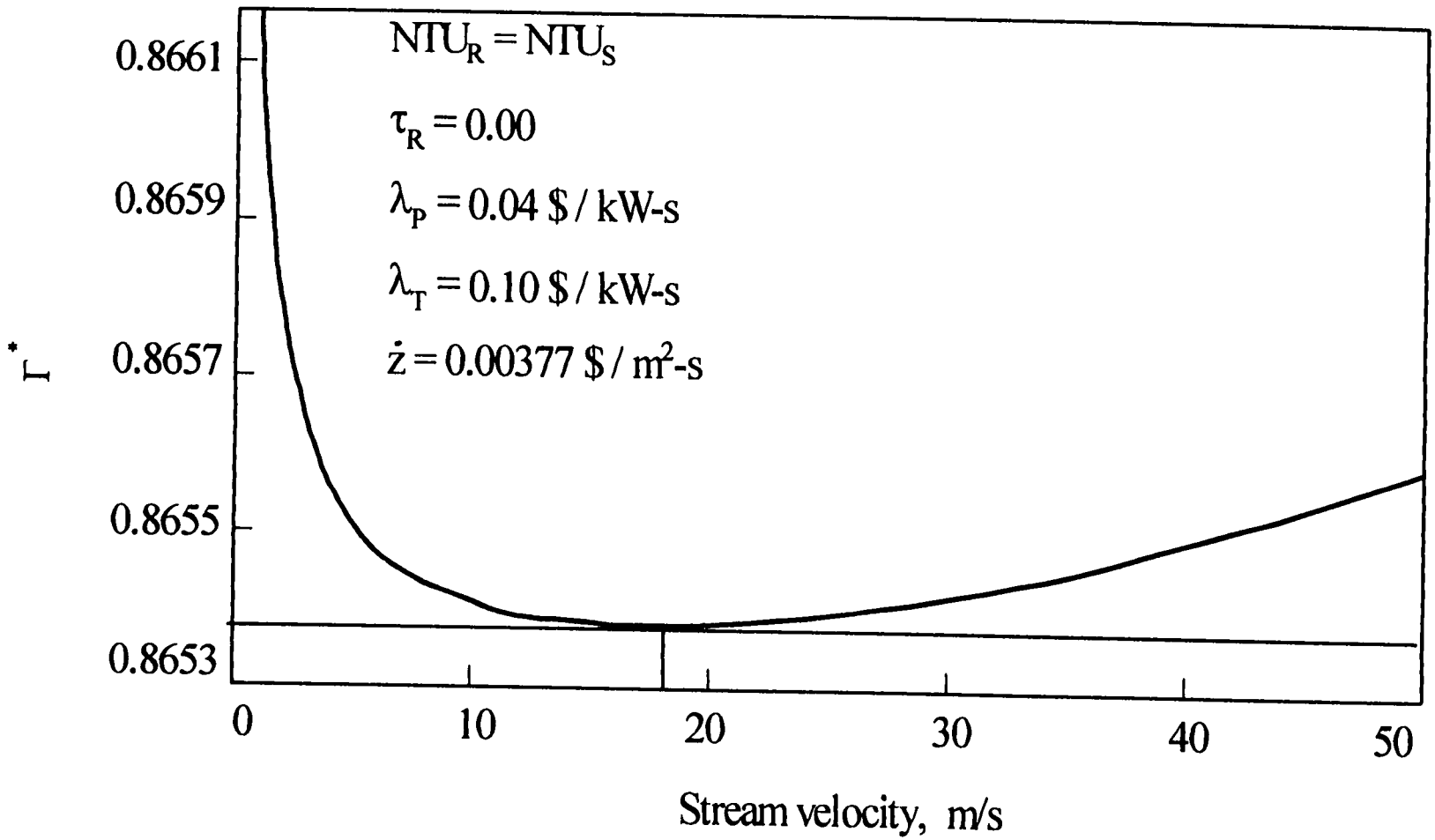

Figure 6.12 Effect of inlet stream gas velocity on the system performance

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

A general scheme of thermoeconomic technique is derived in order to be applied to thermal-energy-storage systems. This method is different from the previous ones by considering a new number (cost rate number), which is defined as the ratio between the cost rate of losses to the cost rate supplied, as the performance criterion of thermal systems in general and for thermal-energy-storage systems in particular that are considered in this study. The importance of the cost rate number is very similar in concept to that of entropy generation number used in second-law-based thermodynamic analysis of thermal energy systems in a way that it also varies from zero to one. The value of zero is considered for an ideal case which means that no losses in the system, while the value of one means that the losses are 100 %.

This general scheme of thermoeconomic design and analysis is then applied to sensibly Joulean-heated, sensible-heat, and latent-heat thermal-energy-storage systems as illustrative examples of the procedure. The effect of the cost parameters on the performance of each system was studied in somewhat more detail to examine the sensitivity of these parameters on number of heat transfer units (NTU_R).

Sensible-heat, thermal-energy-storage system with Joulean heating of the storage element was optimized with respect to the heat exchanger size. The effects of the stream velocity,

the maximum storage element temperature ($\overline{T}_{FS}$), the tareoff capacity (ε) and the cost parameters on the system size and performance are provided. It is found through the analysis that smaller stream velocities, greater tareoff capacity values, higher maximum storage element temperature, larger dimensionless cost ratio, and smaller cost parameter z will result in better system performance that is the cost rate number is decreased. By considering the effect of all the above parameters on the heat exchanger size, it was found that larger heat exchangers (larger NTU values) are needed for optimum operation of the storage unit.

Sensible-heat, thermal-energy-storage systems were optimized with respect to number of transfer units and dimensionless charging time by minimizing the cost rate number. It was found that there is only one optimum point that we can get a minimum cost rate number for each size of the heat exchanger and for each dimensionless charging time we can set. The dimensionless cost ratio and the cost parameter ($\dot{z}$) gave almost the same effect on the system performance and the heat exchanger size as that of sensible-heat, thermal-energy-storage systems with Joulean heating of the storage element. The effect of stream velocity on the heat exchanger size was also considered and it was found that there is a point were the optimum number of transfer units is maximum that will result in a minimum cost rate number.

Latent-heat, thermal-energy-storage systems were optimized with respect to number of transfer units and the phase change temperature. The effect of the dimensionless inlet storage gas temperature on the system performance, optimum number of heat transfer

units, and optimum phase change temperature was also demonstrated. It was found that as the dimensionless inlet storage gas temperature increases it results in a better system performance, a larger optimum heat exchanger size and a higher optimum phase change temperature. It was also found that the optimum phase change temperature decreases as the number of transfer units increases until we reach the optimum size of the heat exchanger.

The influence of the cost parameters on latent-heat storage systems was found to be similar to that studied earlier on the other two systems analyzed in this study. That is, higher dimensionless cost ratio will result in larger heat exchanger and better system performance, while higher cost parameter ($\dot{z}$) will result in smaller heat exchanger and higher cost rate number. The effect of inlet gas stream velocity on the size of heat exchanger and on the system performance was also considered for latent-heat-storage systems. It is found that the performance trends are similar to that discussed earlier in the case of sensible-heat, thermal-energy-storage systems.

From the study of thermal-energy-storage systems using thermoeconomic techniques and by comparing the results of this study with the available data, it is shown that the effect of cost parameters play an important role on the system design. It therefore gives us an indication of the importance of including all the required and available factors (thermodynamic and economic) to get a better design of a storage system from both the thermodynamic as well as economic aspects.

It would be a very good idea to extend thermoeconomic analysis to include distributed system analysis of thermal-energy-storage systems to check if there are major changes in the optimization results or not.

Appendix - A

APPENDIX A

SOME USEFUL RELATIONS

Relation Between U_R and U_S

We make the reasonable assumption that the overall thermal resistance to heat transfer between the gas in the heat exchanger and the storage element is dominated by the resistance between the gas and the heat exchanger duct wall. Under this assumption the overall heat transfer coefficient is closely approximated by the gas side heat transfer coefficient such that (Krane, 1985a; Bejan, 1982a)

$$\frac{U_R}{U_S} = \frac{\dot{m}_R}{\dot{m}_S}\frac{(St)_R}{(St)_S} \tag{A1}$$

Also if we considered the flow to be turbulent for all systems analyzed in this study then

$$\frac{(St)_R}{(St)_S} = \left(\frac{\dot{m}_R}{\dot{m}_S}\right)^{-.2} \tag{A2}$$

Substituting equation (A2) into equation (A1) gives

$$\frac{U_R}{U_S} = \left(\frac{\dot{m}_R}{\dot{m}_S}\right)^{.8} \tag{A3}$$

Relation Between NTU_S and NTU_R

For the same wetted area of the heat exchanger and equal c_P's, and from the definition of the number of transfer units (NTU) we have

$$NTU_R = \frac{U_R}{U_S} NTU_S \tag{A4}$$

Applying equation (A3) into equation (A4) for turbulent flow then

$$NTU_R = \left(\frac{\dot{m}_R}{\dot{m}_S}\right)^{-.2} NTU_S \tag{A5}$$

Calculating the Pressure ratio

We may write for the storage process

$$\frac{P_{iS}}{P_O} = .5 + \left\{ .25 + \left[\frac{f_S}{8(St)_S} \right] \overline{G}^2 (1+\tau_S)(NTU_S) \right\}^{1/2} \tag{A7}$$

Where

$$\overline{G} = \frac{\rho . V}{P_O / \sqrt{RT_O}} \tag{A8}$$

From Reynolds' analogy

$$\frac{f_S}{8(St)_S} = \Pr^{2/3} \tag{A9}$$

Substituting into equation (A7) we get

$$\frac{P_{iS}}{P_O} = .5 + \left\{ .25 + \left[\Pr^{2/3} \right] \overline{G}^2 (1+\tau_S)(NTU_S) \right\}^{1/2} \tag{A10}$$

Similarly for the removal process we can show that

$$\frac{P_{iR}}{P_O} = .5 + \left\{ .25 + \left[\Pr^{2/3} \right] \overline{G}^2 \left(\frac{\dot{m}_R}{\dot{m}_S} \right)^{1.8} (1+\tau_R)(NTU_S) \right\}^{1/2} \tag{A11}$$

Relation Between the Stream Velocity and the Overall Heat Transfer Coefficient

We can write the overall heat transfer coeffecient as a function of the stream velocity of the flowing stream by using the imperical equation (Badar and Zubair, 1995)

$$\frac{1}{U} = (A1) + (A2) V^{-n} \tag{A12}$$

where A1, A2 and n are positive constants.

NOMENCLATURE

A	area (m^2)
C	specific heat of the storage element (kJ / kg-°C)
c_P	constant pressure specific heat of the gas (kJ / kg-°C)
f	coefficient of friction
$\overline{G}$	dimensionless mass flux
h	enthalpy per unit mass (kJ / kg)
I	irreversibility rate (kW)
K	intercept of entropy generation approximation curve
K_0	fixed annual cost applied to the storage system as a whole ($ / s)
L_{ph}	latent heat of fusion (kJ / kg)
$\dot{m}_{S(R)}$	mass flowrate of gas in the heat exchanger duct during the storage (removal) process (kg / s)
M	mass of the storage element (kg)
N_S	entropy generation number
NTU	number of transfer units
P	pressure or power supplied (Pascal - kW)
Pr	Prandtle number
$\dot{Q}$	rate of heat transfer (kW)

R	gas constant (kJ / kg-K)
s	entropy per unit mass of gas (kJ / kg-K)
S	entropy (kJ / K)
$\dot{S}$	entropy per unit time (kW / K)
St	Stanton number
t_R	cycle time (s)
t_S	storage process time (s)
T	temperature (K)
U	overall heat transfer coefficient (kW / m^2-K)
V	stream velocity (m / s)
W	availability supplied (kJ)
x	fraction of melted material
y	heat exchanger parameter
$\dot{z}$	annualized capital cost of owning and maintaining the energy storage system per unit area (\$ / m^2-yr.)
$\dot{Z}$	annualized capital cost of owning and maintaining the energy storage system (\$ / yr.)

Greek Symbols

β	dimensionless cost ratio
ε	parameter characterizing the 'tare capacity' of the system in order to deliver thermal energy to the load
γ_{UA}	cost per unit overall heat conductance (\$ °C / kW-s)

$\dot{\Gamma}$	total annual cost of owning and operating the storage system ($ / yr)
Γ^*	cost rate number
η	parameter defined by equation (5.11)
λ	unit cost of lost work ($ / kW-s)
θ_R	dimensionless cycle time
θ_S	dimensionless storage time
τ	dimensionless gas inlet temperature

Subscripts

as	availability supplied
CV1	control volume 1
CV2	control volume 2
e	exit
E	electric heating
eR	exit conditions for the removal process
FR	final state of the removal process
FS	final state of the storage process
GEN	generation
i	inlet
iR	inlet of the removal process
IR	initial state of the removal process
iS	inlet of the storage process
IS	initial state of the storage process

opt	optimum value
out	gas outlet condition during storage process
P	pressure
ph	phase change condition
R	removal process
Ri	instantaneous during removal
S	storage process
SE	storage element
Si	instantaneous during storage
T	temperature
o	ambient condition

Superscripts

-	normalized

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